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Final Report On

The Launching and Landing of Carrier Aircraft

Contract ONR 583 (01)
December 1952

Part III
Of Four Parts

Part I General Report
Part II Limitations of Cable-Drive Catapults
Part III A Multi-Jet Driven Catapult (Hydrapult)
Part IV Barricades

By

A University of Kansas Research Group
University of Kansas
Lawrence, Kansas

A MULTI-JET DRIVEN CATAPULT (HYDRAPULT)

Part III of the Final Report on

THE LAUNCHING AND LANDING
OF CARRIER AIRCRAFT

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Lawrence, Kansas

PREFACE

In August 1951 a University of Kansas Research group was assigned to study the general problem of the launching and landing of carrier aircraft. The work was done under contract ONR 583 (01). The purpose of the study was to obtain from a well-trained diversified group not too imbued with past and present Navy thinking and procedure, an independent evaluation of the problem and possible methods of solution, emphasis being placed upon development to meet future needs rather than just to solve immediate problems.

It was left to the group to choose those aspects of the problem on which to concentrate. As a result, certain aspects of the problem have been studied intensively while others have been considered only superficially. In analyzing the problem and dividing it into its several aspects, the group asked two questions:- (1) Is this aspect of the problem of decided importance? (2) Can the group make a worthwhile contribution by studying intensively this aspect of the problem? Emphasis was placed upon those aspects for which the answer to each question was affirmative.

The group submits its final report in four parts. The title and general content of each part is as follows:

Part I. General Report.

This section presents in a comprehensive yet understandable manner the problem as the group sees it, and makes clear what the group believes can and/or should be done. This section is relatively free of details but comprehensive as regards general conclusions.

Part II. Limitations of Cable-Drive Catapults.

This section presents a detailed study of the limitations of cable-drive catapults and the relative effects of different modifications of cable drives. It is rather analytical.

Part III. A Multi-Jet Driven Catapult (Hydrapult).

This section presents the results of a study of a multi-jet catapult which the group refers to as a "hydrapult." Although emphasis is placed upon the general features and operation of the proposed hydrapult, numerous details are included.

Part IV. Barricades.

This section presents the results of a model study of barricades. It contains many tabular data giving force distributions among the various elements of typical barricades. Numerous photographs are included.

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A MULTI-JET DRIVEN CATAPULT (HYDRAPULT)

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INTRODUCTION

A catapult must furnish to the aircraft being launched a large amount of energy in a very short time interval. Since a machine which can produce directly great amounts of power for short time intervals is impracticable, the usual procedure is to store a large amount of energy in some potential form and then release this energy during the launching period. The supply of potential energy is then reestablished during the relatively long time interval between launchings. The basic problem with such machines is that of developing a satisfactory method of transmitting the energy from the storage units to the aircraft. Any practical transmission system must be reliable; it must be capable of delivering the required energy in the short time interval available; it must not be too massive; and the mass of material that has to be accelerated with the aircraft must be kept low.

Basic Principle of the Hydrapult

One method of energy transmission is that used in the Pelton water wheel. In the catapult considered here, the water buckets are attached to a shuttle which moves along a straight track rather than to a wheel which rotates. The aircraft is attached to the shuttle in a standard manner. Work has been done elsewhere on a system using a single, large jet of water.* With a single jet serious difficulties arise because of divergence of the stream as the nozzle-to-shuttle distance becomes large. In order to avoid this difficulty it is proposed that many nozzles be placed beneath the shuttle track. Each nozzle will be turned on as the shuttle arrives at the position where the jet from that nozzle will strike one of the buckets mounted on the shuttle. As the shuttle moves to a position where the jet from this nozzle will no longer strike a bucket, the nozzle will be turned off. Thus the thrust of the single stream will be replaced by many short-time impulses as succeeding jets of water (or other fluid) propel the shuttle.#

In Greek mythology the "hydra" was a many-headed sea serpent. The name hydrapult is proposed for a catapult of the multi-jet design. The essential features of a hydrapult are shown by the schematic diagram on the following page.

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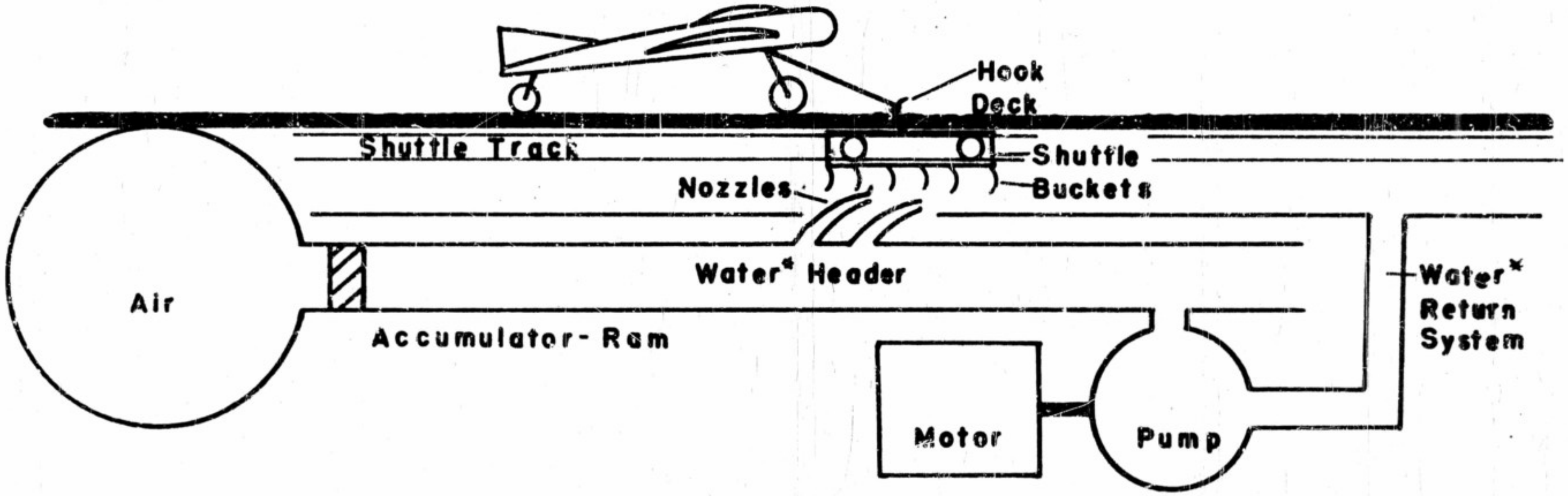
* "Considerations on a Large Hydraulic Jet Catapult," by Langley Aeronautical Laboratory, NACA RM L51B27, April 1951.

This basic idea has been under consideration by this group since September 1951. It has been considered independently by F. O. Ringleb and others, "Dynamics of a Hydraulic Jet Catapult with Automatic Jet Control," NAMC Report No. M-5231, March 1952.



* Or other fluid

Schematic Diagram of the Hydrapult

Basic Components of the Hydrapult

The general features of the several basic components of the hydrapult will now be considered one by one. A number of them will be considered in detail later.

Accumulator-Ram

The accumulator(s) will contain a high pressure gas charge. The pressure will be transmitted to the fluid by means of a diaphragm or piston. Appropriate controls will insure that the pressure is transmitted at the proper time.

High Pressure Piping System

A high pressure piping system will run from the accumulator-ram unit(s) to the nozzles which are located along the accelerated run. Flow of the fluid from the nozzles against the buckets on the shuttle will be controlled by fast acting valves which are activated by the shuttle.

Shuttle

The shuttle will move along the track as a result of the force exerted by the high pressure jets from the nozzles playing on buckets attached to the shuttle. This motion can be transmitted to the plane by an appropriate bridle and hook arrangement to provide a direct drive, or by a cable system to provide an indirect drive. The length of the shuttle should be approximately 10 feet. A height of 1 foot and a width of 2 or 3 feet appear feasible. Thus there is a good possibility that a direct drive arrangement can be utilized, with the shuttle operating in a channel directly under the flight deck. The buckets on the shuttle will be of the Pelton type in which the jet impinges on a sharp dividing line which splits the stream in half, with each half undergoing a change in direction of almost 180°. Since Pelton type buckets in existing water wheels withstand impulsive forces of over 100,000 lbs. per bucket, it would seem that design of satisfactory hydrapult buckets would not be too difficult.

Power Plant

After the fluid leaves the buckets it will impinge on the walls of the channel, go to a collecting duct at the bottom or side, and enter the return piping system. The fluid will then be pumped from the return system into the high pressure system by means of hydraulic pumps. The pressure will return the accumulator-ram to battery position as the fluid is pumped into the high pressure system.

Brake

If a direct drive system is used, the only moving mass which requires a brake is the shuttle with its attached buckets. Because of the comparatively light shuttle weight, a water brake can be used to decelerate the shuttle to standstill in a short distance, say 10 feet. The design of the water brake would present no major problem.

Retrieving Mechanism

Retrieving the shuttle is a simple matter because it is not attached to a cable or piston. A possible retrieving mechanism could consist of an electric motor driving an endless cable. The cable would have a dog attached to it; the dog would engage the shuttle and return it to battery.

Air Compressor and Associated Piping

An air compressor installation, and possibly an associated high pressure piping system, will be necessary to precharge the accumulator-ram(s). However, the capacity of the compressor system need not be large since after the initial air charge is placed in the accumulators, addition is required only to compensate for leakage.

Miscellaneous

The hydrapult may also require other miscellaneous devices such as an initial tensioner, a runaway shot preventer, and a pneumatic-electrical valve operating system. None of these is likely to present a serious development problem.

General Evaluation of the Hydrapult

The hydrapult appears to offer a promising method of catapulting heavy loads at high velocities. The component parts of the hydrapult are of standard design and hence should require a minimum of development. Once a given hydrapult design proves satisfactory, the design can be modified to handle heavier loads at the same terminal velocity by increasing the area available for fluid flow. A nearly linear relationship between catapult weight and dead load weight is obtained for any given terminal velocity.

By causing a number of jets to play on the shuttle at any one time, a ratio of maximum to average acceleration nearly equal to unity can be obtained. For instance, using six jets, and using an initial accumulator pressure which is 1.60 times the final pressure, a ratio of max/av. acceleration of 1.02 can be obtained. If only one jet is used, with the same pressure range, a value 1.11 is obtained for the ratio max/av. acceleration. If a limit is placed upon the maximum acceleration, the fact that the average acceleration is so nearly equal to the maximum allows the use of a shorter accelerated run.

Another advantage of the hydrapult is its adaptibility to launching planes of various weights. There are three obvious methods of adapting it for planes lighter than the designed weight. One of these is to render selected nozzles inoperative. This method alone would provide properly only for plane weights some submultiple of the designed weight. Another method is to reduce the initial gas pressure in the accumulator-ram. This method of adaptation might introduce considerable complication into the valves, pumping, and piping systems if it were exercised over too wide a pressure range. A third method is to reduce the nozzle area. Adaptation through reduction of nozzle area would complicate materially the nozzle design.

Consider, for example, a hydrapult designed to launch a dead load of 50,000 lbs. with six jets playing on the shuttle. If alternate nozzles were rendered inoperative, a dead load of 25,000 lbs. could be launched at the same average acceleration and with but a small increase in the max/av. acceleration ratio. By using 6, 5, 4, 3, 2 or 1 of the possible six jets playing on the shuttle, loads of 50,000, 41,600, 33,400, 25,000, 16,660 or 8330 lbs. respectively could be catapulted with the designed acceleration and an acceptable max/av. acceleration ratio. It would be possible to use in combination with this method the method wherein the initial accumulator pressure is reduced for planes lighter than the maximum designed weight. By selecting nozzles as the major adjustment and varying the pressure as a minor adjustment, it would be possible to launch a plane of any weight between 50,000 lbs. and 8330 lbs. with the designed acceleration.

A further advantage of the hydrapult is the possible variation of average acceleration along different portions of the accelerated run. For instance, if for psychological or physiological reasons it were desired to have a lower average acceleration during the initial part of the run and then to increase the average acceleration in the latter portions, the nozzle areas and/or spacings could be easily designed to yield (within limits) the desired results.

DESIGN FACTORS

In the investigation of hydrapult design and performance, a number of factors must be considered. The following, which are the more important of these factors, will be discussed one by one.

1. Fluid Velocity Build-Up Time
2. Jet Velocity versus Time Relation
3. Ratio of Maximum to Average Acceleration
4. Accumulator-Rams
5. Cavitation and Aeration
6. Jet System
7. Valves and Nozzles
8. Power Plant

Fluid Velocity Build-Up Time

An important factor in the satisfactory operation of the hydrapult is the time required for the fluid (driven by constant pressure) to approach its steady state velocity after being released by a valve. The importance arises from the fact that the fluid from a given nozzle can play upon the rapidly moving shuttle buckets but a short time. For example, if a shuttle having an effective length of 12 feet is moving with an average velocity of 200 ft/sec. during the time the fluid from a particular nozzle plays on the buckets, the total time that the jet from this nozzle can exert an effective force on the shuttle is

$$t = \frac{12}{200} = 0.06 \text{ sec.}$$

Thus, in order that the hydrapult shall conserve high energy fluid, the velocity build-up time must be a small fraction of 0.06 seconds.

An applicable formula for the velocity-time relationship during the build-up period is given in, "Applied Fluid Mechanics," by O'Brien and Hickox, McGraw-Hill, 1937, pages 234 and 238. The relationship is

$$v = v_s \tanh \frac{g H t}{L v_s}$$

where v is the fluid velocity at time t seconds, v_s is the steady state velocity, g is 32.2 ft/sec.^{2}, H is the effective head in feet, and L is the effective length of pipe.

For the purpose of determining numerically a representative velocity-time relation for the velocity build-up, the following arrangement will be assumed. An accumulator-ram supplies fluid at high pressure to a pipe with a nozzle at the far end. The pipe is assumed to be 5 feet long and 7.66 inches in diameter (area 46.2 sq.in.). The nozzle area is taken as 7.1 sq.in., and the velocity coefficient of the nozzle, C_v, as 0.98. The accumulator pressure is assumed to be 4000 psi, which corresponds to a water head of 9230 feet. These assumptions correspond to a steady state velocity of approximately 115 ft/sec. in the pipe. If the supply pipe is considered to be smooth and to include only smoothly rounded curves, the friction drop throughout will be negligible in comparison to the head of 9230 feet. Although the accumulator will be located below the nozzle, the difference in elevation is negligible in comparison to the head. Thus, the only drop in pressure which need be considered is that in the nozzle. Accordingly, the steady state velocity is given by

$$v_s = C_v \sqrt{2\ g\ H} = 0.98 \sqrt{2 \times 32.2 \times 9230} = 755 \text{ ft/sec.}$$

The effective length L of the pipe applicable in calculation of the velocity build-up is given by

$$L = L_p \frac{A_n}{A_p} = 5 \times \frac{7.1}{46.2} = 0.77 \text{ ft.}$$

where L_p is the pipe length, A_p the pipe area, and A_n the nozzle area. Thus the velocity-time relationship for the assumed conditions becomes

$$v = v_s \tanh \frac{g\ H\ t}{L\ v_s} = 755 \tanh \frac{32.2 \times 9230}{0.77 \times 755}\ t$$

$$= 755 \tanh(511\ t) \qquad \text{ft/sec.}$$

The following table gives calculated values of the velocity attained by the fluid at various times after the fluid was started in motion.

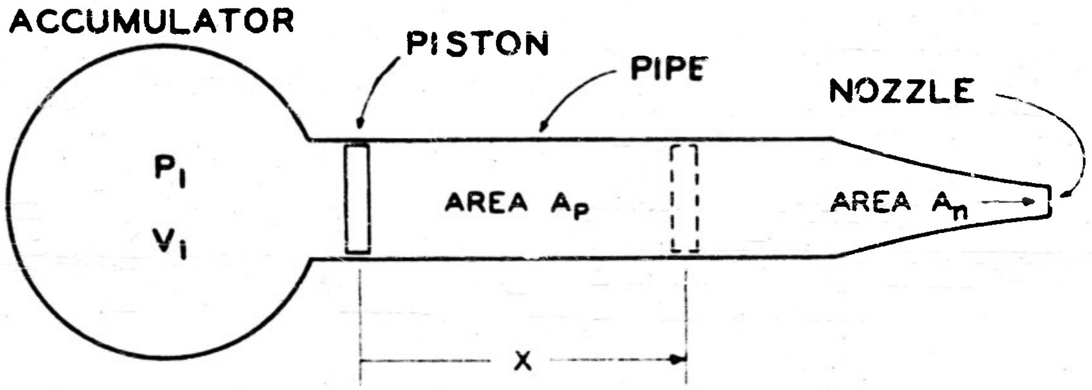
t (in sec.)	v (in ft/sec.)
0.000	0
.001	355
.002	582
.003	688
.004	730
.005	746
∞	755

The above calculations indicate that if the control valve is assumed to operate instantly, the jet will build up to 91.1 % of its steady state velocity in 3 milliseconds, and to 98.8 % of its steady state velocity in 5 milliseconds. This indicated fast velocity build-up would be entirely satisfactory for hydrapult operation. However, it would be extremely desirable to have experimental verification of the calculated values.

Jet Velocity versus Time Relation

A constant pressure accumulator is not feasible; the pressure will fall significantly as the volume of fluid delivered by the nozzle increases. Hence, the jet velocity will never attain the steady state value corresponding to the initial pressure. After reaching a maximum at the end of the initial build-up, it will decrease thereafter. Consideration of this variation of jet velocity with time is important in evaluating the performance of the hydrapult. The velocity-time relationship will now be developed, after several simplifying assumptions are made. These assumptions will not influence the results significantly.

Consider an accumulator-ram supplying high pressure fluid to a pipe and nozzle as shown below.



Initially the weightless piston is at the left-hand position shown; the accumulator is filled with a gas of volume V_1 at pressure P_1; the pipe of area A_p is filled with fluid at rest. Upon release of the piston, the gas pressure acting upon the fluid through the piston will force the fluid out through the nozzle of area A_n. As shown in a previous section, the transient build-up time of the jet velocity will be small, 3 to 5 milliseconds if the initial pressure P_1 is high and if the pipe is not too long. Since the transient build-up time is small (5 % of the total time the jet is in operation), it will be neglected in what follows. During the time t the piston will move through the distance x. If P and V represent respectively the gas pressure and volume corresponding to this new position of the piston, and if one assumes that the rapid expansion is adiabatic (the total flow time is less than 1 second),

$$V = V_1 + A_p x \quad \text{and} \quad P V^k = P_1 V_1^k .$$

Combining these two equations,

$$P\,(V_1 + A_p\,x)^k = P_1\,V_1^{\,k} \;. \qquad (A)$$

The actual jet velocity v_j will be

$$v_j = C\,\sqrt{P} = \frac{A_p}{A_n}\,\frac{dx}{dt} \qquad (B)$$

from which

$$\sqrt{P} = \frac{A_p}{C\,A_n}\,\frac{dx}{dt} \qquad (C)$$

where C is a constant. For water $C = C_v\,\sqrt{2g/0.434} = 12.18\,C_v$, where C_v is the velocity coefficient of the nozzle, where P is in psi, and where v_j is in ft/sec.

All frictional and inertial effects, with the exception of nozzle friction, have been neglected in writing the above relationships. It has been pointed out previously that the neglected effects are small. Combining (A) and (C) above leads to the equation

$$(V_1 + A_p\,x)^{k/2}\,dx = \frac{C\,A_n}{A_p}\,\sqrt{P_1\,V_1^{\,k}}\;dt \;.$$

Integrating this expression,

$$\frac{1}{(k/2 + 1)\,A_p}(V_1 + A_p\,x)^{k/2 + 1} = \frac{C\,A_n}{A_p}\,\sqrt{P_1\,V_1^{\,k}}\quad t + K$$

where K is a constant of integration. Since x = 0 at t = 0,

$$K = \frac{1}{(k/2 + 1)\,A_p}\,(V_1)^{k/2 + 1} \;.$$

Inserting this value of K in the preceding equation,

$$\frac{1}{(k/2 + 1)A_p}(V_1 + A_p x)^{k/2 + 1} = \frac{CA_n}{A_p}\,\sqrt{P_1 V_1^{\,k}}\quad t + \frac{V_1^{\,k/2 + 1}}{(k/2 + 1)A_p}$$

and solving this explicitly for x,

$$x = \frac{1}{A_p}\left[C\ A_n(k/2 + 1)\ \sqrt{P_1\ V_1^{\ k}}\ t + V_1^{\ k/2 + 1}\right]^{2/(k+2)} - \frac{V_1}{A_p}\ .$$

Differentiating with respect to t yields

$$\frac{dx}{dt} = \frac{2}{k+2}\ \frac{1}{A_p}\left[CA_n(\frac{k}{2} + 1)\sqrt{P_1 V_1^{\ k}}\ \ t + V_1^{\ \frac{k}{2} + 1}\right]^{(\frac{2}{k+2} - 1)} \cdot \left[CA_n\frac{k+2}{2}\ \sqrt{P_1 V_1^{\ k}}\right].$$

But from (B)

$$v_j = \frac{A_p}{A_n}\ \frac{dx}{dt}\ .$$

Substituting the above value of dx/dt into this and rearranging terms,

$$v_j = C\sqrt{P_1\ V_1^{\ k}}\left[C\ A_n(\frac{k+2}{2})\sqrt{P_1\ V_1^{\ k}}\ \ t + V_1^{\ (k+2)/2}\right]^{-k/(k+2)}\ .$$

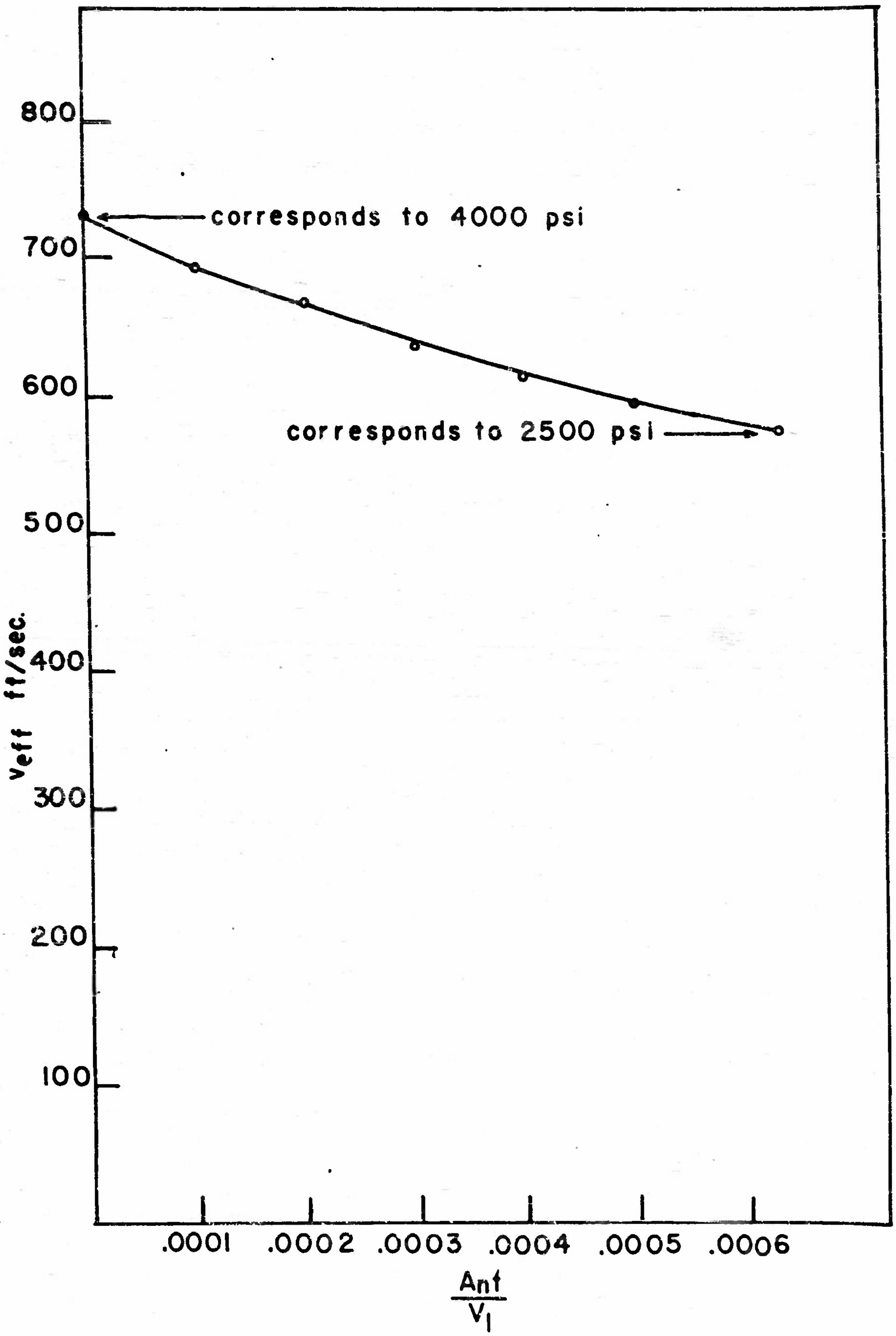
In the specific case for which the accumulator gas is air with $k = 1.4$, the fluid is water for which $C = 12.18\ C_v$, the initial pressure P_1 is 4000 psi, and C_v is taken as 0.98, the last equation above reduces to the following simply through substitution of numerical values.

$$v_j = 755\left[1283\ \frac{A_n}{V_1}\ \ t + 1\right]^{-0.412}$$

This shows the manner in which the jet velocity depends upon time, the nozzle area and the initial volume under the conditions represented by the specific case. If the axis of the nozzle is inclined at an angle δ with the direction of motion of the shuttle, the effective velocity is given by

$$v_{eff} = v_j \cos\delta = 755\left[1283\ \frac{A_n}{V_1}\ \ t + 1\right]^{-0.412} \cos\delta\ .$$

The curve on the following page represents a plot of this equation, v_{eff} versus $A_n t/V_1$, for the particular case where the angle δ is 15°. Inspection of the curve shows that within the pressure range 4000 to 2500 psi, the jet velocity-time relationship is linear within a maximum divergence of 2.3 %.



Jet Velocity — Time Relationship

Ratio of Maximum to Average Acceleration

In this section a simplified analysis of the maximum to average acceleration will be made for a typical hydrapult. The shuttle will be assumed to be 10 feet long, with six buckets at 2-foot intervals. The nozzle spacing will also be taken as 2 feet. An accelerated run of 200 feet is assumed. The shuttle is assumed to be in such a position that the nozzles at 104, 106, 108, 110, 112 and 114 feet are playing upon the buckets. A step by step analysis of the required nozzle area for an average acceleration of 4g and a force of 230,000 lbs. shows that the nozzle area required would be 4.52 sq.in. for the nozzle at 104 feet and 4.70 sq.in. for the nozzle at 114 feet. Intervening nozzles would require intermediate areas. Thus it appears that a nozzle area of say 4.65 sq.in. would be satisfactory for each of the six nozzles.

When the nozzle at 114 feet first plays upon the first shuttle bucket, the effective jet velocities at the nozzles, as taken from the curve on the preceding page, are:

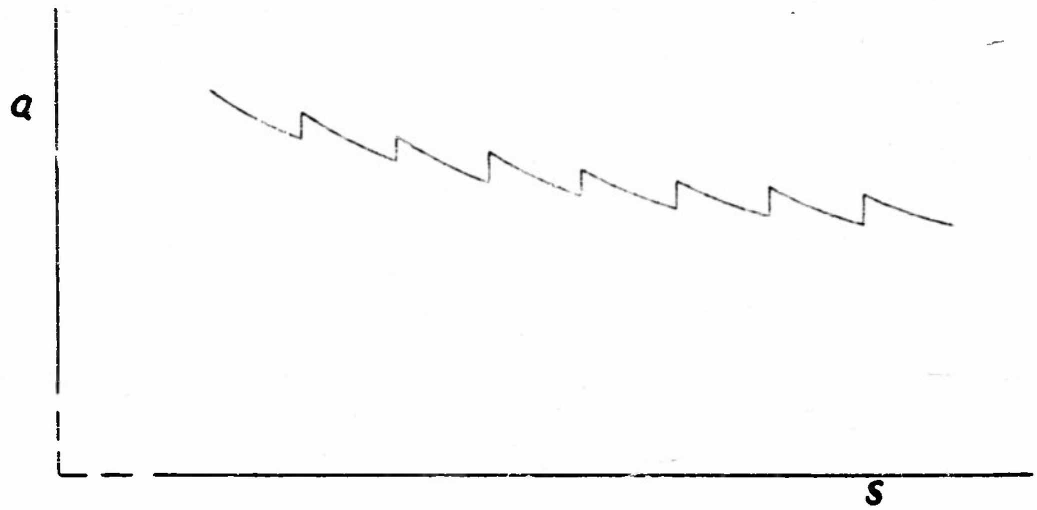
$$v_{114} = 725 \text{ ft/sec.}$$
$$v_{112} = 690$$
$$v_{110} = 660$$
$$v_{108} = 635$$
$$v_{106} = 612$$
$$v_{104} = 591$$
$$\Sigma v_{eff} = 3913$$

After the shuttle travels 2 feet, the nozzle at 104 feet ceases to play upon the last bucket on the shuttle. At the instant just prior to this the effective jet velocities are:

$$v_{114} = 690 \text{ ft/sec.}$$
$$v_{112} = 660$$
$$v_{110} = 635$$
$$v_{108} = 612$$
$$v_{106} = 591$$
$$v_{104} = 574$$
$$\Sigma v_{eff} = 3762$$

Since the nozzle areas were assumed to be equal, and since the shuttle velocity in this 2-foot interval has changed only from 171.3 to 172.9 ft/sec., the shuttle acceleration is readily obtainable as a function of the summation of jet velocities.

This idealized acceleration versus distance relation is shown in the figure below.



The ratio of maximum to minimum acceleration in this 2-foot interval is

$$\text{max/min. acceleration} = \frac{3913}{3762} = 1.04$$

while the ratio of maximum to average acceleration for the interval is

$$\text{max/av. acceleration} = \frac{2 \times 3913}{3913 + 3762} = 1.02 \; .$$

If in order to launch a lighter plane, only one nozzle were allowed to play on the shuttle at any time, the ratio of maximum to minimum acceleration would be

$$\text{max/min. acceleration} = \frac{725}{574} = 1.26$$

and the ratio of maximum to average acceleration would be

$$\text{max/av. acceleration} = \frac{2 \times 725}{725 + 574} = 1.11 \; .$$

It would seem from the foregoing analysis that highly satisfactory ratios of maximum to average acceleration would be feasible with the hydrapult, and that by proper nozzle control a wide variation in dead loads could be accommodated without resulting in a too unfavorable ratio of max/av. acceleration.

It should be pointed out that the above acceleration ratios have been calculated for only a 10-foot section of the accelerated run. The ratio of the peak acceleration encountered within the entire launching run to the average acceleration for the entire run will depend upon whether the average acceleration is maintained constant throughout the run. The average can be maintained constant by use of unit accumulators. Even were a large accumulator feeding many nozzles to be used, the average acceleration could be maintained constant by some means such as proper variation of nozzle area. If the average acceleration is kept constant the acceleration ratios calculated above for a 10-foot section would pertain equally well to the entire run.

It should also be pointed out that in order to minimize shock effects as the jets change from bucket to bucket, it may be desirable to stagger the bucket and nozzle spacings slightly so that all jets will not change at the same time.

Accumulator-Rams

There are two basic types of accumulator-rams which might be used in the hydrapult. These two types are:- (1) The small or unit accumulator-ram in which there is one small accumulator for each nozzle; (2) The large or multiple nozzle accumulator where one accumulator feeds a number of nozzles. It appears that the hydrapult would operate equally well with either type of accumulator. The choice of accumulator type will therefore depend primarily upon the development of a satisfactory valving system and upon the space available for the installation. The unit type requires a large number of small volume spaces whereas the multiple type requires a small number of large volume spaces. The unit accumulator will deliver a metered amount of water. The valves used will have to open quickly, but do not have to close quickly. On the other hand, the multiple type requires valves which both open and close quickly. It may also require some means of reducing water hammer shocks.

It is entirely possible that with some modification the accumulators used in aircraft hydraulic systems would be satisfactory for a unit type installation. The multiple type accumulator-rams might be similar to those used in die casting machines such as the model 400-A manufactured by the Hydraulic Press Manufacturing Company, Mount Gilead, Ohio. This machine uses an 8000 cu.in. nitrogen accumulator to obtain very high pressure for die casting work. Calculations show that the total accumulator size is well within practical limits when air is used as the gas. If the initial air pressure is 2400 psi and the final pressure is 1130 psi, a 40×10^6 ft.lb. hydrapult would require a total accumulator volume of 726 cu.ft.

Some consideration should be given to the use of a gas charge other than air. If P_1 and V_1 represent the initial pressure and volume of the gas charge, P_2 and V_2 the final pressure and volume, and k the coefficient of adiabatic expansion,

$$P_1 V_1^k = P_2 V_2^k .$$

Now the difference $(V_2 - V_1)$ is V_d, where V_d is the displacement volume which is fixed for a given design. Hence

$$V_2 = (P_1/P_2)^{1/k} V_1 = \frac{V_d}{1-(P_2/P_1)^{1/k}}.$$

Accordingly, if V_d and the ratio P_2/P_1 are fixed, the total accumulator volume V_2 will depend upon the value of k. Thus if a gas with a value of say k = 1.1 could be used instead of air, for which k = 1.4, the total accumulator volume would be decreased by about 25 %. There is some possibility of designing a gas mixture with this lower value of k. The possibility appears worthy of further investigation. An interesting article on this aspect of accumulator design is one entitled, "Hydraulic Accumulator for Constant Pressure Operation," by Kurlovits and Svindenkor, Engineers Digest, Vol. 3, No. 10, October 1946, page 516. However, the process described in this article takes place over a comparatively long time and thus is isothermal rather than adiabatic.

It is also possible to use steam near the critical point as the charge medium. Its use would reduce the accumulator volume required. However, the disadvantage of insulation, heaters, etc., seem to outweigh the saving in accumulator size, Nevertheless, the possible use of using high pressure, through-type steam generators as the hydrapult power source has considerable merit.

In the design of the accumulator-rams and associated piping systems, allowance must be made for the fact that the forces due to the nozzle reaction and due to pipe bends must be taken by the accumulator-ram piping system.

Cavitation and Aeration

The effects of cavitation may become very important in any hydraulic system. Thus in the design of a hydrapult, all components where cavitation might occur should be considered carefully. It would seem that the liquid sides of the ram surface and the Pelton type buckets on the shuttle would be the most vulnerable to cavitation effects. Further work would have to be done on this aspect of the problem.

Since the jet stream leaves the buckets at a fairly high velocity, considerable air may be entrained in the fluid. If this air were still in the fluid during the succeeding shot, the performance and efficiency of the nozzle would be reduced greatly. It is possible that if the exhaust fluid were allowed to stand for a time before re-entering the high pressure pumps, the entrained air would leave the fluid. This might entail use of a larger amount of fluid than would otherwise be required. Other possibilities are the use of centrifuges or heating to remove the air from the fluid. Further study on this aspect of the problem would also be desirable.

Jet System

In the direct drive hydrapult, the force driving the shuttle forward is exerted by jets playing on buckets attached to the shuttle. The character of the driving force will depend upon the nature of the jet stream. In order to insure a satisfactory jet stream, the nozzles must be carefully designed. Calculations have shown that the nozzle exit diameter required will be between 2 and 5 inches, depending upon the design chosen. Since the nozzles are not used to vary the fluid flow, and since nozzles with velocity coefficients of 0.98 have been built, the nozzle design should be along conventional lines. Nozzles up to 12 inches in diameter have been used in hydraulic turbines; and quite high jet velocities have been used. For example, in the Chandeline Plant in Switzerland, there are three Pelton type turbines, each developing 50,000 horsepower and operating at a head of 5740 feet.* This head corresponds to a pressure of 2490 psi.

The buckets should be of the Pelton type in which the jet impinges on a sharp edge which splits the stream into halves. Each half-stream then undergoes a change in direction of almost 180° before leaving the bucket. The buckets in present day Pelton turbine installations sustain impulsive forces of over 100,000 lbs. and have satisfactory resistance to cavitation effects. Thus the design of satisfactory buckets should not be too difficult.

The hydrapult shuttle is a force transmitting member only and can therefore be fairly light in weight. It must support the buckets and the bridle hook and be capable of withstanding the forces involved. The shuttle may also have attached to it certain simple mechanisms necessary to activate the valves which control the jets. It may be necessary to offset the deck slot from the center line of the nozzles so that there is no possibility of a jet stream coming through the slot.

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*"Handbook of Applied Hydraulics" - Davis, McGraw-Hill, page 617.

As the exhaust streams leave the buckets, they will have considerable velocity. It may therefore be necessary to use louvers of some sort in the sides of the channel so that the fluid does not interfere with the shuttle or the jet streams.

For cold weather operation, the fluid used may be some liquid with a low freezing point and with the density for which the hydrapult was designed. Heaters may be used to warm exposed surfaces if necessary.

Valves and Nozzles

Early in the considerations of the hydrapult it was realised that the turning on and off of the many jets of water or other fluid must be done so precisely as to make the specifications of valves severe. The valves must seal against high pressure when they are closed. They must be positive in action, never opening at an undesired time and always opening at the proper time. They must open quickly and completely, so that the jet of fluid can exert its full force on the shuttle for the proper length of time. The flow must cease at the proper instant. If the flow ceases ahead of time, the full impulse will not be given to the shuttle. If the flow lasts too long, large quantities of fluid will be wasted. The enlargement of equipment required to handle and store this extra volume of fluid could become serious.

The four major factors that must be considered in designing valves suitable for the hydrapult are listed below. They will be considered in order.

1. Velocity coefficient
2. Pressure
3. Timing
4. Cross-sectional area

The valve must not reduce the velocity of fluid flow drastically, either through causing turbulence or the introduction of other restraints. The open valve should present a smooth hole of the proper cross section through which the fluid can flow. While design of a valve which is satisfactory in this respect will certainly require careful consideration, it is certainly possible.

The pressure at which the valves must operate is well within the range of pressures now employed in standard engineering procedure.

The timing of the movement of the valves is a matter of great importance. The fluid flow must start as soon as the first bucket on the shuttle is in a position to intercept the jet, and it should reach full momentum as soon as possible. The flow should cease as soon as the jet can no longer efficiently engage the last bucket on the shuttle. Since the shuttle is not of great length, the time ("on-time") for which the fluid flows from a given nozzle is quite short.

It is particularly short after the shuttle reaches a high velocity. The on-time can be computed from simple kinematic considerations. Assuming the use of water as a fluid, assuming a 10-foot shuttle with six buckets spaced at 2-foot intervals, and assuming a 200-foot run with a constant acceleration of 4g, one obtains the values shown in the table below for the on-times of nozzles at different positions along the run.

Position of nozzle*	On-time
0 feet	432 milliseconds
50	100
100	73
150	60
190	42

Since the on-time is of the order of 50 milliseconds in some instances, it seems desirable to use a valve which will open in 5 milliseconds or less. Valves having 5 millisecond opening times are apparently feasible in the pressure range desired. The following information quoted from a letter dated 26 June 1952 to Warren E. Snyder from M. N. States of the Chicago ONR office, indicates that commercial valves approaching the requirements of the hydrapult exist or can be made.

1. "Atkomatic Valve Company, Inc., 400 N. Michigan Avenue, Chicago. This firm has available as stock items 3000 psi fast acting valves in 1/2, 3/4 and 1 inch diameters. Representative price on the 1 inch valve is $250 each. They do not have information on the speed of opening or closing except to claim that they are 'very fast'. They claim that the A. O. Smith Company in Milwaukee recently tested valves of this type, using 3000 psi hydrogen and found reliable lifetimes extending to 1 1/2 million operations. Atkomatic feels that they could design and build a valve to meet your 4000 psi 50 milliseconds normally closed valve requirement."

2. "Norman Engineering Company, 2115 W. Marquette, Chicago. This firm is the Chicago representative for the Skinner Electric Valve Company, Norwalk, Connecticut. They state that your requirements are certainly within design possibility and that they recently processed an order for Armour Research Foundation for a 1200 psi 2 sq. inch opening, 700 gal. mineral oil per minute application with a 5 millisecond opening and closing time but with a limited lifetime of about 1000 cycles. They stated that any request for pricing on development valves should be addressed to the home office of the Skinner firm."

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* Measured from the front end of the shuttle when the shuttle is in the ready position.

3. "Marine and Industrial Products Company, 1150 W. Marquette, Chicago. Mr. Vincent McCaffery, Sales Manager, advised that this firm routinely furnishes extremely fast acting large valves for AEC, many of which have specifications for up to 20,000 operations per minute. Mr. McCaffery stated that he would be pleased to quote on the design of a suitable valve if you will furnish him complete specifications. He is of the opinion that your requirements would necessitate the use of an air assisted solenoid valve operating on 110 or 230 volts. He felt that his company could design and build valves to meet your specification."

The closing time of the valve should also be of the order of 5 milliseconds, simply to conserve fluid. Since it appears feasible to construct a valve with a 5 millisecond opening time, it would appear feasible also to design it with an equal closing time were it not for the fact that one would like to close it against a high fluid pressure. This requirement will make the design difficult. One way of avoiding the difficulty would be to meter by independent means the amount of fluid flowing through each jet. The valves can then be closed easily and leisurely.

Activation of the valves at the proper instant is highly important. It must be done by the shuttle. Satisfactory activation can certainly be accomplished, though it may require an undesirable complexity of gadgets.

This group has made no attempt to design an appropriate valve and control system. It does wish to call attention, however, to one possible method of controlling the fluid flow without using a conventional valve. The method is simple and has certain distinct advantages, but it also has disadvantages. The method consists of using a "trigger cap" to turn the fluid jet on, and an accumulator to furnish a metered amount of fluid. When the metered amount has been furnished the jet ceases, and the trigger cap can be reset at atmospheric pressure.

The trigger cap might consist of a flat metal plate covering the end of the nozzle, one edge of the plate being hinged to a side of the nozzle and the opposite edge of the plate being latched to the opposite side of the nozzle. As the shuttle reaches that position at which a given jet should come into play, a projection on the shuttle releases the latch holding the cover plate in place. The high pressure fluid in the nozzle pushes the cover plate open, and the jet operates until a metered amount of fluid has been spent. Some mechanism will then return the caps to the closed position. This mechanism could be controlled manually or activated by the arrival of the shuttle at the end of its run.

One of the major difficulties in such a system would be the design of gaskets which would not allow the cap to leak when subjected to high pressure. A hollow, flexible, pressurized gasket has been used successfully in similar situations. A tubular gasket made of reinforced rubber is embedded in the face of the nozzle against which the cap fits. The cap is closed while the hollow gasket is at atmospheric pressure; the force required would be small. After the cap is latched the gasket is inflated to a pressure slightly larger than the maximum pressure to be exerted by the fluid inside the nozzle. The inflated gasket, if properly mounted, would provide a good seal.

The method of controlling the fluid flow suggested above has several distinct advantages over the use of conventional valves. It provides for quick opening and for a proper seal. It eliminates the necessity of closing and opening against a large pressure. It is recognized, however, that the method presents several difficulties. Both the hinge and the latch which hold the cap on the nozzle must be heavy. Dashpots would probably be necessary to absorb the energy of the cap after it opens. A reliable mechanism would be necessary to relatch the caps. Alternate inflation and deflation of the hollow gaskets would have to be provided for. Many of these difficulties would be eliminated if a conventional valve with the required operating characteristics can be made.

The fourth consideration in valve and nozzle design is the cross-sectional area. The aggregate jet stream area which must play on the buckets will now be computed approximately. Let F (lbs.) be the total force exerted on all shuttle buckets as fluid of density ρ (slugs/ft.^{3}), aggregate area A (ft.^{2}), and jet velocity v_j (ft/sec.) strikes the buckets on the shuttle which is moving with a velocity v_{sh} (ft/sec.). Also, let δ be the angle between the direction of the jet velocity and the direction of the shuttle travel. Furthermore, let it be assumed that the jet emerges from the bucket at an equal angle δ.

The force exerted by the jet streams upon the shuttle buckets is equal to the aggregate rate of change of momentum in the jet streams. Hence, if there is no loss of stream velocity during contact with the bucket,

$$F = 2 \rho A v_j (v_j \cos \delta - v_{sh}) .$$

If allowance is made for an assumed 5 per cent loss of stream velocity during contact with the bucket, the force is given by

$$F = 1.95 \rho A v_j (v_j \cos \delta - v_{sh}) .$$

Thus the aggregate jet area required is

$$A = \frac{F}{1.95 \rho v_j (v_j \cos \delta - v_{sh})} .$$

Consider a hydrapult operating between pressures of 4000 and 2500 psi and accelerating a 50,000-lb. load at 4g for a run of 200 feet. The aggregate nozzle area A from which fluid plays upon the buckets at any instant will depend upon whether the design employs unit accumulators or one large accumulator. If unit accumulators are employed the jet velocity used to calculate the area should be that corresponding to the average pressure 3250 psi. The calculated aggregate nozzle area A ranges from 17 sq.in. at the start of the run to 26 sq.in. near the end of the run. If a single large accumulator were used the jet velocity at the beginning of the run would be that corresponding to a pressure of 4000 psi, whereas the velocity near the end of the run would be that corresponding to 2500 psi. The corresponding aggregate areas range from 14 to 43 sq.in. In either case, if the aggregate area were divided equally among six circular cross section jets, the jet diameters would fall within the range from 1.7 to 3.0 inches.

The operating pressure range (P_1 = 4000 psi and P_2 = 2500 psi) for which these calculations have been carried out is not the optimum indicated by the equation derived in a later section for the minimization of weight. Larger nozzle diameters would result if the calculations were carried out for the optimum pressure range, but even then they would be less than 5 inches.

Power Plant

The hydrapult would require a high pressure pumping system with a suitable power supply. Indicated below are two types of pumps, one small and one large, which are currently available. The small one would be useful only with small unit accumulators. The capacity of the large one is comparable to the capacity required if but a single large accumulator is used.

1. Dowty Equipment Ltd., Cheltenham, England, manufactures a small two-stage Vardel pump# which has the following specifications:

5 gpm	13.5 lbs. weight
4000 psi	85 % efficiency
3000 rpm	

2. The Worthington Pump Co. manufactures a large pump which has the following specifications:

2140 gpm	8000 lbs. weight
4000 psi	6300 horsepower
10,000 rpm	

Thus it seems that there is available at least one pump which, with some modification, would be satisfactory for hydrapult service.

The choice of power supply will depend upon the pump used as well as upon such factors as convenience, resistance to battle damage, ease of repair, weight, simplicity and reliability.

"Applied Hydraulics," July 1952, page 90.

Possible power supplies are the electric motor, the steam turbine, the gas turbine and the piston type aircraft engine. From the standpoint of convenience, resistance to battle damage, ease of repair, and reliability, the electric motor is the most desirable. The chief disadvantage of the electric motor is the weight per horsepower. For instance, the electric motors used to supply power to the present H-8 catapult weigh 20 lbs/horsepower. A 40 x 10^6 ft.lb. hydrapult firing at 30-sec. intervals would require about 7500 horsepower. If 20 lb/horsepower electric motors were used the weight of the motors would be 150,000 pounds. Possibly by using evaporative cooling techniques the weight to power ratio could be reduced to 4 or 5 lbs/horsepower. This would reduce the weight of the required power unit materially.

If the high pressure pump operates at a high speed, say 10,000 rpm, gas or steam turbines might be used to advantage because of their light weight and their relatively high efficiency at high speeds.

A PROPOSED 40 x 10^6 ft.lb. HYDRAPULT

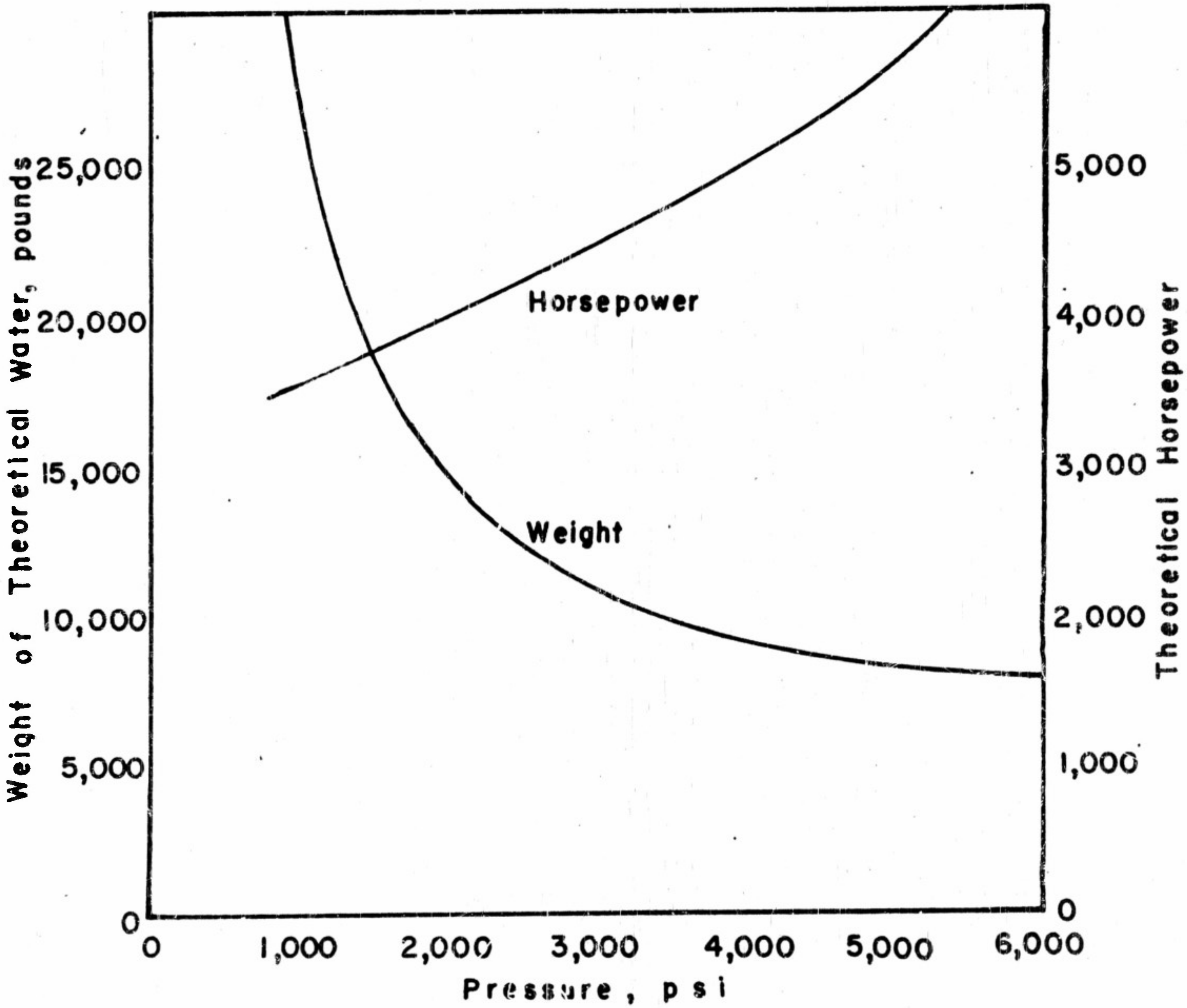
This section will deal with the general design of a proposed 40 x 10^6 ft.lb. hydrapult capable of accelerating a dead load of 50,000 lbs. at 4g over a 200-foot power run. This acceleration and run lead to an end speed of 227 ft/sec. (155 mi/hr. or 135 knots). With a 30 knot wind over the deck, this speed corresponds to an air speed of 165 knots (189 mi/hr.). The discussion of this proposed hydrapult will deal first with general features of the design, second with the required power plant, and third with the weight of the installation.

General

The mass to be accelerated will consist of a dead load of 50,000 lbs. and a shuttle with buckets of about 4000 lbs., a total mass of 54,000 lbs. Assuming that the frictional force is 14,000 lbs., the jets must exert on the shuttle a force given by

$$F = 54{,}000 \times 4 + 14{,}000 = 230{,}000 \text{ lbs.}$$

Preliminary calculations have indicated that an average accumulator pressure of 3000 psi, corresponding to a jet velocity of about 668 ft/sec., represents a good compromise between conflicting fluid volume and power plant requirements. As the jet velocity is increased, the total weight of fluid is decreased, but since the efficiency decreases the size of the power plant will increase. A representative plot of the variation of these two requirements for a slightly smaller capacity hydrapult is shown on the following page.



Weight of Theoretical Water, pounds
25,000
20,000
15,000
10,000
5,000
0
Theoretical Horsepower
5,000
4,000
3,000
2,000
1,000
0
Horsepower
Weight
0
1,000
2,000
3,000
4,000
5,000
6,000
Pressure, psi

Since constant pressure operation is difficult if not impossible to obtain, a pressure range from 4000 psi initial to 2500 psi final was selected. This corresponds to a pressure ratio of 1.6. With air as the working medium, and with an adiabatic expansion, the ratio of final to initial volume will be

$$\frac{V_2}{V_1} = \left[\frac{P_1}{P_2}\right]^{1/1.4} = 1.6^{.714} = 1.4$$

Step by step calculations at one-foot intervals have shown that, using water as the fluid, a theoretical amount of water of 12,700 lbs., or 204 cu.ft., would be required. If a 10 % increase for contingencies is allowed, the total displacement water would have a volume of 224 cu.ft. and a weight of 14,000 lbs. From the same step by step calculations, the total initial air volume in the accumulators was found to be 508 cu.ft. At the end of the run, the total air volume will be the sum of 508 and 224, or 732 cu.ft. These figures check well with the quantities calculated from the more general development in Appendix A.

Power Plant

The pump capacity must be sufficient to force a volume of water equal to the displacement volume, 224 cu.ft., into the high pressure accumulators. The pressure against which the pumps must work is initially 2500 psi and rises to 4000 psi as the accumulator rams are returned to battery. If a 30-second launching interval is used, the volume of fluid pumped per minute will be

$$Q = \frac{224 \times 2 \times 1728}{231} = 3350 \text{ gpm.}$$

Using 3250 psi as the average pressure against which the fluid must be pumped, and assuming a pump efficiency of 85 %, one finds that the power required is

$$\text{Power} = 0.000583 \frac{P\,Q}{E} = 0.000583 \frac{3250 \times 3350}{0.85} = 7470 \text{ horsepower.}$$

The pump must be capable of delivering 3350 gpm against a maximum pressure of 4000 psi. Two of the Worthington pumps mentioned on page 22 would constitute an adequate arrangement.

The pump may be driven by any suitable prime mover. From the standpoint of convenience an electric motor drive would be most satisfactory. However, to produce an output of approximately 7500 horsepower from induction motors weighing even as little as 5 pounds per horsepower would require a motor weight of 37,500 lbs. If the saving in weight is deemed to justify the added complexity, aircraft type engines or gas turbines weighing perhaps 2 pounds per horsepower could be used. Their use would reduce the prime mover weight to about 15,000 pounds.

The over-all efficiency of the hydrapult from prime mover shaft to kinetic energy of the plane would be

$$E = \frac{\text{Average power output}}{\text{Average power input}} \times 100 = \frac{2 \times 40 \times 10^6}{7470 \times 33{,}000} \times 100 = 32\,\% .$$

Weight Estimates

The over-all weight of the proposed 40×10^6 ft.lb. hydrapult will now be estimated by considering individually the probable weights of the components.

Accumulator

In later considerations having to do with the minimization of the weight of the hydrapult, an expression giving the weight in pounds of a spherical steel accumulator of volume V cu.ft. and maximum pressure P_1 psi is developed on the assumption that a stress of 26,000 psi is allowable in the steel. This expression (see page 31) is

$$Wt. = 0.028\, P_1\, V .$$

The proposed hydrapult requires an accumulator of 732 cu.ft. volume (see page 25) operating at a maximum pressure of 4000 psi. Substituting those numerical values into the above expression gives the following weight for the accumulator.

$$Wt. = 0.028 \times 4000 \times 732 = 82{,}000 \text{ lbs.}$$

Pump

The Worthington pump mentioned on page 22 operates at 4000 psi, has a capacity of 2140 gpm, and weighs 8000 lbs. Since the proposed hydrapult requires a pump which will furnish 3350 gpm, it is reasonable to suppose that the weight of this larger pump will be given approximately by

$$Wt. = 8000 \frac{3350}{2140} = 12,500 \text{ lbs.}$$

Power Plant

The proposed hydrapult requires a power plant of approximately 7500 horsepower. If electric motors weighing 5 lbs/horsepower are used, the weight of the power plant would be

$$Wt. = 5 \times 7500 = 37,500 \text{ lbs.}$$

Pipe (Including Nozzles)

In later considerations having to do with the minimization of the weight of the hydrapult, an expression is deduced for the weight of steel pipe of density 480 lbs/cu.ft. and maximum allowable stress 26,000 psi which would be required for an installation operating at maximum pressure P_1 psi, having a fluid displacement V_d cu.ft., and providing a terminal velocity v_t. This expression (see page 34) is

$$Wt. = 5.0 \times 10^{-7} v_t^2 P_1 V_d .$$

This expression should provide an adequate allowance for the nozzles also. The proposed hydrapult operates at a maximum pressure of 4000 psi, requires a fluid displacement of 224 cu.ft., and provides a terminal velocity of 227 ft/sec. Substituting these numerical values into the above expression leads to the following weight of the piping.

$$Wt. = 5.0 \times 10^{-7} \times 227^2 \times 4000 \times 224 = 23,000 \text{ lbs.}$$

Control Equipment

The weight of the control equipment needed is estimated as 10,000 lbs.

Shuttle

The weight of the shuttle, including buckets, is estimated as 4000 lbs.

Water Brake

In view of NAMC Report M-5110 entitled, "Performance Analysis for the Proposed XC-7 Catapult," the weight of the water brake required to stop the shuttle is estimated as 5000 lbs.

Retrieving Mechanism

The weight of the shuttle retrieving mechanism is estimated as 3000 lbs.

Air

The air charge in the accumulator is assumed to be at a temperature of 120° F. Since the accumulator contains 508 cu.ft. of air (see page 25) at a pressure of 4000 psi, the weight of the air is given by

$$\text{Wt.} = \frac{P\ V}{R\ T} = \frac{4000 \times 144 \times 508}{53.3 \times 580} = 9{,}500 \text{ lbs.}$$

Fluid

The total volume of fluid in the system is assumed to be twice the displacement volume. The displacement volume is 224 cu.ft. (see page 25). If this fluid is assumed to be water, its weight will be

$$\text{Wt.} = 2 \times 224 \times 62.4 = 28{,}000 \text{ lbs.}$$

Fittings

The weight of fittings is estimated arbitrarily as 10,000 lbs.

Contingencies

An additional 10 per cent is allowed for contingencies.

Total Weight

The estimated weights of the components of the proposed hydrapult are assembled in the following table.

Component	Estimated Weight
Accumulator	82,000 lbs.
Pump	12,500
Power Plant	37,500
Pipe (Including Nozzles)	23,000
Control Equipment	10,000
Shuttle	4,000
Water Brake	5,000
Retrieving Mechanism	3,000
Air	9,500
Fluid	28,000
Fittings	10,000
Subtotal	224,500
Contingencies 10 %	22,500
Total weight	247,000 lbs.

MINIMIZATION OF WEIGHT

Since the choice of a number of parameters enters into the design of a hydrapult, it seems worthwhile to examine the effect of the variation of certain parameters on the total weight of the hydrapult. In this section, therefore, certain definite assumptions concerning the design will be made and the total weight will be expressed in the form of an equation. The relation existing among certain parameters for the condition of minimum weight will then be found.

The components of the hydrapult which will require consideration are assumed to be as follows:

Spherical Accumulator
High Pressure Pump
Prime Mover for High Pressure Pump
High Pressure Piping System, Including Nozzles
Air Charge in Accumulator
Fluid in System
Control Equipment
Water Brake for Shuttle
Shuttle
Retrieving Mechanism
Fittings
Contingencies

A general expression will now be developed for the weight of each individual component. The several expressions will then be added to give the total weight of the hydrapult.

Expression for Weight of Each Component

In order to investigate the possibility of minimizing the weight of the hydrapult by proper selection of certain parameters, it will be necessary to express the weight in terms of these parameters. The parameters chosen are initial pressure, initial volume of the air charge in the accumulator, fluid displacement volume, and, in one instance, the terminal velocity.

Accumulator

The accumulator will be assumed to be spherical in shape. It can be shown that for the same configuration and the same stress in the material, the total weight of accumulator necessary to contain a given volume is independent of the number of accumulators used. One accumulator large enough to contain the necessary volume will therefore be assumed for the purpose of this weight calculation.

For a thin walled spherical vessel of inside diameter D (or inside radius r) and wall thickness t subjected to an interior pressure P_1, the stress S is given approximately by

$$S = \frac{D\ P_1}{4\,t} = \frac{r\ P_1}{2\,t}^{*} .$$

If the allowable stress is taken as 26,000 psi, the required wall thickness is

$$t = \frac{r\ P_1}{2\ S} = \frac{r}{52{,}000}\ P_1 .$$

The weight of a spherical shell of inner radius r, wall thickness t and density ρ is given approximately by

$$Wt. = 4\pi r^2 t \rho^{*} = \frac{4\pi r^3 \rho}{52{,}000}\ P_1 .$$

But

$$4\pi r^3 = 3\ V$$

where V is the volume contained by a spherical shell of inner radius r. If one substitutes this in the equation for the weight, and assumes the material to be steel of density 480 lbs/cu.ft., he obtains

$$Wt. = \frac{3 \times 480}{52{,}000}\ P_1\ V = 0.028\ P_1\ V .$$

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* These expressions are correct only for a vessel with a vanishingly thin wall. More rigorous expressions could be used. They lead to a more complex expression for the weight. Since the entire estimate of weight is only approximate, and since at a pressure of 4000 psi the approximation used here introduces an error of less than 5 per cent, the added simplicity warrants its use. The approximate method yields a weight which is slightly too low.

High Pressure Pump

The equation for the weight of the high pressure pump is based upon information concerning a high pressure pump manufactured by the Worthington Pump Company. This pump (page 22) has an output pressure of 4000 psi, a capacity of 2140 gpm, and a speed of 10,000 rpm. It weighs 8000 lbs. An input power of 6300 horsepower is required to drive the unit.

If a launching cycle of 30 seconds is assumed, and if the volume of the displacement fluid in cu.ft. (volume of fluid through all the nozzles) is denoted by V_d, the following expression gives the pump capacity required.

$$\text{Capacity} = \frac{2\ V_d \times 1728}{231} = 14.95\ V_d \text{ gpm}$$

If the further assumption that the weight of the pump is directly proportional both to the capacity and to the pressure is made, one obtains the following for the weight of the pump.

$$\text{Wt.} = 8000 \times \frac{14.95\ V_d}{2140} \times \frac{P_1}{4000} = 0.014\ P_1\ V_d$$

Prime Mover for High Pressure Pump

The weight of the prime mover to drive the pump will depend greatly upon the type of prime mover chosen. Although the choice will be determined by a number of factors, a turbine drive would offer some distinct advantages if the pump used were one of high speed. It appears that aircraft type gas turbines having a weight to power ratio approaching 1 lb/horsepower may be feasible. The 8000-lb. Worthington pump mentioned previously requires a prime mover of 6300 horsepower. As an approximation, therefore, it will be assumed that the weight of the prime mover is equal to the weight of the pump. Accordingly,

$$\text{Wt.} = 0.014\ P_1\ V_d\ .$$

High Pressure Piping System, Including Nozzles

For a cylindrical pipe of inside diameter D and wall thickness t subjected to an interior pressure P_1, the stress S is given by

$$S = \frac{D\ P_1}{2t}\ .$$

If the maximum allowable stress is taken as 26,000 psi, the required wall thickness is

$$t = \frac{D\,P_1}{2\,S} = \frac{D}{52,000}\,P_1 .$$

The weight of a pipe of length L, inside diameter D and wall thickness t is given approximately by

$$Wt. = \pi D t L \rho^{*} = \frac{\pi D^2 L \rho}{52,000}\,P_1$$

where ρ is the density of the material.

It has been assumed that the total amount of fluid in the system is twice the displacement volume V_d. If one assumes a 30-second interval between hydrapult shots, and designs the piping system so that the maximum velocity of fluid flow is 100 ft/sec., he can write

$$\frac{\pi D^2}{4} \times 100 = \frac{2\,V_d}{30} .$$

Hence

$$\pi D^2 = \frac{4\,V_d}{1500} .$$

Substituting this into the last equation above for the weight, assuming that the pipe is of steel having a density of 480 lbs/cu.ft., and performing the indicated operations, one obtains the following for the weight of the pipe.

$$Wt. = \frac{4 \times 480\ L\ P_1\ V_d}{1500 \times 52,000} = 2.5 \times 10^{-5}\ L\ P_1\ V_d = L'\ P_1\ V_d$$

where

$$L' = 2.5 \times 10^{-5}\ L .$$

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* This expression is correct only for a pipe with a vanishingly thin wall. A more rigorous expression could be used. It leads to a more complex expression for the weight. Since the entire estimate of weight is only approximate, and since at a pressure of 4000 psi the approximation introduces an error of only 8 per cent, the simplicity of the expression to which it leads warrants its use. The approximate method yields a weight which is slightly too low.

The length of pipe L will depend upon the design of the hydrapult. In particular, it will depend upon the length of the run; it will not depend appreciably upon the design acceleration. As a considered estimate, let it be assumed that for a 200-foot run 1000 feet of pipe are required. This amount should provide an adequate allowance for the nozzles also. Hence, $L' = 0.025$ and

$$Wt. = 0.025\ P_1\ V_d \quad .$$

If in order to obtain a higher design terminal velocity one uses the same design acceleration but a longer accelerated run, one can argue with some logic that the length of the pipe would be approximately proportional to the square of the terminal velocity. Let it be assumed that this proportionality exists. Consistent with the assumption of 1000 feet of pipe for a 200-foot run, the length of pipe required for any design velocity v_t obtained with an acceleration of 4g is given by

$$L = 0.020\ v_t^2 \qquad \text{from which} \qquad L' = 5.0 \times 10^{-7}\ v_t^2 \quad .$$

Substituting this value into the expression for the weight,

$$Wt. = 5.0 \times 10^{-7}\ v_t^2\ P_1\ V_d \quad .$$

Emphasis has been placed upon the dependence of pipe weight upon v_t mainly for one reason. The process of minimizing the hydrapult weight depends upon whether the weight is a function of the terminal velocity. By way of example, it is desired to minimize the weight both under the condition where the weight is not a function of v_t and where it is a function of v_t. As will be seen later, the results are somewhat different.

Air Charge in the Accumulator

The weight of the air in the accumulator can be calculated from the general gas formula. If the temperature is taken as 120° F., the pressure as P_1 psi and the volume as V_1 cu.ft., the weight of the air is given by

$$Wt. = \frac{144\ P_1\ V_1}{53.3\ T} = \frac{144\ P_1\ V_1}{53.3 \times 580} = 0.0047\ P_1\ V_1 \ .$$

Fluid in the System

It has been assumed that the total volume of fluid in the system is twice the displacement volume V_d. Assuming that the fluid is water, one obtains the following for its weight.

$$Wt. = 62.4 \times 2\ V_d = 125\ V_d$$

Remaining Components

The weights of other components of the hydrapult will depend but little if any upon the pressure and volume chosen. Some, the water brake for example, will depend upon the terminal velocity. In the general case these weights should really be expressed in terms of v_t and any additional parameters necessary. The weights of these other components are relatively small, however, and the manner in which they are handled will have but little effect upon the results. They will therefore be assumed constant, with the values shown in the following table. The manner in which the weight of the pipe is handled will serve as an example of the procedure to be followed if one should desire to express some of these weights as functions of v_t.

Component	Assumed Weight
Control Equipment	10,000 lbs.
Water Brake	5,000
Shuttle	4,000
Retrieving Mechanism	3,000
Fittings	10,000
Total	32,000

Expression for the Total Weight

An expression has been deduced for the weight of each component of the hydrapult. The total weight of the installation is the sum of the several expressions. Thus,

$$\text{Total Wt.} = 0.028\, P_1 V + 0.014\, P_1 V_d + 0.014\, P_1 V_d + 5.0 \times 10^{-7} v_t^2 P_1 V_d + 0.0047\, P_1 V_1 + 125\, V_d + 32{,}000 \quad .$$

The total accumulator volume V can be expressed in terms of the initial air volume V_1 and the fluid displacement volume V_d. That is, $V = V_1 + V_d$. If one eliminates V from the last equation above and collects similar terms, he obtains the following.

$$\text{Total Wt.} = (125 + 0.056\, P_1 + 5.0 \times 10^{-7} v_t^2 P_1)\, V_d + 0.033\, P_1 V_1 + 32{,}000$$

An addition of 10 per cent will be allowed for contingencies. Thus the final expression for the weight of the hydrapult installation becomes

$$\text{Total Wt.} = (137 + 0.062\, P_1 + 5.5 \times 10^{-7} v_t^2 P_1)\, V_d + 0.036\, P_1 V_1 + 35{,}000 \quad .$$

Minimization of the Total Weight

It is desired to minimize the total weight of a hydrapult installation designed for any specified terminal velocity. In order to do this it is necessary that V_1 and V_d which appear in the expression for total weight be expressed in terms of other quantities. Let $R = P_1/P_2$, the ratio of the initial to the final gas pressure in the accumulator. For an adiabatic expansion from volume V_1 to volume V_2

$$P_1 V_1^k = P_2 V_2^k = P_2 (V_1 + V_d)^k$$

from which

$$R^{1/k} \cdot V_1 = V_1 + V_d$$

and

$$V_1 = \frac{V_d}{R^{1/k} - 1} .$$

In order to evaluate V_d it will be necessary to use the following expression (page 69) which is developed in Appendix A.

$$V_d = - \frac{M}{2\rho} \ln \left[1 - \frac{v_t}{v_{eff}} \right]$$

In this equation M is the load in lbs., ρ the density of the fluid, v_t the terminal velocity of the load, and v_{eff} the effective jet velocity. It will now be necessary to determine the average effective jet velocity to use in this equation. Although the calculation will be carried out on the assumption that unit accumulators each supply only a few jets, the results should not be greatly different were one large accumulator to be used.

The manner in which the jet velocity varies with time has been considered earlier. This consideration led to the following relation (page 11), in which $C = 12.18\ C_v$ where C_v is the nozzle coefficient, and in which A_n is the nozzle area.

$$v_j = C \sqrt{P_1 V_1^k} \left[CA_n \frac{k+2}{2} \sqrt{P_1 V_1^k}\ t + V_1^{(k+2)/2} \right]^{-k/(k+2)}$$

Assuming that $C_v = 0.98$, knowing that $v_{eff} = v_j \cos\delta$ in which δ is assumed to be 15° (in which case $C = 11.9$ and $C \cos\delta = 11.5$), and rearranging terms, one obtains

$$v_{eff} = 11.5 \sqrt{P_1} \left[11.9 \; \frac{k+2}{2} \; \sqrt{P_1} \, \frac{A_n}{V_1} \, t + 1 \right]^{-k/(k+2)} .$$

In order to find the average effective jet velocity during the time interval $t = 0$ to $t = T$, during which the pressure changes from its initial value P_1 to its final value $P_2 = P_1/R$, one sets

$$\text{Av. } v_{eff} = \frac{1}{T} \int_0^T v_{eff} \, dt .$$

Substituting the value of v_{eff} given above, performing the integration, substituting the limits, and inserting the value $k = 1.40$, one obtains

$$\text{Av. } v_{eff} = \frac{0.97 \, V_1}{A_n \, T} \left[\left[20.3 \sqrt{P_1} \, \frac{A_n \, T}{V_1} + 1 \right]^{0.588} - 1 \right] .$$

For convenience write

$$\tau = 20.3 \sqrt{P_1} \, \frac{A_n \, T}{V_1} .$$

Then

$$\text{Av. } v_{eff} = \frac{19.7 \sqrt{P_1}}{\tau} \left[\left[\tau + 1 \right]^{0.588} - 1 \right] .$$

By evaluating the equation for v_j on page 36 to obtain the initial jet velocity v_1 at $t = 0$ and the final jet velocity v_2 at $t = T$, one finds that

$$(\tau + 1) = \left[\frac{v_1}{v_2} \right]^{(k+2)/k} = \left[\frac{v_1}{v_2} \right]^{2.428} .$$

Furthermore,

$$\frac{v_1}{v_2} = \sqrt{\frac{P_1}{P_2}} = \sqrt{R} .$$

Hence

$$(\tau + 1) = \left[\frac{v_1}{v_2}\right]^{2.428} = R^{1.214} .$$

Substituting this value of $(\tau + 1)$ and the corresponding value of τ in the third from the last equation on page 37, one finds that the average effective jet velocity is given by

$$\text{Av. } v_{eff} = 19.7 \sqrt{P_1} \left[\frac{R^{0.714} - 1}{R^{1.214} - 1}\right] .$$

The purpose of finding this average effective jet velocity has been to use it in calculating V_d. If the value found be substituted into the equation for V_d given on page 36, one finds that

$$V_d = - \frac{M}{2\rho} \ln \left[1 - \frac{v_t \left[R^{1.214} - 1\right]}{19.7 \sqrt{P_1} \left[R^{0.714} - 1\right]}\right] .$$

Now that expressions for V_1 (page 36) and V_d (just above) are available, it is possible to express the total weight of the hydrapult installation in terms of quantities which are suitable for investigating the possibility of minimization. The total weight of the hydrapult (page 35) has been shown to be

$$\text{Total Wt.} = (137 + 0.062 P_1 + 5.5 \times 10^{-7} v_t^2 P_1) V_d$$

$$+ 0.036 P_1 V_1 + 35{,}000 .$$

If the values of V_1 and V_d are substituted into this equation, one obtains for the total weight of the hydrapult

$$\text{Total Wt.} = \left[- \frac{M}{2\rho} \ln \left[1 - \frac{v_t \left[R^{1.214} - 1\right]}{19.7 \sqrt{P_1} \left[R^{0.714} - 1\right]}\right]\right] \left[137 \right.$$

$$\left. + 0.062 P_1 + 5.5 \times 10^{-7} v_t^2 P_1 + 0.036 P_1 \frac{1}{R^{0.714} - 1}\right]$$

$$+ 35{,}000 .$$

This expression takes account of the fact that the weight of the pipe required is a function of v_t. As pointed out on pages 33 and 34, the weight of the pipe is given by

$$Wt. = L' \, P_1 \, V_d$$

where $L' = 0.028$ for a 200-foot accelerated run, or $L' = 5.5 \times 10^{-7} \, v_t^2$ for any length of run which results in a terminal velocity v_t with an acceleration 4g. These numerical values of L' include the 10 per cent which was added for contingencies. In order that the minimization of hydrapult weight can be carried out for either case, the term $5.5 \times 10^{-7} \, v_t^2$ appearing in the last equation on page 38 will be replaced by L'. Thus the total weight of the installation becomes

$$\text{Total Wt.} = \left[- \frac{M}{2\rho} \ln \left[1 - \frac{v_t \left[R^{1.214} - 1 \right]}{19.7 \sqrt{P_1} \left[R^{0.714} - 1 \right]} \right] \right] \left[137 \right.$$
$$\left. + (0.062 + L') \, P_1 + \frac{0.036 \, P_1}{R^{0.714} - 1} \right] + 35{,}000 \, .$$

It is desired to minimize this weight for any chosen terminal velocity v_t. Two interesting facts are obvious from the equation. First, the weight of the hydrapult is directly proportional to the total mass M to be accelerated, that is, proportional to the sum of the mass of the plane and the mass of the shuttle. Second, for any design terminal velocity v_t the weight of the installation is an explicit function of the variables P_1 and R. Thus, proper selection of these two variables will lead to a minimum weight. The minimum will occur when

$$\frac{\partial}{\partial P_1} (Wt.) = 0 \qquad \text{and} \qquad \frac{\partial}{\partial R} (Wt.) = 0$$

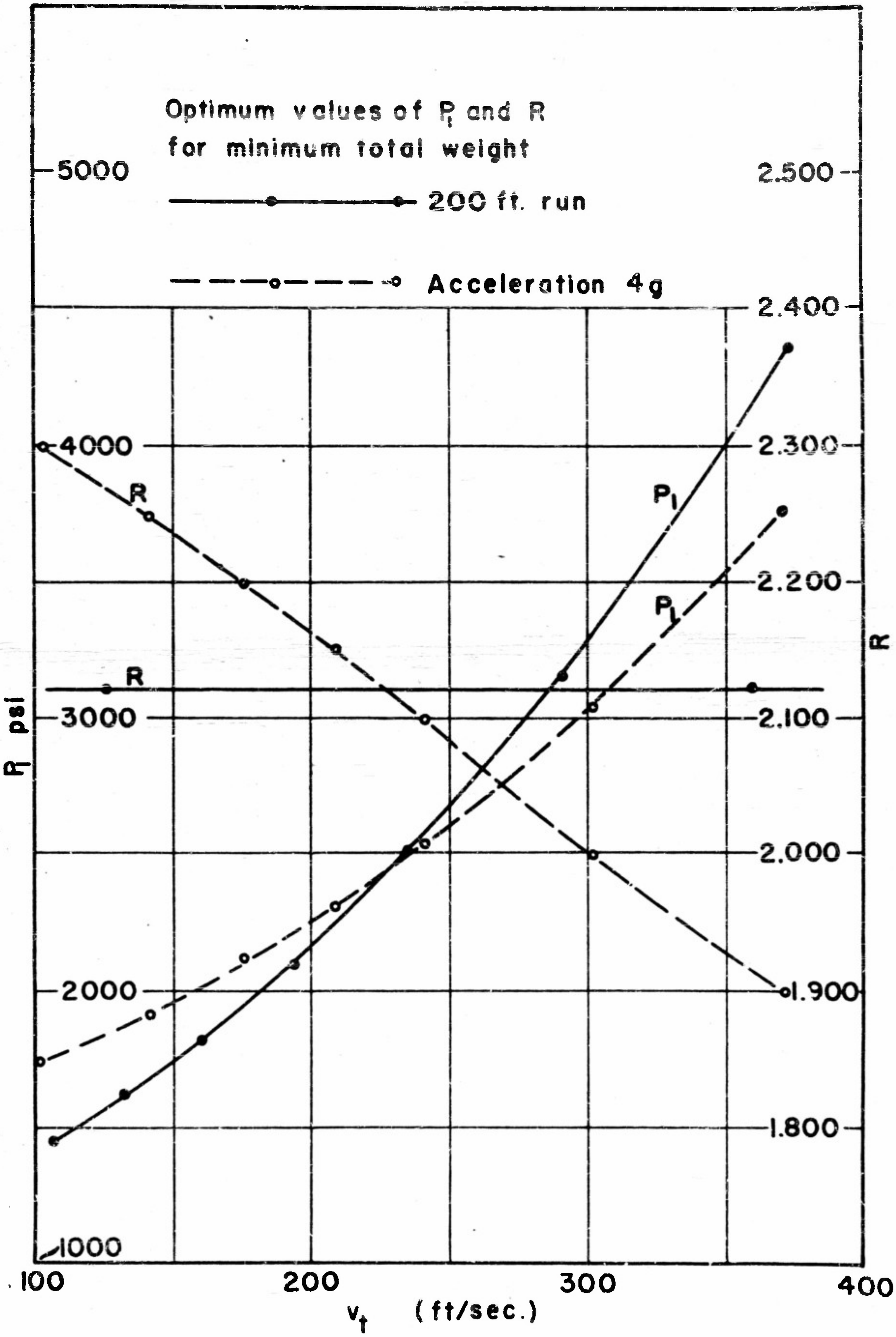
simultaneously. In order to find the values of P_1 and R which make these partial derivatives zero, one differentiates the last written expression for total weight with respect to P_1 and with respect to R, sets each derivative equal to zero, and solves the resulting two simultaneous equations for P_1 and R. The details of the solution are given in Appendix C, page 83, where the solution has been carried out for each of the following cases:- (a) For a hydrapult with a 200-foot accelerated run, regardless of acceleration, in which case $L' = 0.028$; (b) For a hydrapult of any length run but an acceleration of 4g, in which case $L' = 5.5 \times 10^{-7} \, v_t^2$. Minimization for the latter case will serve as an example of the procedure to be followed in case it is desired to express the weights of one or more components as functions of v_t as well as of P_1 and R.

Tables included in Appendix C give the values of P_1 and R required to minimize the weight for various values of terminal velocity v_t. The results are somewhat different for the two cases mentioned above. The optimum values of P_1 and R for each case are shown by the curves on page 41.

It should be emphasized that the equation for the total weight of the hydrapult is based upon a series of assumptions about the several components. If different design assumptions are made, the numerical values in the weight equation will be changed. If rigorous expressions (for example, those mentioned on pages 31 and 33) are used throughout for calculating the weights of components, the expression for total weight will include powers of P_1 other than the first power. Under these circumstances the minimization is more difficult, but it can still be carried through. The important conclusion is that there will always exist for any design terminal velocity v_t an optimum initial pressure P_1 and an optimum ratio R of initial to final pressure which will lead to a hydrapult of minimum weight. The optimum values of P_1 and R corresponding to any terminal velocity which may be required in the foreseeable future are not impracticable.

By obtaining the optimum values of P_1 and R from the curves on page 41, and using the equation for total weight given on page 39, it is possible to calculate the weight of the minimum-weight-hydrapult required to give any design load M any design terminal velocity v_t. For example, let it be desired to accelerate a 50,000-lb. plane 4g in a run of 200 feet, thus giving the plane a terminal velocity of 227 ft/sec. From the curves on page 41, P_1 = 2430 psi and R = 2.123. Assuming a 4000-lb. shuttle, and assuming a frictional force of 14,000 lbs. (which is equal to the force that would be required to accelerate an additional mass of 3,500 lbs. at 4g), the total effective load M would be 57,500 lbs. Calculation of the hydrapult weight from the equation on page 39, in which L' = 0.028 in this instance, leads to a total weight of 183,000 lbs. This figure for the weight of a minimum-weight-hydrapult should be compared with the estimated weight of 247,000 lbs. (see page 29) for a hydrapult designed to accomplish the same job when operating at an initial pressure of 4000 psi and with a ratio R equal to 1.6. It is clear that the weight of an installation can be reduced materially by designing the hydrapult to operate at the most favorable values of P_1 and R.

The weights of minimum-weight-hydrapults to give an effective load of 57,500 lbs. terminal velocities of 100, 200, 227, 300 and 375 ft/sec. have been calculated in the manner illustrated above. Calculations have been made for the two following cases:- (a) For a hydrapult having a 200-foot accelerated run, the acceleration being whatever is necessary to produce the specified terminal velocity; (b) For a hydrapult having an acceleration of 4g, the length of the run being whatever is necessary to produce the specified terminal velocity.



Optimum values of P_1 and R for minimum total weight
200 ft. run
Acceleration 4g
5000
4000
3000
2000
1000
2.500
2.400
2.300
2.200
2.100
2.000
1.900
1.800
R
R
P_1
P_1
P_1 psi
R
100
200
300
400
v_t (ft/sec.)

The results are shown by the curves on page 43. It is important to note that at the higher velocities the weight of an installation required to produce a given terminal velocity is less if this higher velocity is acquired by increasing the acceleration and keeping the length of the run fixed than if it is obtained by using a fixed acceleration and increasing the length of the run.

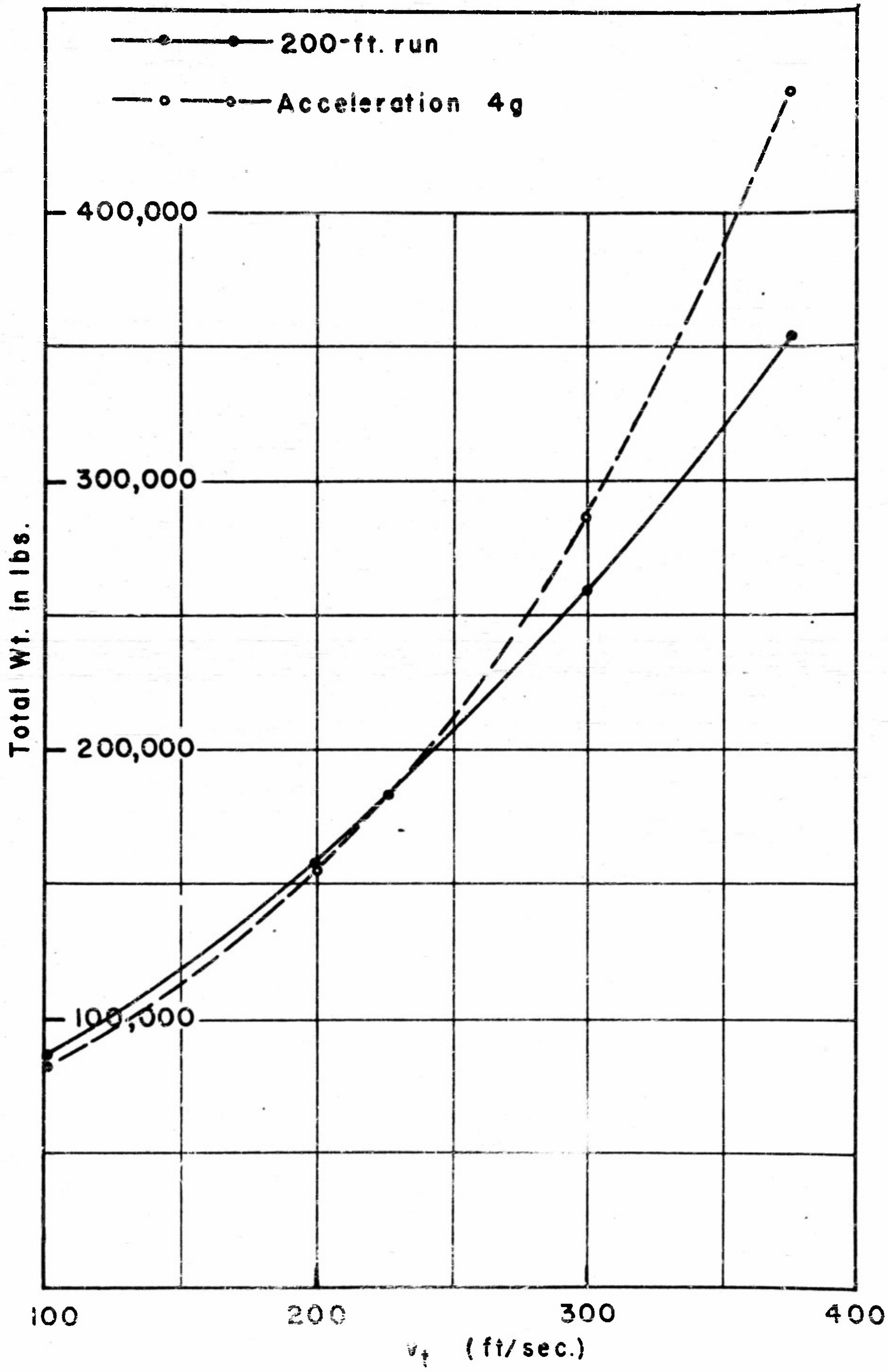
It should be pointed out that although the curves on page 43 are for an effective load of 57,500 lbs., they can also be used to determine the approximate weight of an installation required for any other load. In the development of the expression for the weight it was assumed that the weights of certain components of the hydrapult remain constant and that they total 35,000 lbs. (See page 35, and recall that 10 per cent has been added for contingencies.) If these weights actually remain constant the weight, $Wt._2$, of a hydrapult for an effective load M_2 would be

$$Wt._2 = \frac{M_2}{M_1}(Wt._1 - 35{,}000) + 35{,}000$$

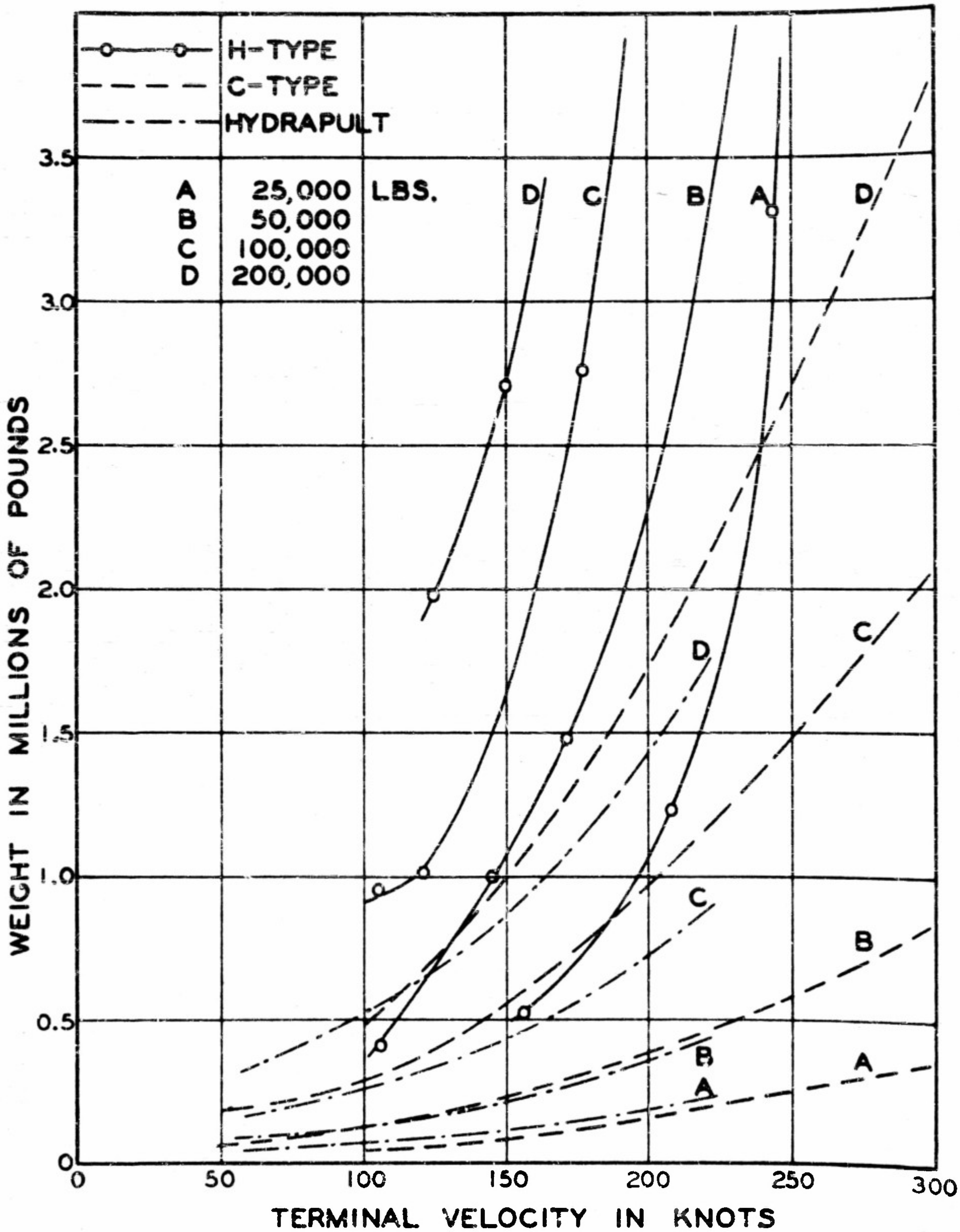
where $Wt._1$ is the weight of the installation for catapulting an effective load M_1 for which the assumed constant weights are presumably correct. Actually, as the mass of the load increases the weights of at least some of these components would increase. They might even increase in such a manner as to be proportional to the mass being catapulted. In this case one could write

$$Wt._2 = \frac{M_2}{M_1} Wt._1 \quad .$$

It is of interest to compare the estimated weights of hydrapults for different terminal velocities with the weights of other types of catapults. Weight versus terminal velocity data for H-type and C-type catapults have been taken from NAMC Report No. M-4805 entitled, "Comparison of Hydraulic and C-Type Catapults," and dated October 1948. They are reproduced by the curves on page 44. On this page also are weight versus terminal velocity curves for hydrapults designed to catapult dead loads of 25,000, 50,000, 100,000 and 200,000 lbs. at various terminal velocities attained with an acceleration 4g. The second expression given above was used in calculating these weights. Inspection of the curves shows that the estimated weight of a hydrapult is comparable throughout to the weight of an equivalent C-type catapult. Furthermore, the weight of a hydrapult increases no more rapidly with increases in terminal velocity than it does for a C-type catapult.



200-ft. run
Acceleration 4g
400,000
300,000
200,000
100,000
Total Wt. in lbs.
100
200
300
400
v_t (ft/sec.)


H-TYPE
C-TYPE
HYDRAPULT
A 25,000 LBS.
B 50,000
C 100,000
D 200,000
WEIGHT IN MILLIONS OF POUNDS
TERMINAL VELOCITY IN KNOTS
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
0
50
100
150
200
250
300
D C B A D
D C B B A A C

PERFORMANCE CURVES OF HYDRAPULTS

The calculation of the performance curve (load versus terminal velocity) for a given hydrapult depends upon the method of calculation used in determining its design; the result is necessarily somewhat approximate. To simplify the calculations of performance, use is made here of the assumptions described and the results obtained in Appendix A for an idealized hydrapult. From the tabular results given (page 68) in Appendix A, a curve of hydrapult load versus terminal velocity can be constructed for either of two operating conditions, constant pressure or constant total energy input. In the following the superscript * shall refer to the value for the design load.

Constant Operating Pressure

At constant pressure the jet velocity v_j is constant for all loads and equal to v_j^*, that for the design load. The terminal velocity v_t is therefore proportional to the quantity v_t/v_{eff} given in the table on page 68 of Appendix A as a function of the load M. In particular, for the hydrapult designated in Appendix A as Case A,*

$$v_t = 750 \times 0.98\, v_t/v_{eff} = 735\, v_t/v_{eff} \text{ ft/sec.}$$

$$= 435\, v_t/v_{eff} \text{ knots .}$$

For the hydrapult designated as Case B,*

$$v_t = 695 \times 0.98\, v_t/v_{eff} = 681\, v_t/v_{eff} \text{ ft/sec.}$$

$$= 403\, v_t/v_{eff} \text{ knots .}$$

The table on page 68 in Appendix A gives corresponding values of load M and the quantity v_t/v_{eff} for both cases. The constant pressure is 3790 psi for Case A and 3250 psi for Case B. The calculated values of load M and terminal velocity v_t at constant pressure are shown for both hydrapults in the table on page 48 and by the curves on page 49.

- -

* Case A represents a hydrapult which will give a total load of 100,000 lbs. an acceleration of 3.5g and a terminal velocity of 300 ft/sec. in a 400-foot run. It has 100 nozzles spaced 4 feet apart, and the fluid displacement is 420 cu.ft. It operates at a pressure of 3790 psi.

Case B represents a hydrapult which will give a total load of 50,000 lbs. an acceleration of 4.0g and a terminal velocity of 227 ft/sec. in a 200-foot run. It has 100 nozzles spaced 2 feet apart, and requires a fluid displacement of 162 cu.ft. It operates at a pressure of 3250 psi.

Constant Energy Input

At constant total energy input equal to that required for the design load, the end speed is calculated from the tabulated value of v_t/v_{eff} corresponding to the arbitrary load M, and the calculated value of the jet velocity v_j required to give the arbitrary load the specified constant energy input. Using Equations 34 and 35 of Appendix A, one finds that the constant total energy input E_i° is given by

$$E_i^{\circ} = - \frac{M^{\circ} {v_j^{\circ}}^2}{4} \ln \left[1 - (v_t/v_{eff})^{\circ} \right] = - \frac{M\, v_j^2}{4} \ln \left[1 - (v_t/v_{eff}) \right] .$$

Likewise, one finds that the required jet velocity v_j is given by

$$v_j = v_j^{\circ} \sqrt{\frac{M^{\circ}}{M} \frac{\ln \left[1 - (v_t/v_{eff})^{\circ} \right]}{\ln \left[1 - (v_t/v_{eff}) \right]}} .$$

Incidentally, since the pressure $P = \frac{1}{2} \rho v_j^2$,

$$P = P^{\circ} \frac{M^{\circ}}{M} \frac{\ln \left[1 - (v_t/v_{eff})^{\circ} \right]}{\ln \left[1 - (v_t/v_{eff}) \right]} .$$

This gives the pressure P required to furnish the constant energy input E_i° to the arbitrary load M.

Now the end speed v_t is given by

$$v_t = v_j \cos \delta \, \frac{v_t}{v_{eff}} = \frac{v_t}{v_{eff}} \cos \delta \; v_j^{\circ} \sqrt{\frac{M^{\circ}}{M} \frac{\ln \left[1 - (v_t/v_{eff})^{\circ} \right]}{\ln \left[1 - (v_t/v_{eff}) \right]}} .$$

The table on page 68 of Appendix A gives for each value of the load M the corresponding value of v_t/v_{eff}. From the equations above one calculates the corresponding end speeds v_t and pressures P. Inserting numerical values for the design load, one obtains for the hydrapult designated as Case A,

$$v_t = \frac{6.57 \times 10^4 \, (v_t/v_{eff})}{\sqrt{M \log \frac{1}{1 - (v_t/v_{eff})}}} \text{ knots,}$$

and

$$P = \frac{8.635 \times 10^7}{M \log \dfrac{1}{1 - (v_t/v_{eff})}} \text{ psi.}$$

For the hydrapult designated as Case B,

$$v_t = \frac{3.78 \times 10^4 \, (v_t/v_{eff})}{\sqrt{M \log \dfrac{1}{1 - (v_t/v_{eff})}}} \text{ knots,}$$

and

$$P = \frac{2.865 \times 10^7}{M \log \dfrac{1}{1 - (v_t/v_{eff})}} \text{ psi.}$$

The calculated values of load M and terminal velocity v_t at constant energy input are shown for both hydrapults in the table on page 48 and by the curves on page 49. The table also lists the corresponding values of the pressure P.

The figure on page 49 gives six performance curves. Curves A_p and B_p are for the two hydrapults operating at constant pressure; curves A_E and B_E are for the two hydrapults operating at constant energy input; curves C_7 and C_{10} are for the C-7 and C-10 slotted tube catapults, the only catapults of comparable capacity.

One further aspect of performance which should be investigated is the behavior of a given hydrapult for pressures too high or too low for the particular load, or for loads too heavy or too light for the particular pressure setting. While the hydrapult does not possess inherent "phase stability," that is, the ability to speed up lagging loads or slow down those accelerating too rapidly, analysis shows that acceleration variations are very small and definitely delimited. For example, too light a load is accelerated more than planned for the design load, but the acceleration is rapidly limited to a value only slightly in excess of that for the design load; the end speed likewise is only slightly greater than that for the design load. This limitation of acceleration, inherent in the basic principles on which an idealized hydrapult is built, is generally enhanced in practice by construction restrictions such as the time lag in valve opening.

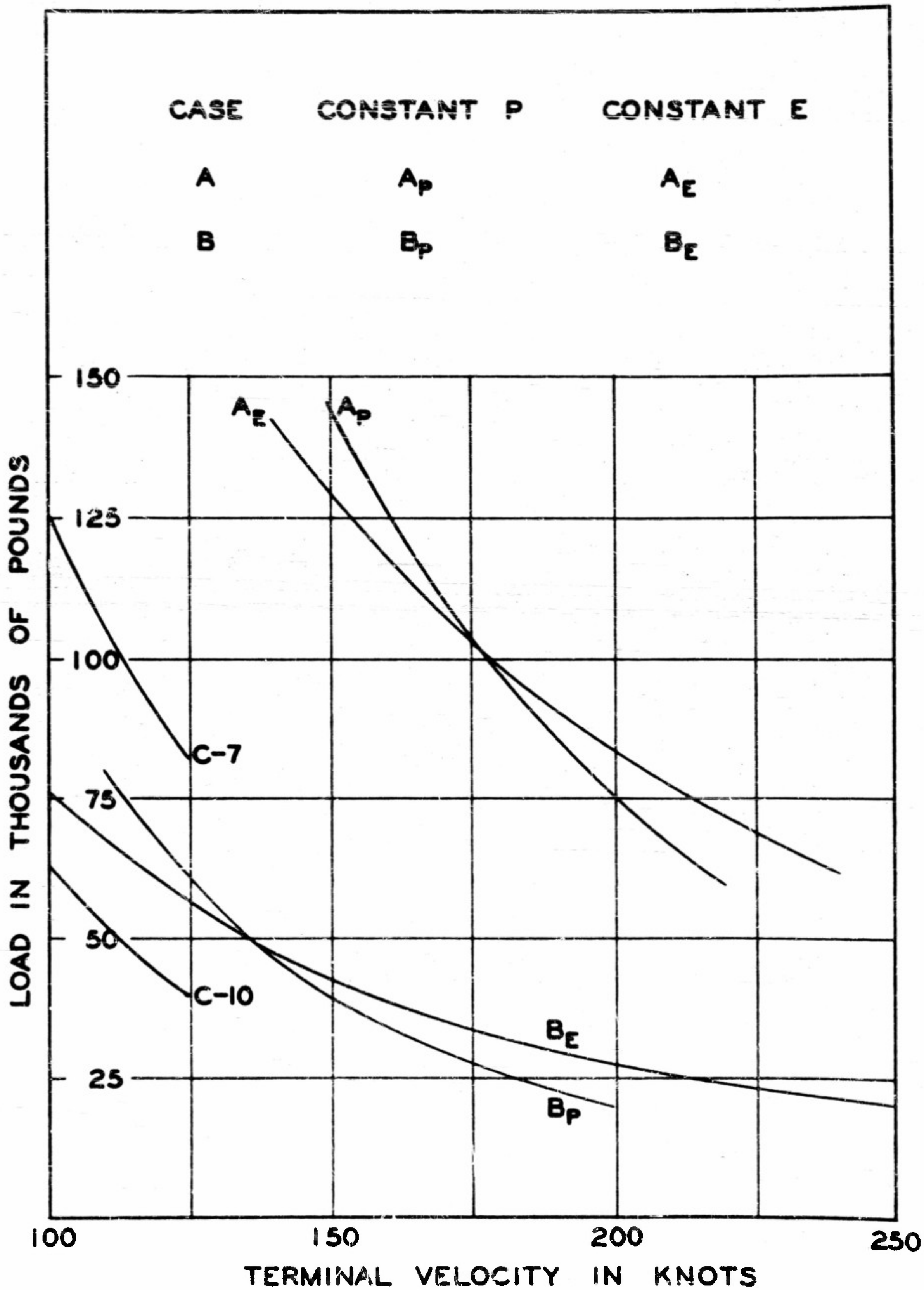
	Case A				Case B			
		Constant Pressure	Constant Energy Input			Constant Pressure	Constant Energy Input	
v_t/v_{eff}	M	v_t	v_t	P	M	v_t	v_t	P
0.50	60,240	218	244	4770	18,670	202	252	5100
0.49	63,460	213	236	4660	19,670	198	245	4980
0.48	66,900	209	229	4550	20,740	194	237	4870
0.47	70,570	205	221	4450	21,870	189	229	4760
0.46	74,500	200	214	4340	23,090	185	221	4640
0.45	78,710	196	207	4240	24,390	181	214	4530
0.44	83,220	191	200	4130	25,790	178	207	4420
0.43	88,070	187	192	4020	27,300	173	200	4310
0.42	93,290	183	186	3910	28,910	169	192	4200
0.41	98,910	178	179	3800	30,660	165	185	4090
0.40	104,990	174	172	3700	32,540	161	178	3970
0.39	111,560	170	165	3600	34,580	157	171	3860
0.38	118,690	165	159	3500	36,790	153	165	3760
0.37	126,430	161	152	3400	39,180	149	158	3650
0.36	134,850	157	146	3300	41,800	145	152	3550
0.35	144,030	152	140	3200	44,640	141	145	3440
0.34					47,760	137	139	3330
0.33					51,170	133	133	3230
0.32					54,920	129	126	3120
0.31					59,060	125	120	3010
0.30					63,630	121	114	2900
0.29					68,700	117	108	2790
0.28					74,350	113	102	2690
0.27					80,650	109	97	2590

v_t = Terminal velocity in knots

v_{eff} = Effective jet velocity = $v_j \cos \delta$

M = Weight of load in pounds

P = Pressure in psi



CASE
CONSTANT P
CONSTANT E
A
A_P
A_E
B
B_P
B_E
150
125
100
75
50
25
LOAD IN THOUSANDS OF POUNDS
A_E
A_P
C-7
C-10
B_E
B_P
100
150
200
250
TERMINAL VELOCITY IN KNOTS

As described earlier for an idealized hydrapult, the basic construction (geometry) determines the relation between load weight M and the quantity v_t/v_{eff}, which quantity is proportional to the end speed and approximately inversely proportional to the square root of the pressure. From the corresponding values of M and v_t/v_{eff} given in the table on page 68 in Appendix A, terminal velocities can be calculated for arbitrary variations of load weight and pressure from design values. The end speeds in knots for variations of 10 per cent and 20 per cent in either direction for both weight and pressure are given in the following table for the hydrapult designated as Case B. For this hydrapult the design load is 50,000 lbs., the design pressure 3250 psi, and the design end speed 134.4 knots.

M in lbs.	Pressure P in psi				
	2600	2930	3250	3580	3900
40,000	132.3	140.3	147.9	155.1	162.0
45,000	125.8	133.5	140.7	147.5	154.1
50,000	120.2	127.5	134.4*	141.0	147.2
55,000	115.4	122.4	129.0	135.3	141.3
60,000	111.0	117.7	124.1	130.2	135.9

It may be noted that even in such an extreme case as a 20 per cent overload with a 20 per cent low pressure setting, the terminal velocity is but 18 per cent less than the design value; or for a 20 per cent underload with a 20 per cent high pressure setting, the terminal velocity is only 21 per cent more than the design value. In general load weight variations and pressure setting errors will be considerably less than 20 per cent, and the effects on terminal velocity will be quite small. Thus neither "runaway" nor "cold" shots are inherently likely to occur except through gross errors in operation.

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* This represents conditions for the design load, pressure and end speed.

VALVES AND NOZZLES FOR EXTRAPOLATED DESIGNS

It is of interest to determine whether the valve and nozzle requirements for a hydrapult become prohibitive as the hydrapult design is extrapolated to higher terminal velocities. The principal factors that must be considered are pressure, fluid on-times, and nozzle areas.

The optimum initial pressure P_1 and the optimum ratio R of initial to final pressure (P_1/P_2) for a specified terminal velocity v_t can be obtained from the curves on page 41. Values for four terminal velocities are shown in the following table, both for the case of a constant 200-foot length of accelerated run and for the case of a constant acceleration of 4g.

v_t	Constant Run of 200 Feet		Constant Acceleration of 4g	
	P_1	R	P_1	R
200 ft/sec.	2130 psi	2.123	2250 psi	2.165
227	2430	2.123	2430	2.123
300	3270	2.123	3020	2.005
400	4820	2.123	4120	1.855

The terminal velocity of 400 ft/sec. is sufficiently high to cover reasonable future requirements. It corresponds to the velocity attained in a 200-foot run with an acceleration of 12.4g, or to that attained with an acceleration of 4.0g in a 621-foot run. The pressure required for minimizing the weight of a hydrapult having this terminal velocity is not unreasonably high.

The time for which any particular jet flows would become rather short for a terminal velocity of 400 ft/sec. A 10-foot shuttle (effective length 12 feet) traveling 400 ft/sec. would be acted on by a given jet for only 30 milliseconds. The desired jet on-time is therefore at least approaching a lower practical limit. It is only six times the time required for the jet stream to build up to 98 per cent of its maximum velocity. Extremely quick acting valves would be required to insure satisfactory operation and conservation of fluid. The on-time could be doubled by doubling the length of the shuttle. At such high speeds a longer shuttle may be desirable for other reasons also.

Consideration of the required nozzle areas is somewhat more involved. Nevertheless, certain approximate calculations can be made. It has been shown (page 21) that the aggregate nozzle area A from which fluid of density ρ must play upon the shuttle buckets in order to exert a force F is given by

$$A = \frac{F}{1.95\,\rho\, v_j\,(v_j \cos\delta - v_{sh})}$$

where v_j is the jet velocity, v_{sh} the shuttle velocity, and δ the angle between the direction of the jet velocity and the direction of travel of the shuttle. The quantity ($v_j \cos\delta$) has been designated as v_{eff}. Since v_j, and hence v_{eff}, vary as the pressure decreases during discharge of the accumulator, a question arises as to what jet velocity should be used in calculating the nozzle area. The value will depend upon whether one large accumulator or a group of unit accumulators is used. Calculations will be made on the assumption that unit accumulators are used. It has been shown (page 38) that the average value of v_{eff} for an adiabatic expansion of air from a pressure P_1 to a pressure P_1/R is given by

$$v_{eff} = 19.7\sqrt{P_1}\left[\frac{R^{0.714} - 1}{R^{1.214} - 1}\right].$$

Using this with appropriate values of P_1 and R obtained from the table on the preceding page, and assuming an angle δ of 15°, the following values of v_j and v_{eff} have been calculated for various specified values of terminal velocity v_t. All velocities are given in ft/sec.

	Constant Run of 200 Feet		Constant Acceleration of 4g	
v_t	v_j	v_{eff}	v_j	v_{eff}
200	449	434	458	442
227	480	463	480	463
300	555	536	544	525
400	670	647	650	628

Assume a load consisting of a 50,000-lb. plane, a 4000-lb. shuttle, and a frictional force of 14,000 lbs. If one assumes the use of unit accumulators and uses the above values of v_1 and v_{eff} to calculate the largest aggregate nozzle areas (those near the end of the run) required for various terminal velocities attained with accelerations a and accelerated runs s, and if one assumes that this aggregate area is divided equally among six nozzles playing simultaneously upon the shuttle buckets, he obtains the jet diameters D given in the following table. Velocities are given in feet/second, distances in feet, areas in square inches and diameters in inches.

	Constant Run, 200 Feet			Constant Acceleration, 4g		
v_t	a/g	A	D	s	A	D
200	3.1	66	3.8	155	78	4.0
227	4.0	77	4.0	200	77	4.0
300	7.0	114	5.0	350	71	3.9
400	12.4	158	5.8	621	59	3.5

The table shows that if a hydrapult is designed for the values of P_1 and R which lead to a minimum weight, the required nozzle diameter does not increase too rapidly with increases in terminal velocity even in the case for which the length of the run is kept fixed. In case the acceleration is kept fixed, the diameter actually decreases slightly with increases in terminal velocity. The decrease results from the fact that the optimum P_1 increases as the terminal velocity increases.

It should be emphasized that these estimates of nozzle area are based upon the assumed use of unit accumulators, in which case the average jet velocity is the same for all nozzles regardless of their locations along the run. Were a single accumulator to be used, the jet velocities near the end of the run would be lower (due to the decrease of pressure) than those near the beginning of the run. Areas of nozzles near the end of the run would therefore be correspondingly larger than those near the beginning of the run, and also larger than those which are required if unit accumulators are used. For example, with a single accumulator and a terminal velocity of 227 ft/sec., the diameter of the largest nozzle is 5.2 in., whereas it was but 4.0 in. for unit accumulators and the same terminal velocity.

CONCLUSIONS

In this report the University of Kansas group suggests a possible multi-jet driven catapult, herein referred to as a hydrapult, and discusses the feasibility, general features of construction, and performance of such a device. Although considerations of the hydrapult are necessarily approximate, certain significant conclusions are warranted. They will be stated briefly.

1. Basically the hydrapult consists of a series of high pressure fluid jets distributed uniformly along the catapult track. These jets impinge upon a series of Pelton-type buckets attached to a moving shuttle. The jets are controlled so that they flow only while engaging the shuttle buckets. The thrust upon the shuttle is in general provided by a series of jets acting simultaneously on the shuttle; there is sufficient overlapping of jet impulses to provide a continuous thrust.

2. The general design features of the basic components of the hydrapult have been considered individually. Most of the components are of fairly standard design and should require a minimum of development work. Certain flow-versus-time characteristics of high pressure jets and of quick opening valves need further investigation before accurate designs can be made. The valve problem would require an appreciable amount of development work.

3. It appears feasible to build a hydrapult having as large a capacity as will probably be required. The weight of a hydrapult installation for a given capacity appears reasonably low. The weight does not increase too rapidly as the design is extrapolated to higher terminal velocities. As regards the weight of the installation and the rate at which the weight increases with design terminal velocity, the hydrapult is quite comparable to the C-type catapult.

4. The basic design of a 40×10^6 ft.lb. hydrapult has been considered in some detail. This hydrapult would accelerate a 50,000-lb. airplane at 4g and give the load a terminal velocity of 227 ft/sec. (135 knots) in a run of 200 feet. The weight of the major components, accumulator, pumps, prime movers for pumps, pipe, control equipment, shuttle, water brake, retrieving mechanism, compressed air, fluid and fittings, is estimated to be 247,000 lbs. Some optimism may have been exercised in estimating the weight, but it is felt that the figure mentioned is attainable. A separate study of weight minimization indicates that a more judicious choice of operating pressure and expansion ratio would lower the estimated weight of this hydrapult to 183,000 lbs. The over-all efficiency of the proposed 40×10^6 ft.lb. hydrapult is 32 per cent.

5. The choice of operating pressure affects the weight of a hydrapult installation in two ways. For a given input energy, the use of a higher fluid pressure results in a smaller volume of fluid being required, and hence in a smaller total weight. On the other hand, the higher fluid pressure, producing higher jet velocities, results in a lower efficiency. These two competing effects combine to dictate an intermediate operating pressure which will lead to a hydrapult of minimum weight for a given terminal velocity.

6. A general study of weight minimization has been made. The total weight of a hydrapult installation has been expressed in terms of the initial operating pressure P_1 and the ratio R of initial to final pressure. Those values of P_1 and R which lead to a minimum value of the resulting expression for weight have been determined by appropriate mathematical methods. Curves showing the optimum values of P_1 and R for different design terminal velocities have been obtained. Curves are shown (page 41) for two cases:- (a) For a hydrapult having a 200-foot accelerated run, and whatever acceleration is required to produce the specified terminal velocity; (b) For a hydrapult having an acceleration 4g and whatever length of run is necessary to produce the specified terminal velocity. The minimization that has been carried out has been largely by way of example. The specific results obtained will depend upon the manner in which the weights of such components as the pipe, the control equipment, the shuttle, the water brake, the retrieving mechanism and the fittings are judged to vary with terminal velocity. The specific results of the example presented here are qualitatively correct over the entire range of end speed considered; they should be reliable quantitatively up to end speeds of 250 to 300 ft/sec. Beyond this speed the obtaining of reliable results would necessitate a more thorough evaluation of the weights of certain components in terms of terminal velocity.

7. Included in the report are curves which show the weights of hydrapults to give a 50,000-lb. plane various terminal velocities, the hydrapult being designed in each instance to operate at the optimum values of P_1 and R. Hydrapult weight versus terminal velocity curves are shown (page 43) for each of the two cases mentioned in 6.

8. Performance curves (load versus terminal velocity) for the hydrapult have been calculated both for constant pressure and for constant energy input. These curves (page 49) are similar to those for the C-10 and the C-7 catapults. Investigation has shown that although the hydrapult does not possess inherent "phase stability," operation with loads and/or pressures as much as 20 per cent higher or lower than the design load and/or pressure will not cause objectionable variations from the design acceleration and terminal velocity. The variations are definitely delimited. Neither "runaway" nor "cold" shots are likely to occur except through gross errors in operation.

9. The hydrapult offers a number of distinct advantages, and has some disadvantages. Some of its advantages are:

a. It is a direct drive catapult, and therefore not subject to the limitations introduced by the use of cables. If it is desired, the hydrapult can of course be used as a cable drive catapult up to those velocities for which the limitations of cable drive become prohibitive.

b. It provides a good card factor, if uniform acceleration is desired. If for psychological or physiological reasons an acceleration pattern other than that corresponding to uniform acceleration should be desired, the hydrapult has a sufficient number of variable parameters to give any reasonable acceleration pattern desired.

c. The hydrapult has an unusual amount of flexibility which allows adjustment for different loads and different terminal velocities. Practicable changes in the number of nozzles in operation and/or in the operating pressure are sufficient to accommodate for a wide range of loads and end speeds.

d. The hydrapult uses fairly standard components. Only a few, in particular the quick acting valves and the mechanism for controlling the valves, would require an appreciable amount of development work. The only conceivably intricate mechanism in the installation is that associated with the valves and their control.

e. The hydrapult has no critical parts subject to excessive wear or to failure as a result of fatigue. This is in contrast, for example, to the sealing strip in any slotted tube catapult.

f. The hydrapult uses ship power, a highly desirable feature. No serious problem of logistics is involved.

g. The hydrapult has a reasonable weight for a given capacity, and the total weight does not increase too rapidly as the design is extrapolated to higher terminal velocities.

Some of the disadvantages of a hydrapult are:

a. It has a fairly heavy shuttle, a characteristic feature of direct drive catapults. The shuttle weight will certainly be greater than that of the H-type catapult, probably somewhat greater than that of the C-type catapult, probably somewhat smaller than that of the steam catapult, and certainly smaller than that of any possible electropult.

b. The hydrapult would introduce a considerable top-side weight, again a characteristic feature of direct drive catapults. However, the amount of top-side weight introduced by the hydrapult is probably less than that for any other type of direct drive installation of equal capacity.

c. The fact that the hydrapult requires high pressure accumulators and piping is a disadvantage. However, the high pressure accumulators are no more serious than in the case of the H-type catapult; nor are the high pressure accumulators and piping any more serious than they are for the steam catapult.

d. The large volume of fluid required represents a disadvantage for cold weather operation. The fluid will either have to be some antifreeze solution, or it will have to be protected from freezing by auxiliary heating.

10. In Part I, "General Report," the University of Kansas group has expressed the conviction that major development of catapults for the future should be directed primarily toward direct drive catapults. The group believes that the hydrapult may prove to be one of the two most promising direct drive devices, the other being the steam catapult. The steam driven catapult is already a practicable device, though a great deal of work will be required to extend it to terminal velocities required in the future. Simultaneously with further development of the steam catapult, the group recommends serious further consideration of the advisability of development work on the hydrapult.

APPENDIX A. GEOMETRICAL CONSIDERATIONS OF AN IDEALIZED HYDRAPULT

The geometry of a hydrapult, that is, the spacing and size of the nozzles, is determined by the acceleration pattern, capacity, and terminal velocity desired, and by the pressure available. When the geometry has been established, then the performance, fluid requirement and efficiency are all easily determined. To illustrate the relationships among these quantities, and to calculate several nozzle sizes for hypothetical hydrapults, an idealized hydrapult may be defined as in the following treatment.

Consider a system consisting of N nozzles equally spaced a distance b apart;* there will be nozzles at x = 0, b, 2b, ... nb, ... (N-1)b. Let the nozzle areas be A_0, A_1, A_2, ... A_n, ... A_{N-1} respectively. Let the nozzles be inclined at an angle δ to the horizontal. Finally, let the jets impinge on a single bucket on the shuttle, the height of the bucket being not less than b tan δ, so that just as the bucket leaves one jet it enters the next.

Equation of Motion of the Bucket

Consider the bucket struck by the jet from the nozzle at x = nb. The problem may be treated as one of collision, in which energy and momentum are both conserved. Let the incident jet velocity be w_n, the return jet velocity w_r, the fluid density ρ, the total shuttle mass M, and the shuttle velocity v; let the angle between the return stream and the line of shuttle travel be ϕ. Neglecting the conversion of some kinetic energy into heat through frictional losses, the conservation of energy leads to the first of the three following equations. The conservation of momentum leads to the second and third of the three equations regardless of frictional losses.

$$\frac{1}{2} A_n w_n \rho w_n^2 = \frac{1}{2} A_n w_n \rho w_r^2 + M v \frac{dv}{dt} \qquad (1)$$

$$A_n w_n \rho w_n \cos\delta = - A_n w_n \rho w_r \cos\phi + M \frac{dv}{dt} \qquad (2)$$

$$A_n w_n \rho w_n \sin\delta = A_n w_n \rho w_r \sin\phi \quad . \qquad (3)$$

Equation (3) may be solved to give

$$\sin\phi = \frac{w_n \sin\delta}{w_r}$$

- -

* Units are not specified in this theoretical treatment. Equations are written in such form that any consistent system of units may be used.

from which

$$w_r \cos \phi = \sqrt{w_r^2 - w_n^2 \sin^2 \delta} \quad . \tag{4}$$

From equation (1),

$$M v \frac{dv}{dt} = \frac{A_n w_n \rho}{2} (w_n^2 - w_r^2) \quad . \tag{5}$$

From equations (2) and (4),

$$M \frac{dv}{dt} = A_n w_n \rho \left(w_n \cos \delta + \sqrt{w_r^2 - w_n^2 \sin^2 \delta}\right) \quad . \tag{6}$$

Eliminating $M \frac{dv}{dt}$ and rearranging, one obtains the expression

$$\frac{w_n^2 - w_r^2}{2 v} = w_n \cos \delta + \sqrt{w_r^2 - w_n^2 \sin^2 \delta} \quad . \tag{7}$$

Since in general $(w_n^2 - w_r^2) \neq 0$, equation (7) may be solved to yield

$$w_n^2 - w_r^2 = 4 v (w_n \cos \delta - v) \quad . \tag{8}$$

Substituting this value of $(w_n^2 - w_r^2)$ into (5), one obtains the following for the equation of motion of the bucket. This equation holds within the range $nb < x < (n+1)b$.

$$\frac{dv}{dt} = \frac{2 A_n w_n \rho}{M} (w_n \cos \delta - v) \tag{9}$$

<u>Integration of the Equation of Motion</u>

Let the time at which the bucket arrives at point $x = nb$ and the velocity with which it is moving at that time be designated as

$$v = v_n \qquad \text{and} \qquad t = t_n \quad .$$

As the bucket arrives at point $x = (n+1)b$,

$$v = v_{n+1} \qquad \text{and} \qquad t = t_{n+1} \, .$$

Then by integration of equation (9) between these limits, one obtains the following expression relating the velocities at the beginning and the end of the time interval.

$$w_n \cos \delta - v_{n+1} = (w_n \cos \delta - v_n) \, \epsilon^{-\frac{2 A_n w_n \rho}{M} (t_{n+1} - t_n)} \tag{10}$$

By integrating equation (9) twice between the time limits corresponding to the times of arrival at successive jets, one obtains the distance b between jets. Thus, for n = 0, 1, 2, ... N-1,

$$b = w_n \cos \delta \, (t_{n+1} - t_n) - \frac{(w_n \cos \delta - v_n) M}{2 A_n w_n \rho} \left[1 - \epsilon^{-\frac{2 A_n w_n \rho}{M} (t_{n+1} - t_n)} \right] . \tag{11}$$

With b, ρ, M and all values of w_n and A_n given for n = 0, 1, ... N-1, these relations determine bucket (shuttle) velocity v and time at each nozzle station. The initial velocity v_0 is to be taken as zero. The terminal velocity and total time are those at the end of the N-th section, that is, v_N and t_N respectively.

Note that these two sets of relations involve variables of several kinds. The variables N, A_n, ρ, δ and b are determined by the installation and configuration. The velocity w_n is determined in part by the installation and configuration, but also in part by an experimental variable, the pressure P. The quantity M is determined by the load. The v's and t's are dependent variables, subject, however, to the following conditions:- (a) v_N is the terminal velocity v_t, which is determined by load requirements; (b) The acceleration must not exceed a specified maximum.

Consider N, ρ, δ, b and M fixed. There remain five sets of variables, w_n, v_{n+1}, v_n, A_n and $(t_{n+1} - t_n)$. These five sets of variables are not independent; they are related by equations (10) and (11). In general one might suppose it possible to assign arbitrary values to three of these and calculate the other two from equations (10) and (11). That this is not generally possible may be shown in the following manner. Write $U_n = (t_{n+1} - t_n)$, and substitute equation (10) twice in equation (11). This results in the equation

$$b = U_n w_n \cos\delta - \frac{U_n (v_{n+1} - v_n)}{\ln \dfrac{w_n \cos\delta - v_n}{w_n \cos\delta - v_{n+1}}} \tag{12}$$

which may be written in the form

$$\ln \frac{w_n \cos\delta - v_n}{w_n \cos\delta - v_{n+1}} = \frac{v_{n+1} - v_n}{w_n \cos\delta - b/U_n} \quad . \tag{13}$$

If v_{n+1}, v_n and U_n are chosen arbitrarily, this transcendental equation specifies the value of w_n. The quantity A_n may then be calculated from equation (10) in the form

$$A_n = \frac{M}{2 w_n \rho U_n} \ln \frac{w_n \cos\delta - v_n}{w_n \cos\delta - v_{n+1}} \quad . \tag{14}$$

However, like many transcendental equations, equation (13) has a non-trivial root only in special cases. Define the symbol

$$G_n = \frac{v_{n+1} - v_n}{w_n \cos\delta - v_{n+1}}$$

so that (15)

$$w_n \cos\delta = v_{n+1} + \frac{v_{n+1} - v_n}{G_n} \quad .$$

And define the symbol

$$h_n = \frac{v_{n+1} - b/U_n}{v_{n+1} - v_n} \quad . \tag{16}$$

Then equation (13) becomes

$$\ln (1 + G_n) = \frac{G_n}{1 + h_n G_n} \quad . \tag{17}$$

The roots of this equation are distributed as follows:

Value of h_n	Value of G_n
$h_n \leqq 0$	0
$0 < h_n < 1/2$	0, plus a positive root
$h_n = 1/2$	0
$1/2 < h_n \leqq 1$	0, plus a negative root
$1 < h_n$	0

Since from equation (15) G_n is obviously positive, equation (17) has a physically useful solution only for $0 < h_n < 1/2$. Thus there are restrictions on the allowable values of v_{n+1}, v_n and b/U_n.

Simulated Uniformly Accelerated Linear Motion

Suppose that it is desired to approximate uniformly accelerated linear motion by requiring v and t to have at each nozzle station located at x = nb the values they would have if the acceleration were constant. It is easily shown that this requires that for n = 0, 1, 2, ... N,

$$v_n = \sqrt{2\,a\,b}\,\sqrt{n}$$

and

$$U_n = \sqrt{\frac{2\,b}{a}}\,(\sqrt{n+1} - \sqrt{n}\,) \tag{18}$$

in which a is the hypothetical acceleration (actually the space average). In terms of the terminal velocity v_t,

$$a = \frac{{v_t}^2}{2\,N\,b} \qquad \text{and} \qquad v_n = v_t\sqrt{\frac{n}{N}} = v_1\sqrt{n}\ . \tag{19}$$

Now from equation (16),

$$h_n = \frac{\sqrt{2ab}\,\sqrt{n+1} - \dfrac{b}{\sqrt{\dfrac{2b}{a}}\,(\sqrt{n+1} - \sqrt{n})}}{\sqrt{2ab}\,(\sqrt{n+1} - \sqrt{n})} = \frac{\sqrt{n+1} - \frac{1}{2}(\sqrt{n+1} + \sqrt{n})}{\sqrt{n+1} - \sqrt{n}} = \frac{1}{2} \tag{20}$$

But for $h_n = 1/2$, $G_n = 0$. Hence, it is impossible to build such a device which will produce a uniform (constant) acceleration. Only the trivial case of no acceleration at all can approximate, in these terms, the corresponding uniform acceleration. It is still convenient to retain the restrictions given in equation (19), that is, to retain something approximating uniformly accelerated motion insofar as the velocities are concerned. The calculated time intervals will then determine fluid volume, efficiency, etc.

Geometry of a Constant Jet Velocity Hydrapult

Consider now a simple case in which all jets have the same velocity w. Equations (10) and (11) will then determine the various A_n values and the corresponding U_n values. By substituting equation (12) or (13) into equation (14), one obtains, for n = 1, 2, ... N-1,

$$A_n = \frac{M\cos\delta}{2\rho b}\left[\ln\frac{w\cos\delta - v_1\sqrt{n}}{w\cos\delta - v_1\sqrt{n+1}} - \frac{v_1}{w\cos\delta}(\sqrt{n+1} - \sqrt{n})\right]. \tag{21}$$

From this one can calculate the desired nozzle areas for a given configuration, load and terminal velocity. If the jet velocity w is adjusted for different desired terminal speeds so that

$$\frac{v_1}{w\cos\delta} = \frac{v_t}{\sqrt{N}\,w\cos\delta} = \text{constant}, \tag{22}$$

then all loads of the same mass M will use the same track regardless of the desired end speed. However, and unfortunately, it is not possible to adjust the single parameter w to accommodate all variations in both v_t and M, nor even to accommodate variations in M regardless of v_t. It might be possible to allow for small variations in M by maintaining the quantity $M\cos\delta$ constant, that is, by tilting the nozzles. Perhaps two tracks with different spacing b and variable angle δ might be feasible.

Now for this same special case one obtains from equation (12), for n = 1, 2, ... N-1,

$$U_n = \frac{b}{w\cos\delta - \dfrac{v_1(\sqrt{n+1} - \sqrt{n})}{\ln\dfrac{w\cos\delta - v_1\sqrt{n}}{w\cos\delta - v_1\sqrt{n+1}}}}. \tag{23}$$

From equation (9) one can calculate the acceleration at the beginning of each stage (a_n), and that at the end of each stage (a_n').

$$a_n = \frac{(w \cos \delta)^2}{b}\left[1 - \frac{v_1 \sqrt{n}}{w \cos \delta}\right]\left[\ln \frac{w \cos \delta - v_1 \sqrt{n}}{w \cos \delta - v_1 \sqrt{n+1}} - \frac{v_1 (\sqrt{n+1} - \sqrt{n})}{w \cos \delta}\right] \qquad (24)$$

$$a_n' = \frac{(w \cos \delta)^2}{b}\left[1 - \frac{v_1 \sqrt{n+1}}{w \cos \delta}\right]\left[\ln \frac{w \cos \delta - v_1 \sqrt{n}}{w \cos \delta - v_1 \sqrt{n+1}} - \frac{v_1 (\sqrt{n+1} - \sqrt{n})}{w \cos \delta}\right] \qquad (25)$$

Calculations of A_n, U_n, a_n and a_n' for selected values of n have been made for two hypothetical cases. The configurations chosen for the two cases have the following specifications:

Case A: $N = 100$; $b = 4$ feet; $\cos \delta = 0.98$; $w = 750$ ft/sec.; $P = 3792$ psi; $M = 100{,}000/g$ slugs; $v_t = 300$ ft/sec. = 204.5 mph; $\rho = 62.5$ lbs/cu.ft.; $g = 32.2$ ft/sec.^{2}; average acceleration = 3.494g; fluid displacement = 419.6 cu.ft.

Case B: $N = 100$; $b = 2$ feet; $\cos \delta = 0.98$; $w = 694.9$ ft/sec.; $P = 3255$ psi; $M = 50{,}000/g$ slugs; $v_t = 227$ ft/sec. = 154.8 mph; $\rho = 62.5$ lbs/cu.ft; $g = 32.2$ ft/sec.^{2}; average acceleration = 4.000g; fluid displacement = 162.2 cu.ft.

The calculated values are shown in the table on the following page.

n	Case A A_n	Case A a_n/g	Case A a_n'/g	Case A U_n	Case B A_n	Case B a_n/g	Case B a_n'/g	Case B U_n
0	0.1673	3.58	3.43	0.266	0.1114	4.09	3.96	0.175
1	0.1718	3.53	3.46	0.110	0.1135	4.03	3.97	0.0730
2	0.1745	3.52	3.47	0.0847	0.1149	4.02	3.98	0.0560
3	0.1768	3.51	3.47	0.0714	0.1161	4.02	3.98	0.0472
4	0.1787	3.51	3.48	0.0630	0.1171	4.02	3.98	0.0416
5	0.1805	3.51	3.48	0.0569	0.1181	4.02	3.99	0.0376
6	0.1822	3.51	3.48	0.0523	0.1190	4.02	3.99	0.0346
7	0.1838	3.51	3.48	0.0487	0.1198	4.01	3.99	0.0322
49	0.2290	3.50	3.49	0.0189	0.1423	4.01	4.00	0.0125
99	0.2754	3.50	3.49	0.0134	0.1631	4.00	4.00	0.0088

A_n is given in sq.ft.

U_n is given in sec.

Performance of the Constant Jet Velocity Hydrapult

Suppose now a track is designed to give simulated uniformly accelerated linear motion to a dead load of mass M°, with terminal velocity v_t° and jet velocity w°. It is now necessary to determine the behavior of different loads on this same track; more specifically, it is necessary to determine whether w can be varied in such a way as to give any arbitrary mass M an arbitrary terminal velocity v_t. It is to be noted first that in any case, the space average acceleration a is given by

$$a = v_t^2/2\,N\,b \quad .$$

Hence, if there exists a limiting maximum value of this space average acceleration, then there also exists for a given catapult configuration a limiting maximum value of v_t which is given by

$$v_{t(max)} = \sqrt{2\,N\,b\,a_{max}} \quad .$$

In practice, since there will be fluctuations in the acceleration, the allowed limit for v_t may be somewhat lower than this.

The relation between the two cases now under consideration arises from the fact that the same configuration has been used for both; although the jet velocity w may be different, the two have the same values of N, b and A_n. Suppose the hydrapult has been designed specifically for a load of mass M° and terminal velocity v_t°. From equations (19) and (21) one obtains, for values of n = 0, 1, 2, ... N-1,

$$A_n = \frac{M^\circ \cos\delta}{2\rho\, b}\left[\ln\frac{1-\sqrt{n}\,\dfrac{v_t^\circ}{\sqrt{N}\,w^\circ\cos\delta}}{1-\sqrt{n+1}\,\dfrac{v_t^\circ}{\sqrt{N}\,w^\circ\cos\delta}} - \left(\sqrt{n+1}-\sqrt{n}\right)\frac{v_t^\circ}{\sqrt{N}\,w^\circ\cos\delta}\right] \quad . \tag{26}$$

But also, for any load of mass M, from equations (12) and (14), or equation (21), one finds that for values of n = 0, 1, 2, ... N-1,

$$A_n = \frac{M \cos\delta}{2\rho b}\left[\ln \frac{1 - \dfrac{v_n}{w \cos\delta}}{1 - \dfrac{v_{n+1}}{w \cos\delta}} - \frac{v_{n+1} - v_n}{w \cos\delta}\right] . \qquad (27)$$

If A_n is eliminated from equations (26) and (27), one obtains

$$M^\circ \left[\ln \frac{1 - \sqrt{n}\, \dfrac{v_t^\circ}{\sqrt{N}\, w^\circ \cos\delta}}{1 - \sqrt{n+1}\, \dfrac{v_t^\circ}{\sqrt{N}\, w^\circ \cos\delta}} - (\sqrt{n+1} - \sqrt{n})\, \frac{v_t^\circ}{\sqrt{N}\, w^\circ \cos\delta}\right]$$

$$= M\left[\ln \frac{1 - \dfrac{v_n}{w \cos\delta}}{1 - \dfrac{v_{n+1}}{w \cos\delta}} - \frac{v_{n+1} - v_n}{w \cos\delta}\right] . \qquad (28)$$

Now, if one writes out equation (28) for $n = 0$, noting that $v_0 = 0$, substitutes the result in the equation for $n = 1$, and repeats the process for successive n values, it is found that, for $n = 1, 2, 3, \ldots N$,

$$M^\circ \left[\ln\left[1 - \sqrt{n}\, \frac{v_t^\circ}{\sqrt{N}\, w^\circ \cos\delta}\right] + \sqrt{n}\, \frac{v_t^\circ}{\sqrt{N}\, w^\circ \cos\delta}\right]$$

$$= M\left[\ln\left[1 - \frac{v_n}{w \cos\delta}\right] + \frac{v_n}{w \cos\delta}\right] . \qquad (29)$$

Note first that equation (29) enables one to calculate the value of $v_n/(w \cos\delta)$ at each stage (that is, at $x = n\,b$ where $n = 1, 2, \ldots N$) for the arbitrary load on the standard track. Note second that equation (29) for $n = N$ enables one to calculate the required value of w to give the desired $v_t = v_n$. Thus

$$\ln\left[1 - \frac{v_t}{w \cos\delta}\right] + \frac{v_t}{w \cos\delta}$$

$$= \frac{M^\circ}{M}\left[\ln\left[1 - \frac{v_t^\circ}{w^\circ \cos\delta}\right] + \frac{v_t^\circ}{w^\circ \cos\delta}\right] . \qquad (30)$$

The track was designed for M°, v_t° and w°. By solving the transcendental equation (30) one obtains the appropriate jet velocity w required to give the desired arbitrary terminal velocity v_t to the desired arbitrary load M on this same track. For computational purposes, it is simpler to calculate M°/M for specific values of $v_t/(w \cos \delta)$. The table below gives calculated values of M (in lbs.) for arbitrarily selected values of the quantity $v_t/(w \cos \delta)$, for the two tracks previously designed and designated as Cases A and B on page 64. Case A represents a track 400 feet long with 100 jet nozzles, designed for a load of 100,000 lbs. and a terminal velocity of 300 ft/sec. (204.5 mph); it has a jet velocity of 750 ft/sec. corresponding ideally to a pressure ($P = \rho w^2/2$) of 3792 psi. Case B represents a track 200 feet long with 100 jet nozzles, designed for a load of 50,000 lbs. and a terminal velocity of 227 ft/sec. (154.8 mph); it has a jet velocity of 694.9 ft/sec., corresponding ideally to a pressure of 3255 psi.

$\frac{v_t}{w \cos \delta}$ *	M Case A	M Case B	$\frac{v_t}{w \cos \delta}$ *	M Case A	M Case B
0.50	60,240	18,670	0.38	118,690	36,790
0.49	63,460	19,670	0.37	126,430	39,180
0.48	66,900	20,740	0.36	134,850	41,800
0.47	70,570	21,870	0.35	144,030	44,640
0.46	74,500	23,090	0.34		47,760
0.45	78,710	24,390	0.3333		50,000
0.44	83,220	25,790	0.33		51,170
0.43	88,070	27,300	0.32		54,920
0.42	93,290	28,910	0.31		59,060
0.41	98,910	30,660	0.30		63,630
0.4082	100,000		0.29		68,700
0.40	104,990	32,540	0.28		74,350
0.39	111,560	34,580	0.27		80,650

The table includes no calculations for values of $v_t/w \cos \delta$ greater than 0.5; higher values might give lower efficiencies. However, since it is the total integrated efficiency that is important, larger values might be feasible. (See later section on efficiency.)

It is obvious from the table that a track designed for a given load may still be suitable for a wide range of different loads, provided first that the desired end speed does not impose an unacceptably high acceleration for a track of constant length, and second that the required jet velocities for heavier loads and higher terminal velocities do not require unacceptably high pressures. For the same average acceleration, 3.494g, one finds in Case A the following consistent combinations:

---

* $w \cos \delta = v_{eff}$

$M = 60,240$ lbs.	$M^\circ = 100,000$ lbs.	$M = 144,030$ lbs.
$w = 612.2$ ft/sec.	$w^\circ = 750$ ft/sec.	$w = 874.6$ ft/sec.
$P = 2527$ psi	$P = 3792$ psi	$P = 5157$ psi

In Case B, for an average acceleration of 4.000g, one finds the following:

$M = 18,670$ lbs.	$M^\circ = 50,000$ lbs.	$M = 80,650$ lbs.
$w = 463.3$ ft/sec.	$w^\circ = 694.9$ ft/sec.	$w = 857.9$ ft/sec.
$P = 1447$ psi	$P = 3255$ psi	$P = 4961$ psi

If higher accelerations can be tolerated, then higher velocities may be considered. For a given load on a given track, w is directly proportional to v_t; P is proportional to the square of v_t.

In order to calculate the volume of fluid required and the over-all efficiency of the hydrapult, it is convenient to assume "perfect cutoff," that is, to assume that each jet is open only while its stream impinges on the bucket. Under these conditions the volume V_n used by the jet of area A_n is given by

$$V_n = A_n \, w \, U_n \quad . \tag{31}$$

Equation (14) may be solved to give

$$V_n = \frac{M}{2\rho} \ln \frac{w \cos \delta - v_n}{w \cos \delta - v_{n+1}} \quad . \tag{32}$$

The total displacement volume V_d of fluid used, with perfect cutoff, is given by

$$V_d = \sum_{n=0}^{N-1} V_n \quad . \tag{33}$$

From equation (32) one then obtains the following expression for V_d. This expression holds for any load and any terminal velocity.

$$V_d = - \frac{M}{2\rho} \ln \frac{w \cos \delta - v_N}{w \cos \delta} = - \frac{M}{2\rho} \ln \left[1 - \frac{v_t}{w \cos \delta} \right] \tag{34}$$

In calculating the over-all efficiency on an energy basis it is to be noted that the jet velocity w is the same for all jets. The energy input E_i is therefore given by

$$E_i = \frac{1}{2} \rho V_d w^2 \quad . \tag{35}$$

The energy output is given by

$$E_0 = \frac{1}{2} M v_t^2 \quad . \tag{36}$$

The efficiency E is therefore given by

$$E = \frac{\frac{1}{2} M v_t^2}{\frac{1}{2} \rho V w^2} = \frac{-2 v_t^2}{w^2 \ln \left[1 - \frac{v_t}{w \cos \delta}\right]} \quad . \tag{37}$$

Note that E is a maximum for

$$v_t/(w \cos \delta) = 0.715 \quad . \tag{38}$$

The corresponding maximum value of the efficiency is given by

$$E_{max} = 0.815 \cos^2 \delta \quad . \tag{39}$$

It is interesting that the total or integrated efficiency is greatest for a jet velocity considerably less than twice the terminal velocity. It would seem advisable to design the track and jet nozzles to give this maximum efficiency for the load most often encountered.

Now consider the accelerations for a load different from the standard load for which the track has been designed. For an arbitrary load, equations (24) and (25) become

$$a_n = \frac{(w \cos \delta)^2}{b} \left[1 - \frac{v_n}{w \cos \delta}\right] \left[\ln \frac{w \cos \delta - v_n}{w \cos \delta - v_{n+1}} - \frac{v_{n+1} - v_n}{w \cos \delta}\right] \tag{40}$$

$$a_n' = \frac{(w\cos\delta)^2}{b}\left[1 - \frac{v_{n+1}}{w\cos\delta}\right]\left[\ln\frac{w\cos\delta - v_n}{w\cos\delta - v_{n+1}} - \frac{v_{n+1} - v_n}{w\cos\delta}\right] \qquad (41)$$

For the particular track with standard load M° one obtains $a_n{}^\circ$ and $a_n{}'^\circ$ by substituting w° for w and $v_1{}^\circ \sqrt{n} = v_t{}^\circ \sqrt{n}/\sqrt{N}$ for v_n in these equations. Combination of equation (28) with equations (40) and (41) yields

$$a_n = \left[\frac{w}{w^\circ}\right]^2 \frac{M^\circ}{M}\left[\frac{1 - \frac{v_n}{w\cos\delta}}{1 - \frac{\sqrt{n}\, v_t{}^\circ}{\sqrt{N}\, w\cos\delta}}\right] a_n{}^\circ \qquad (42)$$

$$a_n' = \left[\frac{w}{w^\circ}\right]^2 \frac{M^\circ}{M}\left[\frac{1 - \frac{v_{n+1}}{w\cos\delta}}{1 - \frac{\sqrt{n+1}\, v_t{}^\circ}{\sqrt{N}\, w\cos\delta}}\right] a_n{}'^\circ \ . \qquad (43)$$

In order to determine actual values of a_n and a_n' the transcendental equations (29) may be solved for the v_n values and then these values may be inserted in either equations (40) and (41) or in (42) and (43). However, some conclusions may be drawn from equations (42) and (43) without detailed calculation. For the smaller values of n, where the larger variations in $a_n{}^\circ$ and $a_n{}'^\circ$ from their space average are observed, both $v_n/(w\cos\delta)$ and $v_t{}^\circ \sqrt{n}/(\sqrt{N}\, w\cos\delta)$ are very small relative to unity. Hence to a good degree of approximation,

$$a_n \sim \left[\frac{w}{w^\circ}\right]^2 \frac{M^\circ}{M} a_n{}^\circ \qquad \text{and} \qquad a_n' \sim \left[\frac{w}{w^\circ}\right]^2 \frac{M^\circ}{M} a_n{}'^\circ \ . \qquad (44)$$

Since at constant v_t (and hence constant space average acceleration) w^2 is approximately proportional to M, the various accelerations are not changed greatly by changing the load. The several accelerations increase with the square of the terminal velocity, since w is proportional to v_t at constant load mass.

Rewriting equation (42) for n+1, and then dividing by equation (43), one obtains,

$$\frac{a_{n+1}}{a_n'} = \frac{a_{n+1}{}^\circ}{a_n{}'^\circ} \qquad \text{or} \qquad \frac{a_{n+1}}{a_{n+1}{}^\circ} = \frac{a_n'}{a_n{}'^\circ} \ . \qquad (46)$$

The first part of equation (46) shows that the ratio of the accelerations at the beginning of one section and at the end of the previous section is independent of both mass and terminal velocity. Since the results shown for both Case A and Case B in the table on page 65 indicate only relatively slight fluctuations in these accelerations, no further calculations need be made for other loads.

It may be concluded that a hydrapult designed for most efficient operation with the most commonly encountered load may also be used for other loads, either heavier or lighter, and with different terminal velocities, by proper choice of the jet velocity. The jet velocity desired may be obtained by selecting the proper pressure. Furthermore, the acceleration will exhibit almost the same nearly uniform pattern in all cases. Also, the efficiency will not decrease unreasonably as the load changes. Finally, it appears that extrapolation to very heavy loads and high terminal velocities is entirely feasible. The real limitations are the length of the power run possible, the allowable maximum acceleration, and the allowable maximum pressure.

Considerations on Variable Jet Velocities

Now consider briefly the possibility of using nozzle designs which make the individual jet velocities w_n variable in some convenient way. The simplest and probably the most reasonable way would be to have each $w_n = f_n B$, where f_n is determined by the construction of the n-th nozzle and its input system, and where B is a single constant for all nozzles, a constant determined by M and v_t. Such an arrangement is physically possible. The question now arises, how shall the quantities w_n, or B and f_n, be determined?

An obvious choice would be to require that the efficiency in each stage shall be a maximum. It is easily shown, however, using equations (12), (14) and (19), that this requirement leads to impossible results for low n values. In order to obtain high efficiency at low bucket velocities, the jet velocity must be low and the nozzle area correspondingly large. For example, for Case A the calculated jet velocity from the first nozzle is only 42 ft/sec., and the nozzle area required is about 250 sq.ft.

Alternative requirements which suggest themselves are also either inconvenient or impossible. For example, one might require that the fluid consumption be a minimum in each stage, or that the acceleration fluctuations be minimum. Since the uniform jet velocity system seems feasible, little further investigation of variable w_n systems has been made.

Note on Maximum Efficiency Configuration

As is to be expected, a configuration designed for maximum efficiency achieves that result by using low jet velocities and high jet cross sections. In any design the final jet has the largest area. Let attention be focused, therefore, on A_{N-1}, the area of the final jet. From equation (21) one finds, for the standard load,

$$A_{N-1} = \frac{M^{\circ} \cos \delta}{2 \rho b} \left[\ln \frac{1 - \sqrt{\frac{N-1}{N}} \frac{v_t^{\circ}}{w^{\circ} \cos \delta}}{1 - \frac{v_t^{\circ}}{w^{\circ} \cos \delta}} - \left[1 - \sqrt{\frac{N-1}{N}} \right] \frac{v_t^{\circ}}{w^{\circ} \cos \delta} \right] . \tag{47}$$

Define the two quantities q and p as follows.

$$q = \frac{v_t^{\circ}}{w^{\circ} \cos \delta} \quad \text{and} \quad p = \left[1 - \sqrt{\frac{N-1}{N}} \right] \frac{q}{1-q}$$

Then equation (47) becomes

$$A_{N-1} = \frac{M^{\circ} \cos \delta}{2 \rho b} \, p \left[q - 1 - \frac{\ln (1 + b)}{p} \right] . \tag{48}$$

Now q is given by equation (38) for the maximum efficiency configuration. Since N was taken as 100 in both cases previously considered, the same value will be used here. Substitution of these values into (48) gives

$$A_{99} = 0.00893 \, \frac{M^{\circ} \cos \delta}{2 \rho b} \quad .$$

This area may become quite large for a large M° and a small b. The numerical coefficient depends only on N, the number of nozzles. For example, one finds the following results for Case A, Case B, and a newly considered Case C for which M° = 25,000 lbs., b = 2 feet, and $\cos \delta$ = 0.98:

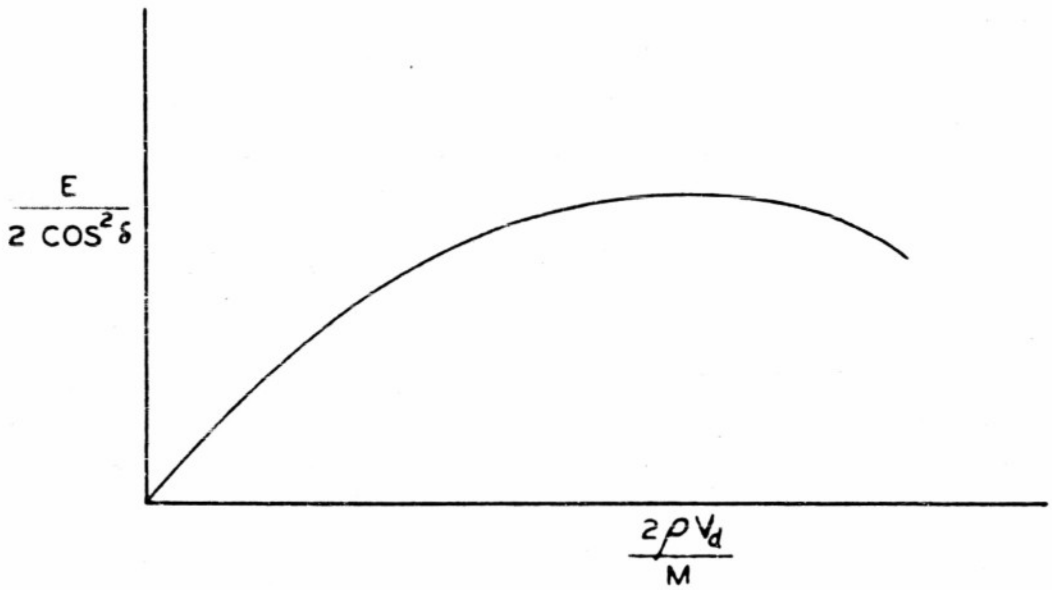
Case A: A_{99} = 1.751 sq.ft. = 252.1 sq.in.; radius = 8.96 in.

Case B: A_{99} = 1.751 sq.ft. = 252.1 sq.in.; radius = 8.96 in.

Case C: A_{99} = 0.875 sq.ft. = 126.0 sq.in.; radius = 6.34 in.

Thus there may be a convenience limit on designing for maximum efficiency.

Consider finally equations (34) and (37). It is apparent that while the efficiency E decreases as $v_t/(w \cos\delta)$ changes from 0.7153 in either direction, the total displacement volume V_d increases for $v_t/(w \cos\delta) > 0.7153$. Hence it is reasonable to design for jet velocities more than the optimum rather than less. This is shown by a graph of $E/(2 \cos^2\delta)$ versus $2\rho V_d/M$, which has the general shape shown below.



APPENDIX B. EFFECT OF DECREASING JET VELOCITY ON HYDRAPULT GEOMETRY

In the idealized treatment in Appendix A, a system of N nozzles equally spaced a distance b apart was considered. The nozzle areas were designated by $A_0, A_1, \ldots A_n, \ldots A_{N-1}$ respectively. The nozzles were inclined at an angle δ to the horizontal. The jet impinged on a single bucket of height not less than b tan δ. It was assumed that a constant pressure was maintained, under which condition the jet velocity would be constant. On this assumption calculations were made of the nozzle areas necessary in order that the shuttle would have at each jet position the velocity that would be observed for truly uniformly accelerated linear motion. With the use of such previously obtained results as are applicable, a treatment will now be given of the effect of a gradually decreasing jet velocity resulting from the decreasing pressure in the adiabatic expansion of a gas in an accumulator driving the jet fluid.

The equation of motion of the shuttle was found to be (page 59), for $nb \leq x \leq (n+1)b$,

$$\frac{dv}{dt} = \frac{2 A_n w \rho}{M} (w \cos \delta - v) \qquad (1)$$

where, as previously, v represents the velocity of the shuttle, t the time, A_n the area of the n-th nozzle, M the mass of the load, ρ the density of the fluid, δ the angle between the direction of the jet and the line of travel of the shuttle, and w the jet velocity. Whereas w was constant in the previous treatment, it will now be a function of t and hence of x. A more convenient independent variable in the present case is the volume V of accumulator gas. Obviously, for $nb \leq x \leq (n+1)b$,

$$\frac{dV}{dt} = A_n w \quad . \qquad (2)$$

Combination of equations (1) and (2) yields

$$\frac{dv}{dV} = \frac{2 \rho}{M} (w \cos \delta - v) \quad . \qquad (3)$$

Now for an adiabatic expansion the pressure P and the volume V are related by the expression

$$P V^k = P_0 V_0^k \qquad (4)$$

where the subscript refers to the initial conditions. Furthermore, one can write that

$$P = \frac{1}{2} \rho w^2 \quad . \tag{5}$$

Therefore, from (4) and (5), one obtains

$$w = w_0 \left[\frac{V_0}{V} \right]^{k/2} \quad . \tag{6}$$

Substituting equation (6) into equation (3) and rearranging, one obtains the relation

$$\frac{dv}{dV} + \frac{2 \rho v}{M} = \frac{2 \rho}{M} w_0 \cos \delta \left[\frac{V_0}{V} \right]^{k/2} \tag{7}$$

which may be written

$$\frac{d}{dV} \left[v \epsilon^{2 \rho V/M} \right] = \frac{2 \rho}{M} w_0 \cos \delta \left[\frac{V_0}{V} \right]^{k/2} \epsilon^{2 \rho V/M} \quad . \tag{8}$$

Integrating this, knowing that $v = 0$ when $V = V_0$, one obtains for v the following expression.

$$v = \epsilon^{-2 \rho V/M} \frac{2 \rho}{M} w_0 \cos \delta \; V_0^{k/2} \int_{V_0}^{V} \epsilon^{2 \rho V/M} V^{-k/2} dV \tag{9}$$

This may be rewritten in the form

$$v = \left[\frac{2 \rho V_0}{M} \right]^{k/2} w_0 \cos \delta \; \epsilon^{-2 \rho V/M} \int_{2 \rho V_0/M}^{2 \rho V/M} \epsilon^{u} u^{-k/2} du \quad . \tag{10}$$

Note that since in the derivation of the differential equation for dv/dV the discontinuous variable A_n was eliminated, equation (10) determines continuously the velocity v as a function of the volume V for every value of V.

Now to determine A_n equations (2) and (6) are used to obtain, for $nb \leq x \leq (n+1)b$,

$$dV = A_n w_0 \left[\frac{V_0}{V}\right]^{k/2} \frac{dx}{v} \tag{11}$$

or

$$A_n \, dx = \frac{v \, V^{k/2} \, dV}{w_0 \, V_0^{k/2}} \quad . \tag{12}$$

This is to be integrated over the entire appropriate range indicated by

$$A_n \, b = \frac{1}{w_0 \, V_0^{k/2}} \int_{V_n}^{V_{n+1}} v \, V^{k/2} \, dV \quad . \tag{13}$$

Hence,

$$A_n = \frac{1}{w_0 \, V_0^{k/2} \, b} \int_{V_n}^{V_{n+1}} v \, V^{k/2} \, dV \quad . \tag{14}$$

When equation (10) is substituted in equation (14) one finds that

$$A_n = \frac{M \cos \delta}{2 \rho b} \int_{\frac{2\rho V_n}{M}}^{\frac{2\rho V_{n+1}}{M}} \epsilon^{-s} s^{k/2} \, ds \int_{\frac{2\rho V_0}{M}}^{s} \epsilon^{u} u^{-k/2} \, du \tag{15}$$

in which V_n is the total gas volume at $x = nb$, that is, the total gas volume corresponding to the shuttle velocity v_n (through equation (10)) at $x = nb$. Now for simulated uniformly accelerated linear motion the velocities are given by

$$v_n = \sqrt{2\, a\, n\, b} \tag{16}$$

in which $n = 0, 1, 2, \ldots N$, and in which a is the space average of the acceleration. If the parameter y be defined by $y = 2\rho V/M$, one can write from equation (15), for $n = 0, 1, 2, \ldots N$,

$$A_n = \frac{M \cos\delta}{2\rho b} \int_{y_n}^{y_{n+1}} \epsilon^{-s}\, s^{k/2}\, ds \int_{y_0}^{s} \epsilon^{u}\, u^{-k/2}\, du \tag{17}$$

And from equation (10) one can write

$$\frac{\sqrt{2\, a\, n\, b}}{y_0^{k/2}\, w_0 \cos\delta} = \epsilon^{-y_n} \int_{y_0}^{y_n} \epsilon^{u}\, u^{-k/2}\, du \tag{18}$$

Equation (17), in conjunction with (18), defines the nozzle area A_n. In principle one now eliminates the parameters y_n from the equations to obtain A_n as a function of n. Actually, it is impossible to do this analytically. Hence, one must solve equation (18) for y_n for each valve of n, and then substitute in equation (17) to obtain A_n.

Before the solution of these equations is considered, several other relations should be noted. First, the total liquid displacement volume V_d is given by $V_d = (V_N - V_0)$. Hence, from the definition of the parameter y,

$$V_d = \frac{M}{2\rho}\left[y_N - y_0\right]$$

where y_N is determined from equation (18) for $n = N$.

Second, the time increment $U_n = (t_{n+1} - t_n)$ for each jet section is easily obtained from equations (2) and (6) as follows.

From (2),

$$dt = \frac{dV}{A_n w} = \frac{1}{w_0 \; V_0^{k/2} \; A_n} V^{k/2} \; dV \tag{19}$$

which yields upon integration,

$$U_n = \frac{1}{A_n \; w_0 \; V_0^{k/2}} \left[\frac{V_{n+1}^{1+k/2} - V_n^{1+k/2}}{1 + k/2} \right] . \tag{20}$$

In terms of the parameter y_n this becomes

$$U_n = \frac{M}{2 \rho A_n \; w_0 \; y_0^{k/2}} \left[\frac{y_{n+1}^{1+k/2} - y_n^{1+k/2}}{1 + k/2} \right] . \tag{21}$$

Third, the total energy input E_1 is likewise obtained easily.

$$dE_1 = \frac{1}{2} w^2 \rho \; dV = P \; dV = P_0 \; V_0^{k/2} \frac{dV}{V^{k/2}} . \tag{22}$$

Upon integration this gives

$$E_1 = P_0 \; V_0^{k/2} \left[\frac{V_N^{1-k/2} - V_0^{1-k/2}}{1 - k/2} \right] . \tag{23}$$

If this be expressed in the usual variables, one obtains

$$E_1 = \frac{M \; w_0^2 \; y_0^{k/2}}{4} \left[\frac{y_N^{1-k/2} - y_0^{1-k/2}}{1 - k/2} \right] . \tag{24}$$

Since the useful energy output is $M \; v_N^2/2$, one finds for the total over-all efficiency E

$$E = \frac{(2 - k)\, v_N^{\,2}}{w_0^{\,2}\, y_0^{\,k/2} \left[y_N^{\,1-k/2} - y_0^{\,1-k/2} \right]} \qquad (25)$$

Note from equations (17) and (18) that, as in the case of an assumed constant pressure (Appendix A), all planes of the same mass M may use the same track if the pressure is adjusted so that for each desired terminal velocity $(v_t/w_0 \cos\delta) =$ constant for the particular track used. Also note that within limits planes of different masses may be accommodated if as M and v_t vary δ and w_0 are adjusted so that

$$M \cos\delta = \text{constant} \qquad \text{and} \qquad \frac{v_t}{w_0 \cos\delta} = \text{constant} \quad .$$

This requires, of course, that the track be designed so that it takes the lightest plane with the minimum angle δ. Then for heavier planes the angle δ is increased, and if the terminal velocity v_t is also larger for the heavier plane, then w_0, and consequently the pressure, must be increased by a large amount to compensate for both the larger v_t and the smaller $\cos\delta$. The bucket height must not be less than b tan δ', where δ' is the angle for the heaviest plane to be launched. Obviously a technique such as this can accommodate only fairly small variations of plane weight, say up to 10 per cent, and is therefore of limited importance.

To return now to the solution of equations (17) and (18), define two new symbols z_n and r as

$$z_n = y_n - y_0 \qquad \text{and} \qquad r = k/2 \quad . \qquad (26)$$

Integration of equations (17) and (18) by series expansions leads to the following series for v_n and A_n; these series converge for reasonable values of y_0, y_N and r.

$$\frac{\sqrt{2\, a\, n\, b}}{w_0 \cos\delta} = z_n \left[1 - \frac{1}{2!}\left[1 + \frac{r}{y_0}\right] z_n + \frac{1}{3!}\left[1 + \frac{r}{y_0} + \frac{r\,(r+1)}{y_0^{\,2}}\right] z_n^{\,2} \right.$$

$$\left. - \frac{1}{4!}\left[1 + \frac{r}{y_0} + \frac{r\,(r+1)}{y_0^{\,2}} + \frac{r\,(r+1)(r+2)}{y_0^{\,3}}\right] z_n^{\,3} + \dots \right] \qquad (27)$$

Thus,

$$A_n = \frac{M \cos\delta}{2\rho b} \left[J_{n+1} - J_n \right] \qquad (28)$$

where

$$J_n = z_n^2 \left[\frac{1}{2!} - \frac{1}{3!}\left[1 - \frac{r}{y_0}\right] z_n + \frac{1}{4!}\left[1 - \frac{2r}{y_0} + \frac{r\,(r-2)}{y_0^2}\right] z_n^2 \right.$$

$$\left. - \frac{1}{5!}\left[1 - \frac{3r}{y_0} + \frac{r\,(3r-5)}{y_0^2} - \frac{r\,(r-2)(r-3)}{y_0^3}\right] z_n^3 + \cdots \right] \quad (29)$$

A typical configuration, Case D similar to Case B (page 64) of the idealized treatment of Appendix A, has been partially considered. Case D represents a hydrapult with the following specifications:

Case D: $M = 50{,}000$ lbs.; $b = 2$ feet; $N = 100$; $\cos \delta = 0.98$;
average $a = 4.00g$; $v_t = 227$ ft/sec. $= 155$ mph;
$w_0 = 694.9$ ft/sec.; $P_0 = 3255$ psi; $k = 1.400$;
$V_0 = 800$ cu.ft.; $\rho = 62.5$ lbs/cu.ft.

From these one obtains $r = 0.70$ and $y_0 = 2\rho V_0/M = 2.00$.

For Case D the first three nozzle areas are listed below, with the corresponding values previously obtained (Appendix A) for Case B on the assumption of constant pressure.

Case	A_0	A_1	A_2
D	0.1132 sq.ft.	0.1169 sq.ft.	0.1195 sq.ft.
B	0.1114	0.1135	0.1149

The time required in the first section is 0.1745 sec., compared to 0.1752 sec. for Case B. The accelerations at the beginning and end of the first section are 4.16g and 3.92g respectively for Case D, as compared to 4.09g and 3.96g for Case B. The total water volume used is 178.6 cu.ft. in Case D, whereas it was 162.2 cu.ft. in Case B. With an initial pressure of 3255 psi, the final pressure is 2455 psi, and the calculated over-all efficiency is 51.4 per cent. This efficiency is to be compared with the 52.6 per cent for Case B.

Because of decreasing jet velocity with increasing expansion, the nozzle areas must increase more rapidly in Case D than in Case B. For Case D the calculated area of the final jet, A_{99}, is 0.2336 sq.ft. This corresponds to a circular jet 6.54 inches in diameter.

The results described here for a particular case are representative of results which would be obtained for other systems. For example, if a smaller value of V_0 were chosen, then the calculated A_n values would be larger. Conversely, Case B represents the limiting case in which V_0 approaches infinity. The conclusions about relative accelerations previously reached in Appendix A need be modified only slightly for Case D. The "sawteeth" in the acceleration versus distance curve are somewhat more prominent for Case D than for Case B, but they are still quite small. The adaptability for different loads is essentially as good in Case D as it was found to be in Case B.

Some improvement could be attained by replacing the single accumulator by several smaller accumulators arranged to operate successive sets of jets. If this were done, an analogous computation would have to be made for each set of jets to determine the necessary areas.

Finally, in case such calculations as these should become useful, either graphical or tabular means may be used to decrease the labor required. For example, in order to make these calculations the right-hand member of equation (27) has been evaluated to nine significant figures for values of z_n from 0 to 0.46 at intervals of 0.02, with $y_0 = 2$ and $r = 0.7$.

APPENDIX C. MINIMIZATION OF THE HYDRAPULT WEIGHT

The total weight of the hydrapult (see second equation on page 39) is given approximately by the expression

$$\text{Wt.} = \left[- \frac{M}{2\rho} \ln \left[1 - \frac{v_t \, (R^{1.214} - 1)}{19.7 \sqrt{P_1} \, (R^{0.714} - 1)} \right] \right] \left[137 + (0.062 + L') \, P_1 + \frac{0.036 \, P_1}{R^{0.714} - 1} \right] + 35{,}000 \qquad (1)$$

in which L' is proportional to the total length of pipe required. Two cases arise. A catapult can be designed for a larger terminal velocity either by keeping the length of the run fixed and increasing the acceleration, or by keeping the acceleration fixed and increasing the length of the run. A third and more likely way of increasing the terminal velocity is to increase both parameters, but this case would be covered by consideration of the two more specialized cases. If the higher terminal velocity is obtained by keeping the length of the run fixed and increasing the acceleration, L' is a constant; its value is 0.028 for a 200-foot run (see page 34, and recall that 10 per cent has been added for contingencies). If the larger terminal velocity is obtained by keeping the acceleration constant and increasing the length of the run, L' becomes a function of the terminal velocity v_t; in the particular instance for which allowance is made only for the increased weight of pipe necessary (see page 34, and recall that 10 per cent has been added for contingencies), L' may be taken as $5.5 \times 10^{-7} \, v_t^2$. The two values mentioned for L' are identical for a terminal velocity of 227 ft/sec. obtained with an average acceleration of 4.0g and a run of 200 feet.

Minimization of the weight expression may be carried out in terms of L'. Computations for either special case may then be made. The independent variables are taken as P_1 and R. The terminal velocity v_t is considered a parameter. The weight expression may be rewritten in the form

$$G = - \left[1 + P_1 F_1(R) \right] \ln \left[1 - \frac{F_2(R)}{\sqrt{P_1}} \right] \qquad (2)$$

in which

$$G = \frac{1}{137} \frac{2\rho}{M} (\text{Wt.} - 35{,}000) \qquad B = \frac{0.062 + L'}{137}$$

$$F_1(R) = \frac{A}{R^{\alpha} - 1} + B \qquad C = \frac{v_t}{19.7}$$

$$F_2(R) = \frac{C(R^{\beta} - 1)}{R^{\alpha} - 1} \qquad \alpha = 0.714 \tag{3}$$

$$A = \frac{0.036}{137} \qquad \beta = 1.214 .$$

When G is differentiated partially with regard to P_1 and R respectively, one obtains (omitting the designation of arguments in the functions F_1, F_2, F_1' and F_2'),

$$-\frac{\partial G}{\partial P_1} = F_1 \ln\left[1 - \frac{F_2}{\sqrt{P_1}}\right] + \frac{1 + P_1 F_1}{1 - \frac{F_2}{\sqrt{P_1}}}\left[\frac{F_2}{2\,P_1^{3/2}}\right] \tag{4}$$

$$-\frac{\partial G}{\partial R} = P_1 F_1' \ln\left[1 - \frac{F_2}{\sqrt{P_1}}\right] + \frac{1 + P_1 F_1}{1 - \frac{F_2}{\sqrt{P_1}}}\left[-\frac{F_2'}{\sqrt{P_1}}\right] \tag{5}$$

To obtain the values of P_1 and R which minimize the weight these derivatives are each set equal to zero and the resulting two equations solved for P_1 and R. Analytically, this procedure insures only that the conditions represent a maximum or a minimum. They actually represent a minimum in this case.

When the quantity

$$\frac{1 - \frac{F_2}{\sqrt{P_1}}}{1 + P_1 F_1} \ln\left[1 - \frac{F_2}{\sqrt{P_1}}\right]$$

is eliminated from equations (4) and (5), the following simple relation is obtained.

$$\frac{F_1'}{F_2'} = -2\,\frac{F_1}{F_2} \tag{6}$$

This relation defines the value of R which gives a minimum value of the weight. When the value of R is determined, then the corresponding minimizing value of P_1 can be determined from either of the original equations in the form of a transcendental relation. If one defines the quantity $x = F_2/\sqrt{P_1}$, so that $P_1 = F_2^2/x^2$, the relation becomes

$$-\frac{1}{x^2}\left[\frac{1-x}{x}\ln(1-x)+\frac{1}{2}\right] = \frac{1}{2\,P_1\,F_2^2} \quad . \tag{7}$$

This equation can be solved for x, and hence for P_1, when R is known.

Consider the first special case, that in which any change in design terminal velocity is accomplished by keeping the accelerated run constant and changing the acceleration. In this case L' is constant and equal to 0.028. Hence equation (6) becomes

$$\frac{B}{A}\left[\frac{\beta}{\alpha}-1\right]R^{\alpha+\beta}+\left[\frac{\beta}{\alpha}-\frac{3}{2}-2\frac{B}{A}\left[\frac{\beta}{\alpha}-\frac{1}{2}\right]\right]R^{\beta}+\frac{B}{A}R^{\alpha}$$
$$-\frac{\beta}{\alpha}\left[1-\frac{B}{A}\right]R^{\beta-\alpha}+\left[\frac{3}{2}-\frac{B}{A}\right]=0 \tag{8}$$

Note that here there is no dependence on terminal velocity v_t. When numerical values are inserted, one finds that

$$1.75\,R^{1.929}-5.80\,R^{1.214}+2.50\,R^{0.714}+2.55\,R^{0.500}-1.00=0\ . \tag{9}$$

The solution of this equation is R = 2.123, a constant for all values of v_t. Since R is a constant, F_1 and F_2/v_t are also constant. They have the values $F_1 = 0.00103$ and $F_2/v_t = 0.107$. It is convenient to express v_t and P_1 in terms of the parameter x. If this is done,

$$v_t = \sqrt{-\frac{4.28\times10^4\,x^2}{\frac{1-x}{x}\ln(1-x)+1/2}}\ \text{ft/sec.} \tag{10}$$

and

$$P_1 = \frac{-485}{\frac{1-x}{x}\ln(1-x)+1/2}\ \text{psi}\ . \tag{11}$$

Restricting attention to a range of acceleration from approximately 1g to 10g, for a 200-foot run (for which R = 2.123) one obtains the results given in the following table.

x	v_t	P_1	a/g
0.30	108 ft/sec.	1460 psi	0.91
0.35	132	1620	1.35
0.40	160	1820	1.99
0.45	194	2100	2.92
0.50	235	2510	4.29
0.55	291	3160	6.57
0.60	373	4370	10.8

The manner of variation of R and of P_1 with v_t is shown graphically by the full-line curves on page 41.

Now consider the second special case, that in which any change in design terminal velocity is accomplished by keeping the acceleration constant and changing the length of the run. In the particular instance for which allowance is made only for the increased weight of pipe necessary, L' may be taken as $5.5 \times 10^{-7}\, v_t^2$. In this case it is found that R as defined by equation (6) is a function of v_t. In order to calculate corresponding values of R and v_t, however, it is more convenient to solve equation (6) for v_t for selected values of R, and then to calculate P_1 in a manner analogous to that used in the first case. Solved for v_t^2, equation (6) becomes

$$v_t^2 = \frac{1}{5.5 \times 10^{-7}}\left[\left[\frac{0.036}{R^{\alpha} - 1}\right]\frac{\left[-\left[\frac{\beta}{\alpha} - \frac{3}{2}\right] R^{\beta} + \frac{\beta}{\alpha} R^{\beta-\alpha} - \frac{3}{2}\right]}{\left[\frac{\beta}{\alpha} - 1\right] R^{\beta} - \frac{\beta}{\alpha} R^{\beta-\alpha} + 1} - 0.062\right] \qquad (12)$$

or

$$v_t^2 = \frac{1}{5.5 \times 10^{-7}}\left[\left[\frac{0.036}{R^{0.714} - 1}\right]\left[\frac{-0.200\, R^{1.214} + 1.700\, R^{0.500} - 1.500}{0.700\, R^{1.214} - 1.700\, R^{0.500} + 1.000}\right] - 0.062\right]. \qquad (13)$$

Using this equation, values of v_t can be calculated for selected values of R. The results are then used to calculate

$$P_1 = \frac{1}{137}\left[\frac{0.036}{R^{0.714} - 1} + 0.062 + 5.5 \times 10^{-7}\, v_t^2\right]$$

and

$$F_2 = \frac{v_t}{19.7}\left[\frac{R^{1.214} - 1}{R^{0.714} - 1}\right] .$$

These define x as follows by equation (7).

$$-\frac{1}{x^2}\left[\frac{1 - x}{x} \ln (1 - x) + \frac{1}{2}\right] = \frac{1}{2 F_1 F_2^2}$$

Having the value of x, P_1 is then given by

$$P_1 = \frac{F_2^2}{x^2} .$$

Restricting attention to approximately the same rang terminal velocities as considered in the first case, com ing to accelerated runs a up to about 500 feet at a cons acceleration of 4.0g, one obtains the results given in t following table.

R	v_t	x	P_1
2.30	103 ft/sec.	0.271	1740 psi
2.25	141	0.350	1910
2.20	176	0.413	2120
2.15	209	0.466	2310
2.10	241	0.509	2530
2.00	302	0.574	3040
1.90	371	0.622	3770

The manner of variation of R and P_1 with v_t is shown by the broken-line curves on page 41.