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EXPERIMENTAL STUDIES OF TRANSPORT PHENOMENA IN HIGHLY
IONIZED GASES

BY

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TECHNICAL REPORT

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EXPERIMENTAL STUDIES OF TRANSPORT PHENOMENA IN HIGHLY IONIZED GASES

CHAPTER I

INTRODUCTION

In 1879 E. H. Hall¹ demonstrated that if an electric current exists in a conductor which is in a magnetic field, a potential difference is developed across the conductor in a direction normal to both the primary current and the magnetic field. The phenomenon, known as the Hall effect, was discovered and first studied in metallic conductors, and in such conductors, it is still of great interest, chiefly because the explanation of the observed phenomena depends upon a quantum theory of the solid state.

In 1886 Boltzmann² investigated the Hall effect in a Geissler discharge, and in 1901 H. A. Wilson³ made a careful study of the effect in different parts of the discharge. A further investigation was carried out in 1907 by Wilson and Martyn⁴, and the effect in mercury

¹E. H. Hall, Am. Jour. of Math. 2, 287 (1879).

²L. Boltzmann, Anz. Acad. Wiss. Wien 24, 217 (1886).

³H. A. Wilson, Proc. Phil. Soc. Camb. 11, 249 (1901).

⁴H. A. Wilson and G. H. Martyn, Proc. Roy. Soc. Lond. A 79, 417 (1907).

vapor was studied by Schenkel¹. An elementary theory² sufficed, however, to explain the observed facts in gases and the effect seemed to give little information of interest. As a consequence, the Hall effect in gases has been almost untouched since the early years of the century.

One of the purposes of this study is the reconsideration of the Hall effect as a possible tool in the study of gas discharges, with particular attention given to the information which may be gained from it because of the advances in instrumentation since the time of Boltzmann and Wilson, by the use, for example, of electronic amplifiers and oscilloscopes in place of the quadrant electrometer used by Wilson.

A distinction might be made between the effect as studied by Hall and as studied by Boltzmann and Wilson. In the first case, current in the metallic conductor consists of electrons drifting through a lattice made up of positive ions; in the second case, a counter-current drift of both positive and negative ions constitutes the current. These might be designated the Hall effect of the first and second kinds. Their common feature is that an electric field produces current through a region in which a magnetic field is applied. The effect used in the present study is not a true Hall effect, for the ions are not moving through the magnetic field because of an electric field; in fact, there is no electric field in the region where the magnetic field is applied. The motion of the ions is produced by actual mass flow of the gas, and consequently a modified or pseudo-Hall effect is being used.

¹W. Schenkel, *Eclair. Electr.* 48, 321 (1906).

²J. J. Thomson, Conduction of Electricity Through Gases (2nd ed.; Cambridge Univ. Press, 1906) p. 434, 562.

Another purpose of the study is to obtain indications of ion concentrations in a pulsed discharge by means of the pseudo-Hall effect for comparison with determinations made by Stark broadening measurement on the lines of the Balmer series. For this reason, all the work has been done with pulsed discharges in which gas streams rapidly through the region in which the magnetic field is applied and in which the current present in the probe circuit depends upon the conductivity of the ionized gas.

Two approaches are possible to the problem of computing the conductivity of a gas. One method is to define a mobility by the equation

$$v = bE$$

in which v is the drift velocity of the ion or electron, b is the mobility, and E is the electric field strength. A complete discussion of the theoretical results of this method before 1939 and of the experimental determinations of b for many ions and for electrons is given by Loeb.¹ The most complete theoretical expressions developed are those of Langevin² and of Hase and Cook.³ In 1939 A. V. Hershey⁴ generalized the theories of Langevin and of Hase and Cook for the mobility of ions in weak fields to permit consideration of fields of any strength. He

¹L. B. Loeb, Fundamental Processes of Electrical Discharge in Gases, (John Wiley and Sons, Inc., 1939) Chap. 1 and 4.

²P. Langevin, Ann. chim. phys. 8 238 (1905).

³Hase and Cook, Phil. Mag. 12, 554 (1931).

⁴A. V. Hershey, Phys. Rev. 56, 916 (1939).

assumed that the random energy of the ions has a Maxwellian distribution and evaluated both the drift and random energies by momentum and energy balances.

Some experimental measurements of mobilities have been made since 1939. In 1940, R. J. Munson and A. M. Tyndall¹ reported values for the mobility of positive ions in their own gases using neon, argon, krypton, and xenon. Also in 1940, W. H. Bennett² published values for the mobilities of free electrons and of H^+ ions in hydrogen at atmospheric pressure in discharges from sharp points and fine wires. W. H. Bennett and L. H. Thomas³ gave mobility coefficients for free electrons and positive ions in hydrogen, deuterium, and some mixtures for a range of field strengths in 1942. The formula of Langevin and those derived from it involve the dielectric constant of the gas. New determinations of the dielectric constants of helium, argon, neon, hydrogen, oxygen, nitrogen, carbon dioxide, and air were made by L. G. Hector and D. L. Woernley⁴ in 1946.

The second approach which may be made to the problem of computing the conductivity of a gas is setting up and solving Boltzmann's equation for an ionized gas. This has been done with two approximations. Chapman and Cowling⁵ used a free path method to calculate the conductivity

¹R. J. Munson and A. M. Tyndall, Proc. Roy. Soc. Lond. 177, 187 (1940).

²W. H. Bennett, Phys. Rev. 58, 992 (1940).

³W. H. Bennett and L. H. Thomas, Phys. Rev. 62, 41 (1942).

⁴L. G. Hector and D. L. Woernley, Phys. Rev. 69, 191 (1946).

⁵S. Chapman and T. G. Cowling, The Mathematical Theory of Non-Uniform Gases (Cambridge University Press, 1939) pp. 177-179.

for total ionisation, but the method is subject to criticism because it does not properly weight long-range interactions. Essentially this same method—free path—without consideration of ion-electron interactions, was used by L. G. H. Huxley¹ in 1951 to derive a general formula for the conductivity of a gas containing free electrons. R. Landshoff² extended the treatment of Chapman and Cowling in 1941 to include a magnetic field, considering only a completely ionized gas. A similar treatment was given in 1945 by T. G. Cowling³ with application to the solar atmosphere. A slightly different method of attack was used by J. H. Cahn⁴ in 1949 to treat the electrostatic interaction in discharges of high current density and low field strength, but he based his work on a paper by Landau⁵ which was later shown to be in error.⁶

Using a method due to Chandrasekhar⁷ to compute the change in the velocity distribution for the ions and electrons produced by many small interactions, Cohen, Spitzer, and Routley⁸ obtained a more accurate solution of Boltzmann's equation for an ionized gas than is

¹L. G. H. Huxley, Proc. Phys. Soc. 64-B, 844 (1951).

²R. Landshoff, Phys. Rev. 76, 904 (1949).

³T. G. Cowling, Proc. Roy. Soc. Lond. 183, 453 (1945).

⁴J. H. Cahn, Phys. Rev. 75, 293 (1949)

⁵E. Landau, Physik. Zeits. Sowjetunion 10, 154 (1936).

⁶W. P. Allis, Phys. Rev. 76, 146 (1949).

⁷S. Chandrasekhar, Astrophys. J. 27, 255, 263 (1943).

⁸R. C. Cohen, L. Spitzer, and P. M. Routly, Phys. Rev. 80, 230 (1950).

possible with the method of Chapman and Cowling, which considers the close encounters of more importance in changing the distribution function than the long-range interactions. The same method was extended with some simplifications by Spitzer¹ in 1952. Spitzer and Härn² have recently improved the calculation made by Cohen, Spitzer, and Routly by the inclusion of an interaction term previously neglected. The resulting expression for the conductivity differs only by a small constant from the one originally derived.

As will be explained later, it is believed that double-layer formation and similar considerations related to probe theory are unimportant in the work to be discussed; however, a brief discussion of the present state of knowledge in regard to probe theory will be included for the sake of completeness.

The use of probes as tools for the study of gas discharges was initiated by Langmuir, and an adequate discussion of the theory and techniques of the Langmuir probe is given by Loeb³ along with a full bibliography. It was demonstrated by Van Berkel⁴ in 1938 that errors may be introduced into measurements made with the Langmuir probe by apparent changes in the work function of the probe metal caused, perhaps, by adsorption of gas onto the surface. In 1947 T. A. Anderson⁵, using alternating potentials applied to the probe, found that the effect

¹L. Spitzer, *Astrophys. J.* 116, 299 (1952).

²L. Spitzer and R. Härn, *Phys. Rev.* 89, 979 (1953).

³Loeb, *Op. cit.*, chap. v, Part B.

⁴Van Berkel, *Physica* 5, 230 (1938).

⁵T. A. Anderson, *Phil. Mag.* 38, 179 (1947).

described by Van Berkel depends upon the gas and upon the frequency of the applied potential.

Experiments with probes in a non-steady discharge were described by Barnes and Eros¹ in 1950; they used a constant-current probe system and found it more convenient than constant-potential circuits. Data were obtained in a discharge subjected to square-wave changes in current.

A probe system essentially different from that used by either Langmuir or Barnes and Eros was suggested in 1950 by E. O. Johnson and L. Malter.² Instead of applying a potential difference between the probe and one of the discharge electrodes as Langmuir did, they used two probes and recorded the current to the probes as a function of the potential difference between the two. The advantage of the double probe system is that the probes follow the potential of the plasma when it is changing, as during decay. Because some of the computations in the analysis of data taken by this method are long and tedious, Malter and Webster³ have published nomographs and other aids to calculation.

Another paper, by Boyd,⁴ which has appeared recently and is applicable to ordinary probe work discusses the effect of negative probes on the positive ion current in gases at low pressure. It shows

¹B. T. Barnes and S. Eros, J. Appl. Phys. 21, 1275 (1950).

²E. O. Johnson and L. Malter, Phys. Rev. 80, 58 (1950).

³L. Malter and W. M. Webster, RCA Rev. 12, 191 (1951).

⁴R. L. F. Boyd, Proc. Roy. Soc. Lond. 201, 329 (1950).

that at low pressures, the field of a negative probe may penetrate the plasma sufficiently to give the same positive ion current as if the ion temperature were near that of the electrons.

In an ionized gas, atoms radiate in the electric field produced by ions; consequently their spectral lines may show Stark splitting. Since the fields at the radiating atoms are randomly distributed, however, the effect is to broaden the lines rather than to split them into discrete components. Debye¹ made a dimensional analysis of the field produced by the surrounding ions in 1919, and Holtsmark² developed a theory relating the half-width of the lines to the ion concentration; his theory has been widely used and verified to some extent for discharges at low temperature. It has been extended by Verweij³ and by van Dien.⁴ There is reason, however, to believe that at high temperatures the theory may fail. Holtsmark assumed a static distribution of ions about the radiating atom and computed the probability that the effective field at the atom has a value F . At high temperatures the static distribution is a poor approximation, and the effect of motion of the ions must be considered. L. Spitzer, Jr.⁵ treated this problem in 1939 from the viewpoint of collisions, as in pressure broadening theory, and concluded that so long as the ions do not approach to within

¹P. Debye, *Phys. Zeits.* 20, 110 (1919).

²Holtsmark, *Phys. Zeits.* 20, 162 (1919); 25, 73 (1924).

³Verweij, *Pub. Ast. Inst. Univ. Amsterdam* 5, (1936).

⁴E. van Dien, *Astrophys. J.* 109, 452 (1949).

⁵L. Spitzer, Jr., *Phys. Rev.* 55, 699 (1939); 56, 39 (1939).

ten times the average atomic radius, the matrix elements calculated for the usual Stark effect may be used, but that the effective field produced by moving ions may differ greatly from that of a static distribution. The center of the spectral line is affected more than the wings by the velocity correction obtained from this calculation. M. K. Krogdahl¹ considered this same problem from a slightly different viewpoint and found that although the Holtsmark distribution describes the higher members of the Balmer series quite well, corrections must be applied in order to discuss the broadening of H_{α} , H_{β} , Lyman α , etc. Her approach was to recompute the Holtsmark distribution function $\sum \frac{K_i}{r_i^4}$ with the r 's time dependent. The effect of the time dependence is to increase the probability of small fields and to make the most probable perturbing field smaller than in the static configuration. Again the wings of the lines are almost unaffected, whereas the center is changed appreciably.

Olsen and Huxford² obtained time-resolved spectra as part of a very thorough study of confined pulsed discharges in 1952 and interpreted the observed broadening of Balmer lines by the Holtsmark-Verweij theory, ignoring the possible effects of high temperature. They determined ion concentrations by fitting the observed wings of H_{α} and H_{β} to theoretical curves. The indications were that the ion concentration reaches a maximum at a time when the current has fallen almost to zero.

Two recent papers on results with ring discharges have some application to the work to be described: C. G. Smith³ used a toroid

¹M. K. Krogdahl, *Astrophys. J.* 110, 355 (1949).

²H. N. Olsen and W. S. Huxford, *Phys. Rev.* 87, 922 (1952).

³C. G. Smith, *Phys. Rev.* 59, 997 (1941).

around a transformer core in which an emf. of about five volts was induced to produce a discharge in mercury vapor at very low pressure. Probe measurements gave an electron temperature of the order of 300,000°K. At peak ionization, approximately thirteen per cent of the atoms present were ionized. Cousins and Ware¹ studied the self-magnetic field of a high current discharge by means of the oscillations which the field produced in a toroidal ring discharge. Currents greater than 10,000 amperes were used.

Since 1948 pulsed gas discharges have been studied at the University of Oklahoma. The general method of producing them is to connect a capacitor (previously charged to a high potential) across a tube containing gas at low pressure. The capacitor is always at a potential greater than the breakdown potential of the gas, and immediately after the connector is made, a discharge with currents of the order of 5,000 amperes occurs, lasting approximately ten microseconds. The discharge heats the gas through which it passes sufficiently to create strong shock waves and a rapid expansion of the hot gas out of the discharge volume along any path provided. The expansion was studied by means of rotating mirrors by Goldstein² in 1948, by Lee³ in

¹S. W. Cousins and A. A. Ware, Proc. Phys. Soc. 64-B, 159 (1951).

²J. S. Goldstein, "Anomalous Radiation Processes in Gases Other Than Hydrogen" (Unpublished M.S. Thesis, Dept. of Physics, University of Oklahoma, 1948).

³R. J. Lee, "Investigation of Afterglow in Hydrogen Discharges" (Unpublished M. S. Thesis, Dept of Physics, University of Oklahoma, 1949).

1949, and by Clotfelter¹ in 1949. The early work was reported in two papers.²

In 1950 Atkinson³ investigated the apparent ion concentrations given by application of the Holtsmark theory to the Stark broadening of the Balmer lines. This work was continued by Marks⁴ and by Rose.⁵ These measurements and calculations indicated that the ion concentration reaches a maximum at a point in the discharge tube several centimeters outside the discharge region at a time after the main discharge current has fallen to a very low value or to zero.

In connection with studies of line broadening, measurements of the intensities of O III, H and Si I lines were used with the Milne-Fowler color-temperature theory to estimate temperatures in the discharge; in H₂ at 3 mm Hg pressure and 4500 volts potential, the values arrived at are 26,000° K. at the head of the expansion region, 10,000° K. six centimeters down the tube, and 7,000° K. fifteen centimeters from the head.

¹B. E. Clotfelter, "Anomalous Radiation Processes in Pulsed Gas Discharges" (Unpublished M.S. Thesis, Dept. of Physics, University of Oklahoma, 1949).

²R. J. Lee and R. G. Fowler, Phys. Rev. 81, 457 (1951).
R. G. Fowler, J. S. Goldstein, and B. E. Clotfelter, Phys. Rev. 82, 879 (1951).

³W. R. Atkinson, "Half Intensity Breadths of the Balmer Lines in Pulsed Gas Discharges" (Unpublished M. S. Thesis, Dept of Physics, University of Oklahoma, 1950).

⁴L. W. Marks III, "Ion Concentration Versus Intensity in Pulsed Gas Discharges" (Unpublished M. S. Thesis, Dept of Physics, University of Oklahoma, 1951).

⁵H. C. Rose, "Ion Concentration and Recombination in Electrically Excited Shock Tubes" (Unpublished M. S. Thesis, Dept of Physics, University of Oklahoma, 1952).

STATEMENT OF THE PROBLEM

The present research has three purposes: (1) To investigate these ion concentrations by another method to determine whether the result obtained from line broadening methods is correct or whether the need exists for a modification of Holtzmark's equation under these conditions; (2) To measure particle velocities in the shock wave and the following contact surface; (3) To reconsider the Hall effect or pseudo-Hall effect as a possible research tool in the study of gas discharges.

CHAPTER II

THEORY

Consider a block of ionized gas of finite but considerable length moving down a glass tube between diametrically opposed probes. Because of the greater velocities of the electrons, they will diffuse to the walls faster than the positive ions so that the walls and probes (assumed ungrounded) will take a negative potential with reference to the plasma. The outflow of electrons will be checked by the field produced by the surplus of positive charge throughout the plasma. If the difference of potential between the probes is measured, it will be found to be zero, because each will have taken the same potential with respect to the plasma. If, now, a magnetic field is applied across the moving column of ionized gas in a direction perpendicular to the line joining the probes, the probes will show a potential difference because of the deflection of positive ions to one probe and the deflection of electrons to the other.

We may consider two cases. First, suppose that the probes are connected to ground by resistors whose impedance is so much greater than that of the plasma that it may be considered infinite. As the gas passes with velocity u through the field of strength B , individual charged particles experience a force of magnitude uB normal to the direction

of the field and to the direction of flow. This force will cause particles to flow to the walls and probes until the excess of positive charge in the plasma is sufficient to produce electrostatic fields which retard further charge migration. This process is the same as the charging of a capacitor through a resistor from a source of constant emf. Since the capacitance from either probe to ground is $17 \mu\text{f}$ and the maximum resistance of the plasma measured is of the order of 300 ohms, the time constant of the charging process is of the order of 10^{-9} seconds. At equilibrium, the force on a charged particle must be zero, hence

$$e(uB \times 10^{-8} - E) = 0,$$

where E is the retarding electric intensity. Since this must be true at every point in the gas, E must be constant across the tube if u and B are the same everywhere. The potential difference developed between the probes will be Ed if d is the separation between probes, or

$$u = \frac{V}{Bd} \times 10^8$$

if V is the observed potential difference in volts between probes. Actually u is not constant across the tube because of viscous effects in the gas, and so precisely

$$V = 10^{-8} \times \int_0^d u(x) B \, dx.$$

This relation permits one to compute the velocity with which the gas moves through the probe space if B , d , and V are known.

The second case is that in which a resistor of the same order of magnitude as the resistance of the plasma is used as a shunt to ground each probe circuit. Theory is not available to carry out the

analysis of this situation so exactly as in the case of the infinite shunt. The quantity which is measured and discussed in this case is the resistance of the plasma, and in order to show that the resistance can be related to temperature and to ion concentration, two limiting cases will be considered.

If the ion concentration is small, ordinary mobility calculations may be applied. Let b be the mobility of the positive ions expressed in cm/sec/volt/cm so that the velocity of motion of the ion through the neutral gas is b cm/sec when the electric field strength is one volt/cm. The force ueB is exactly equivalent to a force eE , and one can use uB as if it were an electric intensity. If n is the ion particle density, the current drawn from the plasma per unit area will be $enbuB$ provided that the external resistance of the circuit is small or of the same order of magnitude as the internal resistance. The internal resistance can be expressed by changing from current density to total current as

$$I = enbuBA = enbVA/d$$

when A is the area of the probes and V is the total potential difference applied across the plasma. From this the resistance is

$$R = V/I = d/(enbA).$$

If the ion concentration is quite high, the analysis given above is inapplicable, because b as reported in the literature from theoretical computations or from measurements is determined for conditions in which all collisions made by ions drifting under the influence of the field are made with neutral atoms so that the momentum and energy transferred from the ions during such encounters is lost to the ion cloud.

If an appreciable part of the atoms in the gas are ionized, such will not be the case, and interactions between the ions will modify and finally, for high ionizations, override the ion-neutral atom interactions.

In the discussion above of mobility motion, the fact that ions of two signs are present was neglected and the mobility of positive ions only was considered. That this is essentially correct can be seen from the following argument: When the field is first placed across the gas, electrons will be urged out of the gas in the preferred direction, and they will leave the gas much faster than the positive ions because of the greater electron mobility. An equilibrium condition will be reached quickly, however, such that at every point in the gas electrostatic forces will limit the outflow of electrons so that positive ions and electrons will leave at the same rate. When this situation exists, the limiting factor in the current will be the positive ion mobility. This will actually cause the current to be twice the value derived above and will cause the resistance to be one-half as much, or

$$R = d/(2enbA).$$

When the level of ionization becomes so high that the interactions between charged particles become more important than those between the ions and neutral atoms, one may try as a first approximation the use of an electron-positive ion cross section with an elastic, solid sphere mobility equation. To this degree of approximation, the electron-electron and positive ion-positive ion collisions may be neglected, for neither makes any change in the momentum of the charged particles; only electron-positive ion collisions are effective in

impeding the flow of charge. Goudsmit and Saunderson¹ have shown that in the case of Rutherford scattering with a sharp cut-off of the Coulomb field at the radius a , for multiple scattering the effective radius of total scattering cross section is a . For hydrogen, $a = a_0$, the Bohr radius. It will be shown later that the use of this value in the elastic solid sphere theory does not give satisfactory results.

The most exact treatment of the case with the gas totally ionized is that given by Cohen, Spitzer, and Routly.² They derive as an expression for the conductivity

$$\sigma = \frac{.890 (2/3\pi)^{3/2} m C^3}{e^2 \ln(qC^2)},$$

where

$$qC^2 = \frac{3}{\pi^2 n^2 e^3} \left(\frac{KT}{2}\right)^{3/2},$$

and σ is conductivity, m is the mass of the electron, C^2 is the mean-squared velocity, and n is the number density of electrons or protons. For high densities and low kinetic temperature, this formula fails, and the authors suggest use of an expression taken from Chapman and Cowling.³ In the notation of Cohen, Spitzer, and Routly, this is

$$\sigma = \frac{1.156 (2/3\pi)^{3/2} m C^3}{e^2 \ln(q^1 C^2)}$$

where

$$q^1 C^2 = \frac{4KT}{n^{1/3} e^2}.$$

¹S. Goudsmit and J. L. Saunderson, Phys. Rev. 57, 24 (1940).

²Cohen, Spitzer, Jr., and Routly, op. cit.

³Chapman and Cowling, op. cit., Section 10.33.

This last equation is the one which will be used in the analysis of results.

CHAPTER III

DESCRIPTION OF APPARATUS

Vacuum System

The vacuum system used in this work was of standard construction; it included a Welch Duo-Seal fore pump, a two-stage mercury diffusion pump, two cold traps(which were surrounded by liquid nitrogen during operation), a McLeod gauge, and a tilted manometer accurate to less than one-fourth of a millimeter of mercury.

Tube

The tube is shown in figure 1. The upper electrode was a solid nickel cylinder, and the lower electrode was a nickel ring. All the glass tubing in the actual discharge path was precision bore tubing, .6000 - .0005 inches internal diameter. The nickel ring lower electrode was machined with shoulders inside onto which the tubing fitted so that the inside wall of the ring and the inside of the tubing made a smooth surface. Current was led to the lower electrode by a brass tube external to the primary discharge region and concentric with it; the brass and the ionized gas inside the inner glass tube formed a coaxial conductor. The probe section, comprising the shields and probes, was of copper. Two shields were machined like the electrode with shoulders to hold the tubing, and the section of tubing between the electrode and the probe

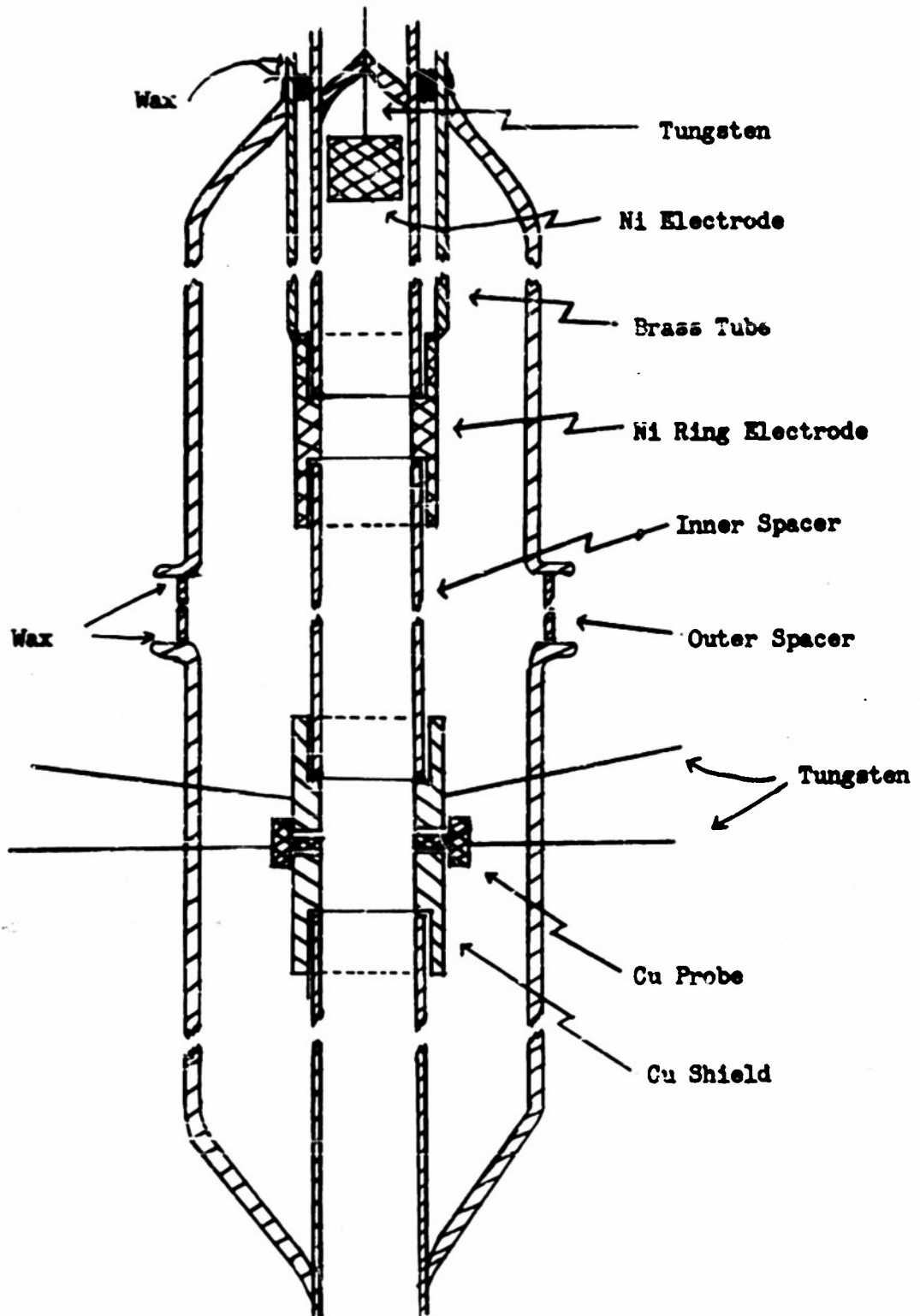


Fig. 1—Discharge Tube

section was removable so that different lengths could be used in order to vary the distance down the expanding discharge at which the probes were placed. The shields were two centimeters long, shoulder to shoulder, with a hole three millimeters in diameter in the center of each. The probes, two millimeters in diameter, were inserted through the hole in the shields so that their inner surfaces were flush with the inner shield surfaces, being carefully placed so that they did not touch the shields. At either side, the shields, which were half cylinders, were separated by a space of one millimeter. Each shield and each probe was held in place and connected to the external circuit by a tungsten lead (.080 inches in diameter) sealed through the glass. Each shield was connected to ground by a resistor of the same size as the one in the probe circuit; hence, the shields acted as guard rings. It was necessary to use four wax joints: between the inner discharge tube and the brass tube, between the brass and the outer vacuum jacket, and at each end of the spacer in the vacuum jacket. Hard deKhotinsky wax was used for all wax joints.

Spacers of length 16.1, 13.1, 9.0, and 5.4 cm., were used in the discharge region, making the distances from the probes to the lower discharge electrode 17.1, 14.1, 10.0, and 6.4 cm.

The copper pieces were cleaned electrolytically in a mixture of perchloric acid and acetic anhydride before they were placed in the tube, and later they were cleaned by sputtering in an atmosphere of hydrogen.

Hydrogen was admitted to the vacuum system through a palladium leak. Helium and argon flasks were sealed to the system and gas was admitted through a dosing stopcock.

Charge and Discharge Circuit

The capacitor bank used was composed of twelve Westinghouse Inerteen one microfarad capacitors connected in parallel. It was charged from a transformer with maximum output of 6500 volts, whose actual output was controlled by a Varitran in the primary circuit. Between the transformer and the capacitor bank were two 2X2 vacuum tubes, one on either side. In addition to rectifying the current to the capacitors, these served to isolate the bank from the transformer during firing. The switches for charging and firing were interconnected in such a way that wiring was impossible until the primary of the transformer and the filaments of the 2X2's were disconnected.

During the charging of the capacitors, the bank was disconnected from the tube electrodes by means of two relays. The firing operation consisted of two steps: first the transformer primary and rectifier filaments were disconnected and simultaneously a relay was closed connecting one electrode to one side of the capacitor bank; second, a second relay was closed completing the circuit and firing the tube. The first relay was closed in air, but the second was placed in an atmosphere of hydrogen in order to reduce arcing, burning of the electrodes, and consequent erratic firing behaviour. Both firing relays were high-current relays with silver contacts. The circuit was critically damped by a resistance of approximately 0.4 ohms. A mechanically driven switch repeated the charging and discharging cycle three times per minute. The complete discharge circuit had 1.5×10^{-6} henries inductance and, before damping, approximately .1 ohm resistance.

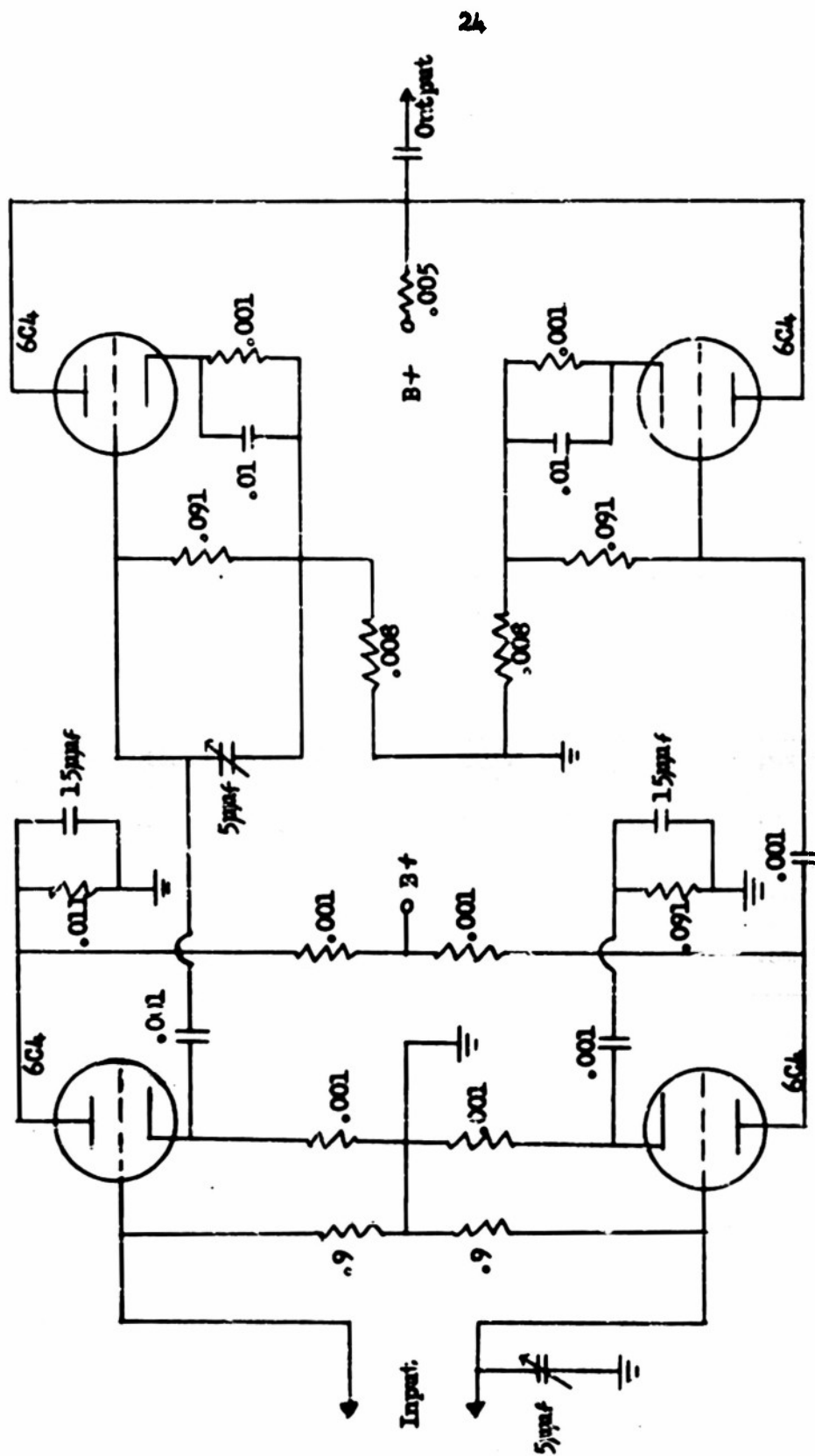
Amplifier

Because it was necessary that both probes be ungrounded in order to avoid unbalanced currents produced by electrostatic effects, a double-ended input difference amplifier was constructed. The circuit diagram is given in figure 2 and the performance is indicated by table 1.

TABLE 1
DIFFERENCE AMPLIFIER PERFORMANCE

Frequency	Difference Gain	Sum Gain
10 kc.	.250	.003
50 kc.	.255	.002
100 kc.	.250	.002
200 kc.	.235	.002
300 kc.	.215	.002
400 kc.	.190	.001
500 kc.	.170	.001
750 kc.	.130	0.000
1 mc.	.105	0.000
2 mc.	.060	.001
3 mc.	.047	.002
4 mc.	.040	.002
5 mc.	.036	.002

The difference gain, the factor which gives the output when multiplied by the difference between input signals whose sum is zero, was measured by putting a signal from a General Radio Beat Frequency Signal Generator between either input and ground. The gain so measured is the same as that obtained by putting the same signal onto each input with the two out of phase by 180° . The sum gain, the factor which gives the output signal when multiplied by the sum of input signals whose difference is zero and which ideally is zero, was measured by putting a signal between ground and both inputs connected together. Both gains were checked with



Unlabeled Capacitances in μ f; Resistances in Megohms

Figure 2 -- Difference Amplifier

input signals up to five volts, and the response was linear to that point; the data listed in the table were obtained with an input signal of two volts. A trimmer capacitor was used to make the input capacitances exactly the same on each input when the amplifier was connected to the probe system, and another trimmer was used to give the best possible balance of the two sides of the amplifier at one megacycle. The difference gain is seen to be essentially constant at .25 from ten kilocycles to two hundred kilocycles, and this range easily covers the fundamental and the second harmonic frequency of the signals studied; consequently, a gain of .25 was used in all calculations. In order that shunts might be put into the circuit easily without forming large loops to serve as antennae, nuts were soldered to the input leads within the amplifier case and matching screws were soldered to the shunting resistors. The resistors were inserted through openings in the case and one end was screwed fast, after which the other end was soldered to the case externally.

Magnetic Field Coils

The magnetic field was produced by coils constructed as shown in figure 3. The construction of the tube prohibited placing coils of the largest size used near enough to the tube wall to give large fields at the center with reasonable currents, and so the smaller coils were added although their field probably was not so homogeneous as would be produced by coils of larger diameter. Currents of 13, 9, and 5 amperes were used giving fields of 310, 220, and 120 gauss respectively. The currents were measured with a Weston D. C. ammeter. A reversing switch was placed in the circuit to facilitate the reversal of the field

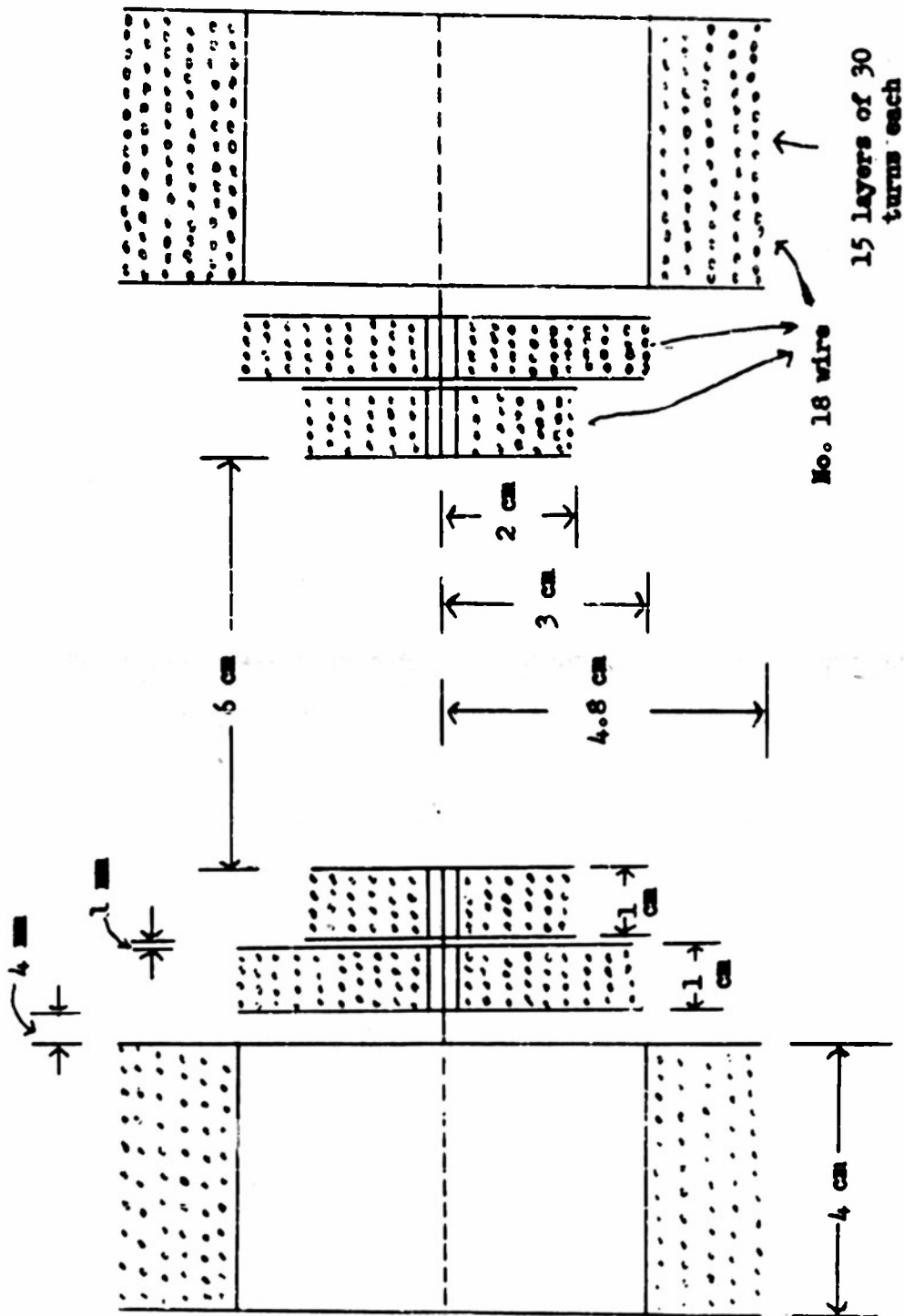


Figure 3 — Magnetic Field Coils

direction. The field was calculated to be given by $B = 24.3 I$ when I is measured in amperes. It was also measured with a search coil and a ballistic galvanometer. Measurements gave a value $B = 23.8 I$ and showed the field to be uniform to within 10% throughout a volume of 36 cc, much more than the volume occupied by the probe section. Since measurements of coil dimensions were made to only two significant figures, fields were computed from $B = 24 I$.

Oscilloscope

The output of the difference amplifier was displayed on a model 514D Tekronix Oscilloscope. The oscilloscope was triggered by the potential drop across a copper wire about five inches long in the discharge circuit. Sweep speeds of 3.0 to 20 microseconds/centimeter were used, and sensitivity varied from .04 to .8 volts/centimeter.

Rotating Mirror

In order to correlate present work with past work and with work being done now using rotating mirror pictures, a set of six pictures was made, one for each pressure and firing potential combination by the use of a four-sided rotating mirror and lens system as described by Compton.¹

Shielding

Because the primary current of the discharge is so large (approximately five thousand amperes at peak) and because the capacitor

¹W. D. Compton, "Investigation of Reflected Luminescence in Pulse Gas Discharges" (Unpublished M.S. Thesis, Dept. of Physics, University of Oklahoma, 1951).

bank oscillates with a natural frequency of approximately thirty megacycles, presumably because of the inductance of the capacitor foils, it was necessary to shield all the equipment very carefully to prevent extraneous signals from tripping the oscilloscope erratically and completely masking the signals desired. Since the successful elimination of such interference would be a major problem in repeating the work, a discussion is given here of the methods found useful.

The difference amplifier eliminated signals arising from the discharge of stray capacitances through unequal paths to ground when one probe was grounded and the other was connected directly to the input of the oscilloscope. The difficulty and solution were pointed out by Barnes and Eros,¹ and early work here indicated that the effect could cause spurious signals. To avoid any trace of sixty cycle signal through the amplifier, the filaments were battery-heated, and the battery was placed in a closed metal box to avoid pick-up of stray radiation. Leads to the amplifier were a shielded coaxial conductor. An RC-filtered power supply was used for the amplifier, and again the connector was coaxial.

The main power supply and capacitor bank were placed above an aluminum roof which extended several feet in all directions and was about eighteen inches above the tube at its longest. Leads down to the switches and the tube were coaxial, and a pi-section RC filter composed of two .2 ohm non-inductive resistances (the damping resistance) and two .1 microfarad capacitors just above the roof blocked

¹Barnes and Eros, op. cit.

most of the high frequency oscillation of the capacitor bank. The side of the circuit going to the brass tube and ring electrode was grounded at the tube.

It was found necessary to shield the tube itself, and consequently it was wrapped with copper screen wire, which was grounded.

The closing of switches operating relays caused so much spurious signal that it was necessary to shield all switch wires and to place the switch contacts in a grounded, metal box.

To prevent the oscilloscope's being triggered early and erratically by signals from the discharge riding on the power line, it was fed from an outlet away from the rest of the equipment. The trigger lead to the oscilloscope was coaxial and a filter comprising a 47 ohm resistor and a .001 microfarad capacitor was placed on that line to block the high frequency signal which passed the filter above the roof.

The leads to the magnetic field coils were shielded to a point near the coils, but shielding the coils themselves proved to be unnecessary.

When the equipment was first built, before most of these shielding measures were instituted, signals of the order of one hundred volts appeared at the oscilloscope from the radiation of the discharge, but the shielding described reduced these signals below the threshold of detectability.

CHAPTER IV

RESULTS

One would expect that with no magnetic field impressed, the oscilloscope trace would be a straight line along the axis, and in some cases this is true. In many cases, however, there is a signal even without a magnetic field. The zero-field signals can be divided into two classes depending upon the time of appearance. What is referred to as the "photo-electric signal," appears within one microsecond of the initiation of the primary discharge and disappears before the streaming gas reaches the probe section. The true zero-field signal comes at the time when the ionized gas reaches the probes and lasts for all or part of the time the gas is streaming past. The two effects will be discussed in chronological order, the photo-electric signal first.

The form of the photo-electric signal observed with infinite shunt is quite different from that observed with finite shunt. With infinite shunt, the signal is approximately half of a square wave, having a very sharp initial rise, a slow decay for a period of three to five microseconds, and then a rather quick return to the base line. This signal is always of the same polarity, is of the same form in hydrogen, helium, and argon, and is field independent. It is illustrated

by figure 4. With finite shunt, the signal is almost invisible in hydrogen, appearing only as an almost unnoticeable widening of the trace near its starting point, as in figure 5. In both helium and argon, the signal with finite shunt is rather large, appearing as a pulse with a polarity opposite to that with infinite shunt, and showing field sensitivity. Under normal conditions, reversal of the magnetic field does not reverse the polarity of the pulse, but merely increases and decreases slightly the height of the pulse. Figure 7 shows this effect. (If the shields are grounded and an infinite shunt is used, no photoelectric signal is in evidence for zero field, but a field produces a signal which reverses polarity when the field is reversed.) The signal with finite shunts rises to its maximum more slowly than in the case of infinite shunts.

This early signal has been ascribed to a photoelectric effect because it appears at the same time as the primary discharge, and its behavior cannot be explained by electromagnetic induction. This interpretation is supported by the results of an experiment in which the inside of the internal spacer was silvered, increasing the amount of light reaching the probe region. The result was an increase in the magnitude of the photoelectric signal.

The true zero-field signal, which appears at the time the flowing gas reaches the probes, is usually small in hydrogen and appears as a short pulse in most cases and as a damped oscillation occasionally. It appears to be random in both magnitude and direction. In proportion to the signal produced by the magnetic fields under identical

conditions, it is greater with shunting resistors, the magnitude increasing as the size of the shunte is decreased.

In both helium and argon, the zero-field signals are larger than in hydrogen (see figures 6, 7, and 8), but in both these gases they show much less randomness. In helium, particularly, the signals reproduce perfectly, as figure 8 illustrates. The exact explanation of these signals is unknown, but since it has been demonstrated that placing the upper part of the tube approximately five degrees out of line with the lower part so that the flowing gas is directed somewhat toward one electrode or placing a small amount of wax just above one probe to disturb the gas flow makes the signals larger and changes their character, it is presumed that the source of most and perhaps all of the zero-field signals in helium and argon is irregularities in the tube. The wax has much more effect than the misalignment, and it is believed that unavoidable irregularities in the tube surface near the probes are responsible for most of the observed signal. Undoubtedly this effect is also present in hydrogen; however, it probably is not the dominant feature, for the periodicity which was noticeable in hydrogen is always missing in both helium and argon.

In all the work, five traces were recorded on each film under identical conditions; if magnetic fields were being used, ten traces were recorded, five with the field in each direction. In hydrogen the traces with magnetic field coincide only during the rise, and from the peak throughout the decay they show random variations. These variations, presumed to be related to the zero-field signal, show the periodicity

mentioned above in some cases, but are so erratic in general that they place a limit on the accuracy of measurement possible.

All measurements from which velocities and resistances were calculated were made between traces produced by reversal of the magnetic field. Effects produced by tube irregularities and possible variations in the probes (differing capacitances to the shield, for example) were thus eliminated.

In both hydrogen and helium, the maximum signal with shunts in the circuit comes at the same time as the maximum potential observed without shunts. In argon, however, the current signal maximum lags behind the potential maximum. Since the potential maximum indicates the velocity maximum and the current indicates the ion concentration or some function of it, the implication is that the greatest ion concentration does not exist in the highest velocity portion of the gas flow.

In argon at 5.6 mm pressure two crossings of the traces on to reversed fields were observed, indicating two reversals in flow direction of the gas. These occur approximately three hundred and seven hundred fifty microseconds after the initiation of the discharge. The effect is shown in figure 9.

An experiment was performed in hydrogen in which 1.5 volt batteries were put in the probe circuit, from the shunts to ground. When the batteries were in opposition, they had no effect. When they were additive so that an emf of three volts was applied across the tube, a potential difference of 2.1 volts was measured. This is interpreted to mean that any sheaths which form over the probes have



Fig. 4
 H_2 ; 2.8 mm.; 4500 V.
 4.5 μ sec./cm.; ∞ shunt
 Pos. 17.1 cm.



Fig. 5
 H_2 ; 2.8 mm.; 4000 V.
 3.0 μ sec./cm.; 52 Ω shunt
 Pos. 6.4 cm.



Fig. 6
 A ; 5.6 mm.; 3500 V.
 8.0 μ sec./cm.; ∞ shunt
 Pos. 6.4 cm.



Fig. 7
 A ; 5.6 mm.; 4000 V.
 8.0 μ sec./cm.; 99 Ω shunt
 Pos. 6.4 cm.



Fig. 8
 He ; 2.8 mm.; 4500 V.
 3.0 μ sec./cm.; ∞ shunt
 Pos. 6.4 cm.

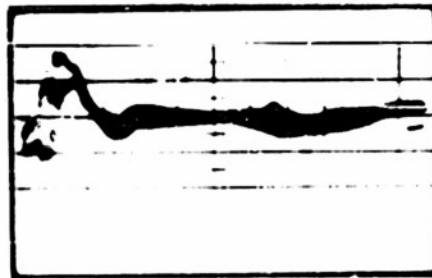


Fig. 9
 A ; 5.6 mm.; 4000 V.
 20 μ sec./cm.; ∞ shunt
 Pos. 14.1 cm.

very little effect on the signal recorded. When magnetic fields were used in conjunction with the batteries, the deflections produced by the two agents were superimposed additively; this indicates that the effect of the magnetic field on this flowing gas of high ion density is equivalent to that of an electric field.

If no shunt is used in the probe circuit and the shields are allowed to float instead of being tied to ground through a large resistor, the potential signal recorded is not affected. If the shields are grounded directly, the main signal is not affected, but the photoelectric signal is reduced to zero in the absence of a magnetic field; the presence of a field causes the photoelectric signal to appear, and reversal of the field reverses its polarity. If the probes are shunted by 99 ohms, floating the shields increases the magnitude of the signals (both photoelectric and normal) very slightly, whereas grounding the shields reduces the signals slightly and causes the photoelectric signal to be in the direction normal for infinite shunt.

Velocity Measurements

Tables 2 and 3 give the maximum velocities in H_2 computed by the formula

$$u = \frac{V}{Bd} \times 10^8$$

The two or three values given in each entry are from different runs of data. The quality designation is based solely on the appearance of the traces and the accuracy with which measurements could be made. The symbols have the following meaning: g--good; f--fair; v.p.--very poor; q--questionable.

TABLE 2

Maximum Velocities in H₂ at 2.8 mm.

Position	V (Volts)	B (Gauss)	Quality	u x 10 ⁻⁵ cm./sec.
6.4	4500	310	g,g	13.4, 13.4
6.4	4500	215	f,f	15.6, 16.4
6.4	4000	310	g,g	12.8, 13.0
6.4	4000	215	p,p	12.8, 13.4
6.4	3500	310	f,f	9.6, 10.0
6.4	3500	215	f,f	11.2, 11.8
10.0	4500	310	g,g	11.6, 11.6
10.0	4500	215	f,p	8.4, 13.2
10.0	4000	310	f,f	7.4, 9.0
10.0	4000	215	p	11.4
10.0	3500	310	f,p	5.6, 7.8
10.0	3500	215	f,p	5.2, 9.2
14.1	4500	310	f,f	5.2, 6.8
14.1	4500	215	f,f	8.4, 6.0
14.1	4500	120	f,f	8.0, 6.0
14.1	4000	310	f,f	7.4, 5.4
14.1	4000	215	p,f	8.4, 6.6
14.1	4000	120	f,f	9.0, 5.4
14.1	3500	310	f,f	6.4, 5.2
14.1	3500	215	f,f	8.0, 6.4
14.1	3500	120	f	8.4
17.1	4500	310	g,g,g	5.8, 7.8, 8.0
17.1	4500	215	f,g,g	6.6, 10.2, 8.2
17.1	4500	120	f,p,g	6.6, 8.6, 8.0
17.1	4000	310	g,g,g	5.6, 6.8, 7.0
17.1	4000	215	f,g,f	6.0, 9.2, 6.2
17.1	4000	120	f,p	5.8, 8.0
17.1	3500	310	g,g,g	4.6, 6.8, 6.4
17.1	3500	215	f,f,f	5.2, 6.8, 6.2
17.1	3500	120	fp	7.2

TABLE 3

Maximum Velocities in H₂ at 5.6 mm.

Position	V (Volts)	B (Gauss)	Quality	u x 10 ⁻⁵ cm./sec.		
6.4	4500	310	g	8.0		
6.4	4500	215	f		7.8	
6.4	4000	310	g,f	7.0,	7.4	
6.4	4000	215	g,f	6.6,	7.8	
6.4	3500	310	q,q	4.4,	6.2	
6.4	3500	215	f,q	5.2,	5.2	
10.0	4500	310	g,f	5.4,	5.0	
10.0	4500	215	f,f	4.4,	5.4	
10.0	4000	310	g	3.4		
10.0	4000	215	g,f	6.4,	6.2	
10.0	3500	310				
10.0	3500	215	f,g	5.8,	6.2	
14.1	4500	310	g,g	6.2,	7.0	
14.1	4500	215	g,f	6.0,	6.6	
14.1	4500	120	f	6.6		
14.1	4000	310	g,f	5.6,	6.6	
14.1	4000	215	f,f	5.2,	7.4	
14.1	4000	120	f	6.0		
14.1	3500	310	g,g	5.4,	5.2	
14.1	3500	215	f,f	5.0,	7.0	
14.1	3500	120	f	4.6		
17.1	4500	310	g,g,g	6.2,	6.6,	6.8
17.1	4500	215	g,f,g	6.8,	7.2,	7.4
17.1	4500	120	f,q,p	4.8,	5.8,	7.2
17.1	4000	310	g			6.0
17.1	4000	215	p,f,g	6.8,	7.2,	7.4
17.1	4000	120	f,f	4.4,	6.0	
17.1	3500	310	g,g,g	4.6,	5.0,	5.0
17.1	3500	215	f,q,g	4.8,	4.8,	5.0
17.1	3500	120	f,f	4.2,	4.4	

For purposes of comparison, six rotating mirror pictures were made, one for each firing potential and pressure used. Table 4 gives the velocities of the contact surface measured from the mirrorgrams and

TABLE 4
Comparison of Velocity Measurements

Magnetic	Mirror
8.2 Km/sec.	8.1 Km/Sec.
7.1	7.8
6.2	6.8
7.4	6.5
7.8	6.2

from magnetic signals under identical conditions. These magnetically determined velocities are the weighted averages of values given in Table 2 for the corresponding pressure and potential at the 14.1 cm position. It is apparent that the agreement is satisfactory. Figure 15 gives typical velocity profiles in time.

Resistance Measurements

When shunts were in the circuit, charge flowed through a circuit comprising the plasma and tv shunts. If the values of the shunts are denoted by r , the potential drop measured across the plasma when no shunts are in the circuit is designated by V_0 , the resistance of the plasma is given by

$$R = \left(\frac{V_m}{V_r} - 1 \right) 2r.$$

To determine V_m and V_r , the maximum separations between traces produced by field reversal (as those in figures 4 and 5) were measured, and from the distances on the film, the known magnification of the camera, and the known amplifier gain, the input potentials were computed. Each value of R used was computed from the measurements of V_m and V_r taken during the same run, so that the effect of possible changes in the equipment between runs should have been minimized. Plots of plasma resistance as a function of tube positions in He, and H_2 , and A are given in figures 11-16. In figures 13, 14, and 15, the resistance at 6.4 cm is less than one ohm. As can be seen from the curves, the apparent plasma resistance is smaller in some cases when a smaller shunt is used.

The standard deviation of the velocity measurements at each firing potential in hydrogen is less than 10% of the mean at 6.4 cm and rises to approximately 15% (never more than 20%) at more distant positions. So few values of resistance were computed at each position that the calculation of any such measure of precision from them has doubtful significance; however, the average deviations from the mean among the plotted points vary from 7% to 60% of the mean, and the average of these percentage values is 24%. So long as an infinite shunt was used, most of the traces reproduced sufficiently well that measurements of trace separation produced by field reversal could be made with reasonable certainty. When shunts were used, however, the signals were frequently so poorly reproducible that measurements were not attempted. This was particularly true with all magnetic fields except the strongest. No

value of the resistance is given on the graphs unless the traces on both pictures which were used to calculate it (pictures which yielded V_{∞} and V_p) reproduced sufficiently well that measurements could be made with assurance. All computed values of resistance which were used as a basis for the curves plotted are shown on the graphs. The lack of reproducibility observed in hydrogen is caused by a randomness in the discharge itself rather than in the operation of the instruments.

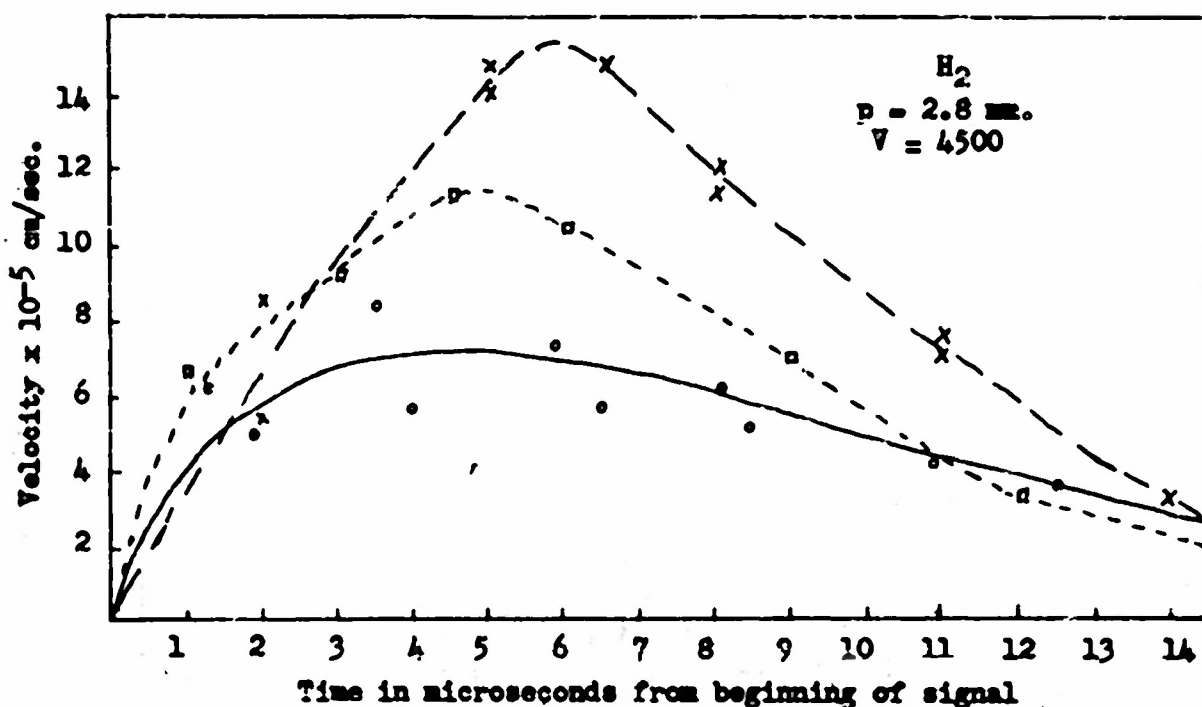


Fig. 10—Velocity Contours

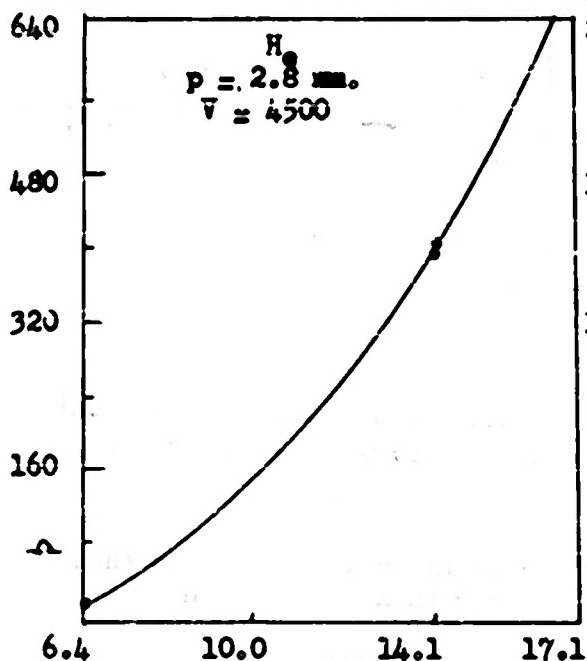


Fig. 11

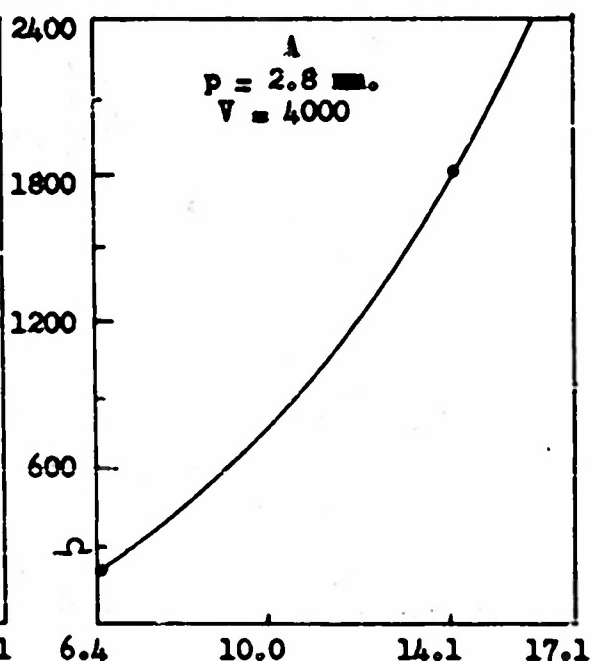


Fig. 12

Plasma Resistance as a Function of Position

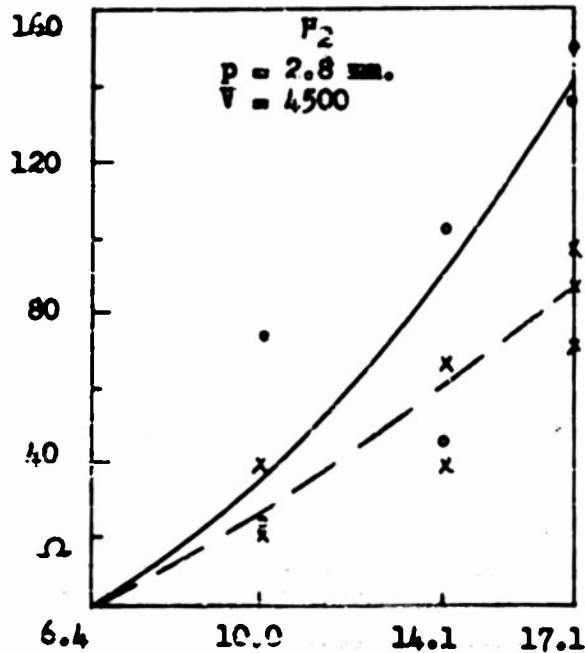


Fig. 13

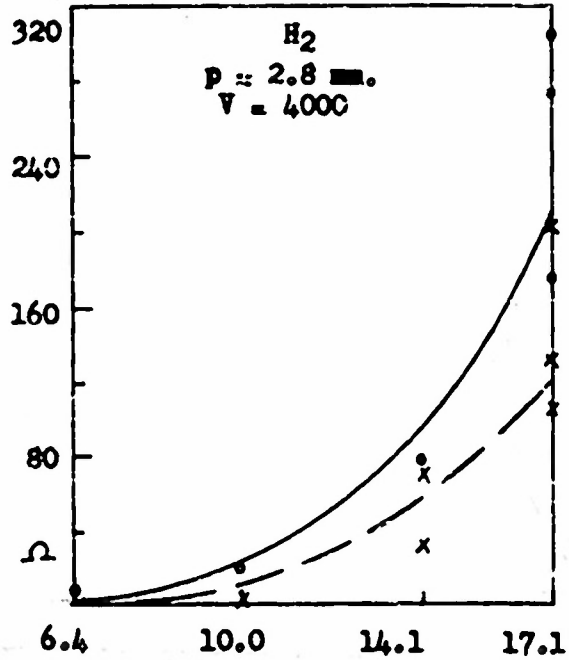


Fig. 14

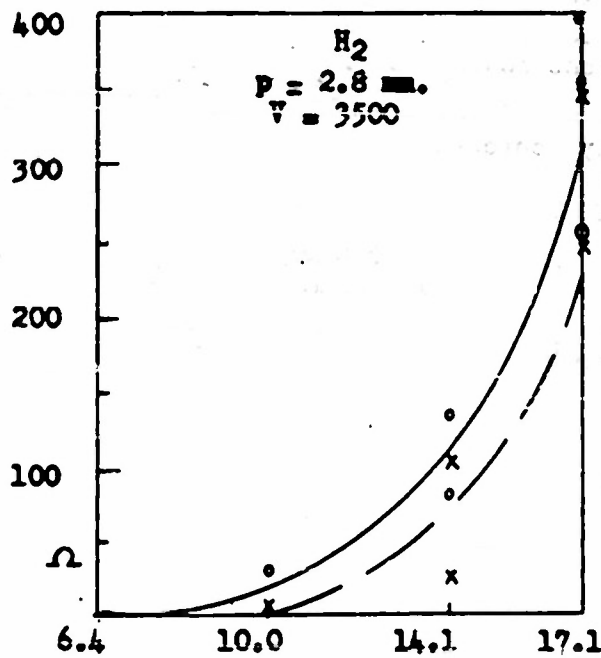


Fig. 15

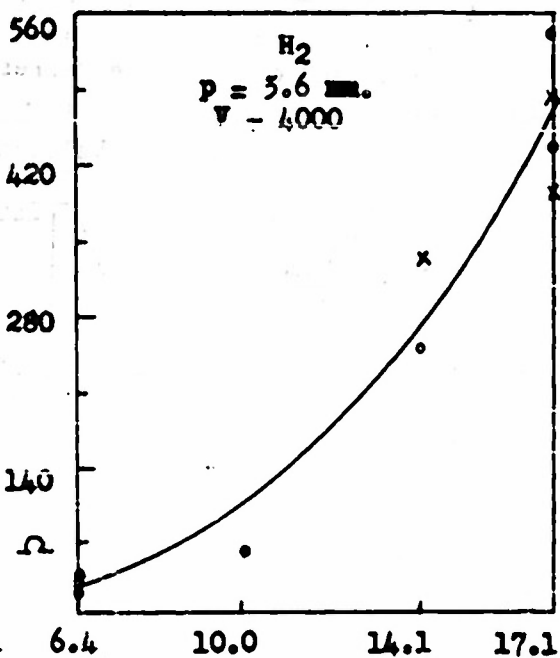


Fig. 16

The circles and solid lines represent data taken with a 99 ohm shunt; x's and dashed lines represent data taken with a 52 ohm shunt.

Plasma Resistance as a Function of Position

CHAPTER V

DISCUSSION

Trace Forms

Since both the photoelectric signal and the zero-field signal were unexpected, they require some consideration. Neither has been investigated exhaustively, but enough is known of their behavior to suggest some possible explanations and to eliminate others.

The photoelectric signal observed with shunts in the probe circuit can be explained as follows: As radiation from the main discharge volume ionizes the gas down the tube, an ion density gradient will be produced because the number of ions produced in a unit volume decreases as the distance from the source of the radiation increases. As a consequence of the density gradient, there will be a diffusion current down the tube. If one of the probes protrudes into the gas a short distance more than the other, it will collect more of the drifting charge than the other, and since the electrons leave the gas more rapidly than the positive ions, that probe will show a potential negative with respect to the other probe. A signal produced by this mechanism should be somewhat field-sensitive, and the photoelectric signal with shunts is. Since argon is less transparent to its radiation than hydrogen is to its radiation, the density gradient will be greater in argon than in hydrogen, and hence the field-sensitivity

will be more pronounced in argon. This is in agreement with observations.

Three different mechanisms have been considered as possible explanations of the photoelectric signal with infinite shunt, but none of the three is satisfactory. They are the following: (1) If one probe extends farther into the gas than the other, radiation from the discharge may cause sufficient photoemission of electrons to make that probe positive with respect to the other one. This explanation fails to explain the effect of the shields on the signal. (2) If one probe is nearer the shield than the other and hence has a larger capacitance to the shields, it will show the greater tendency to follow the potential of the shields, and if they become positive because of photoemission of electrons, that probe which is nearer its shield will become positive with respect to the other probe. This explanation conflicts with the one given of the signal with shunts, because in the one case the signal is attributed to the accumulation of negative charge on the probes (and hence, on the shields), and in the other case, it is attributed to a dearth of negative charges on the shields. (3) If a contact potential exists somewhere in the probe circuits, the signal can be explained by the argument that photo-ionization of the gas permits a current to flow when the discharge begins. This explanation, too, fails to describe the effect of the shields on the observed signal.

The mechanism which best explains the hydrogen zero-field signal, which is coincident with the arrival of the shock wave at the probe section, is a spiraling motion of the streaming gas. (The zero-field signal in argon and helium has already been discussed in connection

with the experiments which indicate that it is related to surface irregularities.) Such a spiraling has been observed photographically in discharge tubes similar to this one in previous work at the University of Oklahoma, and the effect is known in chemical combustion processes, although the mechanisms may not be the same. It is possible that it is related to the "pinch" effect described by Cousins and Ware.¹ When potential is being measured, that is when no shunts are being used, the spiraling effect can cause relatively little unbalance between the probes, affecting them only by changes in wall potential produced by the proximity of the concentration of plasma. When shunts are in the circuit, however, the tendency to develop differing wall potentials causes currents, and since the concentration of plasma is much greater near one probe than the other, a considerable unbalance in current results. Some of the signals appear to arise from a periodic disturbance, and this fact supports the explanation of the zero-field signal in terms of a spiraling of the plasma. Several records have been studied in which two complete cycles can be observed; the periods vary from nine to twelve microseconds and appear to be constant for constant firing potential and pressure. When the signals show more than one half loop, they oscillate about a base line which is displaced from the axis.

The experiments in which the shields were grounded, floated, and shunted by resistors of various sizes while the condition of the probe circuit was unchanged have great significance, for they indicate that the signal recorded from the probes is quite insensitive to the

¹Cousins and Ware, op. cit.

condition of the shields. Normally, the shields and probes were shunted by resistors of the same value, but their potentials with respect to ground may not have been the same because the total currents which they collected from the plasma may have been different. Ideally, resistors should be used in the shield circuit which would produce the same potential drop as that produced by the shunts used in the probe circuit, but this goal probably is unattainable. The current in the shield circuit begins before that in the probe circuit and continues longer; because of their length, the shields draw current from parts of the column of ionized gas moving at different speeds and with different ion densities; hence, choosing resistors of such value as to equalize the potential drops in both shield and probe circuits for all times during the passage of the ionized gas probably is impossible. The very small effect upon the signal of the shield bias indicates that the probes draw charge from cylindrical volumes of plasma extending directly from their surfaces, and therefore, that the shields adequately serve the purpose of guard rings to insure that the probes behave as planes. The small effect also indicates that the capacitive coupling between shield and probe is small, a fact verified by direct measurement which gave a value of $4.4 \mu\text{mf}$ capacitance between a probe and its shield.

Resistance Measurements

Before the measured resistance values can be used with confidence that they are significant, the role of ion sheaths must be considered. If an electric field is placed across a stationary plasma, ion clouds form near the surface of the electrodes and prevent the

field's penetration into the plasma. If such an effect is present in this work, the resistance measurements must be interpreted with caution.

Two pieces of experimental evidence reveal something about the sheaths which are present. First, the fact that differing plasma resistances are measured with different shunts implies that sheath formation is present to some extent, caused by the electrostatic field produced between electrodes by the potential drop across the shunting resistors. The difference between the resistances measured in hydrogen increases as the point of observation is moved down the tube, perhaps because the gas is flowing more slowly so that sheaths can form more easily. The difference is undetectable in argon, however, although the velocity is low. The difference may be caused by the effect of viscosity on the boundary layer. The fact that a difference is observed supports the interpretation that sheath formation is present in some degree. However, the experiments with batteries in the shunt circuit indicate that the degree must be slight. If sheaths can form, they should have a greater effect when a signal is produced by electric fields than when it is caused by magnetic fields. The signal recorded when batteries were in the circuit was $2 Ir$ if r is the value of one shunt and I is the current flowing in the shunt-plasma circuit. Since r was 99 ohms, I was slightly larger than ten milliamperes. The impressed emf was approximately three volts, and consequently, the plasma resistance was approximately 100 ohms. The value obtained for the plasma resistance under the same conditions by means of magnetic fields is approximately 160 ohms. The two are of the same order of magnitude, and the difference is in the opposite direction to that expected if sheath formation is

important, for the effect of sheaths is to increase the plasma resistance. An additional fact supporting the conclusion that sheath formation is slight is the slow return to the axis of the trace produced by electric field compared to that produced by a magnetic field. If the plasma resistance had become very great at any time during the passage of the ionized gas, the signal would have decreased rapidly; no such quick drop is seen. The fact that the electric field signal returns to zero more slowly than that produced by magnetic fields suggests also that the gas is left somewhat ionized after mass flow has become very small.

Although formation of sheaths probably is present to some slight degree, it cannot be a major factor in the production of the signals recorded. Since the sheaths arise because of the electrostatic field produced by the IR drop in the shunts, the best value of resistance to use as the plasma resistance is that measured with smallest shunt. In most of the measurements the difference between the values obtained with various shunts is very small. It is believed that to neglect this correction is not serious.

In order to relate the resistance as measured to the ion concentration, the quantity whose value is desired, three calculations are possible. The first that will be considered is the mobility analysis; a reasonable value for the mobility taken from the literature will be used.

The mobility of the positive hydrogen ion in neutral hydrogen gas is of the order of $10 \text{ cm}^2/\text{sec}/\text{volt}/\text{cm}$. If this value is used for b , the number of positive ions per cubic centimeter is

$$n = \frac{1.6 \times 10^{19}}{R} .$$

Since R varies from less than one ohm to approximately three hundred and since the particle density of the neutral gas is of the order of $10^{17}/\text{cc}$, it is apparent that the theory of ordinary mobility gives results of questionable validity at even the highest resistance measured, that at 18.1 cm from the lower firing electrode. If the ion concentration measured by Rose at 18 cm by means of Stark broadening is taken as correct, on the assumption that the temperature has fallen low enough by the time the expanding gas has gone 18.1 cm that Holtmark's theory is adequate, and from those measurements and the resistance measured at the same point under conditions as nearly identical as possible the mobility is calculated, it is found to be 5.95 cm/sec/volt/cm, a not unreasonable value in spite of the fact that the ionization indicated by line broadening is about twenty-eight per cent. If this value of the mobility is used to compute the ionization at positions nearer the discharge, however, the indicated ionization goes over 100% at the next position. The mobility of the positive argon ions in argon is approximately 2 cm/sec/volt/cm, but even in the case of these heavy molecules, the use of a classical mobility value together with the measured resistance fails to give a reasonable value for the ion density.

A theory developed by Goudsmit and Saunderson for multiple scattering of electrons by protons gives a value for the effective electron-proton cross section, and that value may be used with an elastic solid sphere mobility theory. The equation developed by Langmuir,¹

¹Loeb, op. cit., p. 55.

for example, is

$$b = .815 (e/m) (L/C_m) \frac{M+m}{M}$$

where C_m is the rms velocity of thermal agitation of the electron, L is the mean free path, and M and m are the masses of the proton and electron respectively. From kinetic theory, $L = 1/\pi a^2 N$ and $C_m = \sqrt{3kT/m}$, from which b is proportional to $1/\sqrt{T}$ and R is proportional to $\sqrt{T}$. Both b and R are independent of N , the particle density. If the assumption were made, however, that the gas in the expanding discharge is so nearly completely ionized that 100% ionization is a good approximation for some appreciable time as it moves down the tube, the variation of resistance with temperature indicated by this theory would be in the wrong direction. Therefore, this theory is unsatisfactory.

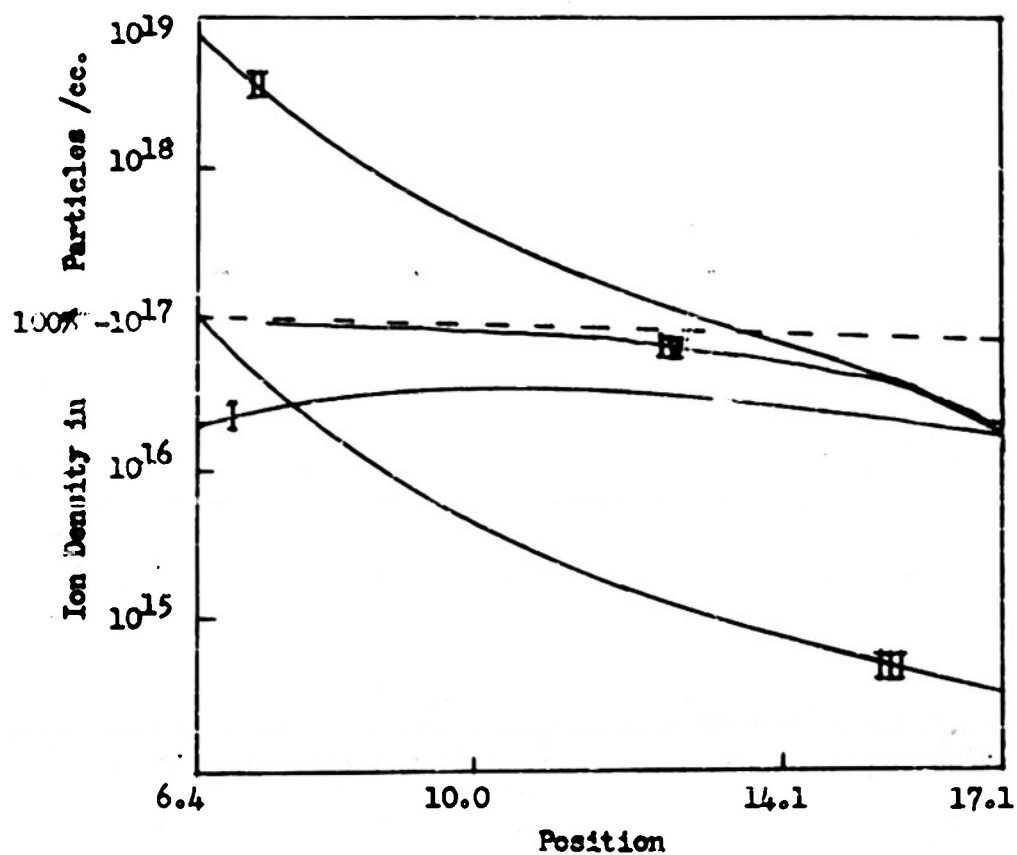
Near the discharge region, at the 6.4 cm position, one might expect the temperature and degree of ionization to be high enough to justify the use of the theory developed by Cohen, Spitzer, and Routly and extended by Spitzer and Härm if consideration is confined to H_2 . Actually, the density is too high and the temperature too low for this theory to be valid, and the formula derived by Chapman and Cowling must be used. This formula is discussed by Cohen, Spitzer, and Routly, and recommended by them for use in the narrow range of densities and temperatures where it is valid and their own is not. When the values of the constants and tube dimensions are put into the expression, the final result for the resistance is

$$R \text{ (ohms)} = (24.0 + 7.13 \log_{10} T - 2.37 \log_{10} n) \frac{10^5}{T^{3/2}} .$$

The best indications of the temperature available are those from color-

temperature methods mentioned in Chapter I. If the value given there for the 6.4 cm position, $10,000^{\circ}\text{K.}$, is used with the resistance measured at that position under similar conditions, approximately one ohm, the value of n is of the order of 10^{17} . This seems to indicate very nearly 100% ionization and suggests that the theory gives a satisfactory relation between resistance, temperature, and ion concentration. When similar calculations are tried at other positions, however, the theory fails completely. At 10 cm, for example, n comes out approximately 10^{-70} . This failure is believed to be caused by the effect of ionizing collisions as the temperature decreases somewhat. Insufficient data are available to permit an attempt to apply this theory to gases other than H_2 .

Figure 17 shows four curves giving information related to estimation of the ion concentration. Curve I is the curve obtained by Rose from Stark broadening methods applying the Holtzmark theory without corrections for temperature. Curve II is the curve obtained by computing a value for the mobility from Rose's ion density value at the 18.1 cm position with the resistance measured there and then using that mobility value with measured resistances to compute the ion concentration at other points. Curve III is obtained by assuming that the gas is 100% ionized at the 6.4 cm position and that the resistance then varies inversely as the ion concentration. Curve IV is suggested as a reasonable compromise which may indicate the approximate shape of the true curve of ion concentration versus position.



- I is curve obtained by Rose from line broadening.
 II is curve computed from mobilities and matching I at 17.1 cm.
 III is curve computed by assuming concentration is 10^{17} at 6.4 and that the concentration varies as $1/R$.
 IV is the curve suggested as most probably of the proper form.

Fig. 17—Ion Concentration Versus Position

CHAPTER VI

CONCLUSIONS

Comparison of particle velocities measured by magnetic deflection with flow velocities measured from pictures taken with a rotating mirror show reasonable agreement. Two inferences may be drawn from this: (1) the magnetic field method offers an easy and convenient way to make measurements of flow velocities in ionized gases, and (2) boundary layers have approximately the same effect on the luminous gas moving down the center of the tube and on the bulk of the gas which is responsible for the potential difference observed. No attempt has been made to use the integral of $u(r)$ to determine the potential difference produced across the tube, and the success of the computations in which u is assumed constant throughout the space indicates that this is a good approximation. The approximation may be satisfactory because boundary effects either are negligible over most of the distance or are quite pronounced over all the distance.

The resistances of the plasma under varying conditions of pressure and firing potential measured as a function of the distance from the discharge region are in every case monotonically increasing functions. The naive interpretation is that the increasing resistance is indicative of decreasing ion concentration, but since measurements

with Stark broadening have shown an apparent increase of the ion concentration to a maximum and a subsequent decay in the region considered, it will be necessary to consider the possible effects of temperature changes in order to eliminate the possibility that temperature changes cause the resistance to increase even though the ion concentration is increasing.

In the preceding chapter it was demonstrated that the formula for the conductivity given by Chapman and Cowling properly relates probable ion concentration, temperature, and resistance at the 6.4 cm position in H_2 . If the ion concentration is to increase, the assumption must be made that at that first position it is less than 100%, but the satisfactory agreement of theory and experiment implies that the ionization is so nearly 100% that the actual condition may be approximated by this state. If the ion concentration increases farther down the tube, this approximation must become even better, and consequently the use of the formula to estimate the temperature at the next position should be legitimate. If the order of magnitude of the ion concentration is assumed to remain the same, the temperature must fall from 10,000°K. at 6.4 cm to less than 1,000°K. at 10 cm in order to cause the plasma to have the measured resistance, and then in order that the ion concentrations may decrease at the indicated rate, the temperature must remain approximately constant. Such a behavior of the temperature seems extremely unlikely and is in sharp disagreement with estimates of it made by Fowler, Atkinson, and Marks from color-temperature theory. Unless there are factors entering into the determination of the resistance other than the ion concentration and temperature, therefore, the

experimental results indicate that the ion concentration is a monotonically decreasing function down the tube.

In the type of discharge and the region of the discharge studied, the Hall effect is an excellent tool for the investigation of the plasma because rapid mass motion of the gas is present. The great advantage of magnetic fields over electric fields in the study of the plasma is that magnetic fields penetrate the ionized gas, and the plasma cannot protect itself from them by means of sheaths.

This ability of the magnetic field to penetrate the plasma suggests another variation of the present work. A steady discharge in which mass flow is not present might be investigated by means of a changing magnetic field. The changing field would produce an effective electric field which could act on all ions equally, and again the possibility of sheaths would be eliminated. Perhaps the use of this technique would yield no information which could not be obtained by micro-wave techniques, but it deserves consideration.

The author believes that in a steady discharge, in which the only cause of ion motion is the impressed electric field, the Hall effect has little to recommend it over the probe methods developed by Barnes and Eros and by Johnson and Malter. In any discharge in which there is gas flow, however, or in a case in which ionized gas flows, as in shock tubes producing luminous shocks, consideration of the Hall effect as a possible tool is indicated.

Three questions raised by this work deserve consideration in future work. One concerns the theory of the conductivity of a partially ionized gas, which is completely lacking at the present time. The

second concerns the origin of the zero-field signals. The explanation given of these probably is essentially correct, but further work to prove that surface irregularities in the tube have an effect and to investigate the periodic phenomena in hydrogen is desirable. It is possible, also, that greater accuracy could be attained by this method through the use of a tube so constructed that no irregularities were present, if such a tube could be devised. The third question is that of the cause of the photoelectric signals. The explanations suggested in this thesis are all unsatisfactory. It is believed, however, that the explanation of a charge flow down the tube because of a density gradient in the ions produced by photo-ionization processes is correct. Once the entire effect is understood, it may offer a means of investigating other properties of the gas and of the discharge. The gradient in ion density, for example, must depend upon the cross section for photo-ionization, and in a suitably designed probe system, the time at which the radial current became space-charge limited might be determined, giving a method of estimating the necessary density of the ion cloud and the relative mobilities of the ions involved.

In anticipation, it may be mentioned that in the next chapter another question, the origin of the continuous radiation in such discharges, will be raised, and it may well be the most important of all the slightly understood processes.

CHAPTER VII

CRITIQUE

As has been pointed out, the most rigorously derived equation for the conductivity of an ionized gas, that of Cohen, Spitzer, and Routly, is inapplicable in the interpretation of the results of this work because it fails at density-temperature ratios as high as those encountered here. The next best formula, that of Chapman and Cowling, correctly relates the conductivity, temperature, and ion density at the one experimental point where all three are highest, but it fails completely at all other points. At an experimental position where the ion concentration probably has decreased only a few per cent and the temperature has decreased by about twenty per cent, the predicted conductivity is many times larger than the measured value. The discrepancy between prediction and experiment is so great that one must assume that some process becomes operative when the ionization falls below 100% which reduces the conductivity far more than the removal of a relatively small number of ions should indicate. This process probably is ionizing collisions. The theory of conductivity of a totally ionized gas at high temperatures is derived from solutions of Boltzmann's equation under the assumption that the only process changing the velocity distribution function of the ions and electrons is inverse square law interactions in which no single encounter makes a

great change in the energy of any particle. If an appreciable number of neutral atoms are present and the temperature is high enough that the rate of ionization by impact is high, many particles will lose most of their energy in single impacts and many more electrons will be released into the gas by ionizing events with energies far below those of the neighboring particles. As a consequence, a relatively small number of neutral atoms will produce a great effect on the conductivity. The development of a theory for the conductivity of partially ionized gases is greatly needed.

It is customary, in discussions of Stark broadening by ion fields in an ionized gas, to ignore the electrons, excusing this procedure by the argument that electron velocities are so high that their effect is that of a smear of negative charge, and consequently they contribute nothing to the broadening. By exactly the same reasoning, one must conclude that at a sufficiently high temperature, for example at a temperature forty-five times as high as that at which the electron effect becomes negligible, the ion fields become that of a smear of positive charge so that the effective field at the radiating atom is zero. It follows, therefore, that for cases of high temperature, corrections should be made to the Holtmark distribution to take account of the velocities of the ions, and that the effect of these corrections should be to reduce the broadening observed for a given ion concentration as the temperature is increased. If such corrections are not made and ion concentration is estimated from the half-widths or from the contour of the hydrogen lines in any region except at the extreme outer wings of the lines, at which point the temperature correction becomes

small, the apparent ion concentration is low.

Theoretical work on the correction of the Holtsmark theory for velocity has been done by Krogdahl and by Spitzer. At the densities and temperatures involved in this work, the results derived by Krogdahl predict almost no change from the static theory originally suggested by Holtsmark. Experimental evidence seems to indicate, however, that the deviations are rather large. The work by Spitzer seems to predict rather large deviations, and it is felt that probably his approach to the problem is more promising when gases of relatively high densities are considered, that is, particle densities of the order of $10^{17}/\text{cc}$ or higher.

The deduction drawn from the present work that the ion concentration is a monotonically decreasing function from the discharge region down the expansion tube conflicts with interpretations previously given of three facts.

In the past work at the University of Oklahoma, a maximum in the broadening of Balmer lines occurring some distance down the expansion tube has been interpreted to mean that ion concentration reached a maximum at the same position. From what has been said above, it is plausible that the dip in the broadening at the very head of the expansion region is caused by the extremely high temperatures (of the order of $26,000^\circ\text{K.}$) present there. The interpretation of the broadening was made on the basis of measurements of half intensity widths, and these must be quite sensitive to the peaking of the line contour produced by high temperature.

The second point of difficulty is the extremely careful work of

Olsen and Huxford, who found an apparent maximum in the ion concentration delayed in time after the peak of the discharge current. This result seems to support the interpretation given of the maximum of line broadening in space as being caused by a maximum of ion concentration. The description given of the method used to determine ion concentrations from the observed broadened lines suggests that their results should be free from the fault criticised above, for they fitted their experimental curves to the theoretical curves on the skirts of the lines and did not compute a half width. Careful study of the sample curve fittings which they offer reveal two interesting facts: (1) the best fit was made at a part of the profile where the slope is rather large, that is, at approximately the region of half intensity, whereas the fit at the extreme outer wings is not so good as might be hoped, and (2) the data are least precise at the place where they are most important, namely, on these outer wings. These observations indicate that their work, too, may have been interpreted incorrectly because of the temperature of the gas. The conclusion of Olsen and Huxford that the ion concentration reaches a maximum after the maximum discharge current is substantiated in their published work by one measured point in neon and two points in argon. Since these points were taken while the gas was very hot, and since the effect of this temperature upon the broadening would be to make the concentration appear low, it is not unreasonable to suppose that deviations from Holtmark's theory account for the apparent increase in ion concentration in early stages of the discharge.

The third point of conflict is in the interpretation of the

maximum of the intensity of continuous radiation observed in space by Atkinson and in time by Olsen and Huxford. On the assumption that the gas reaches a state of almost 100% ionization before entering the region at which it is observed or before the end of the discharge current and that the temperature is so high that the degree of ionization falls slowly for the first few microseconds because of the temperature dependence of the recombination coefficient, a maximum in the intensity of quantized radiation and in the intensity of radiation from recombination continua can be explained. Olsen and Huxford have suggested that this unquantized radiation may arise chiefly from Bremsstrahlung processes, which should give an intensity proportional to the square of the ion concentration. The belief that this continuous radiation is primarily a recombination continuum is supported, however, by the remarkable agreement found by Jurgens¹ between temperature in a hydrogen arc calculated by the application of Kramer's formula to the measured intensity of the continuum and temperature calculated by several other independent methods.

One reason that Olsen and Huxford considered recombination an unlikely cause of the recombination is that they obtained a very small value for the recombination coefficient. If their measurements of ion concentrations need corrections for temperature, their determinations of the recombination coefficient also need corrections. The effect of the consideration of this factor is to increase somewhat the values given in their paper.

An even larger deviation from the true recombination coefficient

¹G. Jurgens, *Zeits. f. Physik* 134, 21 (1952).

may have been introduced, however, by the assumption that production of ions is zero during the plasma decay period when the recombination coefficient is measured. The recombination coefficient, α , is defined by the equation

$$dN/dt = -\alpha N^2,$$

in which N is the electron density, assumed equal to the density of positive ions. If, as a first approximation, production processes are neglected, the solution of this equation is

$$1/N = 1/N_0 + \alpha t,$$

if N_0 is the electron density at time $t = 0$. If $1/N$ is denoted by X and $1/N_0$ by X_0 , this becomes

$$X = X_0 + \alpha t,$$

the equation of a straight line with slope α . This is the equation used by Olsen and Huxford to determine α from their measurements of ion concentrations as a function of time.

If production processes are included, the major ones may be included in an equation of this form:

$$dN/dt = -\alpha N^2 + \beta N(N_a - N) + \gamma N^2(N_a - N) + \delta N^3(N_a - N).$$

N_a is the number of atoms originally present in the gas, so that $(N_a - N)$ is the number of neutral atoms at any instant. These terms represent the following processes: The first term is simply recombination. The second term represents ionization by impact of electrons on neutral atoms, and hence is proportional to the number of electrons present and to the number of neutral atoms. The third term describes the process of photo-ionization. Since the production of radiation is proportional to N^2 if those atoms which are merely excited without being

ionized are ignored, and the production of ions must be proportional to the number of neutral atoms present, the term has the form indicated. In a slightly ionized gas this coefficient would be small, but in a highly ionized gas, the ionic fields lower the ionizing potential sufficiently that photons of sufficient energy to raise the atom to a highly excited state may ionize it. Because of the intense high energy radiation present in such discharges, this effect may be quite significant. The last term describes the production of ions by photo-excitation (proportional to $N^2(N_a - N)$) followed by collision with an electron.

The equation may be rewritten as

$$dN/dt = N(\beta N_a) + N^2(\gamma N_a - \alpha - \beta) + N^3(\delta N_a - \gamma) - \delta N^4,$$

or

$$dN/dt = gN + hN^2 + jN^3 - \delta N^4.$$

If all terms containing N^3 and N^4 are neglected, the equation becomes

$$dN/dt = gN + hN^2,$$

whose solution is

$$1/g \log N/(g + hN) = t + C,$$

or

$$(g + hN)/N = C'e^{-gt}.$$

If N_0 is the value of N at the time $t = 0$, that is, the initial ionization, and if X and X_0 represent $1/N$ and $1/N_0$ respectively,

$$gX + h = (gX_0 + h)e^{-gt},$$

and if g is quite small and only a small interval of time is considered, this may be approximated by

$$gX = gX_0 - g(gX_0 + h)t$$

or

$$X = X_0 - (gX_0 + h)t .$$

If g and h are replaced by their values, this becomes

$$X = X_0 - (\beta N_a X_0 + \gamma N_a - \alpha - \beta)t ,$$

and if the gas is assumed to be 100% ionized initially, so that

$N_a = N_0$, this is

$$X = X_0 + (\alpha - \gamma N_a)t .$$

This, too, is the equation of a straight line, having the same intercept as the line obtained when production is ignored, but having a smaller slope. As was pointed out above, under the conditions of intense discharges which produce almost 100% ionization, the coefficient γ may be rather large, and ignoring it will yield a value for α much too small.

A process which may supplement recombination in the production of a continuum is the formation of negative hydrogen ions. If the gas is initially almost 100% ionized and remains in that condition for several microseconds because the temperature is so high that the recombination coefficient is small, the recombination continua should show a maximum at a time after the discharge current has passed a maximum. By the same argument, formation of negative hydrogen ions should be proceeding most rapidly at the time when recombination is most important. Chandrasekhar¹ has suggested that the negative hydrogen ion may be responsible for much of the absorption shown by the solar atmosphere, and he indicates that an atmosphere containing many such ions should show a continuous absorption coefficient, with a maximum in the

¹S. Chandrasekhar, Rev. Mod. Phys. 16. 301 (1944).

vicinity of 8,000 to 12,000 Angstroms. Possibly the presence of negative hydrogen ions contributes to the continuum which is spread across the spectrum as far as investigations have been made in either direction, but recombination, possibly enhanced by the ionic fields, probably contributes most of the radiation observed associated with the line series.

One other piece of evidence which supports the interpretation given in this thesis of decreasing ion concentration as the cause of increasing resistance of the plasma is the gradation of the lines of ionized silicon and oxygen observed in similar discharges at the University of Oklahoma. Silicon IV lines are most intense at the head of the expansion chamber, disappearing five centimeters down the tube. Silicon III is also graded, and it persists eight centimeters. Silicon II is visible throughout the fourteen centimeter region studied with a maximum at six centimeters, and silicon I is observed in only the lower nine centimeters of the region under investigation. These facts are explained nicely by the assumption that initially most of the silicon atoms are ionized, singly, doubly, or triply; then, as the gas cools, triply ionized systems disappear first, followed soon by those doubly ionized. As the level of ionization and the temperature fall, neutral atoms become numerous enough to make their radiation visible. If, initially, more ions are ionized doubly and triply than singly, the maximum in silicon II lines is explained.

CHAPTER VIII

SUMMARY

Recent literature pertaining to the conductivity of ionized gases has been reviewed, particularly the work of Chapman and Cowling, Landshoff, and of Cohen, Spitzer, and Routly. These theories were designed for astronomical application. No experimental work to test them has appeared prior to the present report. The results of Chapman and Cowling in the form discussed by Cohen, Spitzer, and Routly have been taken as most suitable for the analysis of our data, although none of the theories applies to partially ionized gases. Conductivity in a completely ionized gas depends on ion concentration and ion temperature.

Because numerous workers have used Stark broadening to estimate ion concentrations in highly ionized gases, reference has been made to new literature concerned with corrections to the Holtsmark theory of Stark broadening by random fields. The major contributions have been made by L. Spitzer and by M. K. Krogdahl. The effect of ion motion upon Stark broadening is shown by these workers to give low values of apparent concentration at high ion temperatures when half widths are used for estimation.

The research described in this thesis has used a pseudo Hall effect in which deflections of ions in a gas flowing between probes through a region free of electric fields are produced by the application

of a magnetic field to measure the particle velocities of the gas ions and to measure the resistivity of the plasma at points along the expansion path of a pulsed discharge. Particle velocities observed agree well with the flow velocity of the expanding gas determined from pictures made with a rotating mirror.

The resistance of the plasma is shown to decrease monotonically as the point of observation is moved from 13.1 cm to 6.4 cm from the discharge region. This is believed to be inconsistent with the interpretation made of Balmer line broadening on the basis of the Holtmark theory by Fowler, Atkinson, and Marks and by Olsen and Huxford. Studying a discharge similar to the one used in this work, Fowler, Atkinson, and Marks found that the line broadening indicated that the ion concentration rises to a maximum within the range 18 to 6.4 cm. Olsen and Huxford, making a time-resolved study of a confined discharge, found a maximum of broadening of the Balmer lines in time which they interpreted to mean that the ion concentration reaches a maximum at a time after the discharge current has fallen almost to zero.

All available theory is shown to be inadequate to calculate the ion concentration from the measured resistances. Calculations made with mobility values given in the literature for hydrogen ions in hydrogen indicate an ionization far above 100% at all except the greatest resistance. The cross section for electron-proton multiple scattering calculated by Goudsmit and Saunderson has been used in an elastic solid sphere mobility equation for mobility, but the resulting expression is independent of ion concentration and gives the wrong dependence on temperature to explain the data.

When the formula derived by Chapman and Cowling is used to compute the particle density at the point nearest the discharge region, where the lowest resistance is measured, an approximately correct value for complete ionization is obtained if a temperature of $10,000^{\circ}\text{K}$. is used. This temperature is the one estimated from spectral observations using the Fowler-Milne color-temperature theory. It may be assumed, therefore, that at this one point the gas is nearly 100% ionized and the theory gives the proper relation between the particle density, temperature, and resistance. The theory fails completely at all other points and resistances. It is suggested that the reason is that as the ionization drops below 100%, so much energy is lost by the ions in ionizing impacts that the neutral gas has an effect on the resistance far out of proportion to its relative abundance. Modification of the theory of Chapman and Cowling to consider this effect is needed.