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NOTE ON A PREDICTION EQUATION FOR THE SURFACE LAYER OF A TWO LAYER OCEAN

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Project 29

NOTE ON A PREDICTION EQUATION FOR THE SURFACE LAYER OF A TWO LAYER OCEAN

(Technical Report No. 5)

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ABSTRACT

Time changes in the thickness of a thin surface layer of a two layer ocean model are proportional to the curl of the wind stress if accelerations are sufficiently small.

INTRODUCTION

On page 500 of "The Oceans", Sverdrup, in speaking of the transport of water by the wind stress, says "...converging and diverging wind systems must, in certain regions lead to an accumulatio of light surface waters and, in other regions, to a rise of denser water from subsurface depths." We have exploited this idea in finding the response to be expected of a two layer ocean to wind stresses which persist. It is well known that the wind stress causes a transport which is proportional to the square of the wind speed and directed to the right of the wind vector. It can be seen that if we have a region of weak winds to the right of a region of strong winds, as for example in Figure 1, there will be a tendency for water to accumulate between these regions.

In a two layer ocean under certain conditions the accumulation leads to a change in the thickness of the surface layer which follows the equation:

$$\frac{\partial H}{\partial t} = \frac{1}{\rho' f}\left(\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y}\right).$$

In this equation:

H = thickness of a thin homogeneous top layer of a two layer ocean.

f = coriolis parameter.

τ = wind stress.

ρ' = density of surface fluid.

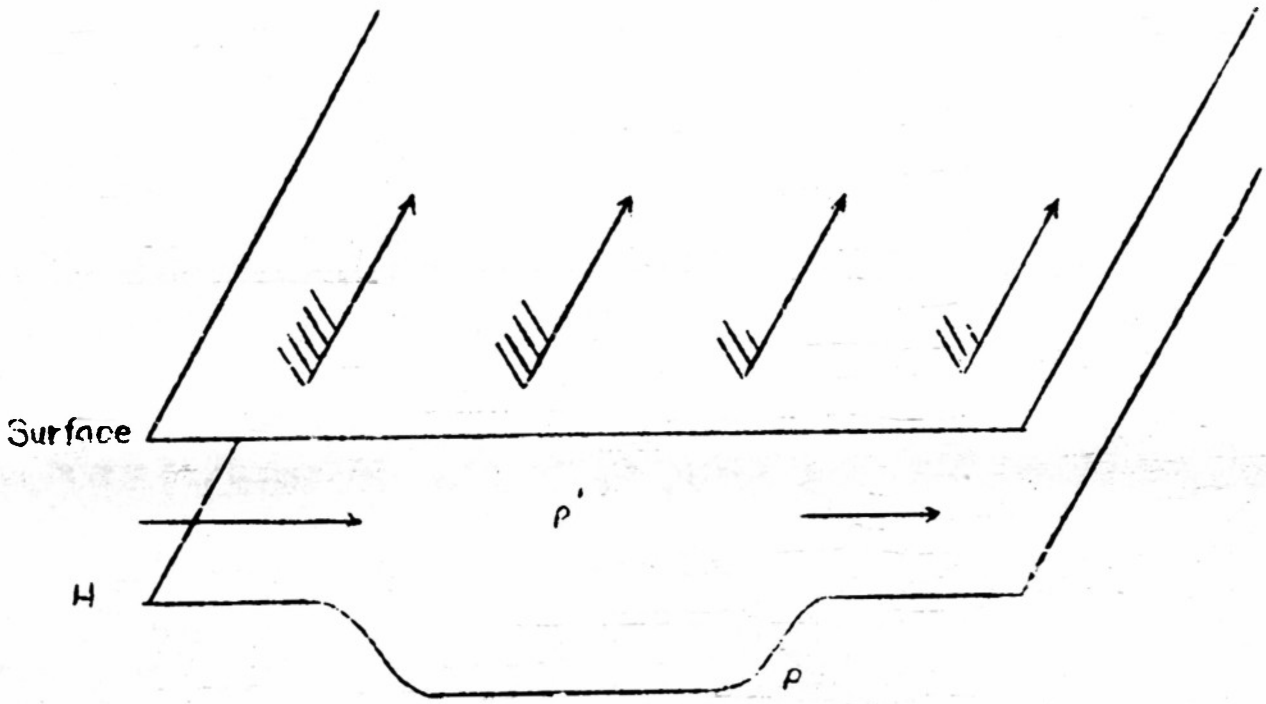

FIGURE 1

The surface layer with density ρ' is transported to the right of the winds. The strong winds transport more water into the region in the center of the drawing than is transported out by the weak winds. The resulting accumulation of water pushes the deep water of density ρ away from the center of the drawing.

EQUATIONS OF MOTION

The equations of motion for the homogeneous upper layer with density ρ' can be written:

$$\frac{Du}{dt} = -\frac{1}{\rho'}\frac{\partial p}{\partial x} + fv - \frac{1}{\rho'}\frac{\partial \tau_x}{\partial z} \qquad (1)$$

$$\frac{Dv}{dt} = -\frac{1}{\rho'}\frac{\partial p}{\partial y} - fu - \frac{1}{\rho'}\frac{\partial \tau_y}{\partial z} \, . \qquad (2)$$

We concern ourselves with the portion of the pressure and wind field that persists. We assume that the currents, set up by these pressures and the winds, change with time in such a way that we can say in equations (1) and (2)

$$\left|\frac{Du}{dt}\right| \ll \left|fv\right|$$

$$\left|\frac{Dv}{dt}\right| \ll \left|fu\right|$$

and equations (1) and (2) become:

$$fv = \frac{1}{\rho'}\frac{\partial p}{\partial x} + \frac{1}{\rho'}\frac{\partial \tau_x}{\partial z} \qquad (3)$$

$$fu = \frac{1}{\rho'}\frac{\partial p}{\partial y} - \frac{1}{\rho'}\frac{\partial \tau_y}{\partial z} \, . \qquad (4)$$

We assume that all currents are insignificant at and below the interface. These small currents result in negligible values of the stresses near the interface. (We cannot say the currents are zero near the interface because the water moves to bring about adjustments in the interface, following equation (3), but we assume these motions near the interface are primarily vertical in the upper layer of water and the total horizontal transport in the lower layer is spread over a very deep layer and the resulting currents are extremely small.)

CONTINUITY OF MASS

Though changes in the mean level of the ocean can possibly bring about quite large pressure gradients in the homogeneous water layer, the error introduced in the thickness of the upper layer by using the water surface as an origin rather than some fixed level line is small. Changes in the thickness of this layer are observed to be in the order of tens of feet* and water level changes are observed to be in the order of feet, at least as measured on coasts. We can say from observation, therefore, that our model should have such small density differences that when water in the upper layer accumulates in an area that it pushes the interface down and bottom water out of the way rather than raising the water surface by any significant amount.

The integral form of the continuity equation then becomes:

$$\frac{\partial H}{\partial t} = \frac{\partial}{\partial x}\int_{-H}^{S} u\,dz + \frac{\partial}{\partial y}\int_{-H}^{S} v\,dz .$$

We define Tx and Ty such that this can be written:

$$\frac{\partial H}{\partial t} = -\left(\frac{\partial T_x}{\partial x} + \frac{\partial T_y}{\partial y}\right). \qquad (5)$$

* See the values computed in Technical Report No. 4, Project 29 Texas A & M Research Foundation, ONR Contract N7onr-48703, NR 083-061, "Heat Budget of a Water Column-Autumn-North Atlantic," 1953.

COMPUTATION OF THE DIVERGENCE OF WATER TRANSPORT

We wish to find suitable values of the divergence, $\frac{\partial T_x}{\partial x} + \frac{\partial T_y}{\partial y}$ We do this by eliminating the pressure from equations (3) and (4) and integrating the resulting equation in the vertical direction. To eliminate the pressure we differentiate equation (3) with respect to y and equation (4) with respect to x and we find

$$f\frac{\partial v}{\partial y} = \frac{1}{\rho'}\frac{\partial^2 p}{\partial x \partial y} + \frac{1}{\rho'}\frac{\partial^2 \tau_x}{\partial z \partial y}$$

$$f\frac{\partial u}{\partial x} = -\frac{1}{\rho'}\frac{\partial^2 p}{\partial x \partial y} - \frac{1}{\rho'}\frac{\partial^2 \tau_y}{\partial z \partial x}.$$

We add these equations to get

$$f\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = -\frac{1}{\rho'}\left(\frac{\partial^2 \tau_y}{\partial z \partial x} - \frac{\partial^2 \tau_x}{\partial z \partial y}\right).$$

This equation is integrated from z = -H to z = s resulting in

$$f\int_{-H}^{s}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)dz = -\frac{1}{\rho'}\int_{-H}^{s}\left(\frac{\partial^2 \tau_y}{\partial x \partial z} - \frac{\partial^2 \tau_x}{\partial y \partial z}\right)dz,$$

This can be rewritten, using Leibnitz rule for differentiating integrals,

$$f\frac{\partial}{\partial x}\int_{-H}^{s} u\,dz + f\frac{\partial}{\partial y}\int_{-H}^{s} v\,dz + f u_H \overset{2}{\frac{\partial H}{\partial x}} + f v_H \overset{3}{\frac{\partial H}{\partial y}}$$

$$- f u_s \overset{4}{\frac{\partial s}{\partial x}} - f v_s \overset{5}{\frac{\partial s}{\partial y}}$$

$$= -\frac{1}{\rho'}\left(\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y}\right)\Bigg|^{s} \overset{6}{+ \frac{1}{\rho'}\left(\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y}\right)\Bigg|^{H}}.$$

We now use some of our assumptions:

Items 2 and 3 can be neglected because velocities are very small at the bottom of the surface layer and slopes due to large scale wind systems are small.

Items 4 and 5 can be neglected because the slope of the water surface is very small.

Item 6 can be neglected because velocities and stresses are very small at the bottom of the surface layer.

Using these facts and the definitions in equation (5) we are now left with the equation

$$f\left(\frac{\partial T_x}{\partial x} + \frac{\partial T_y}{\partial y}\right) = -\frac{1}{\rho'}\left(\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y}\right). \tag{6}$$

We substitute this result in equation (5) and find

$$\frac{\partial H}{\partial t} = \frac{1}{\rho' f}\left(\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y}\right). \tag{7}$$

Thus we have derived the prediction equation mentioned in the introduction.

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