TABLES FOR THE DISTRIBUTION OF THE NUMBER OF EXCEEDANCES

by

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TECHNICAL REPORT NO. 6

July 15, 1953

PREPARED UNDER CONTRACT Nonr-451(00)
(NR-042-017)

FOR

OFFICE OF NAVAL RESEARCH

DEPARTMENT OF MATHEMATICS
WAYNE UNIVERSITY
DETROIT,, MICHIGAN

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1. Introduction and Summary.

Consider a random sample of size n taken from a continuous distribution f(x). Let another random sample, independent of the first sample and also of size n, be drawn from the same population. Let U_r^n be the random variable associated with the number of values in the second sample which exceed the rth smallest value in the first sample. Similarly let V_s^n be the random variable associated with the number of values in the second sample which exceed the sth largest value in the first sample. Due to the fact that the rth smallest value in a sample of size n is at the same time the sth largest value in the sample with s = n-r+1, it follows that

(1) $$\Pr(U_r^n = x) \equiv \Pr(V_s^n = x), \text{ where } s = n-r+1,\ r = 1, 2, \ldots, n, \text{ and } x = 0, 1, 2, \ldots, n.$$

The probability distribution of U_r^n (and hence of V_s^n) is given by:

(2) $$\Pr(U_r^n = x) = \binom{n-x+r-1}{r-1}\binom{n-r+x}{x} \Big/ \binom{2n}{n}, \quad x = 0, 1, 2, \ldots, n.$$

Formula (2) can be proved by combinatorial methods. Details are left to the reader. An alternative formula, derived in another way, is given in [1]. If the values of $Pr(U_r^n \leqslant x)$, for $x = 0, 1, 2, \ldots, n-1$[1], $r = 1(1)n$, are written (for fixed n) in matrix form, one notes certain useful symmetries, which can be expressed by the following identities

$$Pr(U_r^n \leqslant x) = Pr(U_x^n \leqslant r-1) \tag{3}$$

and

$$Pr(U_r^n \leqslant x) + Pr(U_{n-r+1}^n \leqslant n-x-1) = 1. \tag{4}$$

If one takes $x = n-r$ in (4) and uses the relation (3), it is readily verified that

$$Pr(U_r^n \leqslant n-r) = 1/2. \tag{5}$$

Proofs[2] of (3), (4), and (5) can be obtained by using the results of pages 257-258 of [1]. Because of these symmetries, the complete matrix can (for any fixed r) be constructed if one knows only the quantities, $Pr(U_r^n \leqslant x)$, $r = 1(1)[n/2]$, $x = r-1, r, r+1, \ldots, n-r-1$. In Table 1 these values are given for $n = 2(1)20(5)50$.[3] To see how the complete matrix is obtained from Table 1, it is interesting to verify using (3), (4), and (5) that the complete matrix is, in the special case $n = 5$, given by:

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(1) $Pr(U_r^n \leqslant n) = 1$ for any n and any r.

(2) We wish to acknowledge with thanks a communication from Dr. E. J. Gumbel on this point.

(3) The case where the two samples may be of unequal size is of practical importance. Formulae for this case and tables for selected pairs of unequal values of the sample size will appear in a subsequent report.

$r \backslash x$	0	1	2	3	4
1	$.0^2397$	.0238	.0833	.2222	.5000
2	.0238	.1032	.2619	.5000	.7778
3	.0833	.2619	.5000	.7381	.9167
4	.2222	.5000	.7381	.8968	.9762
5	.5000	.7778	.9167	.9762	$.9^2303$

Table 2: Values of $Pr(U_r^5 \leq x)$

A somewhat different, but related exceedance problem, is to take two random samples of size n from a continuous distribution f(x). Let us for convenience attach the letter x to one of the samples and the letter y to the other sample. Further let $x_{r,n}$ and $y_{r,n}$ be respectively the rth smallest observations in each of the samples. Let us define $z_{r,n} = \max(x_{r,n}, y_{r,n})$. If $z_{r,n} = x_{r,n}$, count the number of y's which are $\geqslant x_{r,n}$; if $z_{r,n} = y_{r,n}$, count the number of x's which are $\geqslant y_{r,n}$ (4). Denoting the number of exceedances as W_r^n, it is readily seen from (1) that the probability distribution of W_r^n is given by

(6) $$Pr(W_r^n = x) = 2\binom{n-x+r-1}{r-1}\binom{n-r+x}{x} \Big/ \binom{2n}{n}, \quad x = 0,1,2,\ldots, n-r.$$

It is evident from the definition that,

(7) $$Pr(W_r^n \leq x) = 1, \qquad \text{for } x \geqslant n-r.$$

Clearly one can find the values of $Pr(W_r^n \leq x)$ by using Table 1. Thus,

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(4) Since f(x) is continuous, it is assumed that $Pr(x_{r,n} = y_{r,n}) = 0$.

e.g., in the special case n = 5 one gets

r \ x	0	1	2	3	4
1	$.0^2794$	.0476	.1667	.4444	1.0000
2	.0476	.2064	.5238	1.0000	
3	.1667	.5238	1.0000		
4	.4444	1.0000			
5	1.0000				

Table 3: $Pr(W_r^5 \leqslant x)$

2. Applications of Exceedance Theory

There are three principal uses of exceedance theory. These are:

(a) Floods and droughts. This theory has been used by H. A. Thomas, Jr. [4] in making predictions about the occurrences of floods and droughts in the future on the basis on what is known from past data.

(b) Non-parametric tests for slippage. The functions U_r^n, V_s^n and W_r^n can be used to give two-sample non-parametric tests for slippage of the mean. There are close connections between the results in this paper and recent tests for slippage by Mosteller and Tukey [2,3].

(c) Life Testing. It is a characteristic feature of life tests that data become available in order of size. Thus it becomes very natural to apply exceedance theory, which is based purely on order statistics. By so doing it is possible in many cases to shorten both the average time and average number of items destroyed in order to reach a decision as to whether or not the items in one population are in some sense superior to the items in another population.

3. Numerical Examples

Example 1: What is the probability that the third largest flood during the past 20 years will be exceeded at least once during the next 20 years? Answer. The probability is $p = 1-\Pr(V_3^{20} = 0) = 1-\Pr(U_{18}^{20} = 0)$ $= 1-.1154 = .8846$.

Example 2: During a period of 20 years the lowest observed annual rainfall in a certain locality was 8.6 inches. What is the probability that in the next 20 years at least two of the years have rainfall $\leqslant$ 8.6 inches? Answer. The probability is $p = \Pr(U_1^{20} \leqslant 18) = .2436$.

Example 3: (a one-sided test)

We are now interested in making a choice between two lots A and B. In particular we are interested in some characteristic such as life or strength where data become available in order of magnitude. Let it be known a priori that the probability density function associated with lot B is either the same as that of lot A or is displaced to the left (e.g., is inferior). Put in another way we are thinking of a case where the only relevant parameter is some measure of slippage. We wish to test the hypothesis H_0 (no displacement) against the alternative H_1 (B is displaced to the left of A). The Type I error is taken to be $\leqslant .05$. Let 10 items be drawn from each of the lots and placed on life test. It is decided in advance that a decision will be based on how many failures occur in the sample from B before the second failure occurs in the sample from A. The two samples are put on test simultaneously and one gets the following pattern (a denotes a failure in the sample drawn from A, b denotes a failure in the sample drawn from B): bbbabbb... The experiment is stopped at the seventh failure with the rejection of the null hypothesis. This is because $\Pr(U_2^{10} \leqslant 4) = .0286 < .05$. If, however, we had obtained a pattern like babba... we would have stopped experimentation after the fifth failure with the acceptance of H_0.

Example 4: (two-sided test). Given two lots A and B. We wish to test the null hypothesis that the life distribution of A and B are the same against the alternative that they are different. As in example 3 let 10 items be drawn at random from each of the two lots and placed on life test. It is decided in advance that our decision will be based on the statistic W_2^{10}. Suppose e.g., the failure pattern observed is aaaaabaa... , then the experiment would be terminated on the eighth trial with the rejection of the null hypothesis (on the .05 level of significance). This is because $Pr(W_2^{10} \leq 3) = .0198$. On the other hand a pattern like babba... would lead to acceptance of the null hypothesis on the fifth trial.

It might be remarked that fairly extensive random sampling experiments have shown that the statistics W_1^{10}, W_2^{10}, and W_3^{10} are more effective than the run test and somewhat less effective than the Wilcoxon rank test for detecting slippage of the mean in the case where the underlying distributions are normal, all with the same variance. Since the improvement in power obtained by using W_2^{10} or W_3^{10} rather than W_1^{10} is minor in this case there are sound practical reasons for preferring W_1^{10}. Decisions based on this statistic can be made at a great saving in average time to decision as well as average number of items destroyed. It should be noted in Example 4 that if decisions were based on W_1^{10}, we would have truncated testing on the fifth trial with the rejection of H_0 since $Pr(W_1^{10} \leq 5) = .0325$.

A detailed discussion of the points raised in the last paragraph will appear elsewhere.

4. Acknowledgement.

We wish to thank John Lay for his work in computing the tables.

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3. F. Mosteller and J. W. Tukey, "Significance levels for a k-sample slippage test", Annals of Math. Stat., 21, 120-123, 1950.

4. H. A. Thomas, Jr., "Frequency of Minor Floods", Journal of the Boston Society of Civil Engineers, 35, 425-442, 1948.

TABLE 1

$Pr(U_r^n \leq x)$

n	r \ x	0	1	2	3	4	5	6	7	8	9	10	11	12
2	1	.1667												
3	1	.0500	.2000											
4	1	.0143	.0714	.2143										
	2		.2429											
5	1	$.0^{2}397$	.0238	.0833	.2222									
	2		.1032	.2619										
6	1	$.0^{2}108$	$.0^{2}758$	.0303	.0909	.2273								
	2		.0400	.1212	.2727									
				.2835										
7	1	$.0^{3}291$	$.0^{2}233$	.0105	.0350	.0962	.2308							
	2		.0146	.0513	.1329	.2797								
	3			.1431	.2960									

Table 1 (Continued)

n	r \ x	0	1	2	3	4	5	6	7	8	9	10	11	12
8	1	$.0^4777$	$.0^3699$	$.0^2350$	.0128	.0385	.1000	.2333						
	2		$.0^2505$	.0203	.0594	.1410	.2846							
	3			.0660	.1573	.3042								
	4				.3096									
9	1	$.0^4206$	$.0^3206$	$.0^2113$	$.0^2452$	.0147	.0412	.1029	.2353					
	2		$.0^2169$	$.0^2761$	.0249	.0656	.1471	.2882						
	3			.0283	.0767	.1674	.3100							
	4				.1735	.3186								
10	1	$.0^5541$	$.0^4595$	$.0^3357$	$.0^2155$	$.0^2542$	.0163	.0433	.1053	.2368				
	2		$.0^3547$	$.0^2274$	$.0^2988$	.0286	.0704	.1517	.2910					
	3			.0115	.0349	.0849	.1749	.3142						
	4				.0894	.1849	.3250							
	5					.3281								
11	1	$.0^5142$	$.0^4170$	$.0^3111$	$.0^3516$	$.0^2193$	$.0^2619$	.0175	.0451	.1071	.2381			
	2		$.0^3173$	$.0^3953$	$.0^2376$	.0119	.0317	.0743	.1554	.2932				
	3			$.0^2446$	.0150	.0402	.0913	.1807	.3176					
	4				.0431	.0992	.1935	.3297						
	5					.1974	.3350							

Table 1 (Continued)

n	r \ x	0	1	2	3	4	5	6	7	8	9	10	11
12	1	$.0^{6}370$	$.0^{5}481$	$.0^{4}337$	$.0^{3}168$	$.0^{3}673$	$.0^{2}229$	$.0^{2}686$	.0186	.0466	.1087	.2391	
	2		$.0^{4}536$	$.0^{3}322$	$.0^{2}138$	$.0^{2}471$	.0136	.0343	.0775	.1584	.2950		
	3			$.0^{2}166$	$.0^{2}614$	.0180	.0447	.0965	.1854	.3202			
	4				.0196	.0498	.1069	.2002	.3334				
	5					.1102	.2068	.3401					
	6						.3421						
13	1	$.0^{7}961$	$.0^{5}135$	$.0^{4}101$	$.0^{4}538$	$.0^{3}229$	$.0^{3}824$	$.0^{2}261$	$.0^{2}745$	.0196	.0478	.1100	.2400
	2		$.0^{4}163$	$.0^{3}106$	$.0^{3}491$	$.0^{2}180$	$.0^{2}558$	.0151	.0365	.0801	.1609	.2965	
	3			$.0^{3}601$	$.0^{2}242$	$.0^{2}771$	.0207	.0484	.1008	.1891	.3224		
	4				$.0^{2}847$	.0236	.0554	.1131	.2055	.3364			
	5					.0576	.1189	.2142	.3441				
	6						.2169	.3475					
14	1	$.0^{7}249$	$.0^{6}374$	$.0^{5}299$	$.0^{4}170$	$.0^{4}763$	$.0^{3}290$	$.0^{3}966$	$.0^{2}290$	$.0^{2}797$	.0204	.0489	.1111
	2		$.0^{5}491$	$.0^{4}344$	$.0^{3}171$	$.0^{3}$[illegible]	$.0^{2}221$	$.0^{2}638$	.0164	.0384	.0824	.1630	.297[illegible]
	3			$.0^{3}211$	$.0^{3}919$	$.0^{2}316$	$.0^{2}915$	.0230	.0516	.1043	.1923	.3242	
	4				$.0^{2}351$	.0[illegible]07	.0271	.0601	.1182	.2099	.3388		
	5					.0285	.0642	.1259	.2200	.3473			
	6						.1284	.2247	.3518				
	7							.3532					

Table 1 (Continued)

n	r \ x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
15	1	$.0^{8}645$	$.0^{6}103$	$.0^{6}877$	$.0^{5}526$	$.0^{4}250$	$.0^{4}100$	$.0^{3}350$	$.0^{2}110$	$.0^{2}316$	$.0^{2}843$	.0211	.0498	.1121	.2414	
	2		$.0^{5}146$	$.0^{4}109$	$.0^{4}579$	$.0^{3}242$	$.0^{3}850$	$.0^{2}260$	$.0^{2}710$	.0176	.0400	.0843	.1648	.2989		
	3			$.0^{4}725$	$.0^{3}339$	$.0^{2}125$	$.0^{2}389$	.0105	.0251	.0543	.1074	.1949	.3257			
	4				$.0^{2}141$	$.0^{2}461$	.0127	.0302	.0641	.1225	.2135	.3408				
	5					.0134	.0328	.0697	.1318	.2249	.3499					
	6						.0716	.1362	.2311	.3552						
	7							.2331	.3576							
16	1	$.0^{8}166$	$.0^{7}283$	$.0^{6}255$	$.0^{5}161$	$.0^{5}806$	$.0^{4}339$	$.0^{3}124$	$.0^{3}408$	$.0^{2}122$	$.0^{2}340$	$.0^{2}884$	.0217	.0506	.1129	.2419
	2		$.0^{6}428$	$.0^{5}342$	$.0^{4}193$	$.0^{4}854$	$.0^{3}318$	$.0^{2}103$	$.0^{2}296$	$.0^{2}775$	.0186	.0415	.0860	.1663	.2998	
	3			$.0^{4}244$	$.0^{3}122$	$.0^{3}483$	$.0^{2}159$	$.0^{2}457$	.0117	.0269	.0567	.1100	.1972	.3270		
	4				$.0^{3}546$	$.0^{2}192$	$.0^{2}567$	.0145	.0329	.0675	.1262	.2166	.3425			
	5					$.0^{2}606$	.0160	.0366	.0744	.1367	.2289	.3521				
	6						.0378	.0778	.1426	.2363	.3580					
	7							.1445	.2397	.3612						
	8								.3622							

Table I (Continued)

n	x / r	0	1	2	3	4	5	6	7
17	1	$.0^{9}429$	$.0^{8}771$	$.0^{7}733$	$.0^{6}489$	$.0^{5}256$	$.0^{4}113$	$.0^{4}433$	$.0^{3}148$
	2	.	$.0^{6}124$	$.0^{5}106$	$.0^{5}630$	$.0^{4}296$	$.0^{3}116$	$.0^{3}395$	$.0^{2}120$
	3			$.0^{5}805$	$.0^{4}430$	$.0^{3}181$	$.0^{3}634$	$.0^{2}193$	$.0^{2}522$
	4				$.0^{3}206$	$.0^{3}777$	$.0^{2}245$	$.0^{2}668$	.0162
	5					$.0^{2}263$	$.0^{2}746$	.0183	.0399
	6						.0190	.0422	.0832
	7							.0847	.1514
	8								.2468
18	1	$.0^{9}110$	$.0^{8}209$	$.0^{7}209$	$.0^{6}147$	$.0^{6}806$	$.0^{5}371$	$.0^{4}143$	$.0^{4}530$
	2		$.0^{7}358$	$.0^{6}322$	$.0^{5}203$	$.0^{4}100$	$.0^{4}414$	$.0^{3}148$	$.0^{3}472$
	3			$.0^{5}262$	$.0^{4}148$	$.0^{4}661$	$.0^{3}245$	$.0^{3}790$	$.0^{2}226$
	4				$.0^{4}760$	$.0^{3}305$	$.0^{2}102$	$.0^{2}297$	$.0^{2}763$
	5					$.0^{2}111$	$.0^{2}335$	$.0^{2}880$	.0205
	6						$.0^{2}920$	.0219	.0461
	7							.0472	.0906
	8								.1588

Table I (Continued)

n	r \ x	8	9	10	11	12	13	14	15	16
17	1	$.0^{3}463$	$.0^{2}134$	$.0^{2}362$	$.0^{2}920$	.0222	.0513	.1136	.2424	
Cont'd.	2	$.0^{2}330$	$.0^{2}834$	.0195	.0427	.0874	.1676	.3006		
	3	.0128	.0285	.0588	.1123	.1992	.3281			
	4	.0354	.0705	.1294	.2192	.3440				
	5	.0785	.1409	.2323	.3540					
	6	.1480	.2406	.3603						
	7	.2453	.3641							
	8	.3659								
18	1	$.0^{3}172$	$.0^{3}516$	$.0^{2}145$	$.0^{2}381$	$.0^{2}953$	.0227	.0519	.1143	.2429
Cont'd.	2	$.0^{2}136$	$.0^{2}362$	$.0^{2}888$	.0204	.0438	.0887	.1688	.3013	
	3	$.0^{2}582$	.0137	.0300	.0606	.1142	.2009	.3291		
	4	.0177	.0375	.0732	.1321	.2215	.3453			
	5	.0429	.0821	.1445	.2353	.3556				
	6	.0878	.1526	.2443	.3623					
	7	.1573	.2499	.3666						
	8	.2525	.3690							
	9	.3698								

Tablo 1 (Continued)

n	r \ x	0	1	2	3	4	5	6	7	8	9
19	1	$.0^{10}283$	$.0^{9}566$	$.0^{8}594$	$.0^{7}436$	$.0^{6}251$	$.0^{5}120$	$.0^{5}501$	$.0^{4}186$	$.0^{4}628$	$.0^{3}195$
	2		$.0^{7}102$	$.0^{7}973$	$.0^{6}646$	$.0^{5}335$	$.0^{4}145$	$.0^{4}545$	$.0^{3}$[illegible]82	$.0^{3}549$	$.0^{2}152$
	3			$.0^{6}838$	$.0^{5}503$	$.0^{4}237$	$.0^{4}928$	$.0^{3}314$	$.0^{3}946$	$.0^{2}257$	$.0^{2}638$
	4				$.0^{4}274$	$.0^{3}117$	$.0^{3}415$	$.0^{2}127$	$.0^{2}347$	$.0^{2}852$	.0191
	5					$.0^{3}453$	$.0^{2}146$	$.0^{2}407$	.0101	.0224	.0455
	6						$.0^{2}428$	.0109	.0244	.0496	.0918
	7							.0251	.0516	.0957	.1623
	8								.0969	.1650	.2574
	9									.2586	.3729
20	1	$.0^{11}725$	$.0^{9}152$	$.0^{8}168$	$.0^{7}128$	$.0^{7}771$	$.0^{6}385$	$.0^{5}167$	$.0^{5}644$	$.0^{4}225$	$.0^{4}727$
	2		$.0^{8}291$	$.0^{7}291$	$.0^{6}203$	$.0^{5}110$	$.0^{5}501$	$.0^{4}197$	$.0^{4}685$	$.0^{3}216$	$.0^{3}624$
	3			$.0^{6}265$	$.0^{5}168$	$.0^{5}832$	$.0^{4}343$	$.0^{3}122$	$.0^{3}386$	$.0^{2}110$	$.0^{2}287$
	4				$.0^{5}969$	$.0^{4}438$	$.0^{3}164$	$.0^{3}532$	$.0^{2}153$	$.0^{2}396$	$.0^{2}935$
	5					$.0^{3}180$	$.0^{3}616$	$.0^{2}182$	$.0^{2}477$	.0112	.0242
	6						$.0^{2}192$	$.0^{2}519$	.0124	.0268	.0527
	7							.0128	.0281	.0555	.1001
	8								.0564	.1025	.1703
	9									.1715	.2636
	10										.3762

Table I (Continued)

n	r \ x	10	11	12	13	14	15	16	17	18
19 -	1	$.0^3567$	$.0^2155$	$.0^2399$	$.0^2983$	.0232	.0525	.1149	.2432	
Cont'd	2	$.0^2391$	$.0^2938$	.0211	.0448	.0899	.1699	.3019		
	3	.0146	.0312	.0622	.1160	.2024	.3299			
	4	.0394	.0755	.1345	.2235	.3464				
	5	.0852	.1476	.2378	.3570					
	6	.1566	.2475	.3641						
	7	.2538	.3687							
	8	.3716								
	9									
20 Cont'd	1	$.0^3218$	$.0^3614$	$.0^2164$	$.0^2416$	.0101	.0236	.0530	.1154	.2436
	2	$.0^2167$	$.0^2418$	$.0^2983$	.0218	.0457	.0909	.1708	.3025	
	3	$.0^2691$	.0155	.0324	.0637	.1176	.2037	.3307		
	4	.0204	.0412	.0776	.1367	.2253	.3474			
	5	.0479	.0880	.1504	.2401	.3582				
	6	.0954	.1601	.2503	.3655					
	7	.1666	.2572	.3705						
	8	.2616	.3738							
	9	.3756								
	10									

Table 1 (Continued)

n	r \ x	0	1	2	3	4	5	6	7	8	9	10	11
25	1	$.0^{14}791$	$.0^{12}206$	$.0^{11}278$	$.0^{10}259$	$.0^{9}188$	$.0^{8}113$	$.0^{8}582$	$.0^{7}266$	$.0^{6}110$	$.0^{6}415$	$.0^{5}145$	$.0^{5}475$
	2		$.0^{11}495$	$.0^{10}619$	$.0^{9}535$	$.0^{8}359$	$.0^{7}199$	$.0^{7}951$	$.0^{6}401$	$.0^{5}152$	$.0^{5}530$	$.0^{4}170$	$.0^{4}510$
	3			$.0^{9}717$	$.0^{8}574$	$.0^{7}357$	$.0^{6}183$	$.0^{6}809$	$.0^{5}315$	$.0^{4}111$	$.0^{4}355$	$.0^{3}105$	$.0^{3}289$
	4				$.0^{7}426$	$.0^{6}245$	$.0^{5}117$	$.0^{5}477$	$.0^{4}172$	$.0^{4}559$	$.0^{3}166$	$.0^{3}452$	$.0^{2}115$
	5					$.0^{5}131$	$.0^{5}577$	$.0^{4}219$	$.0^{4}733$	$.0^{3}220$	$.0^{3}604$	$.0^{2}153$	$.0^{2}358$
	6						$.0^{4}236$	$.0^{4}832$	$.0^{3}258$	$.0^{3}721$	$.0^{2}183$	$.0^{2}429$	$.0^{2}931$
	7							$.0^{3}272$	$.0^{3}784$	$.0^{2}203$	$.0^{2}480$	.0104	.0210
	8								$.0^{2}210$	$.0^{2}506$	.0111	.0225	.0423
	9									.0114	.0232	.0438	.0768
	10										.0444	.0782	.1281
	11											.1289	.1981
	12												.2861

Table 1 (Continued)

n	r \ x	12	13	14	15	16	17	18	19	20	21	22	23
25	1	$.0^4147$	$.0^4428$	$.0^3119$	$.0^3318$	$.0^3815$	$.0^2201$	$.0^2481$	.0111	.0251	.0549	.1173	.2449
	2	$.0^3143$	$.0^3381$	$.0^3961$	$.0^2231$	$.0^2529$	.0116	.0244	.0491	.0947	.1743	.3046	
(Cont.)	3	$.0^3744$	$.0^2181$	$.0^2415$	$.0^2904$	.0187	.0369	.0692	.1234	.2087	.3336		
	e	$.0^2272$	$.0^2606$	.0127	.0253	.0477	.0853	.1445	.2317	.3510			
	5	$.0^2783$	.0161	.0311	.0568	.0982	.1604	.2481	.3626				
	6	.0189	.0359	.0642	.1083	.1726	.2601	.3708					
	7	.0396	.0699	.1160	.1818	.2690	.3768						
	8	.0741	.1218	.1886	.2756	.3812							
	9	.1258	.1934	.2804	.3844								
	10	.1965	.2836	.3866									
	11	.2855	.3881										
	12	.3888											

Table 1 (Continued)

n	r \ x	0	1	2	3	4	5	6	7	8	9	10	11	12	13
30	1	$.0^{17}846$	$.0^{15}262$	$.0^{14}419$	$.0^{13}461$	$.0^{12}392$	$.0^{11}274$	$.0^{10}165$	$.0^{10}871$	$.0^{9}414$	$.0^{8}179$	$.0^{8}717$	$.0^{7}267$	$.0^{7}935$	$.0^{6}309$
	2		$.0^{14}762$	$.0^{12}114$	$.0^{11}118$	$.0^{11}939$	$.0^{10}616$	$.0^{9}346$	$.0^{8}171$	$.0^{8}760$	$.0^{7}307$	$.0^{6}115$	$.0^{6}398$	$.0^{5}130$	$.0^{5}398$
	3			$.0^{11}161$	$.0^{10}155$	$.0^{9}116$	$.0^{9}714$	$.0^{8}376$	$.0^{7}174$	$.0^{7}723$	$.0^{6}274$	$.0^{6}954$	$.0^{5}309$	$.0^{5}937$	$.0^{4}268$
	4				$.0^{9}141$	$.0^{9}988$	$.0^{8}569$	$.0^{7}281$	$.0^{6}122$	$.0^{6}475$	$.0^{5}168$	$.0^{5}549$	$.0^{4}166$	$.0^{4}471$	$.0^{3}125$
	5					$.0^{8}649$	$.0^{7}351$	$.0^{6}162$	$.0^{6}662$	$.0^{5}242$	$.0^{5}803$	$.0^{4}245$	$.0^{4}695$	$.0^{3}184$	$.0^{3}458$
	6						$.0^{6}178$	$.0^{6}774$	$.0^{5}296$	$.0^{4}101$	$.0^{4}316$	$.0^{4}905$	$.0^{3}240$	$.0^{3}597$	$.0^{2}139$
	7							$.0^{5}316$	$.0^{4}113$	$.0^{4}365$	$.0^{3}107$	$.0^{3}287$	$.0^{3}715$	$.0^{2}166$	$.0^{2}364$
	8								$.0^{4}382$	$.0^{3}115$	$.0^{3}317$	$.0^{3}802$	$.0^{2}188$	$.0^{2}411$	$.0^{2}843$
	9									$.0^{3}328$	$.0^{3}847$	$.0^{2}201$	$.0^{2}443$	$.0^{2}912$	.0176
	10										$.0^{2}206$	$.0^{2}460$	$.0^{2}955$	.0185	.0336
	11											$.0^{2}969$	.0189	.0346	.0594
	12												.0349	.0602	.0977
	13													.0982	.1507
	14														.2195

Table 1 (Continued)

n	r \ x	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
30	1	$.0^{6}972$	$.0^{5}292$	$.0^{5}838$	$.0^{4}232$	$.0^{4}618$	$.0^{3}159$	$.0^{3}399$	$.0^{3}968$	$.0^{2}229$	$.0^{2}527$	.0119	.0261	.0562	.1186	.2458
	2	$.0^{4}116$	$.0^{4}321$	$.0^{4}849$	$.0^{3}216$	$.0^{3}525$	$.0^{2}123$	$.0^{2}279$	$.0^{2}609$	.0128	.0262	.0514	.0973	.1766	.3060	
(Cont.)	3	$.0^{4}724$	$.0^{3}186$	$.0^{3}455$	$.0^{2}106$	$.0^{2}238$	$.0^{2}512$	.0106	.0210	.0399	.0727	.1271	.2119	.3354		
	4	$.0^{3}316$	$.0^{3}753$	$.0^{2}171$	$.0^{2}370$	$.0^{2}767$	.0152	.0287	.0521	.0903	.1495	.2358	.3532			
	5	$.0^{2}108$	$.0^{2}239$	$.0^{2}506$	.0102	.0195	.0358	.0626	.1046	.1667	.2531	.3653				
	6	$.0^{2}305$	$.0^{2}635$	.0125	.0235	.0420	.0716	.1163	.1802	.2660	.3740					
	7	$.0^{2}749$	.0146	.0269	.0473	.0790	.1258	.1908	.2760	.3805						
	8	.0163	.0298	.0517	.0851	.1334	.1993	.2837	.3855							
	9	.0320	.0551	.0899	.1395	.2059	.2898	.3894								
	10	.0577	.0936	.1441	.2110	.2944	.3923									
	11	.0962	.1475	.2148	.2980	.3946										
	12	.1497	.2174	.3004	.3962											
	13	.2189	.3021	.3974												
	14	.3029	.3981													
	15	.3983														

TABLE 1 (Continued)

n	r \ x	0	1	2	3	4	5	6	7	8	9	10	11	12	13
35	1	$.0^{20}891$	$.0^{18}321$	$.0^{17}594$	$.0^{16}752$	$.0^{15}733$	$.0^{14}587$	$.0^{13}401$	$.0^{12}240$	$.0^{11}129$	$.0^{11}632$	$.0^{10}284$	$.0^{9}119$	$.0^{9}466$	$.0^{8}172$
	2		$.0^{16}109$	$.0^{15}191$	$.0^{14}229$	$.0^{13}211$	$.0^{12}160$	$.0^{11}103$	$.0^{11}585$	$.0^{10}297$	$.0^{9}137$	$.0^{9}581$	$.0^{8}229$	$.0^{8}844$	$.0^{7}293$
	3			$.0^{14}317$	$.0^{13}359$	$.0^{12}313$	$.0^{11}224$	$.0^{10}137$	$.0^{10}733$	$.0^{9}352$	$.0^{8}153$	$.0^{8}614$	$.0^{7}228$	$.0^{7}792$	$.0^{6}259$
	4				$.0^{12}385$	$.0^{11}318$	$.0^{10}215$	$.0^{9}124$	$.0^{9}630$	$.0^{8}286$	$.0^{7}118$	$.0^{7}445$	$.0^{6}156$	$.0^{6}512$	$.0^{5}158$
	5					$.0^{10}248$	$.0^{9}159$	$.0^{9}869$	$.0^{8}416$	$.0^{7}179$	$.0^{7}697$	$.0^{6}249$	$.0^{6}826$	$.0^{5}256$	$.0^{5}743$
	6						$.0^{9}964$	$.0^{8}499$	$.0^{7}226$	$.0^{7}920$	$.0^{6}339$	$.0^{5}115$	$.0^{5}359$	$.0^{4}105$	$.0^{4}289$
	7							$.0^{7}244$	$.0^{6}105$	$.0^{6}404$	$.0^{5}141$	$.0^{5}451$	$.0^{4}134$	$.0^{4}370$	$.0^{4}962$
	8								$.0^{6}428$	$.0^{5}156$	$.0^{5}514$	$.0^{4}156$	$.0^{4}438$	$.0^{3}115$	$.0^{3}282$
	9									$.0^{5}537$	$.0^{4}168$	$.0^{4}483$	$.0^{3}128$	$.0^{3}319$	$.0^{3}743$
	10										$.0^{4}498$	$.0^{3}136$	$.0^{3}342$	$.0^{3}805$	$.0^{2}178$
	11											$.0^{3}350$	$.0^{3}838$	$.0^{2}187$	$.0^{2}392$
	12												$.0^{2}190$	$.0^{2}403$	$.0^{2}802$
	13													$.0^{2}811$	.0153
	14														.0275
	15														
	16														
	17														

Table 1 (Continued)

n	r \ x	14	15	16	17	18	19	20	21	22	23	24	25	26	27
35	1	$.0^{8}602$	$.0^{7}201$	. 640	$.0^{6}196$	$.0^{6}576$	$.0^{5}164$	$.0^{5}450$	$.0^{4}120$	$.0^{4}311$	$.0^{4}784$	$.0^{3}193$	$.0^{3}463$	$.0^{2}109$	$.0^{2}249$
(Cont.)	2	$.0^{7}963$	$.0^{6}301$	$.0^{6}898$	$.0^{5}257$	$.0^{5}704$	$.0^{4}186$	$.0^{4}475$	$.0^{3}117$	$.0^{3}279$	$.0^{3}646$	$.0^{2}145$	$.0^{2}316$	$.0^{2}669$	.0138
	3	$.0^{6}800$	$.0^{5}235$	$.0^{5}657$	$.0^{4}176$	$.0^{4}451$	$.0^{3}111$	$.0^{3}264$	$.0^{3}604$	$.0^{2}133$	$.0^{2}285$	$.0^{2}588$	.0117	.0226	.0420
	4	$.0^{5}459$	$.0^{4}127$	$.0^{4}333$	$.0^{4}836$	$.0^{3}201$	$.0^{3}464$	$.0^{2}103$	$.0^{2}219$	$.0^{2}450$	$.0^{2}890$	.0170	.0312	.0552	.0936
	5	$.0^{4}204$	$.0^{4}531$	$.0^{3}131$	$.0^{3}106$	$.0^{3}700$	$.0^{2}151$	$.0^{2}314$	$.0^{2}626$	.0120	.0221	.0391	.0667	.1091	.1710
	6	$.0^{4}748$	$.0^{3}184$	$.0^{3}429$	$.0^{3}750$	$.0^{2}203$	$.0^{2}412$	$.0^{2}803$	.0150	.0269	.0464	.0767	.1218	.1853	.2700
	7	$.0^{3}236$	$.0^{3}547$	$.0^{2}120$	$.0^{2}232$	$.0^{2}506$	$.0^{2}970$	.0178	.0313	.0528	.0853	.1324	.1969	.2806	[illegible]
	8	$.0^{3}654$	$.0^{2}144$	$.0^{2}299$	$.0^{2}573$	.0112	.0203	.0351	.0583	.0926	.1411	.2063	.2891	.3883	
	9	$.0^{2}163$	$.0^{2}339$	$.0^{2}668$	.0123	.0225	.0385	.0630	.0987	.1484	.2139	.2958	.3925		
	10	$.0^{2}370$	$.0^{2}729$	.0136	.0241	.0412	.0669	.1038	.1544	.2202	.3013	.3959			
	11	$.0^{2}774$	.0145	.0257	.0434	.0700	.1079	.1592	.2252	.3057	.3986				
	12	.0150	.0267	.0451	.0724	.1111	.1631	.2292	.3092	.4008					
	13	.0273	.0462	.0742	.1135	.1659	.2323	.3119	.4025						
	14	.0467	.0752	.1150	.1680	.2345	.3139	.4038							
	15	.0755	.1158	.1691	.2360	.3154	.4047								
	16		.1695	.2367	.3162	.4053									
	17			.3165	.4056										

Table 1 (Continued)

n	r \ x	28	29	30	31	32	33
35	1	$.0^2561$	.0124	.0268	.0571	.1196	.2464
(Cont.)	2	.0274	.0530	.0990	.1782	.3069	
	3	.0753	.1297	.2141	.3366		
	4	.1529	.2386	.3548			
	5	.2565	.3672				
	6	.3762					
	7						
	8						
	9						
	10						
	11						
	12						
	13						
	14						
	15						
	16						
	17						

TABLE 1 (Continued)

n	r \ x	0	1	2	3	4	5	6	7	8	9	10	11	12
40	1	$.0^{23}930$	$.0^{21}381$	$.0^{20}801$	$.0^{18}115$	$.0^{7}126$	$.0^{16}114$	$.0^{16}871$	$.0^{15}585$	$.0^{14}351$	$.0^{13}191$	$.0^{13}955$	$.0^{12}443$	$.0^{11}192$
	2		$.0^{19}149$	$.0^{18}298$	$.0^{17}407$	$.0^{16}426$	$.0^{15}365$	$.0^{14}266$	$.0^{13}170$	$.0^{13}971$	$.0^{12}503$	$.0^{11}239$	$.0^{10}105$	$.0^{10}433$
	3			$.0^{17}567$	$.0^{16}738$	$.0^{15}736$	$.0^{14}601$	$.0^{13}417$	$.0^{12}254$	$.0^{11}138$	$.0^{11}679$	$.0^{10}307$	$.0^{9}128$	$.0^{9}502$
	4				$.0^{15}914$	$.0^{14}868$	$.0^{13}675$	$.0^{12}447$	$.0^{11}259$	$.0^{10}134$	$.0^{10}627$	$.0^{9}270$	$.0^{8}107$	$.0^{8}399$
	5					$.0^{13}786$	$.0^{12}582$	$.0^{11}367$	$.0^{10}202$	$.0^{10}997$	$.0^{9}445$	$.0^{8}182$	$.0^{8}690$	$.0^{7}244$
	6						$.0^{11}411$	$.0^{10}247$	$.0^{9}130$	$.0^{9}608$	$.0^{8}259$	$.0^{7}104$	$.0^{7}364$	$.0^{6}122$
	7							$.0^{9}141$	$.0^{9}707$	$.0^{8}316$	$.0^{7}128$	$.0^{7}475$	$.0^{6}163$	$.0^{6}523$
	8								$.0^{8}337$	$.0^{7}144$	$.0^{7}555$	$.0^{6}196$	$.0^{6}643$	$.0^{5}196$
	9									$.0^{7}584$	$.0^{6}215$	$.0^{6}725$	$.0^{5}226$	$.0^{5}658$
	10										$.0^{6}754$	$.0^{5}242$	$.0^{5}722$	$.0^{4}200$
	11											$.0^{5}744$	$.0^{4}211$	$.0^{4}558$
	12												$.0^{4}571$	$.0^{3}144$
	13													$.0^{3}347$

Table 1 (Continued)

n	r \ x	13	14	15	16	17	18	19	20	21	22	23	24	25
40	1	$.0^{11}783$	$.0^{10}302$	$.0^{9}111$	$.0^{9}387$	$.0^{8}130$	$.0^{8}419$	$.0^{7}130$	$.0^{7}390$	$.0^{6}113$	$.0^{6}319$	$.0^{6}874$	$.0^{5}233$	$.0^{5}606$
(cont.)	2	$.0^{9}167$	$.0^{9}612$	$.0^{8}212$	$.0^{8}703$	$.0^{7}223$	$.0^{7}677$	$.0^{6}198$	$.0^{6}559$	$.0^{5}152$	$.0^{5}403$	$.0^{4}103$	$.0^{4}256$	$.0^{4}620$
	3	$.0^{8}184$	$.0^{8}639$	$.0^{7}210$	$.0^{7}659$	$.0^{6}197$	$.0^{6}567$	$.0^{5}157$	$.0^{5}417$	$.0^{4}107$	$.0^{4}265$	$.0^{4}637$	$.0^{3}148$	$.0^{3}335$
	4	$.0^{7}139$	$.0^{7}458$	$.0^{6}143$	$.0^{6}425$	$.0^{5}121$	$.0^{5}328$	$.0^{5}856$	$.0^{4}215$	$.0^{4}520$	$.0^{3}122$	$.0^{3}275$	$.0^{3}599$	$.0^{2}127$
	5	$.0^{7}808$	$.0^{6}253$	$.0^{6}750$	$.0^{5}212$	$.0^{5}570$	$.0^{4}147$	$.0^{4}363$	$.0^{4}862$	$.0^{3}197$	$.0^{3}435$	$.0^{3}924$	$.0^{2}190$	$.0^{2}377$
	6	$.0^{6}386$	$.0^{5}115$	$.0^{5}324$	$.0^{5}868$	$.0^{4}222$	$.0^{4}542$	$.0^{3}127$	$.0^{3}286$	$.0^{3}619$	$.0^{2}129$	$.0^{2}259$	$.0^{2}502$	$.0^{2}939$
	7	$.0^{5}157$	$.0^{5}445$	$.0^{4}119$	$.0^{4}304$	$.0^{4}739$	$.0^{3}172$	$.0^{3}382$	$.0^{3}815$	$.0^{2}167$	$.0^{2}330$	$.0^{2}628$	.0115	.0203
	8	$.0^{5}562$	$.0^{4}151$	$.0^{4}387$	$.0^{4}938$	$.0^{3}217$	$.0^{3}479$	$.0^{2}101$	$.0^{2}205$	$.0^{2}400$	$.0^{2}749$	.0135	.0234	.0392
	9	$.0^{4}179$	$.0^{4}461$	$.0^{3}112$	$.0^{3}259$	$.0^{3}571$	$.0^{2}120$	$.0^{2}241$	$.0^{2}466$	$.0^{2}862$	.0153	.0263	.0433	.0687
	10	$.0^{4}520$	$.0^{3}128$	$.0^{3}296$	$.0^{3}651$	$.0^{2}137$	$.0^{2}274$	$.0^{2}525$	$.0^{2}964$	.0170	.0288	.0469	.0736	.1111
	11	$.0^{3}139$	$.0^{3}324$	$.0^{3}716$	$.0^{2}151$	$.0^{2}301$	$.0^{2}575$	.0105	.0184	.0310	.0500	.0777	.1162	.1674
	12	$.0^{3}341$	$.0^{3}762$	$.0^{2}161$	$.0^{2}323$	$.0^{2}616$	.0112	.0196	.0328	.0526	.0812	.1205	.1722	.2372
	13	$.0^{3}785$	$.0^{2}167$	$.0^{2}337$	$.0^{2}646$	.0118	.0205	.0343	.0548	.0840	.1240	.1762	.2412	.3184
	14	$.0^{2}169$	$.0^{2}345$	$.0^{2}664$	.0122	.0212	.0354	.0564	.0863	.1269	.1794	.2444	.3211	.4075
	15		$.0^{2}670$	.0123	.0216	.0361	.0576	.0880	.1290	.1819	.2469	.3233	.4088	
	16			.0218	.0365	.0583	.0891	.1305	.1837	.2489	.3250	.4099		
	17				.0585	.0896	.1314	.1849	.2502	.3262	.4106			
	18					.1317	.1855	.2510	.3271	.4112				
	19						.2513	.3275	.4115					
	20							.4116						

Table 1 (Continued)

n	r \ x	26	27	28	29	30	31	32	33	34	35	36	37	38
40	1	$.0^4154$	$.0^4382$	$.0^4927$	$.0^3221$	$.0^3515$	$.0^2118$	$.0^2265$	$.0^2587$	.0128	.0274	.0578	.1203	.2468
	2	$.0^3146$	$.0^3335$	$.0^3747$	$.0^2163$	$.0^2346$	$.0^2716$	.0144	.0284	.0542	.1004	.1794	.3077	
(cont.)	3	$.0^3733$	$.0^2156$	$.0^2322$	$.0^2647$	.0126	.0238	.0436	.0772	.1317	.2158	.3316		
	4	$.0^2259$	$.0^2514$	$.0^2983$	.0184	.0330	.0574	.0963	.1555	.2407	.3559			
	5	$.0^2724$	.0134	.0241	.0417	.0697	.1123	.1741	.2589	.3685				
	6	.0169	.0295	.0497	.0805	.1258	.1890	.2729	.3777					
	7	.0346	.0568	.0899	.1371	.2012	.2839	.3848						
	8	.0631	.0980	.1467	.2112	.2928	.3903							
	9	.1051	.1548	.2196	.3000	.3948								
	10	.1616	.2265	.3060	.3984									
	11	.2323	.3109	.4014										
	12	.3150	.4038											
	13	.4058												

Table 1 (Continued)

n	r \ x	0	1	2	3	4	5	6	7	8	9	10
45	1	$.0^{26}963$	$.0^{24}443$	$.0^{22}104$	$.0^{21}167$	$.0^{20}204$	$.0^{19}204$	$.0^{18}173$	$.0^{17}129$	$.0^{17}854$	$.0^{16}512$	$.0^{15}282$
	2		$.0^{22}195$	$.0^{21}439$	$.0^{20}673$	$.0^{19}789$	$.0^{18}755$	$.0^{17}614$	$.0^{16}437$	$.0^{15}277$	$.0^{14}159$	$.0^{14}835$
	3			$.0^{20}946$	$.0^{18}139$	$.0^{17}156$	$.0^{16}143$	$.0^{15}111$	$.0^{15}757$	$.0^{14}459$	$.0^{13}252$	$.0^{12}127$
	4				$.0^{17}195$	$.0^{16}210$	$.0^{15}184$	$.0^{14}137$	$.0^{14}893$	$.0^{13}518$	$.0^{12}272$	$.0^{11}131$
	5					$.0^{15}242$	$.0^{14}181$	$.0^{13}129$	$.0^{13}807$	$.0^{12}449$	$.0^{11}226$	$.0^{10}104$
	6						$.0^{13}146$	$.0^{13}998$	$.0^{12}596$	$.0^{11}317$	$.0^{10}153$	$.0^{10}674$
	7							$.0^{12}654$	$.0^{11}374$	$.0^{10}191$	$.0^{10}879$	$.0^{9}371$
	8								$.0^{10}205$	$.0^{9}100$	$.0^{9}442$	$.0^{8}179$
	9									$.0^{9}468$	$.0^{8}198$	$.0^{8}767$
	10										$.0^{8}803$	$.0^{7}298$
	11											$.0^{6}106$

Table 1 (Continued)

n	r \ x	11	12	13	14	15	16	17	18	19	20
45	1	$.0^{14}143$	$.0^{14}681$	$.0^{13}304$	$.0^{12}128$	$.0^{12}512$	$.0^{11}195$	$.0^{11}712$	$.0^{10}249$	$.0^{10}840$	$.0^{9}273$
	2	$.0^{13}406$	$.0^{12}184$	$.0^{12}785$	$.0^{11}316$	$.0^{10}120$	$.0^{10}437$	$.0^{9}152$	$.0^{9}506$	$.0^{8}162$	$.0^{8}500$
(cont.)	3	$.0^{12}589$	$.0^{11}255$	$.0^{10}104$	$.0^{10}399$	$.0^{9}145$	$.0^{9}503$	$.0^{8}167$	$.0^{8}528$	$.0^{7}161$	$.0^{7}472$
	4	$.0^{11}583$	$.0^{10}242$	$.0^{10}940$	$.0^{9}345$	$.0^{8}120$	$.0^{8}397$	$.0^{7}125$	$.0^{7}378$	$.0^{6}110$	$.0^{6}307$
	5	$.0^{10}443$	$.0^{9}176$	$.0^{9}653$	$.0^{8}229$	$.0^{8}760$	$.0^{7}240$	$.0^{7}724$	$.0^{6}209$	$.0^{6}578$	$.0^{5}154$
	6	$.0^{9}274$	$.0^{8}104$	$.0^{8}371$	$.0^{7}124$	$.0^{7}395$	$.0^{6}119$	$.0^{6}343$	$.0^{6}945$	$.0^{5}250$	$.0^{5}633$
	7	$.0^{8}145$	$.0^{8}526$	$.0^{7}179$	$.0^{7}575$	$.0^{6}175$	$.0^{6}505$	$.0^{5}139$	$.0^{5}365$	$.0^{5}921$	$.0^{4}223$
	8	$.0^{8}668$	$.0^{7}232$	$.0^{7}758$	$.0^{6}233$	$.0^{6}677$	$.0^{5}187$	$.0^{5}492$	$.0^{4}124$	$.0^{4}299$	$.0^{4}691$
	9	$.0^{7}275$	$.0^{7}916$	$.0^{6}286$	$.0^{6}842$	$.0^{5}234$	$.0^{5}620$	$.0^{4}156$	$.0^{4}376$	$.0^{4}866$	$.0^{3}192$
	10	$.0^{6}102$	$.0^{6}327$	$.0^{6}978$	$.0^{5}276$	$.0^{5}735$	$.0^{4}186$	$.0^{4}449$	$.0^{3}103$	$.0^{3}228$	$.0^{3}484$
	11	$.0^{6}349$	$.0^{5}107$	$.0^{5}306$	$.0^{5}827$	$.0^{4}211$	$.0^{4}513$	$.0^{3}119$	$.0^{3}262$	$.0^{3}553$	$.0^{2}112$
	12	$.0^{6}110$	$.0^{5}322$	$.0^{5}886$	$.0^{4}229$	$.0^{4}562$	$.0^{3}131$	$.0^{3}290$	$.0^{3}613$	$.0^{2}124$	$.0^{2}241$
	13		$.0^{5}906$	$.0^{4}239$	$.0^{4}593$	$.0^{3}139$	$.0^{3}311$	$.0^{3}661$	$.0^{2}134$	$.0^{2}260$	$.0^{2}485$
	14			$.0^{4}604$	$.0^{3}144$	$.0^{3}324$	$.0^{3}694$	$.0^{2}141$	$.0^{2}275$	$.0^{2}513$	$.0^{2}918$
	15				$.0^{3}329$	$.0^{3}710$	$.0^{2}146$	$.0^{2}285$	$.0^{2}534$	$.0^{2}955$	.0164
	16					$.0^{2}147$	$.0^{2}291$	$.0^{2}546$	$.0^{2}980$	.0169	.0278
	17						$.0^{2}550$	$.0^{2}993$	.0171	.0283	.0449
	18							.0172	.0286	.0454	.0694
	19								.0456	.0698	.1026
	20									.1028	.1459
	21										.1996

Table 1 (Continued)

n	r \ x	21	22	23	24	25	26	27	28	29	30	31	32
45	1	$.0^{9}858$	$.0^{8}261$	$.0^{8}772$	$.0^{7}222$	$.0^{7}622$	$.0^{6}170$	$.0^{6}453$	$.0^{5}118$	$.0^{5}301$	$.0^{5}753$	$.0^{4}185$	$.0^{4}444$
	2	$.0^{7}149$	$.0^{7}430$	$.0^{6}120$	$.0^{6}326$	$.0^{6}862$	$.0^{5}221$	$.0^{5}555$	$.0^{4}136$	$.0^{4}323$	$.0^{4}753$	$.0^{3}172$	$.0^{3}382$
(cont.)	3	$.0^{6}134$	$.0^{6}366$	$.0^{6}969$	$.0^{5}249$	$.0^{5}621$	$.0^{4}151$	$.0^{4}355$	$.0^{4}816$	$.0^{3}182$	$.0^{3}398$	$.0^{3}845$	$.0^{2}175$
	4	$.0^{6}825$	$.0^{5}215$	$.0^{5}540$	$.0^{4}131$	$.0^{4}310$	$.0^{4}711$	$.0^{3}158$	$.0^{3}343$	$.0^{3}721$	$.0^{2}147$	$.0^{2}293$	$.0^{2}567$
	5	$.0^{5}394$	$.0^{5}974$	$.0^{4}233$	$.0^{4}538$	$.0^{3}121$	$.0^{3}262$	$.0^{3}551$	$.0^{2}113$	$.0^{2}223$	$.0^{2}430$	$.0^{2}803$	.0146
	6	$.0^{4}155$	$.0^{4}364$	$.0^{4}828$	$.0^{3}182$	$.0^{3}387$	$.0^{3}798$	$.0^{2}159$	$.0^{2}308$	$.0^{2}577$	.0105	.0185	.0316
	7	$.0^{4}520$	$.0^{3}117$	$.0^{3}253$	$.0^{3}529$	$.0^{2}107$	$.0^{2}210$	$.0^{2}397$	$.0^{2}729$	.0130	.0223	.0372	.0599
	8	$.0^{3}154$	$.0^{3}330$	$.0^{3}681$	$.0^{2}136$	$.0^{2}262$	$.0^{2}488$	$.0^{2}800$	.0154	.0259	.0423	.0669	.1021
	9	$.0^{3}408$	$.0^{3}835$	$.0^{2}165$	$.0^{2}314$	$.0^{2}577$	.0103	.0176	.0293	.0470	.0730	.1098	.1595
	10	$.0^{3}984$	$.0^{2}193$	$.0^{2}363$	$.0^{2}662$	.0116	.0197	.0324	.0513	.0786	.1165	.1669	.2312
	11	$.0^{2}218$	$.0^{2}409$	$.0^{2}739$	.0129	.0216	.0351	.0551	.0835	.1223	.1733	.2375	.3147
	12	$.0^{2}450$	$.0^{2}809$	.0140	.0233	.0376	.0584	.0877	.1274	.1789	.2429	.3192	.4060
	13	$.0^{2}869$	.0150	.0248	.0397	.0613	.0914	.1317	.1836	.2475	.3230	.4082	
	14	.0158	.0261	.0415	.0637	.0945	.1354	.1876	.2514	.3262	.4101		
	15	.0271	.0430	.0658	.0971	.1385	.1909	.2546	.3288	.4117			
	16	.0441	.0674	.0992	.1410	.1936	.2573	.3311	.4130				
	17	.0686	.1008	.1430	.1958	.2595	.3329	.4140					
	18	.1019	.1444	.1975	.2611	.3343	.4149						
	19	.1454	.1987	.2624	.3354	.4156							
	20	.1994	.2632	.3362	.4161								
	21	.2636	.3366	.4164									
	22	.3368	.4166										

Table 1 (Continued)

n	r \ x	33	34	35	36	37	38	39	40	41	42	43
45	1	$.0^{3}105$	$.0^{3}244$	$.0^{3}558$	$.0^{2}125$	$.0^{2}278$	$.0^{2}607$	.0131	.0278	.0583	.1208	.2472
	2	$.0^{3}832$	$.0^{2}177$	$.0^{2}369$	$.0^{2}753$	.0150	.0291	.0551	.1014	.1804	.3082	
(cont.)	3	$.0^{2}353$	$.0^{2}695$	.0133	.0248	.0448	.0786	.1332	.2170	.3383		
	4	.0107	.0195	.0345	.0592	.0983	.1574	.2423	.3568			
	5	.0256	.0437	.0721	.1148	.1765	.2608	.3695				
	6	.0522	.0834	.1288	.1918	.2751	.3789					
	7	.0934	.1407	.2044	.2864	.3861						
	8	.1508	.2149	.2956	.3918							
	9	.2237	.3031	.3964								
	10	.3094	.4002									
	11	.4033										

Table 1 (continued)

n	r \ x	0	1	2	3	4	5	6	7	8	9	10	11
50	1	$.0^{29}991$	$.0^{27}505$	$.0^{25}131$	$.0^{24}232$	$.0^{23}313$	$.0^{22}345$	$.0^{21}322$	$.0^{20}262$	$.0^{19}190$	$.0^{18}125$	$.0^{18}747$	$.0^{17}411$
	2		$.0^{25}248$	$.0^{24}620$	$.0^{22}105$	$.0^{21}137$	$.0^{20}145$	$.0^{19}130$	$.0^{18}101$	$.0^{18}707$	$.0^{17}445$	$.0^{16}257$	$.0^{15}137$
	3			$.0^{22}149$	$.0^{21}243$	$.0^{20}304$	$.0^{19}309$	$.0^{18}266$	$.0^{17}200$	$.0^{16}134$	$.0^{16}812$	$.0^{15}450$	$.0^{14}230$
	4				$.0^{20}382$	$.0^{19}459$	$.0^{18}449$	$.0^{17}372$	$.0^{16}269$	$.0^{15}173$	$.0^{14}101$	$.0^{14}536$	$.0^{13}264$
	5					$.0^{18}530$	$.0^{17}498$	$.0^{16}397$	$.0^{15}276$	$.0^{14}171$	$.0^{14}956$	$.0^{13}489$	$.0^{12}331$
	6						$.0^{16}450$	$.0^{15}345$	$.0^{14}231$	$.0^{13}138$	$.0^{13}740$	$.0^{12}364$	$.0^{11}165$
	7							$.0^{14}255$	$.0^{13}164$	$.0^{13}938$	$.0^{12}485$	$.0^{11}230$	$.0^{10}100$
	8								$.0^{12}101$	$.0^{12}559$	$.0^{11}278$	$.0^{10}126$	$.0^{10}531$
	9									$.0^{11}296$	$.0^{10}142$	$.0^{10}619$	$.0^{9}250$
	10										$.0^{10}652$	$.0^{9}274$	$.0^{8}107$
	11											$.0^{8}111$	$.0^{8}415$
	12												$.0^{7}149$

Table 1 (continued)

n	r \ x	12	13	14	15	16	17	18	19	20	21	22
50	1	$.0^{16}214$	$.0^{15}104$	$.0^{15}474$	$.0^{14}206$	$.0^{14}848$	$.0^{13}334$	$.0^{12}126$	$.0^{12}458$	$.0^{11}160$	$.0^{11}542$	$.0^{10}178$
(cont.)	2	$.0^{15}678$	$.0^{14}315$	$.0^{13}138$	$.0^{13}574$	$.0^{12}227$	$.0^{12}856$	$.0^{11}310$	$.0^{10}108$	$.0^{10}360$	$.0^{9}116$	$.0^{9}363$
	3	$.0^{13}110$	$.0^{13}489$	$.0^{12}206$	$.0^{12}820$	$.0^{11}311$	$.0^{10}112$	$.0^{10}389$	$.0^{9}129$	$.0^{9}414$	.0[illegible]128	$.0^{8}382$
	4	$.0^{12}121$	$.0^{12}517$	$.0^{11}209$	$.0^{11}799$	$.0^{10}290$	$.0^{9}101$	$.0^{9}334$	$.0^{8}107$	$.0^{8}326$	$.0^{8}964$	$.0^{7}275$
	5	$.0^{11}102$	$.0^{11}418$	$.0^{10}162$	$.0^{10}596$	$.0^{9}208$	$.0^{9}692$	$.0^{8}220$	$.0^{8}673$	$.0^{7}198$	$.0^{7}559$	$.0^{6}153$
	6	$.0^{11}698$	$.0^{10}276$	$.0^{9}103$	$.0^{9}363$	$.0^{8}122$	$.0^{8}389$	$.0^{7}119$	$.0^{7}348$	$.0^{7}979$	$.0^{6}265$	$.0^{6}694$
	7	$.0^{10}407$	$.0^{9}155$	$.0^{9}555$	$.0^{8}188$	$.0^{8}605$	$.0^{7}186$	$.0^{7}545$	$.0^{6}153$	$.0^{6}414$	$.0^{5}107$	$.0^{5}269$
	8	$.0^{9}207$	$.0^{9}759$	$.0^{8}261$	$.0^{8}851$	$.0^{7}263$	$.0^{7}776$	$.0^{6}219$	$.0^{6}590$	$.0^{5}153$	$.0^{5}381$	$.0^{5}917$
	9	$.0^{9}940$	$.0^{8}331$	$.0^{7}110$	$.0^{7}343$	$.0^{6}102$	$.0^{6}289$	$.0^{6}783$	$.0^{5}203$	$.0^{5}505$	$.0^{4}121$	$.0^{4}279$
	10	$.0^{8}385$	$.0^{7}130$	$.0^{7}415$	$.0^{6}125$	$.0^{6}358$	$.0^{6}975$	$.0^{5}254$	$.0^{5}632$	$.0^{4}151$	$.0^{4}348$	$.0^{4}771$
	11	$.0^{7}144$	$.0^{7}470$	$.0^{6}144$	$.0^{6}418$	$.0^{5}115$	$.0^{5}301$	$.0^{5}754$	$.0^{4}181$	$.0^{4}415$	$.0^{4}919$	$.0^{3}196$
	12	$.0^{7}500$	$.0^{6}157$	$.0^{6}462$	$.0^{5}129$	$.0^{5}341$	$.0^{5}860$	$.0^{4}207$	$.0^{4}477$	$.0^{3}106$	$.0^{3}224$	$.0^{3}460$
	13	$.0^{6}161$	$.0^{6}486$	$.0^{5}138$	$.0^{5}370$	$.0^{5}942$	$.0^{4}229$	$.0^{4}530$	$.0^{3}118$	$.0^{3}250$	$.0^{3}512$	$.0^{2}101$
	14		$.0^{5}141$	$.0^{5}385$	$.0^{5}995$	$.0^{4}244$	$.0^{4}570$	$.0^{3}127$	$.0^{3}271$	$.0^{3}556$	$.0^{2}110$	$.0^{2}208$
	15			$.0^{4}101$	$.0^{4}252$	$.0^{4}595$	$.0^{3}134$	$.0^{3}287$	$.0^{3}591$	$.0^{2}117$	$.0^{2}221$	$.0^{2}404$
	16				$.0^{4}603$	$.0^{3}137$	$.0^{3}297$	$.0^{3}615$	$.0^{2}122$	$.0^{2}232$	$.0^{2}$[illegible]24	$.0^{2}745$
	17					$.0^{3}301$	$.0^{3}627$	$.0^{2}125$	$.0^{2}239$	$.0^{2}438$	$.0^{2}771$	.0131
	18						$.0^{2}126$	$.0^{2}242$	$.0^{2}446$	$.0^{2}789$	.0134	.0219
	19							$.0^{2}449$	$.0^{2}797$	.0136	.0223	.0352
	20								.0137	.0225	.0356	.0543
	21									.0357	.0545	.0804
	22										.0806	.1150
	23											.1587

Table 1 (continued)

n	r \ x	23	24	25	26	27	28	29	30	31	32	33	34	35
50 (cont.)	1	$.0^{10}564$	$.0^{9}174$	$.0^{9}521$	$.0^{8}152$	$.0^{8}435$	$.0^{7}121$	$.0^{7}330$	$.0^{7}879$	$.0^{6}230$	$.0^{6}589$	$.0^{5}148$	$.0^{5}366$	$.0^{5}858$
	2	$.0^{8}410$	$.0^{8}323$	$.0^{8}921$	$.0^{7}256$	$.0^{7}692$	$.0^{6}183$	$.0^{6}471$	$.0^{5}119$	$.0^{5}292$	$.0^{5}705$	$.0^{4}166$	$.0^{4}385$	$.0^{4}873$
	3	$.0^{7}110$	$.0^{7}309$	$.0^{7}840$	$.0^{6}222$	$.0^{6}571$	$.0^{5}143$	$.0^{5}350$	$.0^{5}835$	$.0^{4}194$	$.0^{4}442$	$.0^{4}982$	$.0^{3}213$	$.0^{3}453$
	4	$.0^{7}759$	$.0^{6}203$	$.0^{6}527$	$.0^{5}133$	$.0^{5}325$	$.0^{5}774$	$.0^{4}180$	$.0^{4}406$	$.0^{4}896$	$.0^{3}193$	$.0^{3}404$	$.0^{2}827$	$.0^{2}165$
	5	$.0^{6}403$	$.0^{5}103$	$.0^{5}255$	$.0^{5}613$	$.0^{4}143$	$.0^{4}325$	$.0^{4}717$	$.0^{3}154$	$.0^{3}322$	$.0^{3}657$	$.0^{2}130$	$.0^{2}252$	$.0^{2}476$
	6	$.0^{5}175$	$.0^{5}429$	$.0^{4}101$	$.0^{4}233$	$.0^{4}519$	$.0^{3}112$	$.0^{3}236$	$.0^{3}483$	$.0^{3}962$	$.0^{2}186$	$.0^{2}350$	$.0^{2}641$	.0114
	7	$.0^{5}652$	$.0^{4}153$	$.0^{4}346$	$.0^{4}759$	$.0^{3}162$	$.0^{3}334$	$.0^{3}670$	$.0^{2}131$	$.0^{2}248$	$.0^{2}456$	$.0^{2}815$	.0142	.0239
	8	$.0^{4}213$	$.0^{4}477$	[illegible]	$.0^{3}218$	$.0^{3}443$	$.0^{3}876$	$.0^{2}168$	$.0^{2}313$	$.0^{2}565$	$.0^{2}991$	.0169	.0279	.0448
	9	$.0^{4}621$	$.0^{3}134$	$.0^{3}278$	$.0^{3}560$	$.0^{2}109$	$.0^{2}207$	$.0^{2}379$	$.0^{2}674$	.0116	.0195	.0317	.0500	.0765
	10	$.0^{3}165$	$.0^{3}340$	$.0^{3}679$	$.0^{2}131$	$.0^{2}245$	$.0^{2}444$	$.0^{2}781$	.0133	.0220	.0352	.0548	.0826	.1207
	11	$.0^{3}402$	$.0^{3}796$	$.0^{2}153$	$.0^{2}283$	$.0^{2}508$	$.0^{2}883$	.0149	.0243	.0385	.0591	.0880	.1271	.1779
	12	$.0^{3}907$	$.0^{2}173$	$.0^{2}318$	$.0^{2}567$	$.0^{2}978$	.0163	.0264	.0414	.0630	.0928	.1327	.1839	.2473
	13	$.0^{2}191$	$.0^{2}351$	$.0^{2}622$	.0106	.0177	.0283	.0440	.0664	.0970	.1376	.1891	.2522	.3264
	14	$.0^{2}380$	$.0^{2}670$	.0114	.0188	.0300	.0464	.0695	.1008	.1419	.1937	.2565	.3299	.4120
	15	$.0^{2}711$	.0121	.0199	.0315	.0485	.0721	.1041	.1456	.1976	.2603	.3329	.4138	
	16	.0126	.0207	[illegible]	.0502	.0744	.1069	.1488	.2009	.2634	.3355	[illegible]		
	17	.0214	.0338	.0517	.0763	.1092	.1515	.2039	.2661	.3377	.4165			
	18	.0346	.0528	.0779	.1112	.1537	.2062	.2684	.3395	.4176				
	19	.0537	.0791	.1127	.1555	.2081	.2703	.3410	.4185					
	20	.0799	.1138	.1569	.2096	.2718	.3422	.4192						
	21	.1146	.1579	.2108	.2729	.3432	.4198							
	22	.1585	.2115	.2737	.3440	.4202								
	23	.2119	.2742	.3444	.4206									
	24	.2744	.3447	.4207										
	25		.4208											

Table 1 (continued)

n	r \ x	36	37	38	39	40	41	42	43	44	45	46	47	48
50 (cont.)	1	$.0^4212$	$.0^4499$	$.0^3116$	$.0^3264$	$.0^3593$	$.0^2132$	$.0^2289$	$.0^2624$	.0133	.0281	.0587	.1212	.2475
	2	$.0^3194$	$.0^3423$	$.0^3904$	$.0^2189$	$.0^2389$	$.0^2783$	.0154	.0297	.0559	.1022	.1811	.3087	
	3	$.0^3941$	$.0^2191$	$.0^3379$	$.0^2734$	.0139	.0256	.0458	.0798	.1343	.2180	.3389		
	4	$.0^2322$	$.0^261$[illegible]	.0113	.0204	.0357	.0606	.0999	.1589	.2435	.3575			
	5	$.0^2870$	.0155	.0269	.0453	.0739	.1168	.1783	.2623	.3703				
	6	.0198	.0332	.0542	.0857	.1312	.1940	.2768	.3798					
	7	.0392	.0624	.0961	.1434	.2070	.2883	.3871						
	8	.0698	.1050	.1540	.2178	.2977	.3928							
	9	.1135	.1631	.2270	.3055	.3976								
	10	.1710	.2348	.3121	.4016									
	11	.2415	.3176	.4048										
	12	.3224	.4076											
	13	.4100												