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FORCE BALLISTOCARDIOGRAPHY

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AERO MEDICAL LABORATORY

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Statement A
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ERRATA - February 1954

The following corrections are applicable to WADC Technical Report 52-340, "Force Ballistocardiography," dated November 1952:

Page 2

In the third line of the third paragraph, add "to the measuring system" after the word "network."

Page 7

In the caption for Figure 3, change "chin" to "shin."

In the first line of text below Figure 3, change "chin" to "shin."

Page 8

Change the first line of text to read, "We do not obtain v_B/F_B but the ratio v_B/v_T ."

Page 12

Add to the caption for Figure 8, "I is equivalent to v_B , U is equivalent to F_B ."

Page 15

At the left side of the schematic diagram, change $\bar{R}$ to R_B .

Wright Air Development Center
Air Research and Development Command
United States Air Force
Wright-Patterson Air Force Base, Ohio

WADC TECHNICAL REPORT 52-340

FORCE BALLISTOCARDIOGRAPHY

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November 1952

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Wright Air Development Center
Air Research and Development Command
United States Air Force
Wright-Patterson Air Force Base, Ohio

FOREWORD

This research was conducted in the Bio-Acoustics Section of the Aero Medical Laboratory, Directorate of Research, Wright Air Development Center, Wright-Patterson Air Force Base, Ohio, under RDO 695-63, entitled "Effects of Vibration, Mechanical and Sonic, on Air Force Personnel" with W. W. von Wittern serving as Project Scientist.

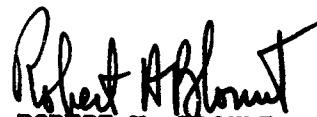
ABSTRACT

This report introduces a method of compensating for the influence of the mechanical properties of the body on the ballistocardiogram. Using this method one obtains a ballistocardiogram in which the recorded amplitudes are directly proportional to the forces produced by heart action which act upon the body mass. These forces can be directly correlated with the heart action. A method for the absolute calibration of this record is presented. It is demonstrated that, using certain reasonable assumptions about the properties of internal mechanical network of the body, the ballistocardiogram could be produced by a simple impulse acting only during systole.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDING GENERAL:



ROBERT H. BLOUNT
Colonel, USAF (MC)
Chief, Aero Medical Laboratory
Directorate of Research

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INTRODUCTION

The ballistocardiogram (BKG) is a record of the oscillatory body motion which occurs with every heart beat and is therefore attributed to forces produced by the heart action. Usually the component parallel to the longitudinal body axis of the deflection (D) or velocity (V) is recorded when the body is either lying free on a rigid surface (Dock (2) method) or fixed on an elastically suspended table (Starr (1) method). In the first case, the body is movable owing to the elasticity of the external tissues, and deflection or velocity of various points of the body surface is recorded. In the second case deflection or velocity of the table is recorded. These records are usually obtained by means of electrical systems.

In each case a force produces the body motion. Only the force is directly connected with the heart action whereas the motion of the body or the table depends on the impedance of the mechanical system as well as on the force. (Impedance is the complex ratio: force/velocity in a steady state of harmonic oscillation). It will be shown later that the influence of the body network on the time course of the body motion can be large. A certain time function of the force can produce almost any time function of the motion, depending on the mechanical network between force and motion. Unequivocal conclusions about the heart action can therefore be drawn from the force record only.

Because we can "pick up" only the body motion the problem is to determine from the function of the motion the magnitude and time course of the force. One method of solving this problem is to place in the recording system an electrical network which compensates for the influence of the body mechanical network so that one obtains a force record directly. This is possible when the body network is known. This network is the mechanical system composed of the body masses and the elasticities and dampings which connect these masses to each other and to the surface on which the body lies.

The body mechanical network can be separated for study into two parts:

- (1) The "external network", formed by the masses of the body parts (head, legs, arms, trunk) and the elasticities and dampings which connect these parts to each other, to the skeleton, and to the surface on which the body lies. In the Starr method the mass of the table plate and the elasticity and damping of its suspension can also be treated as components of the external network.
- (2) The "internal network" which as a first approximation may be assumed to be formed by the masses of the heart and liver and the elasticities and dampings which connect them to each other and to the skeleton.

The division of the body mechanical network into these two parts is justified because (a) the coupling between the two systems is certainly very weak and the combined action of the two networks can therefore be obtained by the product of the actions of both, and (b) the external network can be determined by relatively simple measurements.

The force F_H acting upon the heart is transmitted through the internal network producing the force F_B which acts upon the external network and causes the body motion (velocity V_B). F_B is F_H as altered by the action of the internal network and V_B is produced by F_B and depends on the action of the external network. The amplitude H of the record depends on V_B and on the action of the measuring and recording system.

We shall call:

The complex ratio F_B/F_H as a function of frequency (in the following we will use the complex vector presentation of oscillation problems): "The frequency response characteristic of the internal network (FRC_I)"

The complex ratio V_B/F_B as a function of frequency: "The frequency response characteristic of the external network (FRC_B)"

The complex ratio H/V_B as a function of frequency: "The frequency response characteristic of the measuring system (FRC_M)".

Because the energy transfer from the internal to the external network can be assumed to be relatively small and the energy transfer from the external network can be made negligible by its design, the "overall" frequency response characteristic (FRC_O) is obtained by multiplication of the frequency response characteristics of the components. Then:

For H/F_B is obtained $FRC_O = FRC_B \cdot FRC_M$

For H/F_H is obtained $FRC_O = FRC_I \cdot FRC_B \cdot FRC_M$

F_B and F_H are periodic time functions. They contain the same frequencies and their fundamental frequency is the heart rate. Both functions can be described by a Fourier series which is a sum of sine functions in our case consisting of the frequency of the heart beat as the lowest frequency and its integer multiples (harmonics). The amplitude (H) of the record is a true picture of the force only if FRC_O is constant in the range between fundamental and highest harmonic of the time function of the force.

It was possible to measure FRC_B and the result shows that it agrees for the average man with the resonance curve of a simple oscillator formed by the total mass of the body and by the elasticity and damping of the external tissues which connect it to its support. By inserting into the recording system a "correcting network" adjusted to produce an FRC_M proportional to $1/FRC_B$ we obtain a flat FRC_O between H and F_B and a record of F_B .

This record gives more information about the heart action than a record of the motion of the body because the influence of the external network is eliminated. Though the final goal is the determination of F_H by eliminating also the influence of the internal network we shall confine ourselves, first, to the determination of F_B using the Dock method (subject lying on his back on a rigid supporting plane) since this method possesses the simplest mechanical conditions.

We call:

The record of the body displacement: Displacement-ballistocardiogram (D-BKG);

The record of the body velocity: Velocity-ballistocardiogram (V-BKG);

The record of the force F_B : (F_B -BKG)

The record of the force F_H : (F_H -BKG)

The usual ballistocardiograms therefore correspond to D-BKG's and to V-BKG's when they are recorded with a measuring device having a flat FRC_M and when the load presented by the pick-up is negligible.

The V-BKG is the D-BKG differentiated with respect to time and therefore can show no basically new facts in contrast to the F_B -BKG which is produced by the application of the operator of the external network on the V-BKG and to the F_H -BKG produced by the application of the operator of the internal network on the F_B -BKG.

Since the F_B -BKG and F_H -BKG are records of forces, they can be calibrated in units of force (dyne or gram) while the D-BKG can correctly be calibrated only in units of displacement (cm) and the V-BKG in units of velocity (cm sec⁻¹).

SECTION I

THE MECHANICAL BODY NETWORK AND THE DETERMINATION OF FRC_B

The difficulty in describing the body as a system formed by point masses and elasticities lies in the fact that the body is not really such a system but one with more or less continuously distributed masses and elasticities. The representation of the body as a system of point masses and elasticities is therefore a simplification. However, the lower the frequencies to which the system is applied the better it represents the actual conditions. We shall, therefore, attempt to represent the body for the low frequencies which characterize the forces of the heart action by a relatively simple mechanical network formed by point masses and elasticities. We shall start with a highly simplified mechanical circuit (Fig. 1A) representing the body lying on a rigid, plane surface. We shall consider only components of force and motion parallel to the longitudinal axis of the body and assume that the force acts upon the mass of the trunk. We shall try to determine whether the simplifications of the network are justified and whether still further simplifications are permissible.

The network in Figure 1A contains the masses M_H (head), M_T

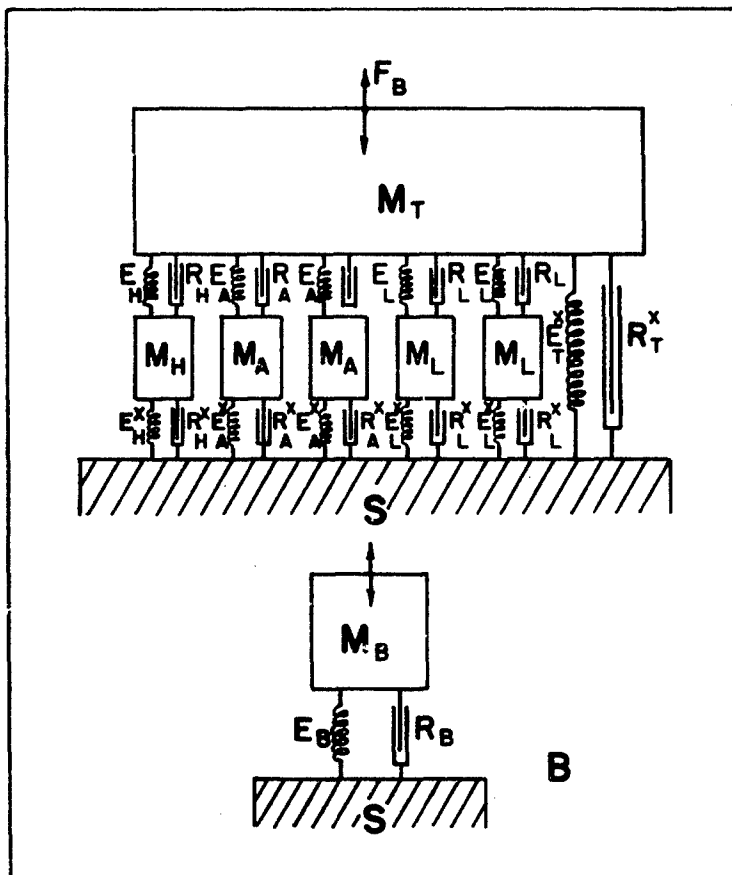


Figure 1. Simplified mechanical network of the body lying on the rigid surface S (A) and its ultimate simplification for low frequencies (B).

(trunk), M_L (legs), M_A (arms), connected to each other by "springs" with the compliances E_H , E_L , E_A , and the damping resistances R_H , R_L , R_A and also connected to the rigid support, S , of the body by means of springs with the compliances E_H^* , E_L^* , E_A^* , E_T^* , and the resistances R_H^* , R_L^* , R_A^* , R_T^* . The impedance, $\bar{R}_M$, of a mass, M , is proportional to the force required to vibrate this mass sinusoidally at unit velocity and equals: $\bar{R}_M = j2\pi fM$ where $j = \sqrt{-1}$ indicates a $+90^\circ$ phase shift between velocity and force and f = frequency. $\bar{R}_M$ increases with frequency. The impedance, $\bar{R}_E$, of a spring is proportional to the force required to expand and compress the spring sinusoidally at unit velocity and equals $\bar{R}_E = 1/j2\pi fE$ where E = compliance of the spring (deflection/force) and $1/j = -j = -\sqrt{-1}$ indicates a -90° phase shift between velocity and force. This impedance decreases with frequency. At a certain frequency the impedances of a given spring-mass system (e.g. M_H , E_H) will be equal. This is the resonance frequency f_0 of the system. For frequencies $f \ll f_0$ the impedance of the spring will be so much larger than the impedance of the mass, that the spring can be considered to represent a rigid connection. Therefore, for frequencies below a certain limiting frequency all springs which interconnect the parts of the body can be considered as rigid connections and all the masses of the body can be combined into one total rigid mass, M_B , provided the impedances of the respective masses are large in comparison to the impedances which connect them with the support. Then the impedances connecting the body to the support can be combined into only one spring with the compliance E_B and one resistance R_B . The resulting mechanical circuit (Fig. 1B) is an oscillator with one degree of freedom. Its FRC, which represents now the FRC_B of the body, is well known and can be represented by the formula:

$$FRC_B = \frac{V_B}{F_B} = \frac{1}{\bar{R}_B} = \frac{1}{j\omega M_B + 1/j\omega E_B + R_B}$$

where $\omega = 2\pi f$
 $F_B =$ amplitude of the force acting upon M_B

$V_B =$ amplitude of the velocity of M_B

$\bar{R}_B =$ impedance of the body system

introducing $x = f/f_B$ (f_B = resonance frequency of the body system)

and the damping constant $k = R_B/\omega_B M_B$

we obtain for the absolute value the equation in dimensionless form:

$$\left| \frac{V_B}{F_B} \right| R_B = \frac{|V_B/F_B|}{|V_B/F_B|_{x=1}} = \frac{k}{\sqrt{(x - \frac{1}{x})^2 + k^2}}$$

The diagram in Figure 2 shows a family of curves of this function with

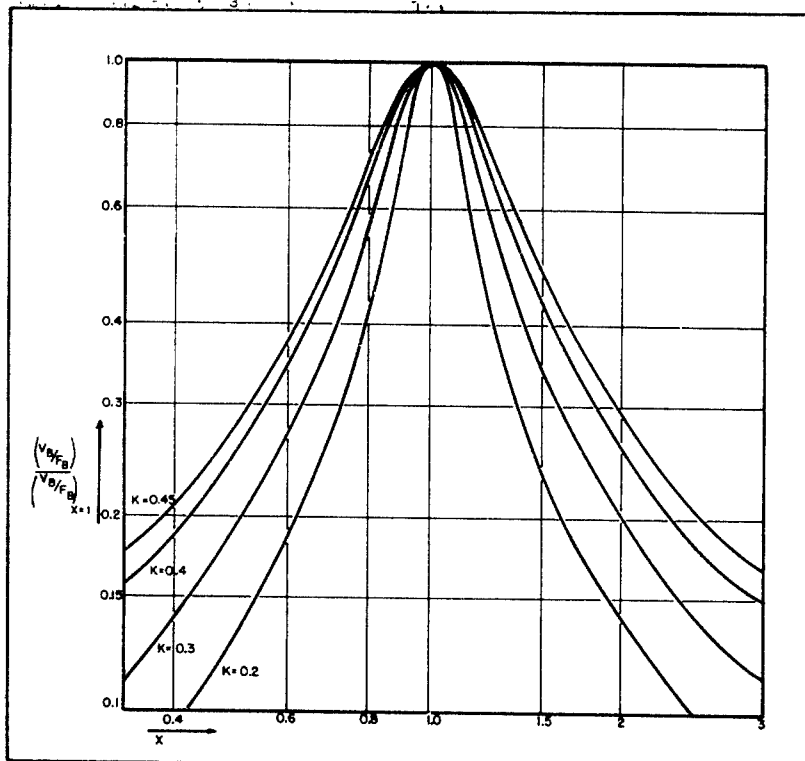


Figure 2. Frequency response characteristic, of the simple oscillator shown in Figure 1 excited by a force acting upon the mass of the system. Parameter: damping constant k .

the parameter k . If we measure the FRC_B of the body in the frequency range of interest and find that the measured function agrees with one of these theoretical curves, it is proof that the body system can be represented by the mechanical circuit of Figure 1B.

To measure FRC_B known external sinusoidal forces must be applied to the body and the velocity V_B imparted to it by these forces must be measured as a function of frequency. The problem is how and where to apply these forces. From Figure 1A it can be concluded that the particular FRC_B obtained will depend upon the place where the force is applied and also upon the place where the velocity is

measured. When the force, for instance, is applied upon M_L , it must travel over E_L and R_L in order to move M_T . Because M_T is much larger than M_L , it is obvious that the application of a certain force upon M_L will cause a larger deflection of E_L than the application of the same force upon M_T . While M_L and M_T might move with practically the same amplitude when the force is applied upon M_T , they will move with different amplitudes when the force is applied upon M_L .

For the measurement of FRC_B , the force must therefore produce the same pattern of motion of the various body parts as is produced by the heart action. If the V-BKG's, shown in Figure 3, which were taken

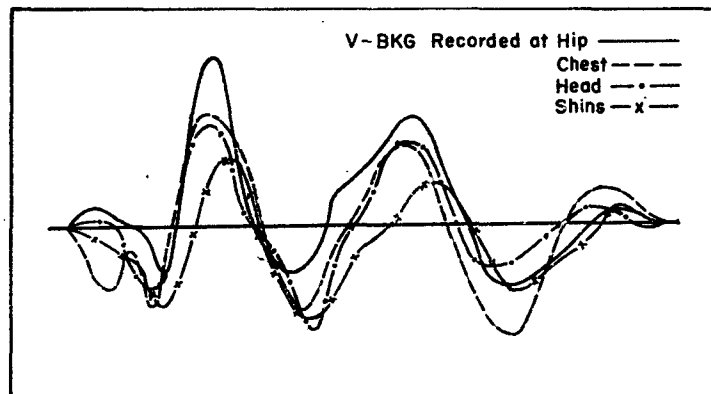


Figure 3. recorded at different points on the body:
 — Hip
 --- Chest
 - - - Head
 - x - Chin

at various parts of the body (head, chest, hip, chin) are compared, one may conclude that all these parts move, to a first approximation, with equal amplitude and phase throughout the frequency range found in these V-BKG's (according to a frequency analysis 1 - 10 cps). When we consider the fact that the records do not represent the same heart beat, but were taken one after the other, and that the connection of the "motion pick up" with the body may not have been equal at the different locations owing to different mechanical qualities of the external tissues, the approximation may be even better.

A way to apply measuring forces which move the parts of the body in the same pattern was required. It was found that such a pattern resulted when a subject was placed on a table plate which oscillates in the longitudinal axis of the body (shake table). The first experiments in this direction were performed at the School of Aviation Medicine, Randolph Field, Texas in 1949 in collaboration with K. R. Reissmann). In this case we have, however, a different kind of mechanical circuit. While the heart action excites the system by a force, F_B , applied to its mass, M_B , (Fig. 4A), by the shake table a certain velocity, V_T , is imparted to the system through the spring and damping. ($V_T = a_T \omega$, where a_T amplitude of the displacement of the shake table).

We do not obtain V_B/F_B but the ratio V_B/F_T . It is, however, possible to determine from this ratio as a function of frequency the constants f_B and k of the system and with these constants we can find FRC in Figure 2.

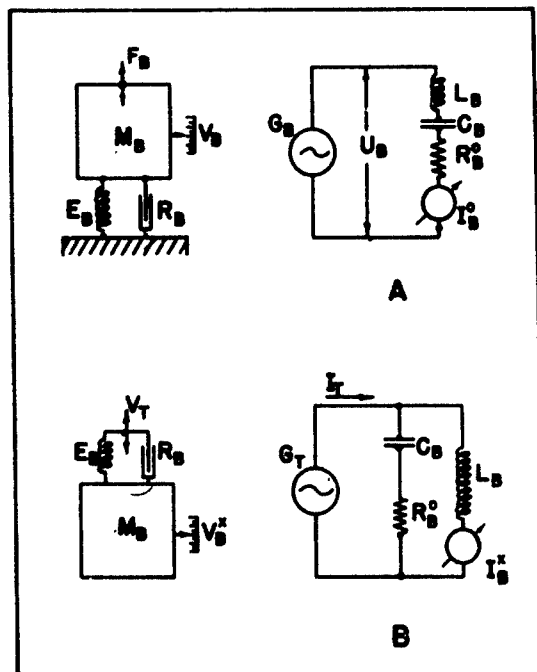


Figure 4. Mechanical circuits and their electrical equivalent.

- A. The system of the body excited by a force (F_B) acting upon the body mass (M_B).
- B. The system of the body excited by a velocity (V_T) imparted through the spring (E_B) and damping (R_B) of the system.

The force, F_B , is equivalent to the voltage, U_B .
 The velocity, V_B , is equivalent to the current, I_B .
 The velocity, V_T , is equivalent to the current, I_T .
 The mass, M_B , is equivalent to the inductance, L_B .
 The compliance, E_B , is equivalent to the capacity C_B .
 The damping, R_B , is equivalent to the ohmic resistance, R_B .

For the ratio V_B/V_T we obtain:

$$\frac{V_B}{V_T} = \frac{1/j\omega E_B + R_B}{1/j\omega E_B + j\omega M_B + R_B}$$

and introducing x and k we obtain for the absolute value:

$$\left| \frac{V_B}{V_T} \right| = \sqrt{\frac{k^2 + 1/x^2}{k^2 + (x - 1/x)^2}}$$

Figure 5 shows $|V_B/V_T|/|V_B/V_T|_{x=1}$ as a function of x calculated for various values of k .

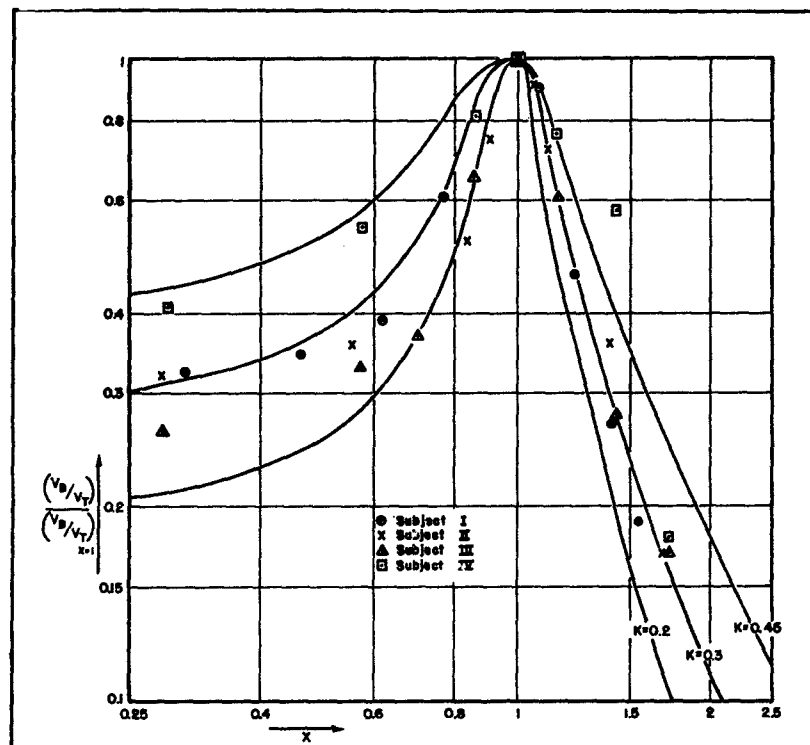


Figure 5. Frequency response characteristic, $|V_B/V_T|/|V_B/V_T|_{x=1}$, of the simple oscillator shown in Figure 1B excited by a velocity, V_T , imparted through elasticity and damping of the system (compare Figure 4B). Parameter: damping constant k ; Solid lines: calculated values; Values measured from various subjects indicated by: $\circ$, $\times$, Δ , $\square$.

Because $V_B/V_T = D_B/D_T$ for a harmonic oscillation and because the shake table works with constant deflection D_T as a function of frequency, not V_B but the deflection of the body, D_B , was measured and D_B as a function of frequency is proportional to the desired ratio V_B/V_T . The apparatus used for the measurement of V_B/V_T is described in Appendix IIa.

Figure 6 shows the record of the body displacement, D_B , of subject No. 1 caused by the shake table at different frequencies. From the mean values of this record the $|V_B/V_T|/|V_B/V_T|_{x=1}$ values in Figure 5 were determined and by comparing them with the theoretical curves we find $k = .3$. $f_B = 3$ cps was determined by extrapolation of the maximal amplitude from the record in Figure 6.

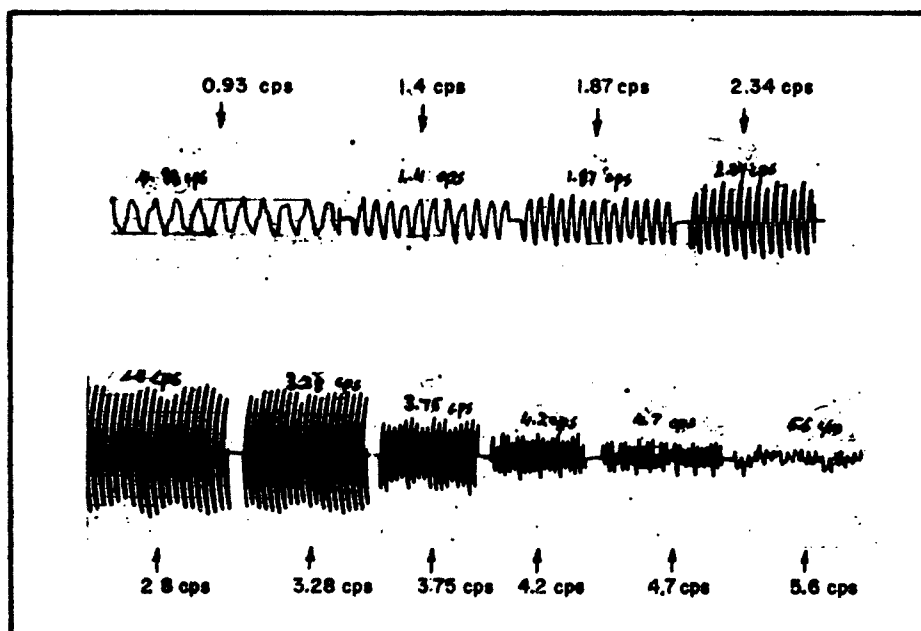


Figure 6. Record of the deflection of the body on the shake table for different frequencies and constant deflection of the shake table.

At times this record shows periodic fluctuations of the amplitude, especially at frequencies $f \geq f_B$. These fluctuations indicate a periodic change of the mechanical body impedance which may account for periodic changes sometimes observed in the BKG. The cause of these fluctuations might be an involuntary tension change of the subject. No connection with the breathing period was found.

In Figure 5 the $|V_B/V_T|/|V_B/V_T|_{K=1}$ values of other subjects are also indicated. Their agreement with the theoretical curves shows that the representation of the body by a simple oscillator is in first approximation justified. With the K values found we can determine in Figure 2 the FRC_B of each subject.

Table 1						
Subj. No.	Size	Weight	Body Build	f_B (cps)	K	$\hat{F}_B$ (gr)
1.	6 ft. 2 in.	70 kg	Tall, slender	3,1	0,3	286
2.	5 ft. 7 in.	63 kg	Slightly less than medium build	3,4	0,3	315
3.	6 ft. 1 in.	85 kg	Tall, moderately heavy	3,3	0,25	540
4.	5 ft. 9 in.	75 kg	Muscular, medium	3,3	0,4	320

Table 1 shows that there is no definite connection between the body build and f_B and k in the case of our more or less "normal" subjects and that the variations of f_B and k with the various subjects are surprisingly small. Slight changes of the position of the subject can produce larger changes of f_B and especially k than appear between different subjects, especially when the external tissues become stressed. Subsequent measurements with the subject having left the table in the interim show different results. To obtain more constant and reproducible conditions, a layer of foam rubber, F (Fig. 7) was placed between the subject and the plate, T , of the shake table, the head was suspended in a free swinging hammock, H , and the arms were stretched upward while the hands gripped a fixed rod, R , at two pieces of insulated metal tubing which served at the same time as EKG electrodes. The suspension of the head was used because the external tissues of the head are relatively stiff and the unsupported head sometimes tends to roll on the table. The stretched up position of the arms diminishes the possibility of changes of position. The layer of foam rubber was used to eliminate the influence of small changes of position on the mechanical body impedance and changed the elasticity of the body system very little.

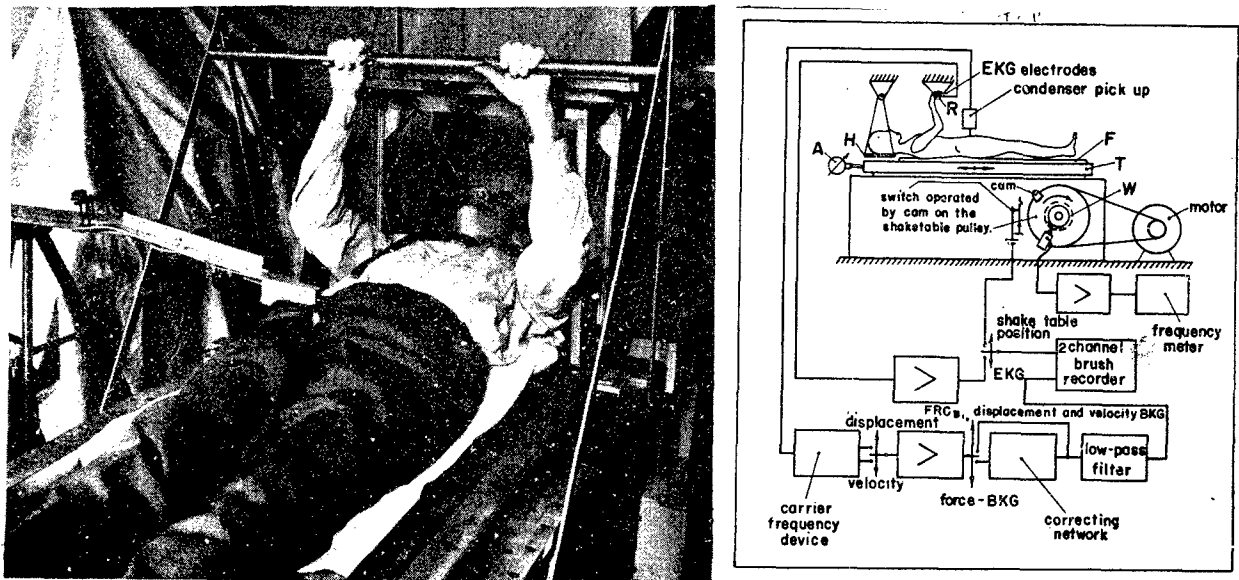


Figure 7. Arrangement of the measuring apparatus for the determination of the FRC_B and for the recording and calibration of the BKG's.

SECTION II

THE INFLUENCE OF THE FRC_B OF THE BODY ON THE BKG .

It is instructive to investigate the response of a system having the measured FRC_B of the body to a force of a certain function of time. This problem can be solved in a relatively simple manner by means of an electrical network with an FRC_{el} proportional to the FRC_B of the body system. This method is equivalent to the solution of the problem by means of an electric analogue computer. The mechanical network of the body and its equivalent electrical circuit are shown in Figure 4A. The FRC_B of the mechanical network, $FRC_B = V_B / F_B$, will be proportional to the FRC_{el} of the electrical network, $FRC_{el} = I_B / U_B$, when the damping constants are equal.

By means of a photoelectric voltage generator (see Appendix III) a simply shaped voltage impulse was applied periodically to the network. The frequency of this period was made $1/3$ of the natural frequency of the electrical circuit since the frequency of the heart beat is $1/3$ of the natural frequency of the body system. The oscillograms (Fig. 8) of the current, I recorded with different dampings, represent the velocities of the body mass under the condition that the force applied by every beat of the heart had the time course of the voltage impulse. U

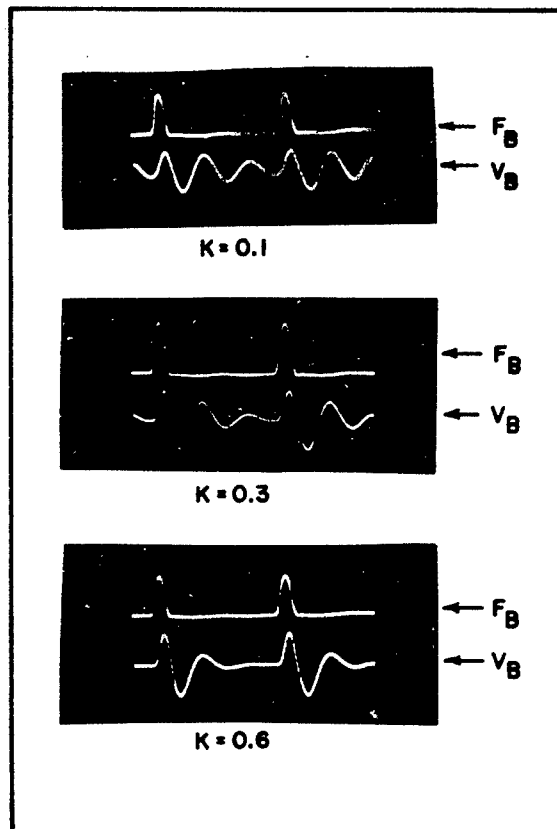


Figure 8. Time function of the current (I) produced by the voltage (U) in an electrical system equivalent to the mechanical system of the body (see Fig. 4A).

The oscillograms are similar to the typical V-BKG or D-BKG recorded by the usual methods which indicates that they could have been produced by only one very simply shaped impulse of force and we see that it is not justified to explain a certain peak or dip of these BKG's as being caused by the occurrence of a certain event in the blood circuit (e.g. expulsion of the blood from the heart, acceleration effects of the blood flowing through the aortic arch etc.).

The D- and V-BKG seem to represent more or less the transient response of the mechanical body system excited by a simply shaped periodic impact. This idea is also suggested when we observe the agreement of a typical FRC_B of the body with the frequency spectra of V-BKG's in Figure 9. Two spectra are shown, the one analyzed

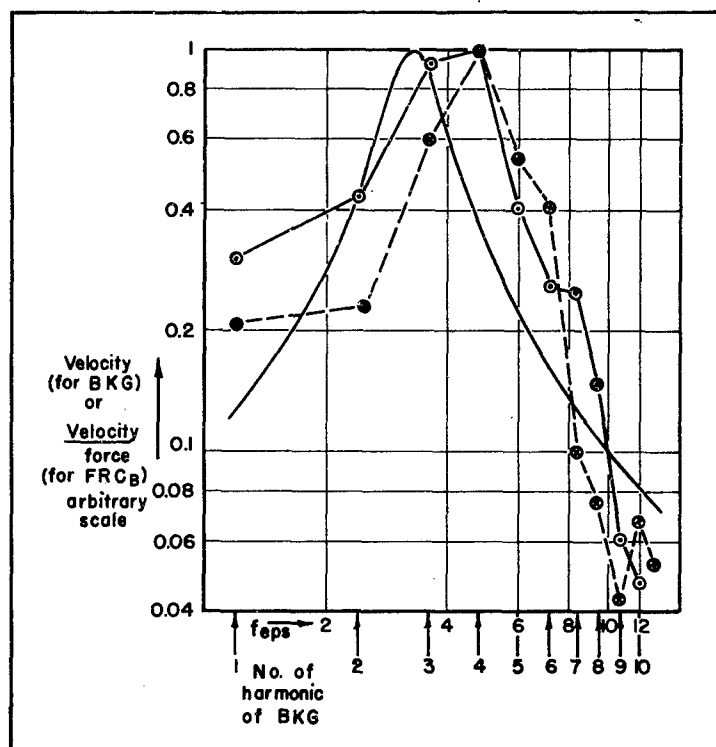


Figure 9. Frequency spectra of V-BKG's and FRC_B .
 —○— Spectrum of a V-BKG taken from a medical journal.
 - - -○- Spectrum of own V-BKG records. Mean values of five subjects.
 — Average FRC_B .

from a record taken from a medical journal, (3), the other representing the mean values of spectra analyzed from five of our own records. (Method described in Appendix III). The maximal amplitudes of the spectra are arbitrarily set to equal 1 and the other harmonics are calculated accordingly.

It is probably because of the effect of the FRC_M that frequencies higher than 10 cps are not present in these V-BKG's. The FRC_M in the case of our measurements was flat up to 20 cps. If, as it is often affirmed, the acceleration and deceleration of the blood, due to the curvature of the aortic arch, have measurable effects, separated from each other and from the effects of expulsion of blood from the heart, they should be separated by a time interval of the order of $1/50$ sec. (distance heart - aortic arch ~ 10 cm, velocity of the pulse wave in the aorta ~ 500 cm/s). Only the change of the velocity of the blood can cause a reaction force and this change represents the pulse wave. The force, F_H , acting upon heart and aorta should then contain frequency components up to 50 cps. These effects can only be discovered when a system having a FRC_0 flat up to this frequency is used. In any case only by means of a spectrum of the F_H -BKG recorded with a system having a FRC_0 known to be flat up to higher frequencies than the highest frequency found in this spectrum, can it be determined whether the complete spectrum has been recorded.

SECTION III

THE F_B -BKG RECORDED WITH THE CORRECTING NETWORK

A. Principle of the Measurement:

The overall FRC_o for the recording of the F_B -BKG is the product of the FRC_B of the body and the FRC_M of the measuring system. If by means of a filter network (correcting network) in the measuring system an FRC_M proportional to $1/\text{FRC}_B$ is produced, the FRC_o will be flat and the record will represent the force F_B . Figure 10 shows the principle of an electrical recording system of this kind.

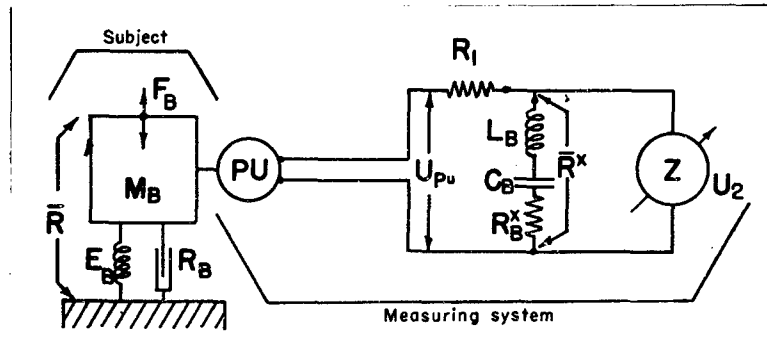


Figure 10. Electro-mechanical circuit of subject and measuring system for the recording of F_B by means of a correcting network, which produces an output voltage, U_2 , proportional to the acting force, F_B . The input voltage, U_{PU} , is produced by the velocity pick-up PU and is proportional to the velocity of M_B .

The output voltage, U_{PU} of the electrical velocity pick up is proportional to the velocity of the body mass, V_B :

$$U_{PU} = C_1 V_B$$

Because: $V_B = F_B / R_B$ we obtain $U_{PU} = C_1 \frac{F}{R_B}$

where: $\bar{R}_B = j\omega M_B + 1/j\omega E_B + R_B$ = impedance of the body system.

The voltage, U_{PU} , drives the current I ,

$$I = U_{PU} / R_1$$

(because $R_1 \gg \bar{R}^*$)

through the impedance of the correcting network $R_1 + \bar{R}^*$,:

and produces the voltage, U_2 , across $\bar{R}^*$:

$$U_2 = \frac{U_{pv}}{R_1} \bar{R}^* = C_1 \frac{F_B}{\bar{R}_B R_1} \bar{R}^* = C_2 \frac{F_B \bar{R}^*}{\bar{R}_B}$$

The record of U_2 represents a record of F_B if we make

$$\bar{R}^* = C_3 \bar{R}_B$$

This condition is fulfilled when the resonance frequency of the electrical system equals the resonance frequency of the system of the body and when the damping constants of both systems are equal. Because it is almost impossible to build a circuit formed by a condenser and an inductance with the correct damping for the low resonance frequency of the body ($f_B \sim 3cps$), an equivalent electronic circuit (see Appendix II) was used.

B. The F_B -BKG:

Before the recording of the F_B -BKG, f_B and k of the body of the subject were first measured on the shake table and the correcting network adjusted accordingly. Then the F_B -BKG was recorded while the subject was still in the same position on the stationary shake table. In Figure 5 we see that the FRC_B agrees very well with the theoretical curves in some cases while in other cases the agreement is only fair. The degree of compensation of the influence of the FRC_B by means of the correcting network depends on this agreement. The most simply shaped F_B -BKG of subject No. 1 (Fig. 11) probably represents most clearly the "ideal" F_B -BKG because in this case the measured FRC_B -function agrees best with the theoretical function of a simple oscillator and thus we can expect the best action of the correcting network. Because the influence of amplitude, and especially phase distortions caused by an incomplete action of the correcting network on the shape of the record can be rather large, we will be cautious in explaining single peaks and dips as being caused by certain events in the blood circuit and will compare more the shape of the F_B -BKG with the shape of the D- and V-BKG. Figure 11 shows these records together with the EKG of our four subjects, all of whom stopped breathing while the records were taken. In contrast to the D-BKG and V-BKG, the F_B -BKG usually shows a simpler and clearer shape and only small waves or none at all towards the end of the heart period.

In the F_B -BKG we can usually distinguish two waves I and II more or less separated from each other, and by comparison with the EKG we discover that the first wave occurs during the systole, while the second occurs in the beginning of the diastole.

Wave I during systole can be explained by the acceleration and deceleration of the stroke volume or more generally expressed, of the center of gravity of the heart-aorta complex. We will not forget that

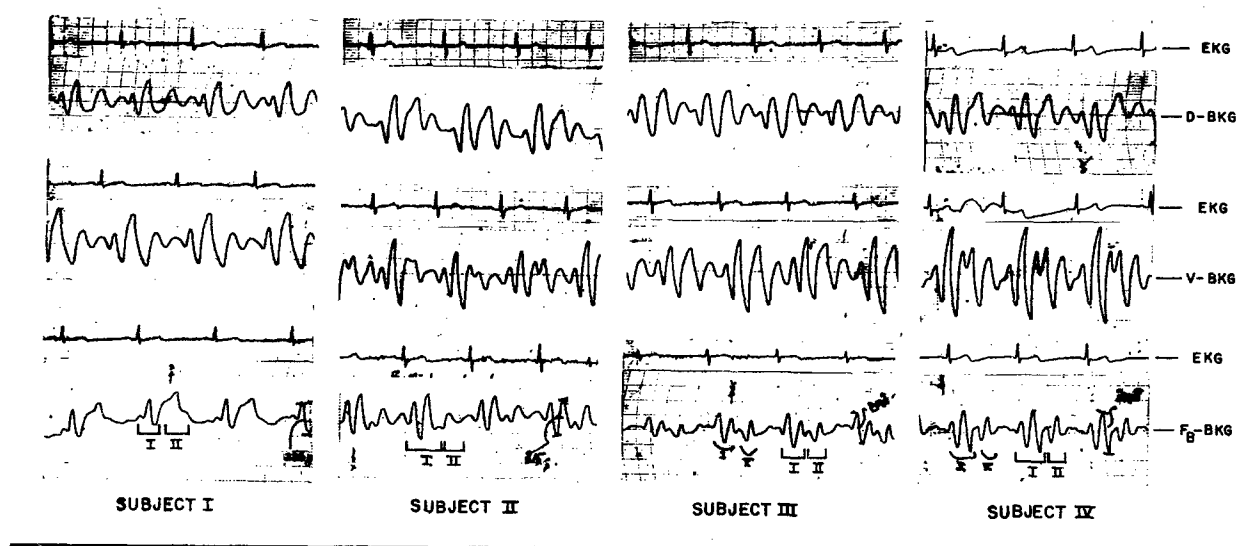


Figure 11. D-BKG, V-BKG, F_g -BKG and EKG of 4 subjects.

the contraction of the heart before the expulsion might also cause a motion of this center of gravity producing an impulse of force which can influence the pattern of wave I. For the explanation of wave II there are several possibilities.

1. A genuine new impulse of force not connected with the systolic action of the heart, produced, for instance, by the filling process of the heart.
2. An impulse of force produced by the energy of the systolic action of the heart but delayed in time by the action of a mechanical network, e.g. the reflection of the pulse wave or, on the other hand, the action of the internal network.

The explanations given under (2.) are the more probable because one can hardly expect that forces described under (1) could be comparable in magnitude to the forces generated by the systolic action of the heart. The reflection of the pulse wave, however, seems less probable because flow processes in the peripheral circuit can hardly be expected to produce forces comparable with the forces of the systolic action since the elastic chamber of the arterial tree should diminish the alternating component (blood mass times velocity) arising there. It is, therefore, understandable that no basic change of the F_g -BKG was observed when the blood flow in the legs was blocked at the thighs. Such a drastic disturbance in the peripheral circuit should have changed at least the timing of wave II if this wave had been produced by the reflection of the pulse wave.

The theory that wave II could have been produced by the action of the internal network was therefore investigated. Because it is possible to explain almost any periodic output function by a certain periodic input function if we assume a certain network between them we will see if we can find a reasonable, simple network which produces from a reasonable input function an output function similar to the typical F_S -BKG. Such an investigation cannot, of course, prove that this explanation of the generation of wave II is correct, but it can prove its possibility.

The problem was investigated by means of an adjustable electrical network the input voltage of which was again produced by the photo-electric voltage generator described in Appendix III. The time function of the input voltage, U_1 , (corresponding to an assumed F_H -BKG) was one cycle of a sine-function of the duration, t , repeated periodically with the period time, corresponding to the heart period, $4t$. It corresponded to an impulse of force of the duration of the systole. The first half-wave may represent the force caused by the acceleration and the second half-wave may represent the force caused by deceleration of the stroke volume. The output load impedance of the network was infinite because the impedance of the body mass upon which the force at the output of the internal network acts can be assumed to be very large in comparison to the output impedance of the internal network. The output voltage, U_2 , of the electrical network corresponds to the force, F_S .

It was possible to adjust a network with two degrees of freedom in such a manner that the input function was changed at the output to the pattern of the typical F_S -BKG. Figure 12A shows the records obtained, Figure 12B the electrical network found and 12C its equivalent mechanical circuit. Curve F_H is the record of the input, F_S the record of the output function.

We will not try at this time to correlate the network to the internal structure of the body but state only that the explanation of wave II by an action of the internal network seems possible and that therefore the F_S -BKG could be produced by only one impulse of force acting during systole.

To give a more popular description of the possible action of the internal network; when the stroke volume is expelled from the heart, the reaction force moves the whole heart downward and its elastic suspension is stressed. After the valves have closed and the reaction force has stopped (i.e. in the diastole) the energy stored in the suspension moves the heart back and thereby generates a reaction force which produces wave I. (this might also be an explanation of the "3rd heart sound").

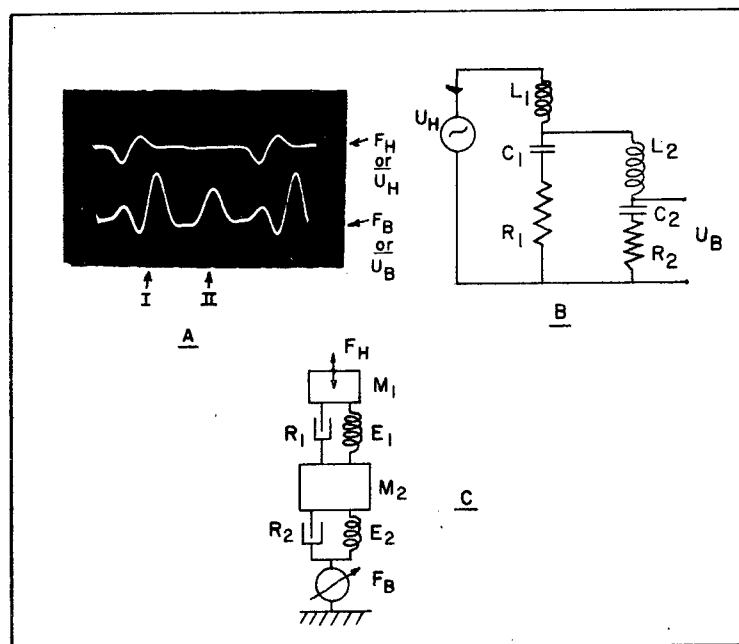


Figure 12. A: Output voltage, U_2 , caused by the input voltage, U_1 , in the electrical network shown in B. C: Shows the mechanical circuit equivalent to the electrical network (B).

The F_B -BKG resembles somewhat the low-frequency recordings of "heart sounds" and when we compare it with recordings of the motion of the apex of Ernsthausen (4), recorded in the frequency range 1 - 20 cps, we can, in some cases, find a surprising similarity which seems too great to be just coincidental. Figure 13 shows a record by Ernsthausen of the apex motion and one of our F_B -BKG's. They were not taken

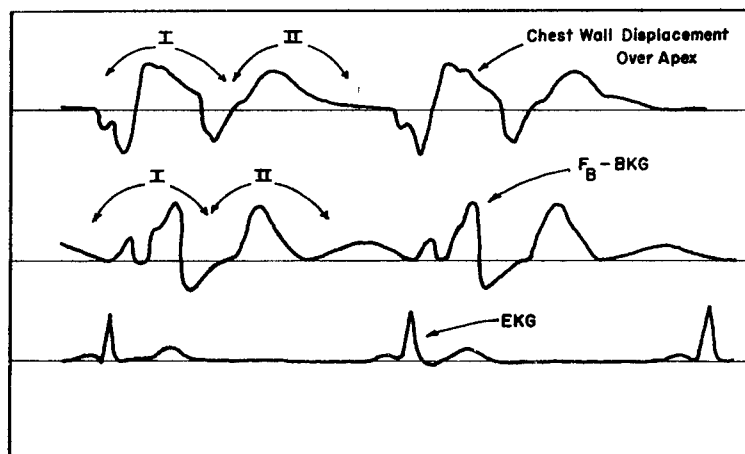


Figure 13. Comparison between a record of the motion of the apex (after Ernsthausen) and F_B -BKG together with the EKG

from the same subject and were matched to each other, as far as the time scale is concerned, by means of the EKG recorded in each case. Ernsthausen explains the first wave, the rise and fall of which produces the first and second of the usual heart sounds, by the change in the shape of the heart due to contraction and expulsion and explains the second wave also by a passive motion of the heart. The explanation of the similarity of the waves I is probably that the change of shape of the heart is closely related to a motion of its center of gravity.

C. Calibration of the F_B -BKG:

For the calibration a known force F_c must be applied to the body, be recorded with the same sensitivity of the measuring apparatus as used for the recording of the F_B -BKG and both records must be compared. For the application of F_c the same considerations apply as for the measurement of FRC_B . F_c must produce the same pattern of motion of the body as the one produced by the force of the heart action. (The application of a calibration force upon e.g. foot or shoulder, as sometimes reported in scientific literature, hardly fulfills this condition). Furthermore, the calibration force must contain only frequencies within the flat range of FRC_0 . Thus the shake table was again used to produce a sinusoidal calibration force.

As we have shown, the shake table imparts a known velocity upon the body system through elasticity and damping of the external tissues and we intend to calibrate the record of the force acting upon the mass of the body. We must therefore determine the force, F_c , acting upon the body mass, which is produced by the velocity of the table. Figure 4 shows, besides the mechanical circuits, the equivalent electrical circuits used when the body system is excited through elasticity and damping by the velocity, V_T , (Fig. 4B) and when the same system is excited by a force, F_B , acting upon the body mass (Fig. 4A).

The generator, G_T , then represents the shake table and drives the current, I_T , (equivalent to the velocity, V_T , of the shake table) through the system. G_B produces the voltage U_B , equivalent to the force F_B . In the calibration and in the F_B -BKG recording the velocity V_B^* produced by G_T and the velocity V_B produced by G_B are recorded over the correcting network. These velocities correspond in the electrical circuits to the currents I_B^* and I_B^0 . We will investigate which voltage, U_B , produces a current, I_B^0 , equal to I_B^* when the latter is produced by I_T .

For I_B^0 and I_B^* we can derive the formulas:

$$I_B^* = I_T \frac{1/j\omega C_B + R_B^0}{j\omega L_B + 1/j\omega C_B + R_B^0}$$

$$I_B^0 = \frac{U_B}{j\omega L_B + 1/j\omega C_B + R_B^0}$$

By setting $I_B^* = I_B^0$ we obtain an equation which gives the desired relation between U_B and I_T . This equation has a very simple form when we choose the resonance frequency of the system, f_B , for calibration. For $\omega_B = 2\pi f_B$ we have $\omega_B L_B = 1/\omega_B C_B$ and $R_B^0 = k \omega_B L_B$ and we obtain: $U_B = I_T \omega_B L_B (k - j)$ and for

the absolute value: $|U_B| = I_T \omega_B L_B \sqrt{k^2 + 1}$. This means

for the mechanical case: $|F_B| = V_T \omega_B M_B \sqrt{k^2 + 1} = \alpha_T \omega_B^2 M_B \sqrt{k^2 + 1}$

where α_T = amplitude of the displacement of the shake table. In case of the calibration F_B is equivalent to the calibration force F_c .

The factor $\sqrt{k^2 + 1}$ equals ~ 1 for the k -values of the body. (for $k=0.4$, $\sqrt{k^2+1}=1.08$). It may be remembered that V_T is the amplitude of the velocity of the shake table and F_c the amplitude of the calibration force. The peak to peak distance, H_c , of the calibration record represents therefore: $H_c = 2 F_c$.

If the calibration is performed with the frequency, f_B , it must be considered that the correcting network has its smallest rate of transmission for this frequency. All other frequencies present (components of the BKG, mechanical vibrations of the building and harmonics of the calibration frequency produced by the table) will be emphasized in the recording. The record of the calibration frequency will, therefore, be distorted. But in the calibration only the frequency, f_B , is of interest. The rate of transmission of the correcting network can, therefore, be diminished for the other frequencies if the rate of transmission of the frequency f_B is not changed. This is done by short circuiting L_B and C_B (Fig. 10) of the correcting network during the

calibration since at this frequency the impedance $j\omega_B L_B + 1/j\omega_B C_B$ is already zero. In this manner and if F_c is large enough, a practically undistorted record of the calibration force, F_c , can be obtained (Fig. 14).

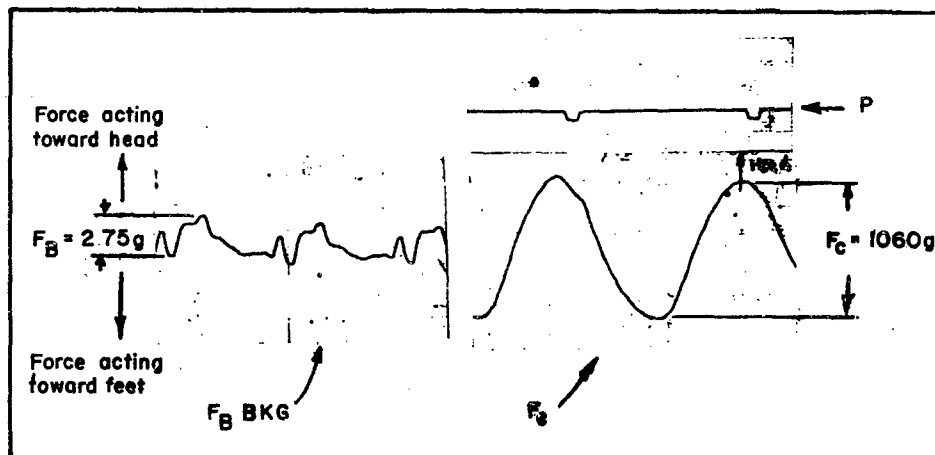


Figure 14. Record of the calibration force, F_c , and F_B -BKG recorded with the same sensitivity of the recording system. The pulses P above the F_c -record indicate that the calibration force at the same moment acts toward the head of the subject. Upward waves of the F_c -record and also of the F_B -BKG therefore indicate a direction of the force towards the head.

By means of the equation: $U_B = I_T \omega_B L_B (k-j)$ the phase relation between U_B and I_T is obtained. This phase relation will be used for the determination of the direction of the calibration force and of the force recorded in the F_B -BKG. With the average k -values of the body the phase shift, φ , between U_B and I_T is about -90° (for $k=4$, $\varphi=-68^\circ$) in the electrical circuit and also between F_B and V_T in the mechanical circuit. (Fig. 4). The maxima of F_B occur $\sim 90^\circ$ later than the maxima of V_T . Therefore, because the maximal displacements of the table occur 90° later than its maximal velocities, the maxima of F_B are in phase with the maximal displacements and have the same direction. By means of a contact operated by the pulley (Fig. 7B) revolving synchronously with the oscillation of the table plate, an electrical impulse, P , (Fig. 14) was produced periodically at the time of maximal F_c in one direction and recorded together with the calibration force. At the maxima synchronous with the pulses, P , the calibration force acts in the direction of the displacement of the table coordinated with the closure of the contact, in our case in the direction of the head of the subject. Amplitudes of the F_B -BKG in the same direction indicate forces of the heart action acting toward the head.

The comparison of H_c of the F_c -record with H_B of the F_B -BKG, recorded with the same adjustment of the measuring apparatus, yields the peak to peak value F_B . The F_B -values measured with different subjects are given in Table 1. Values between 286 gram and 540 gram were measured.

The force F_H , acting upon the heart can be smaller or greater depending on the action of the internal network. One can roughly estimate the amplitude A_H of this force at the first moment that the aortic valve has opened. It should be in the order of: systolic pressure, P_s , minus diastolic pressure, P_d , times cross section of the aorta, q_a .

$$A_H \sim (P_s - P_d) q_a$$

With $P_s - P_d = 50 \text{ mm Hg} = 6.5 \times 10^4 \frac{\text{dyne}}{\text{cm}^2}$ and $q_a = 7 \text{ cm}^2$

we obtain: $A_H = 450 \text{ gr}$. This value is probably too large because the valves cannot open in infinitely short time. On the other hand we can estimate A_H from the peak acceleration necessary to drive the stroke volume, V_s , ($V_s \sim 100 \text{ cm}^3 \sim 100 \text{ gr}$), from the heart into the aorta. If it is assumed that in first approximation the velocity of the blood entering the aorta in $1/4$ heart period, T , ($T \sim 1 \text{ sec.}$) has a time function according to one half of a sine function (i.e. with a frequency $2/T$ c.p.s.), the velocity amplitude, V , would be:

$$V = \frac{V_s}{q_a \int_0^{T/4} \sin 2\pi \frac{2}{T} t dt} = 115 \text{ cm/sec}$$

and we obtain the amplitude of the acceleration: $acc = V \frac{2\pi}{T} = 1450 \frac{\text{cm}}{\text{sec}^2} = 1.5 G$ (G gravitational constant = $981 \frac{\text{cm}}{\text{sec}^2}$). The acceleration of the stroke volume of about 100 gram would then produce an amplitude of the reaction force of 150 gram. This value is probably too small because a

part of the blood already in the aorta must also be accelerated, the amount of blood depending on the elastic qualities of the arterial tree.

If we compare these very rough estimates of the amplitude of the force acting upon the heart with the amplitude ($\hat{F}_B/2$) of the force acting upon the body mass as determined from the F_B -BKG we find that these values are relatively close and that, therefore, the influence of the internal network on the amplitude of the record does not seem significant.

APPENDIX I

The Measurement of the Motion of the Body

For the measurement of the FRC_B and for the recording of the F_B-BKG , we must produce a voltage proportional to the displacement or velocity of the body. This voltage is produced by an electro-mechanical transducer (pick-up) for the design of which any electro-mechanical principle can be used (e.g. motion of a conductor in a magnetic field, change of the reluctance of a magnetic circuit by the motion of an armature, change of the capacity of a condenser by the motion of a condenser plate, deformation of a piezoelectric crystal, modulation of the illumination of a photocell by the motion of a mirror or of a diaphragm etc.). It is not important to which physical magnitude of the motion the output voltage is proportional, because it can be differentiated or integrated by means of electrical networks. The choice of the electro-mechanic principle depends upon the necessary sensitivity and upon the mechanical structure connected with this principle. The mechanical input impedance must be kept as small as possible in comparison to the mechanical impedance of the point at the body surface to be connected with the pick up. If the mechanical impedance of the pick up is too large the motion of this point may be considerably changed. To keep the impedance of the pick up small, its moving mass and the elasticity of the suspension of this mass must be as small as possible. The impedance of a point of the body surface is relatively small because of the great mobility of the external tissues, especially for motions parallel to the surface. The conditions can be improved however, when the pick up is not connected with a point but with a larger area of the surface.

Depending on the problem, one of two methods was used for recording the motion of the body.

1) $V-BKG$ Recorded at Various Parts of the Body.

In order to obtain comparable records it was important that the sensitivity of the measuring apparatus remain constant when the place of the measurement was changed. This is difficult to achieve by a method which measures the motion relative to a fixed point, such as the earth. Therefore, measurements were performed by means of an accelerometer which measures the motion of a point of the body relative to a mass elastically connected with this point. The acceleration of the body caused by the action of the heart was found to be of the order of $1/500 G$. A sufficiently sensitive accelerometer (Figure 15A) was built by connecting (with glue) a mass, M (20 gr.), to the surface of the crystal microphone, C , elastically suspended in the frame, F . The other end of M was suspended in a flat steel spring S . The resonance of the system was damped by filling the narrow adjustable slots between M and the plastic blocks, Q ,

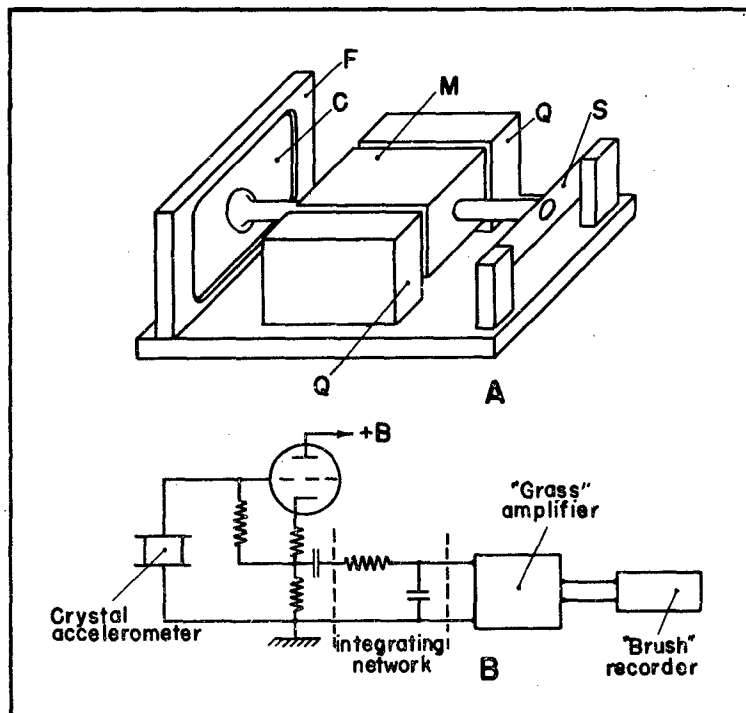


Figure 15. Crystal-Accelerometer (A) and Cathode-Follower amplifier stage (B).

with grease. The output voltage of the accelerometer was found to be proportional to the acceleration in the frequency range .5 to 40cps. The electrical inner impedance of the crystal represents a capacity (ca. 1000 pF). To bring the open circuit voltage at a frequency of .5 cps to the grid of the first amplifier stage, the grid leak resistance must be about 300 Megohms. This resistance was dynamically obtained by means of the cathode follower circuit shown in Figure 15B. A voltage proportional to the velocity was obtained by means of an integrating network following the amplifier stage. After the integrating network a "Grass" amplifier (Model P4), and a direct writing in k-recorder (Brush Recorder BL 902A with amplifier BL905) were used.

2a) Measurement of the Displacement of the Body on the Shake Table for the Determination of FRC_B .

A voltage proportional to the displacement of the body could have been obtained by using the accelerometer described above and integrating its output voltage twice with an electrical network. It was, however, simpler to produce this voltage directly with a condenser pick up (Figure 16) used with a carrier frequency device. One

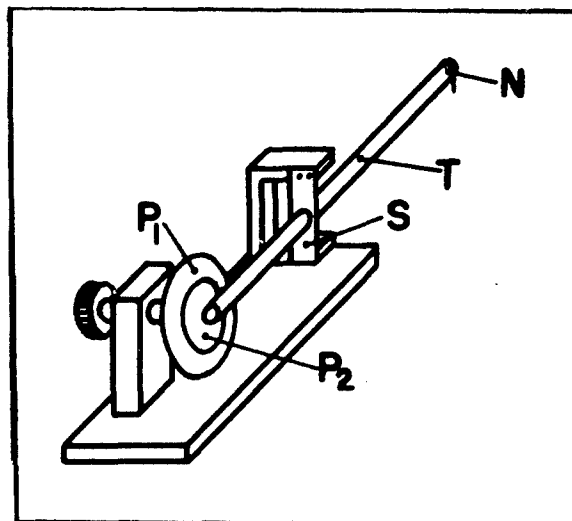


Figure 16. Condenser Pick Up for the Measurement of the Motion of the Body.

condenser plate, P_1 , was rigidly connected with the earth, the other one, P_2 , was connected to one end of a light metal tube, T , which was fixed in the middle by the flat steel spring, S , and was movable in the horizontal and vertical plane. The other end of the tube carried a needle N , which touched a leather belt, B (Fig. 7), buckled around one thigh and hip of the subject. In this way the area of the body surface loaded by the pick up was considerably enlarged and the motion was picked up not too far from the thorax and at a place hardly moved by the subjects breathing. Only components of the motion parallel to the longitudinal axis of the body produced capacity changes by changing the separation of the plates. A motion of the plate, P_2 , parallel to its plane caused no change of capacity because the diameter of the movable plate was smaller than the diameter of the rigid plate. The sensitivity of the measuring system was high enough that a distance between the plates of approximately .5 cm could be used. Small displacements of the body were, therefore, without influence on the sensitivity of the system. The output voltage of the carrier frequency device (proportional to the displacement of the body in the frequency range 0-20 cps) was amplified by a "Grass" amplifier and recorded by a "Brush" recorder.

2b) Recording of the D , V , and F_B -BKG.

For all these recordings the condenser pick up, described above, was used. For the V -BKG and F_B -BKG the output voltage of the carrier system was differentiated by means of an electrical network, the output

voltage of which was then proportional to the velocity of the body. After amplification with the "Grass" amplifier this voltage was recorded, in case of the F_B -BKG, over the correcting network. Because FRC_B was measured up to only 10 cps and the application of the correcting network is not therefore justified for frequencies $f > 10$ cps, the overall frequency response FRC_o was cut off above this frequency. Figure 7 shows the complete arrangement of the measuring system.

APPENDIX II

The "Correcting Network"

The correcting network, shown in Figure 17, produced the FRC_M proportional to $1/FRC_B$ not using inductances. Because: $R_1 \gg 1/\omega C_1$, $R_2 \ll 1/\omega C_2$, and $R_3 \gg R_4$ at the grids of the tubes T_1 , T_2 , T_3 appear the voltages:

$$U_1 = \frac{U_e}{R_1} \frac{1}{j\omega C_1} = U_e \frac{N_1}{j\omega}$$

$$U_2 = U_e j\omega C_2 R_2 k_1 = U_e N_2 j\omega$$

$$U_3 = U_e \frac{R_4}{R_3} k_2 = U_e N_3$$

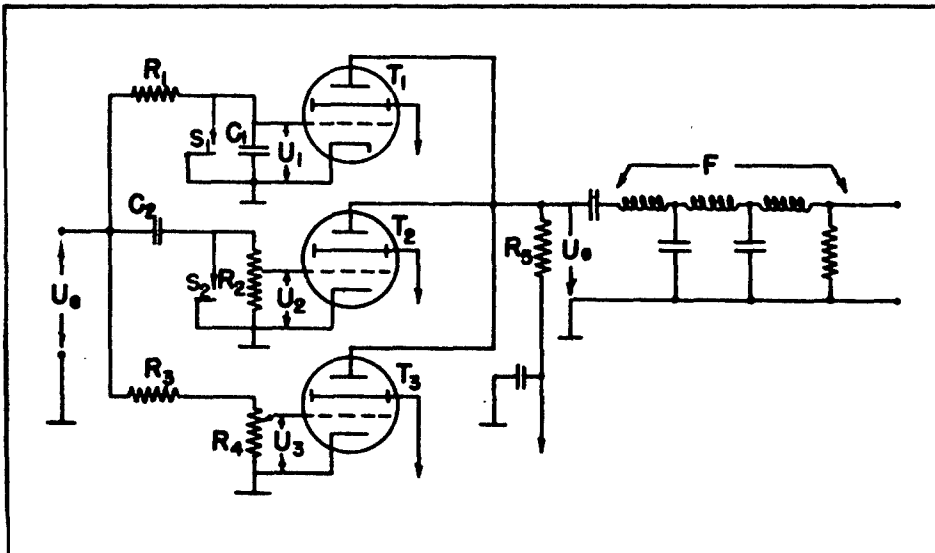


Figure 17. Circuit of the "Correcting Network" used in the measurements which is equivalent to the electrical circuit in Figure 10.

k_1, k_2 = attenuation factors of the potentiometers, R_2, R_4 .
Through R_5 flows the sum of the plate currents of T_1, T_2, T_3 and with all tubes (pentodes to avoid an influence of the plate voltage on the grid voltages) having the same transconductance, T ,

we obtain: $U_q = R_5 U_e T (N_1/j\omega + N_2 j\omega + N_3)$

With the constant factors combined to A_1 , A_2 , and A_3 we obtain the FRC_M of the system: $U_q/U_e = j\omega A_2 + A_1/j\omega + A_3$.

We see that this function is the reciprocal function of frequency as the formula of

$$1/FRC_B = F_B/V_B = 1/(j\omega M_B + 1/j\omega E_B + R_B)$$

We can make FRC_M proportional to $1/FRC_B$ by adjusting the constants A_2 and A_3 by means of the potentiometers R_2 and R_4 .

With R_2 we can adjust the resonance frequency of the system (minimal value of U_q/U_e) and with R_4 we can adjust the damping factor. For the shake table calibration the grids of T_1 and T_2 were shorted to ground by the switches S_1 and S_2 . This is equivalent to the short circuiting of L_B and C_B in the circuit of Figure 10.

APPENDIX III

Generator for the Production of Voltages of any Periodic Function of Time

For the investigation of mechanical circuits by means of equivalent electrical circuits and for the Fourier analysis of the V-BKG's an input voltage was required with the time course of the magnitude to be investigated. This voltage was produced by means of a rotating disc D, (Figure 18) shaped according to the desired time function of the voltage. The disc modulated the area of a light beam produced by a

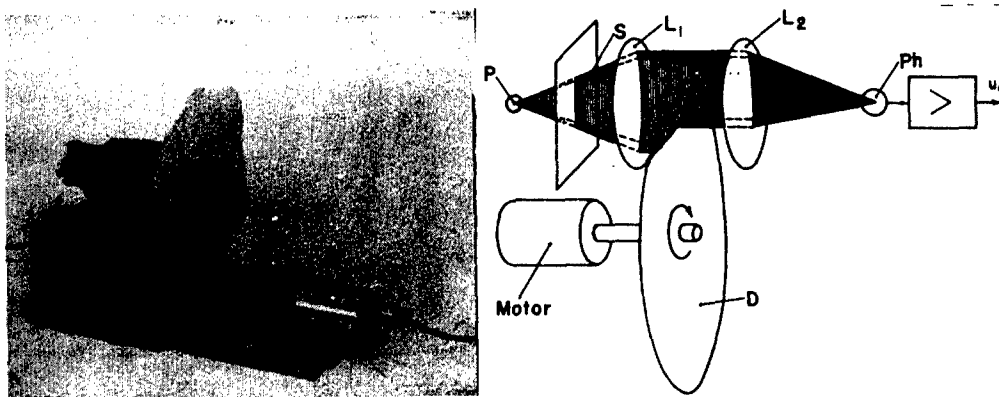


Figure 18. Photoelectric voltage generator (A) and its schematic presentation (B).

light source, P, and the slot S. The lens, L_1 , produced a parallel beam at the place of the disc, and the lens, L_2 , concentrated all the light which had passed the disc on the photocell Ph. The disc was rotated by a synchronous motor 30 times/sec. In case of the investigation of a BKG the frequency of the "heart beat" was, therefore, raised by the factor 30 and the resonance frequency of the electrical circuits was chosen to be 30 times the resonance frequency of the equivalent mechanical circuits. Because all frequency components were raised by the factor 30, for the frequency analysis an analyzer designed for the audio-frequency range (Hewlett Packard Harmonic Wave Analyzer 300A) could be used.

APPENDIX IV

The Shake Table

The shake table (Lab Corporation, Type VU-DM-500) (Figure 7) connected with its motor by a belt and originally designed to produce vibrations in the frequency range 10 - 60 cps, was changed by means of an alteration of the pulleys to produce vibrations in the range 0.5 - 15 cps. To measure the frequency, f_T , of the table, the pulley, rotating with the frequency of the vibration of the table, carried an iron toothed wheel, w , with 100 teeth. The teeth induced in an electromagnetic circuit a voltage with the frequency $100 f_T$ which was measured by an electronic frequency meter (Hewlett Packard 500A). In all the shake table measurements the amplitude of the displacement of the table plate was adjusted to be 0.02 cm as recorded on the dial gauge A. It was found that with constant frequency the amplitudes of the body were proportional to those of the table in this range of amplitudes.

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