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HEAT BUDGET OF A WATER COLUMN AUTUMN—NORTH ATLANTIC OCEAN

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Project 29

HEAT BUDGET OF A WATER COLUMN

AUTUMN - NORTH ATLANTIC OCEAN

(Technical Report No. 4)

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TABLE OF CONTENTS

	Page
List of Tables	
List of Figures	
Abstract	1
Introduction	3
Heat Budget Study	12
Discussion of Components	
A_T : The Total Thermal Change	16
A_S : The Average Seasonal Change in Solar Radiation . . .	16
A_B : The Effective Back Radiation from the Ocean Surface	18
A_E : The Evaporation from the Ocean Surface	19
A_D : The Change of Thermocline Depth in Response to Atmospheric Influences . . .	23
A_{NET} : The Sum ($A_S + A_B + A_E + A_D$)	25
A_P : The Thermal Change Which Is Unexplained	25
A_T^1 : "Best-Fit" Total Thermal Change	27
A_P^1 : The Unexplained Thermal Change Resulting from A_T^1 . .	27
Discussion	
Net Amount of Explained Heat Changes	28
Relation of A_{NET} to A_T^1	29
Relation of A_{NET} , A_T^1 and A_P^1	30
Relation of A_{NET} to Atmospheric Change	31
Suggested Ocean-Atmosphere Relationships	32

TABLE OF CONTENTS (cont'd.)

Discussion (cont'd.)	Page
Summary of the Model	38
Seasonal Total Parameters	40
Conclusions	47
Indicated Future Investigations	50
Acknowledgments	51
References	58
Appendix	
A_T Measurement	i
Solar Radiation	v
Back Radiation	vi
Evaporation	vii
Modifying Jacobs' Formula	viii
Example of A_E Computation	ix
Comperison Between A_E Computations Using Jacobs' Formula and Using the Lake Hefner Formula	xi
Dynamic Effect	xii
Example of A_D Computation	xv

LIST OF TABLES

Table		Page
1	Energy Change Components	5
2	Evaporation Amount Estimates	22
3	Comparison of Lake Hefner Formula, Jacobs' Formula and Total A_T	45
4	Energy Balance Sheet for 1947	52
5	Energy Balance Sheet for 1948	53
6	Energy Balance Sheet for 1949	54
7	Computations of Evaporation Parameters	ix
8	Summing Smooth and Rough Wind Parameters	x
9	Computation of Vertical Advection Parameters	xvi

LIST OF FIGURES

Figure		Page
1	Change in Thermal Energy	5
2	Solar Radiation, A_s , at Station "C", Autumn	17
3	Autumn Season Average Temperature, at Station "C"	41
4	Five-Day Running Mean Temperature vs Depth Curves Centered on 1 September and 12 November, 1947 .	42
5	Five-Day Running Mean Temperature vs Depth Curves Centered on 6 September and 3 November 1948 .	43
6	Five-Day Running Mean Temperature vs Depth Curves Centered on 1 September and 29 November 1949 .	44
7	Time Distribution of A_{NET} and Components	55
8	1947 Time Distribution of A_{NET} , A_T and A_P	55
9	Time Distribution of A_{NET} , A_T and A_P	55
10	Time Distribution of A_{NET} and Components	56
11	1948 Time Distribution of A_{NET} , A_T and A_P	56
12	Time Distribution of A_{NET} , A_T and A_P	56
13	Time Distribution of A_{NET} and Components	57
14	1949 Time Distribution of A_{NET} , A_T and A_P	57
15	Time Distribution of A_{NET} , A_T and A_P	57
16	Five-Day Running Mean Temperature vs Depth Curves Centered on 2 and 7 November 1948	111
17	Atmospheric Pressure Gradient Measurement	xiv
18	Resolution of Atmospheric Pressure Gradient into Components	xiv
19	Surface Isobars on U.S.W.B. Five-Day Mean Northern Hemisphere Surface Chart for 4-8 September 1949	xv

ABSTRACT

Previous heat budget studies have not accounted for all physical processes adequately due to lack of data. The availability of new ocean and weather data made it desirable to reexamine the heat budget of an ocean water column over a short time interval, taking into account more completely than before the effects of all physical processes which can be computed. Such a study is reported here, with the view that a first step toward forecasting such changes is to determine which physical processes have important effects on the heat energy changes of an ocean water column. Such a column, fixed in space, includes a changeable water mass extending from the ocean surface to a level of no thermal change with time.

The effects of solar and back radiation, evaporation and vertical advection of the thermocline probably are important in determining trends of heat energy change in such an ocean column. North Atlantic Ocean Weather Ship "C" data were studied for the autumn seasons of 1947, 1948 and 1949.

The unexplained residual change behaves in a manner suggesting it may include effects of horizontal advection (not estimated here) as a large portion. The residual also includes all the error, effects of smoothing data and inaccuracies of estimating methods.

A superposition of successive weekly bathythermograph traces (BT's) indicates the trend of variation in the area between

the two BT's is related to the explained amount of thermal change in the ocean column. The superposing process represents subtraction of heat change occurring uniformly at all depths in the column. Hence such a subtraction could represent removal of a contribution to energy change from horizontal advection.

A weather-ocean interaction model is suggested by relationships between time variation of ocean parameters and five-day mean sea level pressure patterns for the autumn of 1948. Data from 1947 and 1949 suggest such patterns may persist for a season, but may vary markedly from year to year.

The heat gain of the ocean column for the 1948 autumn almost equalled the heat loss, thus achieving a near heat energy balance. This suggests again that advection of warmer water into the column has compensated for the heat losses due to decreasing solar and back radiation and increasing evaporation as the season progressed. For 1947 and 1949 no such balance was attained, however.

The suggested ocean-weather relationships should be studied with new data to establish possible forecasting techniques. Estimates of the importance of horizontal advection in this problem should be made as sufficient data becomes available.

INTRODUCTION

The heat budget for the earth's hydrosphere has been studied previously for limited intervals of the time scale. Sverdrup (1942), Jacobs (1951) and Cochrane (1950) made studies of seasonal and annual average conditions. Bretschneider (1952) and Schule (1953) have considered the heat budget of an ocean water column over time intervals of one week or less.

Sverdrup gives the heat balance for all ocean surfaces between 70°N and 70°S for average conditions, as computed by Mosby (1936); of the total incoming radiation from sea and sky, 53% is used for evaporation, 41% goes back to the atmosphere as long-wave radiation from the sea surface, and 6% is conducted back to the atmosphere as sensible heat. It is pointed out that for specific regions over short time intervals account must be made of both the heat transported into or out of the region by currents or mixing and the heat used locally to change the temperature of the water.

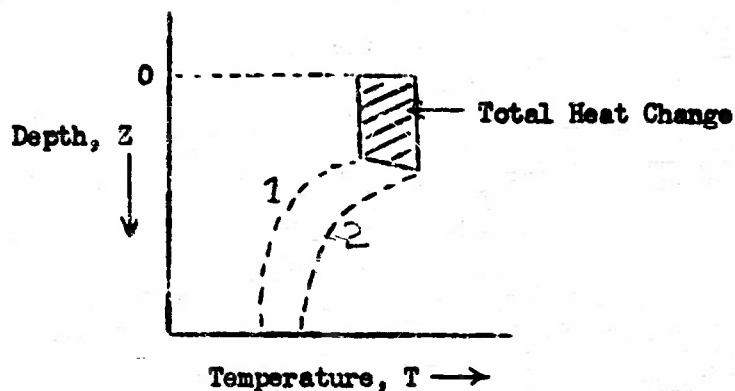
Jacobs examined the energy exchange between ocean and atmosphere due to sensible heat and evaporation. Computations were made for the northern hemisphere oceanic regions on a seasonal and annual basis. Consideration was not made of either the effect of long-wave radiation from the sea surface, or the exchange between ocean heat energy and kinetic energy of current, wave or tidal motions. Large variations from the seasonal or annual average amounts are to be expected on an instantaneous basis, according to Jacobs. Investigation of such brief non-periodic fluctuations in energy exchange needs to be emphasized in future work.

On a smaller scale, both in time and space, Cochrane studied the heat budget of a water column located in the North Pacific Ocean (30° N, 140° W). A nearly balanced budget resulted for the column over a time interval of three to four months. Cochrane found that horizontal advection affecting the water column can be a significant part of the heat budget below a depth of 200 feet during the months of June through October.

Using a limited amount of data, Bretschneider made a study of daily heat balance for a water column in the North Atlantic Ocean. The changes in structure from day to day were measured in a hypothetical water column of unit cross-section, extending from the surface of the ocean throughout the depth of the mixed layer. Such measurements were necessarily of a "local" nature, measuring energy changes of random water particles as they moved through the water column. Consequently, the thermal considerations involved advective effects.

The basic procedure which Bretschneider employed will be of interest in the present study. Bretschneider regarded total energy change within the mixed layer as the sum of components, each of which is independently contributed by a particular physical process. He divided total energy change into components contributed by radiation, evaporation, lateral advection and other physical processes as shown in Table 1.

Plots of temperature versus depth were made for a particular time in the ocean by the bathythermograph (BT). On a single graph,



Bretschneider's Method of
Computing Total Heat Change

FIGURE 1

TABLE 1

Energy Changes from BT_1 to BT_2

TOTAL	= 270 Ft-Deg. F*
RADIATION	= 10 Ft-Deg. F
EVAPORATION	= -60 Ft-Deg. F
ADVECTION	= 20 Ft-Deg. F
OTHER	= 300 Ft-Deg. F
	270 Ft-Deg. F

one may then plot BT traces for the same location, but at two different times, and obtain the area enclosed between the two curves (see Figure 1). This area represents the total change in thermal energy within

* See Appendix for discussion of unit "Ft-Deg. F".

the water present above the thermocline in a vertical column at a geographical location during a day.

For the same time interval, Bretschneider devised methods for computing the contributions to the total energy change due to radiation, evaporation and lateral advection. On the basis of these observations and computations a heat budget balance was sought for the water column over time intervals of one day. However, it appeared no balance was achieved between the contributions of the processes Bretschneider studied for one-day intervals during that particular autumn. This does not preclude consideration for longer periods of time being successful, especially when accounting for contributions of physical factors which Bretschneider did not include.

Schule has considered the change in heat content and the redistribution of heat within the top 100 feet of a water column, over short time intervals. Actual radiation measurements were used. The evaporation and molecular conduction exchange with the atmosphere were estimated through use of a single relationship developed by Jacobs in 1942, depending on the windspeed and on differences between air and ocean in temperature and vapor pressure. The report included estimates of internal waves at the thermocline. A successful trial forecast of redistribution of heat energy change within the column was made for a week in advance, assuming perfect forecasts of meteorological parameters affecting the thermal structure.

Each of these contributions is important in the basic understanding of ocean energy relations, and has provided information within its particular sphere of oceanic region and time interval. Many

questions remain to be answered, however, concerning the behavior of the ocean in the general aspects of both space and time. A discussion of a few of the unanswered questions will illustrate the complexity of the ocean heat balance problem.

The water column under discussion is not a fixed mass of water. The particles of water are in constant motion so that they are piled vertically for only an instant within the chosen fixed boundaries. Then, in response to the unique history of forces affecting it, each particle moves away in its own direction to become a part of the mass in other water columns at later times.

The heat energy changes within such a water column are influenced by several factors. When the thermocline becomes shallow, a large portion of the column contains cold water; thus the warm mixed layer contributes relatively little to the heat energy of the column. As the thermocline deepens, the warm water of the mixed layer plays a more prominent role in the total heat energy of the water column. In the first example, the heat energy of the water column is low; in the second example, it is relatively high.

The effects of solar radiation, back radiation, evaporation, conduction and precipitation are imposed through the top of the water column. Other factors which affect the heat energy of the column include conduction and advection of energy laterally through the water column sides.

In this moving mass of water, affected by so many outside influences, it may be surprising if one can attain any heat energy balance that can be measured. Whether or not such a balance can be

attained depends largely on how accurately the effects of outside influences can be estimated. The space and time variation of these influences, and the interdependence among them are important considerations in this regard. If the space or time variation of an outside influence is small, then estimates obtained from data samples should approximate average conditions. When the influences are related to one another it is possible that an internal smoothing of effects occurs so that the net resulting influence on the heat budget of a water column varies smoothly within a small range of values.

Such a smooth variation would materially aid in forecasting the effects of such outside influences on the heat content of the water column, eventually leading to forecasting time changes in the thermal structure of the column.

It remains to be determined whether previous heat budget studies of the ocean have provided evidence concerning variation and interdependence of outside influences which affect the ocean heat energy.

The basic question before us may be stated:

What is the dominant influence (or influences) which affects the ocean heat energy changes at a fixed location over a short time interval?

Once this is answered, the problem of forecasting changes in the influence, and hence in the ocean heat energy, may be taken up. If more than one influence appears to be important, knowledge

of relationships between the influences may aid the forecasting aspects of the problem.

When one examines the studies above for an answer to the basic question, it appears that much work remains to be done in this regard. Sverdrup and Jacobs used the conservation of energy principle in computing energy budgets for the oceans of the northern hemisphere on an annual basis. This in turn gives a check on seasonal contributions for total oceanic areas. Such data, so well smoothed in time and including all oceanic space within a hemisphere, should indeed give a balanced heat budget annually, with the possible exception of that portion advected into the region from the southern hemisphere. Small accelerations and time lags which could be important terms in short term energy balance considerations are smoothed to become negligible quantities in annual considerations. Hence, such annual heat budget considerations do not answer the question posed by short term energy considerations.

Cochrane has chosen an area for study which apparently lies within a horizontally homogeneous water mass. Thus advective effects on a water column are minimized, and the question remains unanswered concerning a balanced heat budget within regions influenced by advection.

Schule has been concerned with the redistribution of heat within the upper portions of a water column, which is vital of course in direct forecasting applications. The balance of a heat budget for the water column was not directly attacked in this study.

Bretschneider assumed a balanced budget existed within the mixed layer of a water column over a daily interval, and set about to evaluate the contributions of physical processes related to the heat content of the column. Each contribution was estimated from actual data and linear addition provided an explained sum of heat added or subtracted from the water column each day. The remainder was evaluated between this explained amount and the amount of change actually observed, and the remainder was often of the same order as the total heat change within the mixed layer. Processes other than those Bretschneider considered appear to be capable of affecting the heat content of a water column, such as vertical advection. In addition, heat changes do occur below the mixed layer depth (which is usually well above the depth of no appreciable change with time) and these were neglected by Bretschneider.

None of the studies mentioned adequately determines the influence having the most effect on ocean heat energy when dealing with short time intervals and limited oceanic regions. To find that dominant influence (or influences), an investigation seemed warranted to consider the heat balance of a total column of ocean water for periods of a week or less. This investigation should be made from new and comprehensive data and should include the effects of all known physical processes which may be estimated using the best techniques available.

The authors consequently have undertaken such a study, using some modifications of techniques suggested in earlier studies, as well as some techniques which are newly developed and not previously used in this manner. An extensive body of recent data has been investigated.

HEAT BUDGET STUDY

The test station was chosen to be the one having available the largest amount of simultaneous meteorological and oceanographic data. This station is Weather Ship "C" located at 52°45' North Latitude and 35°30' West Longitude in the North Atlantic Ocean. Data records are available for most of the period 1947-49.

The study of the water column was made to include all appreciable thermal change with time; the study thus extended to a depth of about 200 meters. In contrast to Bretschneider's technique, fluctuations of the thermocline here affect the heat budget of the total column. The interval between observed thermal structures was chosen as seven days. Five-day moving average traces were used to obtain curves having "conservative" characteristics through elimination of the apparently random fluctuations occurring in very short time intervals. Since the ocean thermal structure apparently possesses relatively few anomalous features during the autumn season at middle latitudes compared with other seasons of the year, this study was confined to the autumn season.

It is realized that boundary influences (wind, sunshine, atmospheric pressure, etc.) which affect the heat content of a water column are dependent upon one another, possibly to the extent that computations would never account for such dependence. However, the resulting physical processes (i.e., advection, conduction, radiation, etc.) make independent contributions to the water column heat change

Adopting a technique similar to that used by Bretschneider, the effects of the known physical processes which can be evaluated were summed for each week. Then a residual unexplained heat change was evaluated by finding the difference between actual observed heat change and the amount explained by known physical processes. This residual represents the effects of all unevaluated physical processes, accelerations, the summed error in all evaluations and the effects which enter from the nature of the computing process. In particular, smoothing observed BT curves introduces effects from boundary influences which occur outside the period used in evaluating effects of physical processes. While the latter enters into the residual amount, there is little evidence at present to indicate that sizable error is being introduced through this smoothing process. With the reservation that the effect of smoothing deserves additional attention at a later time, it has been assumed negligible for the purposes of this study. An examination of the behavior of this residual amount may offer suggested answers to the question posed above.

The focus of such data analysis should be toward usefulness in forecasting the thermal structure of the ocean. Whether or not the ocean attains a heat balance which can be computed is information useful in short range ocean forecasting problems. Any inferred relationships between the boundary influences likewise is useful. Any new information regarding the manner in which the ocean functions, or new relationships between the parameters which may be detected will aid in this objective.

For the heat budget study, a relationship was used of the form:

$$A_T = A_S + A_B + A_E + A_D + A_P.$$

A_T is the total heat change in the water column during the selected time interval for this study, one week.

A_S is the heating in the column due to solar radiation, computed on an average basis.

A_B is back radiation from the top surface of the water column to the atmosphere and space.

A_E is the cooling of the column due to evaporation of water from the column's top surface.

A_D is dynamic adjustment of the thermocline in response to changes in atmospheric wind shear above the column (a function of air pressure distribution).

A_P is the unexplained residual amount of heat change. Sample computations of each of these components are given in the Appendix.

There are other factors which were not considered specifically in previous heat budget estimates which deserve mention. The addition of water to the surface of the column as precipitation changes the total mass of the column, and the average precipitation for this station is about 40 inches per year (see Plate IX Haurwitz and Austin). As a first approximation assume half of this amount falls in the autumn season; then 20/90 inch (or 0.5 cm) per day would fall; for seven days this would amount to about 4 cm. If this added precipitation is at

20°C, then 80 gm-cal./week would be added, distributed over about 200 feet of water in the mixed layer. Then approximately 0.8 ft-deg. F would be added per week under these maximum conditions. Compared with solar radiation amounts of 50 to 160 ft-deg. F and back radiation amounts of 14 to 33 ft-deg. F, the amount of heat energy transferred by precipitation is negligible.

Conduction of heat from the ocean surface to the atmosphere was assumed to be 6% of the total amount on a yearly basis by Mosby (see above). Bowen Ratio estimates for the ocean are about 0.10 on the average (Sverdrup, 1942, quotes Angstrom, 1920), indicating that energy conducted to the atmosphere represents about 10% of the energy used in the form of evaporation from the ocean surface. Such an estimate then would give 3 to 25 ft-deg. F per week. Thus heat conduction from the ocean surface would also be a negligible amount in comparison with the other large amounts involved. Energy amounts due to precipitation and conduction from the ocean surface probably lie within the magnitude of error involved in estimating the energy amounts resulting from other processes.

Non-advective lateral transfer of energy from the water column into adjacent water cannot be estimated without a dense network of data. Lacking these data, such energy transfer will be included in the residual, A_p , the energy change unexplained by the various physical processes. In addition to these contributions, the residual term will also include the effects of horizontal advection of heat energy, which cannot be satisfactorily estimated with present data.

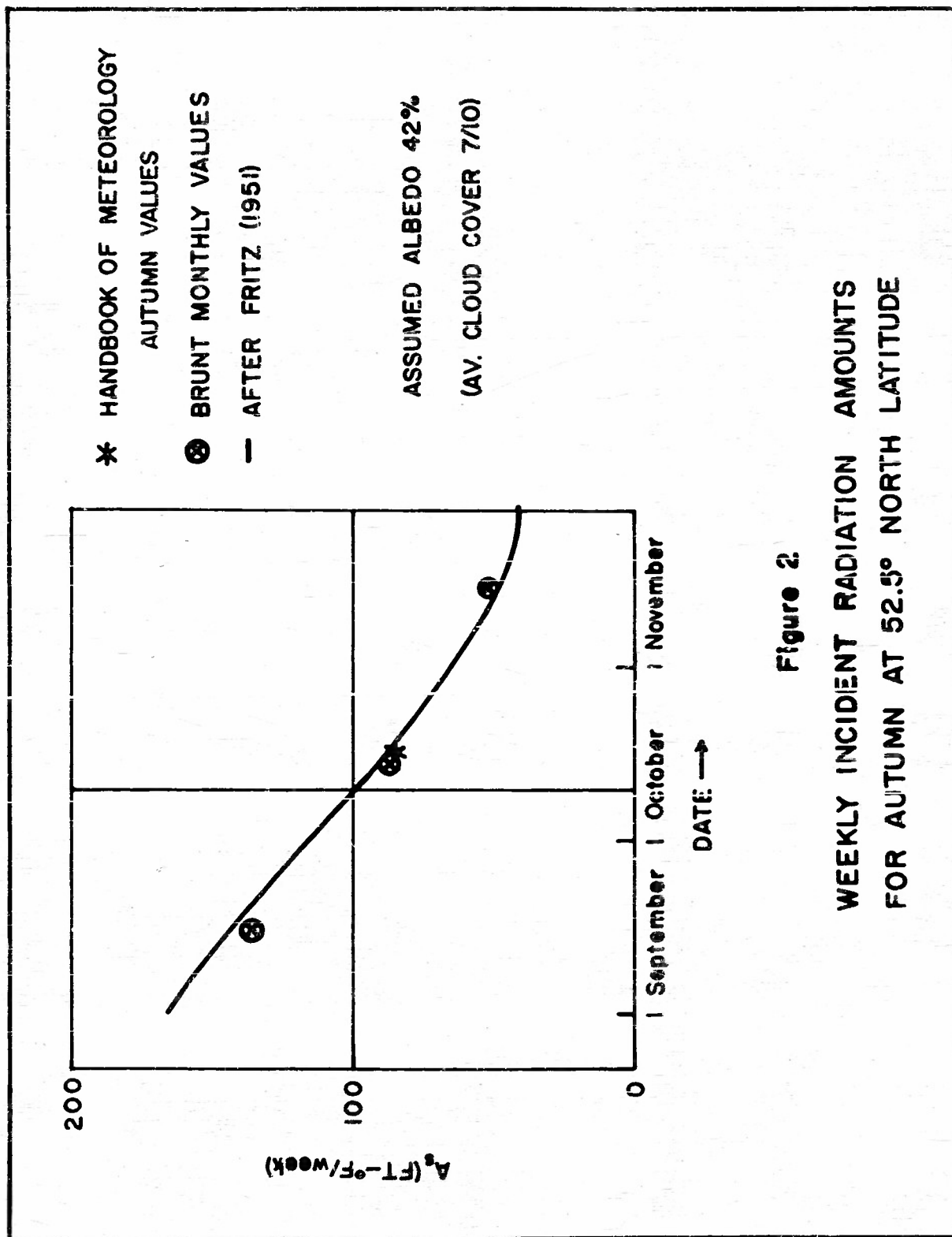
DISCUSSION OF THE COMPONENTS

A_T: The Total Thermal Change

The total thermal change of a water column within a seven-day period was measured in the following way: BT traces were plotted on graph paper for the end points of the time interval. The area enclosed between the two curves, from the ocean surface to the depth where the curves coincided and remained together, (or to a maximum depth of 200 meters in case coincidence was not attained) was measured by planimeter and recorded in ft-deg. F of thermal energy change per seven days. Note this procedure varies from that of Bretschneider, who considered only energy changes above the thermocline. Careful checks of this measurement were made, with strict limits on acceptable tolerance of variation in successive measurements of the area (see Appendix). This procedure was carried out for five-day moving average BT curves centered on the end days of the interval. The resulting areas (in ft-deg. F) are recorded in column 8 of Tables 4, 5 and 6.

A_S: The Average Seasonal Change in Solar Radiation

The average seasonal change in solar radiation minus an average albedo amount of 42% has been estimated for our station from data given by Fritz (1951). The figure of 42% albedo was chosen for use with the reservation that any one albedo constant does not represent the effect of variable cloudiness on solar radiation reflection over short periods of time.



Fritz (1951) suggests an albedo of 35% as an average amount of solar reflection from the earth, assuming a normal cloud cover of 54% for all regions. The Atlas of Climatic Charts of the Oceans (1938) shows an average cloudiness of 7/10 coverage of predominantly middle and low clouds during the autumn season at Station "C". Assuming that the albedo is a function of cloudiness alone (this is almost correct, according to Fritz) then

$$\frac{70}{54} \times 35 = 45.5\%$$

which is nearly equivalent to the albedo figure chosen above. The agreement would be exact if the cloudiness were 65% rather than 70% coverage. The albedo figure which has long been used for the earth is 42%.

The solar radiation amounts computed thus have been checked with seasonal and yearly radiation values given by Brunt (1939) and Landsberg (1945) and all the estimates agree reasonably well (see Figure 2). Bretschneider estimated approximately the same average net radiation. The graph showing the variation of solar radiation in ft-deg. F is given in Figure 2. Values of solar radiation for each time interval are shown in column 2 of Tables 4, 5 and 6.

A_B: The Effective Back Radiation from the Ocean Surface

The effective back radiation from the ocean surface represents the difference between the long-wave radiation going out from the ocean surface and the long-wave radiation received back from the atmosphere. Sverdrup (1942) has prepared a graph to estimate this

radiation as a function of relative humidity and ocean surface temperatures according to results of Angstrom in 1920. Average values of ocean surface temperatures and relative humidities were computed for each of the periods involved. These were used to enter Table 25 of Sverdrup (which assumes clear skies) for estimates of effective back radiation, Q_0 , for the various periods. Sverdrup (1942, p. 113) notes if the sky is completely covered by altostratus clouds, the above computed values of radiation (Q_0) are reduced to $4/10 Q_0$, while for a strato-cumulus overcast the figures are reduced to $1/10 Q_0$. From The Atlas of Climatic Charts of the Oceans (charts 70, 74, 78, 82, 86, 90, 94) these two types of clouds predominate at the "C" Station location in the autumn but only up to $7/10$ coverage rather than overcast. A factor of $3/10 Q_0$ seemed a reasonable value to use to account for average cloud conditions.* The results are shown in column 3 of Tables 4, 5 and 6.

Fig: The Evaporation from the Ocean Surface

Jacobs (1951) has developed a method which estimates evaporation amounts from synoptic weather observations. The theoretical approach used by Montgomery and others in the study of

* This does not account for back radiation in a complete manner. A study is planned which will account for daily cloudiness variation in estimating back radiation amounts, insofar as present information regarding relations of cloud thickness, height, type, and amount to back radiation will permit.

diffusion of water vapor has been incorporated into a formula using readily available weather data:

$$E = 2.8 \times 10^{-6} \left[N(\overline{e_w - e_a})\overline{w_s} + 3.5 M(\overline{e_w - e_a})\overline{w_u} \right] \text{ in gm/cm}^2\text{-hr.}$$

N represents the fraction of hydrodynamically smooth wind observations with windspeeds of 6.5 m/sec. or less, ($\overline{w_s}$); M represents the fraction of hydrodynamically rough wind observations taken with windspeeds greater than 6.5 m/sec., ($\overline{w_u}$); e_w represents vapor pressure at the water surface and e_a represents the vapor pressure at the height of wind measurement. The bars denote average quantities over the time interval involved. By the use of appropriate conversion factors* this transforms into:

$$A_E = -0.5058 \left\{ \sum_0^i (\overline{e_w - e_a})\overline{w_s} + 3.5 \sum_1^j (\overline{e_w - e_a})\overline{w_u} \right\} \text{ in ft-deg. F per time interval.}$$

A_E = energy used for evaporation from the water column for the time interval.

$i + j$ = total observations for the interval.

The usual time interval used is seven days. Values of A_E were computed for each interval and are shown in column 4 of Tables 4, 5 and 6.

Bretschneider used Dalton's evaporation equation with constants as evaluated by Munk in 1947 (see Bretschneider, 1952, p. 9). His equation is similar to the basic equation of Montgomery, which has been modified by Jacobs to the form used in this study. The weekly values of A_E presented here are of the same magnitude or smaller than Bretschneider's daily values.

* See computation of A_E , Appendix.

For comparison purposes, the Lake Hefner empirical evaporation equation from Russel (1952) was used to estimate evaporation amounts, as shown below.

The Lake Hefner equation is surprisingly simple in form:

$$E = 6.25 \times 10^{-4} u_8 (e_0 - e_8)$$

E = evaporation in cm/3 hours

u_8 = windspeed in knots at eight meters height

e_0 = water vapor pressure at the surface

e_8 = water vapor pressure at eight meters height

$$\text{hence } A_E^1 = 0.7312 \Sigma [u (e_0 - e_8)]$$

where A_E^1 is evaporation in ft-deg. F/week; u is in knots; e_0 and e_8 in inches of mercury and Σ is number of three-hourly observations per week.

It is evident that application of the Lake Hefner equation to the ocean may be in serious error when one considers the environmental contrast of Lake Hefner in Oklahoma to the open Atlantic Ocean.

Table 2 shows a comparison of evaporation amounts computed by the Jacobs and by the Lake Hefner formulas:

TABLE 2

Evaporation Amount Estimates

Date	A_E Jacobs Ft-Deg. F	A'_E Lake Hefner Ft-Deg. F	A_E/A'_E
1947			
Sept. 1- 7	- 50	- 25	2.00
8-14	-119	- 55	2.16
15-21	- 72	- 42	1.71
22-28	- 21	- 16	1.31
29-Oct. 5	- 68	- 33	2.06
6-12	-145	- 61	2.38
22-28	- 93	- 41	2.27
29-Nov. 3	-145	- 68	2.13
6-12	-171	- 74	2.31
TOTALS	-884	-415	AVG. 2.13
1948			
Sept. 6-12	- 35	- 17	2.06
13-19	- 63	- 37	1.70
20-26	- 29	- 14	2.07
27-Oct. 3	-103	- 68	1.51
4-10	-260	-112	2.32
11-17	-163	- 68	2.40
18-24	- 71	- 36	1.97
25-31	-208	- 92	2.26
Nov. 2- 7	-153	- 70	2.19
7-13	- 74	- 33	2.24
14-20	-149	- 69	2.16
21-27	-102	- 43	2.37
28-30	- 57	- 27	2.11
TOTALS	-1472	-686	AVG. 2.15
1949			
Sept. 1- 7	-142	- 61	2.33
8-10	- 27	- 14	1.93
17-23	- 18	- 12	1.50
24-30	- 63	- 28	2.25
Oct. 1- 7	- 25	- 10	2.41
8-13	-107	- 45	2.38
Nov. 9-15	-123	- 52	2.37
16-18	-110	- 46	2.39
23-29	- 35	- 19	1.84
TOTALS	-650	-287	AVG. 2.26

A_D: The Change of the Thermocline Depth in Response to Atmospheric Influences

The change of the thermocline depth in response to atmospheric influences (i.e., wind stress) will also affect the heat budget of a water column. Vertical advection of deeper water occurs which compensates mass changes due to horizontal advection processes in the upper layers.

Several unreported methods have been tried to estimate such thermocline changes. Earlier methods attempted to use changes in atmospheric pressure, as well as estimates of the piling-up of upper water by the wind to estimate thermocline change; but these gave results which seemed erratic and arbitrary. The following development by Freeman (1953) relates thermocline adjustment to atmospheric influences.

The effect of wind stress produces a net horizontal transport of mass within the mixed layer of the ocean. Such mass transport has regions of divergence and convergence where the thermocline adjusts to accommodate the mass fluctuation. The thermocline depth change is consequently a function of the horizontal mass transport divergence, hence it is a function of the curl of the wind stress under such circumstances.

The following assumptions enter into the development of the expression below:

- a) no large change of surface water level occurs in response to mass changes within the mixed layer;

- b) no significant motion occurs below the mixed layer (all measurable mass transport occurs within the mixed layer);
- c) complete volume compensation occurs; deep layer movements are slow;
- d) complete pressure compensation occurs by thermocline adjustment;
- e) the rate of time change in the flow is slow and motion appears to depend primarily on wind stresses, pressure forces and Coriolis forces.

These assumptions appear to be compatible with observed ocean conditions.

Freeman shows that such assumptions lead to the relation

$$\frac{\partial H}{\partial t} = \frac{1}{\rho' f} \left(\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y} \right)$$

where H = mixed layer depth

t = time

ρ' = density of salt water

f = Coriolis parameter

τ_x, τ_y = wind stress per unit area on ocean surface in x and y directions respectively.

The thermal change associated with this is given by

$$A_D = \frac{1}{\rho' f} (\bar{T}_{MLD} - T_{200m}) \nabla_h \times \tau$$

where $\nabla_h \times \tau$ = horizontal curl of wind stress per unit area of ocean surface

$\bar{T}_{MLD}$ = average temperature of the mixed layer

T_{200m} = temperature at 200 meter depth.

The curl of the wind stress at the ocean surface may be expressed in terms of the space change in atmospheric horizontal pressure gradients, and the equation becomes

$$A_D = 0.14152 \times 10^{10} \left[\left(\frac{\Delta p}{\Delta n} \right) \left(\frac{\Delta p}{\Delta n} \right) / A - \left(\frac{\Delta p}{\Delta n} \right) \left(\frac{\Delta p}{\Delta n} \right) / B \right] \left[T_{MLD} - T_{200m} \right] \frac{\text{ft-deg. F}}{7 \text{ days}}$$

where $\frac{\Delta p}{\Delta n}$ represents the ocean level atmospheric pressure gradient at points A and B, at a distance $\frac{\Delta n}{2}$ to the left and right, respectively, along the pressure gradient through the station. The notation above is used to preserve algebraic sign in components A and B. The amounts for A_D are shown in column 5 of Tables 4, 5 and 6. No estimate of this effect has been included in any previous heat budget study.

A_{NET} : The Sum ($A_S + A_B + A_E + A_D$)

For comparison purposes, the sum of ($A_S + A_B + A_E + A_D$) was computed for each time interval. These values are shown in column 6 of Tables 4, 5 and 6.

A_P : The Residual

The change which is unexplained above is shown for each weekly period in column 7 of Tables 4, 5 and 6.

Upon examining the magnitude and variation of the weekly residual amounts, it was seen that these amounts often corresponded well with the observed total heat change within the water column. In fact, the total change agreed more often with the residual than

with the net explained change (see Tables 4, 5 and 6). While this is pertinent to our basic questions posed above, it does not indicate successful forecasting of changes in total heat content based on the usual meteorological and oceanographic parameters which affect the physical processes.

As an attempt to improve the forecasting possibilities, it seemed desirable to investigate methods which might eliminate the large residual and still be capable of physical interpretation. One such method was chosen to be included in this report. If the shape of successive BT traces varied only a small amount, then moving one curve parallel to the abscissa into a "best-fit" superposition over the second BT would leave only the resulting small differences between the curves to be explained. The addition or subtraction of a constant amount of energy at all levels (i.e., the "best-fit" process) during the time interval possibly could result from that part of horizontal advection which occurs uniformly at all levels.* Since total horizontal advection is an ocean process which eventually is capable of being forecasted and is independent of other physical processes affecting the thermal structure, such a "best-fit" technique could be useful.

This "best-fit" method thus would allow effort to be concentrated on forecasting the changes in the shape of thermal structure of the BT, by use of a simple procedure which is capable of physical interpretation.

* That component of horizontal advection which is unequally distributed with depth should act to change the shape of the BT.

A_T^i : "Best-Fit" Total Thermal Change

This procedure was used to evaluate a new total heat change (A_T^i) between the "best-fit" positions of BT traces over the weekly intervals studied above, and these are listed in column 10 of Tables 4, 5 and 6.

A_p^i : The Residual from A_T^i

The A_{NET} was subtracted from A_T^i to obtain A_p^i . This quantity is thus similar to the residual A_p previously computed. Values of A_p^i are shown in column 9 of Tables 4, 5 and 6.

DISCUSSION

From the data in Tables 4, 5, and 6 we may prepare plots of the reported average quantities with time (Figs. 7-15).

Such a procedure implies that the averages are continuous functions of time and this may be questioned quite justifiably. With suitable reservations, however, such plots may serve as indicators of relationships existing between the various quantities over the particular time intervals for which the averages apply.

Net Amount of Explained Heat Changes

The fluctuations of the net amount of explained heat content changes, A_{NET} , are seen to depend heavily on variations in the contributions from evaporation, A_E , and the dynamic term, A_D . The quasi-constant effects of solar radiation, A_S , and back radiation, A_B , are imposed on these variations.

It should be remembered that the constancy of A_S is to be expected, considering the average computations on which these values depend (see Figure 3 and the Appendix). A_S may vary actually more than is suspected now; it probably contributes more to the variation in heat content of the water column than is indicated here. Only synoptic radiation measurements during a study such as this will provide a better estimate of actual conditions (Schule had available this type of data).

However, the constant nature of A_B is not necessarily anticipated from its method of computation (see Appendix). It depends on the nature of the variables from which it is computed, ocean surface temperature and relative humidity above the ocean surface; both were computed from observed synoptic conditions during the study. From our results it appears that these variables fluctuate only a small amount, at least over weekly intervals during autumn for this particular location.

Relation of A_{NET} to A_T^I

For the autumn season of 1948, the trend of the "best-fit" total heat change, A_T^I , curve usually agrees with the trend of the total explained heat change, A_{NET} , (see Figure 11). For only two short intervals (23-30 September and 27 October-5 November) the trends do not correspond. However, the magnitude of A_{NET} cannot be brought into coincidence with the magnitude of A_T^I by any simple adjustment procedure applied throughout the season. Nevertheless, considering the significance of the trend agreement could be useful in our overall problem.

The agreement in trends probably indicates that consideration of the primary factors influencing A_T^I has been made in the estimate of A_{NET} . If advection effects as discussed below were incorporated, A_{NET} and A_T^I perhaps would reach near-agreement in magnitude as well as the observed trend agreement. This would not apply, necessarily, to 1947 and 1949 data where the trend relationships are obscure.

The lack of magnitude agreement in such trend relations could arise from summing estimates each of which only approximates the contribution from a significant physical process affecting the heat energy change of a water column. Methods of computing such estimates for this study would permit both positive and negative deviations from the "true" contribution, and hence A_{NET} could vary from under to overestimating the observed "best-fit" heat change.

That contribution from horizontal advection which is non-uniform with depth has not been estimated in the A_{NET} amount. Such advection certainly must occur in the ocean, and would offer a possible explanation for some of the observed differences between A_T and A_{NET} .

Relation of A_{NET} , A'_T and A'_P

In Figures 8, 11 and 14 are shown the distribution of A'_T , A'_P and A_{NET} with time. The features of these distributions will be discussed and tentative explanations will be offered to account for them. In a later section the relation will be shown between these "best-fit" parameters and the total parameters, A_T and A_P .

The agreement between variations in A'_T and A'_P is striking. Such agreement is an argument against having attained a measurable heat balance in the water column using the methods of this study, although trend relationships between A_T and A_{NET} suggest most of the important physical processes affecting the heat budget have been considered in these estimates for 1948. It appears that successful methods

of forecasting A_T^1 will necessarily involve further examination of the residual term A_P^1 which is done in a later discussion.

Another feature of the graphs is the tendency of A_{NET} to change conversely to A_P^1 for certain time intervals (see all of September, 8 October-10 November, 25-30 November 1948; 20 September-11 October 1949). The only relation between this tendency and other graph features apparently is that the A_P^1 line crosses the A_{NET} line near the time when A_D returns to zero. See 19-21 September, 9-10 October, 27-29 October 1948, as illustrations of this tendency.

This invokes some interesting thoughts regarding a check and balance system which could be operating in the ocean under direct influence of changing weather regimes. These are incorporated in a model situation to be described presently.

Relation of A_{NET} to Atmospheric Change

A_D is directly determined by the atmospheric influence on the water column, and in its own way reflects the weather pattern existing above the ocean surface. A period when A_D is zero is likely to indicate an unsettled time when the weather is changing from one given pattern to another (for example, a change from high to low index in the westerlies belt). It may not be mere coincidence that the fluctuations exhibited by A_{NET} , A_T and A_P^1 are similar in nature to the five-day zonal index fluctuations of the atmosphere observed in the westerlies region. The potentialities of such a relation for forecasting usefulness are enormous, if such a relation can be shown.

Undoubtedly the suggestions from these data ought to be further checked in detail with a view toward forecasting application. It appears that a system operating as described below would provide a possible explanation of the observed variation of A_{NET} , A_T^i and A_P^i .

Suggested Ocean-Atmosphere Relationships

The problem before us is to suggest reasons for the observed behavior of A_{NET} and A_T^i for long intervals of the 1948 autumn data. Perhaps a logical set of relationships between these parameters may be developed through a discussion of ocean-atmosphere interactions.

A_{NET} decreases markedly when strong winds and air colder than the ocean remove heat from the water column through strong evaporation. An upward shift of the thermocline (negative A_D) simultaneously removes heat. Why then should A_T^i increase during this period, as observed?

The A_T^i rise could be due to unusually strong horizontal advection in the ocean surface layers. Such advection would be associated with prolonged southerly winds on the east side of the mean low pressure over the region (associated with the negative A_D). Such strong winds could act to bring a warm eddy of the North Atlantic Drift into the region.*

A_T^i then decreases markedly to large negative values, possibly due to winds bringing in tongues of cold East Greenland water. These north winds would result as the mean low shifted eastward over the

* The station location is affected by the North Atlantic Drift.

region. During this period the mean low center has been near the station and A_D has made a negative contribution to the total heat content.

However, continued eastward movement of the mean low pressure allows a mean high pressure ridge to occupy the studied region. This permits A_D to become zero or positive with the diminishing pressure gradient, and A_E becomes small with decreasing winds. Hence A_{NET} becomes a positive amount. A_T' tends to follow the trend of A_{NET} in the absence of abnormal advective influence, which would then remain as a small negative contribution to the heat content until another region of mean low pressure moves into the region with its abnormally strong warm water advection.

The variation in processes visualized thus would depend upon the mean weather pattern for a given season. The mean weather patterns, in turn, change in a manner which is fairly regular for certain years, and highly irregular during others. This peculiarity may provide an explanation for the variation occurring when comparing data for a given season during different years, as is discussed later.

It is now in order to investigate how well the actual data of Figures 8, 11 and 14 correspond with postulated ocean-atmosphere relations. Referring to mean five-day pressure charts prepared by the Weather Bureau for the autumn of 1948 (data not shown) some tentative verification was indicated for processes suggested above.

A cycle began with comparatively stable, calm conditions, apparently around 9 September 1948 (Figure 11). For about a week A_T'

was determined by A_{NET} to a large extent, and A_P^1 hovered near zero. For this particular season A_{NET} appeared to depend heavily on A_D (Figure 10). A_D in turn was a function of changing atmospheric conditions, indicated by the changes in five-day mean atmospheric pressure maps over the water column region (data from the Weather Bureau not shown). Now a mean low pressure system began to move into the region from the west on about 15 September. A_D changed in a negative direction due to the change in pressure gradient, A_{NET} responded in turn, and A_P^1 followed along. At a later date, on 30 September, it is reasonable to postulate that the strong winds associated with the low pressure area increased the evaporation to such an extent that it then became the observed major factor in determining A_{NET} . This strong evaporation persisted and intensified in the region, perhaps due to the temperature contrast between cold air and relatively warm sea even though the wind, which brought in the colder air from the north, was then diminishing and A_D was decreasing. The evaporation then decreased toward zero (beginning 7 October or so) as the cold air mass assumed the character of the warmer ocean surface beneath it. With A_D already decreasing toward zero, the low moved on eastward and was replaced by a ridge of high pressure possessing a smaller pressure gradient and less change in pressure gradient; A_E then decreased and A_{NET} returned toward zero. A_D became a positive contribution in a ridge of high pressure, with relatively low evaporation due to lighter winds and the modified character of the air mass. On 21 October

the cycle above apparently began again with a new low moving rapidly over the region and persisting, bringing strong winds and a new cold air mass over the water column.

Checking of extensive data and much more verification of such tendencies will be necessary to determine the validity of such relationships, of course. Nevertheless, the integration of the ocean-atmospheric physical processes in such a model may be of interest to others.

Comparing the fluctuations of A_{NET} with the fluctuations of A_T , the total observed change of heat content within the column during weekly intervals,* it is seen that the magnitudes hardly ever agree. This could be due to several causes. It could mean that the weekly intervals which were used do not correspond to the "natural" time interval over which the ocean smooths incoming influences to achieve any balanced heat budget which may exist. It could mean that the available techniques for estimating effects of evaporation, back radiation, etc., are inadequate. While the latter may contribute to the lack of agreement to some extent, it probably is not a major item in the discrepancy.

Another possible reason for the lack of agreement may be the neglect of physical processes for which data or techniques for measuring do not exist. Such neglected processes may include the addition

* A few of the intervals are for periods of less than seven days. These are indicated by * on the graph.

of water to the column in the form of precipitation and the conduction of heat laterally from the column by eddy motions, as discussed earlier.

The horizontal advection process should be discussed here as well, for it is one of the important processes of heat transfer within fluids and has not been measured in this study due to lack of suitable data. In atmospheric motions horizontal advection plays an important role at many levels. There is no reason to believe it is not important in the ocean as well, but the necessary current and synoptic temperature data have not been taken to permit actual computation of this term directly. The technique used by Bretschneider involved an estimate of the surface current due to the wind,* which was assumed to advect the mean isotherm pattern for the particular month involved. This procedure permits computation of a value, but it remains to be shown that such a value can be given a physical interpretation. In all likelihood, the actual surface advection over a daily or weekly interval has little correspondence with such a computed value. A completely unanswered question remains concerning horizontal advection into the water column below surface levels.

In Cochrane's study, he has chosen an area where the ocean is almost horizontally uniform in the upper layers of the ocean.

* $\frac{C}{U} = \frac{0.0127}{\sqrt{\sin \phi}}$ (Sverdrup, 1942, p. 494) was used where C is the speed of the ocean surface current and U is the speed of the wind. It was assumed C moved 33° to the right of U.

In spite of the choice of location Cochrane finds that horizontal advection at layers well below the ocean surface is important during parts of the year. The method of computing such advection is not presented, but it may be that the residual in Cochrane's heat budget study was assumed due to horizontal advection. Such deep advection effects also may be the result of variation within internal waves at the thermocline due to changes either in amplitude or period.

In studies on a hemispheric scale the problem of advection in the horizontal is limited to estimates of inflow across the equatorial oceans; in overall energy considerations such as those of Sverdrup and Jacobs, the contribution of such advection is assumed small.

Schule does not evaluate a horizontal advection term in his study, although recognizing the factor as having significance.

The "best-fit" process of obtaining A_T^I would eliminate those energy change effects which are uniformly distributed with depth. Those effects which are irregular in depth distribution, such as a greatly increased ocean surface advective warming (or cooling), or the advection of a deeper (or shallower) thermocline, would not be excluded by the A_T^I matching procedure.

The variation of A_p^I includes such irregularities, and since the variation of A_p is in good accord with variations in advection postulated above, horizontal advection may possibly provide a significant part of A_p^I . It should be recalled from previous discussions

that A'_p also contains all effects due to the smoothing of data, errors in estimation, and effects due to other uncalculated physical processes as well.

The relation of A_T to A'_T now may be discussed. A_T reflects the effect of all temperature change occurring throughout the column depth. To the effect of irregularly distributed vertical variation of heat change as depicted by A'_T , the quantity $(A_T - A'_T)$ may be added to represent the effects of heat change which is uniform with depth occurring in a water column during a particular time interval.

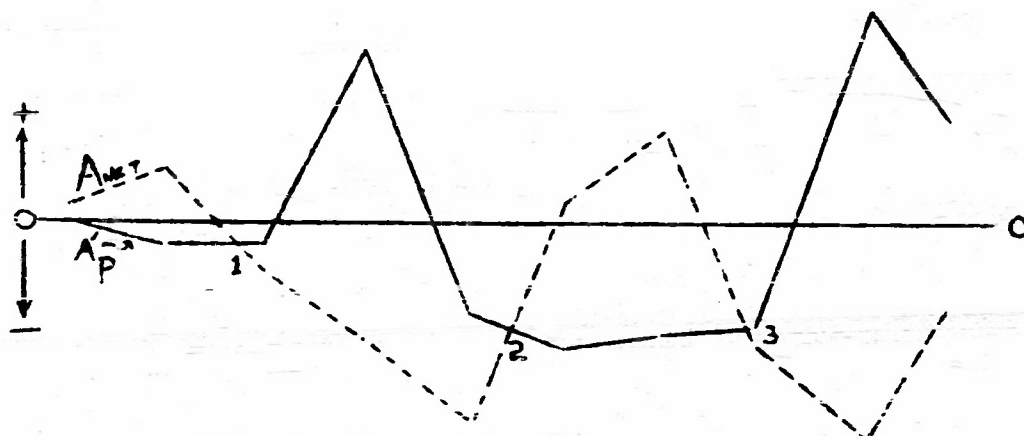
A_p would have a relation to A'_p corresponding to that between A_T and A'_T (compare in Tables 4, 5 and 6). The variation of A_T , A_p and A_{NET} for the three seasons studied is shown in Figures 9, 12 and 15.

Forecasting relations must deal eventually with A_T and A_p , of course. It is suggested from these data, however, that variations of a parameter stand out more clearly in an intermediate stage of relationship, such as the one shown by A'_T and A'_p in the "best-fit" process.

Summary of the Model

It appears that the A_{NET} term (depending mainly on evaporation and the dynamic effect of atmospheric weather) acts to provide compensation in the ocean when A'_p swings toward an unusually high or low value. From the above discussion, it appears that A'_p may be influenced largely by horizontal advective processes which are irregularly distributed with depth. Such advection could be possible through strong

winds persisting at the surface for a prolonged interval (when significantly different water masses, i.e., the Gulf Stream or the East Greenland Current, are near), or through a mean change in the amplitude of internal waves at the thermocline. The "cross-overs" (i.e., points 1, 2 and 3)



may be related to changes in general weather regime indicated by A_P becoming zero. A_T then follows a course which integrates the effects of the variations in A_{NET} and A_P . The absolute value of A_T depends on the positions of A_{NET} and A_P relative to the zero value on the chart.

The above discussion depends heavily on relationships for 1948, when the most complete data were available for this study. It appears that the data for 1949 displayed some of these features but not as extensively. Relationships for 1947 differed from those of 1948 in most of the limited number of intervals studied. It would appear that relationship patterns may be set up and followed in a particular year, but in a succeeding year a different relationship is

established. An analogous situation is the recurrence of certain weather "situations" in a region for a particular season, but with quite a different pattern of recurrence happening from year to year.

Seasonal Total of Parameters

Of interest in relation to previous studies, the seasonal total of each parameter may be formed as the sum of individual weekly totals. These totals may then be compared to determine if a balance exists in the heat budget of a water column on a seasonal basis. In 1948 it appears as if a near-balance is achieved for the autumn season. In 1947 there was a net gain in heat, while in 1949 there was a large net loss, for the entire season.

These total changes for each season were measured by plotting the first and last BT's of each season on a graph and measuring the area between the curves. This is a procedure similar to that used in obtaining the weekly A_T values. The resulting curves are shown in Figures 4, 5 and 6.

Plots of average temperature for the season versus depth correspond roughly to the above losses of heat in 1947, for temperatures are relatively low compared to 1948 and 1949 at levels down to 350 feet in depth. A corresponding relation between 1949 and 1948 does not appear (see Figure 3).

For the entire autumn season of 1948, the evaporation and dynamic process amounts (A_E , A_D) combine to give a large negative total contribution. The slight positive observed total (A_T) is not in the

direction to be expected, considering that the summer oceans cool with the onset of winter; there appears to be a large positive or warming region below the thermocline which more than compensates mixed layer cooling for the season. This is also true for 1947.

The total change due to other causes (A_p) is positive and of the same order of magnitude as the incoming radiation amount (A_s) and the evaporation amount (A_e); it is about 75% of the A_p amount from one particular period, from 20-26 September 1948.*

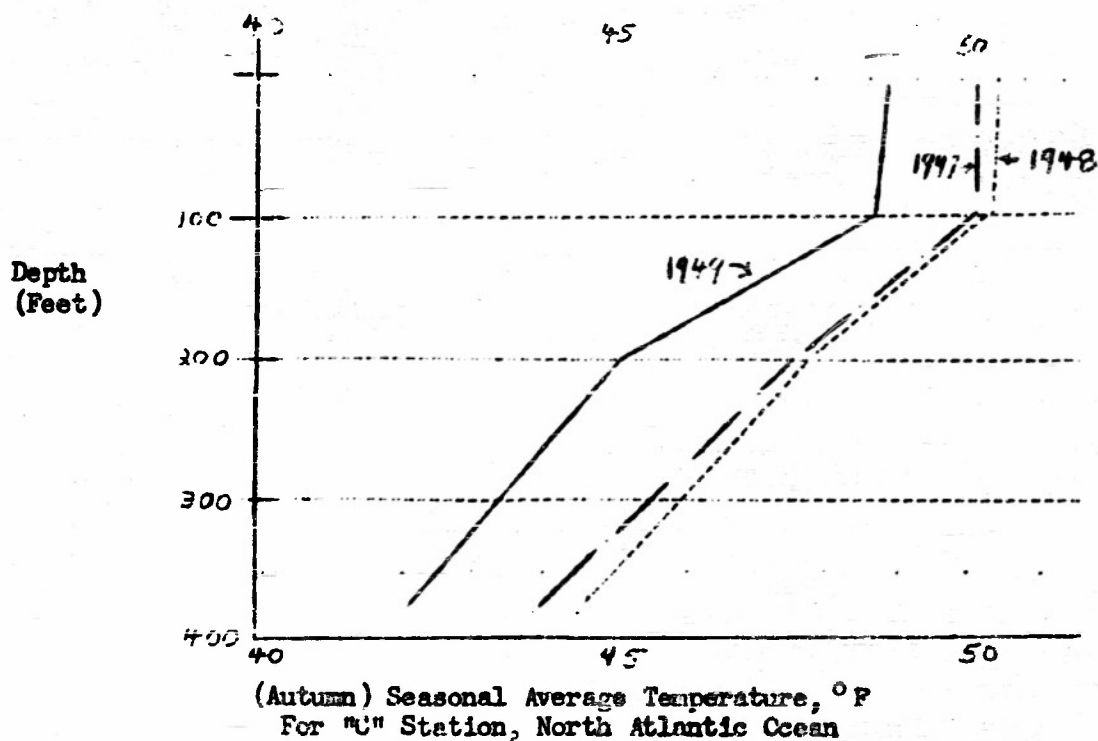
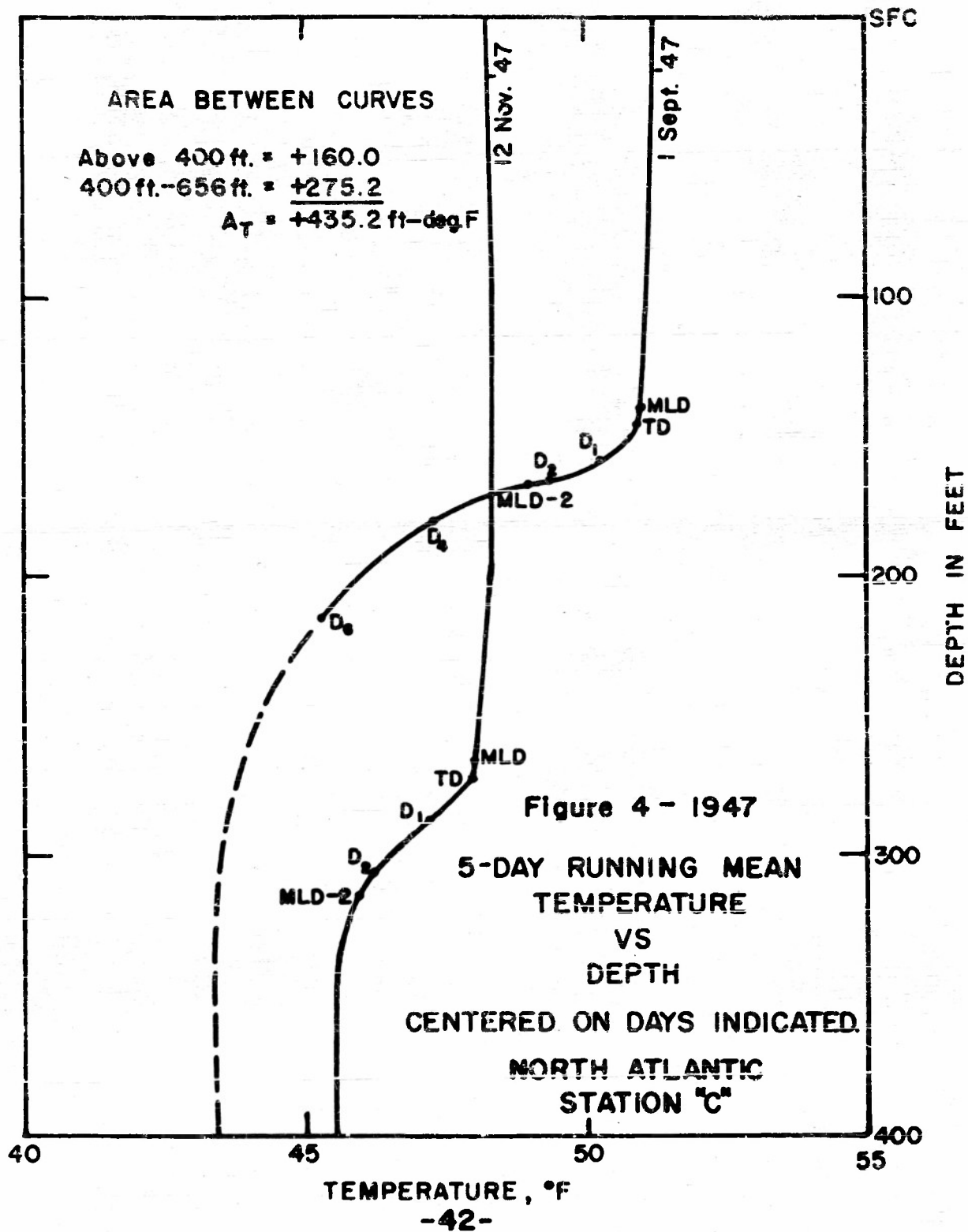
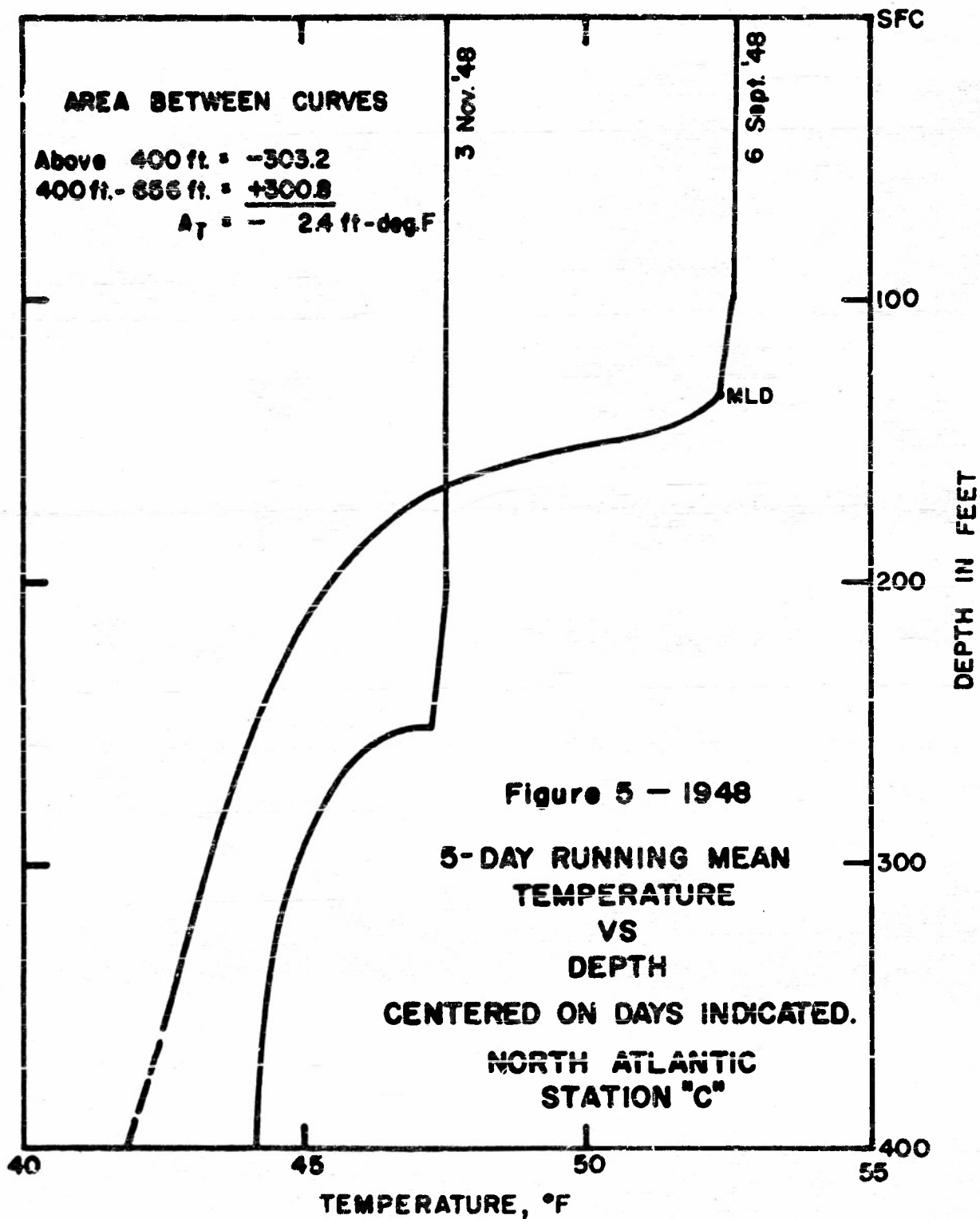
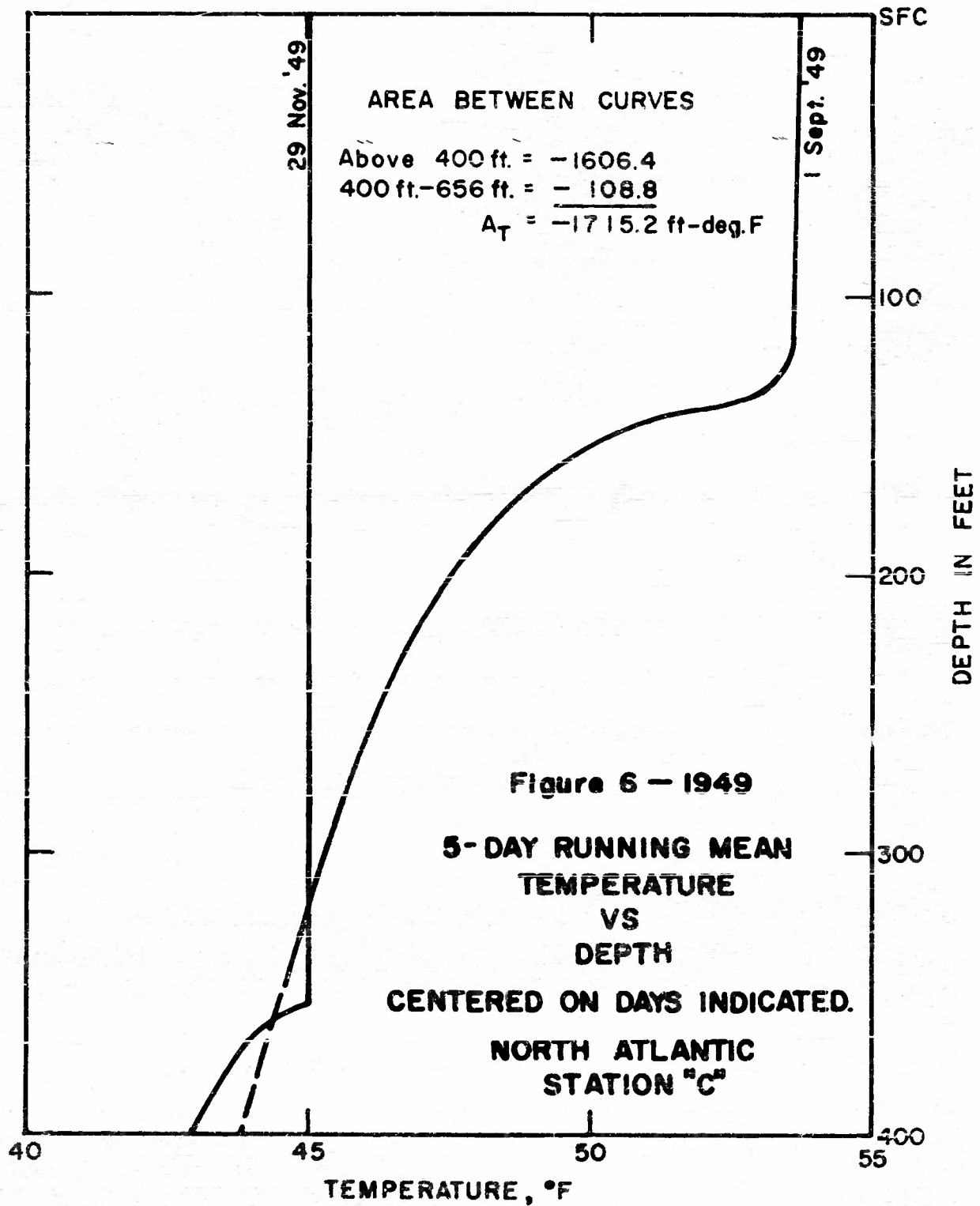


FIGURE 3

* This case has been completely investigated as to validity of data; it appears that a complete and radical change in water mass occurred at the station, reaching all depths down to 450 feet.







The quantities of heat removed by evaporation computed by the two evaporation formulas are obviously well correlated (see Table 2). The agreement of the magnitudes is slightly better than would be expected from a comparison of the constant factors in the two computing equations (those of Jacobs' formula would indicate a maximum possible deviation of about 2.42 times from the Lake Hefner results).

TABLE 3

Comparison of Lake Hefner Formula,
Jacobs' Formula and Total A_T

1948 Totals	$\frac{A}{T}$	Hefner $\frac{A_P + A_D}{T}$	Jacobs $\frac{A_P + A_D}{T}$
in ft-deg. F	107	98	854
	$\frac{A'}{T}$	$\frac{A'_P + A'_D}{T}$	$\frac{A'_P + A'_D}{T}$
	-851	-877	-91

It is of interest to note in the seasonal totals of heat change that both A_T and A'_T agree reasonably well with the respective summed quantities of $A_P + A_D$ and $A'_P + A'_D$ for 1948 (see Table 3), when the Lake Hefner evaporation formula is used for computing A_E (note changes in A_E affect A'_P and A_P). If the residual terms were found to be related to horizontal advection, the sum of $A_P + A_D$ would represent total or three-dimensional advection in a sense. The forecasting possibilities of such a relationship between total advection and total heat change make it highly desirable to investigate this aspect further at some later time.

Measuring the area between the first and last BT for each season in one operation gives a more accurate determination of the total heat energy change than does the sum of the seven-day A_T amounts. However, only in the year having continuous data* does the sum of the A_T 's for seven-day periods permit comparison to a season-long A_T measurement. In 1948, the seasonal A_T amount as determined from Figure 5 is -2 ft-deg. F, whereas the seasonal A_T amount obtained by summing the seven-day amounts is 107 ft-deg. F.

* Data for 1947 and 1949 are discontinuous due to either missing meteorological or BT observations.

CONCLUSIONS

1. There is good agreement between the trends of the "best-fit" total, A_T' , and the explained, A_{NET} , heat changes in many cases studied. The magnitudes and the "periodicity" of the two changes are comparable. Such trend agreement suggests that most processes affecting heat energy change in a water column probably have been included in the estimated contributions.

2. The net variation in heat content within a water column explained by physical processes (solar radiation, back radiation, evaporation, Freeman's dynamic effect) for which estimates or data are available, A_{NET} , appears to depend largely on heat change due to evaporation, A_E , and the dynamic effect, A_D . Which factor predominates is apparently related to the mean weather situation over the surface, in a manner to be determined.

3. The technique of moving successive five-day mean ET traces horizontally into a "best-fit" position allows the variation to be more easily related between total heat change, A_T , net heat change explained, A_{NET} , and the residual amount, A_F .

4. It is suggested that the amount of heat subtracted by such a "best-fit" process may be due to that component of total horizontal advection which acts uniformly at all levels. This does not preclude that part of horizontal advection which is unevenly

distributed with depth from providing a portion of A_p , the unexplained residual remaining after the "best-fit" procedure.

5. The total amount of heat change after the "best-fit" procedure (A_T'), A_{NET} and A_p' (see 3) appear to be related on time sequence charts in a check and balance fashion. When A_{NET} departs appreciably from zero A_p' tends to move in a direction to oppose this tendency during much of the time studied. Each quantity fluctuates in both the positive and negative sense about zero. A_T' integrates the two influences, and varies in magnitude depending on the positions of A_{NET} and A_p' relative to zero.

6. The time when A_{NET} and A_p' "cross-over" appears related to the time when A_p is zero, for much of the data presented. This "cross-over" indicates the time when A_p' , having acted in a negative sense on A_T' , begins acting positively as compared with the effect of A_{NET} .

7. A possible physical relationship is postulated between such behavior of oceanic parameters and changes in the mean atmospheric pressure pattern over the area. This relationship needs verification from independent data.

8. It is suggested that recurrence patterns persisting for at least a season may exist in the ocean heat changes, but these patterns appear to change from year to year.

9. No heat balance is demonstrated to exist in the ocean on a weekly basis from data presented here. If horizontal advection processes could be estimated, the balance possibly could be attained, although the importance of data smoothing effects or unknown physical process effects is not to be minimized. On a seasonal basis it appears that a near-balance is attained for one of the three seasons studied (1948).

10. There are indications from the variations in parameters reported here that temperature may vary significantly in the horizontal as well as clear demonstrations of temperature variation in the vertical and in time. Whether or not ocean currents and other physical features of the ocean demonstrate this same tendency is not indicated.

11. There are indications of interdependence of the boundary influences (note conclusions 5 and 6) which appear to act as a check and balance system on the change in heat content of the water column. Such indications require further investigation.

RECOMMENDATIONS FOR FUTURE STUDY

1. A serious lack of data now exists for computing horizontal advection in the ocean. All possible effort should be exerted to secure adequate sea temperatures and velocities on a synoptic basis to provide this need.
2. An analysis similar to this reported one should be made incorporating effects of measured horizontal advection and measured solar radiation. The remaining residual then should indicate the computing error, lag effects and adequacy of estimating the effects of physical processes.
3. Apparent relations between atmospheric parameters, such as the zonal indices and variation in A_{NET} , A_T^1 and A_T^2 should be investigated with a view toward developing forecasting techniques.

ACKNOWLEDGMENTS

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TABLE 4
Energy Balance Sheet
At "C" Station
For Autumn, 1947

1	2	3	4	5	6	7	8	9	10
Time Interval Dates	A _S	A _B	A _E	A _D	A _{NET}	A _P	A _T	A _P	A _T
(All Values in Ft-Deg. F)									
A Sept. 1- 7	162	- 31	- 50		81	149	230	- 15	66
B Sept. 8-14	149	- 32	-119		- 2	-148	-150	-170	-172
C Sept. 15-21	137	- 32	- 72		32	-127	- 94	-113	- 60
D Sept. 22-28	124	- 31	- 21		72	- 98	- 26	- 82	- 10
E S. 29-Oct. 5	97	- 26	- 68		3	- 7	- 4	- 3	0
F Oct. 6-12	87	- 27	-145		- 85	4	- 81	-178	-263
G Oct. 22-28	65	- 27	- 93		- 55	166	111	- 77	-132
H O.29-Nov. 3	57	- 28	-145		-116	108	- 8	- 22	-138
I Nov. 6-12	56	- 32	-171		-147	912	765	= 2	-149
TOTAL	934	-266	-884		-216	959	743	-662	-878

$$A_D + A_P = 959$$

$$A_D + A_P' = -662$$

$$A_T \text{ SEASON (1st BT - last BT)} = -1715 \text{ ft-deg. F}$$

TABLE 5

Energy Balance Sheet
At "C" Station
For Autumn, 1948

1	2	3	4	5	6	7	8	9	10
Time Interval Dates	A _S	A _B	A _E	A _D	A _{NET}	A _P	A _T	A _P [!]	A _T [!]
(All Values in Ft-deg. F)									
A Sept. 6-12	152	- 26	- 35	- 72	19	- 395	- 376	1	20
B Sept. 13-19	140	- 32	- 63	16	61	- 113	- 52	- 25	36
C Sept. 20-26	128	- 30	- 29	-125	- 56	1720	1664	- 24	- 80
D S. 27-Oct. 3	117	- 32	-103	-110	-128	196	68	174	46
E Oct. 4-10	104	- 32	-260	- 35	-223	117	- 106	-103	-325
F Oct. 11-17	93	- 32	-163	126	24	- 416	- 392	-140	-116
G Oct. 18-24	82	- 32	- 71	130	109	- 255	- 146	-126	- 16
H Oct. 25-31	71	- 32	-208	25	-144	251	107	-118	-263
I Nov. 2- 7	54	- 27	-158	-176	-307	- 359	- 666	240	- 70
J Nov. 7-13	54	- 31	- 74	- 42	- 93	86	- 7	115	22
K Nov. 14-20	46	- 24	-149	- 45	-172	- 327	- 499	35	-137
L Nov. 21-27	42	- 32	-102	- 63	-155	431	276	171	17
M Nov. 28-30	18	- 14	- 57	- 28	- 81	317	236	108	28
TOTAL	1101	-376	-1472	-399	-1146	1253	107	308	-838

$$A_D + A_P = 854$$

$$A_D + A_P^! = -91$$

$$A_T \text{ SEASON (1st BT - last BT)} = -2 \text{ ft-deg. F}$$

TABLE 6

Energy Balance Sheet
At "C" Station
For Autumn, 1949

	1	2	3	4	5	6	7	8	9	10
Time Interval Dates	A_S	A_B	A_E	A_D	A_{NET}	A_P	A_T	A'_P	A'_T	
(All Values in Ft-Deg. F)										
A Sept. 1-7	162	-31	-142	-64	-75	-380	-455	138	63	
B Sept. 8-10	65	-13	-27	-131	-106	138	32	100	-6	
C Sept. 17-23	134	-32	-18	-23	61	-199	-138	-64	-3	
D Sept. 24-30	121	-32	-63	-52	-26	81	55	4	-22	
E Oct. 1-7	109	-31	-25		53	292	345	-109	-56	
F Oct. 8-13	84	-27	-107		-50	911	861	24	-26	
G Nov. 9-15	52	-33	-123		-104	22	-82	39	-65	
H Nov. 16-18	20	-14	-110		-104	-35	-139	-167	-271	
I Nov. 23-29	42	-33	-35		-26	30	4	-132	-158	
TOTALS	789	-246	-650	-270	-377	860	483	-167	-544	

$$A_D + A_P = 590$$

$$A_D + A'_P = -437$$

$$A_T \text{ SEASON (1st BT - last BT)} = 435 \text{ ft-deg. F}$$

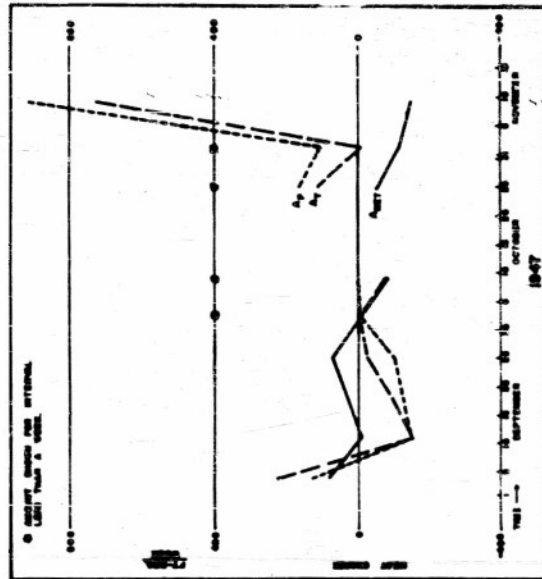


FIGURE 9

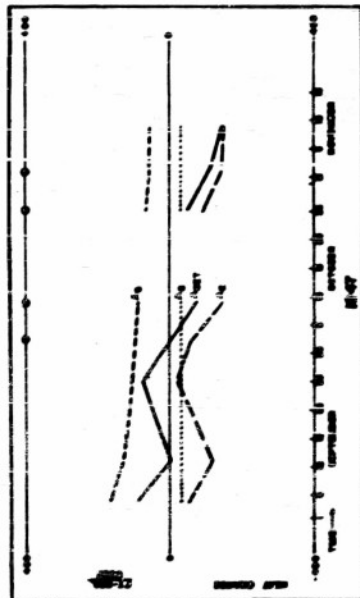


FIGURE 7

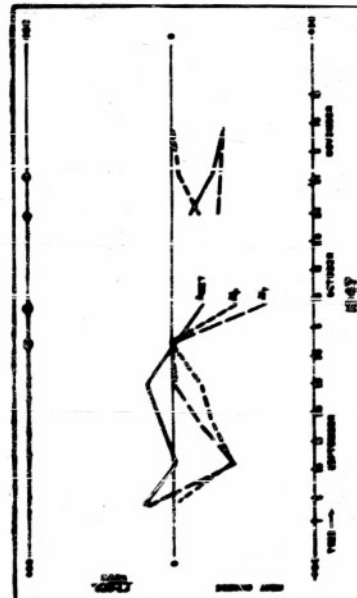
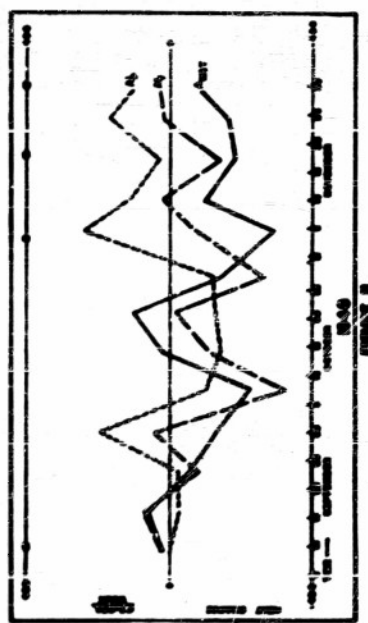
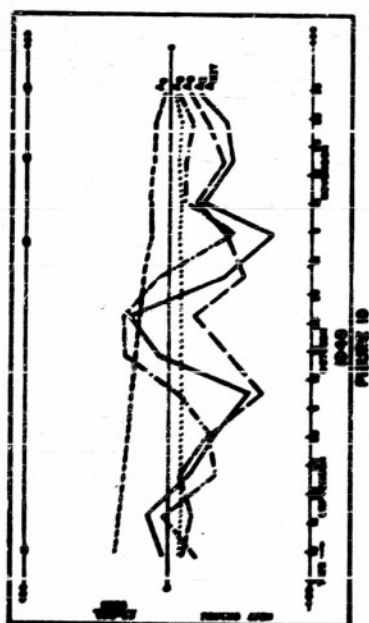
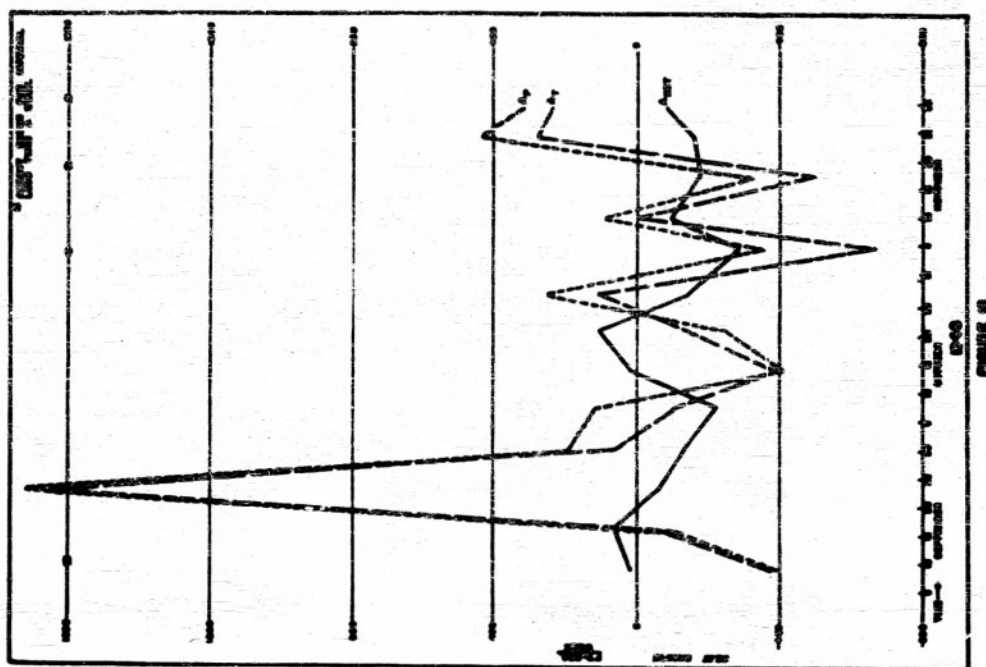


FIGURE 6



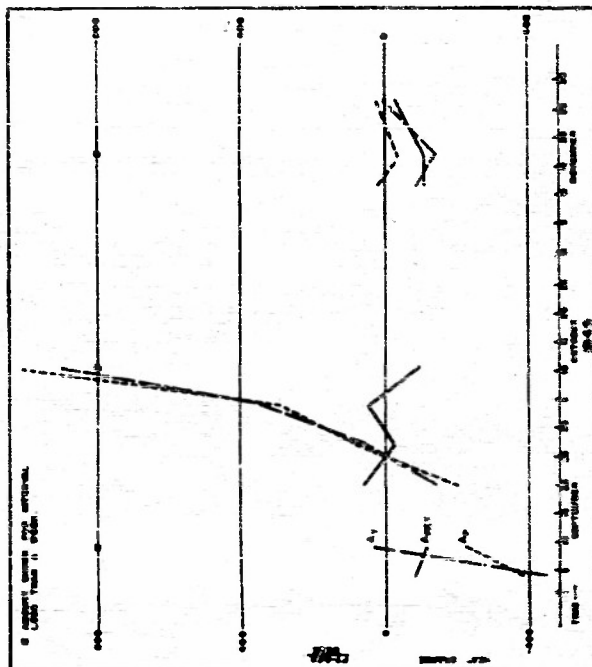


FIGURE 13

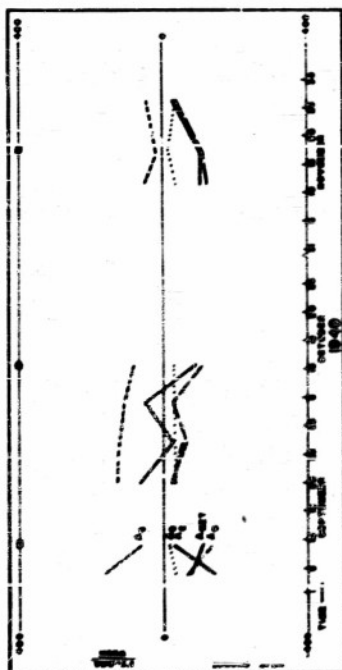


FIGURE 14

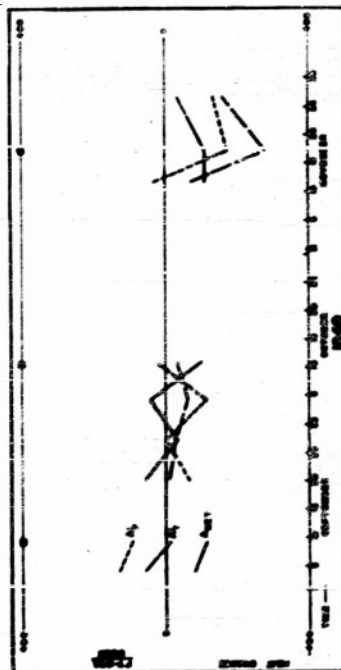


FIGURE 15

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APPENDIX

It is desirable to convert all energy amounts into a common unit which may be used to compare the amounts of energy received through the various physical processes. The unit of foot-degrees Fahrenheit (ft-deg. F) represents the energy necessary to raise the temperature of a column of water one square cm in cross-section and one foot long by one degree Fahrenheit. This is used as the standard energy unit. The system is chosen because Bathymograph readings are reported in units of degrees F and feet.

A_T Measurement

Computation of A_T is shown for the period 2-7 November 1948 at Station "C", in Figure 16. The two curves of Figure 16 are drawn to significant temperature and depth points, listed below, for the second and seventh of November, respectively. Each point represents a five-day running mean centered on the second and seventh of November. These points are as follows:

- T_S = surface temperature
- T₁₀₀ = temperature at 100 feet
- T₂₀₀ = temperature at 200 feet
- T₃₅₀ = temperature at 350 feet
- TD = depth of top of thermocline
- MLD = mixed layer depth
- MLD-2 = depth to temperature 2° lower than that at MLD

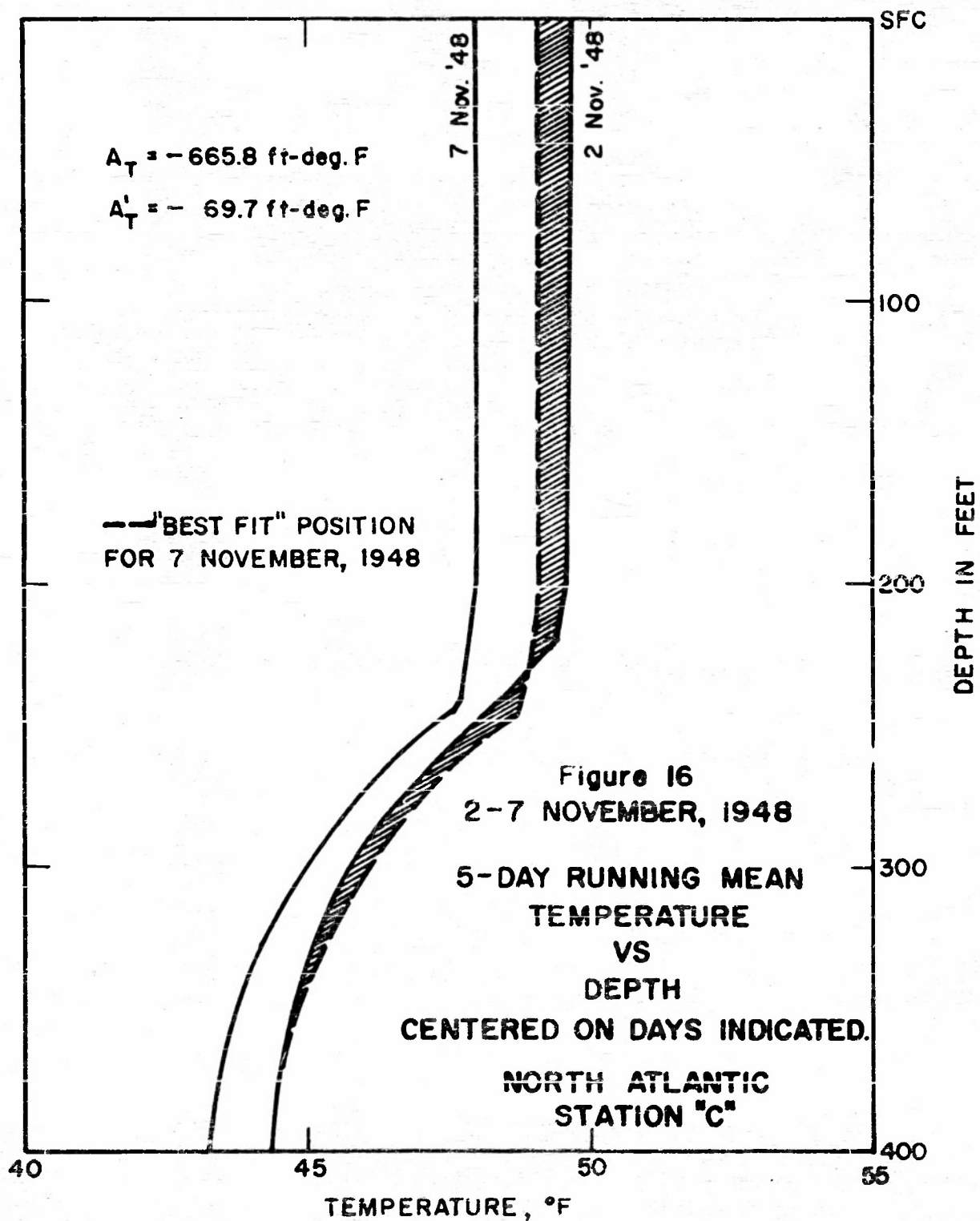
D_1, D_2, D_4 , etc. = depth from the surface to water temperatures 1, 2, 4, etc. degrees lower than the surface water.

All of these quantities are discussed in Technical Report No. 1 of this project (see references), except the quantity TD.

The thermocline depth is represented by this designation, TD, and it often is synonymous with the MLD. A distinction is made between the two items, however, when considering the permanence and prominence of the TD as compared with MLD. While the BT trace may show several small "steps" in the upper regions, each of which is classed as an MLD,* the marked boundary between the region of these small MLD's and the colder lower water is a fairly prominent slope discontinuity which is much more conservative in nature than the small step MLD's. This prominent, quasi-conservative feature is labeled as the thermocline depth, TD.

The area between the two successive curves (including an extension of each from 400' to 656' and 41°F) was measured by means of a planimeter. In the chosen example (Figure 16), the area between the second and seventh of November 1948 was -5.16 square inches. In order to interpret this area, it was then converted into ft-deg. F (the coordinates of the bathythermogram). On the scale which was used for plotting, one square inch equals 129.03 ft-deg. F; thus for the period 2-7 November 1948:

* Except for his internal wave discussion, it is this type of MLD which Schule calls the thermocline.



$$\Delta T = -5.16 \times 129.03 = -665.8 \text{ ft-deg. F}$$

The minus sign is used to indicate loss of thermal energy by the water column during the period.

Since the BT measures temperature to a depth of only 450 feet, the problem of changes in heat content below 450 feet arises. In the case of five-day running mean curves as illustrated in Figure 16, no values below 400 feet were obtained.

Sverdrup (1942) notes:* "In the Kuroshio region where the velocity of the current is great and the turbulence correspondingly intense, the annual variation of temperature becomes perceptible to a depth of about 300 meters, but in the Bay of Biscay it is very small at 100 meters. It can therefore safely be concluded that below a depth of 300 meters the temperature of the ocean is not subject to any annual variation. Since the station being studied is neither in the main current of the Gulf Stream System nor in a quiet bay, it seems reasonable to assume 200 meters as the depth of no significant change in temperature. A value of 5°C or 41°F has been taken from Chart IV of Sverdrup as the average 200 meter temperature at Station "C". In cases where the BT curves coincide, it is assumed there was no temperature change below the depth of coincidence. In cases such as Figure 16, where curves are separated at 400 feet, straight lines were drawn from the temperatures at 400 feet to a point at 41°F at

* "The Oceans", page 137.

666 feet, and the area of the resulting triangle was added to the area above 400 feet to obtain A_T for the period.

The planimeter measurements were checked until three measurements of the areas agreed within 0.04 of a square inch. These three measurements were then averaged to obtain the total used for the study.

A_T is the area enclosed between two successive BT traces when placed in a "best-fit" position. In Figure 16, the 7 November 1948 curve is translated to the position shown by the dashed line, and the area A_T is indicated by the hatched lines. This area is -70 ft-deg. F, contrasted with the corresponding A_T area of -666 ft-deg. F.

Solar Radiation

A_S is used to designate the solar radiation received at the ocean surface in units of ft-deg. F over a given time interval. The Figure 3 given by Fritz (1951) reproduces List's values of solar radiation reaching the outside of the atmosphere as functions of latitude and time. For example, on 1 September the figure is entered at latitude 52°45' north and the value 700 langley/day (gm-cal/cm²-day) is read. Subtracting 42% for albedo leaves 406 langley/day. This C.G.S. value may be expressed in terms of ft-deg. F, for seven days, as

$$406 \times \frac{1.8}{30.48} \times 7 = 168 \text{ ft-deg. F/7 days.}$$

$$\begin{aligned} 1.8 &= \text{cm/ft} \\ 30.48 \text{ cm} &= 1 \text{ foot} \end{aligned}$$

Similar computations were performed for the beginning and middle days of each month, with the values entered on a graph as shown in Figure 2. The value of A_g for any seven-day period may thus be read by entering the graph on the middle day of the interval desired. For any interval of less than seven days, prorating was done. For example, the value for a five-day interval is taken as $5/7$ the appropriate graph values. These values were checked with average values given by Landsberg in the Handbook of Meteorology (1945) and by Brunt (1939); all estimates agree as to order of magnitude.

Back Radiation

Sverdrup (1942) gives a method for estimating back radiation from the ocean as a function of ocean surface temperature (T_s) and relative humidity of the air just above the ocean. Since air temperatures (T), and dewpoint temperatures (T_d) were recorded for three-hourly intervals during the period studied, daily averages of these quantities were used to estimate the relative humidity and the back radiation.

The procedure followed in computing relative humidities is illustrated in the following example. For 1 September 1949, when the average T was 52.7°F and the average T_d was 47°F , the Smithsonian Tables were entered with these temperature values and the respective saturation values of water vapor pressure were read (0.4007 and 0.3240 in.Hg). The relative humidity is computed from the ratio of these values:

$$\frac{e_a}{e_s} \times 100 = \frac{0.3240}{0.4007} \times 100 = 80.9\%$$

The above procedure was followed for each day; averages of the daily relative humidities and T_g were made over each time interval (usually seven days). Thus for 1-7 September 1949 the average relative humidity was 86%, and the average T_g was 53°F. Entering Figure 25, p. 111, in Sverdrup (1942) the value for Q_0 is read, 0.175 gm-cal/cm²-min. $0.3 Q_0$ is 0.0525. When this is multiplied by the number of minutes in a week and converted into the standard energy unit of this study, we have

$$A_B = -0.0525 \times \frac{1.8}{30.48} \times 7 \times 24 \times 60 = -31.2 \text{ ft-deg. F}$$

Evaporation

The equation for evaporation used by Montgomery is similar to equations for vertical flux of momentum and thermal energy:

$$E = \rho K_0 \gamma_a \Gamma_a (q_w - q_a) w_a$$

where E is evaporation in gm/cm²-sec.

ρ is density of air = 1.25×10^{-6} gm/cm³

K_0 is von Karman's Constant = 0.4

γ_a is resistance coefficient = 0.03

Γ_a is evaporation coefficient = 0.085

q_w is saturation specific humidity of sea surface, T_g

q_a is specific humidity in air at anemometer height z

w_a is wind speed at height z

Now $(q_w - q_a)$ can be approximated by $\frac{0.623}{1000} (e_w - e_a)$ where e_w and e_a are the saturation vapor pressure of the ocean surface and vapor pressure of the moisture in the air, respectively; e_w is a function of T_s , and e_a is a function of T_d .

Evaporation can therefore be expressed as $E = C (e_w - e_a) w$ where $C = \rho K_o \delta_a \Gamma_a = 2.8 \times 10^{-6}$ for hydrodynamically smooth flow of wind over water. For hydrodynamically rough flow the above constant is multiplied by a factor of 3.5. This results in Jacobs' formula for evaporation:

$$E = 2.8 \times 10^{-6} \left[N(e_w - e_a) w_s + 3.5 N(e_w - e_a) w_u \right].$$

Modifying Jacobs' Formula: The products $(e_w - e_a) w$ for the "smooth" wind observations were summed separately from the "rough" wind observations each day; conversion factors to transform the computed amount into ft.-dag. F were included; the final converted form of the formula is:

$$A_E = -C \left\{ \sum_0^i (e_w - e_a) w_s + 3.5 \sum_1^j (e_w - e_a) w_u \right\}$$

where $C = 33.86 \times \frac{1}{8} \times 24 \times 51.48 \times 585 \times \frac{1/8}{30.48} \times 2.8 \times 10^{-6} = 0.5053$

$(e_w - e_a)$ inches of Hg; one inch Hg = 33.86 mb

w in knots, one knot = 51.48 cm/sec.

$\frac{24}{8} = \frac{\text{Hours per day}}{\text{Observations per day}} = \text{factor converting to evaporation/interval}$

585 = gm-cal. of heat/gm of ocean water

1.8°F = 1°C

30.48 cm = one foot

Example of A_g Computation: In order to evaluate the terms of the final evaporation formula discussed above, it was necessary to obtain the T , T_d , w and T_g observations for intervals spaced as closely together as possible. The weather ships make observations every three hours, so this time interval was used.

TABLE 7
Computations of Evaporation Parameters
For 1 September 1949

Time	T (°F)	T_g (°F)	T_d (°F)	e_a (° Hg)	e_w (° Hg)	$e_w - e_a$ (° Hg)	w Kts.	$(e_w - e_a)w$
0030Z	51.5	54	47.8	0.3339	0.4203	0.0864	24	2.0736
0330Z	51.0	53	50.6	0.3708	0.4052	0.0344	24	0.8256
0630Z	51.0	53	49.2	0.3519	0.4052	0.0533	24	1.2792
0930Z	52.0	53	46.2	0.3144	0.4052	0.0908	28	2.5424
1230Z	54.5	53	43.5	0.2836	0.4052	0.1216	24	2.9184
1530Z	55.1	53	47.1	0.3252	0.4052	0.0800	28	2.2400
1830Z	54.0	53	48.5	0.3428	0.4052	0.0624	30	1.8720
2130Z	52.5	53	44.7	0.2968	0.4052	0.1084	31	3.3573
								17.1085

Legend: T is surface air temperature

T_g is surface ocean temperature

e_a is surface air vapor pressure

e_w is ocean surface vapor pressure

w is windspeed at anemometer level

Since all eight products involve windspeeds above 6.5 m/sec

(13 knots), all products are added to obtain a "rough" evaporation

factor, $\sum_{1}^8 (e_w - e_a)w = 17.11.$

This procedure is followed for each day in the seven-day period being studied. The resulting "smooth" and "rough" evaporation factors are the following:

TABLE 8

Summing Smooth and
Rough Wind Parameters

Date	"Smooth"	"Rough"
1 Sept. 1949	0	17.11
2 Sept. 1949	0	25.81
3 Sept. 1949	0	16.49
4 Sept. 1949	0	17.89
5 Sept. 1949	3.26	0.68
6 Sept. 1949	0.40	1.43
7 Sept. 1949	0	0*
	<u>3.66</u>	<u>79.41</u>

The "smooth" factor sum plus 3.5 times the "rough" factor sum are added together:

$$3.66 + 3.5 \times 79.41 = 281.60.$$

This amount represents the bracketed part of the formula for A_E :

$$A_E = -0.5058 \left[\sum_0^1 (e_w - e_a) w_s + 3.5 \sum_1^j (e_w - e_a) w_u \right].$$

Then the total evaporation in ft-deg. F for the period is:

$$A_E = -0.5058 \times 281.60 = -142.43 \text{ ft-deg. F per 7-day period.}$$

* When the product $(e_w - e_a)w$ is negative, as occasionally happens, an evaporation factor of zero is taken; the negative terms obviously do not contribute heat to the water column.

Comparison Between A_E Computations Using Jacobs' Formula and Using the Lake Hefner Formula: The Lake Hefner study gives evaporation, E' in cm/3 hrs. as follows:

$$E' = 6.25 \times 10^{-4} u (e_w - e_a)$$

where e_w and e_a are in millibars and u is in knots.

To obtain A_E' in ft-deg. F per three hours, where e_w and e_a are in inches of Hg, and u is in knots, the constant in the above equation may be multiplied by the number of millibars in one inch of Hg, latent heat of vaporization in calories, degrees F per degree C and feet per cm. Thus A_E' in ft-deg. F per three hours is:

$$A_E' = -0.7312 (e_w - e_a) u$$

$$\text{or in } \frac{\text{ft-deg. F}}{7 \text{ days}}, A_E' = -0.7312 \sum [(e_w - e_a) u]$$

where $\sum$ is the number of three-hourly observations per week. Making use of the example of the previous paragraph to compute the seven-day A_E' by the Lake Hefner formula, it is necessary only to sum all of the three-hourly $(e_w - e_a) u$ products for the period 1-7 September 1949 and multiply by 0.7312. Since there is no distinction made here between "smooth" and "rough" flow, the totals of these two columns in Table 2 may be added to give a total $(e_w - e_a) u$ of 83.07. Then for the period 1-7 September 1949

$$A_E' = -0.7312 \times 83.07 = -60.74 \text{ ft-deg. F}$$

as compared with $A_E = -142.43 \text{ ft-deg. F}$

A_E' in Table 2 was computed in this manner.

Dynamic Effect

Using Freeman's equation for change in thermocline depth with time,

$$\frac{\partial H}{\partial t} = \frac{1}{\rho' f} \left(\frac{\partial \zeta_{xy}}{\partial x} - \frac{\partial \zeta_{zx}}{\partial y} \right)$$

the thermal energy change associated with this would be given by

$$A_D = \frac{1}{\rho' f} (T - T_{200m}) v_h \times \zeta$$

$$\text{where } v_h \times \zeta = c \times \frac{1}{(\rho f)^2} \left[\left(\frac{\Delta p}{\Delta n} \middle| \frac{\Delta p}{\Delta n} \right)_A - \left(\frac{\Delta p}{\Delta n} \middle| \frac{\Delta p}{\Delta n} \right)_B \right] / \Delta n,$$

Then the seven-day value converted to ft-deg. F is :

$$A_D = K \left[\left(\frac{\Delta p}{\Delta n} \middle| \frac{\Delta p}{\Delta n} \right)_A - \left(\frac{\Delta p}{\Delta n} \middle| \frac{\Delta p}{\Delta n} \right)_B \right] \times [\bar{T}_{MLD} - T_{200m}].$$

$$\text{where } K = \left[\frac{c}{\rho' \rho^2 f^3} \times 6.048 \times 10^5 \times \frac{1}{\Delta n} \times \frac{1.8}{30.48} \right] = 0.14152 \times 10^{10}$$

$$\rho' = \text{density of ocean water, } 1.025 \text{ gm/cm}^3$$

$$\rho = \text{density of air, } 1.25 \times 10^{-6} \text{ gm/cm}^3$$

$$f = \text{coriolis parameter; at } 52^{\circ}45'N, 1.162 \times 10^{-4} \text{ sec}^{-1}$$

$$c = \text{stress coefficient, } 3.2 \times 10^{-6} \text{ gm/cm}^*$$

$$\Delta n = 3.219 \times 10^7 \text{ cm} = 200 \text{ nautical miles}$$

$$6.048 \times 10^5 = \text{number of seconds in seven days}$$

$$\frac{1.8}{30.48} = \text{conversion from cm-deg. C to ft-deg. F}$$

* Sverdrup (1942), page 49.

The quantity $\frac{\Delta p}{\Delta n}$ is measured at points A and B at a distance $\frac{\Delta n}{2}$ to the left and right of the station on a line perpendicular to the surface isobars through the station (see Figure 17). T_{MLD} can be represented by the ocean surface temperature on the middle day of the period and T_{200m} is about 5°C (the average temperature at 200 meters depth for this station).

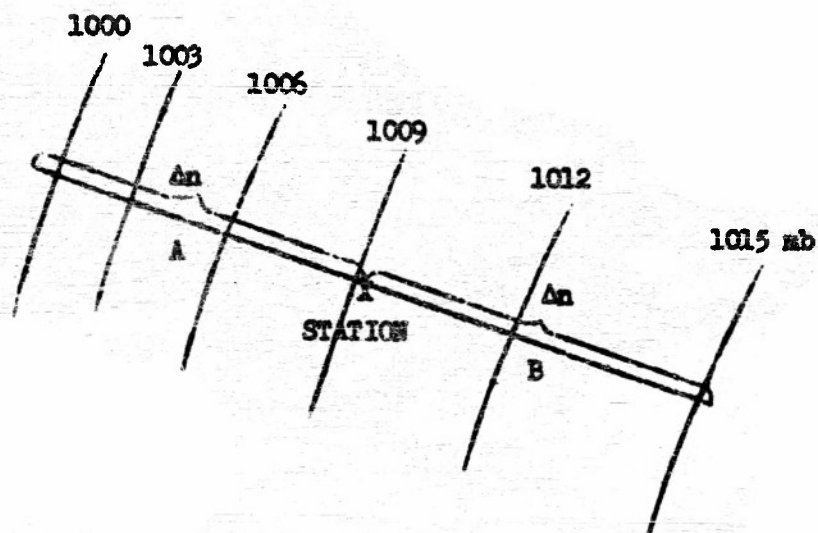
The surface isobars are usually not straight and do not have uniform curvature. This makes it difficult to pick a straight line perpendicular to the isobars. It was decided, therefore, to draw a set of N-S, E-W rectangular coordinates through the station and measure the components of $\frac{\Delta p}{\Delta n}$ on both axes (see Figure 18).

$$\left[\left(\frac{\Delta p}{\Delta n} \middle| \frac{\Delta p}{\Delta n} \right) A - \left(\frac{\Delta p}{\Delta n} \middle| \frac{\Delta p}{\Delta n} \right) B \right] \quad \text{takes the form}$$

$$\left[\left(\frac{\Delta p}{\Delta n} \middle| \frac{\Delta p}{\Delta n} \right) A - \left(\frac{\Delta p}{\Delta n} \middle| \frac{\Delta p}{\Delta n} \right) B + \left(\frac{\Delta p}{\Delta n} \middle| \frac{\Delta p}{\Delta n} \right) C - \left(\frac{\Delta p}{\Delta n} \middle| \frac{\Delta p}{\Delta n} \right) D \right] \quad \text{in which}$$

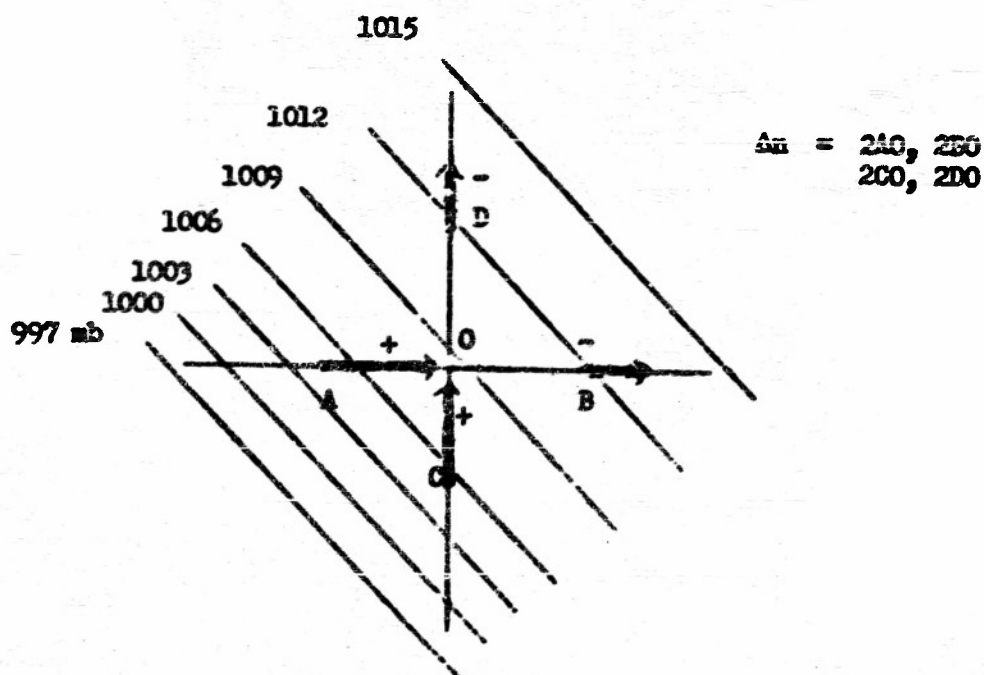
the difference $\left(\frac{\Delta p}{\Delta n} \middle| \frac{\Delta p}{\Delta n} \right) A - \left(\frac{\Delta p}{\Delta n} \middle| \frac{\Delta p}{\Delta n} \right) B$ represents $\frac{dQ_x}{dx}$; and $\left(\frac{\Delta p}{\Delta n} \middle| \frac{\Delta p}{\Delta n} \right) C - \left(\frac{\Delta p}{\Delta n} \middle| \frac{\Delta p}{\Delta n} \right) D$ represents $\frac{dQ_y}{dy}$.^{*} It will be assumed that all transport by wind takes place in the mixed layer (at 90° to the right of the wind direction); and that the surface isobars represent the wind direction as indicated by the arrows. It can be seen that water transport is directed opposite to the atmospheric pressure gradient. For the pressure distribution indicated in Figure 18, the transport would be directed as shown by the vectors at A,

* $1/\Delta n$ has been included in K .



Atmospheric Pressure Gradient Measurement

FIGURE 17



Resolution of Atmospheric Pressure Gradient into Components

FIGURE 18

B, C and D. Bringing water into the column adds heat while taking out water subtracts heat. To calculate A_p , the vectors must be given the signs shown in Figure 18. Thus if Figure 18 is used as a model, and the pressure gradients at A, B, C and D are considered positive, the sign of each quantity $\frac{\Delta n}{\Delta n} \left| \frac{\Delta n}{\Delta n} \right|$ may be determined independently in any given case.

Example of A_p Computation: Computation of A_p for the period 4-8 September 1949, at "C" using five-day mean sea level pressure map for 3-7 September 1949.

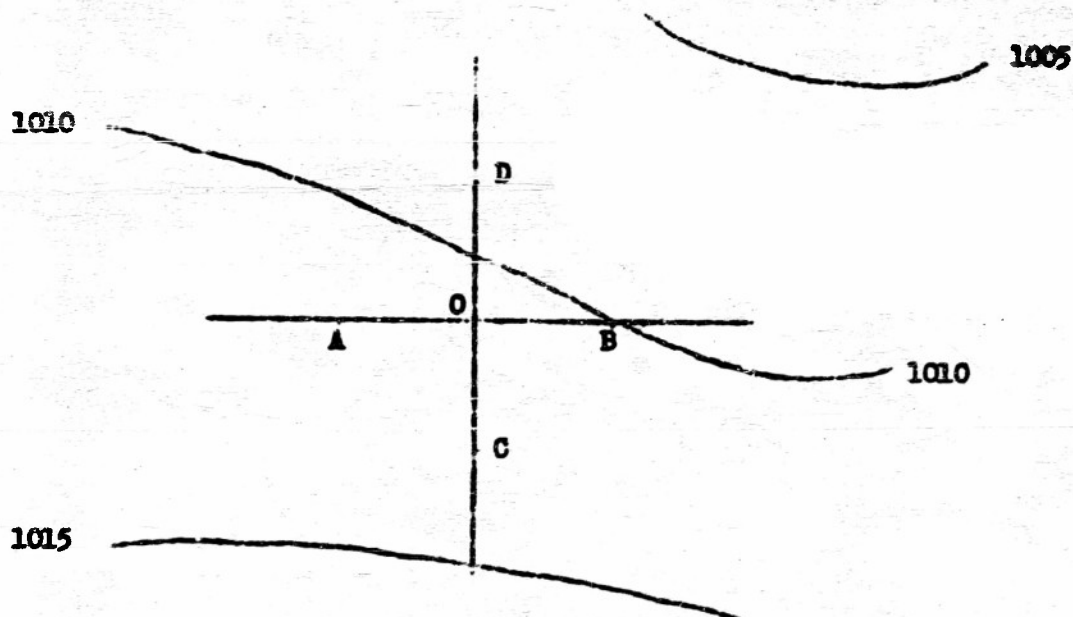


FIGURE 19

Surface Isobars on U.S.W.B. Five-Day Mean
Surface Chart for 4-8 September 1949

Distances Δn (200 Nautical miles) are laid out on the axes through Station "C" (see Figure 19) and Δp (in mb) is measured over the distances 2A0, 2B0, 2C0 and 2D0 and entered in columns 1, 2, 3 and 4 in Table 9. The Δp 's are then divided by Δn and multiplied by 10^3 (conversion of mb to C.G.S. units), each quotient multiplied by its own absolute value, preserving the sign, and entered in columns 5, 6, 7 and 8 of Table 9. The signs of the terms are determined by reference to the model. Enter in column 9 the sum of columns 5, 6, 7 and 8. This is the total volume transport. Multiply each term in column 9 times K and enter in column 10; values in column 10 are multiplied by $(T_8 - T_{200})$ found in column 11 to give the final quantity A_p in column 12.

TABLE 9

Computation of Vertical
Advection Parameters

1949	1	2	3	4	5	6
	ΔP_A mb	ΔP_B mb	ΔP_C mb	ΔP_D mb	$\left(\frac{\Delta p}{\Delta n} \left \frac{\Delta p}{\Delta n} \right \right)_A$	$\left(\frac{\Delta p}{\Delta n} \left \frac{\Delta p}{\Delta n} \right \right)_B$
4-8 Sept.	-0.7	-1.8	-4.0	-3.8	-0.0473	0.3127
					$\times 10^{-8}$	$\times 10^{-8}$
1949	7	8	9	10	11	12
	$\left(\frac{\Delta p}{\Delta n} \left \frac{\Delta p}{\Delta n} \right \right)_C$	$\left(\frac{\Delta p}{\Delta n} \left \frac{\Delta p}{\Delta n} \right \right)_D$	Sum	$\frac{2/5 \text{ ft}}{\text{msec}}$	$(T_8 - T_{200})$ of °C	A_p ft. sec. $\times 10^8$
4-8 Sept.	-1.5440	1.3935	0.1149	1.626	6.6	10.73
	$\times 10^{-8}$	$\times 10^{-8}$	$\times 10^{-8}$			

Since the periods of the five-day mean sea level pressure maps were not the same as those into which the ET data were divided, the A_p results were plotted on a time graph and values of A_p then determined for the periods of ET data.