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**THE OHIO STATE UNIVERSITY
RESEARCH FOUNDATION**

AN R-F ENERGY REGULATOR USED TO CONTROL HIGH TEMPERATURES
IN THERMODYNAMIC MEASUREMENTS

by

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Columbus 10, Ohio

FOREWORD

This work was carried out at The Ohio State University Cryogenic Laboratory under Contract with U. S. Navy, Office of Naval Research Contract Number N6onr-225, Task Order I, ONR Project Number NR 058 031, with The Ohio State University Research Foundation. This report covers information obtained during the study entitled: "An R-F Energy Regulator used to control High Temperatures in Thermodynamic Measurements." It represents the 16th Technical Report of this series.

Director - H. L. Johnston

Editor - Esther R. Fultz

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ABSTRACT

An r-f energy regulator for use in controlling high temperatures in thermodynamic measurements is described. The regulator is an improved version of one used earlier in this laboratory. The new version uses the regulator output to control the position of a phase shifter in an induction heater. The phase shifter in turn controls the sample temperature. This method of coupling eliminates the need for a variometer in series with the r-f work coil and makes it possible to vary the power output of the induction heater from zero to maximum.

INTRODUCTION

The r-f energy regulator described in this report is an improved version of the regulator described in a paper by Speiser, Ziegler and Johnston¹, the essential difference being a new method of coupling the oscillator. The output of the regulator is fed to a small permanent-magnet type of motor which, through an appropriate gear reduction, rotates the shaft of a phase controller. The controller is a 3-phase motor with wound rotor, that provides a-c bias for the thyratrons in induction heater. As the position of the phase controller is changed, the d-c voltage to the oscillator tubes is varied and thus the power output is changed. A block diagram illustrating the operation of the regulator is shown in Figure 1.

INDUCTION HEATER

This regulator was designed specifically for a Westinghouse 450-kc induction heater whose nominal power output is 20 kw if it is suitably connected to a load. A very simplified schematic diagram of the heater is shown in Fig. 2.

Power is supplied to the oscillator from 220-volt, 3-phase, 60-cycle mains with a maximum input of approximately 49 kva at 95 percent power factor. Standby operation requires 3 kva. A delta connection is used for the plate transformer primaries and a star connection for the secondary windings. Secondaries have taps to deliver 6,275 or 8,150 volts line-to-line. This provides a rectifier output voltage up to approximately 11,000 volts.

The rectifier tube circuit consists of six WL-678 thyratrons connected as a 3-phase bridge circuit. Three of the grids are tied to their respective filaments and the three thyratrons are thus converted to phantatrons (diodes). The secondary line-to-line voltages V_{ab} , V_{bc} and V_{ca} are applied to the load through two tubes in series. Current conduction commutates among the various tube combinations, as illustrated in Figure 3. At the instant shown, V_{ca} is a maximum, and cosine components of V_{ab} and V_{bc} are of the same magnitude, and in phase. Thus, this particular instant is a commutation period. A small positive increment of time will see the vectors rotated counterclockwise by an angle δ . In this new position, the magnitude of V_{bc} will be larger than that of V_{ab} and hence will predominate so far as tube conduction is concerned. At this time, thyratrons Nos. 3 and 5 will fire if the grid permits firing. The sequence of commutation by tube numbers is 3-5, 3-4, 2-4, 2-6, 1-6 and 1-5. The commutation periods are depicted by vector diagrams in Figure 4. The set of tubes indicated as conducting current actually start conduction a short time after the commutation period shown. This delay occurs because commutation from one conduction anode to the next

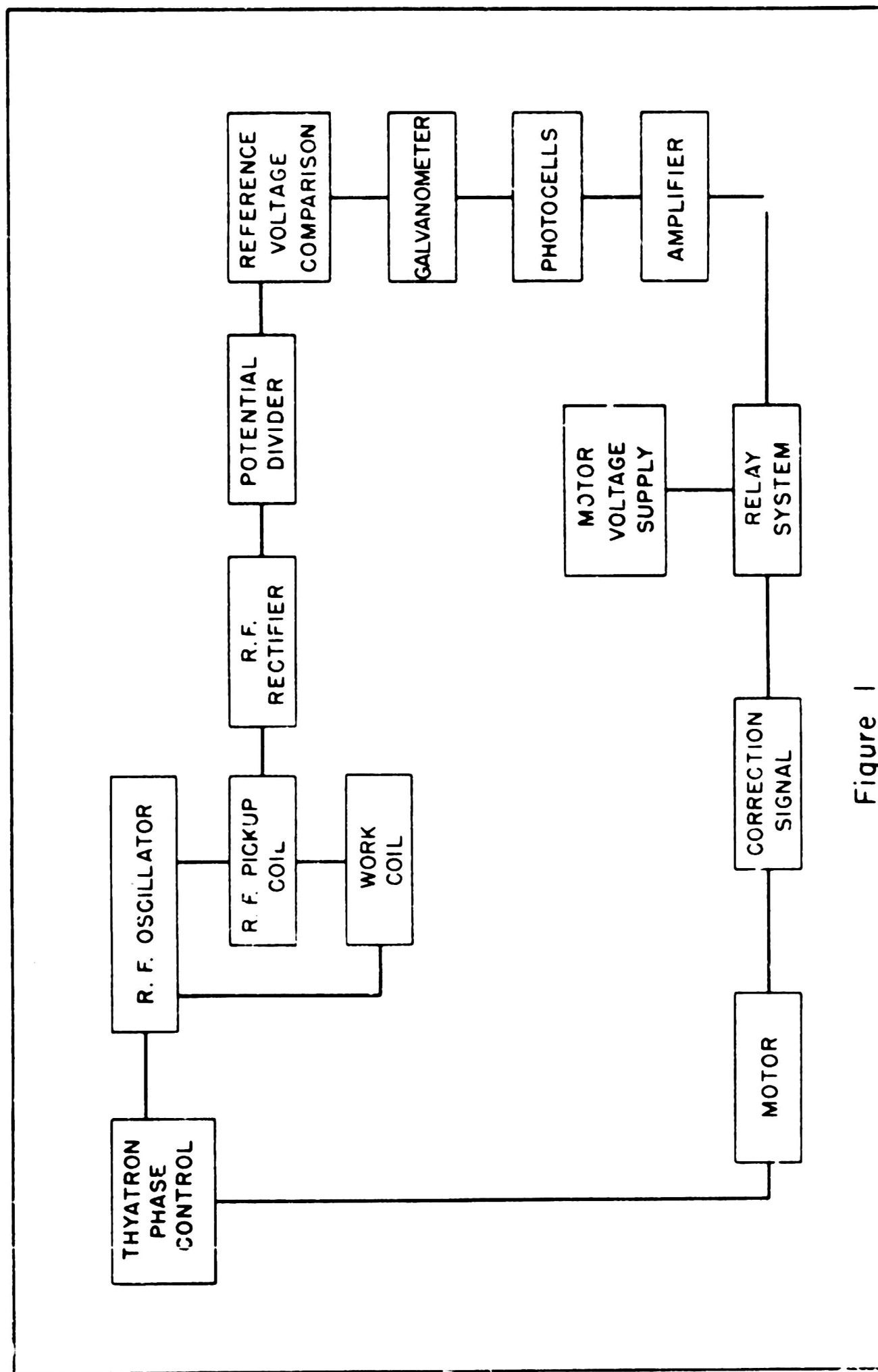


Figure 1

FIGURE 1 - R.F. ENERGY REGULATOR BLOCK DIAGRAM

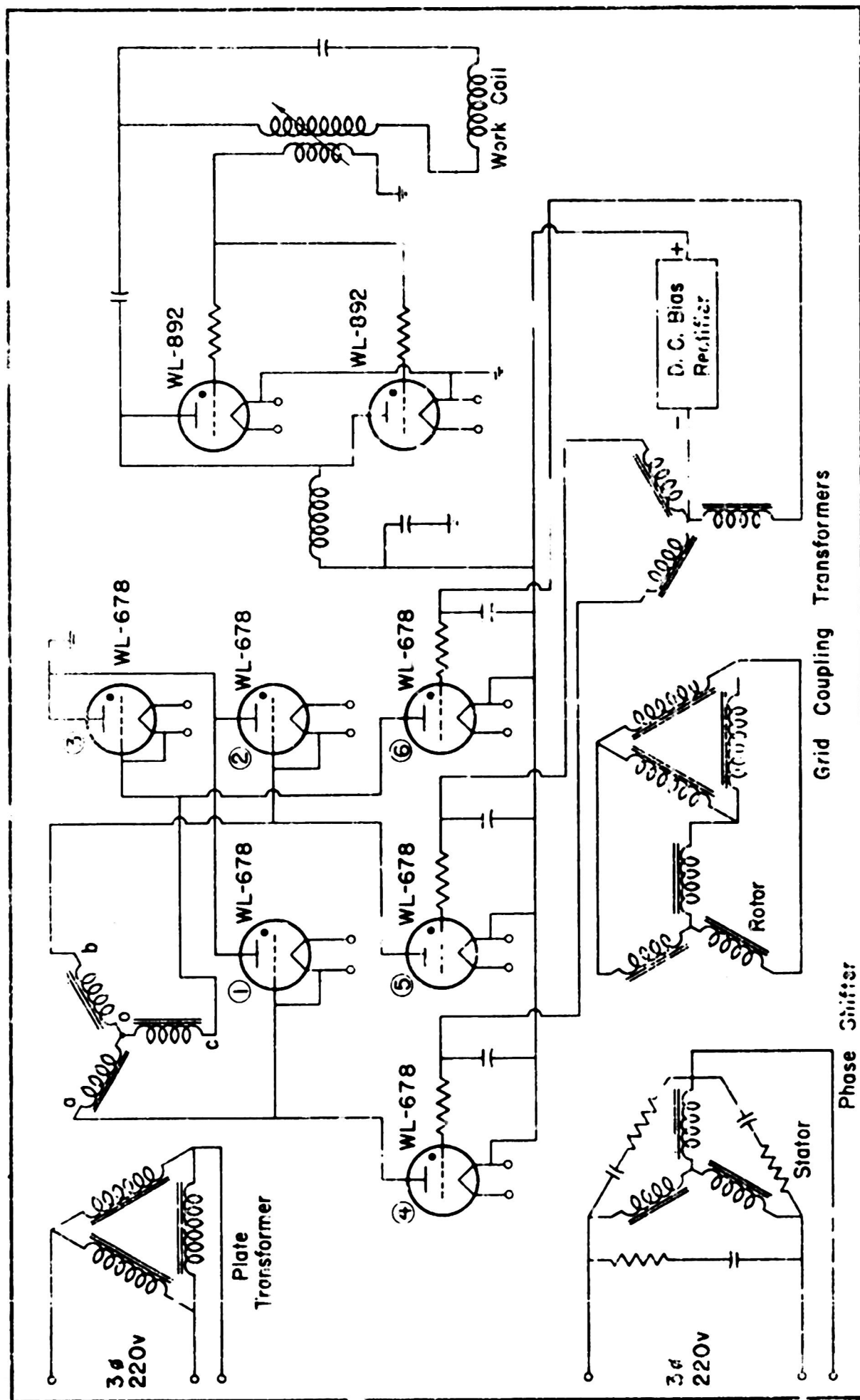
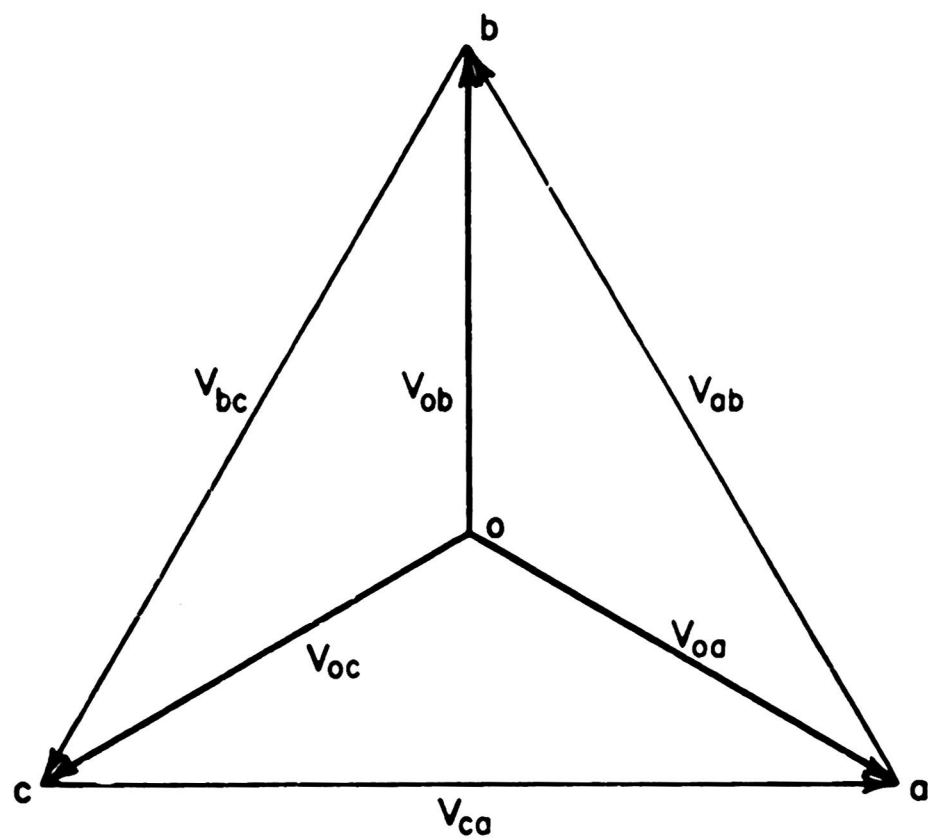


Fig 2 Simplified 20 kW. Westinghouse Induction Heater Schematic



$$|V_{ab}| = |V_{bc}| = |V_{ca}| = \sqrt{3} |V_{ao}| = \sqrt{3} |V_{bo}| = \sqrt{3} |V_{co}|$$

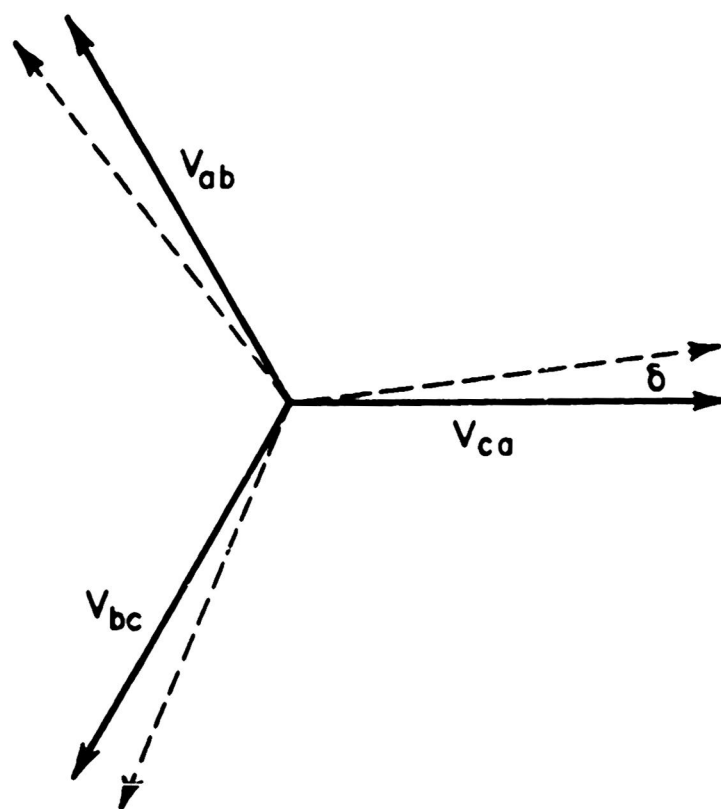


Fig. 3. Plate Transformer Secondary Voltage Phase Diagram

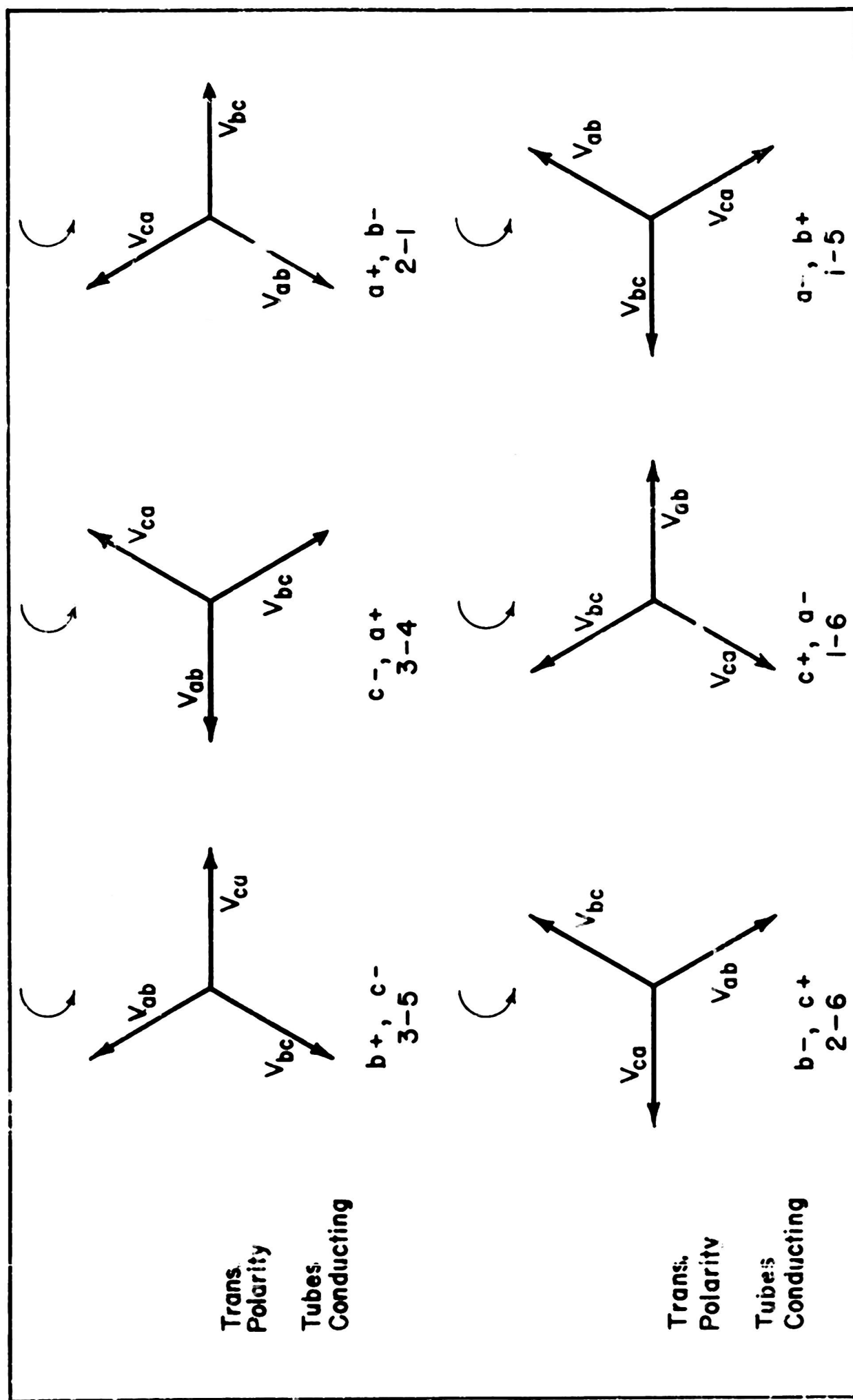


Fig. 4. Vector Diagrams Showing Commutation for Three Phase Bridge Rectifier during One Cycle of Sixty Cycle Operation.

is not instantaneous, owing to the presence of leakage inductance in the plate transformer secondary windings and other stray effects. There is a period of overlap when the current of the anode being extinguished decreases toward zero, while the current of the incoming anode rises to its maximum value.

Each tube in the rectifier circuit conducts for two 60° periods, if overlap is neglected. This circuit has several advantages: The current flow in the transformer secondaries is reversed during the cycle of operation so that core saturation is minimized. Also, the bridge circuit has the wave form of a 6-phase star circuit, together with a desirable secondary utilization factor. Thus, the fundamental frequency component in the rectified voltage wave form is the 6th, and the fundamental ripple fraction is 0.0404, together with a secondary utilization factor of 0.551.*

Since a phanatron and a thyatron are in series during conduction, the three thyatrons can be utilized to completely control the voltage output from zero to 11,000 volts during operation. The method of control used is a combination of d-c bias and a superimposed a-c voltage that can be phase-shifted. The d-c voltage is used to bias the thyatrons below cutoff. The a-c voltage over-rides the d-c bias and causes the thyatrons to fire during a portion of the possible firing cycle. The action of phase-shifting to give control is shown in Figure 5 in simplified form.

An a-c grid bias is derived from a phase-shifter that has a 3-phase stator and a 3-phase rotor. The stator winding is connected to a 220-volt, 3-phase source. The rotor can be manually rotated and thus the voltages induced in the rotor windings from the stator windings are shifted a certain phase angle. This a-c signal is impressed upon the thyatron grids through grid-coupling transformers, which also perform the extra needed function of isolating the high d-c voltages from the phase-shifter windings.

A choke in series with the d-c line and a condenser to ground form a low-pass, L-type r-f filter to prevent r-f currents from getting into the rectifier circuit. Two water-cooled WL-892 tubes are connected in parallel to form a tuned plate-coupled grid oscillator. The condenser between the tank circuit and the WL-892 plates serves to block d-c voltage from the tank components, but at the same time passes r-f current. The tank condenser is of the nitrogen-filled type. The work coil forms part of the tank inductance, so that when a different work coil is used the oscillator will resonate at a slightly different frequency.

* See Appendix 1 for development of these values.

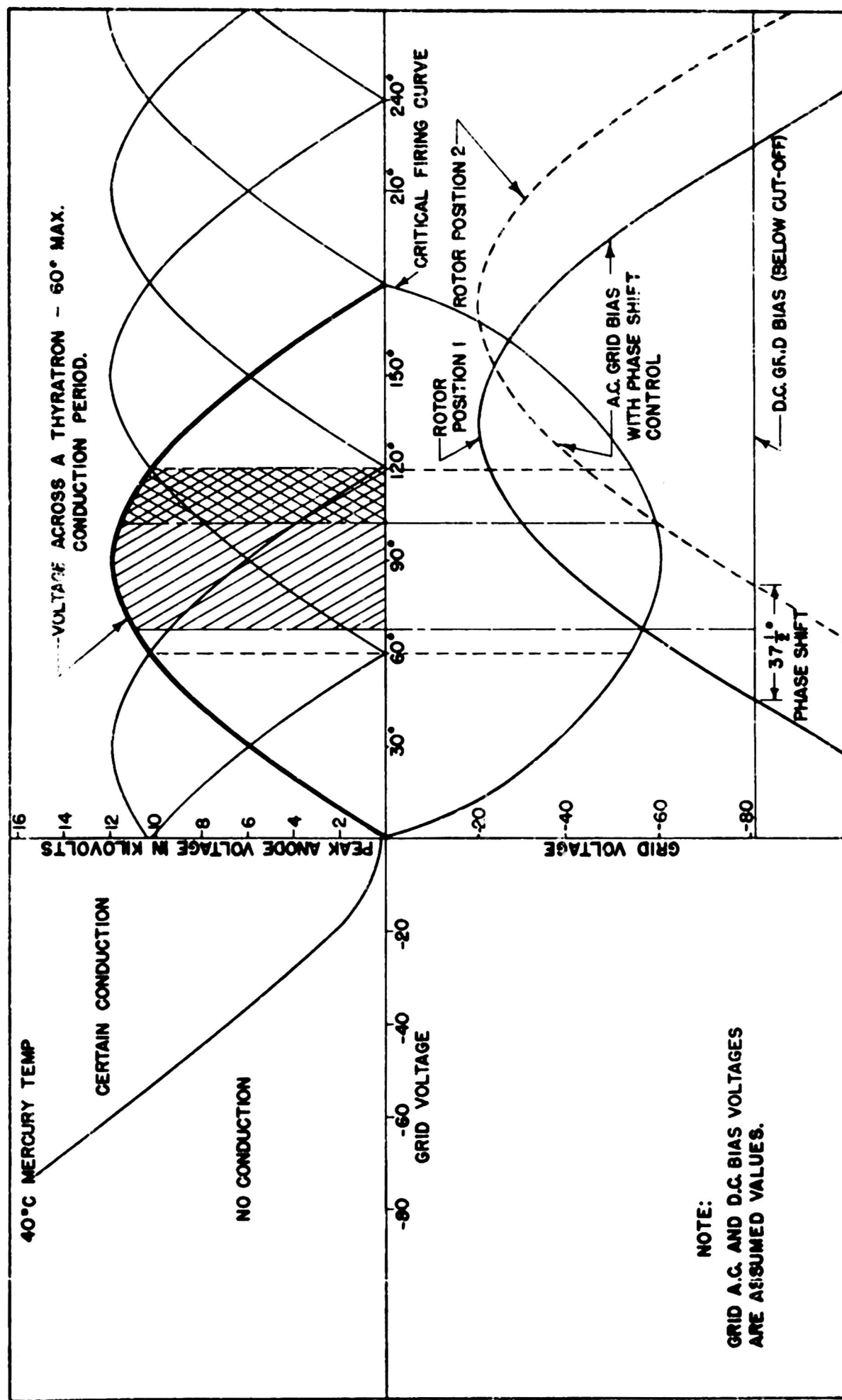


FIG. 5. WL-678 THYRATRON PHASE SHIFT CONTROL CURVES.

REGULATOR

A complete circuit diagram of the r-f energy regulator is shown in Figure 6 and the parts and part functions are listed in Appendix II.

The sensing element of this regulator is a toroid coil, mounted coaxially about one of the r-f conductors emerging from the induction heater. The toroid coil consists of a number of turns of insulated wire wound about a fiber torus. A voltage is generated in this coil by the flux linkage from the a-c current flowing through the r-f conductor to the work coil. This r-f voltage is transmitted through a shielded cable to the chassis containing the regulator. Here, a 6A15 tube, V_1 , functioning as a half-wave rectifier, converts the toroid coil r-f voltage to d-c. The ripple in the d-c voltage is removed by a pi filter formed by two mica condensers and an r-f choke. The reactance-frequency curve of this filter is shown in Figure 7. With this type of filter it is important to operate above the resonant and anti-resonant frequencies. (For our purposes a tuned filter is impractical because the frequency is changed whenever a different work coil is used.) As can be seen from the curve, the region of operation is well above these resonant points. Calculations for the theoretical curve are described in Appendix III. The filter presents an impedance of the order of one-thousandth that of the load resistor to the r-f currents.

R_1 in series with the parallel combination of R_2 and R_3 forms a voltage dividing network for the d-c voltage output from the rectifier. The potentiometer, R_3 , having ten times the resistance of R_2 , acts as a vernier for the output voltage. This d-c output voltage is a function of the r-f current, which in turn is a function of the sample temperature. Therefore, a variation of this voltage represents a change of the sample temperature. The output of R_3 is compared to the voltage output of potentiometer R_5 and rheostat R_6 . Battery B_1 is the reference potential for the regulator. A given setting of R_5 and R_6 represents a certain reference temperature (in terms of voltage) for a given work coil and sample setup. The battery, which is an ordinary lead storage type, is bled constantly by R_7 so as to maintain the battery output voltage on the plateau of the voltage-time curve for a given current.

Switch S_1 is used for rough reference voltage-output settings. R_8 and R_9 are fine adjustments. The adjustment of R_3 , R_5 and R_6 , and S_1 at the start of a run is used to obtain a null balance on the galvanometer G . The galvanometer damping resistor is R_4 . The various combinations of R_8 , R_9 and R_{10} provide sensitivity adjustments for the galvanometer. S_2 is a reversing switch for the galvanometer current.

During initial setting-up operations for null balance, the galvanometer may start to oscillate rather violently. To decrease the galvanometer travel and reduce the waiting time, S_4 can be used to connect an adjustable shunt, R_{11} , across the galvanometer terminals and thus to provide severe damping. This shunt is used only during setting-up operations, and not during the run.

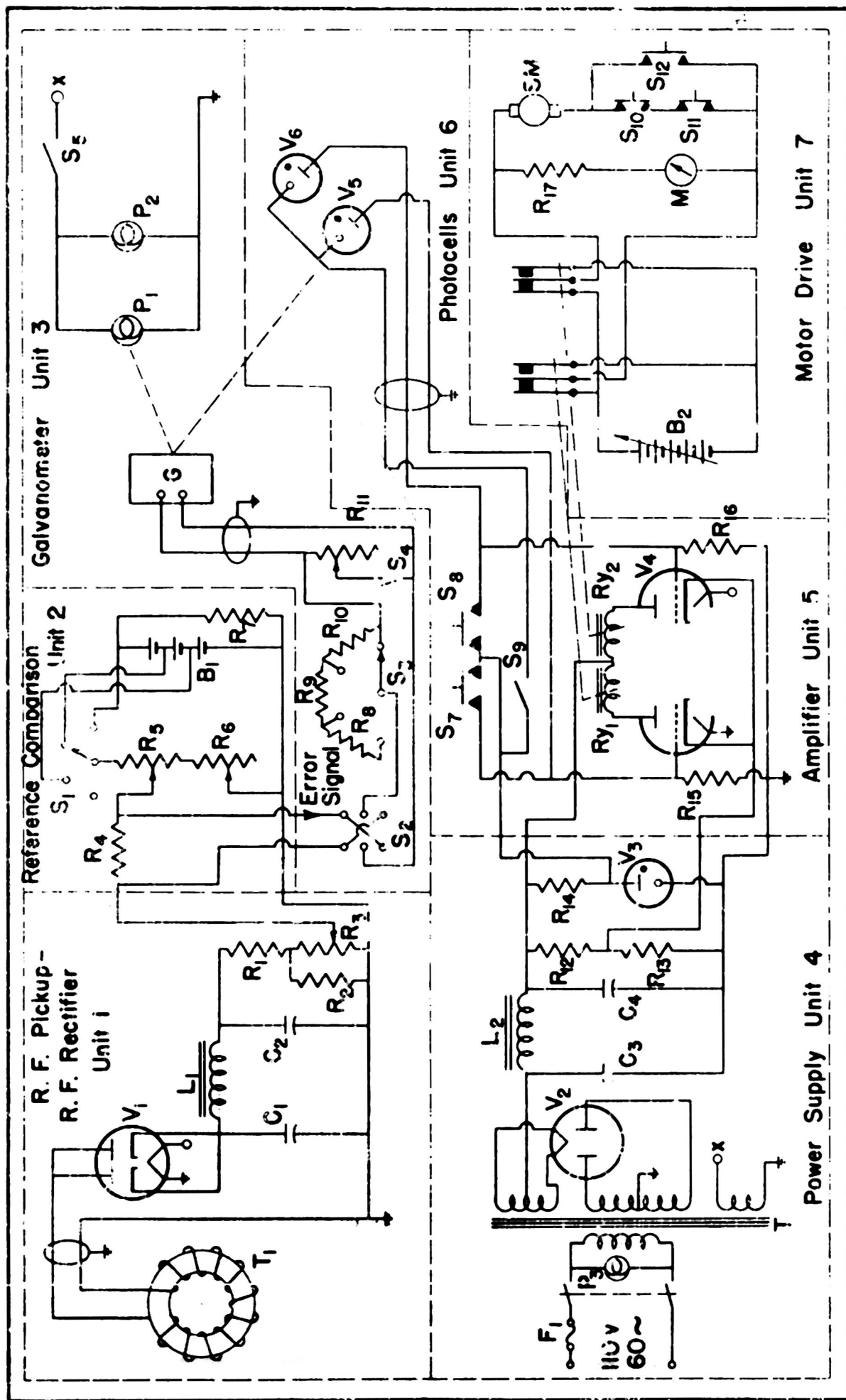
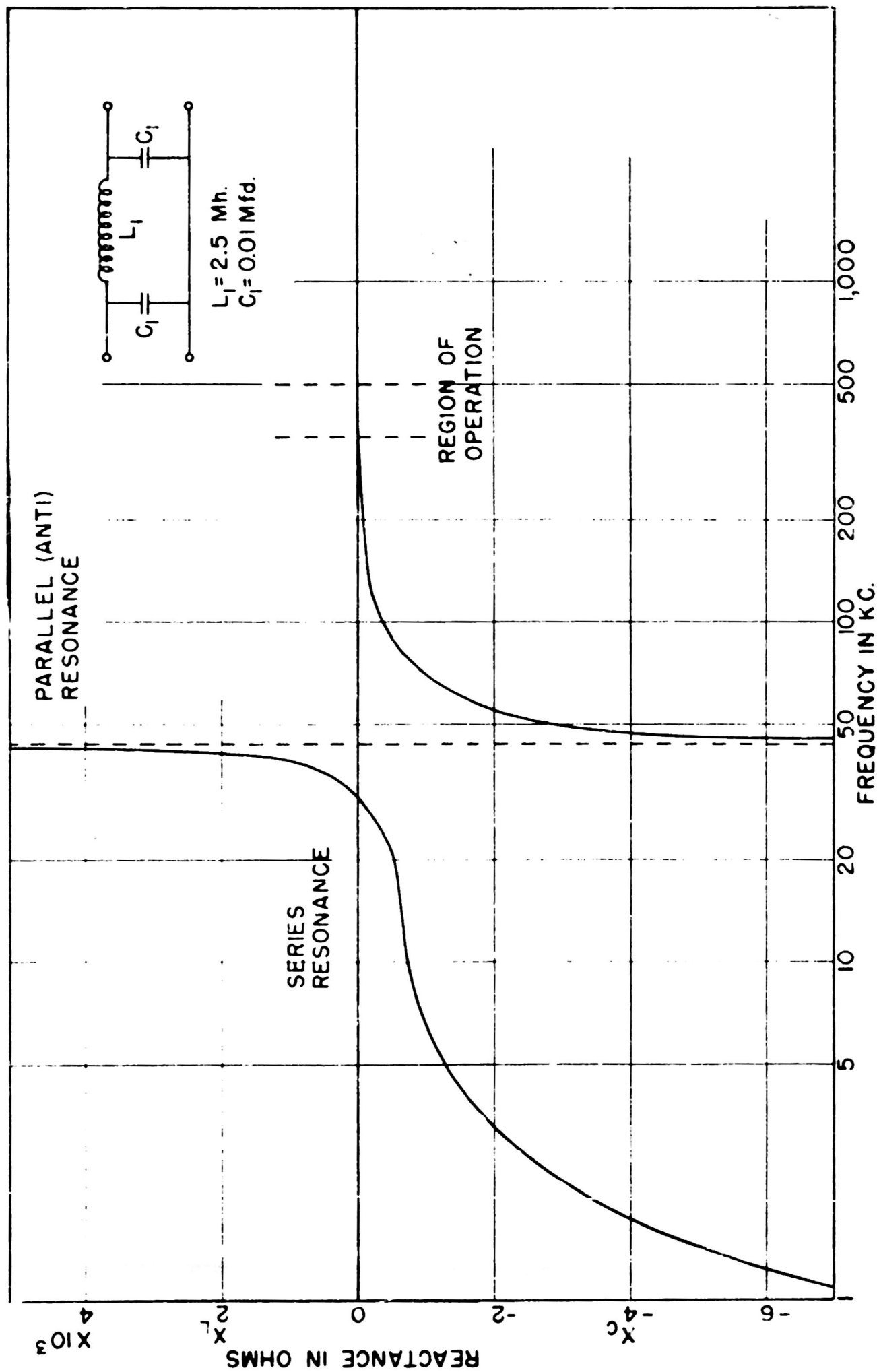


Fig. 6. R. F. Energy Regulator



The galvanometer, which is located in a light-tight housing, is of the portable box-type with a glass scale in the front. This scale is removed and two gas-type photocells are mounted end-to-end in such a manner that the cathodes are in approximately the same plane as the one the glass scale formerly occupied. Thus, light from the lamp, P_1 , reflected from the galvanometer mirror, falls on either of the two phototubes or in the null balance position located between the two tubes (see Figure 8 for their relative positions). The operator adjusts the battery reference voltage and the d-c input voltage (derived from the toroid coil) until the light beam falls in the null position. The regulator is then ready to begin temperature regulation.

A conventional power supply is used to provide d-c voltage for the photocells and the amplifier relay control tubes V_5 , V_6 and V_4 respectively. This supply consists of transformer T_2 , rectifier tube V_3 , filter condensers C_3 and C_4 , choke L_3 , and voltage dividers R_{12} and R_{13} . The voltage drop across R_{13} is used to bias the two grids in V_4 below cutoff. The constant voltage from V_3 , a gas regulator tube, is applied to the anodes of the two photocells V_5 and V_6 .

When light from the galvanometer mirror falls between the two photocells, the null position, neither phototube conducts current. A large voltage drop takes place across the phototube and a small drop across R_{15} or R_{16} . The 'dark' resistance of the phototube is of the order of several hundred megohms. Thus, with the phototube in series with a grid resistor such as R_{15} , very little voltage can be developed to render one triode in V_4 conductive. However, as soon as light falls on either phototube, its effective resistance drops to a lower value (because of current conduction) and there is a relatively large voltage drop across the 10-megohm grid resistor. One triode in V_4 is then brought up to the conducting state. Either relay RY_1 or relay RY_2 closes, depending on which triode is conducting current.

When either RY_1 or RY_2 is closed, voltage is applied from B_2 , a variable voltage source, to a small permanent-magnet-field-type motor, SM . The polarity of voltage determines the direction in which the motor rotates. Relay RY_1 causes the motor to rotate in one direction and RY_2 reverses the rotation. Thus a deviation of sample temperature causes the motor, SM , to rotate in one of two directions through intermediate components, depending upon the direction of change of temperature.

Switches S_7 and S_8 are manual controls, and S_9 is used to put the regulator into operation after the setting-up procedure. Visual indication of motor direction is given by the zero-center meter M . R_{17} is a voltage divider for the meter. The switches S_{10} and S_{11} are limit stops on the motor and S_{12} is a manual limit remover.

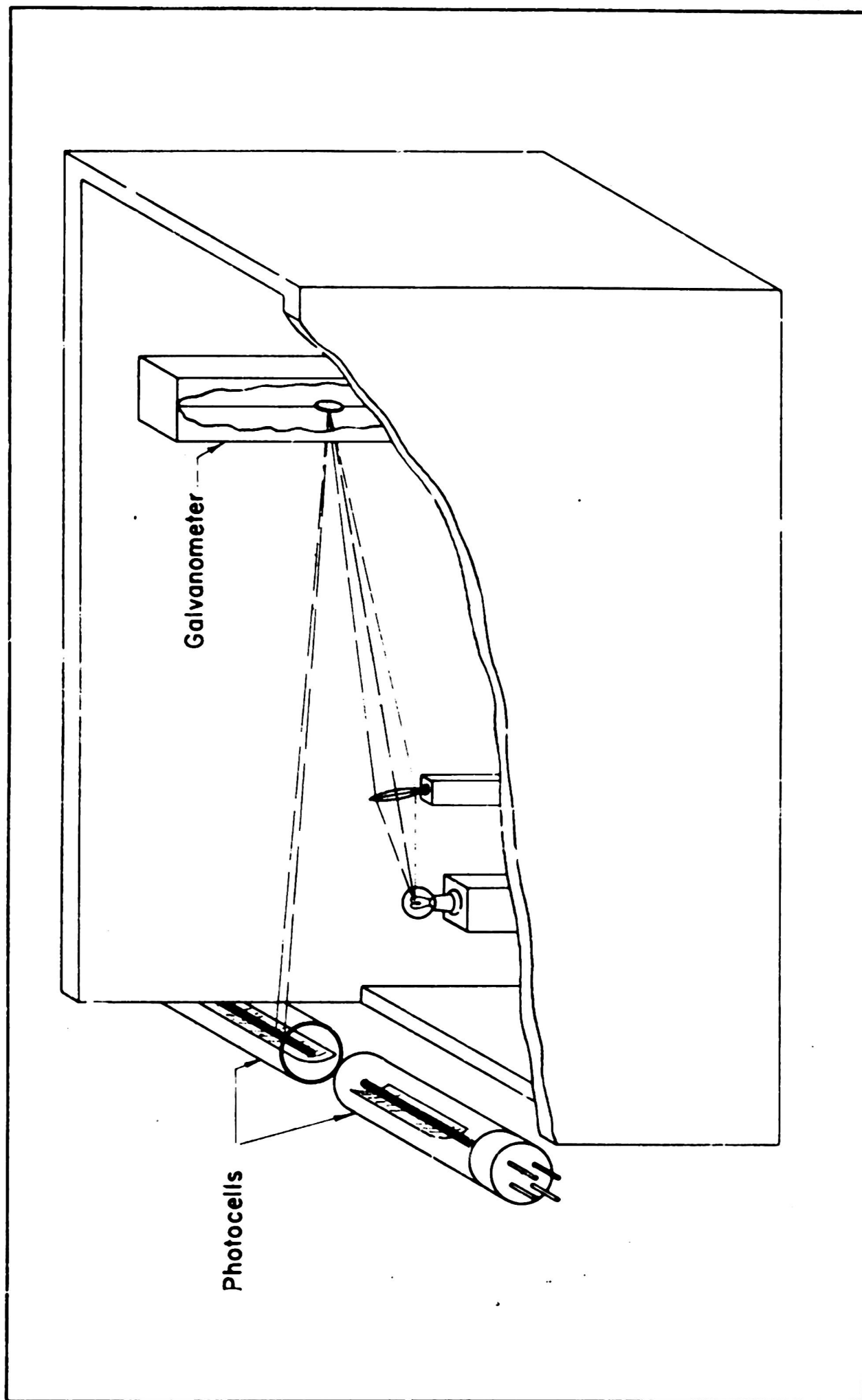


Fig. 8. Photocell Coupling to Galvanometer

The motor, SM, drives the phase-shifter control that determines power output of the induction heater, as previously described.

A sensing element detects any change in the sample temperature and, through the various circuits, restores the original temperature in a null-balance fashion.

APPENDIX I

The amplitude of any harmonic, in a quantity that has a wave form as shown in Figure 9, can be determined by Fourier analysis. Since this wave form has a symmetry about the ordinate, only cosine terms are needed in the Fourier expression. Thus,

$$e_d(\omega t) = E_0 + \sum_{k=1}^{\infty} E_{km} \cos k \omega t, \quad (1)$$

where $e_d(\omega t)$ is the instantaneous voltage output, E_0 is the average value of the rectified voltage, and E_{km} the maximum value of voltage of the k th harmonic. Let p equal the number of anodes in the rectifier circuit. Then, every $\frac{2\pi}{p}$ radians sees a repetition of the wave form.

Therefore,

$$e_d\left(\omega t + \frac{2\pi}{p}\right) = E_0 + \sum_{k=1}^{\infty} E_{km} \cos\left[k\left(\omega t + \frac{2\pi}{p}\right)\right]. \quad (2)$$

Since $\cos(\omega t)$ is the same as $\cos(\omega t + 2n\pi)$, equation (2) is correct only if

$$k\left(\frac{2\pi}{p}\right) = 2\pi n, \quad (3)$$

where n is any integer. Thus,

$$k = np \quad (4)$$

Therefore, the harmonics in a 3-phase bridge rectifier with six anodes ($p = 6$) are the 6th, 12th, etc. The lowest ripple frequency in the d-c output voltage is six times 60 c, or 360 c. This high frequency makes any filtering action very easy (if the high voltage problem is neglected), or filtering may be omitted altogether as was done in the Westinghouse circuit.

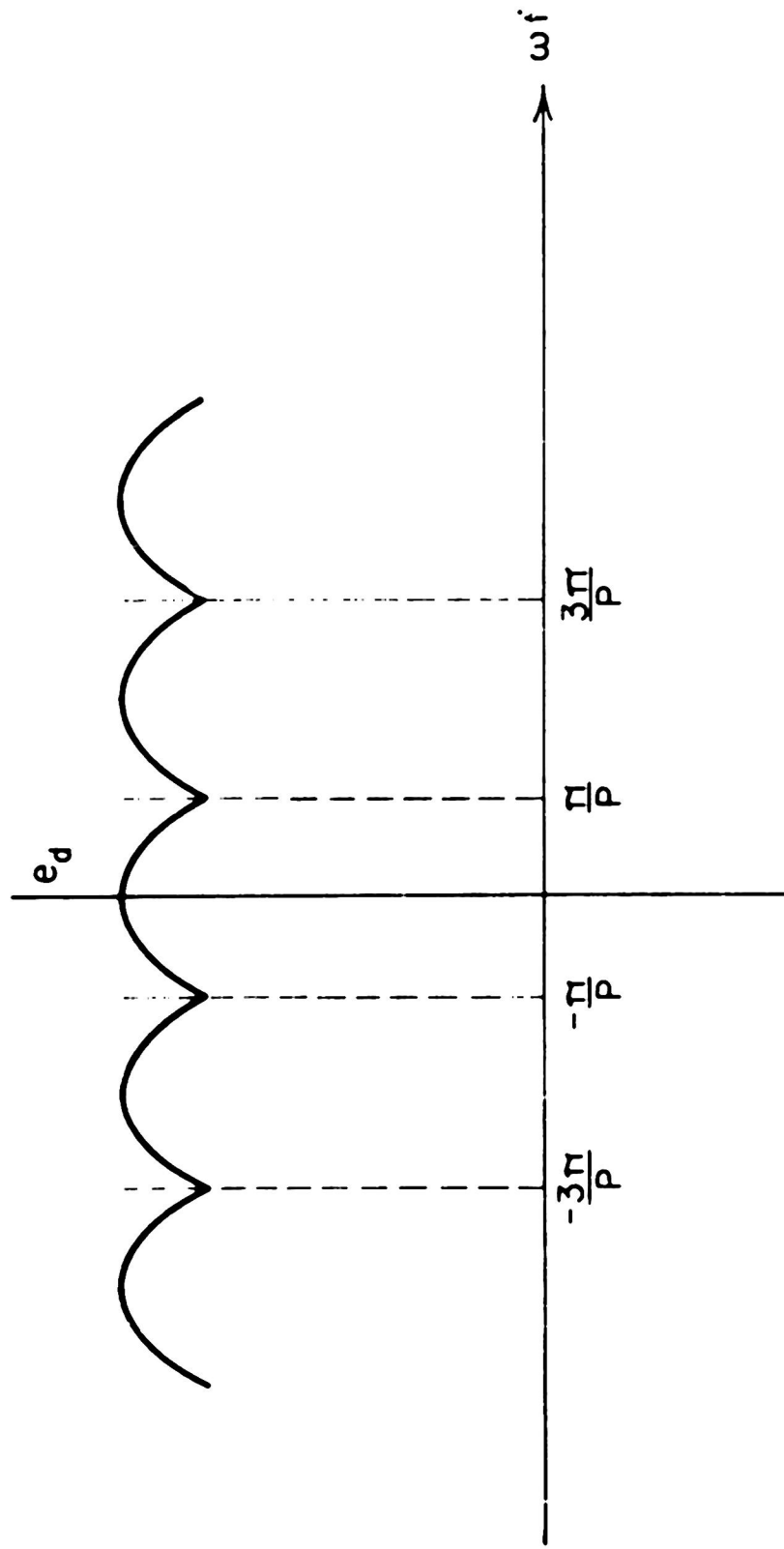


Fig. 9. P-Anode Rectifier Output Voltage Waveform

Continuing with the Fourier analysis,

$$E_{km} = \frac{1}{\pi} \int_{-\pi}^{\pi} e_d \cos k(\omega t) d(\omega t) \quad (5)$$

By symmetry,

$$E_{km} = \frac{p}{\pi} \int_{-\frac{\pi}{p}}^{\frac{\pi}{p}} e_d \cos k(\omega t) d(\omega t) \quad (6)$$

$$= \frac{2p}{\pi} \int_0^{\frac{\pi}{p}} E_m \cos(\omega t) \cos k(\omega t) d(\omega t) \quad (7)$$

$$= \frac{2p E_m \sin \frac{\pi}{p} \cos k \frac{\pi}{p}}{\pi (k^2 - 1)} \quad (8)$$

The average value of the rectified voltage is

$$E_o = \frac{p}{2\pi} \int_{-\frac{\pi}{p}}^{\frac{\pi}{p}} E_m \cos(\omega t) d(\omega t), \quad (9)$$

where E_m is the peak voltage to neutral of the transformer secondary.

$$E_o = \frac{p}{\pi} E_m \sin(\omega t) \Big|_0^{\pi/p} = \frac{p}{\pi} E_m \sin \frac{\pi}{p} \quad (10)$$

The nth harmonic ripple fraction is

$$r_n = \left| \frac{\text{RMS value of harmonic voltage}}{\text{average value of rectified voltage}} \right| = \left(\frac{E_{km} \sqrt{2}}{E_o} \right) \quad (11)$$

$$= \frac{2p E_m \sin \frac{\pi}{p} \cos k \frac{\pi}{p}}{\sqrt{2} \pi (k^2 - 1) (p/\pi E_m \sin \frac{\pi}{p})} = \frac{\sqrt{2} \cos k \frac{\pi}{p}}{(k^2 - 1)} \quad (12)$$

Replacing k by np , we have the relation

$$r_n = \frac{\sqrt{2} \cos n\pi}{(np)^2 - 1} = \frac{\sqrt{2}}{(np)^2 - 1} ; \quad (13)$$

therefore, with $n = 1$ and $p = 6$,

$$r_1 = \frac{\sqrt{2}}{(36-1)} = \frac{\sqrt{2}}{35} = 0.0404 \quad (14)$$

The secondary utilization factor, $(UF)_s$, by definition is the ratio of the d-c power output of the rectifier to the volt amperes in the secondary, where

$$(UF)_s = \frac{E_o I_o}{p E_s I_s} \quad (15)$$

where I_o is the average value of the rectified current, E_s and I_s are the RMS values of secondary voltage and current, respectively. However,

$$I_s = \sqrt{\frac{1}{2\pi} \int_{-\frac{\pi}{p}}^{\frac{\pi}{p}} I_o^2 d(\omega t)} = \sqrt{\frac{I_o^2}{p}} = \frac{I_o}{\sqrt{p}} \quad (16)$$

Therefore,

$$(UF)_s = \frac{E_o I_o}{p E_s I_s} = \frac{E_o I_o}{p E_s I_o / \sqrt{p}} = \frac{E_o}{\sqrt{p} E_s} \quad (17)$$

$$= \frac{\frac{p}{\pi} E_L \sin \frac{\pi}{p}}{\sqrt{p} E_s} = \frac{p/\pi \sqrt{2} E_s \sin \frac{\pi}{p}}{\sqrt{p} E_s} \quad (18)$$

$$= \frac{\sqrt{2} p \sin \frac{\pi}{p}}{\pi} , \quad (19)$$

and when $p = 6$,

$$(UF)_s = \frac{\sqrt{12} \sin \frac{\pi}{6}}{\pi} = 0.551 \quad (20)$$

APPENDIX II

PARTS LIST FOR RADIOFREQUENCY ENERGY REGULATOR

- Units:
1. R.F. pickup - R.F. rectifier
 2. Reference comparison
 3. Galvanometer
 4. Power Supply
 5. Amplifier
 6. Photocells
 7. Motor drive

<u>Part</u>	<u>Description</u>	<u>Unit</u>	<u>Function</u>	<u>Manufacturer</u>
<u>Batteries:</u>				
B ₁	6 volt lead storage battery	2	Reference voltage	Exide Electronic Lab. Inc.
B ₂	Variable d-c voltage supply 5-23 volts	7	Motor drive voltage	
<u>Lamps and Pilot Lights:</u>				
F ₁	2 amp. AGC glass	4	Protects unit 4	Bussman Mfg. Company
P ₁	5-8 volt, bayonet base, G.E. No. 1131	3	Galvanometer lamp	General Elec. Company
P ₂	6.3 volt, 0.25 amp. bayonet base, GE-44	3	Galvanometer lamp indicator	General Elec. Company
P ₃	110 volt, 6 w. Type S6	4	Power supply indi- cator	Sylvania
<u>Capacitors:</u>				
C ₁ , C ₂	0.01 MFD, 600 d-c, W.V. Mica type 1455	1	R-f filter	Aerovox
C ₃	8 MFD, 600 d-c, W.V. electrolytic type GL 600	4	Filter	Aerovox
C ₄	25 MFD, 450 d-c, W.V. electrolytic type GL 450	4	Filter	Aerovox
<u>Chokes and Transformers:</u>				
T ₁	Forcid coil, 120 turns of No. 22 cotton insulated wire wound on a micarta annulus 3" I.D. by 5" O.D.	1	R-f voltage pickup coil	Special
T ₂	Primary 117 volts, 60 cycles Sec. No. 1 - 700 V.C.T. at 110 ma Sec. No. 2 - 6.3 V.C.T. at 1.5 A Sec. No. 3 - 5.0 V.C.T. at 3.0 A	4	Power transformer	Standard Transformer Corporation

<u>Part</u>	<u>Description</u>	<u>Unit</u>	<u>Function</u>	<u>Manufacturer</u>
<u>Chokes and Transformers (cont'd):</u>				
L ₁	2.5 mh., 250 ma. type 34100	1	R.F. choke	Millen Mfg. Company
L ₂	4 Henries, 250 ma., 60 ohms, type C-1412	4	Filter choke	Standard Trans- former Corp.
<u>Electronic Tubes:</u>				
V ₁	6AL5	1	R.F. rectifier	R.C.A.
V ₂	5U4G	4	Rectifier	R.C.A.
V ₃	0A3	4	Regulates phototube anode voltage	R.C.A.
V ₄	6SN7GT	5	Phototube amplifier	R.C.A.
V ₅ , V ₆	CE-1-AB	6	Gas phototube	R.C.A.
<u>Meters:</u>				
M	500-0-500 A. Model 327-T	7	Motor direction indicator	Triplett
G	L and N catalog No. 2420-A	5	Error signal detector	Leeds and Northrup
<u>Motors:</u>				
SM	28 volt d-c field, type 5068571	7	Grid phase control drive	Delco
<u>Relays:</u>				
RY ₁ and RY ₂	5,000 ohm plate relay SPDT, type LC ₅	5	Motor voltage control	Potter and Brunfield
<u>Resistors:</u>				
R ₁	100,000 ohm (2-50,000 in series) 20 w. Type DG	1	Voltage divider	I.R. C.
R ₂	10,000 ohm 10 w. Type AB	1	Voltage divider	I.R. C.
R ₃	100,000 ohm 10 w. Type 471-B	1	Balancing potentiometer	General Radio Company
R ₄	20,000 ohm 10 w. Type AB	2	Galvanometer shunt	I.R. C.
R ₅	200,000 ohm 10 w. Type 471-B	2	Reference voltage potentiometer	General Radio Company

<u>Part</u>	<u>Description</u>	<u>Unit</u>	<u>Function</u>	<u>Manufacturer</u>
<u>Resistors (cont'd):</u>				
R ₆	10,000 ohm 2 w Type W-10000	2	Vernier reference voltage adjustment	I.R. C.
R ₇	560 ohm 1 w. Type BW-1	2	Battery bleeder	I.R. C.
R ₈	22,000 ohm 1 w. Type BTA	3	Galvanometer sensitivity adjustment	I.R. C.
R ₉	51,000 ohm 1 w. Type BTA	3	Galvanometer sensitivity adjustment	I.R. C.
R ₁₀	22,000 ohm 1 w. Type BTA	3	Galvanometer sensitivity adjustment	I.R. C.
R ₁₁	10,000 ohm 1 w. Type DS11-116	3	Galvanometer shunt	I.R. C.
R ₁₂	45,000 ohm (20,000 and 25,000 ohms in series) 10 w Type AB	4	Voltage divider	I.R. C.
R ₁₃	25,000 ohm 10 w Type AB	4	Voltage divider	I.R. C.
R ₁₄	20,000 ohm 20 w Type DG	4	Current limiter	I.R. C.
R ₁₅	10 megohm 1 w Type BTA	5	Grid resistor	I.R. C.
R ₁₆	10 megohm 1 w Type BTA	5	Grid resistor	I.R. C.
R ₁₇	6,800 ohm 1 w Type BTA	7	Voltmeter resistor	I.R. C.

Switches:

S ₁	1 circuit, 4 position Type 3234J	2	Battery voltage selector	Mallory
S ₂	DPDT Type 1333 3A 125V	3	Galvanometer reversing switch	General Cement
S ₃	1 circuit, 4 position Type 3234J	3	Galvanometer sensitivity selector	Mallory
S ₄	SPST, Type 41	3	Galvanometer adjustable shunt on-off control	I.R. C.
S ₅	SPST, Type 1330 3A 125V	3	Galvanometer light control	General Cement
S ₆	DPST, Type 1332 3A 125V	4	Power supply on-off control	General Cement
S ₇	Push button type 1340 3A 125V	5	Manual motor operation	General Cement
S ₈	Push button type 1340 3A 125V	5	Manual motor operation	General Cement
S ₉	SPST Type 1330, 3A 125V	5	Automatic control	General Cement
S ₁₀	Push button, type 1340 3A 125V	7	Motor limit switch	General Cement
S ₁₁	Push button, type 1340 3A 125V	7	Motor limit switch	General Cement
S ₁₂	Push button, type 1340 3A 125V	7	Manual limit remover	General Cement

APPENDIX III

With reference to Figure 10, the input impedance is

$$Z = \frac{\left(-j \frac{1}{\omega C_1}\right) \left(j\omega L_1 - j \frac{1}{\omega C_1}\right)}{\left(-j \frac{1}{\omega C_1} + j\omega L_1 - j \frac{1}{\omega C_1}\right)} \quad (1)$$

The condition for series resonance is that the input impedance be zero. Therefore,

$$Z = 0 = \frac{\left(-j \frac{1}{\omega C_1}\right) \left(j\omega L_1 - j \frac{1}{\omega C_1}\right)}{\left(-j \frac{1}{\omega C_1} + j\omega L_1 - j \frac{1}{\omega C_1}\right)} \quad (2)$$

Thus,

$$\omega L_1 - \frac{1}{\omega C_1} = 0 \quad (3)$$

or

$$f_r = \frac{1}{2\pi \sqrt{L_1 C_1}} = \frac{1}{2\pi \sqrt{25 \times 10^{-12}}} = 31,831 \text{ cycles.}$$

One definition of parallel resonance or antiresonance is that $\frac{1}{Z} = 0$. Therefore,

$$- \frac{1}{\omega C_1} + \omega L_1 - \frac{1}{\omega C_1} = 0 \quad (4)$$

Hence,

$$f_{ar} = \frac{1}{\pi \sqrt{2L_1 C_1}} = \frac{1}{\pi \sqrt{(2)(2.5 \times 10^{-3})(10^{-8})}} \\ = 45,016 \text{ cycles.}$$

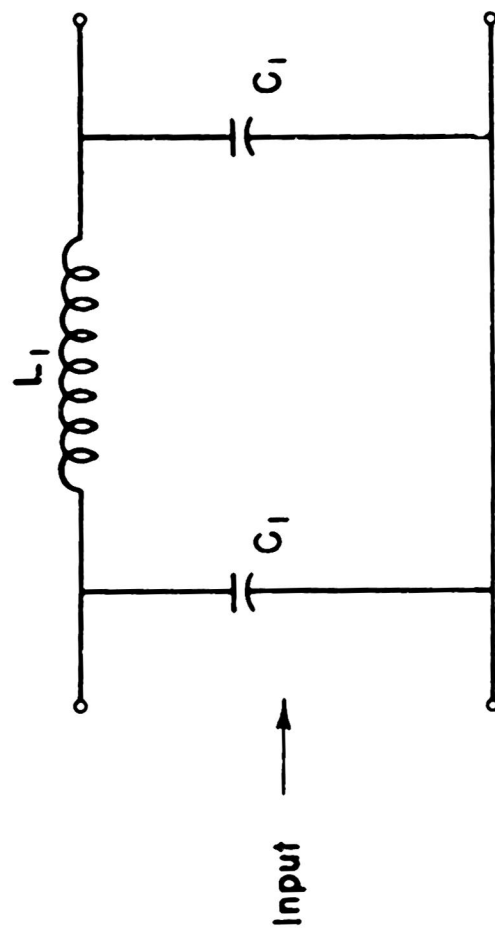


Fig. 10. Symmetrical π Filter

To obtain a plot of reactances for various frequencies, values are inserted in equation 1. Table I gives values for various quantities.

TABLE I

FILTER REACTANCE

Frequency KC	Reactance ohms
0	- ∞
1	- j 7,950
10	- j 721
100	- j 367
200	- j 81
300	- j 54
400	- j 40
500	- j 32
600	- j 26

REFERENCE

1. Speiser, R., Ziegler, G.W. and Johnston, H.L. Rev. Sci. Instruments, 20, 385 (1949)