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**ATOMIC ENERGY  
RESEARCH ESTABLISHMENT**

**NEUTRON DIFFUSION AT LARGE  
DISTANCES FROM THE SOURCE**

An A. E. R. E. Report

**BY**

**K.T. SPINNEY**

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**MINISTRY OF SUPPLY  
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NEUTRON DIFFUSION AT LARGE DISTANCES  
FROM THE SOURCE.

by

K. T. Spinney

Summary.

A theory of neutron diffusion by H.A. Bethe is generalised to include the case of capture and a mixed medium.

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1. Introduction.
2. The Boltzmann Equation.
3. Fourier - Laplace Transform.
4. Solution by expansion in spherical harmonics.
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## 1. Introduction

Bethe<sup>(1)</sup> has provided a theory which deals with neutron densities at large distances from the source and covers the "gap" between age theory and the asymptotic theory of Wick. This theory has obvious applications for calculating the efficiency of neutron shields, except that, as presented in the above paper, it considers only a single type of nucleus in the scattering medium and takes no account of capture. However it may be easily generalized to include the case of a mixed medium and constant capture and it is the purpose of this paper to do so.

It is assumed that the reader is familiar with Bethe's paper and the argument is not elaborated in detail wherever it follows closely that of Bethe. We use the same notation except where differences are noted in the text.

All scattering and capture cross sections are taken independent of neutron energy and we consider only elastic collisions.

As in Bethe's paper we shall limit ourselves to an isotropic monoenergetic plane source in an infinite homogeneous medium.

(1) H.A. Bethe Declassified report A.E.C.D. 2105

## 2. The Boltzman Equation

Suppose the medium contains nuclei of  $m$  different types with masses  $M_j$ ,  $j = 1 \dots m$  and  $M_j \gg 1$  for all  $j$ .

Let the scattering cross sections of the nuclei be  $\sigma_{sj}$ ,  $j = 1 \dots m$  and the capture cross sections  $\sigma_{cj}$ ,  $j = 1 \dots m$

Let the nuclei be distributed uniformly through the medium with  $N_j$  as the number of nuclei of type  $j$  per  $\text{cm}^3$ .

Then the total mean free path,  $l$ , is defined by

$$\frac{1}{l} = \sum_{j=1}^m N_j (\sigma_{sj} + \sigma_{cj})$$

In order to derive the modified form of Bethe's eq<sup>n</sup>(1) we start from eq<sup>n</sup>(83) in the review by Marshak<sup>(2)</sup> on which he bases his work. That eq<sup>n</sup> was written down for a non-capturing medium but it is obvious that it also holds for a capturing medium if the function  $f$  is defined properly.

The equation is

$$l \mu \frac{\partial \psi}{\partial z} + \psi(z, \mu, u) = \int_0^u du' \int d\Omega' \psi(z, \mu', u') f(\mu_0, u-u') + \frac{\delta(z) \delta(u)}{4\pi} \dots (1)$$

(2) Review of Modern Physics Vol.19 3rd July 1947

where  $f(\mu_0, u-u')$  is the relative probability that a collision will change the logarithm of the inverse neutron energy from  $u'$  to  $u$  with deflection through an angle  $\theta$  given by  $\mu_0 = \cos \theta$ .  $u$  is taken to be zero at the source and  $\psi$  is the neutron collision density.

In our case we must have

$$f(\mu_0, u-u') = \sum_j c_j f_j(\mu_0, u-u') \quad \dots (2)$$

$$\text{where } c_j = \lambda N_j \sigma_{sj},$$

this being the relative probability that a neutron will be scattered by a nucleus of type  $j$  in a collision.

For elastic collisions the functions  $f_j$  are given by

$$f_j(\mu_0, u-u') = \frac{(M_j+1)^2}{8\pi M_j} e^{-(u-u')} \delta \left[ \mu_0 - \left\{ \frac{(M_j+1)}{2} e^{-\frac{(u-u')}{2}} - \frac{(M_j-1)}{2} e^{\frac{(u-u')}{2}} \right\} \right] \quad \dots (3)$$

$$\text{for } u' \geq u - q_j$$

$$= 0 \text{ for } u' < u - q_j$$

$$\text{where } q_j = \ln \left( \frac{M_j+1}{M_j-1} \right)^2$$

We may write (1) as follows, taking the total mean free path as our unit of length

$$\mu \frac{\partial \psi}{\partial z} + \psi(z, \mu, u) = \int_{j=1}^m \int_{u-q_j}^u du' \psi(z, \mu', u') c_j f_j(\mu_0, u-u') + \frac{\delta(z) \delta(u)}{4\pi} \quad \dots (4)$$

The capture of neutrons is expressed by the fact that the sum of the coefficients  $c_j$  is less than unity.

3. Fourier - Laplace transform.

The convolution theorem for Laplace transforms states

$$L \int_0^u F_1(u-u') F_2(u') du' = L F_1(u) \cdot L F_2(u)$$

$$\therefore L \int_0^u \psi(z \mu' u') f_j(\mu_0, u-u') du' = L \psi(z \mu' u) L f_j(\mu_0, u)$$

Now

$$L f_j(\mu_0, u) = \int_0^{\infty} e^{-\eta u} f_j(\mu_0, u) du = \int_0^{q_j} e^{-\eta u} f_j(\mu_0, u) du$$

and this =  $g_j(\mu_0, \eta)$

where

$$g_j(\mu_0, \eta) = \frac{1}{4\pi} \frac{(M_j + 1)^2}{M_j (\mu_0^2 + M_j^2 - 1)^{\frac{1}{2}}} \left[ \frac{\mu_0 + (\mu_0^2 + M_j^2 - 1)^{\frac{1}{2}}}{M_j + 1} \right]^{2(\eta + 1)} \dots (5)$$

provided that  $u_0$ , as defined by  $\mu_0 = \frac{(M_j + 1)}{2} e^{-\frac{u_0}{2}} - \frac{(M_j - 1)}{2} e^{u_0/2}$ ,

satisfies  $0 \leq u_0 \leq q_j$  and this is true from the definition of  $q_j$  as the maximum possible logarithmic energy loss in one collision.

Now apply a Fourier Laplace transformation to (4) and we get

$$(1 - iy\mu)\varphi(\eta y \mu) = \int d\Omega' \varphi(\eta y \mu') \sum_j c_j g_j(\mu_0, \eta) + \frac{1}{4\pi} \dots (6)$$

where

$$\varphi(\eta y \mu) = \int_{-\infty}^{+\infty} e^{izy} dz \int_0^{\infty} du e^{-\eta u} \psi(z u \mu).$$

Then writing the last factor of (5) as an exponential we have

$$g_j(\mu_0 \eta) \approx \frac{1}{4\pi} \left( \frac{M_j + 1}{M_j} \right)^2 e^{-\frac{2(1 - \mu_0)(\eta + 1)}{M_j}} \text{ where we have} \dots (7)$$

neglected terms of order  $\frac{1}{M_j^2}$  and above.

4. Solution by expansion in spherical harmonics.

Now expand in spherical harmonics so that

$$g_j(\mu_0, \eta) = \frac{1}{4\pi} \sum_l g_{jl}(\eta) P_l(\mu_0) \quad \dots (8)$$

where

$$g_{jl}(\eta) = (2l+1) \int g_j(\mu_0, \eta) P_l(\mu_0) d\Omega$$

$$= \frac{2l+1}{2} \left( \frac{M_j+1}{M_j} \right)^2 \int_{-1}^{+1} e^{-\frac{2(1-\mu_0)(\eta+1)}{M_j}} P_l(\mu_0) d\mu_0 \quad \dots (9)$$

Thus

$$g_{j0}(\eta) = \left( \frac{M_j+1}{M_j} \right)^2 \frac{M_j}{4(\eta+1)} \left[ 1 - e^{-\frac{4(\eta+1)}{M_j}} \right] \quad \dots (10)$$

$$g_{j1}(\eta) = \left( \frac{M_j+1}{M_j} \right)^2 \frac{3M_j}{8(\eta+1)^2} \left[ \frac{2(\eta+1)}{M_j} - 1 + \left\{ \frac{2(\eta+1)}{M_j} + 1 \right\} e^{-\frac{4(\eta+1)}{M_j}} \right] \quad \dots (11)$$

etc.

Now expand  $\varphi(y, \eta, \mu)$  in spherical harmonics so that

$$\varphi(y, \eta, \mu) = \sum_l (2l+1) P_l(\mu) \varphi_l(y, \eta) \quad \dots (12)$$

where

$$\varphi_l(y, \eta) = \frac{1}{4\pi} \int \varphi(y, \eta, \mu) P_l(\mu) d\Omega \quad \dots (13)$$

Then (6) becomes, on substitution from (8) and (12)

$$(1-iy\mu)\varphi(y, \eta, \mu) - \frac{1}{4\pi} = \int d\Omega' \sum_l (2l+1) P_l(\mu') \varphi_l(y, \eta) \sum_j c_j \frac{1}{4\pi} \sum_{l'} g_{jl'}(\eta) P_{l'}(\mu_0)$$

$$= \frac{1}{4\pi} \sum_l (2l+1) \varphi_l(y, \eta) \sum_j c_j g_{jl}(\eta) \int P_l(\mu) P_l(\mu_0) d\Omega$$

$$\begin{aligned}
 &= \sum_l \varphi_l(y\eta) \sum_j c_j g_{jl}(\eta) P_l(\mu) \\
 &= \sum_l \varphi_l(y\eta) g_l(\eta) P_l(\mu) \quad \dots (14)
 \end{aligned}$$

$$\text{where } g_l(\eta) = \sum_j c_j g_{jl}(\eta) \quad \dots (15)$$

We now look for the value  $y = -i\nu$  for which the homogeneous equation corresponding to (14) can be satisfied, i.e. we consider

$$\varphi(\nu, \eta, \mu) = \sum_{l=0}^N \varphi_l(\nu \eta) g_l(\eta) \frac{P_l(\mu)}{1-\nu\mu} \quad \dots (16)$$

where we consider only a finite number of terms of the infinite series since it is rapidly convergent.

We thus find  $\nu$  in terms of  $\eta$  given by the determinant:-

$$\left. \begin{array}{c}
 \left| \begin{array}{cccc}
 g_0 A_{00}^{-1} & g_1 A_{01} & \dots & g_N A_{0N} \\
 g_0 A_{01} & g_1 A_{11}^{-1} & \dots & g_N A_{1N} \\
 & & & \\
 g_0 A_{0N} & & & g_N A_{NN}^{-1}
 \end{array} \right| = 0 \\
 \dots (17)
 \end{array} \right\}$$

$$\text{where } A_{ln} = \frac{1}{2} \int_{-1}^{+1} \frac{P_l(\mu) P_n(\mu)}{1-\nu\mu} d\mu \quad \dots (18)$$

$$\text{Thus } A_{00} = f(\nu)$$

$$A_{01} = \frac{f-1}{\nu}$$

etc.

$$\text{and } f(\nu) = \frac{1}{\nu} \tanh^{-2} \nu = \frac{1}{2\nu} \ln \frac{1+\nu}{1-\nu} \quad \dots (19)$$

The zero order approximation gives  $A_{00} = \frac{1}{g_0}$

$$\text{i.e. } \frac{1}{2\nu} \ln \frac{1+\nu}{1-\nu} = \frac{1}{\sum_j c_j \frac{(M_j + 1)^2}{4M_j(\eta + 1)} \left[ 1 - e^{-\frac{4(\eta + 1)}{M_j}} \right]} \quad \dots (20)$$

5. Inversion of Fourier - Laplace Transform.

Bethe's equation 16, is, in our notation

$$\int_0^{\infty} d\eta \psi(z, u, \mu) e^{-u\eta} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dy \phi(y, \eta, \mu) e^{-iyz} \quad \dots (21)$$

where we retain the variable  $u$  instead of introducing  $u'$ .

We may now proceed to solve the inhomogeneous equation (14) at the same time carrying out the Fourier inversion of  $\phi$  as in Bethe's para 3.

$$\int_0^{\infty} d\eta w_0(z, u) e^{-u\eta} = \frac{c^{-\nu z}}{g_0^2 \lambda} \quad \dots (22)$$

where  $w_0(z, u) = \int \cdot d\Omega \psi(z, \mu, u)$

and  $\lambda$  is given by  $\lambda = \left. \frac{d\Lambda(\eta\nu')}{d\nu'} \right|_{\nu = \nu'} \quad \dots (23)$

$$\Lambda(\eta\nu') = A_{00}(\nu') - \frac{1}{g_0} + \frac{\sum_{l=1}^N g_l A_{0l}(\nu') \phi_l(\nu'\eta)}{g_0 \phi_0(\nu'\eta) + \frac{1}{4\pi}} \quad \dots (24)$$

To obtain the Mellin inversion we may argue as follows:

The contribution to the integral in (22) will arise from the neighbourhood of some value  $u_0$ , say, of  $u$  and  $u_0$  will be a function of  $\eta$  and  $z$ . We may therefore consider  $\eta$  as a function of  $u_0$  and  $z$  so that using (22) we have

$$\int_0^{\infty} du w_0(z, u) e^{-u\eta(u_0)} = \frac{e^{-\nu(u_0)z}}{g_0^2(u_0) \lambda(u_0)} \quad \dots (25)$$

Now assume

$$w_0(z, u) = \frac{\chi(z, u) e^{u\eta(u) - z\nu(u)}}{g_0^2(u) \lambda(u)} \quad \dots (26)$$

where  $\chi(z, u)$ ,  $g_0(u)$ ,  $\lambda(u)$  are all slowly varying functions of  $u$ .

Then we may write (25) in the form

$$1 = \int_0^{\infty} du \chi(zu) \frac{g_0^2(u_0)\lambda(u_0)}{g_0^2(u)\lambda(u)} e^{u[\eta(u)-\eta(u_0)] - z[\nu(u)-\nu(u_0)]} \dots (27)$$

and  $u[\eta(u)-\eta(u_0)] - z[\nu(u)-\nu(u_0)]$  must have a maximum at  $u = u_0$ .

Hence

$$u[\eta(u)-\eta(u_0)] + u \frac{d\eta}{du} - z \frac{d\nu}{du} = 0 \quad \text{for } u = u_0 \quad \dots (28)$$

whence

$$\left(\frac{d\eta}{d\nu}\right)_{\eta = \eta_0} = \frac{z}{u_0} \quad \text{and we may use this as a}$$

general relation to define  $\eta$  for a given  $z$  and  $u$ .

$$\text{thus} \quad \frac{d\eta}{d\nu} = \frac{z}{u} \quad \dots (29)$$

Now expand the exponent of  $e$  in (27) in powers of  $(u-u_0)$

Thus  $u[\eta(u) - \eta(u_0)] - z[\nu(u) - \nu(u_0)]$

$$= \frac{(u-u_0)^2}{2} \left[ 2 \frac{d\eta}{du} + u \frac{d^2\eta}{du^2} - z \frac{d^2\nu}{du^2} \right] + \dots \quad \dots (30)$$

Hence

$$\chi(z, u_0) = \int_0^{\infty} du e^{(u-u_0)^2 \left[ \frac{d\eta}{du} + \frac{u}{2} \frac{d^2\eta}{du^2} - \frac{z}{2} \frac{d^2\nu}{du^2} \right]}$$

$$\approx \frac{\sqrt{\pi}}{\left[ -\frac{1}{2} \left( u \frac{d^2\eta}{du^2} - z \frac{d^2\nu}{du^2} \right) - \frac{d\eta}{du} \right]^{\frac{1}{2}}} \quad \dots (31)$$

$$\psi_0(u, z) = \frac{1}{\sqrt{\pi}} \frac{\left[ \frac{1}{2} \left( z \frac{d^2 \nu}{du^2} - u \frac{d^2 \eta}{du^2} \right) - \frac{d\eta}{du} \right]^{\frac{1}{2}}}{g_0^2 \lambda} e^{u\eta - z\nu} \dots (32)$$

But we have from (29)  $u \frac{d\eta}{du} = z \frac{d\nu}{du}$

so that

$$\frac{d\eta}{du} + u \frac{d^2 \eta}{du^2} = z \frac{d^2 \nu}{du^2}$$

whence

$$\psi_0(u, z) = \frac{1}{\sqrt{\pi}} \frac{\left[ -\frac{1}{2} \frac{d\eta}{du} \right]^{\frac{1}{2}}}{g_0^2 \lambda} e^{u\eta - z\nu} \dots (33)$$

Now from (29)  $u = z \frac{d\nu}{d\eta}$

and therefore

$$\frac{du}{d\eta} = z \frac{d^2 \nu}{d\eta^2} = u \frac{d\eta}{d\nu} \cdot \frac{d^2 \nu}{d\eta^2} = u \frac{d}{d\nu} \left[ \frac{d\nu}{d\eta} \right]$$

Finally we have

$$\psi_0(z, u) = \frac{1}{\sqrt{2\pi} u} \frac{1}{\left[ -\frac{d}{d\nu} \left( \frac{d\nu}{d\eta} \right) \right]^{\frac{1}{2}}} \frac{e^{u\eta - z\nu}}{g_0^2 \lambda} \dots (34)$$

Where  $\eta$  is given by  $\frac{u}{z} = \frac{d\nu}{d\eta}$  for given  $u$  and  $z$  since  $\nu$  is a known function of  $\eta$

$$g_0 = \sum_j c_j \frac{(M_j + 1)^2}{4M_j(\eta + 1)} \left[ 1 - e^{-\frac{4(\eta + 1)}{M_j}} \right]$$

$\lambda$  is given by (23) and (24)

Hence the result is not altered in any essential way by taking into account capture and a mixed medium. We merely get a more complicated expression for the  $g^S$  and hence for the functions  $\nu(\eta)$  and  $\lambda(\eta)$ .

## 6. Discussion

The chief advantage of the method lies in its ability to give results when  $\nu$  and  $\frac{\eta}{M_j}$  are not small, i.e. at distances greater than those covered by age theory or the spherical harmonic method. However, it is interesting to apply it to the case of small  $\nu$  and  $\frac{\eta}{M_j}$  since the other methods then appear as successive approximations.

### 1. Zero order approximation.

This is given by equation (20) as

$$\frac{1}{2\nu} \ln \frac{1+\nu}{1-\nu} = \frac{1}{\sum_j c_j \frac{(M_j+1)^2}{4M_j(\eta+1)} \left[ 1 - e^{-\frac{4(\eta+1)}{M_j}} \right]}$$

if we now assume that  $\nu$  and  $\frac{\eta}{M_j}$  are small for all  $j$  and equate lowest powers of this equation we have

$$1 + \frac{\nu^2}{3} = \frac{1}{\sum_j c_j \left(1 + \frac{2}{M_j}\right) \left(1 - \frac{2(\eta+1)}{M_j}\right)} = \frac{1}{\sum_j c_j \left(1 - \frac{2\eta}{M_j}\right)} \quad \text{for large } M_j \quad \dots (35)$$

We have  $\sum_j c_j = c$ , say where  $c$  is the number of scattered neutrons per collision, and we may write

$$\begin{aligned} \sum_j \frac{c_j}{M_j} &= \sum_j c_j \cdot \frac{\sum_j \frac{c_j}{M_j}}{\sum_j c_j} \\ &= c \overline{M^{-1}} \end{aligned}$$

where  $\overline{M^{-1}}$  is the average inverse mass taken over the scattering collisions.

Thus

$$1 - \frac{\nu^2}{3} = c - 2c\eta \overline{M^{-1}} \quad \dots (37)$$

We must therefore assume  $(1-c)$  small for this approximation  
i.e. small capture.

From (37)

$$- \frac{2\nu}{3} = - 2 c \overline{M^{-1}} \frac{d\eta}{d\nu} \quad \dots (38)$$

$$\therefore \frac{d\eta}{d\nu} = \frac{\nu}{3 c \overline{M^{-1}}} = \frac{z}{u} \quad \dots (39)$$

Hence the approximation is valid for the case where  $\frac{z c \overline{M^{-1}}}{u}$  is small  
i.e. age theory.

Now consider the term  $e^{u\eta - z\nu}$  in equation (34)  
we have  $u\eta =$

$$\frac{u}{2c \overline{M^{-1}}} \left[ c - 1 + \frac{\nu^2}{3} \right] = \frac{u}{2c \overline{M^{-1}}} \left[ c - 1 + \frac{3c^2 \overline{M^{-1}^2} z^2}{u^2} \right] \quad \dots (40)$$

Now from (38) and (39)  $z\nu = 3c \overline{M^{-1}} \frac{z^2}{u}$

$$\therefore u\eta - z\nu = \frac{u}{2c \overline{M^{-1}}} [c - 1] - \frac{3c \overline{M^{-1}} z^2}{2u} \quad \dots (41)$$

$$\text{Now } \lambda = \frac{df}{d\nu} = \frac{2}{3} \nu \quad \dots (42)$$

$$\left[ - \frac{d}{d\nu} \left( \frac{d\nu}{d\eta} \right) \right]^{\frac{1}{2}} = \left[ \left( - \frac{d}{d\nu} \frac{3c \overline{M^{-1}}}{\nu} \right) \right]^{\frac{1}{2}} = \frac{\sqrt{3c \overline{M^{-1}}}}{\nu} \quad \dots (43)$$

So  $\frac{1}{\lambda} = c - 2 c \overline{M^{-1}} \eta$  which we may take as unity since both  $\eta \overline{M^{-1}}$  and  
 $(1-c)$  are assumed small.

Thus we have

$$\psi(u, z) = \sqrt{\frac{3}{8\pi u}} \frac{1}{(c \overline{M^{-1}})^{\frac{1}{2}}} \exp \left[ - \frac{3z^2}{2u} c \overline{M^{-1}} - \frac{u}{2c \overline{M^{-1}}} (1-c) \right] \quad \dots (44)$$

We may compare this expression with the result given by age velocity theory by introducing the symbolic age  $\tau$  and the mean logarithmic energy loss  $\xi$ . Under our assumptions with respect to the cross sections we find that

$$\tau = \frac{u}{8c M^{-1}} + O(1) \quad \dots (45)$$

$$\xi = 2c M^{-1} + O\left(\frac{1}{M^2}\right) \quad \dots (46)$$

and we may therefore rewrite equation (44) as

$$\psi(u, z) = \frac{1}{\xi} \frac{1}{(4\pi\tau)^{\frac{1}{2}}} \exp \left[ -\frac{z^2}{4\tau} - \frac{u}{\xi} (1-c) \right] \quad \dots (47)$$

The factor  $\exp \left[ -\frac{u}{\xi} (1-c) \right]$  is the usual capture correction obtained by standard methods (See Marshak op. cit pg. 221) while the remaining factors give the usual age-velocity solution without capture.

Thus the age-velocity solution is equivalent to our equation (44) when we take account of the dependence of  $\tau$  and  $\xi$  on capture and approximate by neglecting terms of order  $\left(\frac{1}{M_j}\right)$  in their expansions.

### 2.1<sup>st</sup> order approximation.

The first order approximation to equation (17) is

$$f(\nu) = \frac{1}{g_0} - \frac{[f(\nu) - 1]}{\nu^2} \left( \frac{1}{g_0} - 1 \right) g_1$$

and it may be shown that this equation gives  $\eta$  in terms of  $\nu$  correctly up to order  $\nu^6$ . If we expand the equation up to terms in  $\nu^4$  and  $\left(\frac{1}{M_j}\right)$

we get the following expression for the exponential factor in (34)

$$\exp \left[ -\frac{u p}{2c M^{-1}} \left\{ 1 - \frac{p}{3}(1+\kappa) \right\} - \frac{3}{2} \left\{ 1 + \frac{p}{3}(1+\kappa) \right\} \frac{z^2 c M^{-1}}{u} + \frac{9}{2} \left\{ \frac{4}{5} + \frac{\kappa}{3} \right\} \frac{z^4}{u^3} c^3 M^{-\frac{3}{2}} \right] \quad \dots (48)$$

where  $p = (1 - c)$  and is of the same order of magnitude as  $\nu^2$

$$\kappa = 1 + \frac{2 \sum_j \frac{c_j}{M_j^2}}{[c \overline{M^2}]^2} = 1 + \frac{2 \overline{M^2}}{c [\overline{M^2}]^2} \quad \dots (49)$$

$$\text{where } \overline{M^2} = \frac{\sum_j \frac{c_j}{M_j^2}}{\sum_j c_j} \quad \dots (50)$$

This may be written, neglecting the less important factors,

$$\psi = \psi_{age} e^\epsilon$$

where

$$\epsilon = \frac{up^2}{6c \overline{M^2}} (1+\kappa) - \frac{p}{2} (1+2\kappa) c \overline{M^2} \frac{z^2}{u} + \frac{9}{2} \left[ \frac{4}{5} + \frac{\kappa}{3} \right] c^3 \overline{M^2} \frac{z^4}{u^3} \quad \dots (51)$$

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A. E. R. E.,  
Harwell Berks.

12th January 1950



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**[dstl]** Porton Down,  
Salisbury  
Wiltshire  
SP4 0JQ  
22060-6218  
Tel: 01980 613753  
Fax 01980-613970

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