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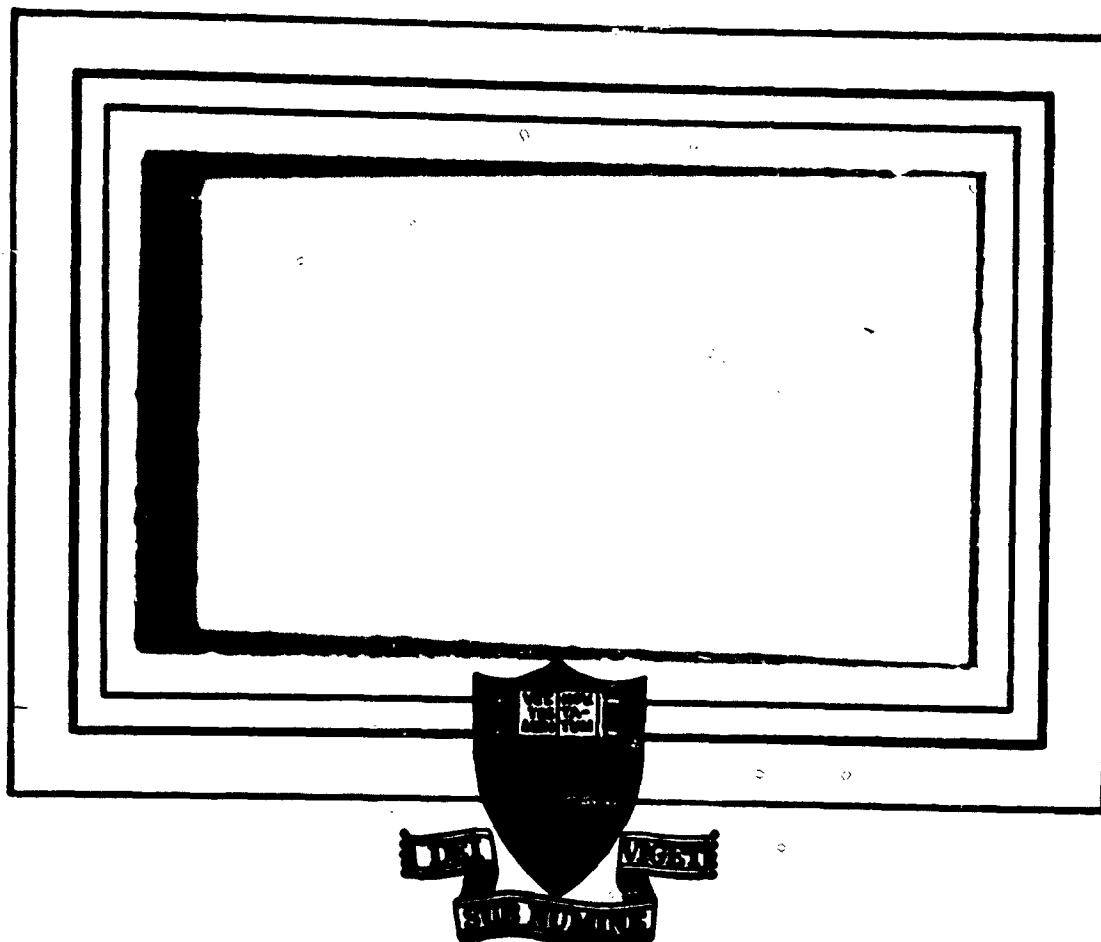
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By direction of *K. E. Wright*
Chief of Naval Research (Code 461)

PRINCETON UNIVERSITY
DEPARTMENT OF AERONAUTICAL ENGINEERING

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DEPARTMENT OF THE NAVY

OFFICE OF NAVAL RESEARCH

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TANDEM HELICOPTER LONGITUDINAL
STABILITY AND CONTROL

Aeronautical Engineering Laboratory

Report No. 233

June 1953

Prepared by:

John G. Gebhard
JOHN G. GEBHARD

Approved by:

Edward Seckel
EDWARD SECKEL

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1. SUMMARY

The equations of motion for the tandem helicopter are formulated for the longitudinal case. The equations are reduced to the static conditions and solved for the stick position gradient vs. speed and stick position gradient vs. normal acceleration at constant power setting. The dependence of the so-called "static stability" and "maneuver stability" margins upon the stability derivatives is shown. Theoretical expressions for the derivatives are given, based upon standard rotor aerodynamic theory, and dynamic solutions are carried out for a number of different trim speeds.

The effects of large variations in the magnitudes of the derivatives, on the solution of the equations are noted, and from them the more important derivatives are determined. The dependence of these derivatives upon c.g. position and other parameters is pointed out.

A preliminary investigation of the effects of rotor interference on these derivatives and solutions is made. Solutions are also carried out for the case of the rotors equipped with a differential δ_3 hinge configuration.

Using the equations of motion as a guide, a series of steady-state flight tests is developed from which it is expected that the static stability and control derivatives may be obtained. Additional tests are devised through which the individual rotor and fuselage contributions to the overall derivatives, may be expected to be obtained. Included in these are tests designed to evaluate the rotor interference effects.

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2. INTRODUCTION

The past ten years have brought about great advancements in the field of helicopter design and performance. The increased utility of the helicopter achieved through these advances has focussed attention upon the stability and control aspects of helicopters, more generally referred to as flying qualities.

Helicopter design practice has favored the advancement in particular of the single rotor and dual rotor tandem arrangements. Theoretical stability and control studies for the single rotor helicopter in forward flight have been presented in Refs. 1 and 3. However, in these works little emphasis was put on the relation of the stability derivatives to the static stability and control characteristics, such as stick position vs. speed and stick position vs. normal acceleration, characteristics which are directly apparent to the pilot.

By contrast, relatively little research has been published on the stability and control aspects of the tandem helicopter configuration. Since the tandem configuration presents one of the most practical avenues of advance for the large cargo-carrying helicopter, it would seem that an understanding of the stability and control aspects of this type would be very useful in future design and development work.

Accordingly, an investigation of the stability and control of the tandem configuration is undertaken in this report. The work

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here presented deals only with motions in the longitudinal plane. It is intended to deal with the lateral and directional stability characteristics in a later report.

The work program for this contract outlines a development of the theory for tandem helicopter stability and control, and a correlation of this theory with first static, and then dynamic flight test data. This report covers the development of the theory and its application to the design of the static flight tests. Since no data is yet available with which to correlate the theory, the nature of this report should be considered preliminary. In view of the above considerations no sample calculations have been presented herein. Sample calculations will be presented in a later report containing the correlation of the theory to the flight test data obtained.

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3. LIST OF SYMBOLS

a) Forces and Moments

- L Lift force, perpendicular to relative wind, positive up.
- D Drag force, parallel to relative wind, positive to rear.
- M Pitching moment, positive nose up.
- T Thrust of front rotor, parallel to axis of no-feathering of front rotor, positive up.
- T' Thrust of rear rotor, parallel to axis of no-feathering of rear rotor, positive up.
- H "Horizontal" force of front rotor, perpendicular to axis of no feathering of front rotor, positive to rear.
- H' "Horizontal" force of rear rotor, perpendicular to axis of no-feathering of rear rotor, positive to rear.

$$C_{L\alpha} = \frac{1}{\sigma \pi R^2 \rho (R\Omega)^2} \frac{dL}{d\alpha}$$

$$C_{L\mu} = \frac{1}{\sigma \pi R^2 \rho (R\Omega)^2} \frac{dL}{d\mu}$$

$$C_{L\dot{\phi}} = \frac{1}{\sigma \pi R^2 \rho (R\Omega)^2} \frac{dL}{d\dot{\phi}} \frac{1}{\bar{v}}$$

$$C_{L\delta_1} = \frac{1}{\sigma \pi R^2 \rho (R\Omega)^2} \frac{dL}{d\delta_1}$$

$$C_{L\delta_2} = \frac{1}{\sigma \pi R^2 \rho (R\Omega)^2} \frac{dL}{d\delta_2}$$

$$C_{L\eta} = \frac{1}{\sigma \pi R^2 \rho (R\Omega)^2} \frac{dL}{d\eta}$$

non-dimensional

lift derivatives

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$$\begin{aligned}
 C_{D\alpha} &= \frac{1}{\sigma \pi R^2 \rho (\Omega R)^2} \frac{\partial D}{\partial \alpha} \\
 C_{D\dot{\alpha}} &= \frac{1}{\sigma \pi R^2 \rho (\Omega R)^2} \frac{\partial D}{\partial \dot{\alpha}} \\
 C_{D\delta} &= \frac{1}{\sigma \pi R^2 \rho (\Omega R)^2} \frac{\partial D}{\partial \delta} \\
 C_{D\dot{\delta}} &= \frac{1}{\sigma \pi R^2 \rho (\Omega R)^2} \frac{\partial D}{\partial \dot{\delta}} \\
 C_{D\alpha_1} &= \frac{1}{\sigma \pi R^2 \rho (\Omega R)^2} \frac{\partial D}{\partial \alpha_1} \\
 C_{M\alpha} &= \frac{1}{(\pi R^2 \rho (\Omega R)^2) l_d} \frac{\partial M}{\partial \alpha} \\
 C_{M\dot{\alpha}} &= \frac{1}{\sigma \pi R^2 \rho (\Omega R)^2 l_d} \frac{\partial M}{\partial \dot{\alpha}} \\
 C_{M\delta} &= \frac{1}{\sigma \pi R^2 \rho (\Omega R)^2 l_d} \frac{\partial M}{\partial \delta} \\
 C_{M\dot{\delta}} &= \frac{1}{\sigma \pi R^2 \rho (\Omega R)^2 l_d} \frac{\partial M}{\partial \dot{\delta}} \\
 C_{M\alpha_1} &= \frac{1}{\sigma \pi R^2 \rho (\Omega R)^2 l_d} \frac{\partial M}{\partial \alpha_1} \\
 C_{M\dot{\alpha}_1} &= \frac{1}{\sigma \pi R^2 \rho (\Omega R)^2 l_d} \frac{\partial M}{\partial \dot{\alpha}_1}
 \end{aligned}$$

non-dimensional drag
derivatives

non-dimensional pitching
moment derivatives

b) Physical Dimensions of the Helicopter

m = Mass of helicopter (slugs)

J_y Helicopter pitching moment of inertia parameter = $\left(\frac{L}{l_d}\right)^2 \frac{J}{m}$

h = Height of front rotor hub above helicopter center of gravity (ft.)

h' = Height of rear rotor hub above helicopter center of gravity. (ft.)

l_d Distance between rotor masts

l = Distance from helicopter c.g. to front rotor mast, measured perpendicular to mast (ft.)

l' = Distance from helicopter c.g. to rear rotor mast

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- b = Number of blades
- Ω = Rotor speed, (rad/sec)
- R = Rotor radius (ft.)
- c = Blade chord ft.
- I_1 = Blade moment of inertia about the flapping hinge (slug ft.)
- M_n = Blade mass moment about hub center-line
- M_s = Blade mass moment about flapping hinge (slug ft.)
- e_1 = Helicopter blade flapping hinge offset
- K_y = Helicopter radius of gyration
- γ = Lock's blade inertia coefficient
- S_R = Total disc area
- $\left. \begin{array}{l} A \\ C \\ E \\ F \\ G \\ J \\ K \\ Q \\ S \end{array} \right\}$ Blade integration constants dependent upon flapping hinge offset and tip loss factor

c) Velocities and Angles

- U = Speed of helicopter along flight path (ft/sec)
- α = Angle of attack of normal to axis of no-feathering of front rotor
- α_r = Rotor blade angle of attack
- α' = Angle of attack of normal to axis of no-feathering of rear rotor.

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β Blade flapping angle measured relative to the no-feathering plane, positive up.

$$\beta = \beta_0 - a_1 \cos \psi - b_1 \sin \psi - a_2 \cos 2\psi - b_2 \sin 2\psi - \dots$$

The second harmonics are neglected in the analysis

a_1 Longitudinal flapping angle of front rotor blades, positive if tip path plane is tilted rearward.

a_1' Longitudinal flapping angle of rear rotor blades, positive if tip path plane is tilted rearward.

b_1 Lateral flapping angle of front rotor blades, positive if advancing blade flaps downward.

b_1' Lateral flapping angle of rear rotor blades, positive if advancing blade flaps downward.

ϵ Downwash angle of front rotor; positive downward.

ϵ' Downwash angle of rear rotor, positive downward.

γ Flight path angle to the horizon, positive in climb.

ϕ Longitudinal component of pitch angle of fuselage, measured relative to horizon, positive when fuselage reference line is pitched up.

θ_s Blade pitch angle of front rotor, positive for increased pitch

$$\theta_s = A_{\theta_s} - A_{\theta_s}' \cos \psi - B_{\theta_s}' \sin \psi$$

A_{θ_s} Collective pitch of front rotor $\equiv \theta$

A_{θ_s}' Lateral cyclic pitch of front rotor, positive stick right.

B_{θ_s}' Longitudinal cyclic pitch of front rotor, positive stick forward.

θ_s' Blade pitch angle of rear rotor, positive for increased pitch

$$\theta_s' = A_{\theta_s}' - A_{\theta_s}'' \cos \psi - B_{\theta_s}'' \sin \psi$$

A_{θ_s}' Collective pitch of rear rotor $\equiv \theta'$

A_{θ_s}'' Lateral cyclic pitch of rear rotor, positive stick right.

B_{θ_s}'' Longitudinal cyclic pitch of rear rotor positive stick forward

θ_d Differential collective pitch of the rotors.

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- ψ Control stick angle
 $\dot{\psi} = -\dot{\psi} \cos \psi - \dot{\psi} \sin \psi$
- ψ Longitudinal control stick angle simultaneously producing differential collective pitch and longitudinal cyclic pitch.

d) Aerodynamic Parameters

- μ Advance ratio

$$\mu = \frac{V_{\text{cyclic}}}{\Omega R} \approx \frac{V}{\Omega R}$$

- λ Inflow ratio of resultant velocity along the axis of no-feathering to the tip speed, of the front rotor.

- λ' Inflow ratio of resultant velocity along the axis of no-feathering to the tip speed, of the rear rotor.

- $\delta_0, \delta_1, \delta_2$ Coefficients in the expression for profile drag coefficient

$$C_{d0} = \delta_0 + \delta_1 \alpha_r + \delta_2 \alpha_r^2$$

- a Slope of the rotor blade lift curve.

- σ Solidity ratio = $\frac{b c}{\pi R}$

- C_T Thrust coefficient of front rotor

$$C_T = \frac{T}{\pi R^2 \rho (\Omega R)^2}$$

- C_T' Thrust coefficient of rear rotor

$$C_T' = \frac{T'}{\pi R^2 \rho (\Omega R)^2}$$

- B Blade tip loss factor

- C_L Helicopter lift coefficient = $\frac{W}{\sigma S_R \rho (\Omega R)^2}$

- τ Factor with dimensions of time = $\frac{\eta}{\rho \sigma S_R \Omega R^2}$

- μ Helicopter relative density parameter = $\frac{\eta}{\rho \sigma S_R b}$

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e) Miscellaneous

b_4, b_3, b_2, b_1, b_0

Coefficients of the characteristic equation

λ

Root of stability characteristic equation

s

LaPlace operator

a_3, a_2, a_1, a_0

Coefficients of numerator of transfer function

$d = d(t/\tau)$

Differential operator

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4. ANALYSIS

a) Formulation of the Equations of Motion

The development of the theory presented here follows the general pattern of the linearized theory of small disturbances in which it is assumed that the change in force or moment from some steady-state condition can be represented in the general form

$$\Delta T = \frac{\partial T}{\partial \alpha} \Delta \alpha + \frac{\partial T}{\partial V} \Delta V + \dots$$

Body axes, aligned in the initial direction of the wind, are used in this development since they are convenient for flight test work and further, are quite common in airplane stability literature. It is assumed that the lateral and directional motions are uncoupled from motions in the longitudinal plane.

Referring to Fig. 1, the equations of motion for a body in space may be written as follows, regarding motions only in the longitudinal plane.

$$\begin{aligned} 1) \quad \Sigma L &= m V (\dot{\phi} - \dot{\alpha}_s) \\ 2) \quad \Sigma D &= -m \dot{u} \\ 3) \quad \Sigma M &= I_y \ddot{\phi} \end{aligned}$$

L and D are the resultant aerodynamic forces on the helicopter perpendicular to and along the flight path and M is the aerodynamic pitching moment about the c.g.

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Since the expressions for rotor forces are usually written with reference to the no-feathering axis it will be convenient then to resolve these forces along the flight path axes. Referring to Fig. 1 again

$$\begin{aligned} 4) \quad L &= T \cos(B_s - \alpha_s) + H \sin(B_s - \alpha_s) \\ &\quad + T' \cos(B'_s - \alpha_s) + H' \sin(B'_s - \alpha_s) \\ &\quad + L_f - W \cos \gamma \end{aligned}$$

$$\begin{aligned} 5) \quad D &= -T \sin(B_s - \alpha_s) + H \cos(B_s - \alpha_s) - T' \sin(B'_s - \alpha_s) \\ &\quad + H' \cos(B'_s - \alpha_s) + D_f + W \sin \gamma \end{aligned}$$

$$\begin{aligned} 6) \quad M &= l(T \cos B_s + H \sin B_s) + h(-T \sin B_s + H \cos B_s) \\ &\quad - l'(T' \cos B'_s + H' \sin B'_s) + h'(-T' \sin B'_s + H' \cos B'_s) \\ &\quad + M_f \end{aligned}$$

For small disturbances, making the approximation for small angles that

$$\sin () \approx ()$$

$$\cos () \approx 1.0$$

we have

$$4a) \quad L \approx T + H(B_s - \alpha_s) + T' + H'(B'_s - \alpha_s) + L_f - W$$

$$5a) \quad D \approx -T(B_s - \alpha_s) + H - T'(B'_s - \alpha_s) + H' + D_f + W\gamma$$

$$6a) \quad M \approx l(T + HB_s) + h(-TB_s + H) - l'(T' + HB'_s) + h'(-TB'_s + H') + M_f$$

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It will be convenient to non-dimensionalize the equations by dividing the forces by $\sigma \pi R^2 \rho (\Omega R)^2$, and the moment equations by $\sigma \pi R^2 \rho (\Omega R)^2 l_d$, where l_d is the distance between rotors.

$$4b) \quad \frac{L}{\sigma \pi R^2 \rho (\Omega R)^2} = \frac{C_T}{\sigma} + \frac{C_T'}{\sigma} - 2C_L$$

$$5b) \quad \frac{D}{\sigma \pi R^2 \rho (\Omega R)^2} = \frac{C_T}{\sigma} (\alpha_s - B_s) + \frac{C_H}{\sigma} + \frac{C_T'}{\sigma} (\alpha_s' - B_s') + \frac{C_H'}{\sigma} \\ + \frac{D_f}{\sigma \pi R^2 \rho (\Omega R)^2} + 2C_L \gamma$$

$$6b) \quad \frac{M}{\sigma \pi R^2 \rho (\Omega R)^2 l_d} = \frac{l}{l_d} \left(\frac{C_T}{\sigma} + \frac{C_H}{\sigma} B_s \right) + \frac{l}{l_d} \left(-\frac{C_T}{\sigma} B_s + \frac{C_H}{\sigma} \right) \\ - \frac{l}{l_d} \left(\frac{C_T'}{\sigma} + \frac{C_H'}{\sigma} B_s' \right) + \frac{l}{l_d} \left(-\frac{C_T'}{\sigma} B_s' + \frac{C_H'}{\sigma} \right) \\ + \frac{M_f}{\sigma \pi R^2 \rho (\Omega R)^2 l_d}$$

The increments in these forces and moments from a steady-state condition may accordingly be written as follows

$$4c) \quad \frac{\Delta L}{\sigma \pi R^2 \rho (\Omega R)^2} = \frac{\Delta C_T}{\sigma} + \frac{\Delta C_T'}{\sigma}$$

$$5c) \quad \frac{\Delta D}{\sigma \pi R^2 \rho (\Omega R)^2} = \frac{\Delta C_T}{\sigma} (\alpha_s - B_s) - \frac{C_T}{\sigma} \Delta B_s + \frac{\Delta C_H}{\sigma} + \frac{\Delta C_T'}{\sigma} (\alpha_s' - B_s') \\ - \frac{C_T'}{\sigma} \Delta B_s' + \frac{\Delta C_H'}{\sigma} + \frac{\Delta D_f}{\sigma \pi R^2 \rho (\Omega R)^2} + 2C_L \Delta \phi$$

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$$\begin{aligned} 6c) \quad \frac{\Delta M}{\sigma \pi R^2 \rho (\Omega R)^2 L_u} = & \frac{\Delta C_T}{\sigma} \left(\frac{L}{L_u} - \frac{1}{L_u} B_z \right) - \frac{C_T}{\sigma} \frac{L}{L_u} \Delta B_z \\ & + \Delta \frac{C_H}{\sigma} \left(\frac{1}{L_u} + \frac{L}{L_u} B_z \right) + \frac{C_H}{\sigma} \frac{L}{L_u} \Delta B_z \\ & - \frac{\Delta C_T'}{\sigma} \left(\frac{L'}{L_u} + \frac{1}{L_u} B_z' \right) - \frac{C_T'}{\sigma} \frac{L'}{L_u} \Delta B_z' \\ & + \Delta \frac{C_H'}{\sigma} \left(\frac{1}{L_u} - \frac{L'}{L_u} B_z' \right) - \frac{C_H'}{\sigma} \frac{L'}{L_u} \Delta B_z' + \frac{\Delta M_F}{\sigma \pi R^2 \rho (\Omega R)^2 L_u} \end{aligned}$$

For the complete solution to the problem one must also write the equations describing the motion of the blades, which will include an additional three degrees of freedom for each rotor; the longitudinal and lateral flapping angles, and the coning angle. The effect of the blade degree of freedom in the plane of rotation due to the drag hinge may be neglected for constant power setting.

The equations of motion for the rotor blades may be found in Appendix I of this report along with expressions for the rotor thrust and H forces.

The complexity of solving 9 differential equations in 9 unknowns is so great as to preclude any attempt at solution before some reasonable simplification can be made. The complete equations of motion are given in Appendix I of this report. Following is a short description of the assumptions made in the development to reduce the equations to a convenient working form.

Since blade motions are very highly damped, the blades may be considered to respond instantaneously when compared with the response of the helicopter.

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The above assumption allows us to drop all flapping acceleration terms $\ddot{a}, \ddot{b}, \ddot{\beta}$ in the equations for the rotor flapping and coning angles. Further, it would appear that the equation for lateral flapping can be neglected, since the effect of lateral tip path plane motion on forces in the longitudinal plane is small. Now, if we also assume that there is no lag in the response of the coning angle, terms containing $\dot{\beta}$ may be dropped. Making Hohenhemser's quasi-static assumption, allows us to solve for β, a, β', a' and substitute the expressions obtained into the lift, drag and moment equations. This procedure reduces our original equations into three differential equations in three unknowns. The equations may be found in Appendix I and are rewritten here.

$$7) (C_{L\alpha} + 2\mu_0 d) a_s + C_{L\mu} \mu + (C_{L\phi} - 2\mu_0) d\phi_i = -C_{L\alpha} u_1$$

$$8) C_{D\alpha} a_s + (C_{D\mu} + 2d)\mu + (C_{D\phi} d + 2C_L)\phi_i = -C_{D\alpha} u_1$$

$$9) C_{m\alpha} a_s + C_{m\mu} \mu + [C_{m\phi} d - 2\left(\frac{K_y}{\mu}\right)^2 \frac{1}{\mu} d^2] \phi_i = -C_{m\alpha} u_1$$

The notation $C_{L\alpha}, C_{m\alpha}$ is a shorthand form for the partial derivative of the total non-dimensional force or moment with respect to the subscript α . It must be remembered that the variables a, μ, ϕ , are actually small variations from trim, $\Delta a, \Delta \mu, \Delta \phi$. The parameter u_1 represents motion of the control stick producing differential collective pitch and cyclic pitch simultaneously.

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The operator d signifies the derivative with respect to the time ratio t/τ where

$$\tau = \frac{m}{\sigma S_R \rho \Omega R}$$

$$J_Y = \left(\frac{K}{L_d}\right)^2 \frac{2}{\mu_{\star}}$$

and the relative density parameter of the helicopter is expressed as

$$\mu_{\star} = \frac{m}{\rho \sigma S_R L_d}$$

The characteristic equation for this set of differential equations is a quartic of the form

$$10) \quad b_4 \lambda^4 + b_3 \lambda^3 + b_2 \lambda^2 + b_1 \lambda + b_0 = 0$$

The expressions for the coefficients are as follows

$$\begin{aligned} 11) \quad b_4 &= -4\mu_0 J_Y \\ b_3 &= -2C_{L\alpha} J_Y - 2\mu_0 (C_{D\mu} J_Y - 2C_{m\dot{\alpha}\alpha}) \\ b_2 &= -C_{L\alpha} (C_{D\mu} J_Y - 2C_{m\dot{\alpha}\alpha}) + 2\mu_0 (C_{D\mu} C_{m\dot{\alpha}\alpha} - \underline{C_{m\mu} C_{D\dot{\alpha}\alpha}}) \\ &\quad + \underline{C_{D\mu} C_{L\mu} J_Y} - C_{m\alpha} (2C_{L\dot{\alpha}\alpha} - 4\mu_0) \\ b_1 &= C_{L\alpha} (C_{D\mu} C_{m\dot{\alpha}\alpha} - \underline{C_{m\mu} C_{D\dot{\alpha}\alpha}}) - 4C_{m\mu} C_L \mu_0 \\ &\quad - \underline{C_{D\alpha} (C_{L\mu} C_{m\dot{\alpha}\alpha} - C_{m\mu} C_{L\dot{\alpha}\alpha} + 2\mu_0 C_{m\mu})} \\ &\quad + C_{m\alpha} (\underline{C_{L\mu} C_{D\dot{\alpha}\alpha}} - \underline{C_{D\mu} C_{L\dot{\alpha}\alpha}} + 2\mu_0 C_{D\mu}) \\ b_0 &= 2C_L (-C_{L\alpha} C_{m\mu} + C_{m\alpha} C_{L\mu}) \end{aligned}$$

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The underlined terms are small and for computational purposes may be neglected. To obtain a particular solution such as the time response of α , μ , or ϕ to a step function, use is made of the LaPlace transform to obtain transfer functions in the following form.

$$12) \quad \frac{\alpha(s)}{u_1} = \frac{\begin{vmatrix} -C_{\alpha\alpha} & C_{\alpha\mu} & (C_{\alpha\phi} - 2\mu_0)s \\ -C_{\alpha\mu} & (C_{\mu\mu} + 2s) & (C_{\mu\phi}s + 2C_\phi) \\ -C_{\alpha\phi} & C_{\mu\phi} & (C_{\phi\phi}s - J_Y s^2) \end{vmatrix}}{\Delta}$$

where

$$\Delta = b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0$$

Expanding the numerator results in

$$13) \quad \frac{\alpha(s)}{u_1} = \frac{\bar{a}_{3\alpha} s^3 + \bar{a}_{2\alpha} s^2 + \bar{a}_{1\alpha} s + \bar{a}_{0\alpha}}{\Delta}$$

similarly the transfer functions for μ and ϕ for a step input are as follows

$$14) \quad \frac{\mu(s)}{u_1} = \frac{\bar{a}_{3\mu} s^3 + \bar{a}_{2\mu} s^2 + \bar{a}_{1\mu} s + \bar{a}_{0\mu}}{\Delta}$$

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$$15) \quad \frac{\phi_1(s)}{u_1} = \frac{\bar{a}_{2\phi} s^2 + \bar{a}_{1\phi} s + \bar{a}_{0\phi}}{\Delta}$$

The coefficients in the numerators of (13), (14), (15) are given below

$$16) \quad \bar{a}_{2u} = 2C_{Lu}, J_4$$

$$\bar{a}_{2u} = C_{Lu}, (C_{Du} J_4 - 2C_{mD\phi_1}) - C_{Du}, C_{L\mu} J_4 \\ + C_{mL}, (2C_{L\phi_1} - 4\mu_0)$$

$$\bar{a}_{1u} = -C_{Lu}, (C_{Du} C_{mD\phi_1} - C_{m\mu} C_{D\phi_1}) + C_{Du}, (C_{L\mu} C_{mD\phi_1} \\ - C_{m\mu} C_{L\phi_1} + 2\mu_0 C_{m\mu}) - C_{mL}, (C_{L\mu} C_{D\phi_1} \\ - C_{D\mu} C_{L\phi_1} + 2\mu_0 C_{D\mu})$$

$$\bar{a}_{0u} = -2C_L (-C_{Lu}, C_{m\mu} + C_{mL}, C_{L\mu})$$

$$17) \quad \bar{a}_{3\mu} = 2\mu_0 C_{Du}, J_4$$

$$\bar{a}_{2\mu} = C_{La} C_{Du}, J_4 - 2\mu_0 (C_{Du}, C_{mD\phi_1} - C_{mL}, C_{D\phi_1}) \\ - C_{Du} C_{Lu}, J_4$$

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$$\begin{aligned} a_{\mu} = & -C_{da}(C_{du}, C_{md\phi} - C_{mu}, C_{dd\phi}) + 4C_L\mu_0 C_{mu} \\ & + C_{du}(C_{Lu}, C_{md\phi} - C_{mu}, C_{dd\phi} + 2\mu_0 C_{mu}) \\ & - C_{md}(C_{Lu}, C_{dd\phi} - C_{du}, C_{dd\phi} + 2\mu_0 C_{du}) \\ a_{\mu} = & 2C_L(C_{mu}, C_{da} - C_{md} C_{Lu}) \end{aligned}$$

$$18) \quad a_{\phi} = -4C_{mu}, \mu_0$$

$$\begin{aligned} a_{\phi} = & 2C_{md} C_{Lu} + 2\mu_0 C_{du}, C_{m\mu} \\ & - C_{mu}(2C_{da} + 2\mu_0 C_{d\mu}) \end{aligned}$$

$$\begin{aligned} a_{\phi} = & -C_{Lu}(C_{m\mu}, C_{da} - C_{md} C_{d\mu}) + C_{du}(C_{da} C_{m\mu} \\ & - C_{\mu} C_{md}) - C_{mu}(C_{da} C_{d\mu} - C_{\mu} C_{da}) \end{aligned}$$

If it is desired to calculate the normal acceleration response to a pull and hold maneuver, it may be easily found from the expression

$$19) \quad \Delta n = \frac{1}{g} \frac{V}{g} (d\phi_i - d\alpha_s)$$

To save time and labor, the transfer function for $\frac{(\phi_i - \alpha_s)(s)}{\omega_i}$ may be expressed as follows, thus necessitating the calculation of only one inverse transform.

$$20) \quad \frac{(\phi_i - \alpha_s)(s)}{\omega_i} = \frac{-\bar{a}_{3u} s^3 + (-\bar{a}_{2u} + \bar{a}_{2\phi}) s^2 + (-\bar{a}_{1u} + \bar{a}_{1\phi}) s + (-\bar{a}_{0u} + \bar{a}_{0\phi})}{\Delta}$$

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The responses may be calculated from the transfer functions following the method of Ref. 1, chapter 5, or according to the methods given in any standard differential equations text.

For the case of near hovering flight the coupling of the vertical motion to the translational and pitching motion may be neglected. Equation (7) is then disregarded and the derivatives $C_{\dot{x}}$ and $C_{\dot{m}}$ may be considered negligible. The equations of motion may then be written in the following form.

$$8a) \quad (C_{D\mu} + B d)\mu + (C_{Dd\phi} d + 2C_L)\phi = -C_{D\dot{x}} \dot{x}_1$$

$$9a) \quad C_{m\mu} \mu + (C_{m\dot{\phi}} d - J_Y d^2)\phi = -C_{m\dot{x}} \dot{x}_1$$

The characteristic equation for this set of differential equations is a cubic of the form.

$$21) \quad B_3 \lambda^3 + B_2 \lambda^2 + B_1 \lambda + B_0 = 0$$

$$22) \quad B_3 = -2 J_Y$$

$$B_2 = -C_{D\mu} J_Y + 2 C_{m\dot{\phi}} d$$

$$B_1 = C_{D\mu} C_{m\dot{\phi}} - C_{m\mu} C_{Dd\phi}$$

$$B_0 = -2 C_L C_{m\mu}$$

The transfer function for μ and ϕ are

$$23) \quad \frac{\mu(s)}{\dot{x}_1} = \frac{\bar{a}_{2\mu} s^2 + \bar{a}_{1\mu} s + \bar{a}_{0\mu}}{B_3 s^3 + B_2 s^2 + B_1 s + B_0}$$

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$$24) \quad \frac{\phi(s)}{u_1} = \frac{\bar{a}_{1\phi} s + \bar{a}_{0\phi}}{b_3 s^3 + b_2 s^2 + b_1 s + b_0}$$

The coefficients of the numerators of equations (23) and (24) are given below

$$\begin{aligned} 25) \quad \bar{a}_{2\mu} &= C_{D\mu_1} J_4 \\ \bar{a}_{1\mu} &= -C_{D\mu_1} C_{m\dot{\phi}} + C_{m\mu_1} C_{D\dot{\phi}} \\ \bar{a}_{0\mu} &= 2 C_L C_{m\mu_1} \end{aligned}$$

$$\begin{aligned} 26) \quad \bar{a}_{1\phi} &= -2 C_{m\mu_1} \\ \bar{a}_{0\phi} &= -C_{m\mu_1} C_{D\mu} + C_{D\mu_1} C_{m\mu} \end{aligned}$$

The expressions for the rotor derivatives reduce directly from the expressions for forward flight.

b) Reduction of the Equations of Motion to the Static Cases

1. Stick Position vs. Speed at Constant Power Setting

If Equations 4d, 5d, 6d, are written for the one "g" steady-state flight condition, all derivatives with respect to nondimensional time drop out, and the equations become

$$7b) \quad C_{D\alpha} \alpha_s + C_{D\mu} \mu = -C_{D\mu_1} u_1$$

$$8b) \quad C_{D\alpha} \alpha_s + C_{D\mu} \mu + 2 C_L \phi_i = -C_{D\mu_1} u_1$$

$$9b) \quad C_{m\alpha} \alpha_s + C_{m\mu} \mu = -C_{m\mu_1} u_1$$

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Equations (7), (8), (9) are now linear algebraic equations and may be treated as such.

Solving for the change in tip speed ratio, which for constant tip speed is a measure of forward velocity, for a given change in control angle we have

$$27) \quad \frac{\mu}{u_1} = \frac{\begin{vmatrix} C_{L\alpha} & -C_{L\mu} & 0 \\ C_{D\alpha} & -C_{D\mu} & 2C_L \\ C_{m\alpha} & -C_{m\mu} & 0 \end{vmatrix}}{\begin{vmatrix} C_{L\alpha} & C_{L\mu} & 0 \\ C_{D\alpha} & C_{D\mu} & 2 \\ C_{m\alpha} & C_{m\mu} & 0 \end{vmatrix}}$$

$$\frac{\mu}{u_1} = \frac{-C_{m\mu} C_{L\alpha} + C_{m\alpha} C_{L\mu}}{-C_{m\alpha} C_{L\mu} + C_{m\mu} C_{L\alpha}}$$

Equation (27) checks exactly with the steady state part of the solution calculated from (14). The denominator represents the static stability margin and is a measure of the resultant moment applied to the helicopter due to the changes in speed and angle of attack. For stability its sign is positive, i.e. an increase in forward velocity producing a nose up pitching moment tending to reduce the speed.

Physically, such a situation would be encountered in changing the speed from a trimmed level flight value by motion of the cyclic control stick, while keeping the collective pitch setting constant. To maintain the "lg" flight condition, the change in thrust due to a change in velocity must be equalized by the effect of the angle of attack change. The resultant

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moment on the helicopter is made up of that due to the speed change plus that due to the change in angle of attack. Depending upon the sign of the derivatives $C_{m\dot{\mu}}$, $C_{m\dot{\alpha}}$ and $C_{\dot{\mu}\dot{\alpha}}$, the resultant moment may be either restoring or destabilizing. If $C_{\dot{\mu}\dot{\alpha}}$ positive an increase in speed will be accompanied by a decrease in angle of attack. Under this condition, which occurs at the very low speeds, an unstable value of $C_{m\dot{\alpha}}$ will result in a nose down contribution of this term. If $C_{\dot{\mu}\dot{\alpha}}$ is negative an increase in speed will be accompanied by an increase in angle of attack, and an unstable value of $C_{m\dot{\alpha}}$ will result in a nose up contribution. If $C_{m\dot{\mu}}$ is positive its contribution will be nose up for an increase in speed.

The boundary at which the resultant moment due to the angle of attack and speed changes is zero is written

$$28) \quad C_{m\dot{\mu}} C_{\dot{\alpha}} = C_{m\dot{\alpha}} C_{\dot{\mu}}$$

and is the condition of vanishing static stability.

If an increase in speed is accompanied by a resultant nose down moment, as would be the case when $C_{m\dot{\mu}} C_{\dot{\alpha}} < C_{m\dot{\alpha}} C_{\dot{\mu}}$, the stick must be moved aft to trim, and the stick position velocity gradient would be unstable. Equation (28) is simply the divergence criterion for the equation (10).

If the derivatives are written in terms of the non-dimensional rotor forces and the c.g. position the expression for the static margin becomes

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$$(29) \quad S.M. = -C_{\mu} \left[\frac{\partial G'_{\theta}}{\partial \mu} \left(\frac{L}{d} \right) - \frac{\partial G'_{\theta}}{\partial \mu} \left(1 - \frac{L}{d} \right) + \left(\frac{\partial C_m}{\partial \mu} \right)_{r+f} \right] \\ + C_{\mu} \left[\frac{\partial G'_{\theta}}{\partial \alpha} \left(\frac{L}{d} \right) - \frac{\partial G'_{\theta}}{\partial \alpha} \left(1 - \frac{L}{d} \right) + \left(\frac{\partial C_m}{\partial \alpha} \right)_{r+f} \right]$$

Where $\left(\frac{\partial C_m}{\partial \mu} \right)_{r+f}$ and $\left(\frac{\partial C_m}{\partial \alpha} \right)_{r+f}$ are the contributions of the fuselage and rotor "H" forces to the derivatives. Remembering that

$C_{\alpha} = \frac{\partial G'_{\theta}}{\partial \alpha} + \frac{\partial G'_{\theta}}{\partial \alpha}$ and $C_{\mu} = \frac{\partial G'_{\theta}}{\partial \mu} + \frac{\partial G'_{\theta}}{\partial \mu}$ the expression for the static margin becomes

$$(30) \quad S.M. = -C_{\alpha} \left[-\frac{\partial G'_{\theta}}{\partial \mu} + \left(\frac{\partial C_m}{\partial \mu} \right)_{r+f} \right] + C_{\mu} \left[-\frac{\partial G'_{\theta}}{\partial \alpha} + \left(\frac{\partial C_m}{\partial \alpha} \right)_{r+f} \right]$$

The terms containing c.g. position have vanished, and it appears that the static margin is independent of this parameter.

Further considerations reveal, however, that as the c.g. is moved further forward the trim thrust on the rear rotor is reduced thus decreasing the value of $\frac{\partial G'_{\theta}}{\partial \mu}$. This decrease in the derivative arises because of the fact that 1) collective pitch is less, 2) the rate of change of inflow with forward velocity is less, for the lightly loaded rotor. Since C_{α} and C_{μ} along with the fuselage and individual rotor contributions remain approximately the same if the helicopter is trimmed at the same angle of attack, the net result of forward c.g. position is an improvement in the static margin. Aft movement of the c. g. has the opposite effect.

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From numerical calculations it becomes apparent that the expression for stick position gradient can be approximated as follows

$$(31) \quad \frac{\dot{\alpha}_1}{\mu} = - \frac{1}{C_{m\dot{\alpha}_1}} (C_{m\ddot{\alpha}_1} - C_{m\dot{\alpha}_1} \frac{C_{L\ddot{\alpha}_1}}{C_{L\dot{\alpha}_1}})$$

The solution for the "steady state" angle of attack at constant collective pitch is

$$(32) \quad \frac{\alpha_s}{\mu} = \frac{-C_{L\dot{\alpha}_1} C_{m\ddot{\alpha}_1} + C_{m\dot{\alpha}_1} C_{L\ddot{\alpha}_1}}{C_{L\dot{\alpha}_1} C_{m\ddot{\alpha}_1} - C_{m\dot{\alpha}_1} C_{L\ddot{\alpha}_1}}$$

The rate of change of angle of attack with tip speed ratio is

$$(33) \quad \frac{\dot{\alpha}_s}{\mu} = \frac{-C_{L\dot{\alpha}_1} C_{m\ddot{\alpha}_1} + C_{m\dot{\alpha}_1} C_{L\ddot{\alpha}_1}}{-C_{m\dot{\alpha}_1} C_{L\ddot{\alpha}_1} + C_{m\ddot{\alpha}_1} C_{L\dot{\alpha}_1}}$$

Since the sign of the product $-C_{m\dot{\alpha}_1} C_{L\ddot{\alpha}_1}$ is, in practical cases, always positive and large enough to keep the denominator positive, the rate of change of angle of attack with tip speed ratio is dependent not only upon velocity stability but also the derivative $C_{L\ddot{\alpha}_1}$. Even for the case of positive velocity stability an increase in tip speed ratio can be accompanied by either an increase or decrease in angle of attack according

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to the sign and magnitude of $C_{L\mu}$. At the low speeds when $C_{L\mu}$ is large and positive a decrease in tip speed ratio will result in an increase in angle of attack whereas in the high speed range where $C_{L\mu}$ is negative, the converse is true.

2. Stick Position vs. "g" at Constant Velocity

If the velocity is assumed constant the equations of motion reduce as follows, for constant collective pitch setting

$$7c) \quad (C_{L\alpha} + 2\mu_0 d) d_s + (C_{L\dot{\phi}} - 2\mu_0) d\phi_i = -C_{L\mu} u_i$$

$$9c) \quad C_{m\alpha} \quad (C_{m\dot{\phi}} d - J_y d^2) \phi_i = -C_{m\mu} u_i$$

Solving for the steady-state incremental pitching velocity, which is a measure of normal acceleration, for a given change in control angle.

$$(34) \quad \frac{d\phi_i}{u_i} = \frac{\begin{vmatrix} C_{L\alpha} & -C_{L\mu} \\ C_{m\alpha} & -C_{m\mu} \end{vmatrix}}{\begin{vmatrix} C_{L\alpha} & (C_{L\dot{\phi}} - 2\mu_0) \\ C_{m\alpha} & C_{m\dot{\phi}} \end{vmatrix}}$$

$$\frac{d\phi_i}{u_i} = \frac{-C_{m\mu} C_{L\alpha} + C_{m\alpha} C_{L\mu}}{C_{L\alpha} C_{m\dot{\phi}} - C_{m\alpha} (C_{L\dot{\phi}} - 2\mu_0)}$$

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Here the denominator represents the maneuver margin and is a measure of the moment applied to the helicopter as a result of the change in angle of attack and pitching velocity at constant tip speed ratio. For maneuvering stability its sign is negative, i.e. an increase in nose up pitching velocity producing a nose down moment tending to decrease the rate of pitch. Physically, such a situation could be realized in steady turns during descent at constant collective pitch setting and tip speed ratio, where the increased thrust required is produced by the change in angle of attack. In such a maneuver, the moments produced by the damping in pitch and angle of attack stability may be either additive or cancellative according to the sign of $C_{m\alpha}$. Since for the tandem configuration $C_{m\dot{\alpha}}$ is always negative, an unstable value of $C_{m\alpha}$ will result in a nose up contribution of this term tending to overcome the nose down moment due to the damping in pitch. If $C_{m\alpha}$ is sufficiently unstable the moment produced by the increased angle of attack will overcome the moment produced by the pitching velocity, and the resultant effect will be a moment tending to increase the pitching velocity. In such a case forward motion of the stick would be required to complete the turn and the stick position vs. "g" gradient would be unstable.

The stability boundary condition, where the effect of the angle of attack term just offsets the damping in pitch, and there is neither a restoring or destabilizing resultant moment, can be written

$$(35) \quad C_{m\dot{\alpha}}(C_{L\dot{\alpha}} - 2\mu_0) = C_{L\alpha} C_{m\dot{\alpha}}$$

which is the condition of vanishing maneuver margin.

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We can write the expression for maneuver margin in terms of the individual rotor derivatives and c.g. position as follows, if $C_{L\alpha}$ and the fuselage and rotor "H" force

$$(36) \quad M.M. = C_{L\alpha} \left[-\frac{1}{\mu_0 \mu_H} \left(\frac{l}{l_H} \right)^2 \frac{\partial G'_{H\sigma}}{\partial \alpha} - \frac{1}{\mu_0 \mu_H} \left(1 - \frac{l}{l_H} \right)^2 \frac{\partial G'_{f\sigma}}{\partial \alpha} \right] \\ + 2\mu_0 \left[\frac{l}{l_H} \frac{\partial G'_{f\sigma}}{\partial \alpha} - \left(1 - \frac{l}{l_H} \right) \frac{\partial G'_{H\sigma}}{\partial \alpha} + \left(\frac{\partial C_m}{\partial \alpha} \right)_{r+f} \right]$$

Equation (36) may be written as a quadratic function of the c.g. position

$$(37) \quad M.M. = -\left(\frac{l}{l_H} \right)^2 + K_1 \left(\frac{l}{l_H} \right) - K_2$$

where

$$(38) \quad K_1 = \frac{2 \frac{\partial G'_{f\sigma}}{\partial \alpha} + 2\mu_0^2 \mu_H}{C_{L\alpha}} \\ K_2 = \frac{\frac{\partial G'_{f\sigma}}{\partial \alpha}}{C_{L\alpha}} - \frac{2\mu_0^2 \mu_H}{C_{L\alpha}^2} \left[-\frac{\partial G'_{H\sigma}}{\partial \alpha} + \left(\frac{\partial C_m}{\partial \alpha} \right)_{r+f} \right]$$

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The c.g. position at which the maneuver margin vanishes is given by

$$\frac{L}{L_d} = \frac{K_1}{2} \pm \sqrt{\left(\frac{-K_1}{2}\right)^2 - K_2}$$

The solution of the quadratic equation for values of k_1 and k_2 encountered throughout the speed range, yields two roots, one between $L/L_d = 0-1$ and the other, $L/L_d > 1$. The first is the only one of practical significance and it can be seen that for L/L_d values forward of this boundary value the stick position vs. pitching velocity gradient will be stable.

The stick position vs. pitching velocity gradient is then a direct function of c.g. position becoming more stable at forward c.g.

The expression for stick position vs. "g" gradient in pull ups and turns may be found from the relationship between "g" and pitching velocity in the respective maneuvers.

$$(39) \text{ pull up} \quad \dot{\phi} = g(n-1) \frac{1}{U}$$

$$(40) \text{ turn} \quad \dot{\phi} = g/n(n - \frac{1}{n})$$

$$\frac{u_1}{n} = \frac{u_1}{\dot{\phi}} \frac{d\dot{\phi}}{dn}$$

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c) Numerical Solution of Equations of Motion

Calculations of dynamic response data are made at speeds from hovering to 120 kts, in increments of 15 kts. The helicopter is initially assumed to be in a trimmed level flight condition.

The calculations are made using the physical characteristics of the Navy HUP-1 helicopter with no tail surfaces, at a gross weight of 5750 pounds and c.g. position 5.91" forward of the mid-point of the rotors.

Calculation of Trim Conditions

The calculation of the trim conditions involves the determination of the angle of attack of the no-feathering axis for each rotor for a given value of μ in level flight. The angle of attack is determined in the following way. It is assumed that the value of thrust on each rotor is equal to half the helicopter gross weight. The thrust vectors are assumed to be perpendicular to their respective tip path planes, and tilted forward to overcome the parasite and profile drag of the rotors and fuselage. The inflow is calculated relative to the tip path plane and the collective pitch and first harmonic feathering are then calculated for each rotor according to the expressions in Chapter VII of Ref. 9. Making use of the equivalence of first harmonic feathering and flapping the axis of no-feathering is then located relative to the wind, and the angle of attack is then defined.

This helicopter obtains longitudinal control by simultaneous tilt of the swash plates accompanied by differential application of collective pitch,

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achieved through motion of the cyclic stick. Through a linkage actuated by a trim wheel, the pilot can also apply differential collective pitch independently of the swash plate tilt.

This arrangement allows any unbalanced moment on the helicopter due to c.g. displacement or other effects to be counteracted by the differential collective pitch trim. Thus the cyclic stick is relieved of the trim function and may be positioned to allow ample margin for control under any flight condition. The fuselage angle of attack is then determined by the combination of cyclic and differential collective pitch. It is assumed in the trim calculations that the cyclic and differential collective pitch are manipulated in such a way as to keep the tip path plane perpendicular to the mast.

With the fuselage angle of attack thus defined, the trim values of thrust on the front and rear rotor are calculated, using the moment equation and assuming the fuselage to be covered by the average downwash of the rotors. Trim values of λ , inflow ratio, and Θ , collective pitch may be calculated, and the steady-state coning and flapping coefficients of each rotor may be found.

The fuselage drag and moment data are obtained from Ref. 7. The rotor derivatives are then calculated according to the expressions given in Appendix I.

For these calculations the tip loss factor is assumed equal to unity and the flapping hinge offsets are neglected while calculating the rotor force

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and flapping derivatives, but are included in calculating moment derivatives about the c.g. It is assumed that one degree of cyclic pitch application is accompanied by two-fifths of a degree of differential collective pitch.

Solutions for the equations of motion were carried out for a wide variety of conditions on an electronic analogue computer, since this offered a convenient way of determining the effects on the motion of large variations in the magnitude of the stability derivatives.

Rotor Interference Effects.

In the calculation of the derivatives two different assumptions are made about rotor interference effects.

The first is that the two rotors can be considered as being completely isolated. The second is that the rear rotor is operating completely in the wake of the front rotor and that disturbances are transmitted downstream only, i.e. the upwash in front of the rear rotor has negligible effect on the characteristics of the front rotor. The value of front rotor downwash at the rear rotor is assumed to be twice its value at the front rotor. This assumption results in the following important effects on the stability derivatives.

- 1) The lift curve slope of the rear rotor is reduced by a factor $(1-2 \frac{1}{5})$ from the case of the isolated rotors, resulting in a decrease in angle of attack stability. The contribution of the rear rotor "H" force to this term is also more destabilizing than before since it is now operating in a climb condition.

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- 2) The value of the velocity stability is greatly changed from that of the isolated rotors case, since an increase in forward velocity, by reducing the downwash angle, now effectively increases the rear rotor angle of attack causing a destabilizing nose down moment.

Although other effects were present they were completely masked in the final solutions by the two major ones noted above.

Flight test experience and common sense would seem to indicate that, for most flight conditions, the helicopter is probably operating somewhere between the two extreme cases cited above. In the section on steady-state flight tests of this report, the subject is dealt with in further detail, and a method is presented whereby it is expected that these effects may be determined experimentally.

It should be noted that while interference between rotors is neglected in the calculation of the near hovering control responses, it is quite likely that these effects may be present in overlapped tandem configurations, since any forward or rearward translation will change the flow pattern over the rotors.

Effect of the Stability Derivatives on the Responses

a) Forward Flight

Calculation of the stability derivatives for the tandem rotor helicopter in forward flight reveals the following major differences between it and the single rotor configuration.

- 1) The damping in pitch is of the order of twenty times greater than that for the single rotor machine.
- 2) The angle of attack stability and velocity stability are dependent upon c.g. position.

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The first of these differences results in a major difference between the roots of the characteristic equation for the tandem helicopter as compared to the single rotor machine.

The quartic equation (10) yields four roots. Under the assumption of isolated rotors, in which velocity stability is calculated to be positive, the roots consist of two pure convergences and one poorly damped long period oscillation. The major contribution to the convergences is from the damping in pitch and the angle of attack stability. The large damping in pitch helps to overcome the unstable contribution of the angle of attack term, and consequently larger unstable values of this derivative may be tolerated in the tandem than in the single rotor machine. Very large unstable values of angle of attack stability lead to a rapid divergence in this mode.

The major contributions to the long period oscillation are from the velocity stability, $C_{\dot{m}\mu}$; angle of attack stability, $C_{m\alpha}$; damping in pitch, $C_{m\dot{\theta}}$; the force along the flight path axis due to a change in velocity, $C_{\dot{\theta}\mu}$; and the vertical damping $C_{L\dot{\alpha}}$.

Increasing the velocity stability decreases the period and results in poorer damping. Decreasing the velocity stability but allowing it to remain large enough so that the condition at the divergence boundary

$$C_{m\mu} C_{L\dot{\alpha}} > C_{m\alpha} C_{\dot{\theta}\mu}$$

where the product $C_{m\mu} C_{L\dot{\alpha}}$ is positive, is not violated improves the damping and lengthens the period. If the velocity stability is allowed

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to become small enough so that the divergence boundary is violated, a mild divergence results, becoming more rapid for negative values of $C_{m\dot{\mu}}$.

Increasing the angle of attack stability lengthens the period and improves the damping, while decreasing it results in an initial decrease and final increase of period with the damping becoming largely negative. Very large values, either stable or unstable, of $C_{m\dot{\alpha}}$ could cause the divergence boundary condition to be violated, depending upon the sign of $C_{L\dot{\mu}}$. However, these values are not likely to be encountered in normal configurations.

An increase in the damping in pitch improves the damping and lengthens the period while decreasing it has the inverse effect. Increasing the force along the flight path due to velocity, results in an improvement of the long period damping.

Decreasing the vertical damping, $C_{Z\dot{u}}$, results in poorer damping of the long period oscillation and relatively small change on the period. Large variations in the derivatives, $C_{Z\dot{u}}$, $C_{Z\dot{\phi}}$, $C_{Z\dot{\psi}}$, were found to have but small effect on the dynamic modes of motion. The effect of $C_{L\dot{\mu}}$ on the steady-state values is noted in the previous section.

The important derivatives as listed above may be divided into two categories, those essentially dependent upon trim conditions, and those that are independent.

Dependent Upon Trim Conditions

$C_{L\dot{\alpha}}$
 $C_{m\dot{\alpha}}$

Independent

$C_{m\dot{\mu}}$
 $C_{m\dot{\alpha}}$
 $C_{m\dot{\phi}}$

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Of the three independent derivatives only $C_{m\dot{\mu}}$ and $C_{m\ddot{\mu}}$ may be conveniently varied in the practical cases. An improvement in $C_{m\dot{\mu}}$ can be achieved through swash plate dihedral or forward c.g. position, while improvement in $C_{m\ddot{\mu}}$ can result from the use of a differential δ_3 hinge configuration (Ref. 5) or again, forward c.g. position. Calculations indicate that if positive velocity stability can be obtained along with a reduction in angle of attack instability, the overall stability of the helicopter will be acceptable, both static and dynamic.

Some theoretical responses throughout the speed range using the isolated rotors assumption are shown in Figs. 3-10. The 15 knot trim speed was omitted since there is no adequate rotor theory to cover this range of low μ and large negative rotor angle of attack. At the 90 knot trim speed a response is also calculated assuming the maximum interference between rotors. It is seen that the motion is rapidly divergent and represents an extremely hazardous condition. Comparison between $C_{m\ddot{\mu}}$ and $C_{m\dot{\mu}}$ for the two different assumptions shows that for the case of maximum interference $C_{m\ddot{\mu}}$ is about $3\frac{1}{2}$ times the value for no interference, and $C_{m\dot{\mu}}$ is about $2\frac{1}{2}$ times its former value, with a negative sign, however. The damping in pitch is slightly increased. A response is also calculated under this same assumption but considering the helicopter to be equipped with a differential hinge configuration. The δ_3 angle is assumed to have a value of 35° on the front rotor and 0° on the rear. The derivatives are changed as follows: $C_{m\ddot{\mu}}$ is now negative, i.e. stable, and with a value roughly equal to the value for no interference. $C_{m\dot{\mu}}$ is about $1\frac{1}{2}$ times as great as the value

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for no interference but again with a negative sign. The damping in pitch is somewhat reduced but still has a large stable value. The motion remains divergent, since C_{my} is negative, but the rate of divergence is much slower.

At the 60 kt. trim speed solutions are carried out for wide variations in the stability derivatives and the effects described formerly are apparent.

The unstable tendencies noted at the higher end of the speed range are due primarily to the decrease in angle of attack stability and velocity stability. These trends would be exhibited under either rotor interference assumption.

Hovering Solutions

Solutions to the equations of motion for near hovering flight are presented in Fig. 2. The large damping in pitch improves the damping and increases the period of the longitudinal oscillation over what could be expected for a single rotor machine. Otherwise the derivatives remain essentially the same.

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d. Proposed Static Flight Tests

In the following section the equations of motion are analyzed in order to determine which of the static derivatives may be determined from steady-state flight tests.

1. Derivatives Directly Apparent to the Pilot

The derivatives which are directly apparent to the pilot will be treated first.

a) Control Power

Since longitudinal tilt of the swash plate on this helicopter is accompanied by differential collective pitch of the rotors, it would seem advisable to separate these two types of control.

1. Differential Collective Pitch

If the helicopter is flown at two widely different c.g. positions, but at the same fuselage angle of attack and tip speed ratio, the change in c.g. position can be accommodated by use of the trim wheel. Since the control power derivative is dependent upon the square of RPM at constant μ , it will be convenient to express it in terms of the ratio of thrust coefficient to solidity ratio of the rotors. The equation which applies is as follows

$$(41) \quad 2C_L \Delta l/l = -C_{m\theta} \theta$$

2. Cyclic Pitch

The nondimensional control moment due to the swash plate tilt can be expressed as follows

$$(42) \quad C_{m\theta} = \frac{\partial C_{H0}}{\partial B_1} \left(\frac{l}{l_0} - \frac{l}{l_0} B_3 \right) + \frac{\partial C_{H0}}{\partial B_1} \left(\frac{l}{l_0} + \frac{l}{l_0} B_3 \right) \\ - \frac{\partial C_{H0}}{\partial B_1} \left(\frac{l}{l_0} + \frac{l}{l_0} B_3 \right) + \frac{\partial C_{H0}}{\partial B_1} \left(\frac{l}{l_0} - \frac{l}{l_0} B_3 \right) \\ - C_{H0} \frac{l}{l_0} - C_{H0} \frac{l}{l_0}$$

The individual terms can be evaluated from later tests.

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b) Angle of Attack Stability

In order to test for angle of attack stability, it is necessary to keep the other variables constant. A method of doing this would be to fly at two different forward speeds, keeping μ , the tip speed ratio, and the collective pitch lever constant, and measuring the application of control required to trim. Again writing the derivatives in the non-dimensional form, the governing equation is

$$(43) \quad C_{m_\alpha} \Delta \alpha = -C_{m_{\dot{\alpha}}} \Delta \dot{\alpha}$$

c) Velocity Stability

A test for velocity stability may be conveniently set up as follows. At a stabilized level flight condition, the trim wheel is moved keeping the collective pitch constant, and a new trim speed is obtained. The change in speed will be accompanied by a change in angle of attack, which effect can be accounted for since, from test b) the angle of attack stability is known. The equation governing the test is

$$(44) \quad C_{m_\alpha} \Delta \alpha + C_{m_\mu} \Delta \mu = -C_{m_{\dot{\alpha}}} \Delta \dot{\alpha}$$

Since C_{m_μ} is a function of μ , α , θ , and C_T this test should be made for a number of different power settings and gross weights.

d) Damping in Pitch

The damping in pitch can be obtained from a series of steady turns at constant power, c.g. position, and tip speed ratio, but different "g".

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As the rate of turn increases, the angle of attack of the helicopter increases accordingly, which again necessitates the knowledge of angle of attack stability. The governing equation in this case is

$$(45) \quad C_{m\alpha} \Delta\alpha + C_{m\dot{\alpha}} \Delta\dot{\alpha} = -C_{m\dot{\theta}} \Delta\dot{\theta}$$

$C_{m\dot{\alpha}}$ is a function of RPM independently of μ and the tests should be made over a range of RPM at a given value of μ .

e) Applied Moment Due to Collective Pitch

The moment about the c.g. due to application of collective pitch can be obtained through a test similar to test b). However, in this test, level flight is maintained through the use of the collective pitch.

The equation is

$$(46) \quad C_{m\alpha} \Delta\alpha + C_{m\theta} \Delta\theta = -C_{m\dot{\alpha}} \Delta\dot{\alpha}$$

f) The Helicopter Lift and Drag Derivatives

In order to evaluate the lift and drag derivatives due to angle of attack, forward velocity, and pitching motion, it is first necessary to know the lift and drag derivatives due to the application of control. These may be obtained by setting up three flight conditions. Two of these will be at constant thrust coefficient, one in level flight, and one a shallow climb in which the fuselage angle of attack for level flight is duplicated. The equation for the change in lift and drag forces for the two conditions will be

$$(47) \quad C_{L\theta} \Delta\theta + C_{L\dot{\theta}} \Delta\dot{\theta} = 0$$

$$(48) \quad C_{D\theta} \Delta\theta + C_{D\dot{\theta}} \Delta\dot{\theta} = -2C_{L\dot{\theta}} \Delta\dot{\theta}$$

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The third condition will be level flight at a different gross weight, but again, the same fuselage angle of attack. The change in lift and drag forces from the level flight condition will be

$$(49) \quad C_{L\theta} \Delta\theta + C_{L\phi} \Delta\phi = 2\Delta C_L$$

$$(50) \quad C_{D\theta} \Delta\theta + C_{D\phi} \Delta\phi = -2\Delta C_D$$

Equations (47) and (49) may be solved simultaneously to yield $C_{L\theta}$ and $C_{L\phi}$, while $C_{D\theta}$ and $C_{D\phi}$ may be extracted from equations (48) and (50).

Now, the lift and drag derivatives due to velocity, angle of attack and pitching may be evaluated from the same flight tests as the lumped moment derivatives.

2. Tests to Determine Static Derivatives Individually

In addition to the tests for the lumped derivatives of the previous section it would be desirable to separate these derivatives into their component parts and to design some tests through which these parts may be evaluated. The following presentation is an outline of how these individual derivatives may be expected to be evaluated from flight tests. Special emphasis is placed upon determining the interference effect of the rotors.

a) Thrust Derivatives with Respect to α , θ

The procedure in this test is essentially to load the helicopter in directions along and normal to the flight path. The first of these is accomplished by flying the helicopter in a shallow climb, where the compo-

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ment of weight along the flight path is the applied load, neglecting any change of fuselage drag due to the fuselage angle of attack change. The helicopter is trimmed by use of the swash plate tilt only, to insure that the change in thrust on each rotor will be zero. The equations for the change in thrust on each rotor, from the level flight condition will be as follows. Although the interference effect of the front rotor upon the rear is expected to be quite small, for generality it is included.

$$(51) \quad \Delta C_T = \frac{\partial C_T}{\partial \alpha} \Delta \alpha + \frac{\partial C_T}{\partial \theta} \Delta \theta + \frac{\partial C_T}{\partial E} \frac{\partial E}{\partial C_T} \Delta C_T = 0$$

$$(52) \quad \Delta C_T' = \frac{\partial C_T'}{\partial \alpha} \Delta \alpha + \frac{\partial C_T'}{\partial \theta} \Delta \theta + \frac{\partial C_T'}{\partial E} \frac{\partial E}{\partial C_T} \Delta C_T = 0$$

From which

$$(53) \quad \frac{\partial C_T}{\partial \alpha} = - \frac{\partial C_T}{\partial \theta} \left(\frac{\Delta \theta}{\Delta \alpha} \right),$$

$$(54) \quad \frac{\partial C_T'}{\partial \alpha} = - \frac{\partial C_T'}{\partial \theta} \left(\frac{\Delta \theta'}{\Delta \alpha} \right),$$

The interference derivatives are expressed in the above form since it is expected that interference effects will arise only from a change in direction of the wash of the rotors and not from a change in the dynamic pressure of the wake.

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Now if an additional weight is attached directly under the front rotor in level flight and the fuselage angle of attack of the first level flight run is duplicated, the equations may be written, neglecting the change in downwash over the fuselage

$$(55) \quad \Delta C_{T_2} = \frac{\Delta W_F}{\pi R^2 \rho (\Omega R)^2} = \frac{\partial G}{\partial \alpha} \Delta \alpha_2 + \frac{\partial G}{\partial \theta} \Delta \theta_2 + \frac{\partial G}{\partial \epsilon} \frac{\partial \epsilon'}{\partial G'} \Delta C_{T_2}'$$

$$(56) \quad \Delta C_{T_2}' = 0 = \frac{\partial G'}{\partial \alpha} \Delta \alpha_2 + \frac{\partial G'}{\partial \theta} \Delta \theta_2 + \frac{\partial G'}{\partial \epsilon} \frac{\partial \epsilon}{\partial G} \Delta C_{T_2}$$

Equations (53), (55) may be solved for $\frac{\partial G}{\partial \theta}$ yielding

$$(57) \quad \frac{\partial G}{\partial \theta} = \frac{\Delta C_{T_2} / \Delta \theta_2}{1 - \left(\frac{\partial \theta}{\partial \alpha} \right) \left(\frac{\partial \alpha}{\partial \theta} \right)_2}$$

Similarly, for a weight added directly under the rear rotor

$$(58) \quad \Delta C_{T_3}' = \frac{\Delta W_R}{\pi R^2 \rho (\Omega R)^2} = \frac{\partial G'}{\partial \alpha} \Delta \alpha_3 + \frac{\partial G'}{\partial \theta'} \Delta \theta_3' + \frac{\partial G'}{\partial \epsilon} \frac{\partial \epsilon}{\partial G} \Delta C_{T_3}$$

$$(59) \quad \Delta C_{T_3} = 0 = \frac{\partial G}{\partial \alpha} \Delta \alpha_3 + \frac{\partial G}{\partial \theta} \Delta \theta_3 + \frac{\partial G}{\partial \epsilon} \frac{\partial \epsilon'}{\partial G'} \Delta C_{T_3}'$$

$$(60) \quad \frac{\partial G'}{\partial \theta'} = \frac{\Delta C_{T_3}' / \Delta \theta_3'}{1 - \left(\frac{\partial \theta'}{\partial \alpha} \right) \left(\frac{\partial \alpha}{\partial \theta'} \right)_3}$$

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The interference derivatives may be found now from equations (56) and (59).

$$(61) \quad \frac{\partial G}{\partial \epsilon'} \frac{\partial \epsilon'}{\partial G_T'} = - \frac{\partial G}{\partial \alpha} \frac{\Delta \alpha}{\Delta G_T'} - \frac{\partial G}{\partial \theta} \frac{\Delta \theta}{\Delta G_T'}$$

$$(62) \quad = \frac{\partial G'}{\partial \epsilon} \frac{\partial \epsilon}{\partial G_T} = - \frac{\partial G'}{\partial \alpha} \frac{\Delta \alpha}{\Delta G_T} - \frac{\partial G'}{\partial \theta} \frac{\Delta \theta}{\Delta G_T}$$

b) Thrust Derivatives With Respect to Tip Speed Ratio

At constant gross weight and c.g. position, if a change in speed is trimmed out by use of the cyclic pitch alone while the collective pitch remains constant, the following equations can be written for the thrust coefficients of the front and rear rotor

$$(63) \quad \Delta G = 0 = \frac{\partial G}{\partial \alpha} \Delta \alpha + \frac{\partial G}{\partial \mu} \Delta \mu + \frac{\partial G}{\partial \epsilon'} \frac{\partial \epsilon'}{\partial \mu} \Delta \mu$$

$$(64) \quad \Delta G' = 0 = \frac{\partial G'}{\partial \alpha} \Delta \alpha + \frac{\partial G'}{\partial \mu} \Delta \mu + \frac{\partial G'}{\partial \epsilon} \frac{\partial \epsilon}{\partial \mu} \Delta \mu$$

This test should also serve as a check upon the interference effects of the rotors, and should be made at a number of different power settings, and gross weights.

c) "H" Force Derivatives With Respect to α, θ

If the rotor "H" forces are measured by means of strain gages, from the tests of part a) the following equations may be written. It should be remembered that the derivatives $\frac{\partial C_H}{\partial \alpha}$, $\frac{\partial C_H}{\partial \theta}$ etc., are not partial

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derivatives in the strict sense, since they include the effects of blade flapping. The change in "H" force coefficient due to climbing

$$(65) \quad \Delta C_H = \frac{\partial \bar{C}_H}{\partial \theta} \Delta \theta + \frac{\partial \bar{C}_H}{\partial \alpha} \Delta \alpha + \frac{\partial \bar{C}_H}{\partial E'} \frac{\partial E'}{\partial G_T} \Delta C_T$$

$$(66) \quad \Delta C_H' = \frac{\partial \bar{C}_H'}{\partial \theta'} \Delta \theta' + \frac{\partial \bar{C}_H'}{\partial \alpha} \Delta \alpha + \frac{\partial \bar{C}_H'}{\partial E} \frac{\partial E}{\partial G_T} \Delta C_T$$

from which

$$(67) \quad \frac{\partial \bar{C}_H}{\partial \theta} = \left(\frac{\Delta C_H}{\Delta \theta} \right) - \frac{\partial \bar{C}_H}{\partial \alpha} \left(\frac{\Delta \alpha}{\Delta \theta} \right)$$

$$(68) \quad \frac{\partial \bar{C}_H'}{\partial \theta'} = \left(\frac{\Delta C_H'}{\Delta \theta'} \right) - \frac{\partial \bar{C}_H'}{\partial \alpha} \left(\frac{\Delta \alpha}{\Delta \theta'} \right)$$

From loading the front rotor we can write

$$(69) \quad \Delta C_{H_2} = \frac{\partial \bar{C}_H}{\partial \theta} \Delta \theta_2 + \frac{\partial \bar{C}_H}{\partial \alpha} \Delta \alpha_2 + \frac{\partial \bar{C}_H}{\partial E'} \frac{\partial E'}{\partial G_T} \Delta C_{T_2}$$

$$(70) \quad \Delta C_{H_2}' = \frac{\partial \bar{C}_H'}{\partial \theta'} \Delta \theta_2' + \frac{\partial \bar{C}_H'}{\partial \alpha} \Delta \alpha_2 + \frac{\partial \bar{C}_H'}{\partial E} \frac{\partial E}{\partial G_T} \Delta C_{T_2}$$

from which

$$(71) \quad \frac{\partial \bar{C}_H}{\partial \alpha} = \frac{\left(\frac{\Delta C_H}{\Delta \alpha} \right)_2 - \left(\frac{\Delta C_H}{\Delta \theta} \right)_2 \left(\frac{\Delta \theta}{\Delta \alpha} \right)_2}{1 - \left(\frac{\Delta \alpha}{\Delta \theta} \right)_2 \left(\frac{\Delta \theta}{\Delta \alpha} \right)_2}$$

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and from loading the rear rotor

$$(72) \quad \frac{\partial \bar{C}_H'}{\partial \alpha} = \frac{\left(\frac{\partial \bar{C}_H'}{\partial \alpha}\right)_3 - \left(\frac{\partial \bar{C}_H'}{\partial \theta'}\right)_3 \left(\frac{\partial \theta'}{\partial \alpha}\right)_3}{1 - \left(\frac{\partial \alpha}{\partial \theta'}\right)_3 \left(\frac{\partial \theta'}{\partial \alpha}\right)_3}$$

The interference effects on the "H" force coefficients may be found from

$$(73) \quad \frac{\partial \bar{C}_H'}{\partial \epsilon} \frac{\partial \epsilon}{\partial G_2} = - \frac{\partial \bar{C}_H'}{\partial \theta'} \frac{\partial \theta'}{\partial G_2} - \frac{\partial \bar{C}_H'}{\partial \alpha} \frac{\partial \alpha}{\partial G_2} + \frac{\partial \bar{C}_H'}{\partial G_2}$$

$$(74) \quad \frac{\partial \bar{C}_H'}{\partial \epsilon'} \frac{\partial \epsilon'}{\partial G_3'} = - \frac{\partial \bar{C}_H'}{\partial \theta'} \left(\frac{\partial \theta'}{\partial G_3'} \right) - \frac{\partial \bar{C}_H'}{\partial \alpha} \left(\frac{\partial \alpha}{\partial G_3'} \right) + \frac{\partial \bar{C}_H'}{\partial G_3'}$$

d) "H" Force Derivatives With Respect to Tip Speed Ratio

The "H" force derivatives with respect to μ may be obtained from test 2 b)

$$(75) \quad \Delta C_H = \frac{\partial \bar{C}_H}{\partial \alpha} \Delta \alpha + \frac{\partial \bar{C}_H}{\partial \mu} \Delta \mu + \frac{\partial \bar{C}_H}{\partial \epsilon'} \frac{\partial \epsilon'}{\partial \mu} \Delta \mu$$

$$(76) \quad \Delta C_H' = \frac{\partial \bar{C}_H'}{\partial \alpha} \Delta \alpha + \frac{\partial \bar{C}_H'}{\partial \mu} \Delta \mu + \frac{\partial \bar{C}_H'}{\partial \epsilon} \frac{\partial \epsilon}{\partial \mu} \Delta \mu$$

e) Blade Flapping and Coning Derivatives

The equations for obtaining all the blade flapping and coning derivatives are identical to those for the "H" force derivatives except for the substitution of the flapping angle for $\bar{C}_H$ wherever it appears in the equations.

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f) Fuselage Derivatives

The fuselage moment derivative with respect to angle of attack may be obtained from flying at two different trim settings and the same tip speed ratio and c.g. position, in level flight.

$$\begin{aligned} 71) \quad \frac{\Delta C_{m_f}}{\sigma} = & \frac{\Delta C_T}{\sigma} \left(\frac{l}{l_d} - \frac{l}{l_d} B_{13} \right) + \frac{\Delta C_H}{\sigma} \left(\frac{l}{l_d} + \frac{l}{l_d} B_{13} \right) \\ & - \frac{C_T}{\sigma} \frac{l}{l_d} \Delta B_3 + \frac{C_H}{\sigma} \frac{l}{l_d} \Delta B_3 - \frac{C_T}{\sigma} \frac{l}{l_d} \Delta B'_3 \\ & + \frac{C_H}{\sigma} \frac{l}{l_d} \Delta B'_3 - \frac{\Delta C_T}{\sigma} \left(\frac{l}{l_d} + \frac{l}{l_d} B_{13}' \right) \\ & + \frac{\Delta C_H}{\sigma} \left(\frac{l}{l_d} - \frac{l}{l_d} B_{13}' \right) \end{aligned}$$

The change in H forces can be measured and the change in thrust forces calculated from knowledge of the derivatives.

The fuselage drag derivative with angle of attack may be calculated similarly from

$$78) \quad \frac{\Delta C_{D_f}}{\sigma} = -\frac{C_T}{\sigma} \Delta B_3 + \frac{C_T}{\sigma} \Delta \phi - (\Delta \frac{C_T}{\sigma} B_3 + \Delta \frac{C_T}{\sigma} B'_3) + \Delta \frac{C_H}{\sigma} + \Delta \frac{C_H'}{\sigma}$$

The fuselage drag and moment derivative with velocity may be obtained by flying at two different forward velocities, but at the same fuselage angle of attack, and applying the same equations as above.

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5. Concluding Remarks

The results of a theoretical analysis of tandem helicopter stability and control indicate the important stability derivatives to be damping in pitch, velocity stability and angle of attack stability. The analysis indicates that rotor interference affects the latter two of these three adversely, and it is to these adverse effects that tandem helicopter deficiencies in forward flight stability can be traced.

Using the theory developed, it is shown how the important tandem helicopter stability derivatives, including the interference effects, may be extracted from steady-state flight test data.

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6. Appendix I

a) Theoretical Estimates of Rotor Derivatives

The derivations for the thrust and "H" forces and their derivatives are made using the same method of analysis as in Ref. 1, Chapter 7, with the exception that the force and flapping coefficients are calculated relative to the no-feathering axes, and the vertical degree of freedom is expressed by α , rotor angle of attack, rather than μ_g . The distribution of induced velocity is assumed constant over the disc with a magnitude that is at all times in equilibrium with the instantaneous value of the thrust.

The effects of flapping hinge offset on blade flapping, rotor forces, and pitching moments about the c.g. are included. Non-linear terms are treated in the same manner as in Ref. 1, Chapter 7.

The expressions so obtained are as follows, written only for the front rotor. The values for $\beta_0, a, b, \theta, \lambda, \mu$, are the initial trim values.

Thrust:

$$I-1) \quad \frac{C_T}{\sigma} = \frac{a}{2} \left[\theta \left(K + \frac{\mu^2}{2} G \right) - Q \frac{\dot{\beta}_0}{\Omega} - C \frac{\mu}{2} \left(a_1 - \frac{\dot{b}_1}{\Omega} \right) + \mu F \frac{a_1}{2} + \lambda F + \frac{\mu}{2} \frac{\dot{\phi}_2}{\Omega} F \right]$$

$$I-2) \quad \frac{\partial C_T}{\partial \alpha} = \frac{\mu a F}{2 \left(1 + \frac{a \theta F}{\lambda \mu} \right)}$$

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$$I-3) \quad \frac{\partial G/\sigma}{\partial \mu} = \frac{a[\theta \mu G - \frac{a}{2}(C-F) + F(\frac{1}{\mu} + \frac{G}{\mu^2})]}{2(1 + \frac{a\theta F}{4\mu})}$$

$$I-4) \quad \frac{\partial G/\sigma}{\partial \theta} = \frac{a}{2} \frac{(K + \mu^2 G)}{(1 + \frac{a\theta F}{4\mu})}$$

$$I-5) \quad \frac{\partial G/\sigma}{\partial \phi} = -\frac{1}{v} \frac{\partial G/\sigma}{\partial a} - \frac{h}{nR} \frac{\partial G/\sigma}{\partial \mu}$$

$$I-6) \quad \frac{\partial G/\sigma}{\partial \beta} = -\frac{a}{2} \frac{Q}{n}$$

$$I-7) \quad \frac{\partial G/\sigma}{\partial \gamma} = -\frac{a}{4} \mu (C-F)$$

$$I-8) \quad \frac{\partial G/\sigma}{\partial b_1} = \frac{a}{4} \mu \frac{C}{n}$$

"H" Force

$$\begin{aligned} I-9) \quad \frac{C_H}{\sigma} = & \frac{a}{2} \left\{ \frac{a}{2} \left[\theta (K + \mu^2 G) - \frac{Q\beta}{n} - C\mu \left(a_1 - \frac{b_1}{n} \right) + \frac{F\mu\phi}{4} \right. \right. \\ & + 3\mu \frac{Fa_1}{4} + 1F - \mu^2 \frac{G\theta}{4} \left. \right] - \frac{\beta}{2} \left[-Q \left(-\frac{a_1}{n} - b_1 \right) + \frac{\phi}{n} K \right. \\ & - \mu F\beta + \mu^2 \frac{b_1}{4} G \left. \right] + \frac{b_1}{4} \left[-C\mu \left(-\frac{a_1}{n} - b_1 \right) + \frac{\mu F\phi}{2n} \right. \\ & + \mu \frac{Fa_1}{2} - \mu^2 \frac{b_1}{2} G \left. \right] - \frac{1}{2} \left\{ \left[-Q \left(a_1 - \frac{b_1}{n} \right) - C\mu \frac{\beta}{n} \right. \right. \\ & \left. \left. + K \frac{\phi}{n} + \mu^2 \frac{a_1}{4} G + \mu G \right] \left(\theta - \frac{\phi}{a} - 2\theta \frac{\phi}{a} \right) \right\} \end{aligned}$$

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$$\begin{aligned}
 &+ [-2QA_1 \dot{\phi}_1 + F(2a_1 \frac{\dot{\phi}_1}{\dot{\theta}} - 2b_1 \frac{\dot{\phi}_1}{\dot{\theta}}) + \mu C(-a_1^2 \\
 &+ a_1 \frac{b_1}{\dot{\theta}} + a_1 \frac{b_1}{\dot{\theta}} + b_1^2) - 2C\lambda(a_1 - \frac{b_1}{\dot{\theta}}) \\
 &- \mu^2 \beta_0 b_1 \frac{G}{2} + 2\mu \frac{\phi_1}{\dot{\theta}} F \frac{b_1}{4} + 2\mu \frac{\phi_2}{\dot{\theta}} F \frac{a_1}{4} \\
 &+ 2\lambda F \frac{\phi_2}{\dot{\theta}} \left[\left(1 - \frac{\dot{\phi}_1}{\dot{\theta}}\right) - 2J\mu \left(\frac{1}{\dot{\theta}} + \frac{\dot{\phi}_1}{\dot{\theta}} \theta + \frac{\dot{\phi}_1}{\dot{\theta}} \theta^2\right) \right] \}
 \end{aligned}$$

$$\begin{aligned}
 I-10) \quad \frac{\partial C_H/\sigma}{\partial \mu} &= \frac{a_1}{2} \left(\frac{\mu}{1 + \frac{a_1 \sigma F}{4\mu}} \right) \left[\frac{a_1}{2} F - \frac{1}{2} \left[\mu G \left(\theta - \frac{\dot{\phi}_1}{\dot{\theta}} - 2\theta \frac{\dot{\phi}_1}{\dot{\theta}} \right) \right. \right. \\
 &\quad \left. \left. - 2Ca_1 \left(1 - \frac{\dot{\phi}_1}{\dot{\theta}}\right) \right] \right]
 \end{aligned}$$

$$\begin{aligned}
 I-11) \quad \frac{\partial C_H/\sigma}{\partial \mu} &= \frac{a_1}{2} \left\{ \frac{a_1}{2} \left[\mu G \theta + \frac{a_1}{4} (3F - C) \right] - \frac{\beta_0}{2} \left[-F\beta_0 + b_1 G \frac{\mu}{2} \right] \right. \\
 &\quad + \frac{b_1}{4} \left[\frac{b_1}{2} (C + F) - \mu \beta_0 G \right] - \frac{1}{2} \left\{ \left(\mu \frac{a_1}{2} G + \lambda G \right) \left(\theta - \frac{\dot{\phi}_1}{\dot{\theta}} - 2\theta \frac{\dot{\phi}_1}{\dot{\theta}} \right) \right. \\
 &\quad \left. \left. + \left[\frac{C}{2} (-a_1^2 + b_1^2) - \mu \beta_0 b_1 G \right] \left(1 - \frac{\dot{\phi}_1}{\dot{\theta}}\right) \right. \right. \\
 &\quad \left. \left. - 2J \left(\frac{\dot{\phi}_1}{\dot{\theta}} + \frac{\dot{\phi}_1}{\dot{\theta}} \theta + \frac{\dot{\phi}_1}{\dot{\theta}} \theta^2 \right) \right\} + \left\{ \frac{1}{\mu} + \frac{C}{\mu^2} \right. \right. \\
 &\quad \left. \left. - \frac{a_1 \sigma}{4\mu} \left[\frac{\theta \mu G - \frac{a_1}{2} (C - F) + F \left(\frac{1}{\mu} + \frac{C}{\mu^2} \right) \right]}{1 + \frac{a_1 \sigma F}{4\mu}} \right] \right\} \times \\
 &\quad \left\{ a_1 \frac{F}{2} - \frac{1}{2} \left[\mu G \left(\theta - \frac{\dot{\phi}_1}{\dot{\theta}} - 2\theta \frac{\dot{\phi}_1}{\dot{\theta}} \right) - 2Ca_1 \left(1 - \frac{\dot{\phi}_1}{\dot{\theta}}\right) \right] \right\}
 \end{aligned}$$

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$$\begin{aligned}
 I-12) \quad \frac{\partial C_{11}/\sigma}{\partial \theta} = & \frac{a}{2} \left\{ \frac{a}{2} (K + \mu^2 G) - \frac{1}{2} \left\{ [-Qa_1 + \mu^2 a_1 G \right. \right. \\
 & + \mu G] (1 - \frac{2s_2}{a}) - 2J\mu (\frac{\theta}{a} + \frac{2s_2}{a} \theta) \left. \right\} \\
 & - \frac{Q\sigma}{4\mu} \frac{(K + \mu^2 G)}{1 + \frac{Q\sigma F}{4\mu}} \left[a_1 \frac{F}{2} - \frac{1}{2} \left\{ \mu G (\theta \right. \right. \\
 & \left. \left. - \frac{s_1}{a} - 2\theta \frac{s_2}{a}) - 2Ca_1 (1 - \frac{s_2}{a}) \right\} \right] \left. \right\}
 \end{aligned}$$

$$\begin{aligned}
 I-13) \quad \frac{\partial C_{11}/\sigma}{\partial \theta} = & -\frac{1}{\sigma} \frac{\partial C_{11}/\sigma}{\partial \lambda} - \frac{4}{2\sigma} \frac{\partial C_{11}/\sigma}{\partial \mu} + \frac{a}{2} \left\{ -\frac{\beta_2 K}{2\sigma} \right. \\
 & \left. + \frac{4}{\sigma} \frac{F\mu}{2} - \frac{1}{2} \left[\mu \frac{F b_1}{2\sigma} (1 - \frac{s_2}{a}) \right] \right\}
 \end{aligned}$$

$$\begin{aligned}
 I-14) \quad \frac{\partial C_{11}/\sigma}{\partial a_1} = & \frac{a}{2} \left\{ \frac{1}{2} [\theta (K + \mu^2 G) - C\mu a_1 + 3\mu F a_1 \right. \\
 & + \mu F + \mu^2 G \theta] - \frac{1}{2} \left\{ (-Q + \frac{\mu^2}{4} G) (\theta - \frac{s_1}{a} - 2\theta \frac{s_2}{a}) \right. \\
 & \left. + [-\mu C a_1 - 2C\lambda] (1 - \frac{s_2}{a}) \right\} \left. \right\}
 \end{aligned}$$

$$I-15) \quad \frac{\partial C_{11}/\sigma}{\partial a_1} = \frac{a}{2} \left\{ -\frac{\beta_2 Q}{2\sigma} + \frac{4}{\sigma} \frac{F\mu}{2} - \frac{1}{2} \left[\mu \frac{C}{2} \frac{b_1}{\sigma} (1 - \frac{s_2}{a}) \right] \right\}$$

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$$I-16) \quad \frac{\partial C_H/\sigma}{\partial b_1} = \frac{a}{2} \left\{ \frac{A}{2} (-\theta + \mu^2 \frac{G}{4}) + \frac{b_1}{2} \mu (C+F) \right. \\ \left. - \frac{1}{2} \left\{ [\mu C b_1 - \mu^2 A_0 \frac{G}{2}] (1 - \frac{f_1}{2}) \right\} \right\}$$

$$I-17) \quad \frac{\partial C_H/\sigma}{\partial b_1} = \frac{a}{2} \left\{ \frac{a_1 C \mu}{2 \Omega} - \frac{1}{2} \left\{ \frac{a}{\Omega} (\theta - \frac{f_1}{2} - 2\theta \frac{f_1}{2}) + [\frac{\mu C a_1}{2 \Omega} \right. \right. \\ \left. \left. + 2C \frac{A}{\Omega}] (1 - \frac{f_1}{2}) \right\} \right\}$$

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Flapping and Coning Derivatives - Equation for coning angle β_0

$$\begin{aligned}
 \text{I-18)} \quad & \frac{\gamma_1}{2} \left\{ \theta \left[A + C \frac{\mu^2}{2} \right] - S \frac{\beta_0}{\Omega} - E \frac{\mu}{2} \left(a_1 - \frac{b_1}{\Omega} \right) \right. \\
 & \left. + Q \frac{\mu}{2} \frac{\phi_1}{\Omega} + \mu a_1 \frac{Q}{2} + 1 \right\} - \beta_0 - \frac{\beta_1}{\Omega^2} \\
 & - \frac{M_m \beta_0}{I_1} - \frac{M_0}{I_1} \frac{V}{\Omega^2} (\phi_1 - \alpha_3) + \frac{M_0}{I_1} \frac{R}{\Omega} \mu \left(a_1 + \phi_1 \right) \\
 & - \frac{2 M_0 \phi_1}{I_1 \Omega^2} = 0
 \end{aligned}$$

$$\text{I-19)} \quad \frac{d\beta_0}{d\mu} = \frac{\gamma_1 Q \mu}{2 \left(1 + \frac{a_1 \sigma F}{\mu} \right)}$$

$$\text{I-20)} \quad \frac{d\beta_0}{d\alpha_3} = \frac{M_0 V}{I_1 \Omega^2}$$

$$\begin{aligned}
 \text{I-21)} \quad \frac{d\beta_0}{d\mu} = & \frac{\gamma_1}{2} \left\{ \theta C \mu + \frac{a_1}{2} (Q - E) + Q \left(\frac{1}{\mu} + \frac{\sigma}{\mu^2} \right) \right. \\
 & \left. - \frac{\frac{a_1 \sigma}{\mu} \left[\theta \mu G - \frac{a_1}{2} (C - F) + F \left(\frac{1}{\mu} + \frac{\sigma}{\mu^2} \right) \right]}{1 + \frac{a_1 \sigma F}{\mu}} \right\}
 \end{aligned}$$

$$\text{I-22)} \quad \frac{d\beta_0}{d\mu} = \frac{M_0}{I_1} \frac{R}{\Omega} \left(\frac{a_1 + \phi_1}{2} \right)$$

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$$I-23) \quad \frac{dB}{d\theta} = \frac{\gamma}{2} \left\{ A + C \frac{\mu^2}{2} - \frac{Q}{4} \frac{a \sigma}{\mu} \frac{(K + \mu^2/2 G)}{1 + \frac{a \sigma F}{4 \mu}} \right\}$$

$$I-24) \quad \frac{dB}{d\dot{\theta}} = -\frac{I}{U} \frac{dB}{d\dot{\theta}} - \frac{M_3}{I_1} \frac{U}{\Omega^2}$$

$$I-25) \quad \frac{dB}{d\dot{\theta}} = -\frac{\gamma}{2} \frac{S}{\Omega}$$

$$I-26) \quad \frac{dB}{d\dot{\theta}} = -\frac{1}{\Omega^2}$$

$$I-27) \quad \frac{dB}{d\dot{\theta}} = \frac{\gamma}{4} \mu (Q - E)$$

$$I-28) \quad \frac{dB}{d\dot{\theta}} = \frac{\gamma}{4} E \frac{\mu}{\Omega}$$

$$I-29) \quad \frac{dB}{d\dot{\theta}} = -\frac{I}{I_1} \frac{M_3}{\Omega^2}$$

Equation for Longitudinal Flapping:

$$\begin{aligned} I-30) \quad & \frac{A}{S} \frac{\dot{\phi}_2}{\Omega} + 2\mu \theta \frac{Q}{S} - a_1 + \frac{b_1}{\Omega} - \frac{E}{S} \mu \frac{\dot{\phi}_2}{\Omega} + \mu^2 \frac{a_1}{4} \frac{C}{S} \\ & + 1\mu \frac{C}{S} - \frac{4}{5\gamma, \Omega} (\dot{a}_3 + \dot{\phi}_1) + \frac{2}{5\gamma, \Omega^2} (\ddot{a}_3 + \ddot{\phi}_2) \\ & + \frac{2Mm}{5\gamma, I_1} \left(b_3 - 2\frac{\dot{\phi}_1}{\Omega} + \frac{\ddot{\phi}_2}{\Omega^2} \right) = 0 \end{aligned}$$

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$$I-31) \quad \frac{da_1}{d\mu} = \frac{\mu^2 C}{3(1 + \frac{a\sigma F}{4\mu})}$$

$$I-32) \quad \frac{da_1}{d\mu} = \frac{2\theta\theta}{3} + \mu \frac{Q_1}{2} \frac{C}{3} + \frac{1}{3} \frac{C}{3} + \frac{\mu C}{3} \left(\frac{1}{\mu} + \frac{C}{\mu^2} \right) \\ - \frac{a\sigma}{4\mu} \left[\theta \mu G - \frac{a}{2}(C-F) + F \left(\frac{1}{\mu} + \frac{C}{\mu^2} \right) \right] \\ 1 + \frac{a\sigma F}{4\mu}$$

$$I-33) \quad \frac{da_1}{d\theta} = 2\mu \frac{Q}{3} - \frac{C}{3} \frac{a\sigma}{4} \frac{(K + \mu^2/2 G)}{1 + \frac{a\sigma F}{4\mu}}$$

$$I-34) \quad \frac{da_1}{d\eta} = -\frac{1}{v} \frac{da_1}{d\mu} - \frac{4}{5\gamma_1 \Omega} \left(1 + \frac{Mm}{I_1} \right)$$

$$I-35) \quad \frac{da_1}{d\eta} = -\frac{E}{5} \frac{\mu}{\Omega}$$

$$I-36) \quad \frac{da_1}{d\eta} = -\frac{4}{5\gamma_1 \Omega}$$

$$I-37) \quad \frac{da_1}{db_1} = \frac{2 Mm}{5\gamma_1 I_1}$$

$$I-38) \quad \frac{da_1}{d\eta} = \frac{1}{\Omega}$$

$$I-39) \quad \frac{da_1}{d\eta} = \frac{2}{5\gamma_1 \Omega^2}$$

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Equation for Lateral Flapping b_1

$$\begin{aligned} \text{I-40)} \quad & \frac{A}{S} \frac{\dot{\Phi}}{\Omega} + \frac{\dot{a}_1}{\Omega} + b_1 - \mu \beta_0 \frac{Q}{S} + \mu^2 \frac{b_1}{2} \frac{C}{S} + \frac{2(a_1 + \ddot{\Phi})}{S \gamma \Omega^2} \\ & + \frac{4}{S} \frac{(b_{12} + \dot{\Phi}_1)}{\gamma \Omega} + \frac{2Mm}{S \gamma I_1} (a_3 + 2\frac{\dot{\Phi}_1}{\Omega} + \frac{\ddot{\Phi}_1}{\Omega^2}) \\ & - \frac{2M_2 \mu}{S \gamma I_1} \frac{R}{\Omega} \beta_0 = 0 \end{aligned}$$

$$\text{I-41)} \quad \frac{db_1}{d\mu} = -\beta_0 \frac{Q}{S} + \mu \frac{b_1}{2} \frac{C}{S}$$

$$\text{I-42)} \quad \frac{db_1}{d\mu} = -\frac{2M_2 R \beta_0}{S \gamma I_1 \Omega}$$

$$\text{I-43)} \quad \frac{db_1}{d\beta_0} = -\mu \frac{Q}{S}$$

$$\text{I-44)} \quad \frac{db_1}{d\alpha_1} = \frac{2Mm}{S \gamma I_1}$$

$$\text{I-45)} \quad \frac{db_1}{d\alpha_1} = \frac{2Mm}{S \gamma I_1}$$

$$\text{I-46)} \quad \frac{db_1}{d\alpha_1} = \frac{1}{\Omega}$$

$$\text{I-47)} \quad \frac{db_1}{d\alpha_1} = \frac{2}{S \gamma \Omega^2}$$

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$$I-48) \frac{db_1}{d\dot{\phi}_1} = \frac{4}{5\gamma_1 \Omega}$$

$$I-49) \frac{db_1}{d\dot{\phi}_1} = \frac{4}{5\gamma_1 \Omega} \left(1 + \frac{Mm}{I_1} \right)$$

$$I-50) \frac{db_1}{d\dot{\phi}_1} = \frac{4}{3} \frac{1}{\Omega}$$

Moment due to Hinge Offset

$$I-51) M_h = e, b \left\{ \rho a c R^3 \frac{\Omega^2}{4} \left[Q \left(-\frac{\dot{\phi}_1}{\Omega} - b_1 \right) + \mu F \beta_0 - K \frac{\phi}{\Omega} \right. \right. \\ \left. \left. + \mu \frac{b_1 G}{4} \right] + \frac{M_0}{2} \left(\Omega^2 a_0 - \ddot{a}_0 - 2\Omega \dot{a}_0 \right) - \frac{M_0}{2} (2\Omega \dot{\phi}_1 + \ddot{\phi}_1) \right\}$$

$$I-52) \frac{dM_h}{d\mu} = e, \frac{a}{4} \frac{E \beta_0}{h}$$

$$I-53) \frac{dM_h}{d\beta_0} = e, \frac{a}{4} \frac{\mu F}{h}$$

$$I-54) \frac{dM_h}{d\dot{\phi}_1} = -e, \frac{a}{4} \frac{Q}{h \Omega}$$

$$I-55) \frac{dM_h}{d\dot{\phi}_1} = -\frac{e}{h} \frac{M_0}{2\rho c R^3}$$

$$I-56) \frac{dM_h}{d\dot{\phi}_1} = -\frac{e}{h} \frac{M_0}{2\rho c R^3}$$

$$I-57) \frac{dM_h}{d\dot{\phi}_1} = -\frac{e}{h} \frac{a}{4} \frac{K}{\Omega}$$

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$$I-58) \quad A = \left[\frac{x^4}{4} - \epsilon_1 \frac{x^3}{3} \right]_{\epsilon_1}^B$$

$$I-59) \quad C = \left[\frac{x^2}{2} - \epsilon_1 x \right]_{\epsilon_1}^B$$

$$I-60) \quad E = \left[\frac{x^3}{3} - 2\epsilon_1 \frac{x^2}{2} + \epsilon_1^2 x \right]_{\epsilon_1}^B$$

$$I-61) \quad F = \left[\frac{x^2}{2} \right]_{\epsilon_1}^B$$

$$I-62) \quad G = [x]_{\epsilon_1}^B$$

$$I-63) \quad J = \left[\frac{x^2}{2} \right]_{\epsilon_1}^1$$

$$I-64) \quad K = \left[\frac{x^3}{3} \right]_{\epsilon_1}^B$$

$$I-65) \quad Q = \left[\frac{x^3}{3} - \epsilon_1 \frac{x^2}{2} \right]_{\epsilon_1}^B$$

$$I-66) \quad S = \left[\frac{x^4}{4} - 2\epsilon_1 \frac{x^3}{3} + \epsilon_1^2 \frac{x^2}{2} \right]_{\epsilon_1}^B$$

$$I-67) \quad I_1 = R^2 \int_{x=\epsilon_1}^1 (x-\epsilon_1)^2 m dx$$

$$I-68) \quad M_m = \epsilon_1 R^2 \int_{x=\epsilon_1}^1 (x-\epsilon_1) m dx$$

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$$I-69) \quad M_s = R \int_{x=\varepsilon_1}^1 (x - \varepsilon_1) m dx$$

$$I-70) \quad M_n = R \int_{x=\varepsilon_1}^1 x m dx$$

where m is the mass of the blade per unit length
and

$$I_1 = \frac{\rho c a R^4}{I_1}$$

$$\varepsilon_1 = \frac{e_1}{R}$$

b) Equations of Motion

The complete equations of motion according to the theory of small displacements are as follows.

Equations of Fuselage Motion

$$I-71) \quad \frac{\partial G/\sigma}{\partial \alpha} (\alpha_s - \beta_s) + \frac{\partial G'/\sigma}{\partial \alpha} (\alpha'_s - \beta'_s)$$

$$+ \left(\frac{\partial G/\sigma}{\partial \mu} + \frac{\partial G'/\sigma}{\partial \mu} \right) \mu + \left(\frac{\partial G/\sigma}{\partial \phi_1} + \frac{\partial G'/\sigma}{\partial \phi_1} \right) \phi_1$$

$$+ \frac{\partial G/\sigma}{\partial \beta_0} \beta_0 + \frac{\partial G'/\sigma}{\partial \beta'_0} \beta'_0 + \frac{\partial G/\sigma}{\partial \alpha_1} \alpha_1 + \frac{\partial G'/\sigma}{\partial \alpha'_1} \alpha'_1$$

$$+ \frac{\partial G/\sigma}{\partial \beta_1} \beta_1 + \frac{\partial G'/\sigma}{\partial \beta'_1} \beta'_1 - 2\mu_0 \tau(\dot{\phi}_1 - \dot{\alpha}_s) + \frac{\partial G/\sigma}{\partial \theta} \theta$$

$$+ \frac{\partial G'/\sigma}{\partial \theta'} \theta' = 0$$

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$$\begin{aligned}
 \text{I-72)} \quad & \frac{\partial C_{\phi} \sigma}{\partial \alpha} (\alpha_3 - \beta_3) + \frac{\partial C_{\phi} \sigma}{\partial \alpha_3} \alpha_3 + \frac{\partial C_{\phi} \sigma}{\partial \alpha} (\alpha'_3 - \beta'_3) \\
 & + \left(\frac{\partial C_{\phi} \sigma}{\partial \mu} + \frac{\partial C_{\phi} \sigma}{\partial \mu} + \frac{\partial C_{\phi} \sigma}{\partial \mu} \right) \mu + \left(\frac{\partial C_{\phi} \sigma}{\partial \phi_i} + \frac{\partial C_{\phi} \sigma}{\partial \phi_i} \right) \phi_i + 2 C_{\phi} \phi_i \\
 & + \frac{\partial C_{\phi} \sigma}{\partial \beta_0} \beta_0 + \frac{\partial C_{\phi} \sigma}{\partial \beta'_0} \beta'_0 + \frac{\partial C_{\phi} \sigma}{\partial \beta_0} \beta_0 + \frac{\partial C_{\phi} \sigma}{\partial \beta'_0} \beta'_0 \\
 & + \frac{\partial C_{\phi} \sigma}{\partial \alpha_i} \alpha_i + \frac{\partial C_{\phi} \sigma}{\partial \alpha'_i} \alpha'_i + \frac{\partial C_{\phi} \sigma}{\partial \alpha_i} \alpha_i + \frac{\partial C_{\phi} \sigma}{\partial \alpha'_i} \alpha'_i \\
 & + \frac{\partial C_{\phi} \sigma}{\partial b_i} b_i + \frac{\partial C_{\phi} \sigma}{\partial b'_i} b'_i + \frac{\partial C_{\phi} \sigma}{\partial b_i} b_i + \frac{\partial C_{\phi} \sigma}{\partial b'_i} b'_i \\
 & + 2 \tau \mu + \frac{\partial C_{\phi} \sigma}{\partial \theta} \theta + \frac{\partial C_{\phi} \sigma}{\partial \theta'} \theta' = 0
 \end{aligned}$$

$$\begin{aligned}
 \text{I-73)} \quad & \frac{\partial C_m}{\partial \alpha} (\alpha_3 - \beta_3) + \frac{\partial C_m}{\partial \alpha_3} \alpha_3 + \frac{\partial C_m}{\partial \alpha} (\alpha'_3 - \beta'_3) \\
 & + \left(\frac{\partial C_m}{\partial \mu} + \frac{\partial C_m}{\partial \mu} + \frac{\partial C_m}{\partial \mu} \right) \mu + \left(\frac{\partial C_m}{\partial \phi_i} + \frac{\partial C_m}{\partial \phi_i} \right) \phi_i \\
 & + \frac{\partial C_m}{\partial \beta_0} \beta_0 + \frac{\partial C_m}{\partial \beta'_0} \beta'_0 + \frac{\partial C_m}{\partial \beta_0} \beta_0 + \frac{\partial C_m}{\partial \beta'_0} \beta'_0 + \frac{\partial C_m}{\partial \alpha_i} \alpha_i \\
 & + \frac{\partial C_m}{\partial \alpha'_i} \alpha'_i + \frac{\partial C_m}{\partial b_i} b_i + \frac{\partial C_m}{\partial b'_i} b'_i - J_4 \tau^2 \phi_i \\
 & + \frac{\partial C_m}{\partial \theta} \theta + \frac{\partial C_m}{\partial \theta'} \theta'
 \end{aligned}$$

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The Flapping and Coning Equations

$$\begin{aligned} \text{I-74)} \quad & \frac{dB_0}{d\mu} (\alpha_s - B_0) + \frac{dB_0}{d\mu} \mu + \frac{dB_0}{d\mu} \mu + \frac{dB_0}{d\phi} \dot{\phi} - (1 + \frac{M_m}{I}) B_0 \\ & + \frac{dB_0}{d\phi} \ddot{\phi} + \frac{dB_0}{d\beta} \dot{\beta} + \frac{dB_0}{d\beta} \beta + \frac{dB_0}{d\theta} \theta = 0 \end{aligned}$$

$$\begin{aligned} \text{I-75)} \quad & \frac{d\alpha_1}{d\mu} (\alpha_s - B_0) + \frac{d\alpha_1}{d\mu} \mu + \frac{d\alpha_1}{d\phi} \dot{\phi} + \frac{d\alpha_1}{d\beta} \dot{\beta} - (1 - \frac{M_s^2}{3}) \alpha_1 \\ & + \frac{d\alpha_1}{d\theta} \theta + \frac{d\alpha_1}{d\alpha_1} \alpha_1 + \frac{d\alpha_1}{d\beta_1} \beta_1 + \frac{d\alpha_1}{d\beta_1} \beta_1 + \frac{d\alpha_1}{d\beta_1} \beta_1 = 0 \end{aligned}$$

$$\begin{aligned} \text{I-76)} \quad & \frac{d\beta_1}{d\mu} \mu + \frac{d\beta_1}{d\mu} \mu + \frac{d\beta_1}{d\beta} \beta_0 + \frac{d\beta_1}{d\alpha_1} \alpha_1 + \frac{d\beta_1}{d\alpha_1} \alpha_1 + \frac{d\beta_1}{d\alpha_1} \alpha_1 \\ & + \frac{d\beta_1}{d\alpha_1} \alpha_1 + \frac{d\beta_1}{d\alpha_1} \alpha_1 + (1 + \frac{M_s^2}{3}) \beta_1 + \frac{d\beta_1}{d\beta_1} \beta_1 = 0 \end{aligned}$$

Equations (I-74 to I-76 above are written for the front rotor, similar expressions may be written for the rear rotor but will not be done so here.

The problem as stated in the equations above is too complex to effect a solution without any simplification.

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The assumptions made in reducing the equations to a convenient form are as follows:

1. The effect of lateral motions of the tip path plane on the longitudinal motion of the aircraft is small and may be neglected. i.e. $b_1 = \dot{b}_1 = \ddot{b}_1 = 0$
2. All acceleration quantities in the flapping equations may be dropped.
3. During pitching the tip path plane rotates at the same angular velocity as the mast axis but lags behind by an angle ϕ . Thus $a_1 = \dot{a}_1 = 0$. This is the Hohenhemser quasi-static assumption.
4. The coning angle of the rotor responds instantaneously to any disturbance so that terms containing $\dot{\theta}$ may be neglected.
5. For normal values of flapping hinge offset thrust derivatives with respect to a_1 may be neglected.

The equations of motion now read as follows:

$$\begin{aligned}
 \text{I-71a)} \quad & \frac{\partial C_{L1}}{\partial \alpha} (\alpha_s - \beta_s) + \frac{\partial C_{L1}}{\partial \alpha} (\alpha_s' - \beta_s') + \left(\frac{\partial C_{L1}}{\partial \mu} + \frac{\partial C_{L1}}{\partial \mu} \right) \mu \\
 & + \left(\frac{\partial C_{L1}}{\partial \phi} + \frac{\partial C_{L1}}{\partial \phi} \right) \phi - 2\mu_0 \tau (\phi - \alpha_s) + \frac{\partial C_{L1}}{\partial \theta} \theta \\
 & + \frac{\partial C_{L1}}{\partial \theta'} \theta' = 0
 \end{aligned}$$

$$\begin{aligned}
 \text{I-72a)} \quad & \frac{\partial C_{D1}}{\partial \alpha} (\alpha_s - \beta_s) + \frac{\partial C_{D1}}{\partial \alpha} \alpha_s + \frac{\partial C_{D1}}{\partial \alpha} (\alpha_s' - \beta_s') \\
 & + \left(\frac{\partial C_{D1}}{\partial \mu} + \frac{\partial C_{D1}}{\partial \mu} + \frac{\partial C_{D1}}{\partial \mu} \right) \mu + \left(\frac{\partial C_{D1}}{\partial \phi} + \frac{\partial C_{D1}}{\partial \phi} \right) \phi + 2C_{D1} \phi \\
 & + \frac{\partial C_{D1}}{\partial \beta_0} \beta_0 + \frac{\partial C_{D1}}{\partial \beta_0'} \beta_0' + \frac{\partial C_{D1}}{\partial a_1} a_1 + \frac{\partial C_{D1}}{\partial a_1'} a_1' + 2\tau \mu \\
 & + \frac{\partial C_{D1}}{\partial \theta} \theta + \frac{\partial C_{D1}}{\partial \theta'} \theta' = 0
 \end{aligned}$$

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$$\begin{aligned}
 \text{I-73a)} \quad & \frac{\partial C_m}{\partial \alpha} (\alpha_s - \theta_s) + \frac{\partial C_m}{\partial \alpha} a_s + \frac{\partial C_m'}{\partial \alpha} (a_s' - \theta_s') \\
 & + \left(\frac{\partial C_m}{\partial \mu} + \frac{\partial C_m'}{\partial \mu} + \frac{\partial C_m}{\partial \mu} \right) \mu + \left(\frac{\partial C_m}{\partial \phi} + \frac{\partial C_m'}{\partial \phi} \right) \phi \\
 & + \frac{\partial C_m}{\partial \beta_0} \beta_0 + \frac{\partial C_m'}{\partial \beta_0'} \beta_0' + \frac{\partial C_m}{\partial a_1} a_1 + \frac{\partial C_m'}{\partial a_1'} a_1' - J_1 \ddot{\phi} \\
 & + \frac{\partial C_m}{\partial \theta} \theta + \frac{\partial C_m'}{\partial \theta'} \theta' = 0
 \end{aligned}$$

$$\begin{aligned}
 \text{I-74a)} \quad & \frac{\partial \beta_0}{\partial \alpha} (\alpha_s - \theta_s) + \frac{\partial \beta_0}{\partial \mu} \mu + \frac{\partial \beta_0}{\partial \phi} \phi - (1 + \frac{M_0}{I_1}) \beta_0 \\
 & \frac{\partial \beta_0}{\partial \theta} \theta = 0
 \end{aligned}$$

$$\begin{aligned}
 \text{I-75a)} \quad & \frac{\partial a_1}{\partial \alpha} (\alpha_s - \theta_s) + \frac{\partial a_1}{\partial \mu} \mu + \frac{\partial a_1}{\partial \phi} \phi \\
 & - (1 - \frac{\mu^2 C}{4 S}) a_1 + \frac{\partial a_1}{\partial \theta} \theta = 0
 \end{aligned}$$

Again the flapping equations are expressed for the front rotor only.

The flapping equations can now be solved for a_1 , β_0 , a_1' , and β_0' and the expressions so obtained substituted into (I-71.), (I-72a), (I-73a).

The controls on this helicopter are linked so that movement of the cyclic pitch stick applies differential collective pitch and simultaneous swash plate

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tilt on the front and rear rotor. The terms containing Θ and B_3 are then transposed to the right hand side of the equations and compose the forcing function. The equations so obtained are written below.

$$I-76) \quad (C_{L\alpha} + 2\mu_0) \alpha_3 + C_{L\mu} \mu + (C_{L\phi} - 2\mu_0) \phi_1 = -C_{L\alpha_1} u_1$$

$$I-77) \quad C_{D\alpha} \alpha_3 + (C_{D\mu} + 2d) \mu + (C_{D\phi} d + 2C_D) \phi_1 = -C_{D\alpha_1} u_1$$

$$I-78) \quad C_{m\alpha} \alpha_3 + C_{m\mu} \mu + (C_{m\phi} d - J_y d^2) \phi_1 = -C_{m\alpha_1} u_1$$

where

$$I-79) \quad C_{L\alpha} = \frac{\partial G/\sigma}{\partial \alpha} + \frac{\partial G'/\sigma}{\partial \alpha}$$

$$C_{L\mu} = \frac{\partial G/\sigma}{\partial \mu} + \frac{\partial G'/\sigma}{\partial \mu}$$

$$C_{L\phi} = \frac{1}{\sigma} \left(\frac{\partial G/\sigma}{\partial \phi} + \frac{\partial G'/\sigma}{\partial \phi} \right)$$

$$C_{L\alpha_1} = C_{L\alpha} \frac{dB_1}{d\alpha_1} + C_{L\alpha} \frac{d\Theta}{d\alpha_1}$$

$$C_{L\beta_1} = -C_{L\alpha}$$

$$C_{L\alpha d} = -\frac{\partial G/\sigma}{\partial \theta} + \frac{\partial G'/\sigma}{\partial \theta'}$$

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$$I-80) \quad C_{dL} = \frac{\partial C_{L/\sigma}}{\partial L} + \frac{L}{(1+M_m/I_1)} \frac{\partial C_{L/\sigma}}{\partial \beta_0} \frac{d\beta_0}{dL} + \frac{L}{(1-\mu_3^2)} \frac{\partial C_{L/\sigma}}{\partial a_1} \frac{da_1}{dL} \\ + \frac{\partial C_{dL/\sigma}}{\partial L} + \frac{\partial C_{L/\sigma}}{\partial L} + \frac{L}{(1+M_m/I_1)} \frac{\partial C_{L/\sigma}}{\partial \beta_0'} \frac{d\beta_0'}{dL} \\ + \frac{1}{(1-\mu_3^2)} \frac{\partial C_{L/\sigma}}{\partial a_1'} \frac{da_1'}{dL} + C_{dL} (d_s - B_s)_{TRIM}$$

$$C_{dL} = \frac{\partial C_{L/\sigma}}{\partial \mu} + \frac{L}{(1+M_m/I_1)} \frac{\partial C_{L/\sigma}}{\partial \beta_0} \frac{d\beta_0}{d\mu} + \frac{L}{(1-\mu_3^2)} \frac{\partial C_{L/\sigma}}{\partial a_1} \frac{da_1}{d\mu} \\ + \frac{\partial C_{L/\sigma}}{\partial \mu} + \frac{L}{(1+M_m/I_1)} \frac{\partial C_{L/\sigma}}{\partial \beta_0'} \frac{d\beta_0'}{d\mu} + \frac{1}{(1-\mu_3^2)} \frac{\partial C_{L/\sigma}}{\partial a_1'} \frac{da_1'}{d\mu} \\ + \frac{\partial C_{dL/\sigma}}{\partial \mu} + C_{dL} (d_s - B_u)_{TRIM}$$

$$C_{d\phi} = \frac{1}{\phi} \left\{ \frac{\partial C_{L/\sigma}}{\partial \phi} + \frac{L}{(1+M_m/I_1)} \frac{\partial C_{L/\sigma}}{\partial \beta_0} \frac{d\beta_0}{d\phi} + \frac{L}{(1-\mu_3^2)} \frac{\partial C_{L/\sigma}}{\partial a_1} \frac{da_1}{d\phi} \right. \\ \left. + \frac{\partial C_{L/\sigma}}{\partial \phi} + \frac{L}{1+M_m/I_1} \frac{\partial C_{L/\sigma}}{\partial \beta_0'} \frac{d\beta_0'}{d\phi} + \frac{1}{(1-\mu_3^2)} \frac{\partial C_{L/\sigma}}{\partial a_1'} \frac{da_1'}{d\phi} \right\} \\ + C_{d\phi} (d_s - B_u)_{TRIM}$$

$$C_{d\theta} = - \left[\frac{\partial C_{L/\sigma}}{\partial \theta} + \frac{L}{(1+M_m/I_1)} \frac{\partial C_{L/\sigma}}{\partial \beta_0} \frac{d\beta_0}{d\theta} + \frac{L}{(1-\mu_3^2)} \frac{\partial C_{L/\sigma}}{\partial a_1} \frac{da_1}{d\theta} \right] \\ + \left[\frac{\partial C_{L/\sigma}}{\partial \theta} + \frac{L}{(1+M_m/I_1)} \frac{\partial C_{L/\sigma}}{\partial \beta_0'} \frac{d\beta_0'}{d\theta} + \frac{L}{(1-\mu_3^2)} \frac{\partial C_{L/\sigma}}{\partial a_1'} \frac{da_1'}{d\theta} \right] \\ + C_{d\theta} (d_s - B_u)_{TRIM}$$

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$$C_{DB} = -(C_{DB} - \frac{\partial C_{DB}}{\partial a})$$

$$C_{DB} = C_{DB} \frac{dB_1}{dB_1} + C_{DB} \frac{dB_2}{dB_2}$$

I-81)

$$\begin{aligned} C_{DB} = & \frac{\partial G}{\partial a} \left(\frac{1}{L_1} - \frac{1}{L_2} B_1 \right) + \frac{\partial G}{\partial a} \left(\frac{1}{L_1} + \frac{1}{L_2} B_2 \right) \\ & + \frac{\partial G}{\partial a} \left(\frac{1}{L_1} - \frac{1}{L_2} B_1' \right) + \frac{\partial G}{\partial a} \left(\frac{1}{L_1} - \frac{1}{L_2} B_2' \right) \\ & + \frac{\partial C_{DB}}{\partial a_1} \frac{1}{(1-\mu^2)} \frac{da_1}{da} + \frac{\partial C_{DB}}{\partial a_1} \left(\frac{1}{1-\mu^2} \right) \frac{da_1'}{da} \\ & + \frac{1}{(1+\mu^2/I_1)} \left(\frac{\partial C_{DB}}{\partial \beta_0} \frac{d\beta_0}{da} + \frac{\partial C_{DB}}{\partial \beta_0'} \frac{d\beta_0'}{da} \right) \end{aligned}$$

$$\begin{aligned} C_{DB} = & \frac{\partial G}{\partial \mu} \left(\frac{1}{L_1} - \frac{1}{L_2} B_1 \right) + \frac{\partial G}{\partial \mu} \left(\frac{1}{L_1} + \frac{1}{L_2} B_2 \right) + \frac{\partial G}{\partial \mu} \\ & - \frac{\partial G}{\partial \mu} \left(\frac{1}{L_1} - \frac{1}{L_2} B_1' \right) + \frac{\partial G}{\partial \mu} \left(\frac{1}{L_1} - \frac{1}{L_2} B_2' \right) \\ & + \frac{1}{(1-\mu^2)} \left(\frac{\partial C_{DB}}{\partial a_1} \frac{da_1}{d\mu} + \frac{\partial C_{DB}}{\partial a_1'} \frac{da_1'}{d\mu} \right) \\ & + \frac{1}{1+\mu^2/I_1} \left(\frac{\partial C_{DB}}{\partial \beta_0} \frac{d\beta_0}{d\mu} + \frac{\partial C_{DB}}{\partial \beta_0'} \frac{d\beta_0'}{d\mu} \right) \\ & + \frac{\partial C_{DB}}{\partial \mu} + \frac{\partial C_{DB}}{\partial \mu} \end{aligned}$$

$$C_{DB} = \frac{1}{\tau} \left(\frac{\partial G}{\partial \phi} \left(\frac{1}{L_1} - \frac{1}{L_2} B_1 \right) + \frac{\partial G}{\partial \phi} \left(\frac{1}{L_1} + \frac{1}{L_2} B_2 \right) \right)$$

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$$= \frac{\partial G_{10}}{\partial \phi} \left(\frac{h}{l_d} + \frac{h'}{l_d} B'_5 \right) + \frac{\partial G_{10}}{\partial \phi} \left(\frac{h}{l_d} - \frac{h'}{l_d} B'_5 \right) + \frac{1}{1-\mu^2} \left[\frac{\partial G_{10}}{\partial \phi} \frac{d\phi}{d\phi} \right. \\ \left. + \frac{\partial G_{10}}{\partial \phi} \frac{d\phi'}{d\phi'} \right] + \frac{1}{(1+M_{10}/I_1)} \left[\frac{\partial G_{10}}{\partial \phi} \frac{dB_5}{d\phi} + \frac{\partial G_{10}}{\partial \phi} \frac{dB'_5}{d\phi'} \right] \\ + \frac{\partial G_{10}}{\partial \phi} + \frac{\partial G_{10}}{\partial \phi'} \}$$

$$C_{med} = -\frac{\partial G_{10}}{\partial \theta} \left(\frac{h}{l_d} - \frac{h'}{l_d} B'_5 \right) - \frac{\partial G_{10}}{\partial \theta} \left(\frac{h}{l_d} + \frac{h'}{l_d} B'_5 \right) \\ - \frac{\partial G_{10}}{\partial \theta} \left(\frac{h'}{l_d} + \frac{h'}{l_d} B'_5 \right) + \frac{\partial G_{10}}{\partial \theta} \left(\frac{h}{l_d} - \frac{h'}{l_d} B'_5 \right)$$

$$C_{m\theta} = -(C_{m\alpha} - \frac{\partial G_{10}}{\partial \alpha}) - \frac{G}{\tau} \frac{h}{l_d} - \frac{G'}{\tau} \frac{h'}{l_d}$$

$$C_{m\alpha} = C_{m\theta} \frac{dB_5}{d\alpha} + C_{med} \frac{d\alpha}{d\alpha}$$

$\frac{\partial G_{10}}{\partial \alpha}, \frac{\partial G_{10}}{\partial \mu}, \frac{\partial G_{10}}{\partial \phi}, \frac{\partial G_{10}}{\partial \theta}$ etc. are the contributions of the rotor "H"

forces to the moments, i.e.

$$\frac{\partial G_{10}}{\partial \alpha} = \frac{\partial G_{10}}{\partial \alpha} + \frac{1}{1+M_{10}/I_1} \frac{\partial G_{10}}{\partial B_5} \frac{dB_5}{d\alpha} + \frac{1}{1-\mu^2} \frac{\partial G_{10}}{\partial \alpha} \frac{d\alpha}{d\alpha}$$

c) Modifications To Derivatives Assuming Rear Rotor to be Completely Immersed in Downwash of Front Rotor.

If the rear rotor is assumed to be in the downwash of the front rotor, and the average value of front rotor downwash at the rear rotor is assumed to be twice the value at the front rotor, the inflow ratio, λ of the rear rotor will be increased by the value $\frac{G}{\mu}$.

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The collective pitch and flapping coefficient for the rear rotor are then solved for, using the value for λ .

The angle of attack of the rear rotor is now expressed as

$$I-82) \quad \alpha' = \alpha - 2E$$

$$I-83) \quad \frac{\partial \alpha'}{\partial \alpha} = 1 - 2 \frac{\partial E}{\partial \alpha}$$

$$I-84) \quad \frac{\partial \alpha'}{\partial \mu} = - 2 \frac{\partial E}{\partial \mu}$$

$$I-85) \quad \frac{\partial \alpha'}{\partial \theta} = - 2 \frac{\partial E}{\partial \theta}$$

and

$$I-86) \quad \frac{\partial E}{\partial \alpha} = \frac{2GF}{4\mu(1 + \frac{2GF}{4\mu})}$$

$$I-87) \quad \frac{\partial E}{\partial \mu} = \frac{1}{2\mu^2} \frac{\partial G}{\partial \mu} - \frac{G}{\mu^3}$$

$$I-88) \quad \frac{\partial E}{\partial \theta} = \frac{1}{2\mu^2} \frac{\partial G}{\partial \theta}$$

The change in E due to pitching velocity will be neglected, although it results in a small increase in damping in pitch.

The rear rotor derivatives affected may now be written as follows. The subscript I indicates derivatives calculated under the interference assumption.

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$$I-89) \quad \left(\frac{\partial \bar{C}_l}{\partial \alpha} \right)_I = \frac{\partial \bar{C}_l}{\partial \alpha} \left(1 - 2 \frac{\partial \epsilon}{\partial \alpha} \right)$$

$$\left(\frac{\partial \bar{C}_l}{\partial \mu} \right)_I = \frac{\partial \bar{C}_l}{\partial \mu} - \frac{\partial \bar{C}_l}{\partial \alpha} \left(2 \frac{\partial \epsilon}{\partial \mu} \right)$$

$$\left(\frac{\partial \bar{C}_l}{\partial \theta} \right) = - \frac{\partial \bar{C}_l}{\partial \alpha} \left(2 \frac{\partial \epsilon}{\partial \theta} \right) =$$

The lift derivatives of course must be recalculated with the above values

$$I-90) \quad C_{\bar{L}} = \frac{\partial \bar{C}_l}{\partial \alpha} + \frac{\partial \bar{C}_l}{\partial \alpha} \left(1 - 2 \frac{\partial \epsilon}{\partial \alpha} \right) + (C_{\bar{L}})_I (d_s - \theta_s)_{trim}$$

$$C_{\bar{L}} = \frac{\partial \bar{C}_l}{\partial \mu} + \frac{\partial \bar{C}_l}{\partial \mu} - \frac{\partial \bar{C}_l}{\partial \alpha} \left(2 \frac{\partial \epsilon}{\partial \mu} \right) + (C_{\bar{L}})_I (d_s - \theta_s)_{trim}$$

$$+ \frac{\partial \bar{C}_l}{\partial \theta}$$

$$C_{\bar{L}} = - \frac{\partial \bar{C}_l}{\partial \theta} + \frac{\partial \bar{C}_l}{\partial \theta} - \frac{\partial \bar{C}_l}{\partial \alpha} \left(2 \frac{\partial \epsilon}{\partial \theta} \right)$$

The moment derivatives are also appropriately modified using the above expressions.

Modifications to Derivatives Due to Differential δ_3 Hinge Configuration.

If the helicopter is equipped with a differential δ_3 hinge configuration, the effect may be accounted for in the following manner. To the equations of motion must be added two equations governing the collective pitch change on each rotor. The cyclic pitch changes due to the δ_3 hinge will have small effect on the motion of the helicopter, and can be neglected when compared with the effects of the collective pitch change. The equations governing the collective pitch change are;

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$$I-91) \quad \frac{d\theta}{d\beta_0} \beta_0 - \theta + \theta_c = 0$$

$$I-92) \quad \frac{d\theta'}{d\beta_0'} \beta_0' - \theta' + \theta_c' = 0$$

If the expressions for β_0 and β_0' obtained from (I-74a) are substituted into equations, (I-91), (I-92) above the following expressions result.

$$I-93) \quad \theta = K_1 \left[-\frac{d\beta_0}{d\alpha} (\alpha - \beta_0) - \frac{d\beta_0}{d\mu} \mu - \frac{d\beta_0}{d\phi} \phi \right] + \frac{\theta_c}{1 - \frac{1}{(1+M_n/I_1)} \frac{d\theta}{d\beta_0} \frac{d\beta_0}{d\theta}}$$

$$I-94) \quad K_1 = - \frac{\frac{d\theta}{d\beta_0} \frac{1}{(1+M_n/I_1)}}{1 - \frac{d\theta}{d\beta_0} \frac{d\beta_0}{d\theta} \frac{1}{(1+M_n/I_1)}}$$

where θ_c is the pitch applied through control motion.

$$I-95) \quad \frac{d\theta}{d\beta_0} = -\tan \delta_3$$

Substituting the expressions for $\theta, \theta', \beta_0, \beta_0', \alpha, \alpha'$ into equations (I-71a), (I-72a), (I-73a) results in equations of the same form as (I-76), (I-77), (I-78). The derivatives are modified by the following changes however.

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$$I-96) \Delta C_{u} = - \frac{\partial G_{10}}{\partial \theta} K_1 \frac{dB_0}{d\alpha} - \frac{\partial G_{10}}{\partial \theta'} K_1' \frac{dB_0'}{d\alpha}$$

$$\Delta C_{\mu} = - \frac{\partial G_{10}}{\partial \theta} K_1 \frac{dB_0}{d\mu} - \frac{\partial G_{10}}{\partial \theta'} K_1' \frac{dB_0'}{d\mu}$$

$$\Delta C_{dd} = \frac{1}{\theta} \left(- \frac{\partial G_{10}}{\partial \theta} K_1 \frac{dB_0}{d\theta} - \frac{\partial G_{10}}{\partial \theta'} K_1' \frac{dB_0'}{d\theta} \right)$$

$$\Delta C_{\alpha\alpha} = - K_1 \frac{dB_0}{d\theta} \frac{\partial G_{10}}{\partial \theta} + K_1' \frac{dB_0'}{d\theta'} \frac{\partial G_{10}}{\partial \theta}$$

$$I-97) \Delta C_{u} = - \frac{\partial G_{10}}{\partial \theta} K_1 \frac{dB_0}{d\alpha} - \frac{1}{(1+M_0 I_1)} \frac{\partial G_{10}}{\partial \theta} \frac{dB_0}{d\alpha} K_1 \frac{dB_0}{d\alpha}$$

$$- \frac{1}{1-\mu^2 \epsilon} \frac{\partial G_{10}}{\partial \alpha} \frac{d\alpha}{d\theta} K_1 \frac{dB_0}{d\alpha} - \frac{\partial G_{10}}{\partial \theta'} K_1' \frac{dB_0'}{d\alpha}$$

$$- \frac{1}{1+M_0 I_1} \frac{\partial G_{10}}{\partial \theta'} \frac{dB_0'}{d\alpha} K_1' \frac{dB_0'}{d\alpha}$$

$$- \frac{1}{1-\mu^2 \epsilon} \frac{\partial G_{10}}{\partial \alpha'} \frac{d\alpha'}{d\theta'} K_1' \frac{dB_0'}{d\alpha} + \Delta C_{u} (\alpha - \alpha_0)_{TRIM}$$

$$\Delta C_{\mu} = - \frac{\partial G_{10}}{\partial \theta} K_1 \frac{dB_0}{d\mu} - \frac{1}{(1+M_0 I_1)} \frac{\partial G_{10}}{\partial \theta} \frac{dB_0}{d\mu} K_1 \frac{dB_0}{d\mu}$$

$$- \frac{1}{(1-\mu^2 \epsilon)} \frac{\partial G_{10}}{\partial \alpha} \frac{d\alpha}{d\theta} K_1 \frac{dB_0}{d\mu} - \frac{\partial G_{10}}{\partial \theta'} K_1' \frac{dB_0'}{d\mu}$$

$$- \frac{1}{(1+M_0 I_1)} \frac{\partial G_{10}}{\partial \theta'} \frac{dB_0'}{d\mu} K_1' \frac{dB_0'}{d\mu}$$

$$- \frac{1}{(1-\mu^2 \epsilon)} \frac{\partial G_{10}}{\partial \alpha'} \frac{d\alpha'}{d\theta'} K_1' \frac{dB_0'}{d\mu} + \Delta C_{\mu} (\alpha - \alpha_0)_{TRIM}$$

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$$\begin{aligned}\Delta C_{D\phi} &= \frac{1}{2} \left\{ - \frac{\partial C_{L\phi}}{\partial \theta} K_i \frac{dB}{d\phi} - \left(\frac{1}{1+M_\infty/I_1} \right) \frac{\partial C_{L\phi}}{\partial \beta_0} \frac{dB}{d\phi} K_i \frac{dB}{d\theta} \right. \\ &= - \frac{1}{(1-\mu^2 \frac{1}{2})} \frac{\partial C_{L\phi}}{\partial \alpha_1} \frac{d\alpha_1}{d\theta} K_i \frac{dB}{d\phi} - \frac{\partial C_{L\phi}}{\partial \theta} K_i' \frac{dB}{d\phi} \\ &\quad - \frac{1}{1+M_\infty/I_1} \frac{\partial C_{L\phi}}{\partial \beta_0'} \frac{dB}{d\phi} K_i' \frac{dB}{d\theta} \\ &\quad \left. - \frac{1}{(1-\mu^2 \frac{1}{2})} \frac{\partial C_{L\phi}}{\partial \alpha_1'} \frac{d\alpha_1'}{d\theta} K_i' \frac{dB}{d\phi} \right\} + \Delta C_{D\phi} (d_s - B_s)_{TEIM}\end{aligned}$$

$$\begin{aligned}\Delta C_{D\phi} &= -K_i \frac{dB_0}{d\theta} \frac{\partial C_{L\phi}}{\partial \alpha_1} + K_i' \frac{dB_0'}{d\theta} \frac{\partial C_{L\phi}}{\partial \alpha_1'} \\ &\quad - \Delta C_{L\phi} (d_s - B_s)_{TEIM}\end{aligned}$$

The moment derivatives are modified appropriately through the use of the above expressions.

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7. References

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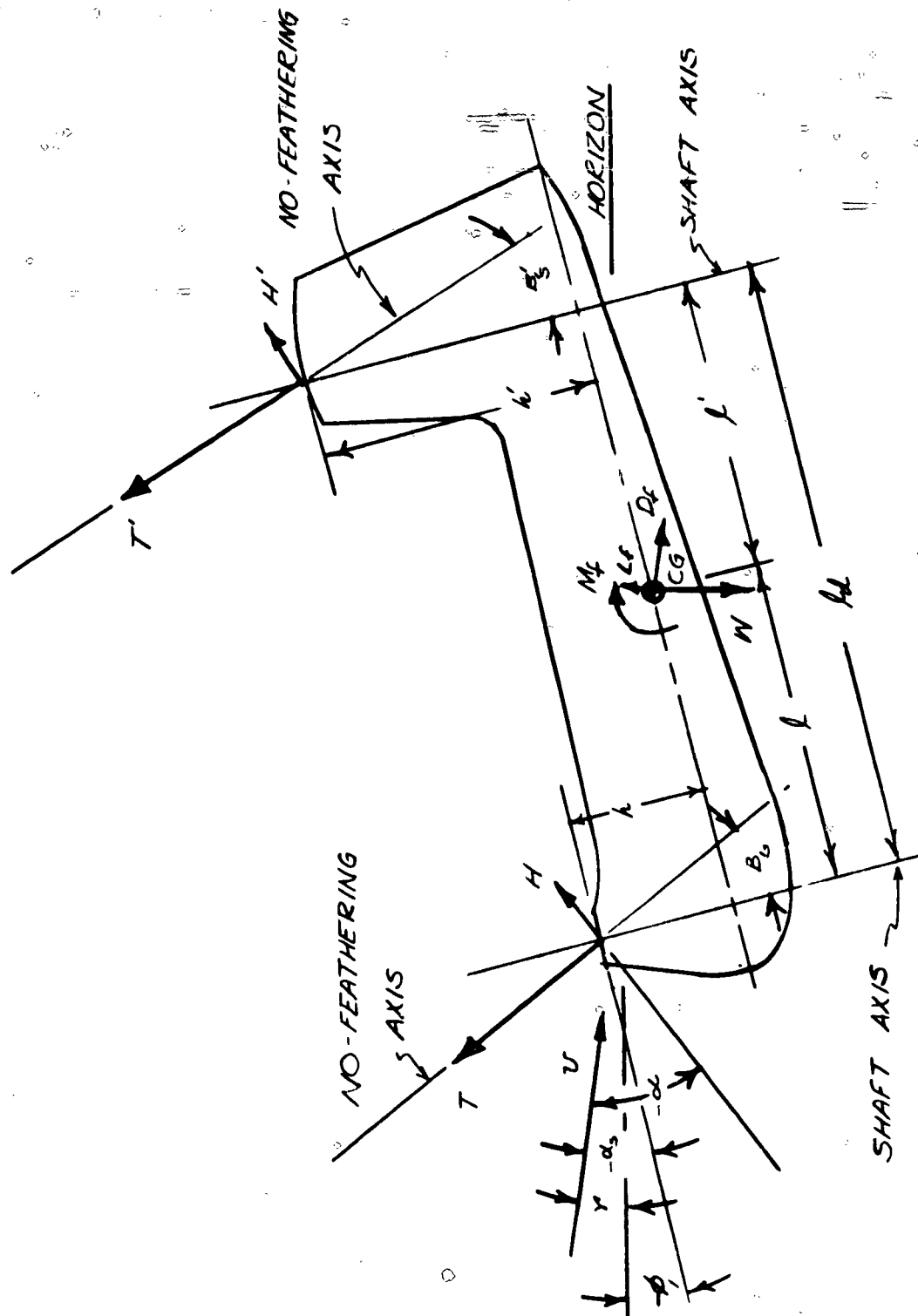


Fig. 1 Schematic Diagram of Helicopter and Coordinate Axes

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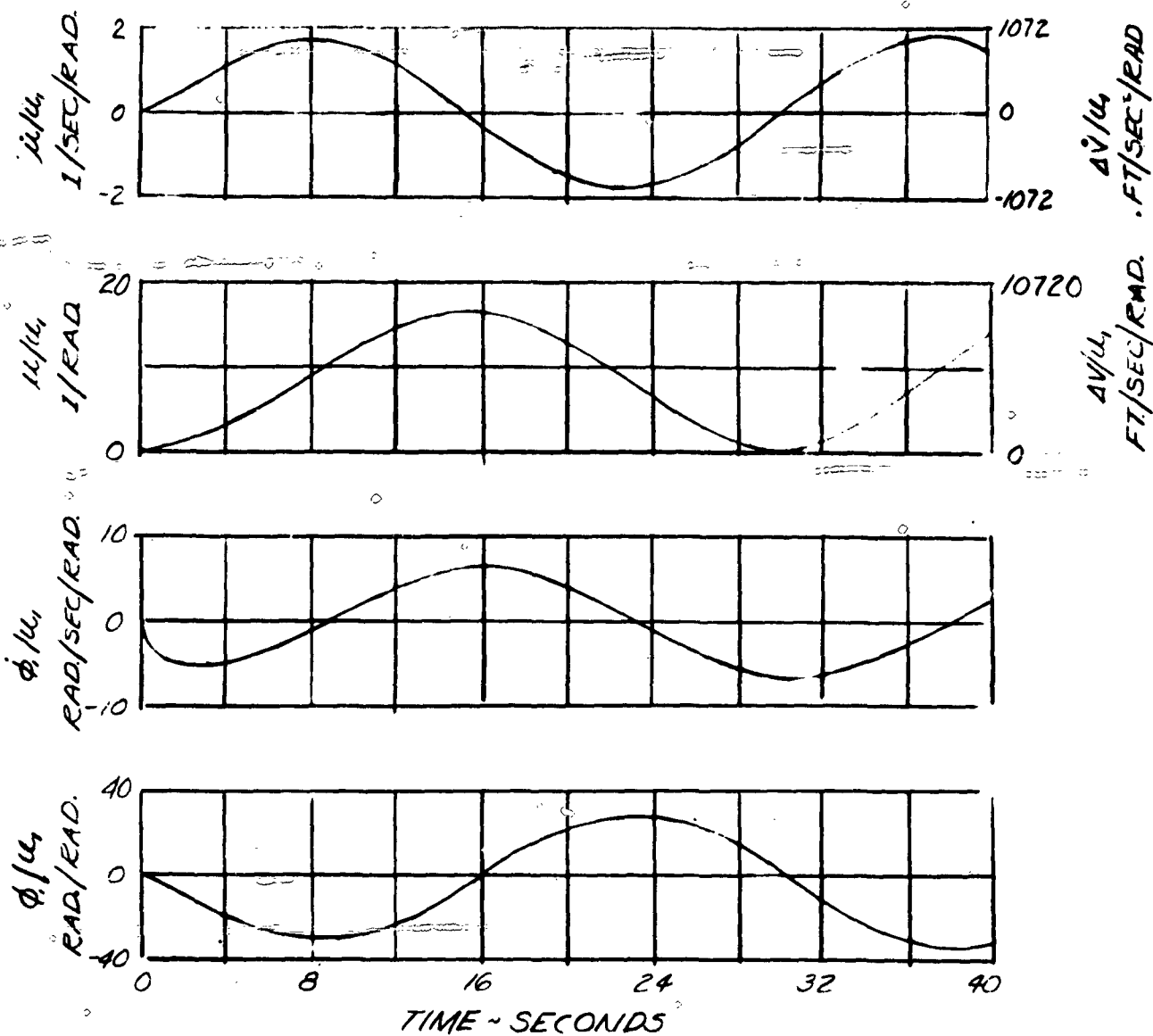


Fig. 2 Response of Tandem Helicopter to Step Input
Trim: Hovering, Sea Level
No Rotor Interference

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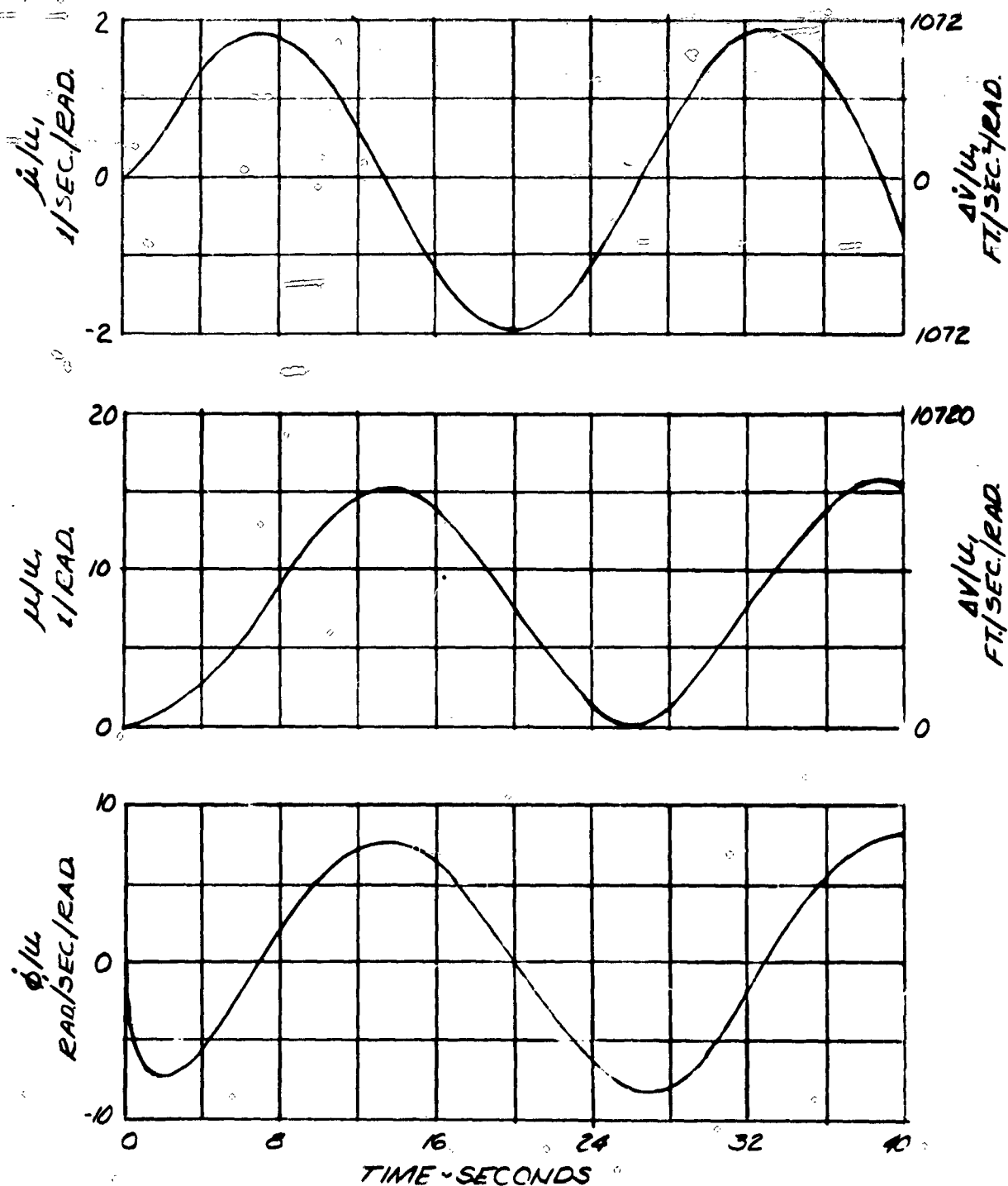


Fig. 3 Response of Tandem Helicopter to Step Input
Trim: 30 Knots Level Flight, Sea Level
No Rotor Interference

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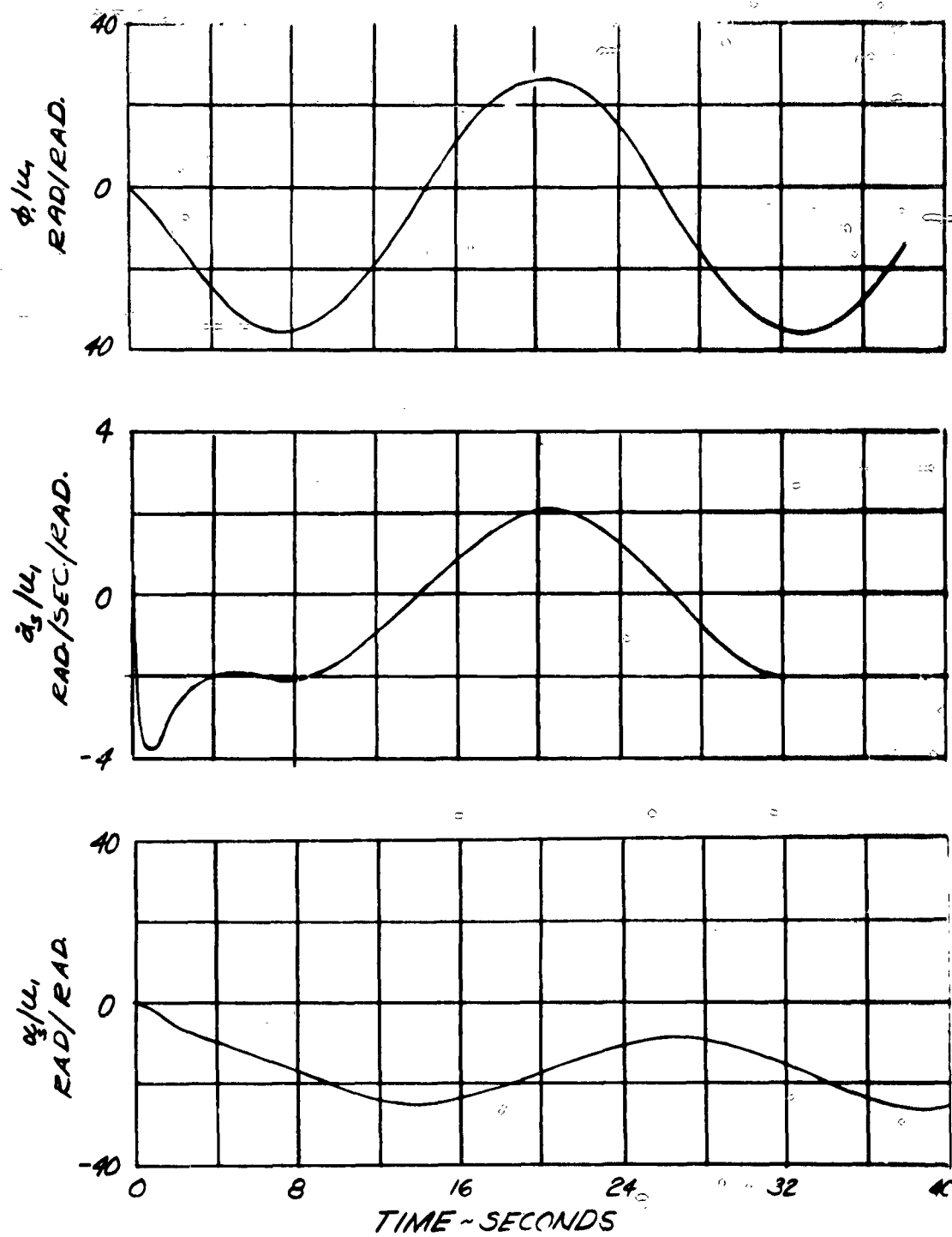


Fig. 3 (Cont'd.)

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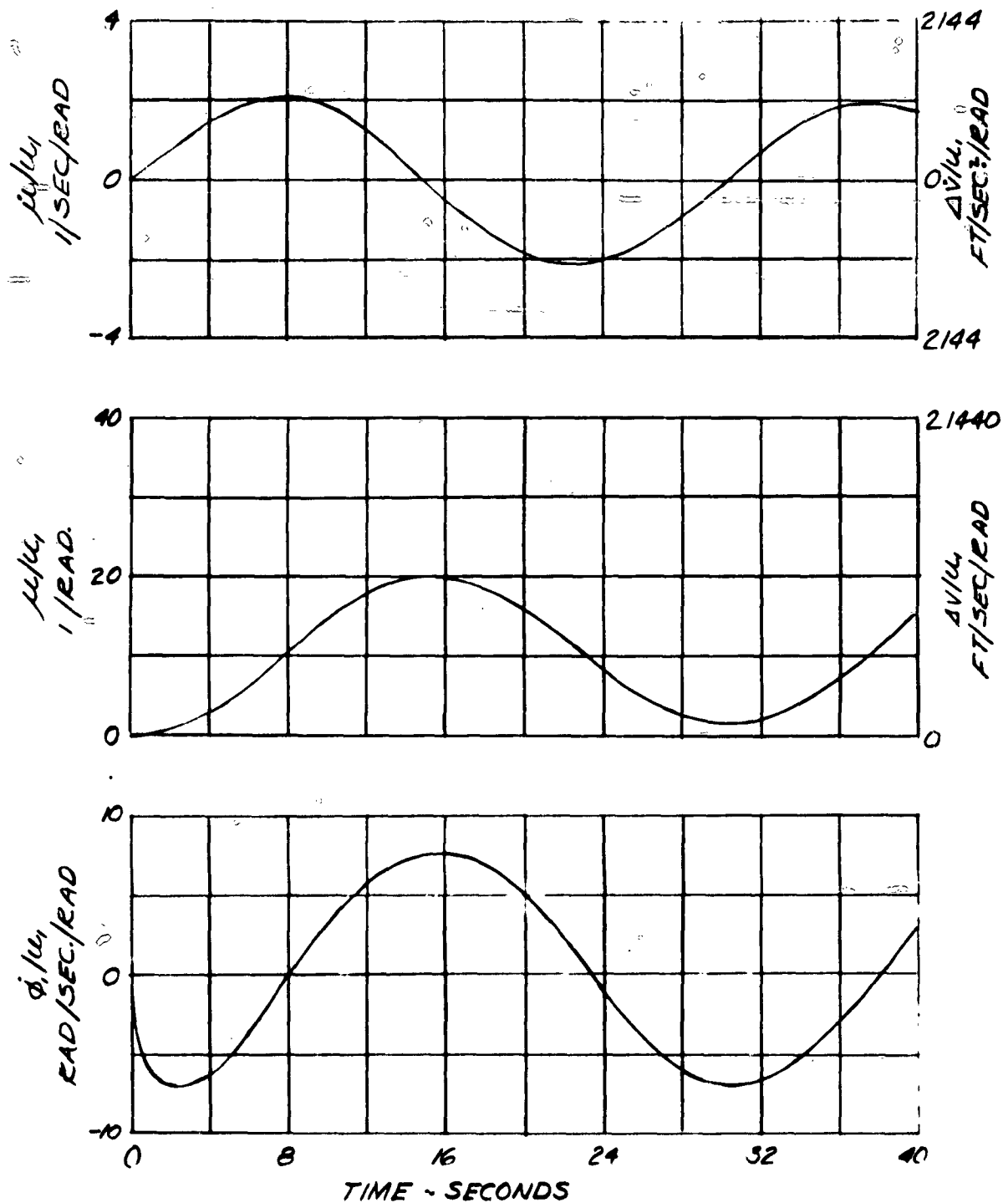


Fig. 4 Response of Tandem Helicopter to Step Input
Trim: 45 Knots Level Flight, Sea Level
No Rotor Interference

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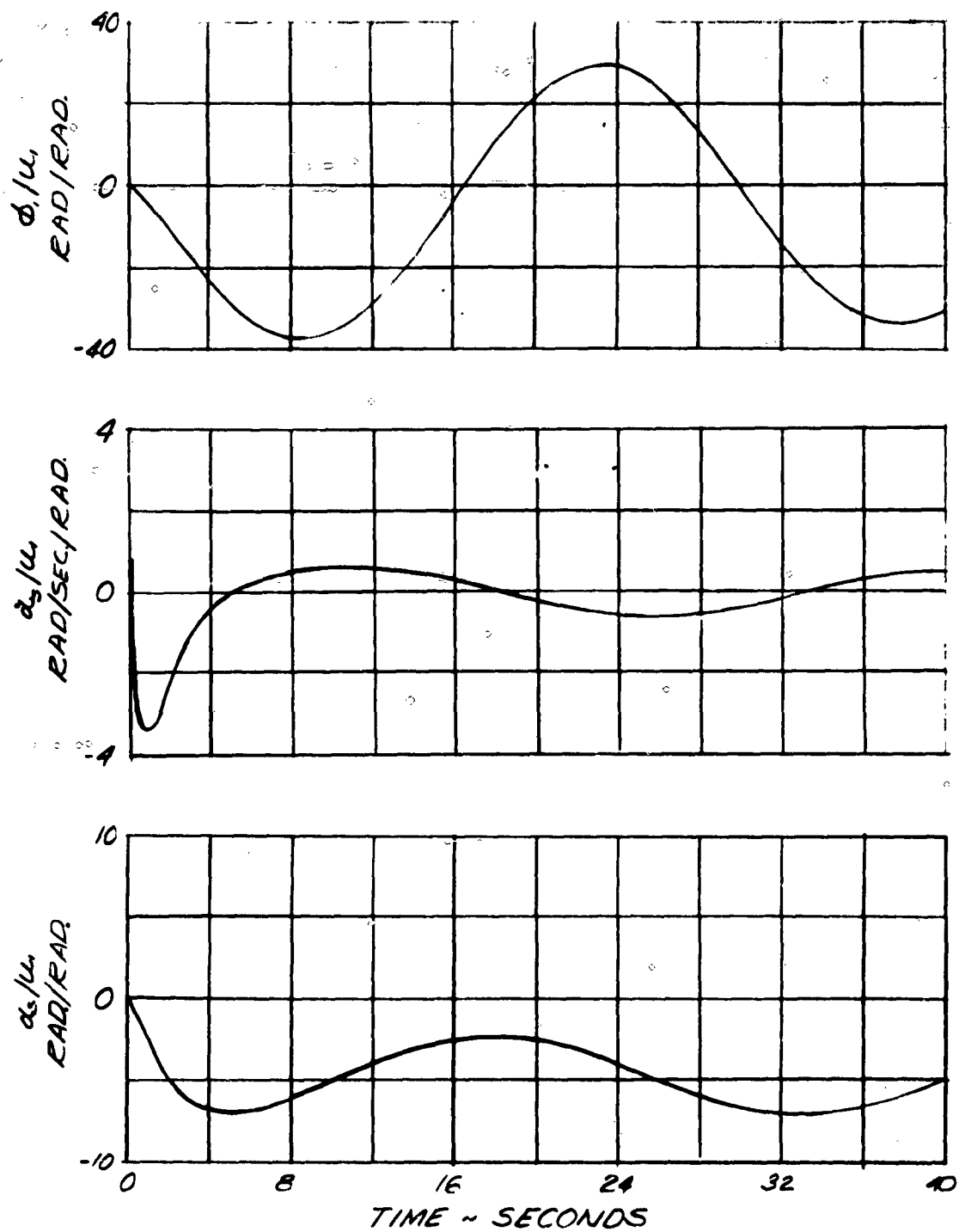


Fig. 4 (Cont'd.)

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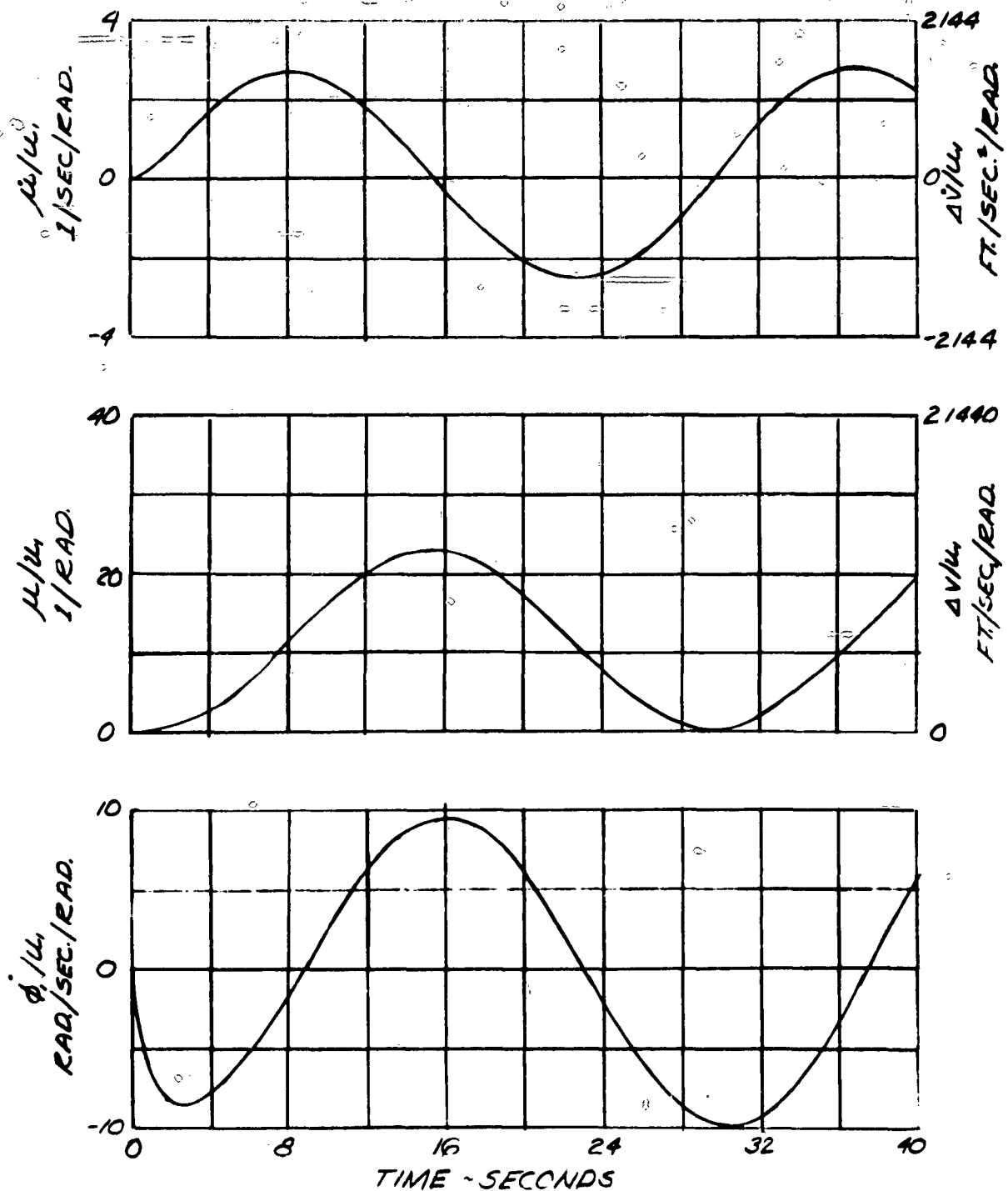


Fig. 5 Response of Tandem Helicopter to Step Input
Trim: 60 Knots Level Flight, Sea Level
No Rotor Interference

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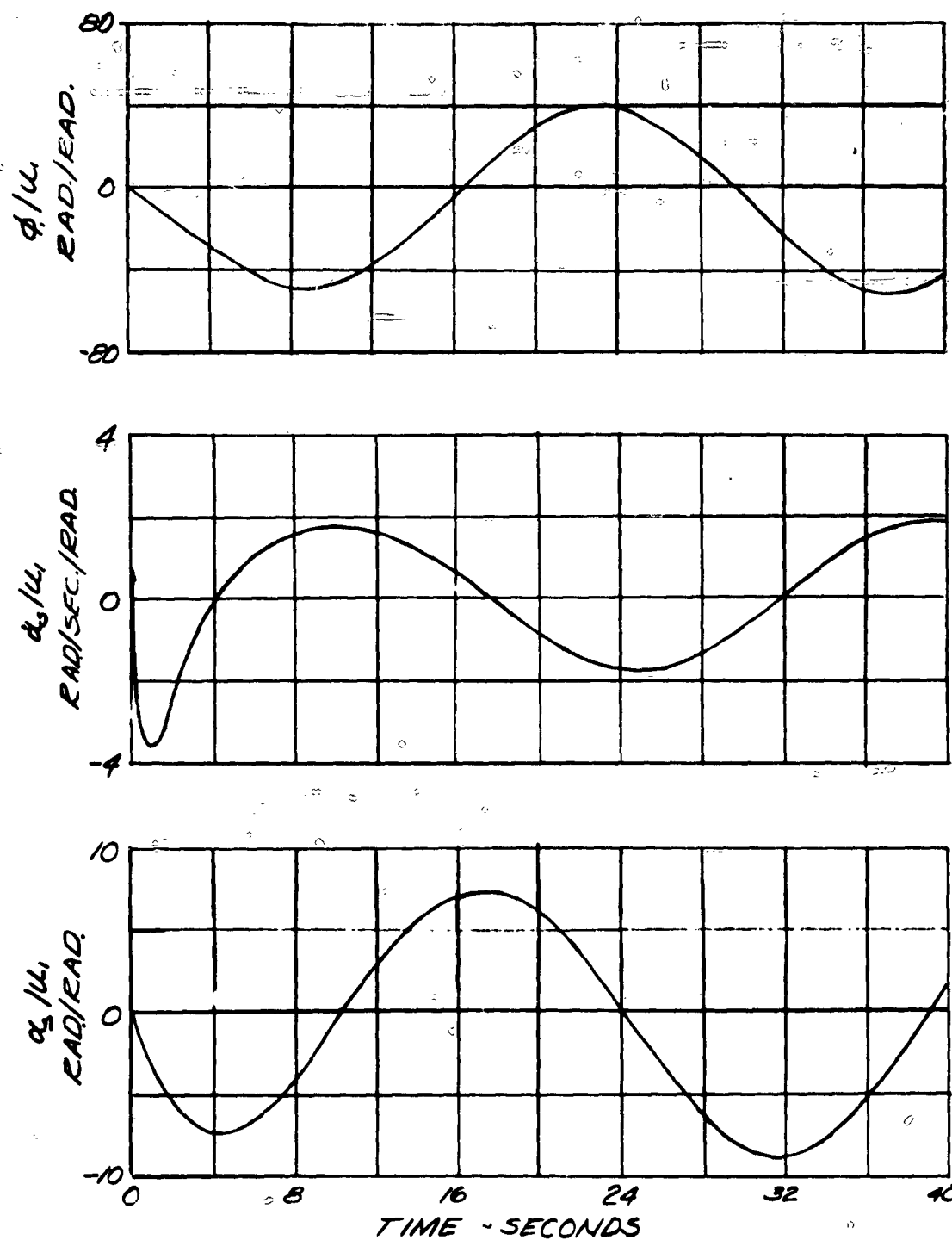


Fig. 5 (Cont'd.)

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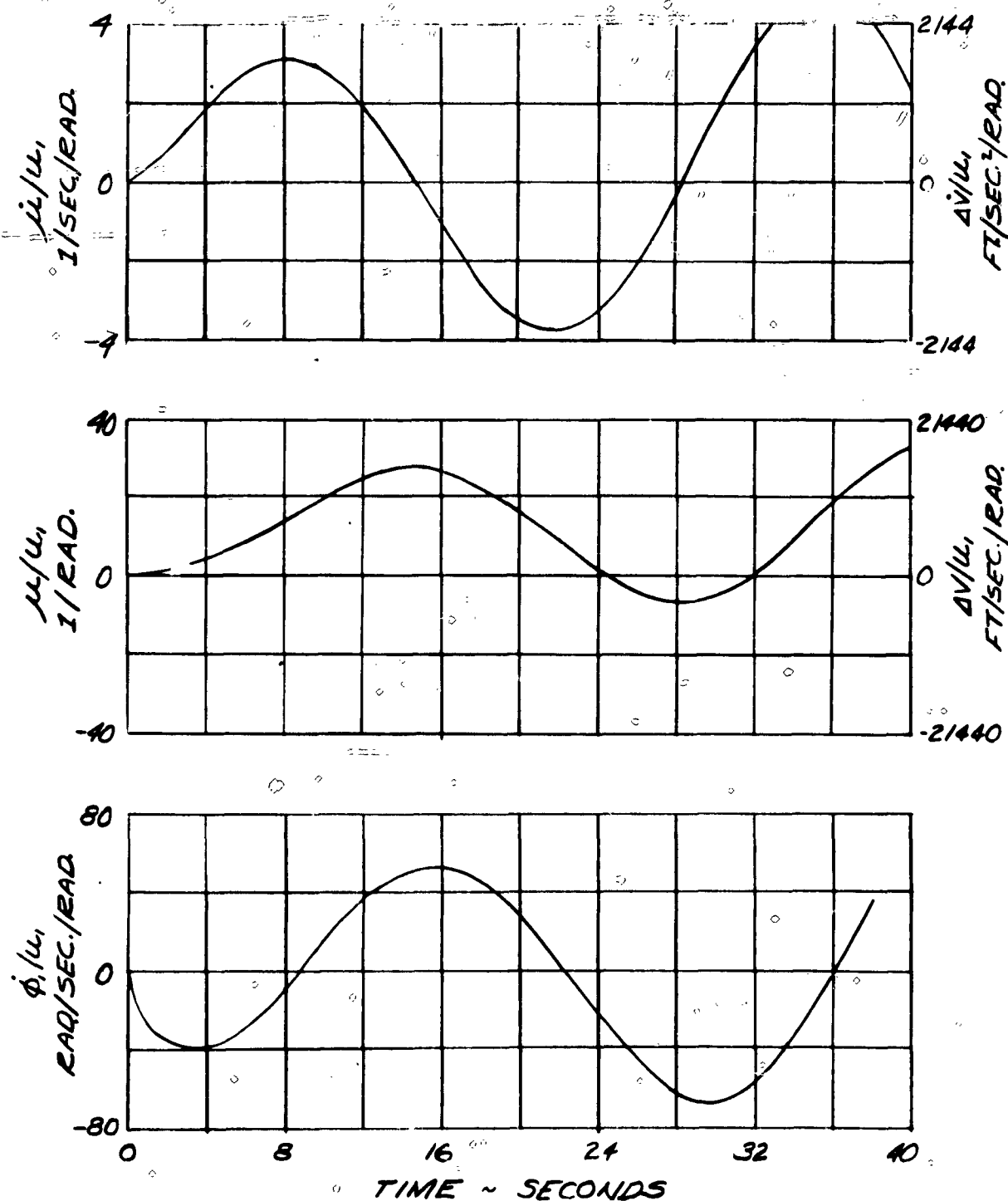


Fig. 6 Response of Tandem Helicopter to Step Input
Trim: 75 Knots Level Flight, Sea Level
No Rotor Interference

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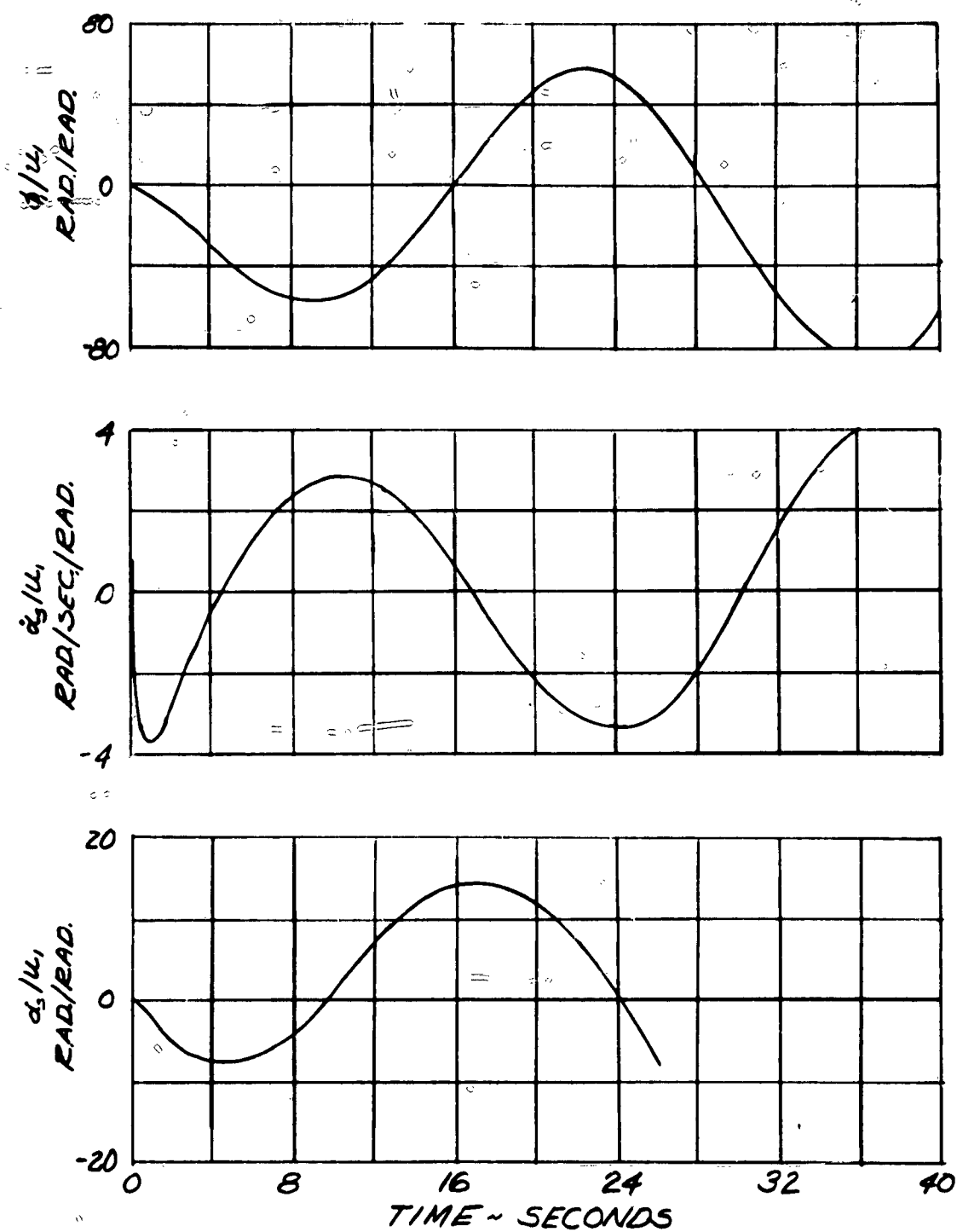


Fig. 6 (Cont'd.)

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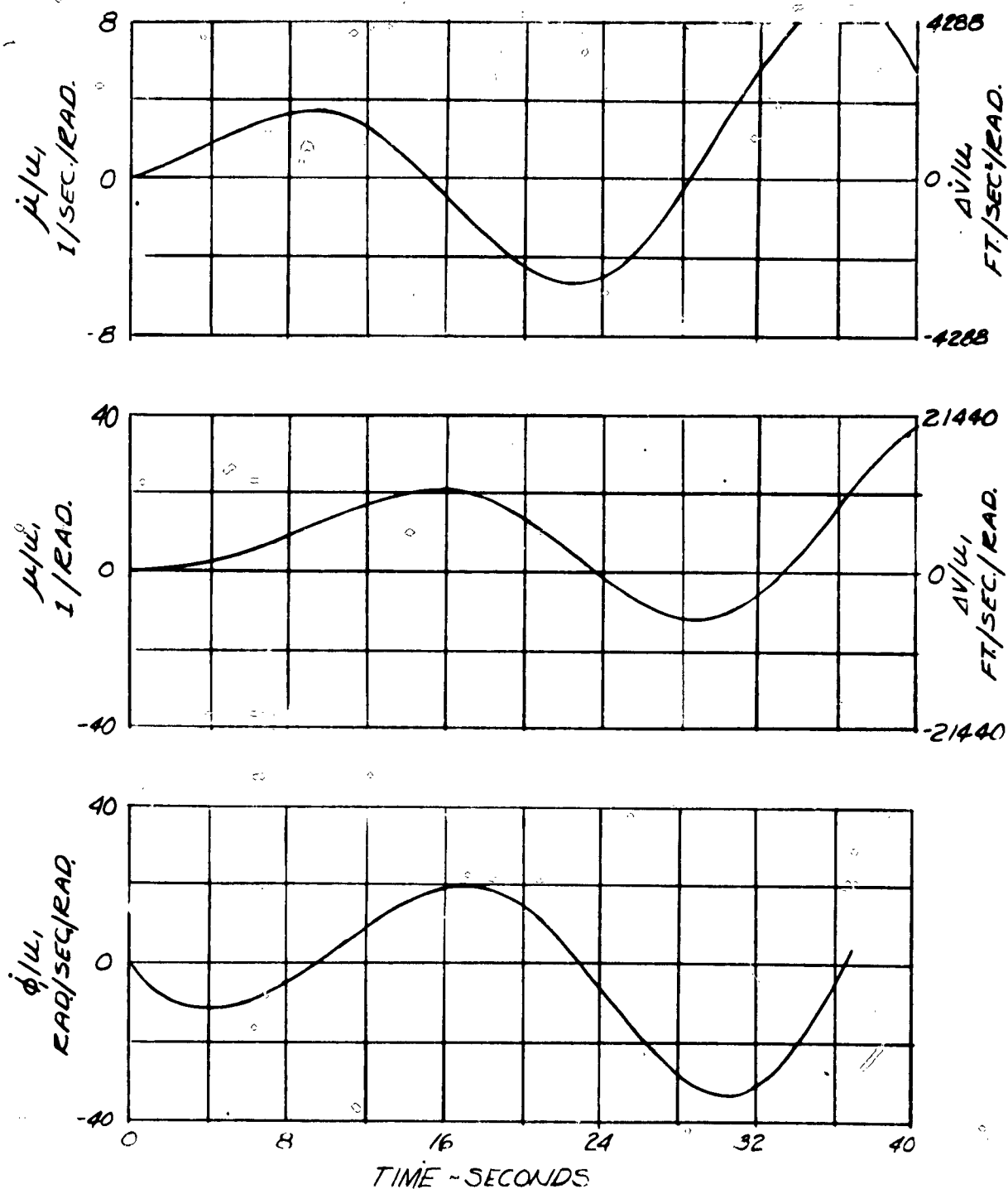


Fig. 7 Response of Tandem Helicopter to Step Input
Trim: 90 Knots Level Flight, Sea Level
No Rotor Interference

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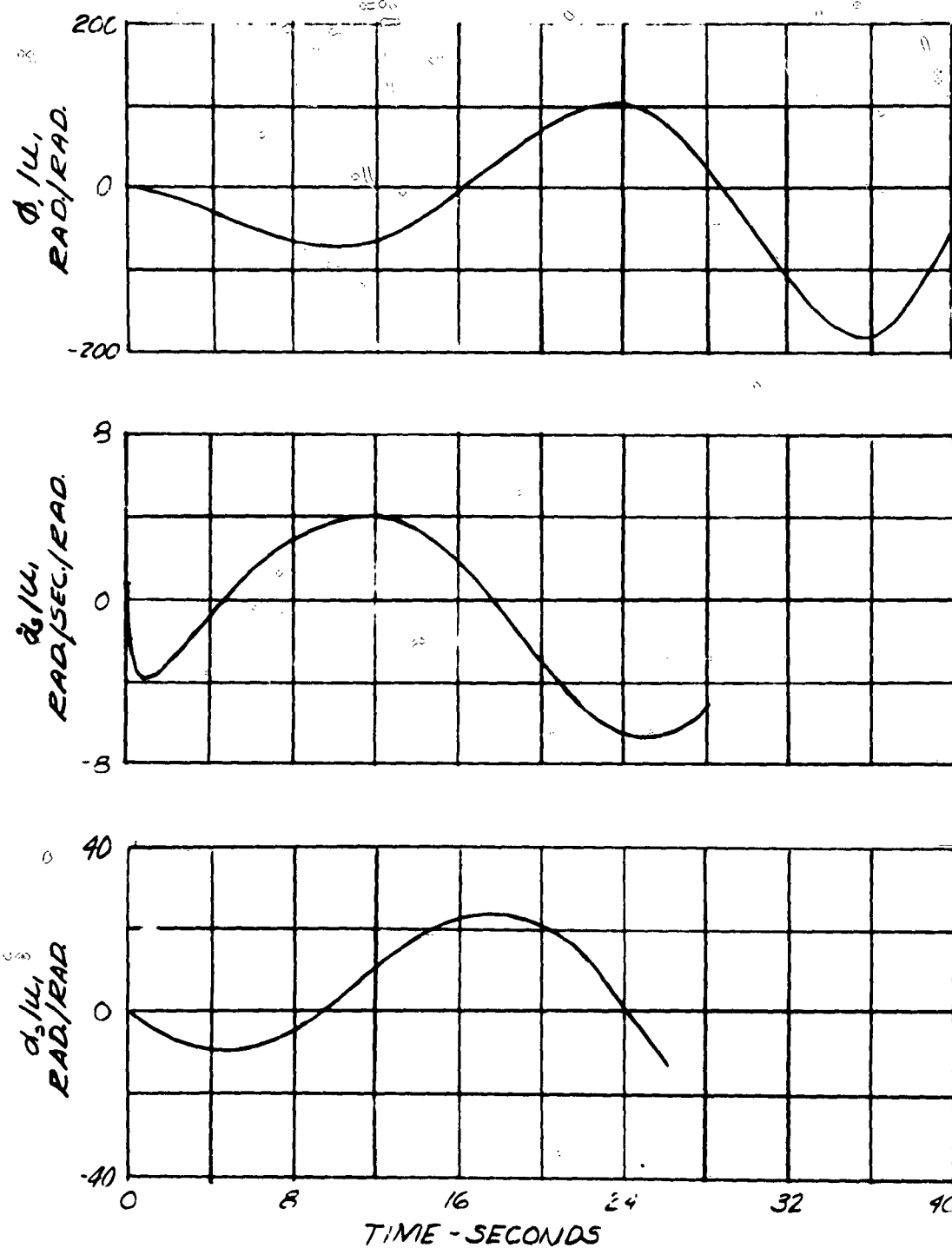


Fig. 7 (Cont'd.)

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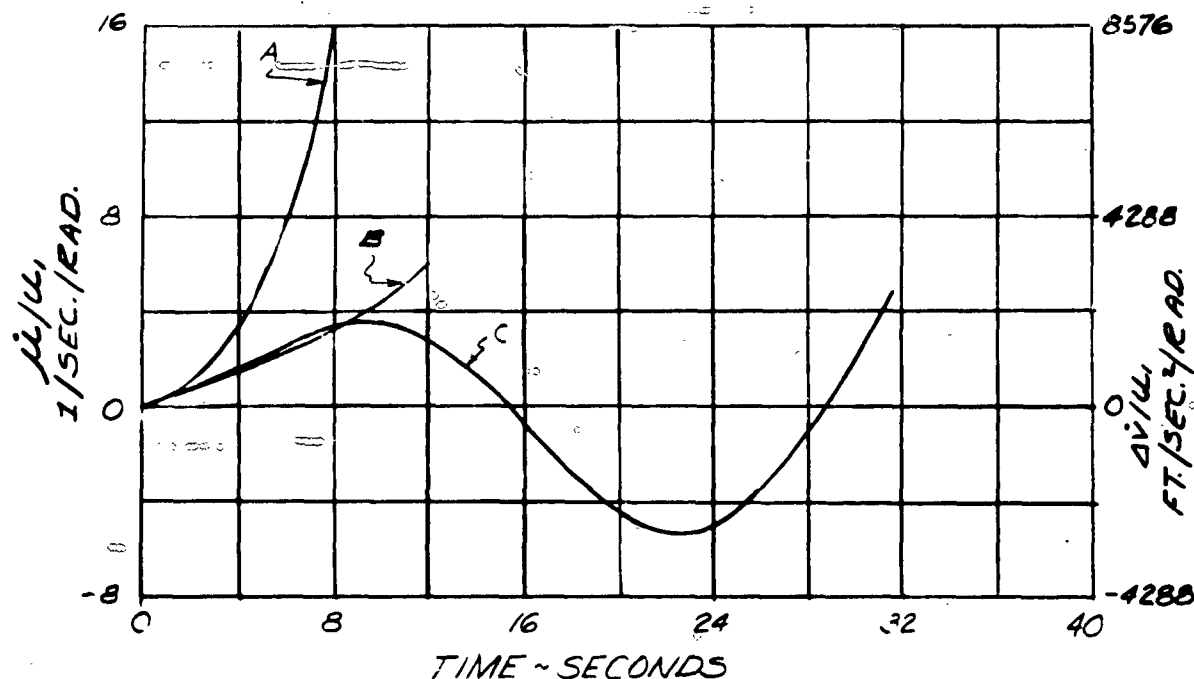


Fig. 8 Response of Tandem Helicopter to Step Input
Effect of Rotor Interference
Trim: 90 Kts. Level Flight, Sea Level
A-Rear rotor in wake of front rotor
B-Same as "A" but helicopter is equipped with a
differential δ_3 hinge, 35° on front rotor and 0°
on rear.
C-No Rotor Interference

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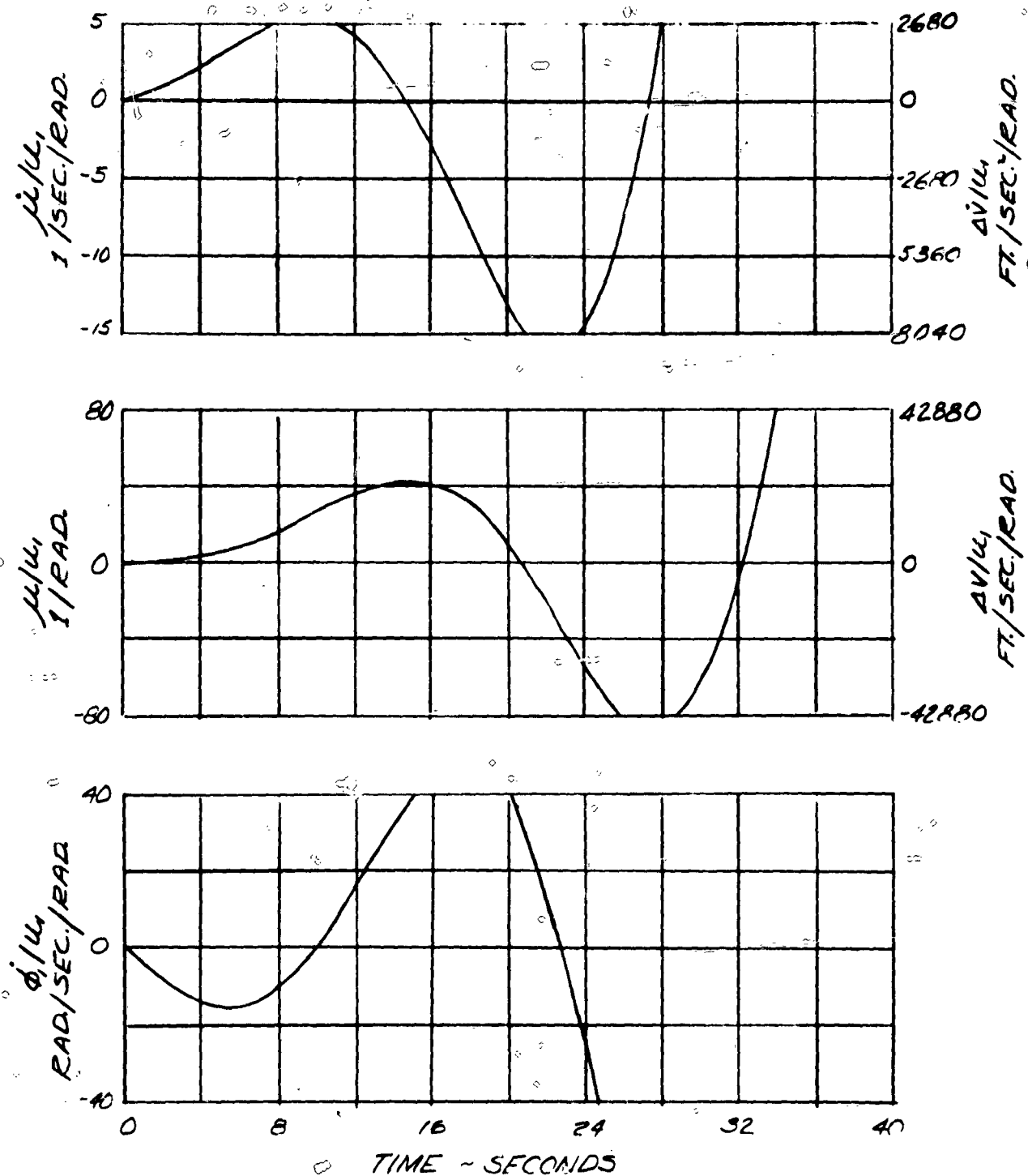


Fig. 9 Response of Tandem Helicopter to Step Input
Trim: 105 Knots Level Flight, Sea Level
No Rotor Interference

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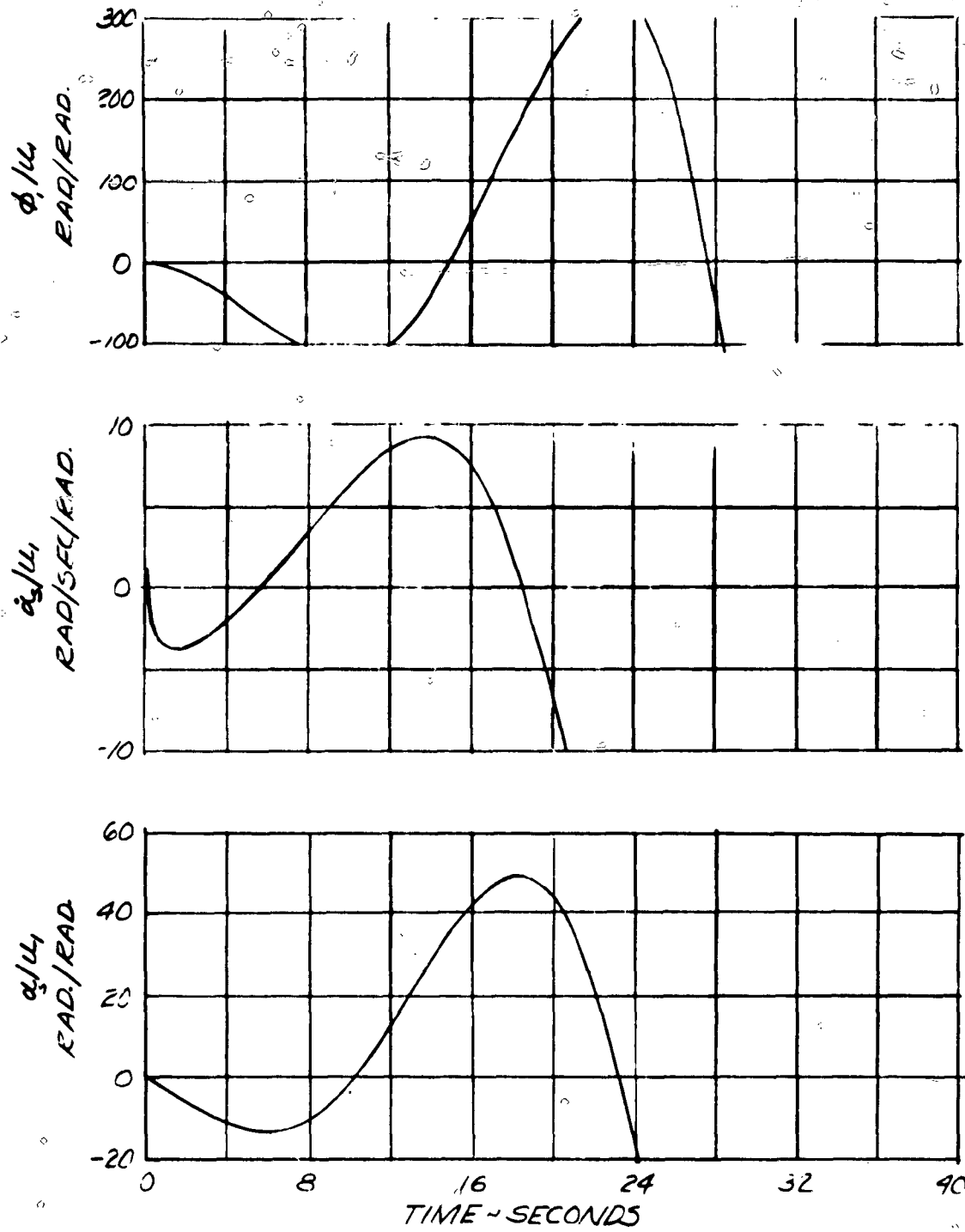


Fig. 9 (Cont'd.)

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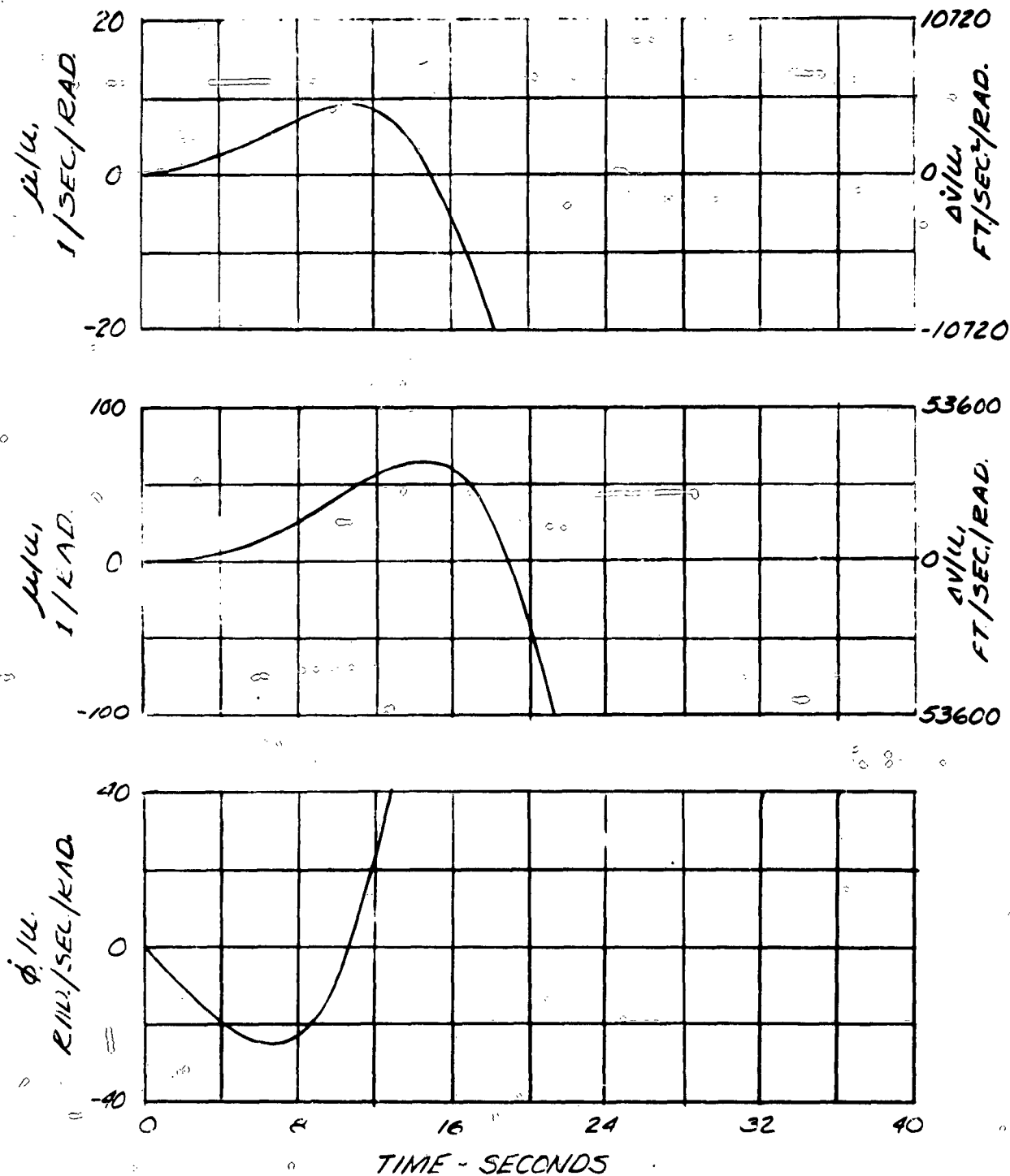


Fig. 10 Response of Tandem Helicopter to Step Input
Trim: 120 Knots Level Flight, Sea Level
No Rotor Interference

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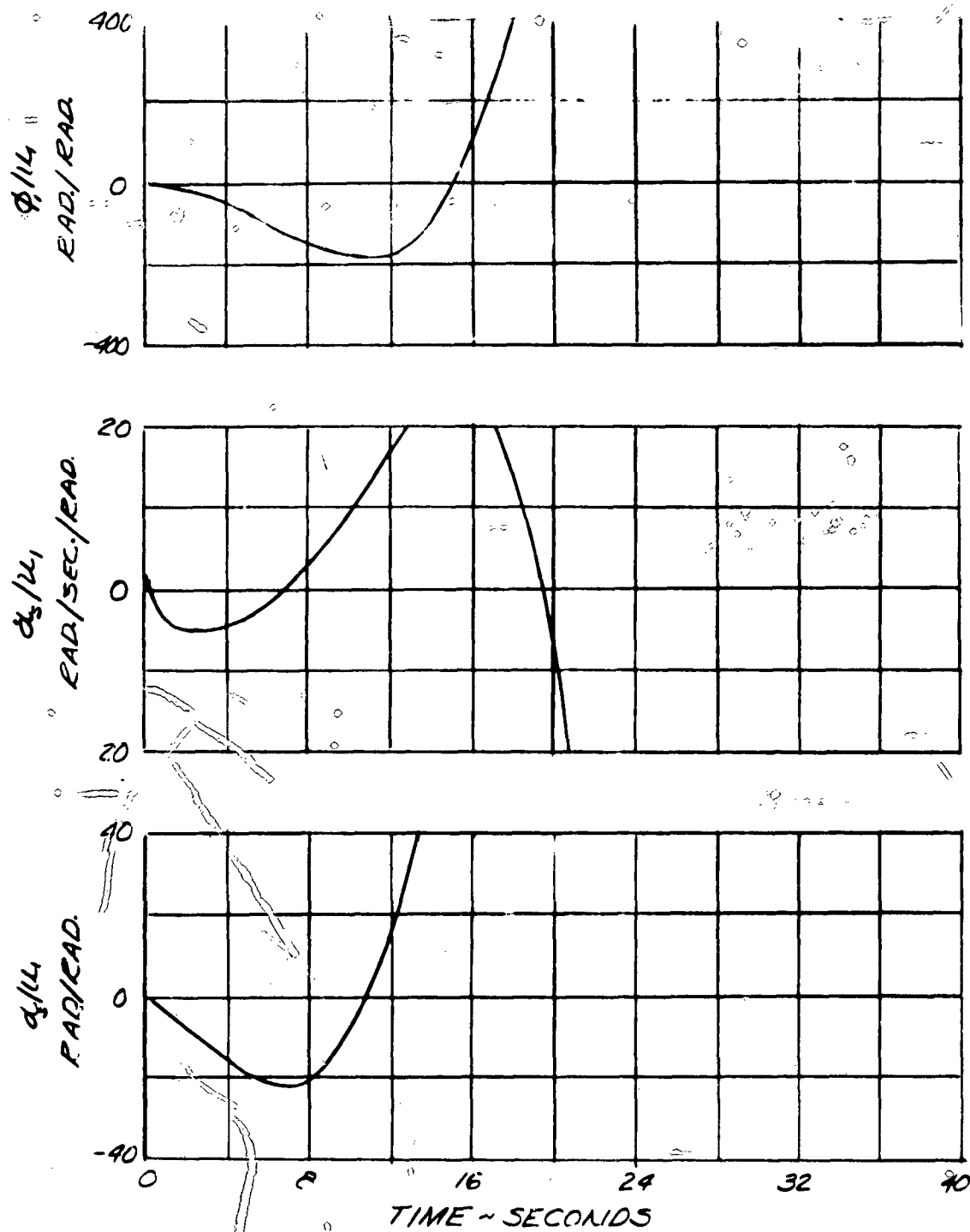


Fig. 10 (Cont'd.)

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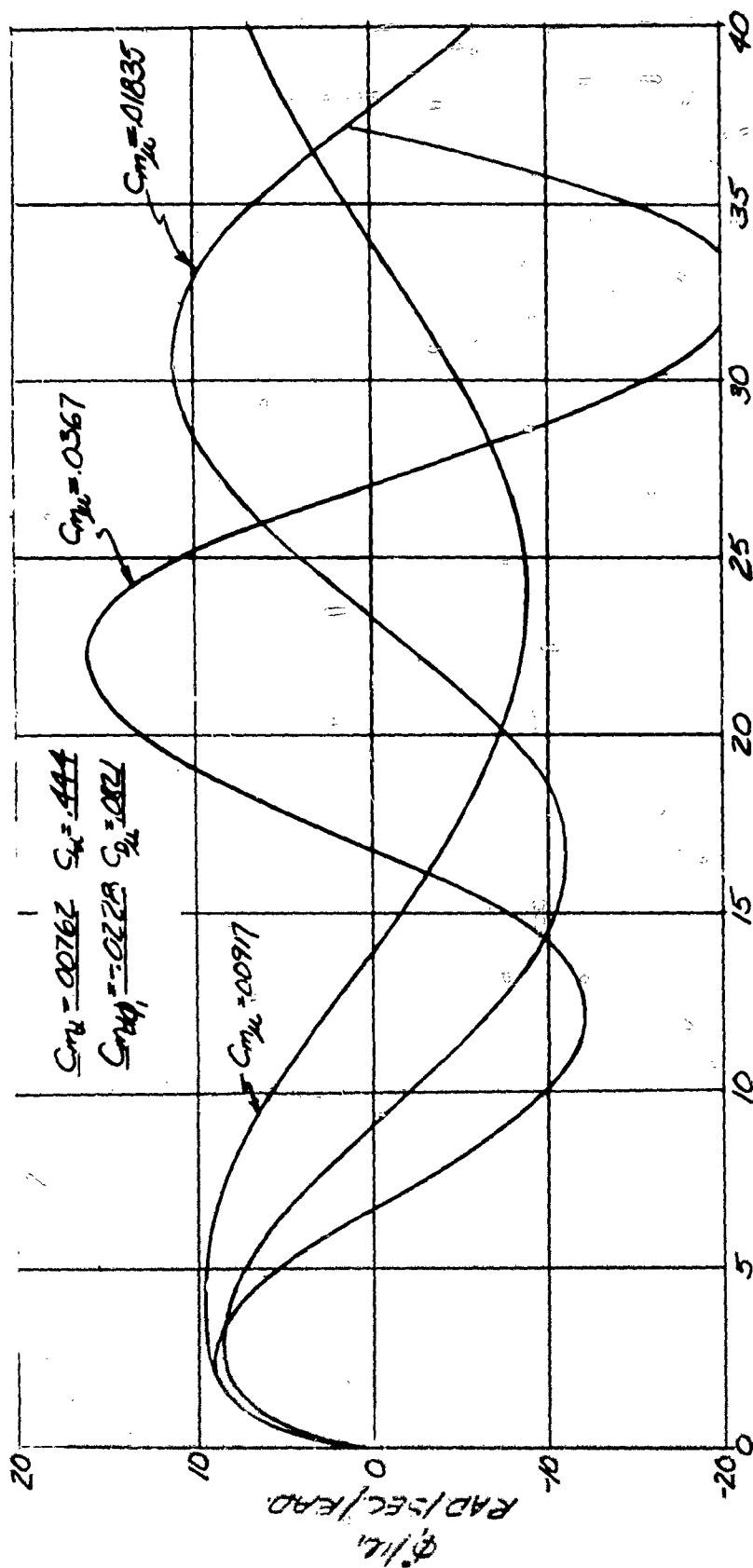


Fig. 11 Effect of Velocity Stability on Response
Trim: 60 Knots Level Flight, Sea Level

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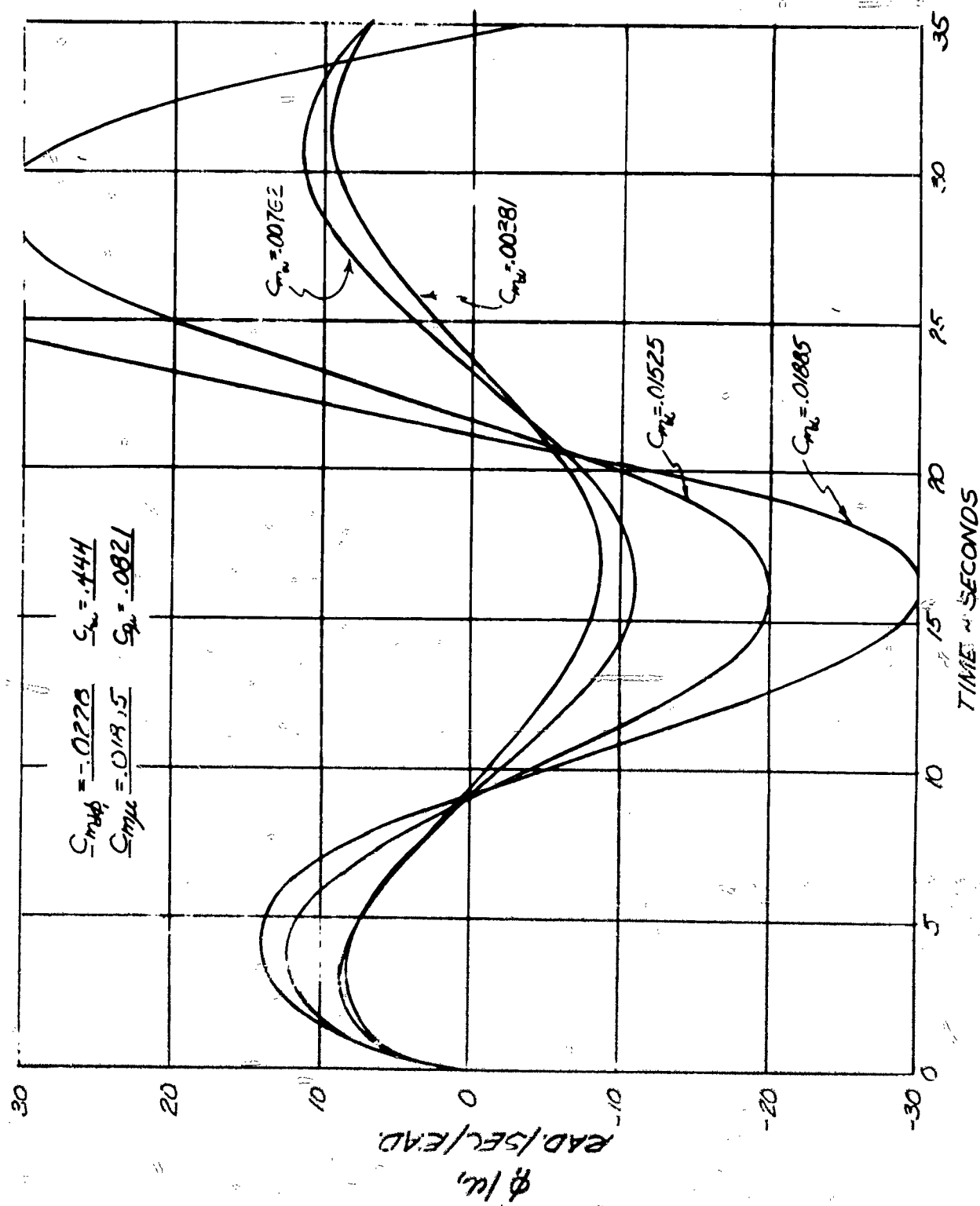


Fig. 12 Effect of Angle of Attack Stability on Response
Trim: 60 Knots Level Flight, Sea Level

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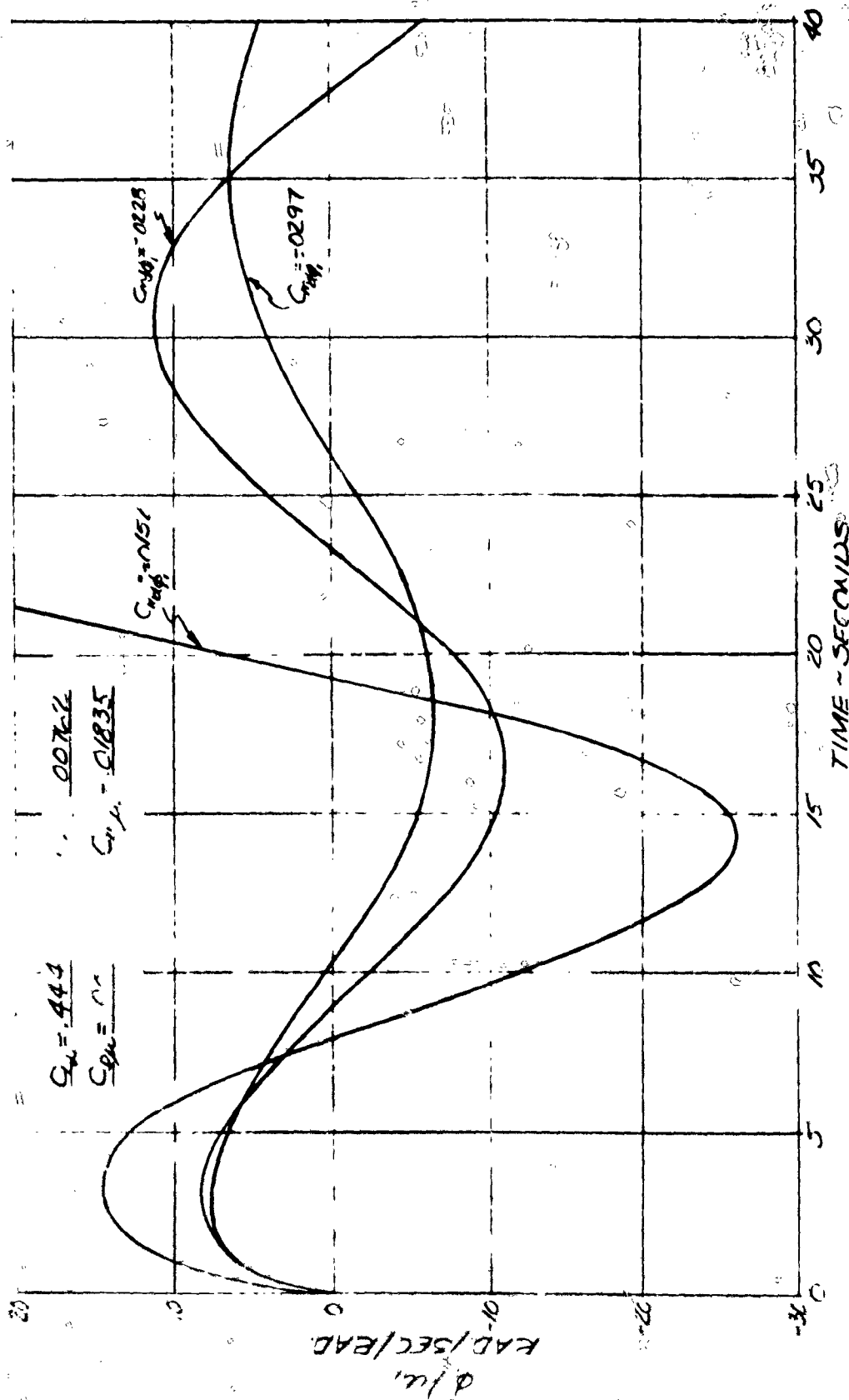


Fig. 13 Effect of Damping in Pitch on Response
Trim: 60 Knots Level Flight, Sea Level

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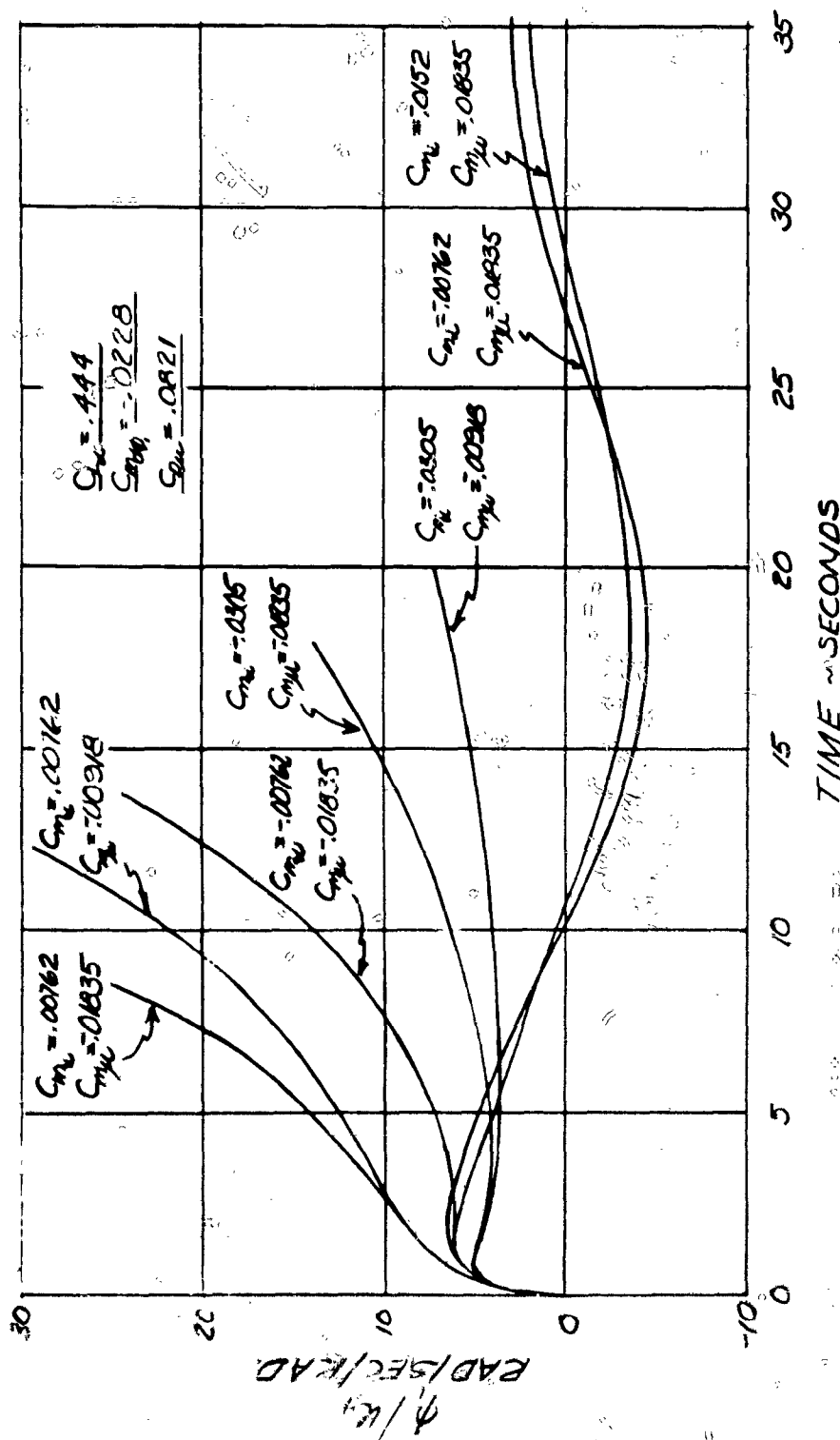


Fig. 14 Effect of Combinations of Velocity Stability and Angle of Attack Stability on Response
Trim: 60 Knots Level Flight, Sea Level

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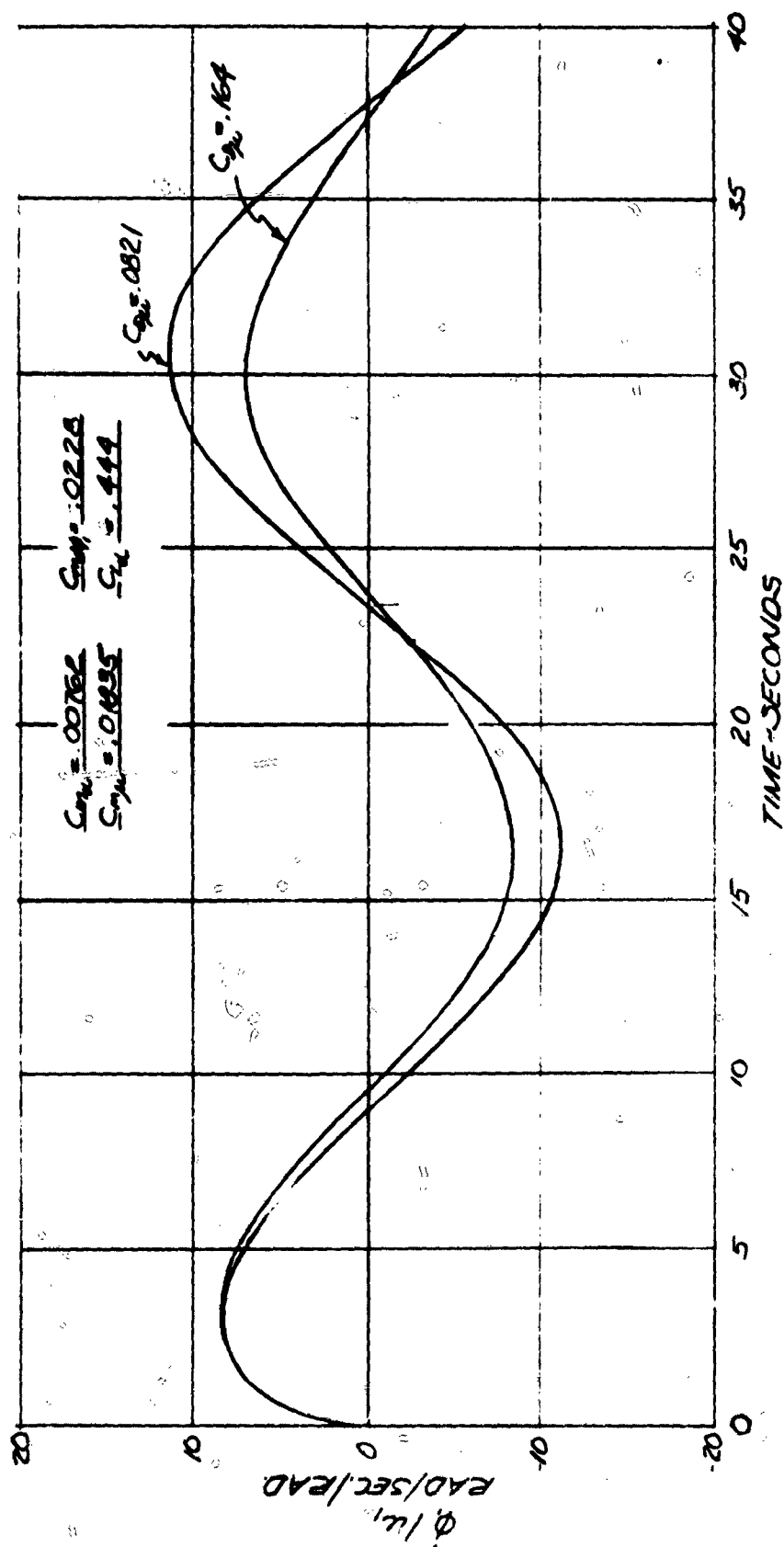


FIG. 15 Effect of C_D on Response
Trim: 60 Knots Level Flight, Sea Level

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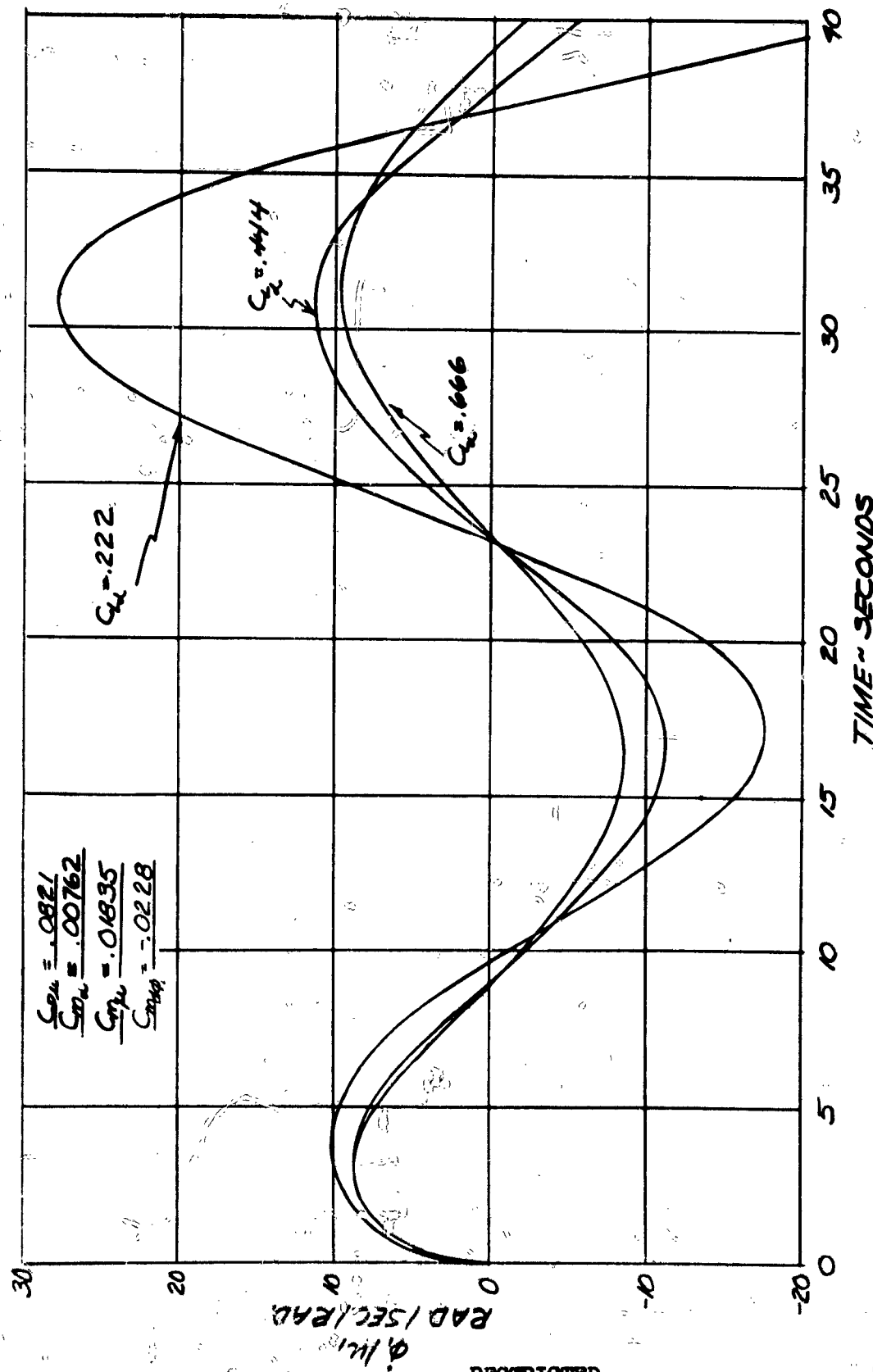


Fig. 16 Effect of Vertical Damping on Response
Trim: 60 Knots Level Flight, Sea Level

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