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AAP YAW MEASUREMENTS BY RF SIGNAL FROM XRAT SONDE

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U. S. NAVAL ORDNANCE LABORATORY
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AAP YAW MEASUREMENTS BY RF SIGNAL FROM XRAT SONDE

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ABSTRACT: A method is presented for analyzing the RF signal received by a vertical antenna from the XRAT sonde in flight. This method of analysis, when coupled with relative amplitude calibration of the receiver and recording mechanism, is capable of giving a very accurate measurement of the vertical component of yaw of the AAP missile, when the angle between the missile axis and the line of sight is small (less than [illegible]). It is also capable of determining the body to loop field strength ratio and phase to a high degree of accuracy. If a horizontal antenna is also used, it will give the horizontal component of yaw in addition, and the complete yaw motion can be reconstructed.

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The method of yaw analysis presented here was developed for analyzing the flight characteristics of full scale Angled Arrow Projectiles (Project NOL-Re4c-117-1-53) and has given satisfactory results when used for this purpose. With proper care in the recording and analysis of data it appears likely that the general method can be successfully applied to spin stabilized projectiles and other rapidly rotating missiles. This report is intended by the Naval Ordnance Laboratory to stimulate interest in this type of analysis and should not be a basis for official action.

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Commander

D. S. Muzzey Jr.

D. S. MUZZEY, JR.
By direction

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REFERENCES

(a) NOLM 10413 - Missile guidance system XRAT, nature of guidance signal L. F. Green (Confidential)

(b) A radio method of studying the yaw of shells - C. C. Gotleib, P. E. Pashler and M. Rubinoff - Canadian Journal of Research - Vol. 26, Sec. A - May 1948

AAP YAW MEASUREMENTS BY RF SIGNAL FROM XRAT SONDE

INTRODUCTION

1. The XRAT sonde, as described in reference (a), was developed to indicate the rotational position of the missile. It consists essentially of an electric dipole along the axis of the missile and a magnetic dipole at right angles thereto. The form of the signal received by a vertical antenna from the rotating sonde is illustrated in Figure 1. The signal received by a vertical antenna from the electric dipole is independent of the rotational position of the missile about its axis, and varies approximately as sin γ, where γ is the illumination or viewing angle, i.e., the angle between the line of sight from the antenna to the missile, and the missile axis. The signal received by the same vertical antenna from the loop varies sinusoidally as the sonde rotates.

2. If we assume that small angle yaw in the horizontal plane has only a second order effect on these signals received by the vertical antenna, that reflections from the ground or water can be ignored, and that the missile rotation rate is considerably more rapid than its yaw rate, then the form of the signal received by the vertical antenna is characteristic of the angle γ, and γ can be determined from the form of the signal. These assumptions are vital to the calculations, and they will be examined later in this report in some detail.

3. At this point it seems worthwhile to draw a comparison between this method of yaw analysis and the Thompson method described in reference (b). In the Thompson method the sonde contains only the dipole signal and changes in the angle γ are detected by changes in the strength of the dipole signal over the yaw cycle. The principal disadvantage of this is that changes in sonde signal amplitude, ground reflection or receiver gain are indistinguishable from changes in amplitude due to yaw. The longer the period of interest the greater are the errors due to this cause. In the limit a fixed angle of yaw (fixed in amplitude and in direction relative to the ground) is not detectable. The present method, on the other hand, compares signal amplitudes only over a single rotation of the projectile, and therefore changes in signal amplitude from whatever cause are of consequence only if they contain appreciable components at the spin frequency. In the limit a fixed angle of yaw is easily measured.

4. In the present method we must obtain an analytic correspondence between the angle γ and the form of the signal. In order to accomplish this, some parameter must be selected to describe the signal form. For this purpose we have selected the ratio of the major peak height to the minor peak height M/m (Figure 1); this ratio is called λ. It is easily determined from the film records, varies rapidly with γ for small values of γ, and is relatively insensitive to antenna impurity and horizontal yaw. It has the disadvantage that γ is related to it by a double valued function;

i.e., as γ increases, λ goes to a maximum, and then decreases, so that corresponding to a given λ, there are two possible values of γ, and distinguishing between them is difficult in the region of the maximum λ. Further, in this region λ is a very insensitive measure of γ. However, as will appear presently, this region corresponds to fairly large values of γ, generally outside the region of interest.

THEORETICAL RELATION λ AND γ

5. Reference (a) presents a graphical derivation of the form of the signal received from the sonde by a vertical antenna. This form is shown in reference (a) to be a function of A_D/A_M and B where B is the R.F. phase difference between the signals from the loop and dipole and A_D/A_M depends on the relative strength of the dipole and loop radiators and the sine of the viewing angle γ. For present purposes we are concerned only with the heights M and m of the signal (figure 1), since λ is the ratio of these two heights. We wish to find directly a relation between B, λ, γ, and A where:

B is the RF phase difference between the radiation from the loop and dipole.

A is the loop to dipole ratio, i.e., the ratio of the maximum loop field strength to the maximum dipole field strength at the same distance from the sonde. This is the ratio of loop to dipole field strength in the plane of the loop at 90° from the missile axis.

γ is the angle between the missile axis, and the line of sight from the antenna to the missile.

λ is the ratio of the field strength of the major maximum to that at the minor maximum (M/m Figure 1).

By a slight change in the notation in reference (a) we can obtain the values of the radiation received from the dipole and loop as:

Dipole $E_d = C\, f(\alpha) \sin \gamma$

Loop $E_l = AC\; f(\alpha) \cos i\; e^{jb}$

Where i is the index position of the loop i.e., the angle between the loop plane and the vertical.

α is the angle between the receiving antenna and the perpendicular to the line of sight in the plane of the trajectory.

C is a function of range, sonde power etc. which varies only slowly and can be considered constant for present purposes.

The major and minor maxima (M and m) occur when the loop is in a vertical plane i.e., when cos i = 1 or -1. Thus M is the magnitude of the sum of the body and loop signals, i.e.:

$$|C\, f(\alpha)(A\, e^{jb} + \sin\gamma)|$$

or

$$C\, f(\alpha)\sqrt{A^2 + \sin^2\gamma + 2A \sin\gamma \cos B}$$

and m is the magnitude of

$$C\, f(\alpha)\ (A\, e^{jB} - \sin\gamma)$$

or

$$C\, f(\alpha)\sqrt{A^2 + \sin^2\gamma - 2A \sin\gamma \cos B}$$

and

$$\lambda^2 = \frac{A^2 + \sin^2\gamma + 2\, A \sin\gamma \cos B}{A^2 + \sin^2\gamma - 2\, A \sin\gamma \cos B}$$

$$\lambda^2 = 1 + \frac{4\, A \sin\gamma \cos B}{A^2 + \sin^2\gamma - 2\, A \sin\gamma \cos B}$$

We have thus found λ as an analytic function of B, γ and A. We can now examine some of the properties of this function. First what is the maximu value of λ, and at what value of γ does it occur

if we let $\frac{\sin\gamma}{A} = \mu$

$$\lambda^2 - 1 = \frac{4\mu \cos B}{1 + \mu^2 - 2\mu \cos B} \qquad \text{Equation (1)}$$

When λ is a maximum $\lambda^2 - 1$ will also be a maximum. This will occur for fixed value of B, when

$$\frac{\partial}{\partial\mu}(\lambda^2 - 1) = 0$$

or

$$\frac{(1 + \mu^2 - 2\mu \cos B)\ 4 \cos B - 4\mu \cos B\ (2\mu - 2 \cos B)}{(1 + \mu^2 - 2\mu \cos B)^2} = 0$$

The denominator cannot be infinite so

$$4 \cos B - 4\mu^2 \cos B = 0$$

or

$$\mu^2 = 1$$

So the maximum value of λ will occur when

$$\sin \gamma = A$$

This maximum value is from Eq. 1:

$$\lambda_{max} = \sqrt{1 + \frac{2 \cos B}{1 - \cos B}}$$

which depends only on B, and not upon A. This maximum value of λ is plotted in Figure 2 as a function of B.

According to the best information available, B for the XRAT 2 turn loop sonde is approximately 60 degrees, and A is roughly 1/8. Figure 3 is a plot of λ vs γ for this condition from Eq. 1.

GENERAL PROCEDURE

6. If we knew A and B (the loop to dipole ratio and phase) with sufficient accuracy, we could make a plot similar to Figure 3 for the AAP missile. We could then determine the values of λ from the calibrated film record of the signal received by the vertical antenna from the missile in flight. By referring each value of λ to our graph, we could find the corresponding value of γ, which is the instantaneous angle the missile axis makes with our line of sight. From the aeroballistic trajectory tables we could find the location of the missile at that time, and the orientation of the line of sight from the antenna to the missile. The instantaneous orientation of the missile could then be obtained from geometrical consideration of these two angles, both in the plane of the trajectory. If there were no yaw, this procedure would give an orientation coincident with that of the trajectory. Yaw would be represented by a periodic fluctuation of the missile orientation about the direction of the trajectory. In fact the yaw oscillations are sufficiently rapid compared to the rate of change of the trajectory that change in trajectory during a yaw cycle is small and the approximate yaw can be determined directly from Figure 3 without reference to the trajectory tables, as angle of yaw $= 1/2 \, (\gamma_{max} - \gamma_{min})$ where the maximum and minimum γ refer to the same yaw cycle.

7. The procedure outlined above for finding the vertical component of yaw presupposes an accurate knowledge of the loop to dipole ratio and phase for the sonde in question. The accurate measurement of these parameters for each sonde before firing is a task of considerable magnitude, and actually need not be done. The plot of λ vs γ can be made directly from the film record of the received signal if the trajectory is known, instead of being derived from B and A. The process is one of plotting an average λ over a yaw cycle against the viewing angle γ' which would exist at that time if the missile pointed along the trajectory. γ' is obtained from the trajectory tables. The average value of λ used should not be an arithmetic average, but an "educated" average based on the assumption that the missile yaws symmetrically above and below the direction of the trajectory. This assumption is quite reasonable since except for aerodynamic forces which are along the trajectory, the missile is a freely falling body. The only

source of asymmetry in the yaw of the missile is the curvature of the trajectory which should have negligible effect on a fin stabilized missile. If this assumption, that the missile yaws equally above and below the direction of the trajectory, is accepted, then γ'' is the mean of γ max and γ min where γ max and γ min are the values of γ corresponding to the maximum and minimum values of γ over the yaw cycle. The "educated average" value of λ which should be plotted on our curve of λ vs γ is that corresponding to γ'' which is not necessarily the arithmetic mean of λ max and λ min.

8. This procedure for determining the "educated average" value of λ for plotting λ vs γ pre-supposes the existence of a λ vs γ plot. This apparent impasse is not serious since the "educated average" depends only on the general shape of the λ vs γ curve, which can be obtained by plotting the average value of λ.

9. The actual mechanism for finding the yaw from the values of λ can be described with the aid of Figure 4 as follows. From the film records λ is obtained as a function of the time of flight. γ', the viewing angle which would exist if the missile pointed along the trajectory is obtained from the trajectory tables, also as a function of the time of flight. λ is then plotted (as in Figure 4) against the value of γ' occurring at the same time. If the upper and lower envelopes of this curve are drawn, a first approximation to the average curve is obtained by averaging the height of the upper and lower envelopes at a single value of γ.

10. This first approximation is sufficiently accurate to permit obtaining from it the "educated average" value of λ. At any γ_0 draw a horizontal line from the upper envelope to the right until it crosses the first approximation average curve, and find the value of γ at this point. Call this γ max. It is, to the accuracy of our first average curve, the value of γ corresponding to the maximum downward yaw angle. From the intersection of γ_0 with the lower envelope draw a horizontal line to the left. The intersection of this with the first average curve gives γ min, the approximate value of γ corresponding to the maximum upward yaw angle. Find $\frac{1}{2}(\gamma \text{ max} + \gamma \text{ min})$ and from the mean curve, the value of λ corresponding to this γ. This value is our "educated average" corresponding to γ_0 and when plotted at γ_0 represents one point on our final λ vs γ curve.

11. From this final λ vs γ curve we can find the yaw angle directly, by a process very similar to that used for finding the final curve. To find the amplitude of yaw at any point on Figure 4, say when $\gamma' = \gamma_1$ find the intersection of the upper envelope with γ_1, and draw a horizontal line to the right until it intersects the final λ vs γ curve. This gives γ_1 max, and the yaw amplitude is γ_1 max $- \gamma_1$. The amplitude could be obtained in the same way from γ_1 min, and will give the same result if the final λ vs γ curve is correct.

ASSUMPTIONS

12. In the introduction several assumptions which are vital to this yaw analysis method were made without justification. The first of these was

that small angle yaw in the horizontal direction could be neglected. Since our analysis is only concerned with the points m and M (Figure 1), when the plane of the loop is vertical, the first order effect of horizontal yaw is to move the receiving antenna from the plane of the loop, and the signal received by it from the loop will be proportional to the cosine of the horizontal yaw angle. For small angles the variation in the cosine function is negligible. The signal arriving from the dipole depends upon the total viewing angle which is the square root of the sum of the squares of γ and the horizontal yaw angle, but the vertical receiving antenna is sensitive only to the vertical component of the arriving signal, which eliminates the dependence on horizontal yaw angle, of the signal from the dipole, so that λ is sensitive to the horizontal yaw angle only to the same order as the cosine of the angle, which is negligible. It should be noted that throughout this analysis the receiving antenna is considered to be in the plane of the trajectory. If the antenna is appreciably removed from this plane much of the present method must be reviewed.

13. The problem of ground and water reflections is not readily subject to theoretical analysis since the reflecting surface is partly ground and partly water, both of variable and uncertain electrical properties. The basis for assuming these reflections negligible is two-fold. First the receiving antennas in use have the vertical pattern of a half-wave dipole cocked up 15 degrees so that the gain in the direction of the missile is greater than the gain in the direction of the image; and second the reflections can cause only minor errors unless their phase variations are rapid, or the angle of the missile between the direct line of sight, and that to the image, is large. Neither of these conditions exists in the Dahlgren configuration after the first few seconds of missile flight.

14. The assumption that the missile rotation rate is considerably more rapid than its yaw rate is actually not justified. For the AAP missiles which have been studied to date, the spin rate is only 4 or 5 times the yaw rate. That this is not a sufficient factor is evident from the fact that if λ is measured as the ratio of a major peak to the following minor peak, the results are substantially different from measuring λ as the ratio of major peak to the preceding minor peak. It is evident that the viewing angle changes appreciably between the time of occurrence of the major and minor peaks. In order to reduce the error from this source, λ is not measured directly from the film. Two graphs are made, one of major peak height (as a function of time), the other of minor peak height. The points on each graph are connected by a smooth curve, and λ is measured as the ratio of the simultaneous heights of the two curves.

15. In this way the accuracy of the analysis depends only on the accuracy with which the proper curve can be drawn between the measured points representing major and minor peak heights.

HORIZONTAL YAW

16. If a horizontal antenna is used in addition to the vertical antenna, the horizontal yaw of the missile is also deducible. The procedure and analysis

is very similar to that described above for the vertical antenna with one essential difference; the λ vs γ curve cannot be obtained from the horizontal antenna unless the antenna is greatly displaced from the missile launching point, and from the plane of the trajectory. For the horizontal antenna the values of λ will depend, not on the value of γ which is measured in the vertical plane, but on the values of γ_H, its horizontal counterpart, i.e., γ_H is the horizontal projection of the angle between the missile axis and the line of sight. If the horizontal antenna is not greatly displaced from the plane of the trajectory, the average value of γ_H over a yaw cycle will be uniformly 0 (for the unsteered missile). Therefore we do not have available in this case a systematic variation of the average value of γ_H from which we can deduce the λ vs γ curve.

17. The λ vs γ curve, however, depends only on the parameters of the missile radiation system, and is the same for a vertical and horizontal antenna. Therefore, although the horizontal antenna record alone cannot be used to deduce the horizontal yaw, when it is used in conjunction with a vertical antenna, the values of λ from the horizontal antenna can be referred to the λ vs γ curve from the vertical antenna, and the horizontal yaw can be deduced.

CALIBRATION AND RESULTS

18. It has been stated that the field strength recordings from which the yaw is to be deduced must be calibrated. The only measurement from these records which is used is λ, the ratio of major to minor peak heights. Since λ is a ratio, only relative and not absolute calibration is required. The accuracy of this relative calibration is the major factor in the accuracy of the final yaw amplitudes, and if the receiver recording system is calibrated after the record is taken, spurious results will sometimes appear due to changes in receiver characteristics, which may or may not be due to changes in signal frequency.

19. The results to be presented in this report were obtained using a system of continuous relative calibration during the recording. A Germanium diode was put across the line between the antenna and the receiver, and adjusted so that a square wave applied to the diode effectively inserted and removed a fixed amount of attenuation. The square wave used was of sufficiently high frequency that in the final record there are effectively two traces, the lower one always representing a fixed fraction of the field strength of the upper one. Thus there is a continuous check of one point on the relative calibration curve, and any fluctuations in system linearity can be detected and corrected for in the analysis.

20. Figure 5 presents a portion of the final yaw data on AAP Round Serial No. 225 fired at the Naval Proving Ground, Dahlgren, Virginia, on January 16, 1952. The round was gun launched at 3600 ft/sec. and steered to the right after 4.50 seconds of flight. Recordings were made of both vertical and horizontal polarization, and analyses of vertical and horizontal yaw were made as described in this report. Figure 5 is a plot of vertical vs. horizontal yaw motion with time indicated by the crosses every .02 seconds along the curve.

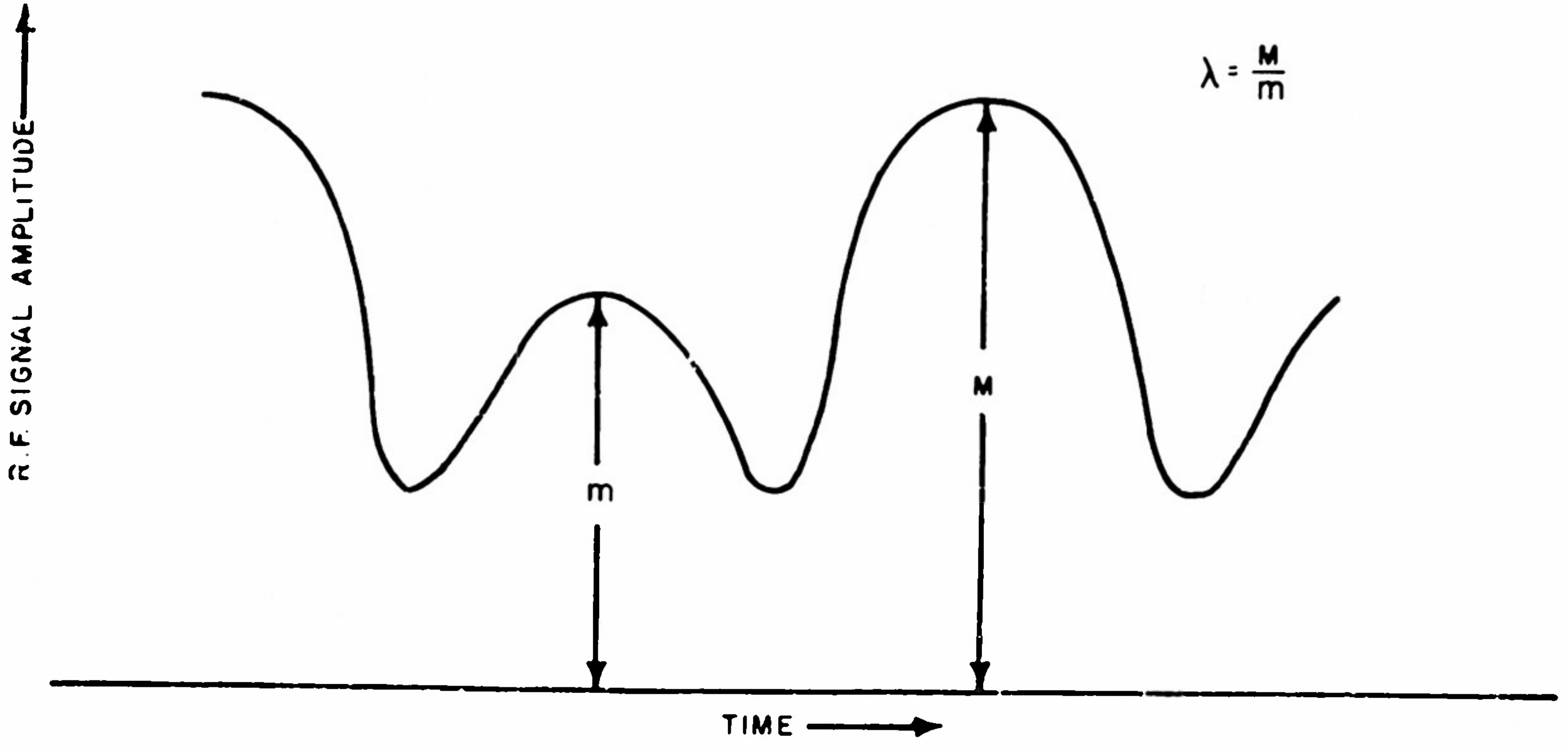


FIG. I FORM OF RECORDED SIGNAL

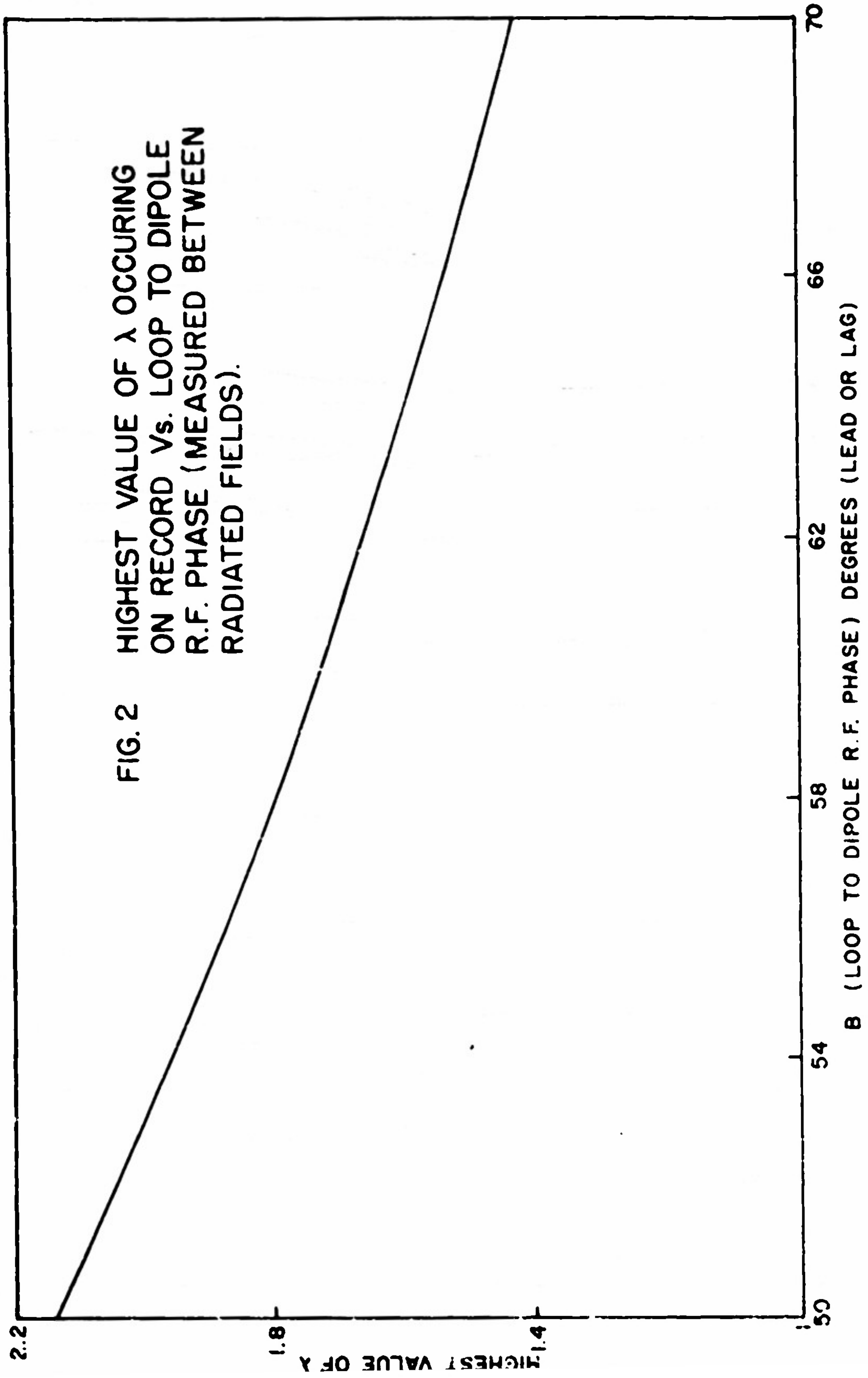


FIG. 2 HIGHEST VALUE OF λ OCCURING ON RECORD Vs. LOOP TO DIPOLE R.F. PHASE (MEASURED BETWEEN RADIATED FIELDS).

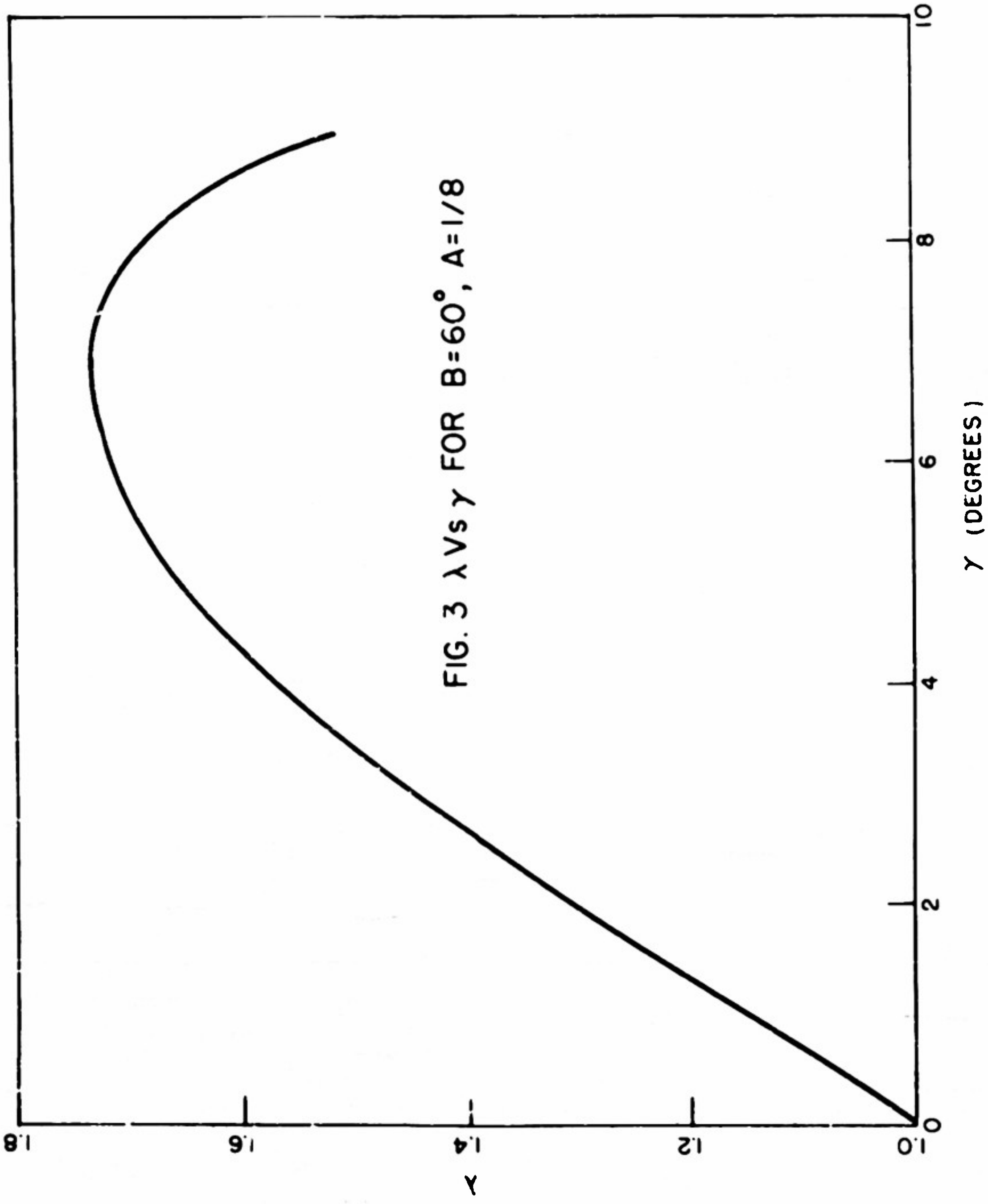


FIG. 3 λ Vs γ FOR B=60°, A=1/8

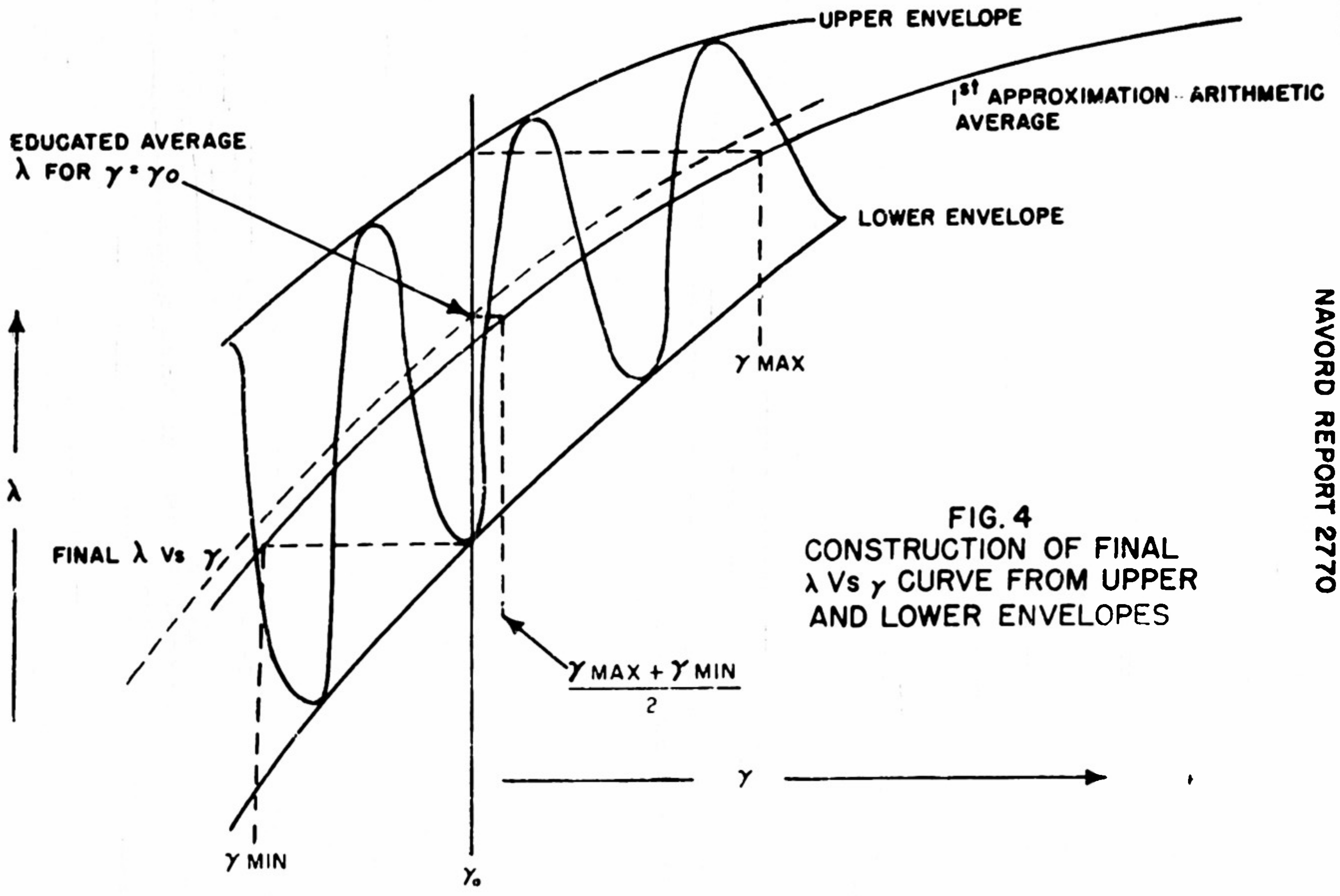
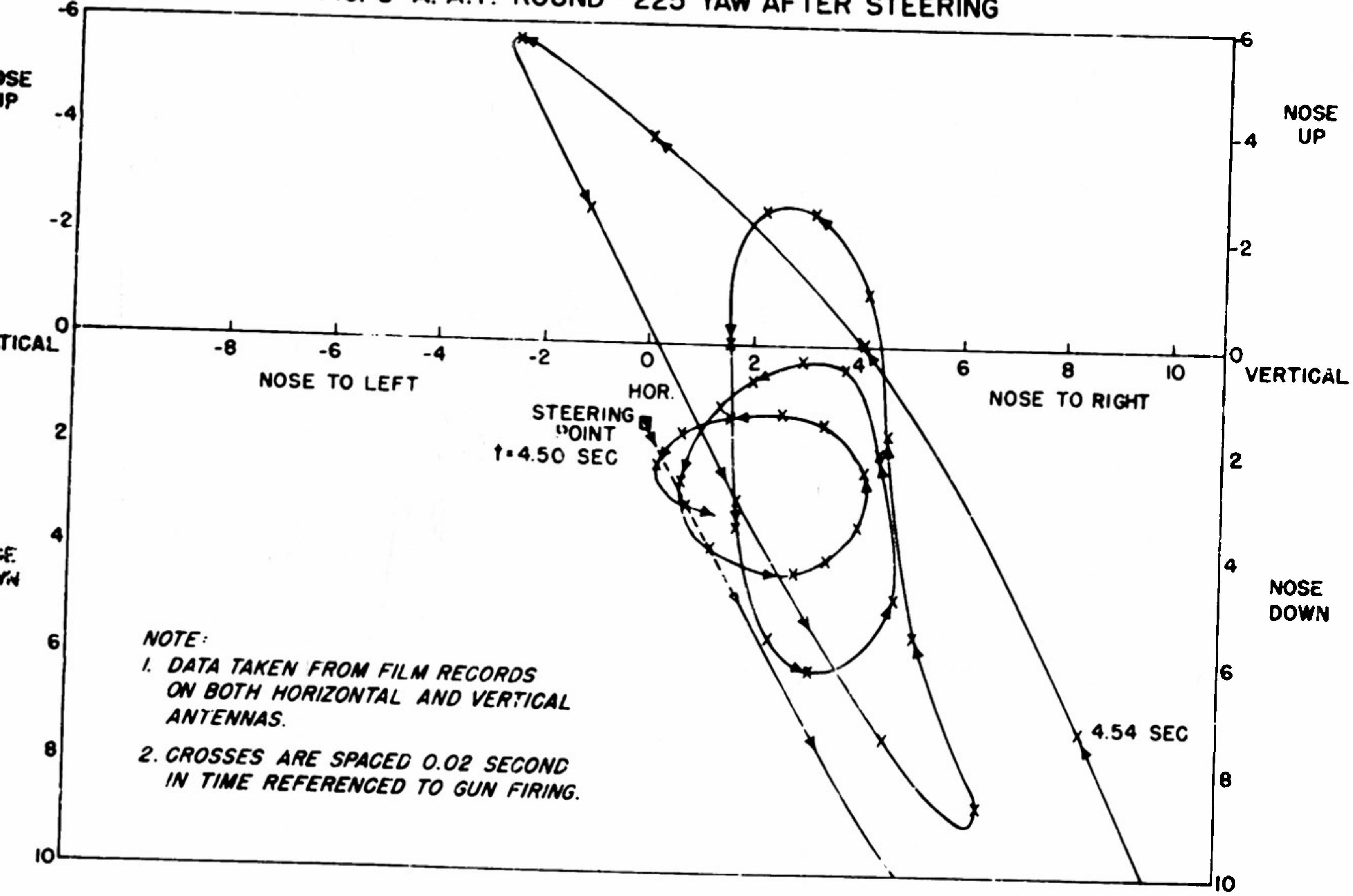


FIG. 4
CONSTRUCTION OF FINAL λ Vs γ CURVE FROM UPPER AND LOWER ENVELOPES

FIG. 5 A. A. P. ROUND #225 YAW AFTER STEERING



NOTE:

1. DATA TAKEN FROM FILM RECORDS ON BOTH HORIZONTAL AND VERTICAL ANTENNAS.
2. CROSSES ARE SPACED 0.02 SECOND IN TIME REFERENCED TO GUN FIRING.

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