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Honeywell

STUDY OF AUTOMATIC CONTROL SYSTEMS
FOR HELICOPTERS

Part III:

ANALYTICAL STUDY OF RPM CONTROL
FOR THE SINGLE SPOOL TURBOPROP ENGINE

AD 5143-TR5

Hugh Albachten RESTRICTED 23 April 1953

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Aeronautical Controls

RESTRICTED

Security Information

STUDY OF AUTOMATIC CONTROL SYSTEMS FOR HELICOPTERS

PART III

ANALYTICAL STUDY OF RPM CONTROL FOR THE SINGLE SPOOL TURBOPROP ENGINE

AD 5143-TR5

23 April 1953

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RESTRICTED

FOREWORD

This interim report was prepared by the Research Department, Aeronautical Division, Minneapolis-Honeywell Regulator Company, under ONR Contract No. Nonr-929(00). The contract was initiated under the research project identified by Expenditure Accounts 46000 (Research Navy) and 46832 (Aircraft and Facilities Navy). This is a contract for research involving the study of helicopter control systems from the point of view of automatic control of attitudes and power. This and succeeding interim reports will be followed by a final report.

ABSTRACT

This report presents the results of a preliminary analytical study of the RPM and temperature control of a helicopter turboprop engine and rotor combination. The system considered is spoken of as being one in which RPM is controlled by the manipulated variable fuel flow, and temperature is controlled by the manipulated variable collective pitch. The following three simple configurations were investigated using root locus procedures:

1) Integral control in the RPM feedback loop and proportional control in the temperature feedback loop.

2) Proportional control in the RPM feedback loop and integral control in the temperature feedback loop.

3) Integral control in the RPM feedback loop and rate control in the temperature feedback loop.

The conclusion reached is that the third configuration listed holds the greatest promise of satisfactory dynamic response of RPM.

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I INTRODUCTION

The fundamental objective of helicopter contract Nonr-929(00) is to investigate control systems in which the engine regulators and attitude stabilizer blend well and complement each other's functions in order that the best result is achieved for the helicopter as a whole. To date only the hovering helicopter has been considered in the analytical work, and in hovering it was found that RPM variations do not couple into attitude variations. Since this report deals with a hovering helicopter, considerations of attitude (pitch and roll) are thereby eliminated. Instead, in regard to displacement, attention must be directed toward the observation of the helicopter's vertical motion and the pilot's effort to maintain it constant.

The engine selected for this initial study is the 280 h.p.-Artouste turbo-prop (single spool) engine. Two engine regulators were considered — a constant RPM governor and a temperature control device. The problem for the hovering helicopter is then one of selecting a constant RPM control and a temperature control device which do not adversely affect the pilot's efforts to maintain his altitude. Altitude is not controlled automatically.

If the helicopter rotor is thought of as a load on the engine, there are available two means of controlling or varying the RPM. One is by the control of the collective pitch of the rotor blades, and the other by the amount of fuel flow to the engine. Each in turn will vary engine temperature as well. The problem then, for purposes of discussion, can be broken down into two systems.

1) System I is termed RPM control by collective pitch and temperature control by fuel flow.*

2) System II is called RPM control by fuel flow and temperature control by collective pitch. The system can be thought of in this way, but in general anything that is done to the temperature control affects the RPM just as variations in the RPM control do, and visa versa. It is perhaps better, therefore, to think of the problem as two controls regulating RPM and the same two controls governing temperature. A similar remark can be made about System I. The reminder of this report will deal solely with System II.

In this comparatively short analytical study the variations of altitude caused by the RPM and temperature controls were not

*A REAC analysis of System I is considered in Reference 2.

investigated. To consider still another controlled variable would at least double the amount of work, and investigation of this variable can best be handled by the REAC. On the REAC, changes in altitude caused by inputs to fuel and collective pitch made by the pilot's stick can be more easily added to variations in these manipulated quantities caused by the controls.

In general the responses of a dynamic system can be investigated for two types of inputs. One is a change in the desired value of the control. In the system considered, this would be initiated by the pilot in changing the set value of either the RPM regulator, the temperature control device, or both. Since these settings are seldom, if ever, made while in flight, these inputs are the least important.

Of more direct concern are the inputs called "disturbances". A gust disturbance is a practical example of this type for the hovering helicopter. A down-gust strikes the helicopter rotor and decreases the aerodynamic angle of attack sufficiently to decrease the load on the engine. The engine under decreased load increases its speed. The controls sense this increase and the RPM control decreases the fuel flow in order to bring the RPM back to its norm position. At the same time, the decreased temperature caused by the increased RPM is combined with the decrease in temperature caused by the fuel flow decrease (to control RPM), and after being compared with the set value of temperature, the temperature deviation is sent to the control to return the temperature to its set value by variations in the collective pitch. These changes in collective pitch again have their effects on the RPM and the cycle is repeated. It is this, the RPM phase of the regulation problem, that is investigated in this report.

II EQUATIONS OF MOTION AND TRANSFER FUNCTIONS

There are six differential equations of dynamics involved in this problem. These are derived in References 1 & 2 and for completeness are listed below. The first three equations describe the helicopter rotor torque, coning, and vertical motion in that order. The last three equations are derived from engine dynamics and unbalanced engine-rotor torque. The nomenclature is defined in Section VII.

1) $$\frac{Q_A}{N} = .915\,W + 13.15\dot{\beta} + 1720\theta_\Delta + .58\Omega_\Delta - .915\,\dot{z}_c$$

2) $$0 = -284.1\,W - 3411.7\dot{\beta} + 122038\theta_\Delta + 40.9\Omega_\Delta + 284.1\,\dot{z}_c - 287382\beta - 224.6\ddot{\beta}$$

3) $$0 = 79.9\,W + 852.3\dot{\beta} - 304858\theta_\Delta - 13.5\Omega_\Delta - 79.9\,\dot{z}_c - 97.1\,\ddot{z}_c$$

4) $$Q_E = \frac{1.45}{1 + .15S}\Delta w_f - .649\Omega_\Delta$$

5) $$T_\Delta = \frac{3.87}{1 + .15S}\Delta w_f - 1.72\Omega_\Delta$$

6) $$Q_E - \frac{Q_A}{N} = 2.22\dot{\Omega}_\Delta$$

The 0.15 sec time lag represents the time to realize the total energy from the change in fuel flow.

If the Laplace transformation of these equations is taken with zero initial conditions, the following transfer functions may be solved for:

$$K_4G_4 = -\frac{\Omega_w}{W} = \frac{.4095S(S - 1.53 \pm 35.74j)}{(S + .69 \pm .198j)(S + 7.6 \pm 34.8j)}$$

$$K_5G_5 = \frac{\Omega_{w_f}}{\Delta w_f} = \frac{.649\,(S + .83)}{(S + .69 \pm .198j)\,(1 + .15S)}$$

$$K_3G_3 = -\frac{\Omega_\theta}{\theta_\Delta} \quad \frac{774\,(S + 1.0)(S + 9.7 \pm 34.2j)}{(S + .69 \pm .198j)(S + 7.6 \pm 34.8j)}$$

The control transfer functions have been taken as

$$K_1G_1 = \frac{\theta_\Delta}{T_\Delta} = \frac{\sum_{-\infty}^{+\infty} K_n' S^n}{1 + T_s S}$$

$$K_2G_2 = \frac{\Delta w_f}{\Omega_\Delta} = \frac{\sum_{-\infty}^{+\infty} K_n S^n}{1 + T_g S}$$

T_s (= .06 sec.) is the time constant in the single-order lag taken to represent the servo motor that actuates the collective pitch lever. T_g (= .05 sec.) is the time constant in the single-order lag representing the RPM sensing device.

The numerators of the control transfer functions are seen to be represented by a general expression of the Laurent's series form. Thus the numerator for K_2G_2 is

$$K_n S^n + \ldots + \frac{K_{-2}}{S^2} + \frac{K_{-1}}{S} + K_0 + K_1 S + K_2 S^2 + \ldots + K_n S^n$$

and for K_1G_1 a similar expression, but with primed K's. In these representations K_1, for example, is entitled the rate gain in the RPM control and K'_{-1}, the integral gain in the temperature control, etc. In a physical sense, however, K_1 represents a rate of change of velocity and, therefore, an acceleration, since RPM is being fed into the K_2G_2 control as represented in this report. A similar remark applies to all the unprimed K's.

It should be pointed out that the Laurent's series form for the control numerators is very general, and that the K's can be given interpretations other than simple integral, rate and accelerations gains, etc. The fact that K_1S is termed rate does not mean that it must come from a rate network. If the lead $(1 + T_1S)/(1 + T_0S)$ where $T_1 > T_0$, is applied to the proportional K_0, the K_1 could be considered the product of K_0T_1.

III BLOCK DIAGRAMS

The block diagram shown in Figure 1(a) presents a means of clarifying the physical picture of the system to be analyzed, and forms the basic first step in the analysis. This diagram is drawn for a gust disturbance input, and points out the system's essential three-loop character with the destabilizing tie between the first two loops making up the third regenerative loop. Figure 1(b) is drawn only to more conveniently handle the system by root locus methods and has no physical significance. Only the overall transfer function $\frac{\Omega_\Delta}{W}$ applies; i.e.,

$$\frac{\Omega_\Delta}{W} = \frac{K_4G_4}{1 + K_2G_2K_5G_5 + 1.72\ K_1G_1K_3G_3 - 3.87\ K_1G_1K_2G_2K_3G_3}$$

IV INVESTIGATION OF DIFFERENT CONTROL CONFIGURATIONS

The first and simplest criterion for this control system is that it be a zero "position" error system. That is, RPM after disturbance is to return quickly to the value set by the pilot. This necessitates integral control in either K_1G_1 or K_2G_2, but not both. It should be stated that with integral control present, the RPM will have zero steady state error only if the input is constant in the steady state. If the input is of a different nature, the system will actually have a steady state error.

From the point of view of the time required to complete an entire design specifying optimum gain setting, etc., the root locus method for a system this involved is outside practical considerations. What is both practical and highly desirable is to determine by root locus methods the principal closed loop pole - zero configuration and its movement in the complex plane as the gains vary. Then the time-consuming task of optimum gain setting can more efficiently be left to the REAC.

The following three simple configurations were investigated using root locus procedures:

1) Integral control in the RPM feedback loop and proportional control in the temperature feedback loop.

2) Proportional control in the RPM feedback loop and integral control in the temperature feedback loop.

3) Integral control in the RPM feedback loop and rate control in the temperature feedback loop.

The complete results of this study are given in the 8 root locus plots contained in Figures 2 to 12. Three summary figures showing the main pole-zero points for all three loops in each configuration are given in Figures 5, 9, and 12.

The general procedure followed for all the configurations investigated was first to choose a type of control for K_2G_2 and draw the first loop root locus. Then choose a value of gain in K_2G_2 and with the results from the first root locus draw the second loop locus. Select a value of gain for K_1G_1 and with the results from the second loop draw the third loop locus. There is unfortunately no way of telling until the final loop is completed whether the choice of control gains will result in a desirable response of RPM. If it does not, by tracing back through the loci of the loops it is possible to determine what happens to the final poles as the gains vary. No matter what gains are chosen, a picture of the final pole-zero configuration is obtained which is invaluable in later REAC investigations.

An example of the procedure used in tracing through the loop loci to determine how the final closed loop poles vary is given below. Configuration One will be used as the example and Figures 2 to 5 are referred to in the explanation.

From the summary (Figure 5) it can be seen that as the gain of the integral control in K_2G_2 varies from a low value to a high value the complex pair of poles vary from the high damping to low damping and then travel over to the right hand plane. At the same time the simple pole varies from 0 to a maximum value of 0.83 (Figure 2). The change of the simple pole is rapid at low gains.

In the second loop the value of the proportional gain less than the maximum permissable of .00018 (see loop 3) has very little effect on the three low frequency poles of the first loop. The simple pole changes the most and moves toward the origin. Again the third loop has a small effect on the second loop, only making the complex pair more highly damped and the simple pole move in toward the origin. What can be concluded from this is that the proportional control in K_1G_1 has little effect on the response of RPM and that the response is mostly determined by the integral control in K_2G_2. It appears that just about any response that is possible with a conjugate pair and a simple pole, all at less than 1 rad/sec, can be achieved with this configuration.

By investigating the other configurations in a similar way further tentative conclusions can be drawn. These are discussed below.

<u>Configuration Two</u>

Much the same type of response can be realized with this configuration as with Configuration One, although at different gains. Except for the possibility of certain gain settings in this configuration being more easily obtained or otherwise more desirable, there does not appear to be any advantage of this configuration over Configuration One.

<u>Configuration Three</u>

This arrangement with comparatively low values of integral control in K_2G_2 and high values of rate in K_1G_1 shows the most promise. This drives out the simple pole from the origin and allows reasonable damping from the complex pair.

In the above nothing has been said about the high frequency closed loop poles and zeros. Those that occur on the real axis are always in pole-zero pairs and have no effect on either the amplitude or shape of the response. The high frequency rotor blade motion (35 rad/sec) that is apparent in the original dynamic equations is carried through the analysis to appear as two closed loop poles. Because in all the configurations the predominate poles are low frequency (less than 1 rad/sec) these blade motions have no effect on the dynamics of the system. This is especially true since the two closed loop right hand plane complex zeros (this is a non-minimum phase system) that appear in the K_4G_4 transfer function nullify even the effect the high frequency poles would alone have on the magnitude of the response. The result is that these high frequency poles make the mechanics of the root locus more difficult because of the wide range of frequencies involved, but have no effect on the results.

A few noteworthy remarks can be made about the stability of various possible control configurations. These are of little value in solving the immediate problem of choosing a practical RPM control, since they will not give any information about the dynamic response. However, they do point out that there is no possibility of stabilizing certain configurations and it is useless to continue the root locus work further.

These results come from consideration of the root locus plots (gain at which the imaginary axis is crossed), the physical aspects of the problem and steady state considerations. They are listed in concise mathematical form, and cover most practical possibilities. Negative K's are not considered unless stated and the input is a step function.

I. If $K_2G_2 = \sum_{n=1}^{a} K_n S^n + \sum_{z=0}^{+\infty} K_z S^z$

And $K_1G_1 = K_0'$

Then

(a) if a = -1, the system will be stable if $K'_0 < .00018$

(b) if a = -2, the system will be neutrally stable if $K_0' < .00018$

(c) if $a < -2$, the system is never stable.

II. If $K_2G_2 = \sum_{n=-a}^{+\infty} K_n S^n$

And $K_1G_1 = K_1'S$

Then

(a) if a = 0, $K_0 < .444$ for stability

(b) if a = 1, $K_{-1}K'_1 < .00017$ for stability

(c) if $a > 1$, can never be stabilized if K's are either positive or negative.

III. If $K_2G_2 = \sum_{n=-a}^{+\infty} K_n s^n$

And $K_1G_1 = K'_2 s^2$

Then

(a) if a = 0, system will be neutrally stable if $K_0 < .444$

(b) if a = 1, system can never be stable.

IV. If $K_2G_2 = \sum_{n=-a}^{+\infty} K_n s^n$

And $K_1G_1 = K_3' s^3$

Then

(a) If a = 0, as one might suspect, it is impossible to stabilize this system.

V. If $K_2G_2 = K_0$

And $K_1G_1 = \sum_{n=-1}^{n=a} K_n' s^n + \sum_{z=0}^{+\infty} K_z' s^z$

Then

(a) if a = -1, $K_0 < .444$ for stability

(b) if a = -2, system will be neutrally stable if $K_0 < .444$

(c) if $a < -2$, never stable

VI. If $K_2G_2 = K_1 s$

And $K_1G_1 = \sum_{n=-a}^{+\infty} K'_n s^n$

Then

(a) if a = 0, $K_0' < .00018$ for stability

(b) if a = 1. $K_1K'_{-1} < .00017$ for stability

(c) if $a > 1$, can never be stabilized

VII. If $K_2G_2 = K_2 s^2$

And $K_1G_1 = \sum_{n=-a}^{+\infty} K'_n s^n$

Then

(a) if a = 0, system will be neutrally stable if $K_0 < .00018$

(b) If a = 1, never stable.

VIII. If $K_2G_2 = K_3 s^3$

And $K_1G_1 = \sum_{n=-a}^{+\infty} K'_n s^n$

Then

(a) if a = 0, system cannot be made stable

V SUMMARY AND CONCLUSIONS

A comparatively short root locus investigation has been made of three configurations useable in the constant RPM control aspect of the hovering helicopter engine RPM and temperature control problem. No specific gain values are recommended but the configuration using integral control in the RPM control and rate control in the temperature control seems likely to result in the most promising dynamic response. Some further rules that point out different configuration stability possibilities and boundries are also listed in the text of the report.

VI FUTURE INVESTIGATIONS

The possibilities for future analytical investigation into a problem of this kind are practically unlimited. It is estimated, however, that complete results with optimum gains, etc., could be reached in about 10 man months, but it is doubtful if it is worth the effort since the REAC is also available for use in this study.

A more practical approach, therefore, is to continue along the lines indicated in this report, not attempting to pin-point gain values, but rather aiming at determining the final closed loop pole-zero picture for different configurations. It is extremely valuable to have this picture in mind when investigating specific gain values on the REAC.

VII SYMBOL NOMENCLATURE

Q_A	=	Aerodynamic torque, ft. lbs.
W	=	Vertical gust, ft. per. sec.
β	=	Flapping angle, radians
θ_Δ	=	Collective pitch angle, radians
Ω_Δ	=	Total change in RPM of engine
Ω subscript	=	Change in RPM due to subscript
Z_c	=	Vertical displacement, ft.
Δw_f	=	Fuel flow, lbs. per. hour
Q_E	=	Engine torque, ft. lbs.
N	=	Gear ratio
S	=	Laplace transform complex variable
T_Δ	=	Total change in engine temperature
$T_{subscript}$	=	Change in Temperature due to subscript
O.L.	=	Open loop
C.L.	=	Closed loop
D.Z.	=	Double zero

VIII REFERENCES

1. Minneapolis-Honeywell Technical Report AD 5143-TR3: Study of Automatic Control Systems for Helicopters. Part I. Aerodynamic Torque of a Helicopter Rotor Experiencing Transients in RPM While Hovering. P. Warsett.

2. Minneapolis-Honeywell Technical Report AD 5143-TR4: Study of Automatic Control Systems for Helicopters. Part II. REAC Study of RPM Control for the Single Spool Turboprop Engine. C. Brown.

BLOCK DIAGRAMS

Control of RPM and Turbine Temperature
Turbo-Prop Engine (Single Spool) on Single-Rotor, Hovering Helicopter

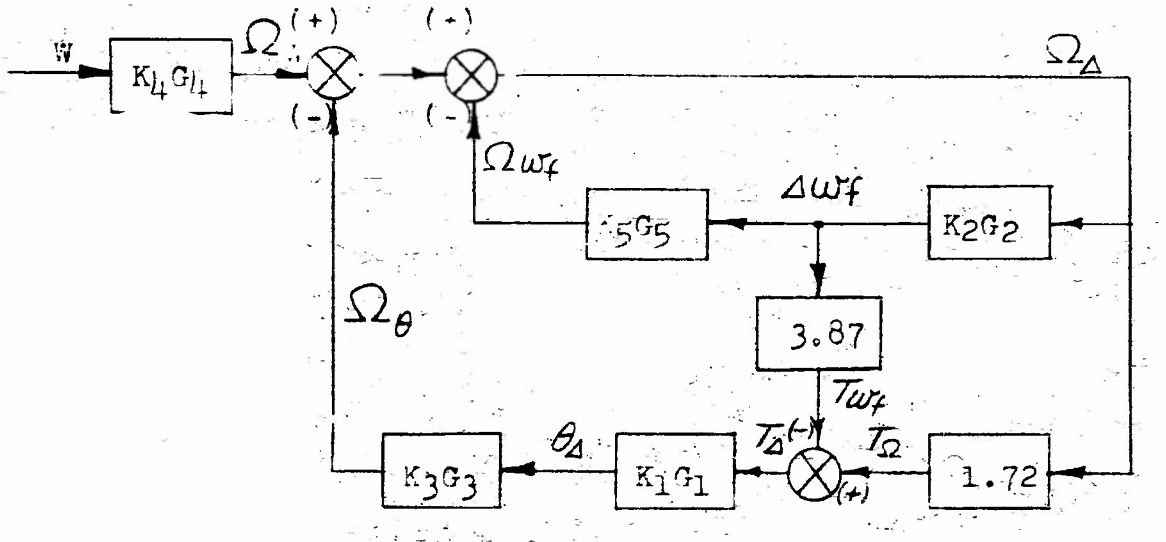


Figure 1a

or for the root locus work :

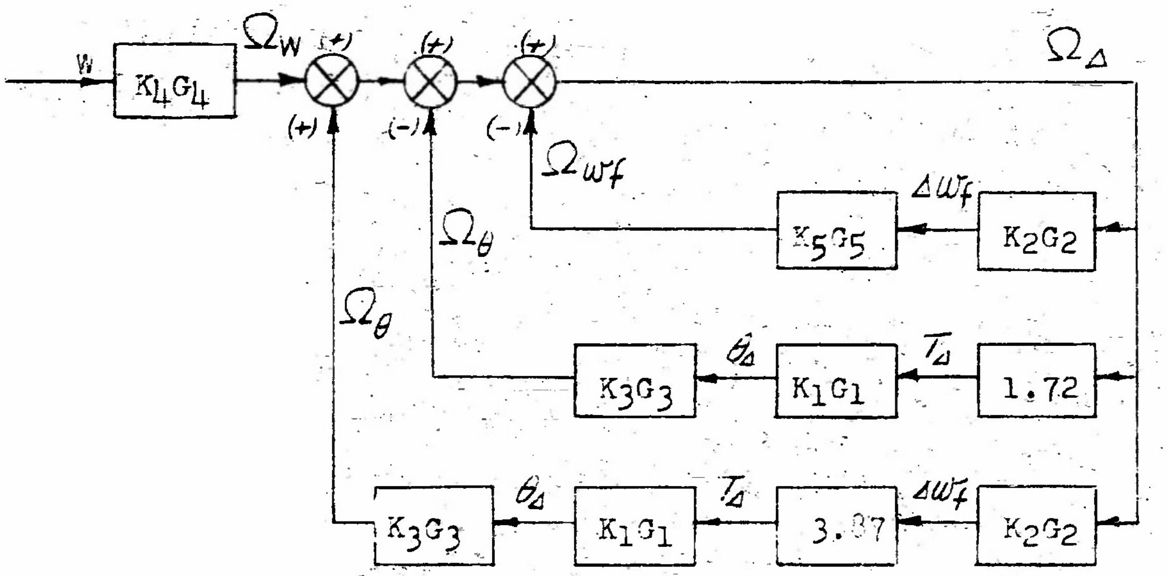


Figure 1b

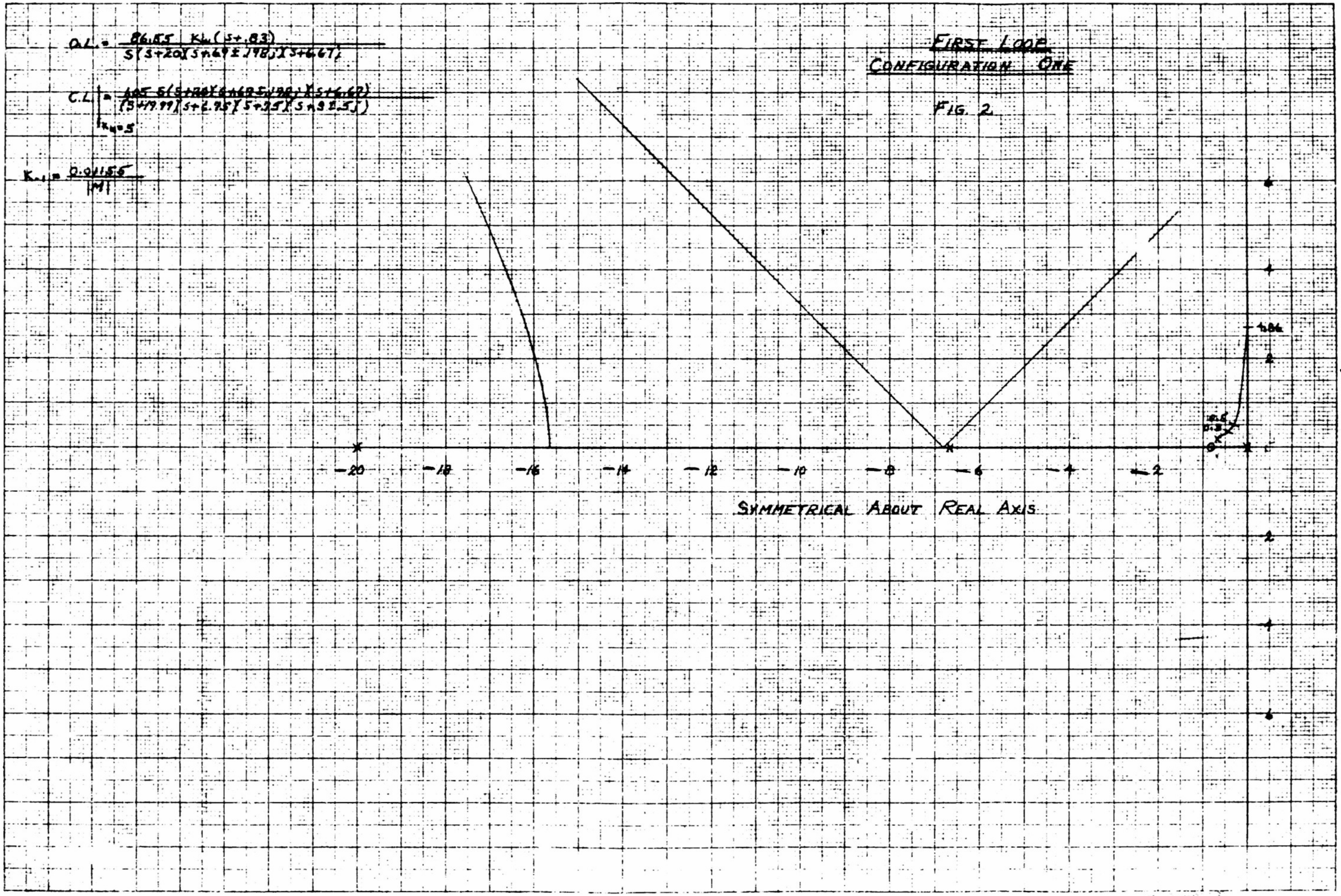
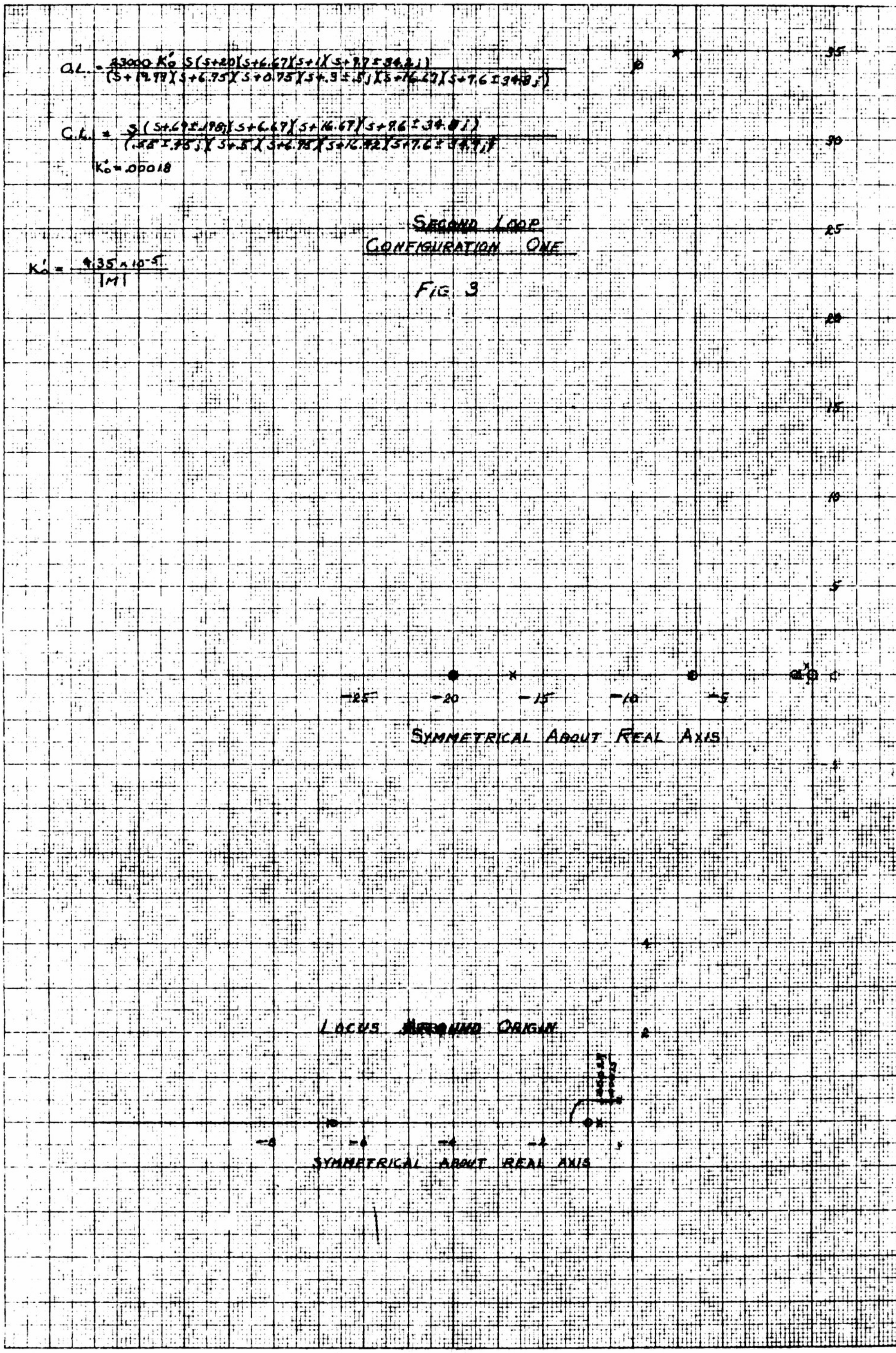
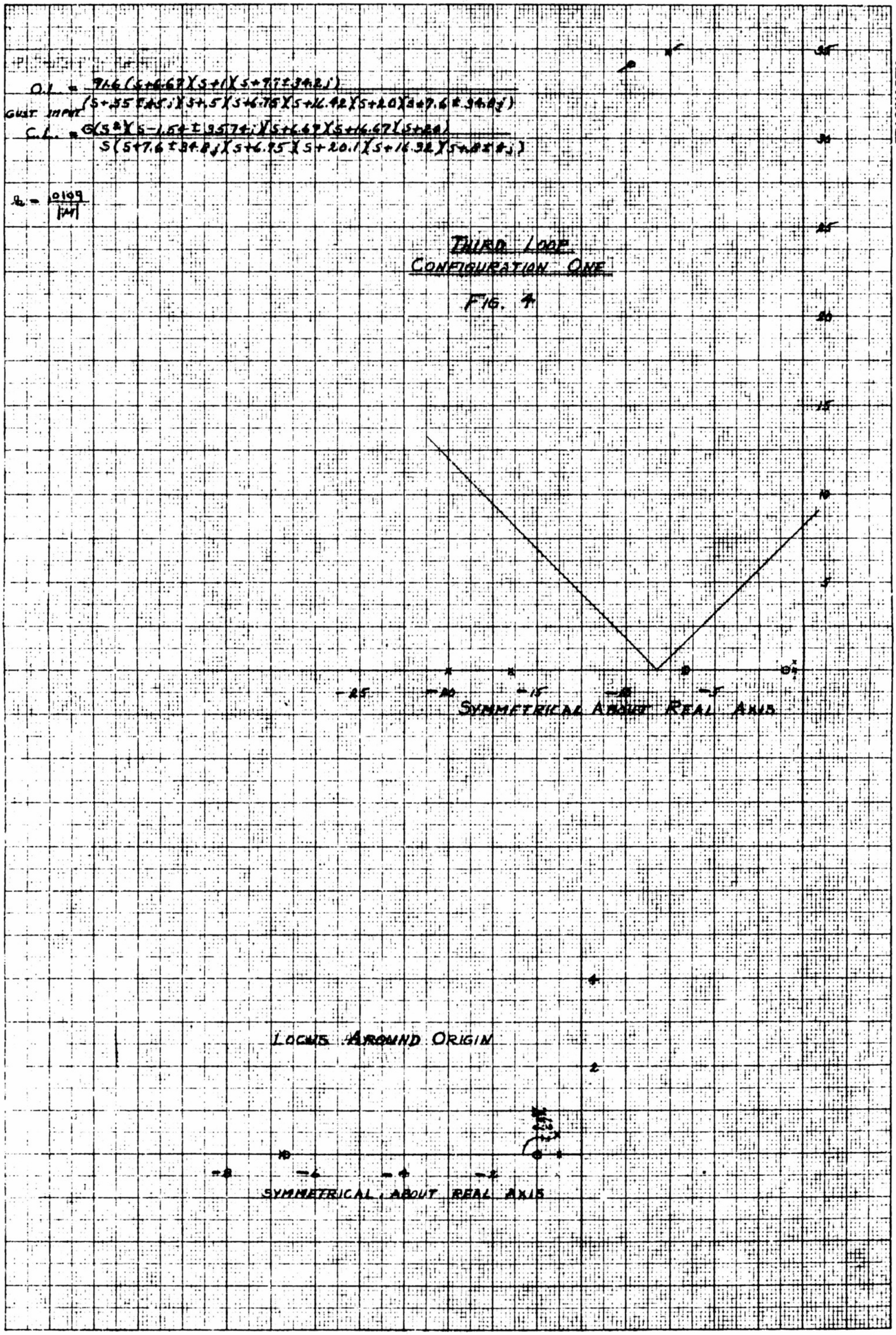
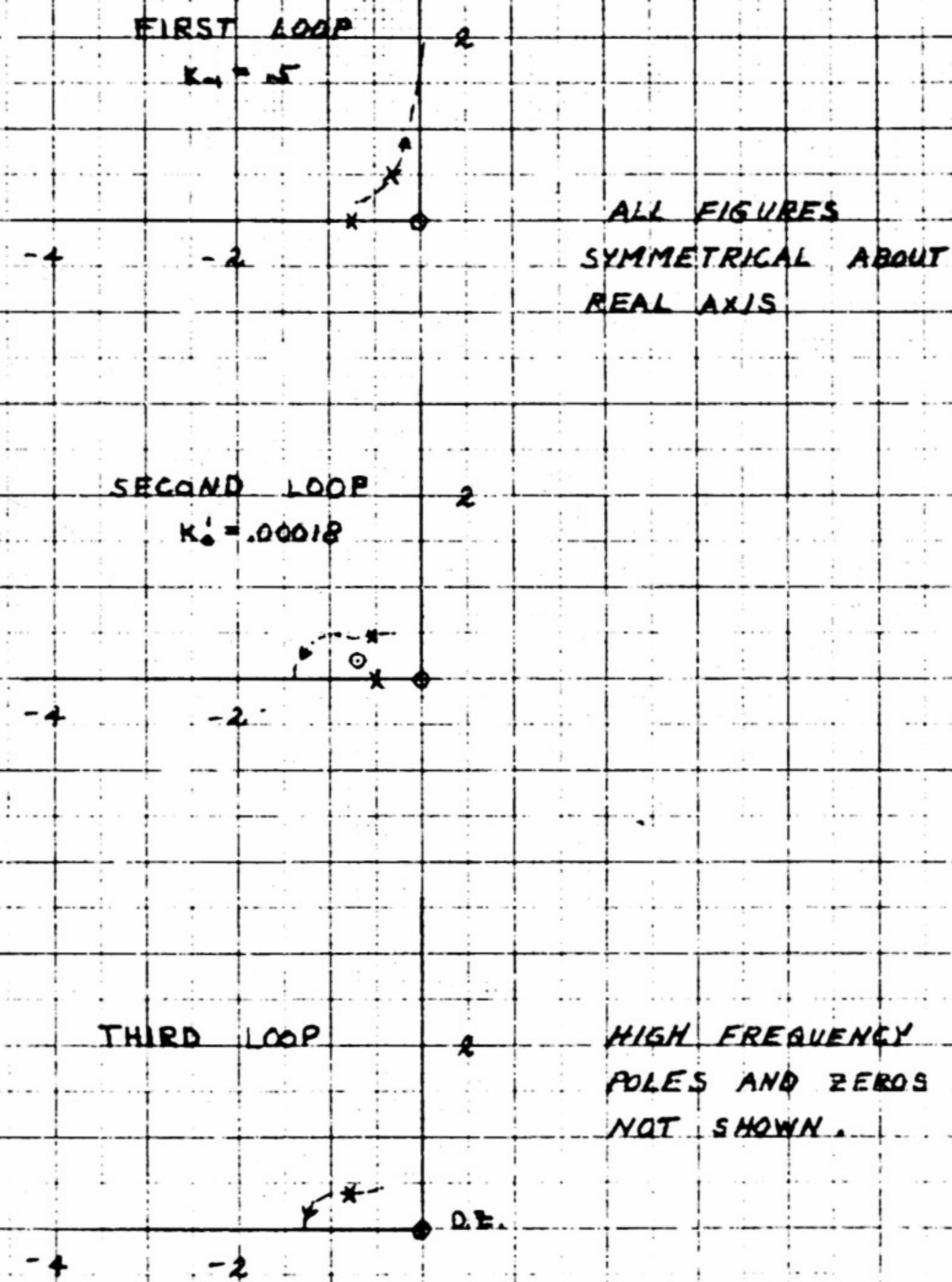

O.L. = 86.55 Kw (S+.83) / S(S+20)(S+.69 ± j.98)(S+6.67)
K = 0.01155 / |M|
FIRST LOOP
CONFIGURATION ONE
-20
-18
-16
-14
-12
-10
-8
-6
-4
-2
SYMMETRICAL ABOUT REAL AXIS

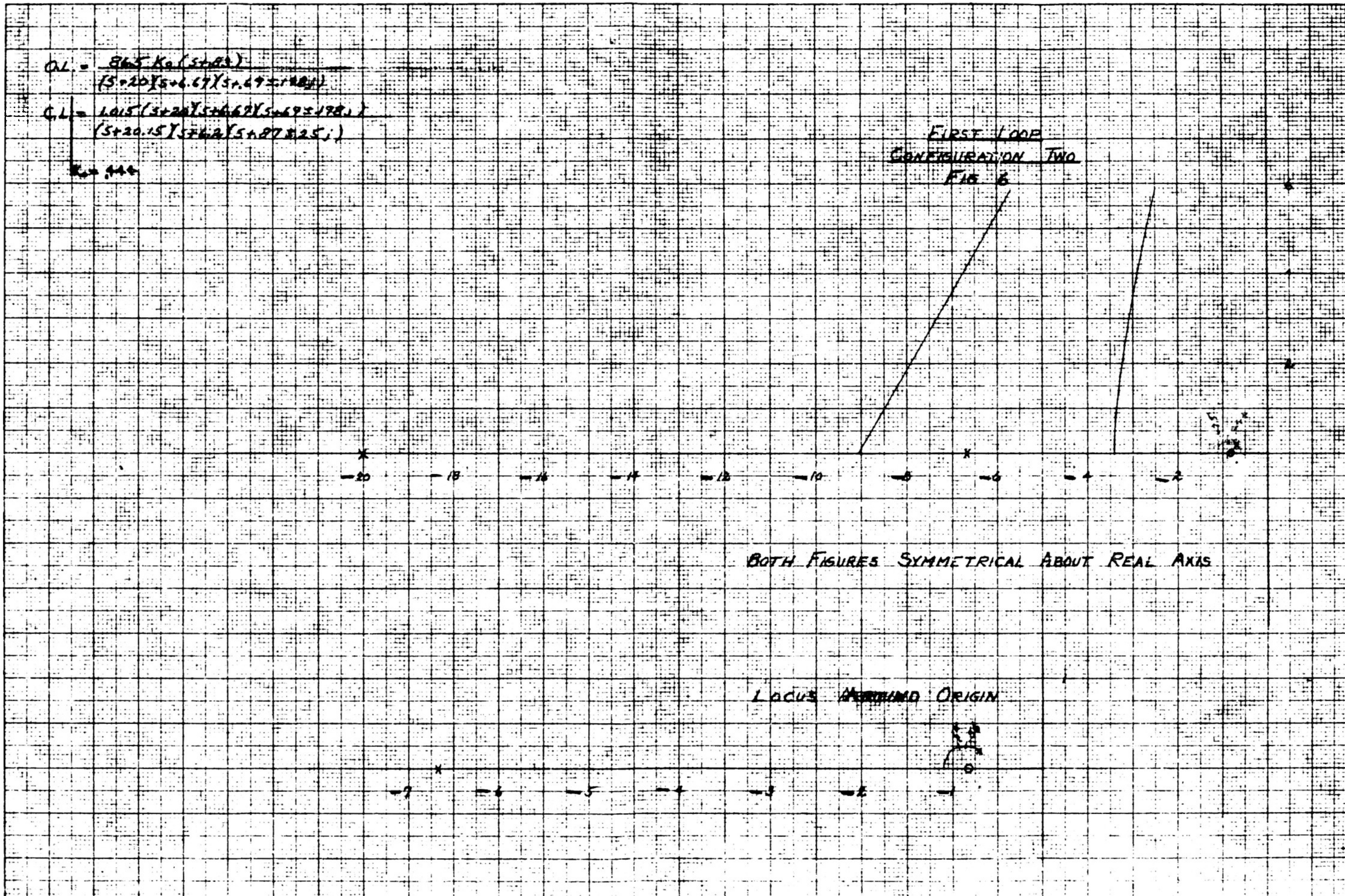
FIG. 2

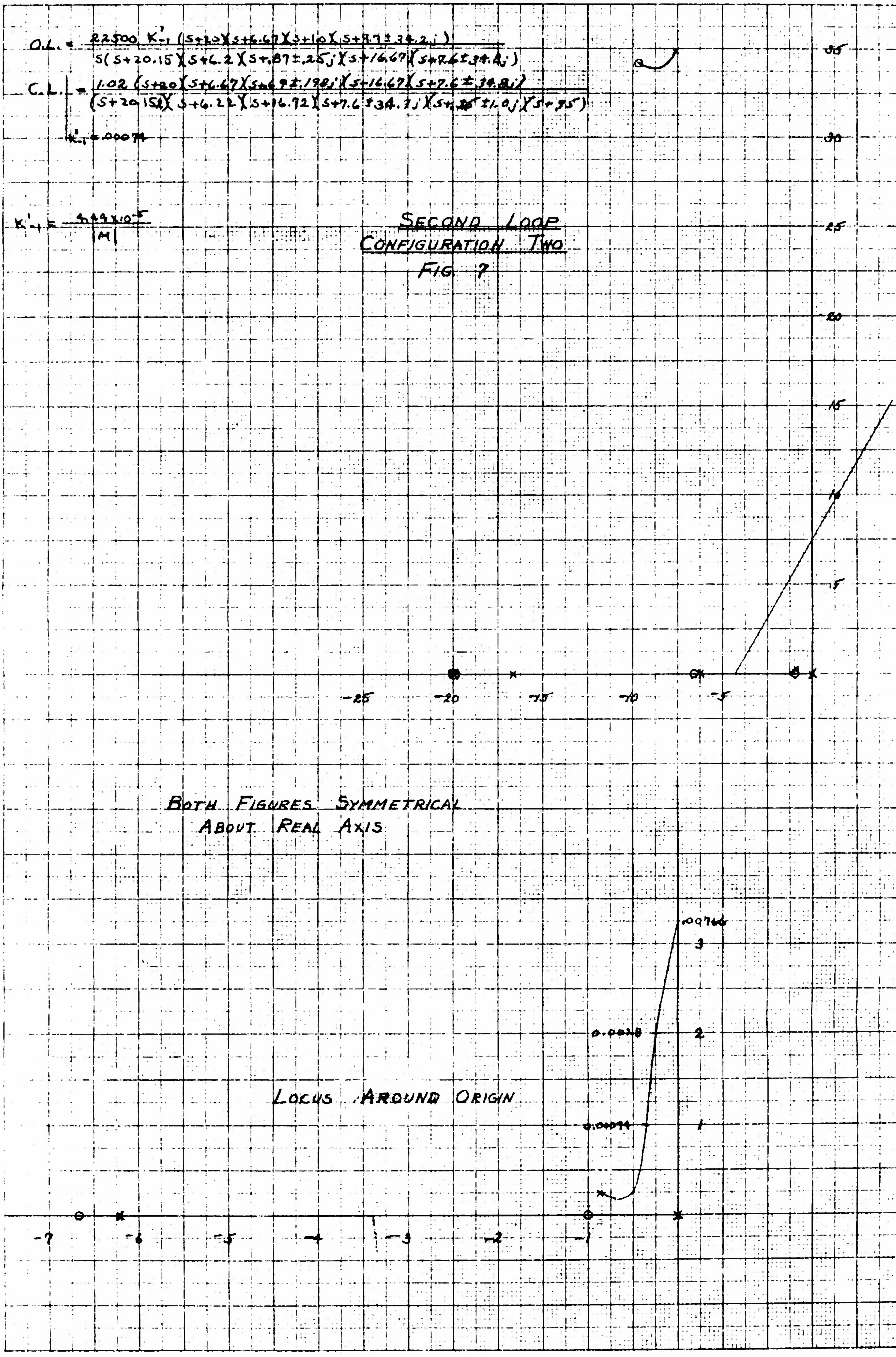

O.L. = 23000 K'o S(S+20)(S+6.67)(S+1)(S+9.7 ± 34.2j) / (S+19.98)(S+6.75)(S+0.75)(S+.9 ± .5j)(S+16.27)(S+7.6 ± 34.8j)
C.L. = S(S+.69 ± .198j)(S+6.67)(S+16.67)(S+9.6 ± 34.8j)
K'o = .00018
K'o = 4.35 x 10^-5 / |M|
SECOND LOOP
CONFIGURATION ONE
FIG 3
35
30
25
20
15
10
5
-25
-20
-15
-10
-5
SYMMETRICAL ABOUT REAL AXIS
4
2
LOCUS AROUND ORIGIN
-8
-6
-4
-2
SYMMETRICAL ABOUT REAL AXIS

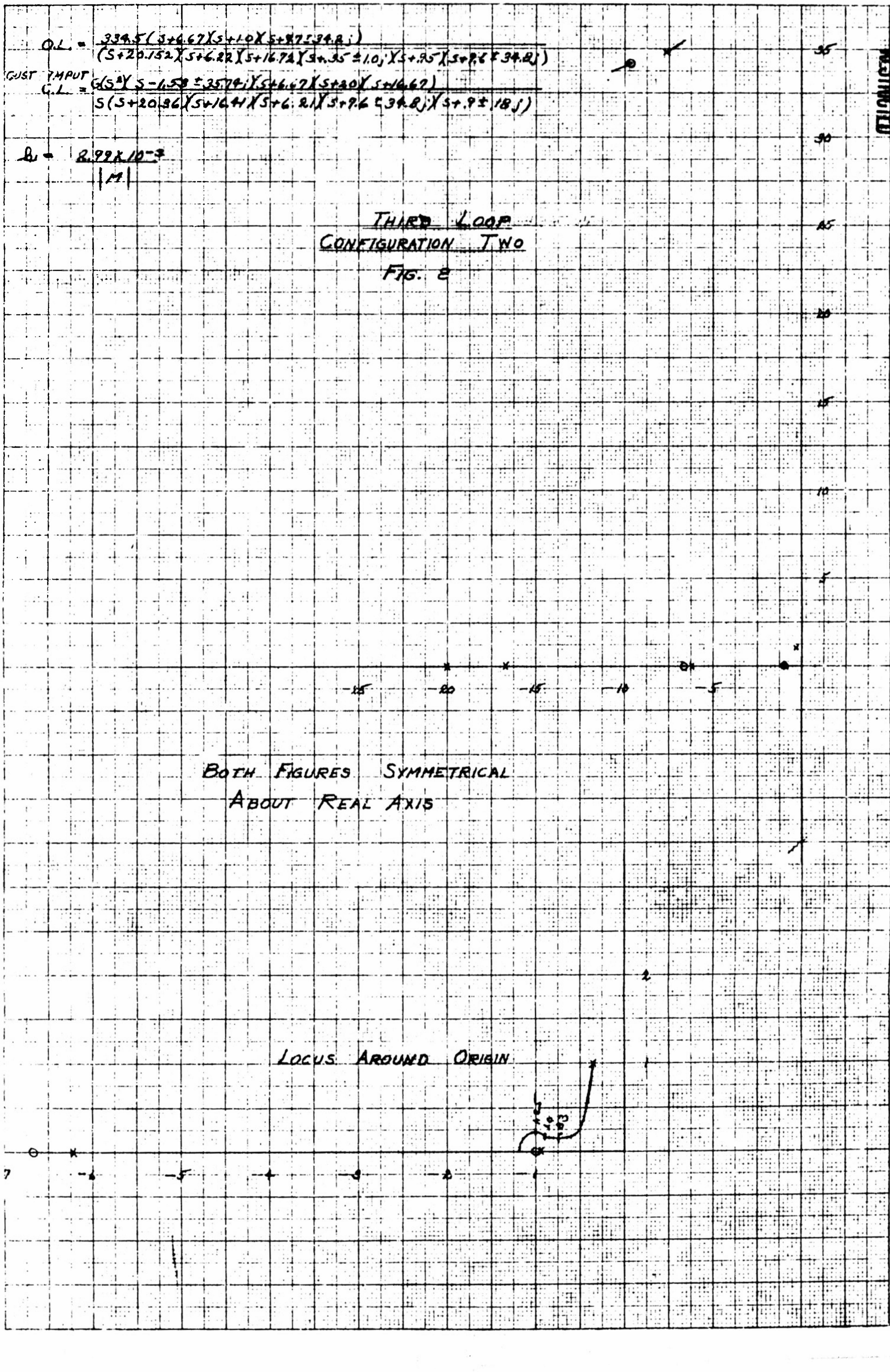

O.L. = 91.6(s+6.67)(s+1)(s+9.7±34.2j) / (s+.35±.45j)(s+.5)(s+6.75)(s+16.42)(s+20)(s+7.6±34.8j)
GUST INPUT
C.L. = G(s²)(s−1.54±9.574j)(s+6.67)(s+16.67)(s+20) / s(s+7.6±34.8j)(s+6.95)(s+20.1)(s+16.32)(s+.8±.8j)
k = .0109/|M|
THIRD LOOP
CONFIGURATION ONE
FIG. 4
35
30
25
20
15
10
5
−25
−20
−15
−10
−5
SYMMETRICAL ABOUT REAL AXIS
4
2
LOCUS AROUND ORIGIN
−8
−6
−4
−2
SYMMETRICAL ABOUT REAL AXIS

ROOT LOCUS SUMMARY
CONFIGURATION ONE
FIG 5



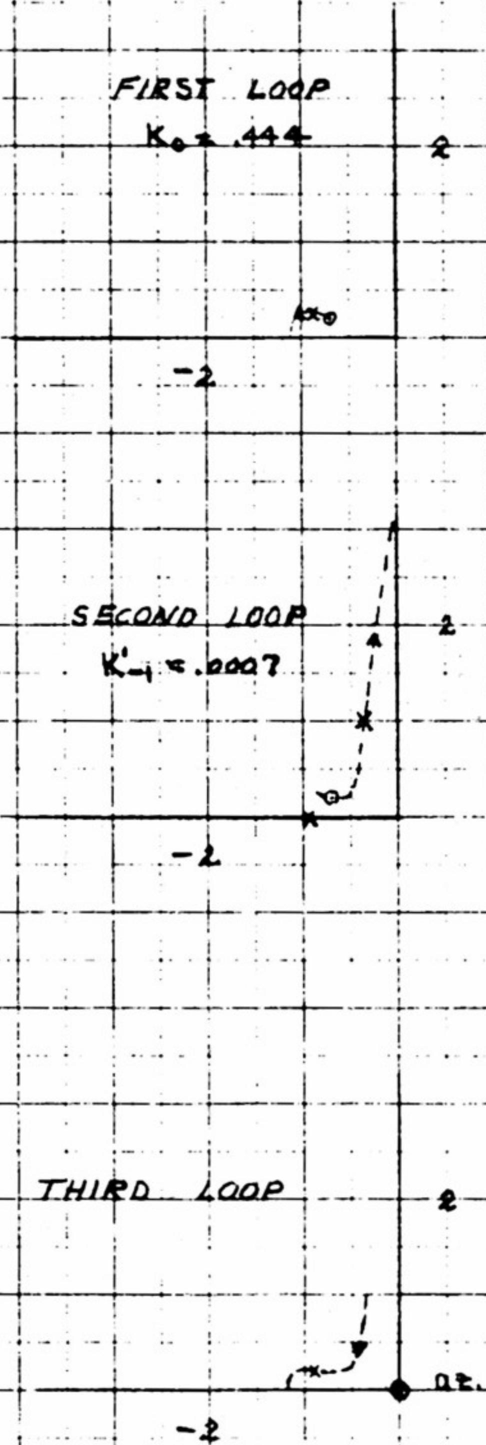

FIRST LOOP
CONFIGURATION TWO
FIG 6
-20
-18
-16
-14
-12
-10
-8
-6
-4
-2
BOTH FIGURES SYMMETRICAL ABOUT REAL AXIS
LOCUS
ORIGIN
-7
-6
-5
-4
-3
-2
-1


O.L. = 22500 K'₋₁ (s+20)(s+6.67)(s+10)(s+8.7 ± 34.2 j) / s(s+20.15)(s+6.2)(s+.87 ± .25 j)(s+16.67)(s+7.6 ± 34.4 j)
C.L. = 1.02 (s+20)(s+6.67)(s+.69 ± .198 j)(s+16.67)(s+7.6 ± 34.8 j) / (s+20.15)(s+6.22)(s+16.72)(s+7.6 ± 34.7 j)(s+[illegible] ± 1.0 j)(s+.95)
K'₋₁ = .00074
K'₋₁ = 3.44 × 10⁻⁵ / |M|
SECOND LOOP
CONFIGURATION TWO
FIG. 7
35
30
25
20
15
10
5
-25
-20
-15
-10
-5
BOTH FIGURES SYMMETRICAL
ABOUT REAL AXIS
.00766
3
0.0038
2
LOCUS AROUND ORIGIN
0.00074
1
-7
-6
-5
-4
-3
-2
-1



THIRD LOOP
CONFIGURATION TWO
FIG. 8

ROOT LOCUS SUMMARY
CONFIGURATION TWO
FIG. 9



ALL FIGURES SYMMETRICAL ABOUT REAL AXIS

HIGH FREQUENCY POLES AND ZEROS NOT SHOWN.

$$O.L. = \frac{29000\, K_1'\, s^2 (s+20)(s+6.67)(s+1)(s+9.7 \pm 34.2j)}{(s+11.99)(s+6.75)(s+.75)(s+.3 \pm .5j)(s+16.67)(s+7.6 \pm 34.8j)}$$

$$C.L. = \frac{973\, s(s+.69 \pm .198j)(s+6.67)(s+16.67)(s+7.6 \pm 34.[illegible])}{(s+.85)(s+6.38)(s+7.6 \pm 34.9j)}$$

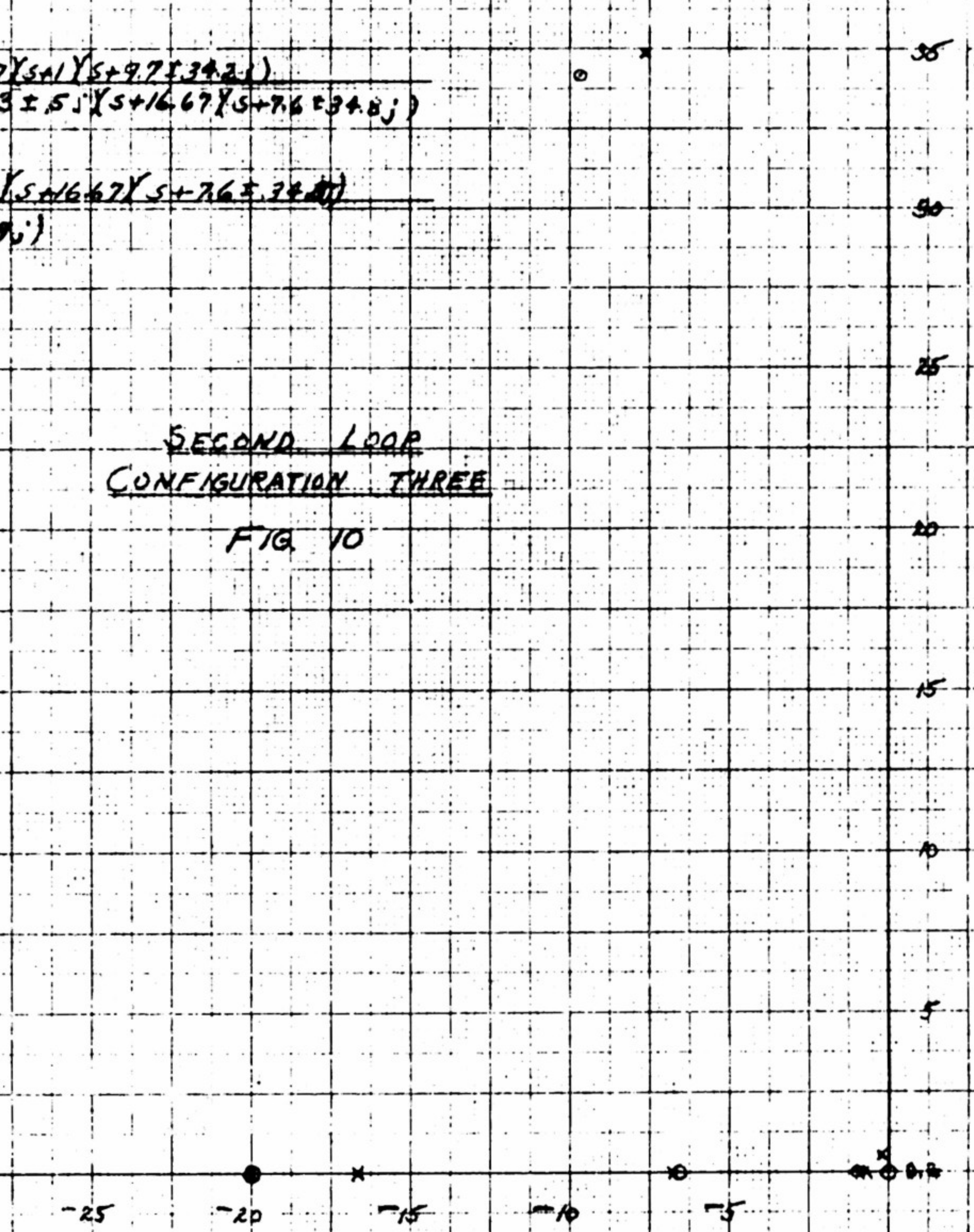
$K_1' = .00034$

$K_1' = \frac{4.34 \times 10^{-5}}{|M|}$

SECOND LOOP
CONFIGURATION THREE

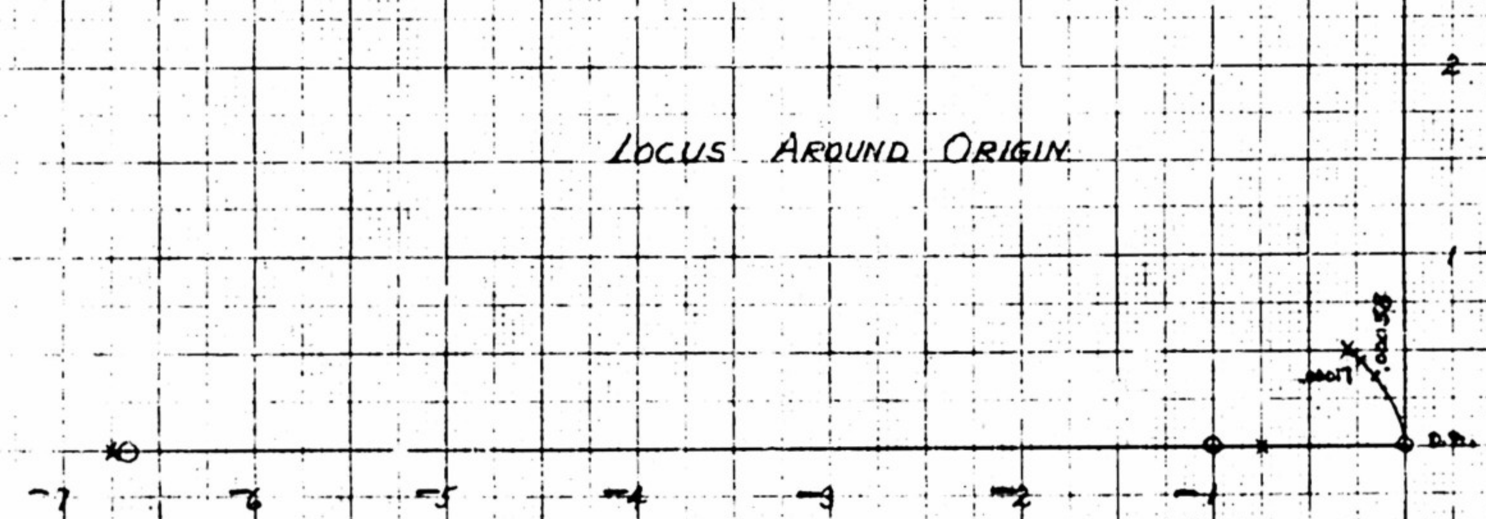
FIG 10

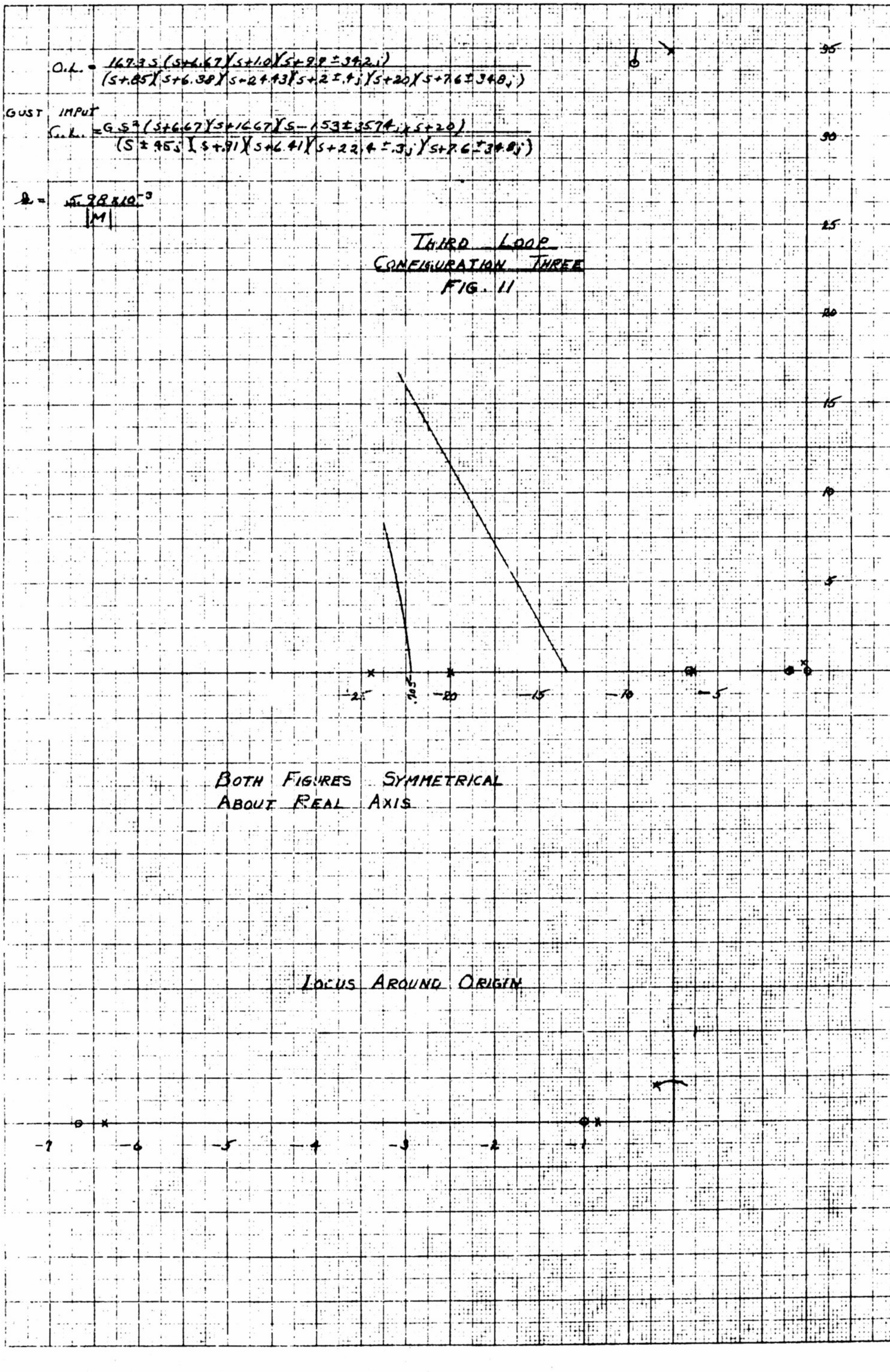
FOR FIRST LOOP
SEE PAGE 15



BOTH FIGURES SYMMETRICAL
ABOUT REAL AXIS

LOCUS AROUND ORIGIN




O.L. = 167.3 S (S+6.67)(S+1.0)(S+9.9 ± 34.2 j) / (S+.85)(S+6.38)(S+24.43)(S+2 ± .4 j)(S+20)(S+7.6 ± 34.8 j)
GUST INPUT
C.L. = G S³ (S+6.67)(S+1667)(S−.153 ± .3574 j)(S+20) / (S ± .95 j)(S+.91)(S+6.41)(S+22.4 ± .3 j)(S+7.6 ± 34.8 j)
k = 5.98 x 10⁻³ / |M|
THIRD LOOP
CONFIGURATION THREE
FIG. 11
35
30
25
20
15
10
5
−25
20.5
−20
−15
−10
−5
BOTH FIGURES SYMMETRICAL
ABOUT REAL AXIS
LOCUS AROUND ORIGIN
−7
−6
−5
−4
−3
−2
−1

ROOT LOCUS SUMMARY
CONFIGURATION THREE
FIG 12