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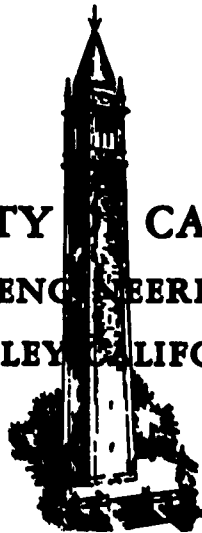
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A LINEAR THEORY OF SHIP MOTION IN IRREGULAR WAVES

By

R. A. Fuchs and R. C. MacCamy

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R. A. Fuchs and R. C. MacCamy

Abstract

This report deals with the heaving and pitching motions of unpropelled ship models in irregular bow or stern seas. A time record of the motions is obtained from a knowledge of the water surface history at one station along the model, using a Fourier integral analysis, as discussed by Fuchs(2)*. Thus, time histories of heaving and pitching of the ship model can be expressed as the convolution-type integral of the recorded wave motion and a kernel function. This kernel is the Fourier transform of the ship model's response to a sinusoidal forcing function. Using the Froude-Kriloff hypothesis, kernels are explicitly computed for oscillations of a rectangular block. The kernel for a model of ship form is determined from the results of a series of experiments using sinusoidal waves, and independently from a single experiment using irregular waves. Predicted time histories agree quite well with recorded time histories, read from 35 mm. motion picture records

* Numbers refer to references listed at end of report

SYMBOLS

A	=	water plane area
A_0	=	equilibrium water plane area
a	=	wave amplitude = semi-wave height
a_k	=	coefficient of Fourier exponents of $x_p(\sigma)$
B	=	beam of ship
b	=	$N_1/2M$
c	=	$N_2/2I$
E	=	general notation for a linear effect
F_1	=	force function for heaving
g	=	acceleration of gravity
h	=	water depth
I	=	virtual moment of inertia = $\bar{I} + \underline{I}$
\underline{I}	=	moment of inertia of object
\bar{I}	=	hydrodynamic moment of inertia of object
I^*	=	hydrodynamic moment of inertia per unit of length
i	=	$\sqrt{-1}$
k	=	wave number = $2\pi / \text{wave-length}$
K_E	=	kernel function for a linear effect E
L	=	semi-length of block
d	=	draft
L_j	=	turning moment function for pitching
M	=	virtual mass = $\bar{M} + \underline{M}$
\underline{M}	=	mass of object
\bar{M}	=	hydrodynamic mass of object
M^*	=	hydrodynamic mass per unit length
N_1^*	=	damping coefficient per unit length

- N_1 = damping coefficient for heaving
 N_2 = damping coefficient for pitching
 n = outward normal to immersed portion of hull
 P_0 = projection of S_0 onto the yz -plane
 p = pressure in incident wave
 r_h, r_p = complex response factors for heaving and pitching respectively
 S = immersed surface of ship
 S_0 = immersed surface of ship in equilibrium
 t = time
 V = immersed volume
 V_0 = immersed volume in equilibrium position in still water
 x = horizontal dimension, positive in the direction of wave travel
 y = vertical dimension, positive upwards with zero at still water level
 X_E, Y_E = real and imaginary parts, respectively, of the associated response factors
 $(X_r, Y_r; X_h, Y_h)$
 z = horizontal dimension perpendicular to xy -plane
 α = $\frac{2 B g \rho}{M}$
 β = $\bar{M}/M L$
 γ = $N_1/M L$
 ϵ = phase angle
 ζ = heaving distance
 η = wave surface elevation
 θ = pitching angle
 μ = $\frac{2 E \rho g}{I}$
 ρ = density of water
 σ = wave frequency = $2 \pi / \text{period}$
 τ = variable of integration, time

iv.

ϕ = velocity potential

$$\omega_n = \sqrt{\frac{\rho g A}{M}}$$

$$\omega_p = \sqrt{\frac{2 \rho g B L^3}{3 I}}$$

A Linear Theory of Ship Motion in Irregular Waves

Introduction

In a report by Fuchs and Einarson⁽¹⁾, the motion of a freely floating form in periodic progressive waves was studied. The motions of heaving and pitching were analyzed into a series of powers of the wave steepness and the first two terms were given explicitly for a rectangular block. The principal effect of the nonlinearity, as given by terms of the order of the square of the wave steepness, is to introduce coupling between heaving, pitching and surging and to introduce small terms of higher frequencies, which cause a shift of maximum values and an additional oscillation in the wave trough.

It is the object of the present paper to extend the previous theory so that it applies to complex wave motions. This practically restricts our efforts to the linear theory, and, hence, the problem is a special case of the linear prediction theory discussed by Fuchs⁽²⁾. Thus the oscillations of a ship in a complex system of progressive waves of small steepnesses can be expressed as the convolution type integral of the recorded wave motion and a kernel function which is the Fourier integral of the response of the ship to a sinusoidal forcing function. Kernels for pitching and heaving are computed explicitly for a freely floating rectangular block using experimental values of added masses and damping coefficients, as determined from oscillations in still water. These kernel functions are applied to recorded irregular wave motions and the predictions are compared with values read from 35 mm. motion picture records of the motion. The experimental work has been reported separately by Sibul⁽³⁾. Similar investigations have been made with a model ship. In order to avoid the laborious numerical integrations involved in computing the response functions, these were determined experimentally.

Theory

We adopt Kriloff's original suggestion and assume that the force exerted by the waves on the ship's hull can be represented as the sum of a virtual mass force, a linear damping force and the force arising from the incident wave acting on the hull in its equilibrium position. It is assumed that the virtual mass and damping coefficients are constants which can be determined from oscillations in still water, although in reality they vary with frequency as has been shown by Havelock⁽⁴⁾. With these assumptions, we can picture the ship in waves as a linear dynamical system for which the forcing function is the sum of the unperturbed wave force and the additional force due to oscillations in still water, with modifications for the relative motion. Thus we explicitly neglect the effect of the diffracted wave. Note that the complete solution of the linear theory problem requires the solution of the diffraction and forced oscillations problems.

Under these conditions the equations of heaving and pitching are respectively:

$$M \frac{d^2 \zeta}{dt^2} + M^* \int_{-L}^L \frac{d^2}{dt^2} (\zeta - n) dx + N_1 \int_{-L}^L \frac{d}{dt} (\zeta - n) dx + g \rho A \zeta = \iint_S \rho \cos \hat{n}_y dS \quad (1)$$

$$I \frac{d^2 \theta}{dt^2} + I^* \frac{d^2 \theta}{dt^2} + N_2 \frac{d\theta}{dt} + \frac{2}{3} \rho g B L^3 \theta = \iint_S (x p \cos \hat{n}_y - y p \cos \hat{n}_x) dS \quad (2)$$

The second and third terms on the left hand sides of these equations represent the vertical mass force or moment of inertia and the damping forces, respectively. The effect of relative motion, indicated by the difference $(\zeta - n)$ in the integrated terms, is considered only for heaving, since the additional calculations introduced are rather tedious and do not appreciably effect the result. Substituting for p the unperturbed wave pressure we show in Appendix I that the heaving and pitching motions of a rectangular block are given in terms of the wave amplitude η by complex response factors r_h and r_p in the form:

$$\zeta = r_h(\sigma) \eta(\sigma, t), \quad \theta = r_p(\sigma) \eta(\sigma, t) \quad (3)$$

The functions $r_h(\sigma)$ and $r_p(\sigma)$ are given explicitly on pages 9 and 10, Appendix II, and their real and imaginary parts are plotted versus σ in Figures 1 and 2. The real and imaginary parts of such a complex response factor represent "in-phase" and "out-phase" components respectively.

For a progressive wave motion which is not purely sinusoidal, the general theory presented by Fuchs⁽²⁾ is applicable and the heaving and pitching motion may be written in the form:

$$E(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \eta(\tau) K_E(t-\tau) d\tau \quad (4)$$

where the kernel is

$$K_E(\tau) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} r_E(\sigma) e^{-i\sigma\tau} d\sigma \quad (5)$$

$r_E(\sigma)$ being the response of the block to a sinusoidal wave of unit amplitude and frequency σ . Note that $r_E(\sigma)$ has real and imaginary parts which are even and odd, respectively, which guarantees that E is

real for a real η . Thus if we write $r_E = X_E + i Y_E$, where X_E, Y_E are real functions of σ , then K_E can be written in the form

$$\begin{aligned} K_E(\tau) &= \sqrt{2\pi} \int_0^{\infty} X_E(\sigma) \cos \sigma \tau d\sigma + \sqrt{2\pi} \int_0^{\infty} Y_E(\sigma) \sin \sigma \tau d\sigma \\ &= K_E^{(C)}(\tau) + K_E^{(S)}(\tau) \end{aligned} \quad (6)$$

The details of the calculation of these kernels are given in Appendix II. The results are plotted in Figures 3 and 4.

The principal advantage of this formulation is that the kernel is independent of the particular wave system encountered and therefore need be calculated only once, thereby eliminating repetitious operations otherwise required. The single operation of Equation (4) combines the three separate operations of the common Fourier analysis consisting of an analysis of $\eta(t)$ into its Fourier components, multiplication of each component by its response factor and finally synthesis to obtain $E(t)$.

The kernel can of course be computed if the response is determined empirically. For our purpose these methods are convenient for ship models, since they replace the laborious calculation of force components due to the unperturbed wave pressure acting on the immersed portion of the hull, with simple laboratory measurements. This calculation can be performed either by a series of tests of the model in sinusoidal waves or by Fourier analysis of records secured in a wave motion having a broad frequency spectrum. The latter method has been carried out for the rectangular block and the results are presented in Figure 5. Run 8-A was used to compute the heaving response, from which the corresponding heaving kernel is obtained directly. This kernel was then applied to Run 8-A as a check, and also to Run 10 which did not enter in the calculations of the kernel.

The results for pitching were much less satisfactory, since the important range of frequencies in the pitching response occurs for higher frequencies (see Figures 1 and 2); and for these frequencies the transforms are difficult to compute accurately. It is suggested therefore that this technique probably could be used to advantage only for heaving. It should be noted, finally, that the calculations will be more accurate, the more irregular the wave form used in the experiment; since only for a fairly irregular wave will components of different frequencies be present in appreciable amounts.

The second method consists of merely placing the model to be tested in uniform wave trains of several different frequencies and recording the heaving and pitching motions. Suppose that a surface profile of the form:

$$\eta = a \cos \sigma t \quad (7)$$

is used and the resulting heaving and pitching motions are

$$\zeta = \zeta_0 \cos(\sigma t - \epsilon), \quad \theta = \phi_0 \cos(\sigma t - \sigma) \quad (8)$$

then X_h, Y_h, X_p, Y_p may be determined by the pairs of equations

$$|r_h| = \sqrt{X_h^2 + Y_h^2} = \zeta_0/a, \quad Y_h/X_h = \tan \epsilon \quad (9)$$

$$|r_p| = \sqrt{X_p^2 + Y_p^2} = \phi_0/a, \quad Y_p/X_p = \tan \tau$$

The ambiguity in sign may be eliminated by the relations

$$X_h = |r_h| \cos \epsilon, \quad Y_h = |r_h| \sin \epsilon, \quad X_p = |r_p| \cos \tau, \quad (10)$$

$$Y_p = |r_p| \sin \tau$$

This analysis was carried out on an actual ship model. The empirical response factors were determined and are plotted in Figures 10 and 11. Then transforms were obtained to give the kernel functions for heaving and pitching. The kernels were then applied to an irregular wave train, and the results are shown in Figure 12.

In considering the effects due to non-uniform wave trains, a method which immediately suggests itself is the treatment "wave by wave". In this, each oscillation of the surface would be considered as part of an infinite wave train of height and period equal to that of the wave in question. The results of this report offer some indications of the validity of this approach. To this end such an analysis was carried out on two of the runs, namely Runs 13 and 8-A. The results are indicated in Figures 6 and 9 by the small circles which represent the amplitude and approximate position of the maximum heaving and pitching. It is seen that while the results are fairly good for Run 13, which is relatively smooth, this analysis does not give even a rough approximation in Run 8-A, which is quite irregular. It is noted, furthermore, that the results for pitching are poorer than the results for heaving. An examination of the kernel function in Figures 3 and 4 will show that this is to be expected, for the pitching kernel dies out more slowly with τ . Generally speaking the rate at which the amplitude of the kernel of a given effect decreases with τ is a measure of the dependence of that effect, at a given instant, on the past and future behavior of the surface elevation.

A second problem which may be discussed in connection with this report is the difference in the central frequency of the Fourier spectra of the surface elevation and the pitching. If one has a given power spectrum, that is the sum of the squares of the amplitudes of the cosine and sine Fourier transforms, of a surface elevation, then the power spectrum for pitching motion may be obtained from it by multiplying by the absolute value of the pitching response $r_p(\sigma)$. Since this latter quantity has

its maximum at a fixed frequency, and the surface spectrum may be centered about different frequencies, the result of this multiplication would be to shift the central frequency toward the center of the response curve; that is, a ship would tend to pitch at its natural frequency. Two examples of this process are presented in Figure 14, using hypothetical spectra for η . The absolute value of the pitching response is plotted in Figure 13. These examples show that if the spectrum of η is sufficiently broad the process may result in an appreciable shift in frequency. The effect would also be present in heaving but would probably be small due to the relatively flat response in heaving. This appears to explain the observations recently reported by Williams⁽⁶⁾ to the effect that under confused sea conditions, the predominant rolling period was the natural rolling period. It does not appear to suggest, however, the absence of forced rolling, as Havelock⁽⁶⁾ has observed.

Acknowledgement

We are indebted to W. Nichol for many of the numerical calculations involved.

APPENDIX ICalculation of Response Factors for a Rectangular Block

In a previous report by Fuchs and Einarson(1), the response factors were computed for the pitching and heaving of a freely floating rectangular block in periodic waves. However, for the range of wave lengths considered in this report a more refined analysis of the relative motion terms is required. Owing to the requirements of the prediction theory, however, we must confine our attentions to the linear theory.

The forcing functions in heave and pitch are then

$$F_i = \int_S p \cos \hat{n}_y ds \quad (\text{I. 1})$$

$$L_i = \int_S (xp \cos \hat{n}_y - yp \cos (nx)) ds \quad (\text{I. 2})$$

where p is the pressure field of the incident wave and the integrations are taken over the immersed portion of the hull when it is in its still water equilibrium position. For the range of frequencies with which we shall be concerned, there is no appreciable lack of accuracy in assuming deep water conditions for all components. Then we have

$$p = \rho g \eta - \rho \frac{\partial \phi}{\partial t} \quad (\text{I. 3})$$

where

$$\phi = i \frac{g a}{\sigma} e^{ky} e^{i(kx - \sigma t)} \quad (\text{I. 4})$$

is the complex velocity potential of the incident waves associated with the surface elevation

$$\eta = a e^{i(kx - \sigma t)} \quad (\text{I. 5})$$

This is a periodic wave of amplitude a with its frequency σ and wave number k related by the equation

$$\sigma^2 = g k \quad (\text{I. 6})$$

This wave moves in the positive x direction with the constant velocity

$$c = \sigma/k = \sqrt{g/k};$$

for convenience we write

$$F_i = \int_S p \cos \hat{n}_y ds = \int_{A_0} p(s) dA_0 \quad (\text{I. 7})$$

where A_0 is the projection of the immersed hull surface S_0 on the xz -plane. Substituting, we find

$$F_i = \int_{A_0} \left[g\rho(d+\zeta+\theta_x) + \rho g a e^{-kd} e^{i(kx-\sigma t)} \right] d A_0 \quad (\text{I. 8})$$

$$= \rho g V_0 + \rho g A_0 \zeta - \rho g B a e^{-kd} \frac{\sin kL}{k} e^{-i\sigma t}$$

where V_0 is the immersed volume of the block in equilibrium, A_0 is the equilibrium value of the water plane area, and B, L, d are the beam, half-length and draft, respectively.

$$L_i = \int_{A_0} x \rho d A_0 - \int_{P_0} y \rho d P_0 \quad (\text{I. 9})$$

where P_0 is the projection of S_0 onto the yz -plane. Substituting, we find

$$L_i = \int_{A_0} \left[g\rho(d+\zeta+\theta_x) + \rho g a e^{-kd} e^{i(kx-\sigma t)} \right] (x d A_0 - y d P_0) \quad (\text{I. 10})$$

$$= \frac{2}{3} \rho g B L^3 \theta - 2i B \rho g a \frac{e^{-kd}}{k^2} \left[\sin kL - kL \cos kL \right] e^{-i\sigma t}$$

$$+ 2i B \rho g a \frac{\sin kL}{k^2} (1 - kd e^{-kd} - e^{-kd}) e^{-i\sigma t}$$

The relative motion terms are

$$M^* \int_{-L}^L \frac{d^2}{dt^2} (\zeta - \eta) dx = \bar{M} \frac{d^2 \zeta}{dt^2} - M^* \int_{-L}^L \frac{d^2 \eta}{dt^2} dx \quad (\text{I. 11})$$

$$= \bar{M} \frac{d^2 \zeta}{dt^2} + M^* a \sigma^2 \frac{2 \sin kL}{k} e^{-i\sigma t}$$

$$N_1^* \int_{-L}^L \frac{d}{dt} (\zeta - \eta) dx = N_1 \frac{d\zeta}{dt} - N_1^* \int_{-L}^L \frac{d\eta}{dt} dx \quad (\text{I. 12})$$

$$= N_1 \frac{d\zeta}{dt} + i\sigma a \frac{2 \sin kL}{k} e^{-i\sigma t}$$

On introducing the notation

$$2b = N_1/M, \quad \omega_h^2 = \frac{\rho g A}{M}, \quad \alpha = \frac{2B \rho g}{M}, \quad \beta = \bar{M}/ML, \quad (I. 13)$$

$$\gamma = \eta_1/ML, \quad 2c = N_2/I, \quad \omega_p^2 = \frac{2\rho g BL^3}{3I}, \quad \mu = \frac{2B\rho g}{I}$$

$$\rho(\sigma) = -\frac{d \cdot ka}{k^2} \left\{ \sin kL - kL \cos kL \right\} + \frac{\sin kL}{k^2} \left\{ 1 - kd e^{-kd} - e^{-kL} \right\}$$

the equations can be written in the form

(I. 14)

$$\frac{d^2 \zeta}{dt^2} + 2b \frac{d\zeta}{dt} + \omega_h^2 \zeta = \left\{ \alpha h(\sigma) e^{-kd} - \frac{\sin kL}{k} \beta \sigma^2 - i \frac{\sin kL}{k} \gamma \sigma \right\} a e^{-i\sigma t}$$

$$\frac{d^2 \theta}{dt^2} + 2c \frac{d\theta}{dt} + \omega_p^2 \theta = \mu \rho(\sigma) a e^{-i\sigma t} \quad (I. 15)$$

neglecting the transient oscillation which dies out exponentially, the forced oscillation can be written in the form

$$\zeta(t) = r_h(\sigma) \eta(t) \quad (I. 16)$$

$$\theta(t) = r_p(\sigma) \eta(\sigma) \quad (I. 17)$$

where r_h and r_p are complex response factors given in Appendix II. These functions are plotted in Figures 1 and 2.

The transition to irregular wave forms is accomplished by representing the surface by its Fourier transform

$$\eta(t) = 1/\sqrt{2\pi} \int_{-\infty}^{+\infty} a(\sigma) e^{-i\sigma t} d\sigma \quad (I. 18)$$

The preceding results may then be applied by making use of the principle of superposition to give, for heaving or pitching,

$$E(t) = 1/\sqrt{2\pi} \int_{-\infty}^{+\infty} r_E(\sigma) a(\sigma) e^{-i\sigma t} d\sigma \quad (I. 19)$$

This last integral may then be transformed into a convolution integral involving the surface elevation and a kernel

$$E(t) = 1/\sqrt{2\pi} \int_{-\infty}^{+\infty} \eta(\tau) K_E(t-\tau) d\tau \quad (I. 20)$$

where

$$K_E(\tau) = 1/\sqrt{2\pi} \int_{-\infty}^{+\infty} r_E(\sigma) e^{-i\sigma\tau} d\sigma \quad (I. 21)$$

It has already been indicated that the response may be determined empirically by measurements in an irregular wave train. To see this we take the Fourier transform of equation (I. 19). Then noting that the response r_E is the inverse transform of the kernel K_E , we obtain

$$r_E(\sigma) = \frac{b_E(\sigma)}{a(\sigma)} \quad (\text{I. 22})$$

where $b_E(\sigma)$ and $a(\sigma)$ are the transforms of the heaving or pitching and surface elevation respectively.

APPENDIX II

Numerical Calculations

For convenience, the equations used to determine the heaving and pitching kernels together with the values of the numerical constants involved are listed below, X and Y representing the real and imaginary parts of r , respectively.

$$K_h(\tau) = \sqrt{2/\pi} \int_0^\infty X_h(\sigma) \cos \sigma \tau d\sigma + \sqrt{2/\pi} \int_0^\infty Y_h(\sigma) \sin \sigma \tau d\sigma = K_h^c(\tau) + K_h^s(\tau)$$

$$X_h(\sigma) = \frac{(ah(\sigma) - \frac{\sin kL}{k} \beta \sigma^2)(\omega_h^2 - \sigma^2) + 2b \frac{\sin kL}{k} \gamma \sigma^2}{(\omega_h^2 - \sigma^2)^2 + 4b^2 \sigma^2} \quad (\text{II. 1})$$

$$Y_h(\sigma) = \frac{2b \sin kL (\sigma) - 2b\beta \frac{\sin kL}{k} \sigma^3 - \gamma \frac{\sin kL}{k} (\omega_h^2 - \sigma^2)}{(\omega_h^2 - \sigma^2)^2 + 4b^2 \sigma^2}$$

$$r_h = \sqrt{X_h^2 + Y_h^2}$$

$$h(\sigma) = e^{-kd} \frac{\sin kL}{k}$$

$L = 0.50 \text{ ft.}, \quad d = 0.109 \text{ ft.}, \quad B = 0.425 \text{ ft.}, \quad M = 0.199 \text{ lbs. ft}^{-1} \text{ sec.}^2,$
 $\underline{M} = 0.097 \text{ lbs. ft.}^{-1} \text{ sec.}^2, \quad N_1 = 0.66 \text{ lbs. ft.}^{-1} \text{ sec.}, \quad \omega_h^2 = 133 \text{ sec.}^{-2},$
 $2b = N_1/M = 3.30 \text{ sec.}^{-1}, \quad a = \frac{2B\rho g}{M} = 267 \text{ ft.}^{-1} \text{ sec.}^{-2}, \quad \beta = \underline{M}/ML = 1.00 \text{ ft.}^{-1},$
 $\gamma = N_1/ML = 6.66 \text{ ft.}^{-1} \text{ sec.}^{-1}.$

$$K_p(\tau) = \sqrt{2/\pi} \int_0^\alpha X_p(\sigma) \cos \sigma \tau d\sigma + \sqrt{2/\pi} \int_0^\alpha Y_p(\sigma) \sin \sigma \tau d\sigma = K_p^c(\tau) + K_p^s(\tau)$$

$$X_p(\sigma) = \frac{-2c\sigma\mu p(\sigma)}{(\omega_p^2 - \sigma^2)^2 + 4c^2\sigma^2} \quad (\text{II. 2})$$

$$Y_p(\sigma) = \frac{(\omega^2 - \sigma^2)\mu p(\sigma)}{(\omega_p^2 - \sigma^2)^2 + 4c^2\sigma^2} \quad r_p = \sqrt{X_p^2 + Y_p^2}$$

$$p(\sigma) = \frac{e^{-k d}}{k^2} \left\{ \sin kL - kL \cos kL \right\} - \frac{\sin kL}{k^2} \left\{ 1 - kd e^{-kd} - e^{-kd} \right\}$$

$$I = 0.014 \text{ lbs. ft. sec.}^2, \quad N_2 = 0.071 \text{ lbs. ft. sec.}, \quad \omega_p^2 = 133 \text{ sec.}^{-2}, \\ \mu = \frac{2B\rho g}{I} = 3780 \text{ ft.}^{-3} \text{ sec.}^{-2}, \quad 2c = N_2/I = 5.26 \text{ sec.}^{-1}.$$

Two methods of computation were used to carry out the integration in equations (II. 1) and (II. 2). In keeping with the approximate nature of the pitching response due to neglect of the relative motion terms, an approximate method was used for equation (II. 2). If the function $X_p(\sigma)$ is extended so as to form an even function of σ , then over the range $-\bar{\sigma} < \sigma < +\bar{\sigma}$ it may be approximated by a finite Fourier sum involving only cosine terms; that is,

$$X_p(\sigma) \cong a_0 + a_1 \cos \frac{\pi\sigma}{\bar{\sigma}} + \dots + a_n \cos \frac{n\pi\sigma}{\bar{\sigma}} \quad (\text{II. 3})$$

where

$$a_k = \frac{2}{\bar{\sigma}} \int_0^{\bar{\sigma}} X_p(\sigma) \cos \frac{n\pi\sigma}{\bar{\sigma}} d\sigma \quad (\text{II. 4})$$

Hence, if $\bar{\sigma}$ is taken sufficiently large that $X_p(\sigma)$ is approximately zero for $\sigma > 0$, the a_k 's give an approximation to $K_p^c(\sigma)$ at the points $\frac{k\pi}{\bar{\sigma}}$, $k = 0, \dots, n$; that is,

$$K_p^c\left(\frac{k\pi}{\bar{\sigma}}\right) \simeq \frac{\bar{\sigma}}{\sqrt{2\pi}} a_k, \quad k = 0, \dots, n \quad (\text{II. 5})$$

In an exactly analogous manner, if $Y_p(\sigma)$ is extended so as to form an even function it may be approximated by a finite sine series, with coefficients b_1, \dots, b_n , and these give

$$K_p^s\left(\frac{k\pi}{\bar{\sigma}}\right) = \frac{\bar{\sigma}}{\sqrt{2\pi}} b_k, \quad k = 1, \dots, n \quad (\text{II. 6})$$

A Fourier analysis was carried out using the standard 24 ordinate scheme, giving the value of K_p^c at thirteen values of σ , and K_p^s at eleven values.

The integrals in equation (II. 1) were evaluated with somewhat more accuracy. For large values of τ asymptotic representations are available for $K_h^c(\tau)$ and $K_h^s(\tau)$ in the form

$$K_h^c(\tau) = -\frac{X_h'(0)}{\tau^2} + \frac{X_h'''(0)}{\tau^4} - \frac{X_h^{(5)}(0)}{\tau^6} + \dots \quad (\text{II. 7})$$

$$K_h^s(\tau) = \frac{Y_h(0)}{\tau} - \frac{Y_h''(0)}{\tau^3} + \frac{Y_h^{(4)}(0)}{\tau^5} - \dots$$

where primes indicate differentiation with respect to σ . The indicated differentiations were carried out and it was found that K_h^c and K_h^s

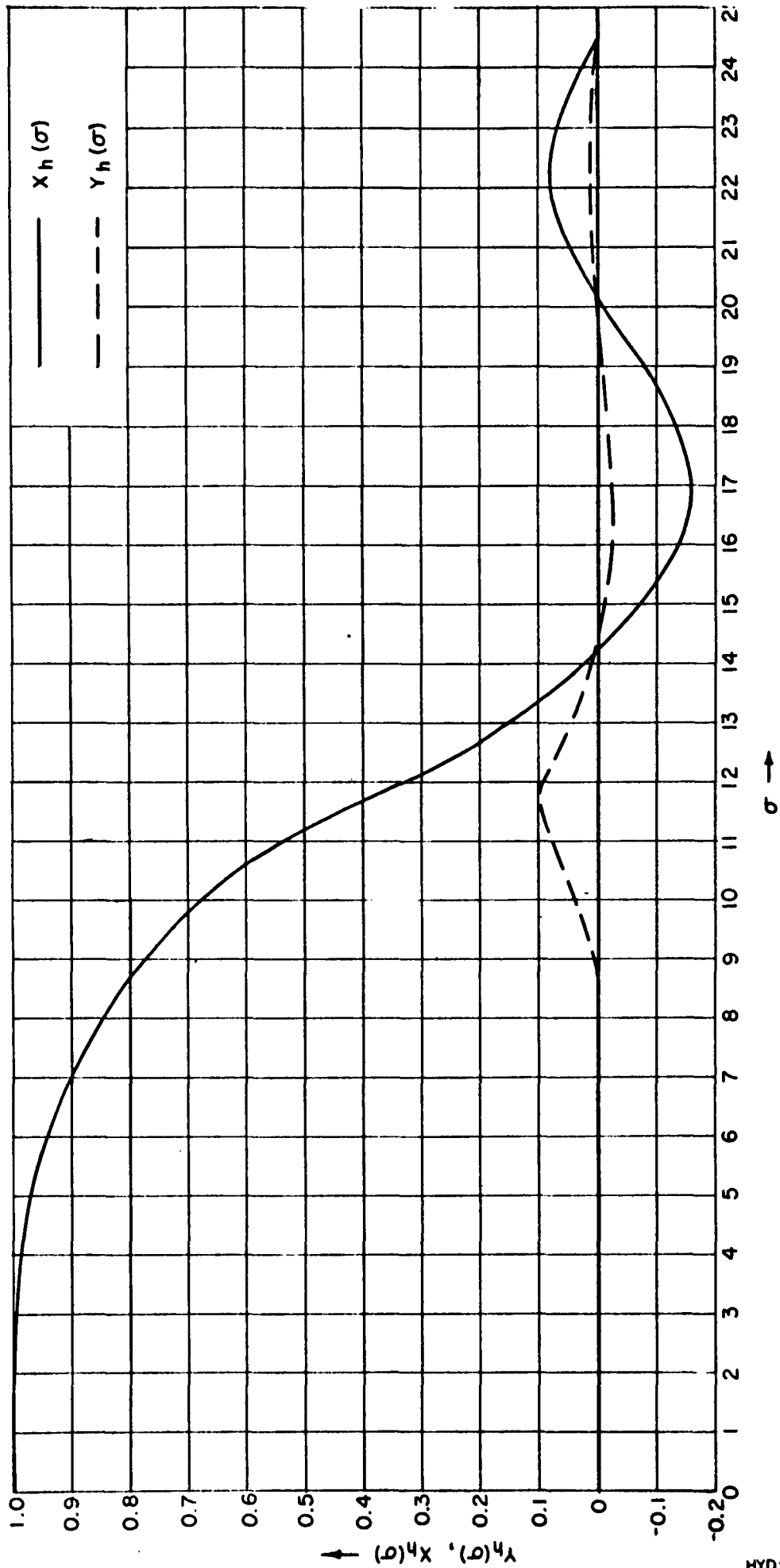
may be taken to be zero for $\tau > 0.8$. In the region $\tau < 0.8$, K_h^c and K_h^s were computed using Filon's formula⁽⁵⁾ for trigonometric integrals.

The errors in the use of this formula can be estimated from the third differences of $X_h(\sigma)$ and $Y_h(\sigma)$, and were kept less than two per cent of the maximum values.

The convolution integrals involving $\eta(t)$ and the kernels for heaving and pitching were carried out using the trapezoid rule with an interval of 0.1 seconds. The integrations involved in determining the transforms of η and ζ , which led to the empirical response factor for heaving were carried out using Filon's formula and the kernel obtained by applying the Fourier series technique already outlined.

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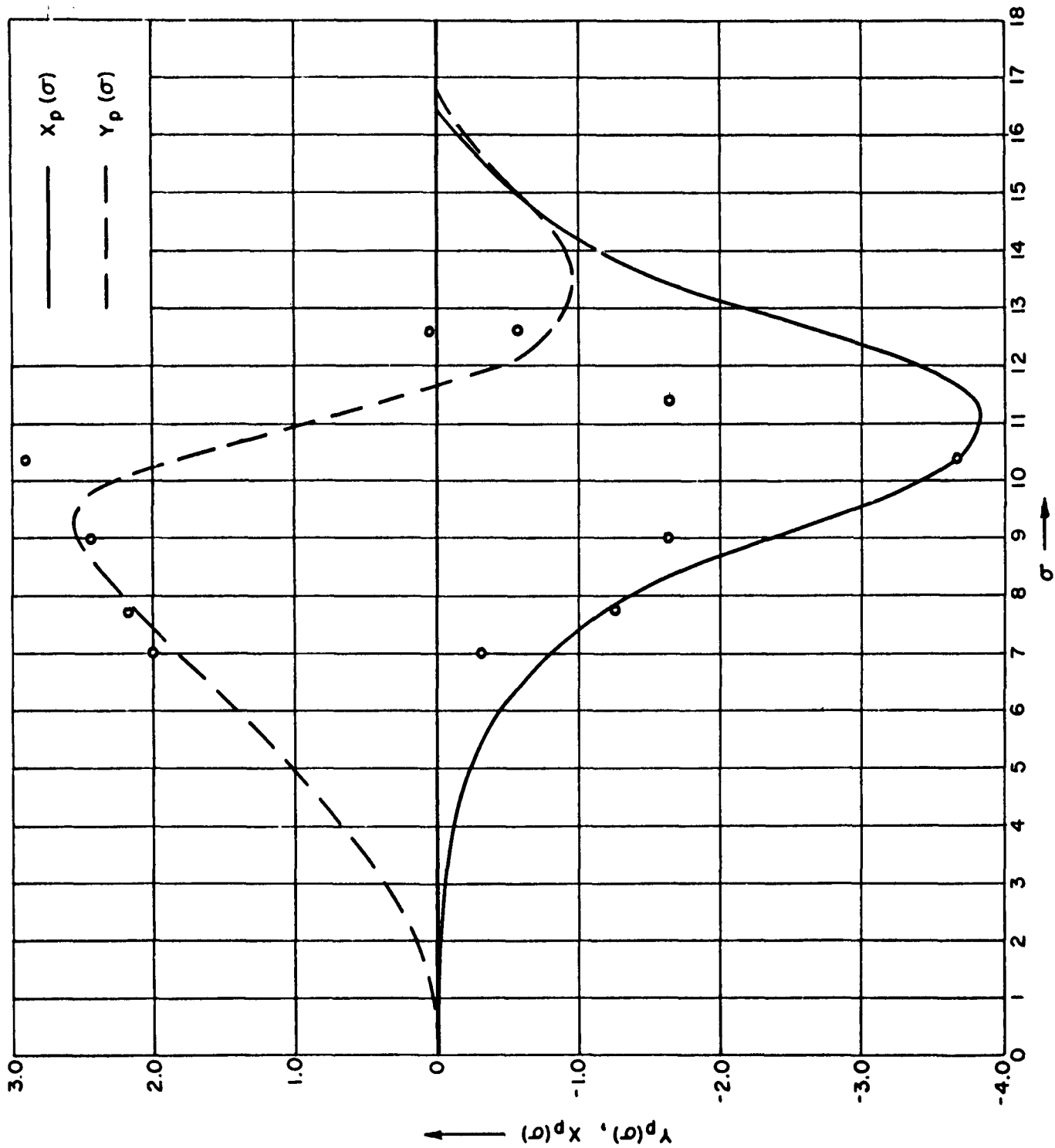
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COMPONENTS OF THE HEAVING RESPONSE OF A RECTANGULAR BLOCK

HYD-6606

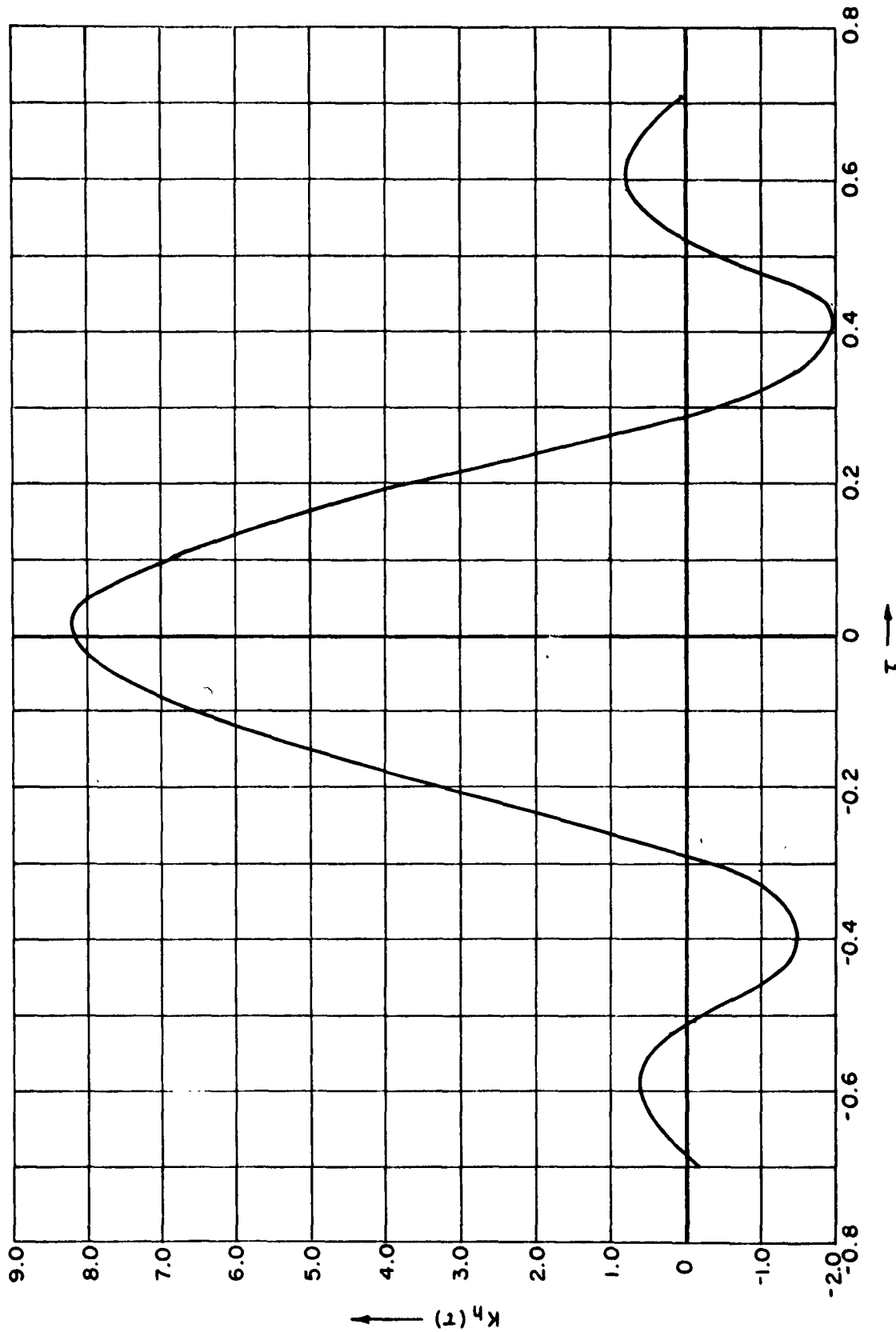
FIGURE 1



HYD-6607

FIGURE 2

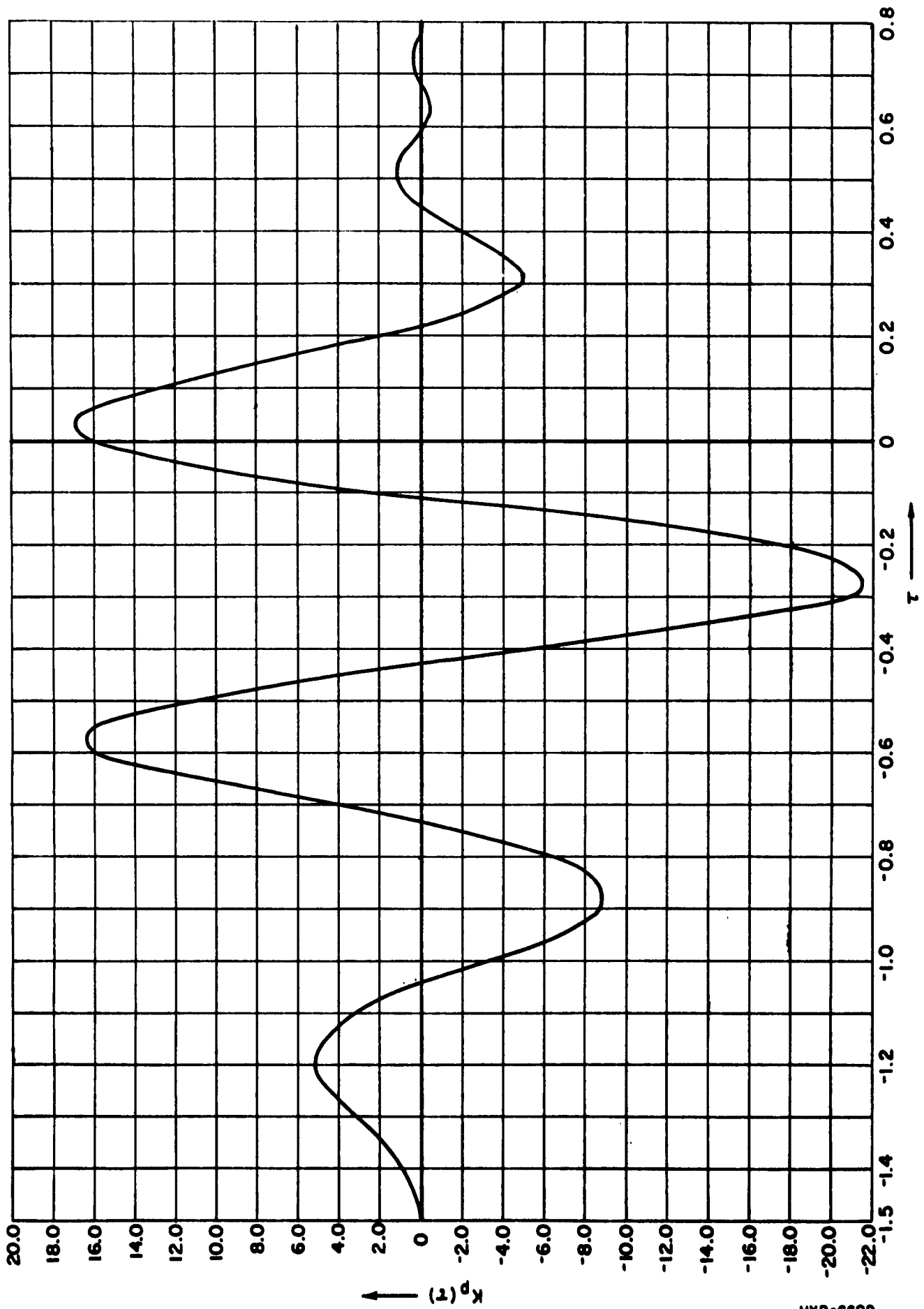
COMPONENTS OF THE PITCHING RESPONSE OF A RECTANGULAR BLOCK



HYD-6608

HEAVING KERNEL FOR RECTANGULAR BLOCK

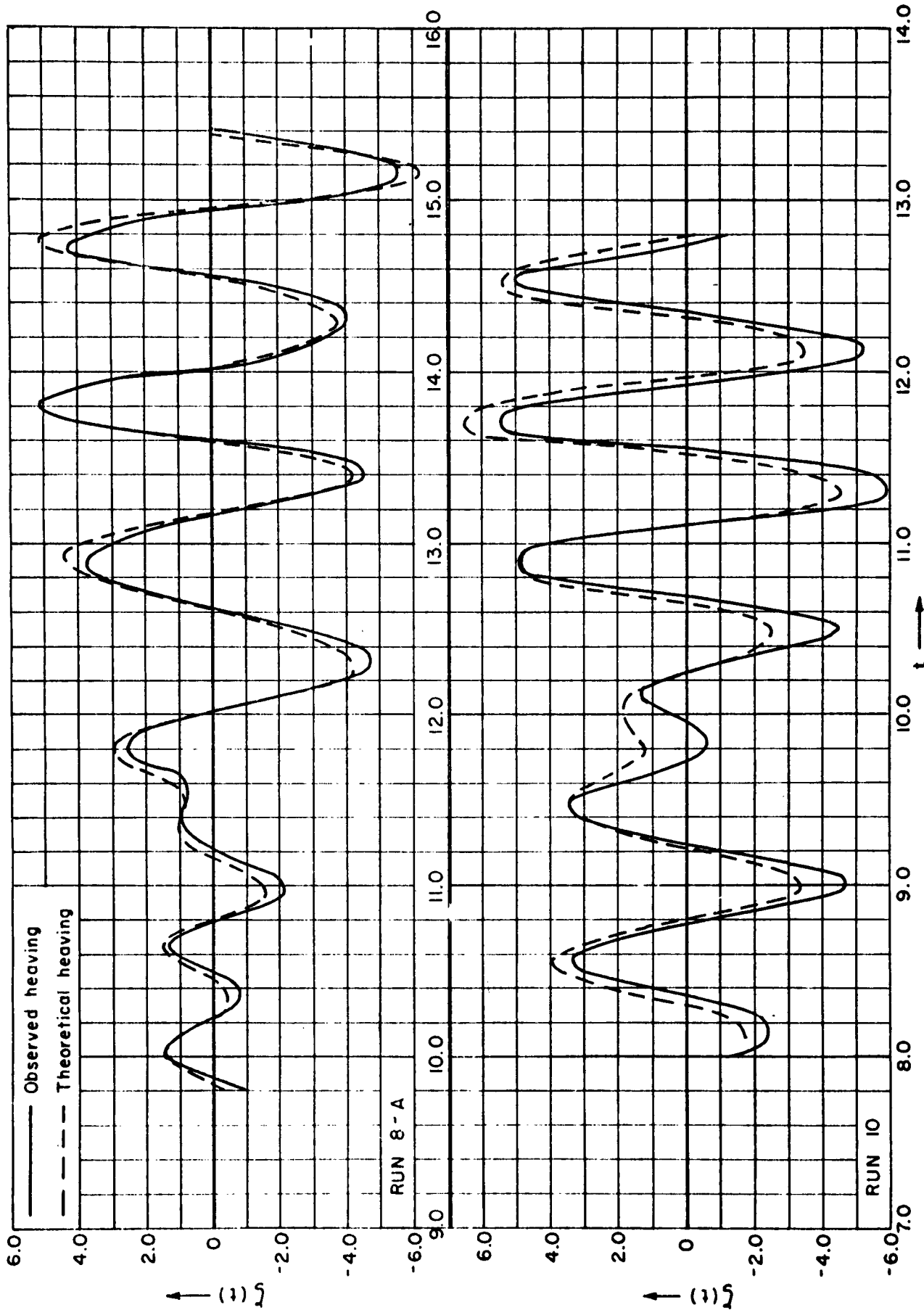
FIGURE 3



HYD-6609

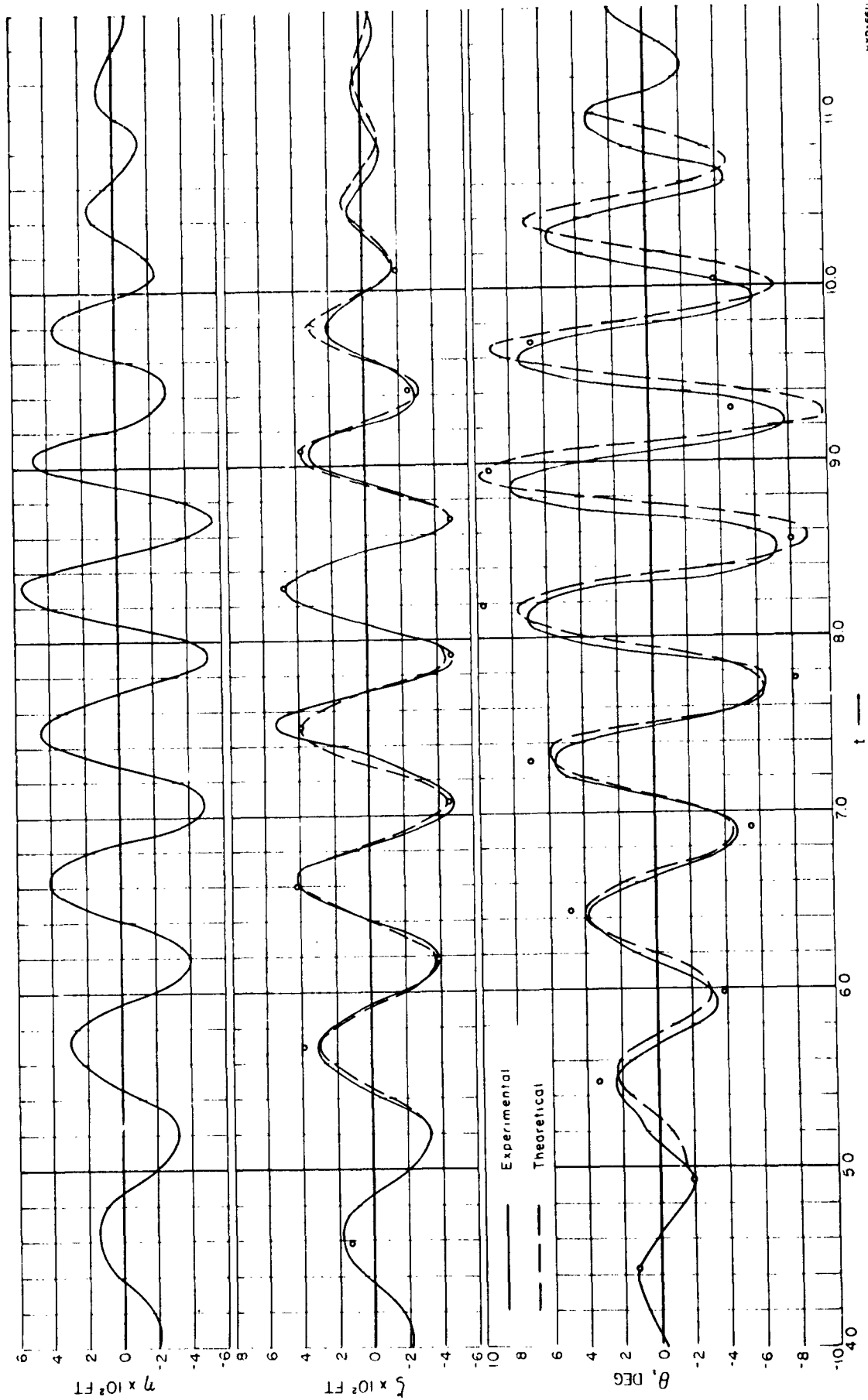
PITCHING KERNEL FOR A RECTANGULAR BLOCK

FIGURE 4



HEAVING MOTION USING AN EMPIRICALLY DETERMINED HEAVING KERNEL

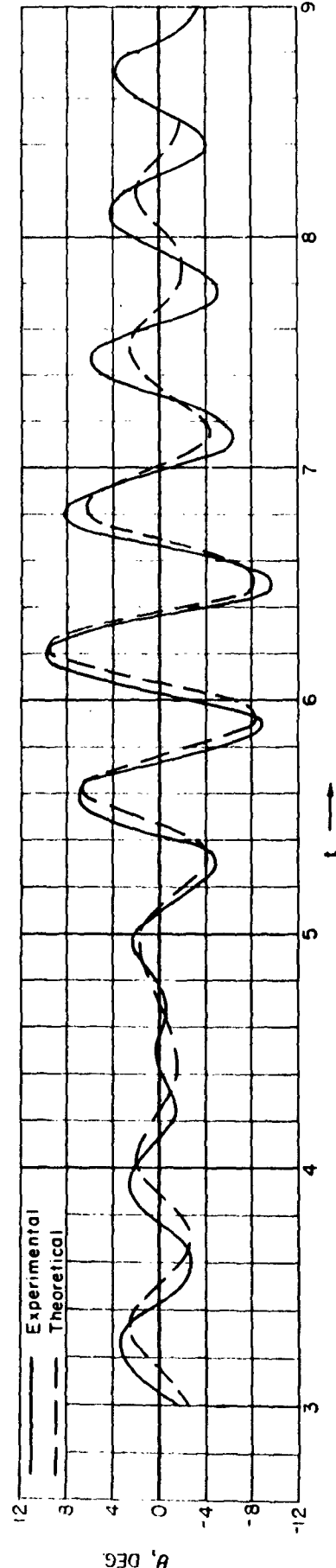
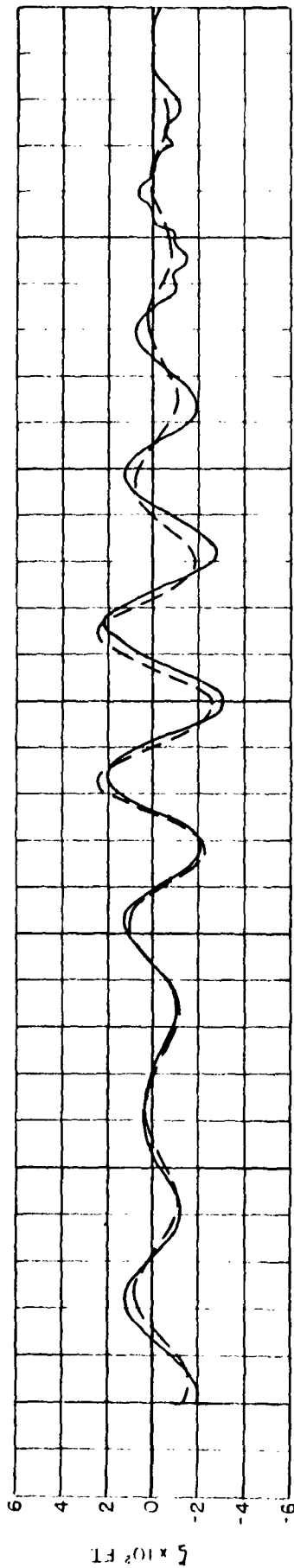
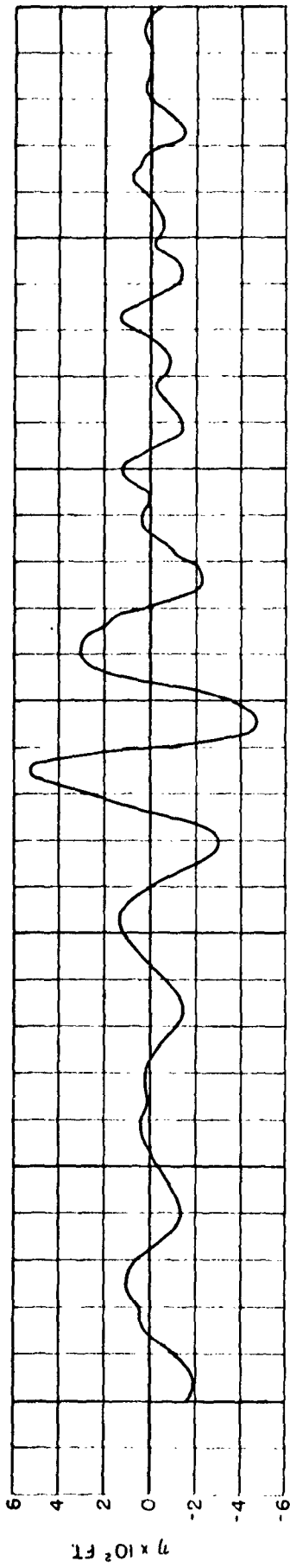
HYD-6610



NYO-6611

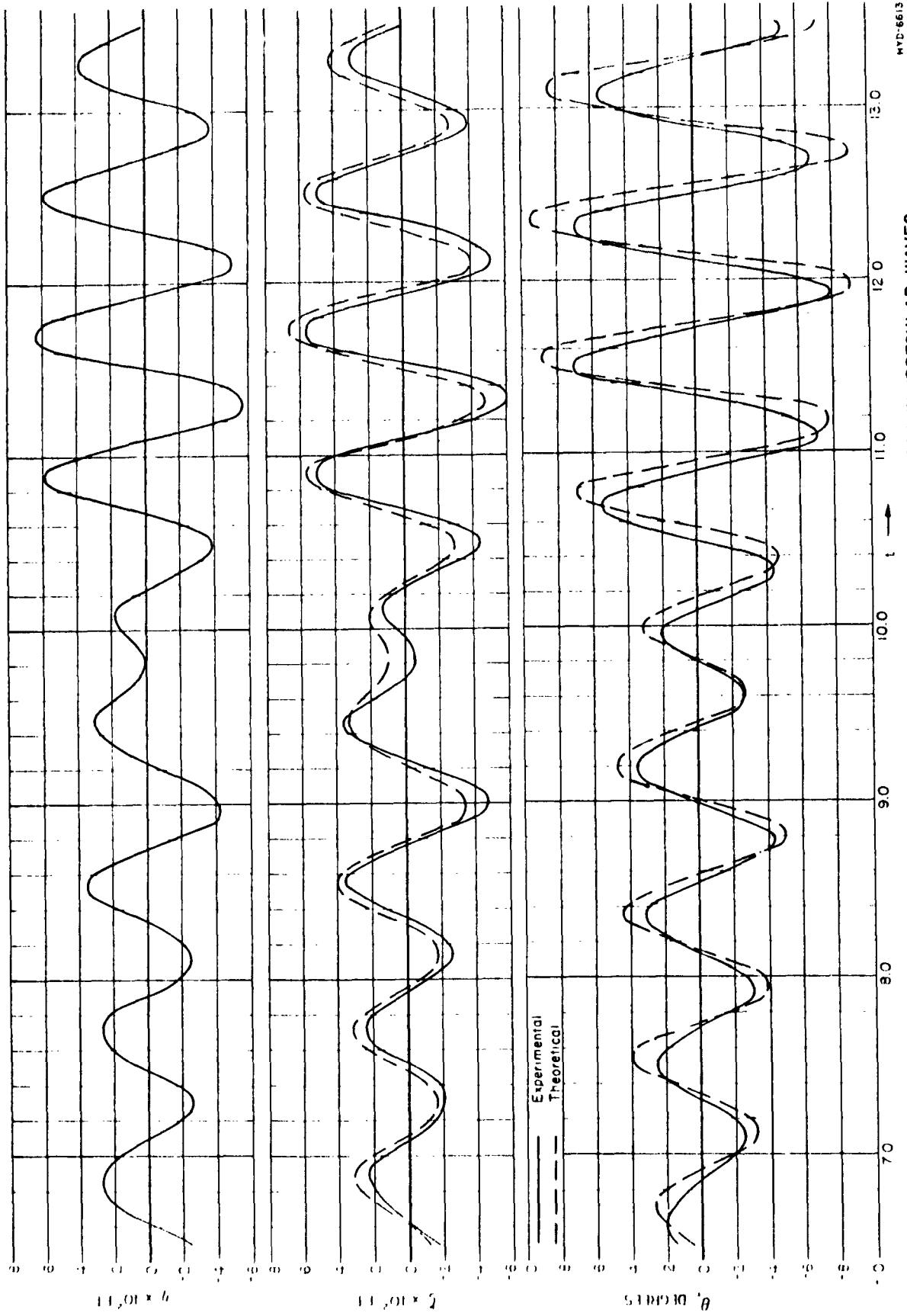
ANALYSIS OF PITCHING AND HEAVING OF A RECTANGULAR BLOCK IN A WAVE GROUP

FIGURE 6



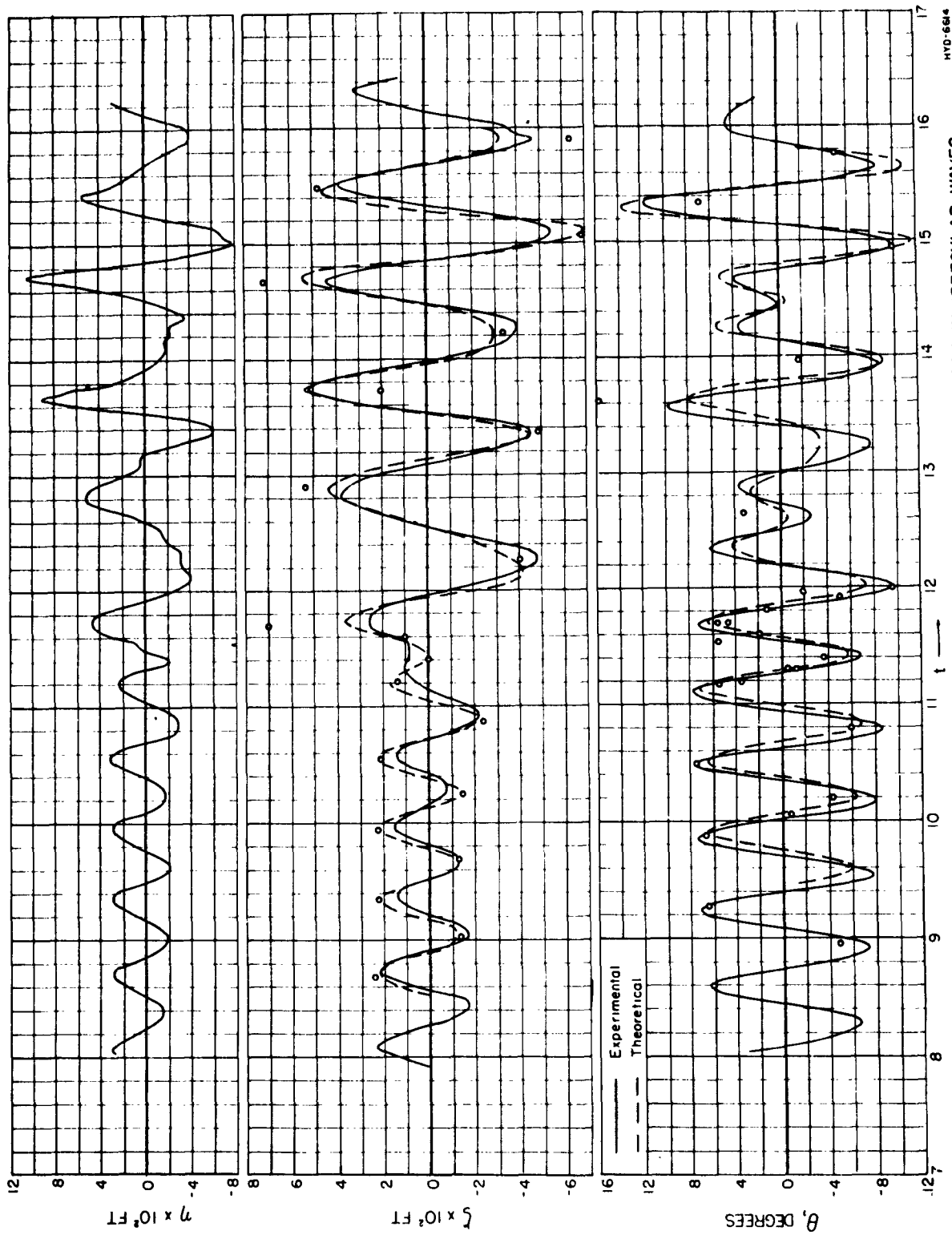
MYD-5612

ANALYSIS OF PITCHING AND HEAVING OF A RECTANGULAR BLOCK IN IRREGULAR WAVES



MYD-6613

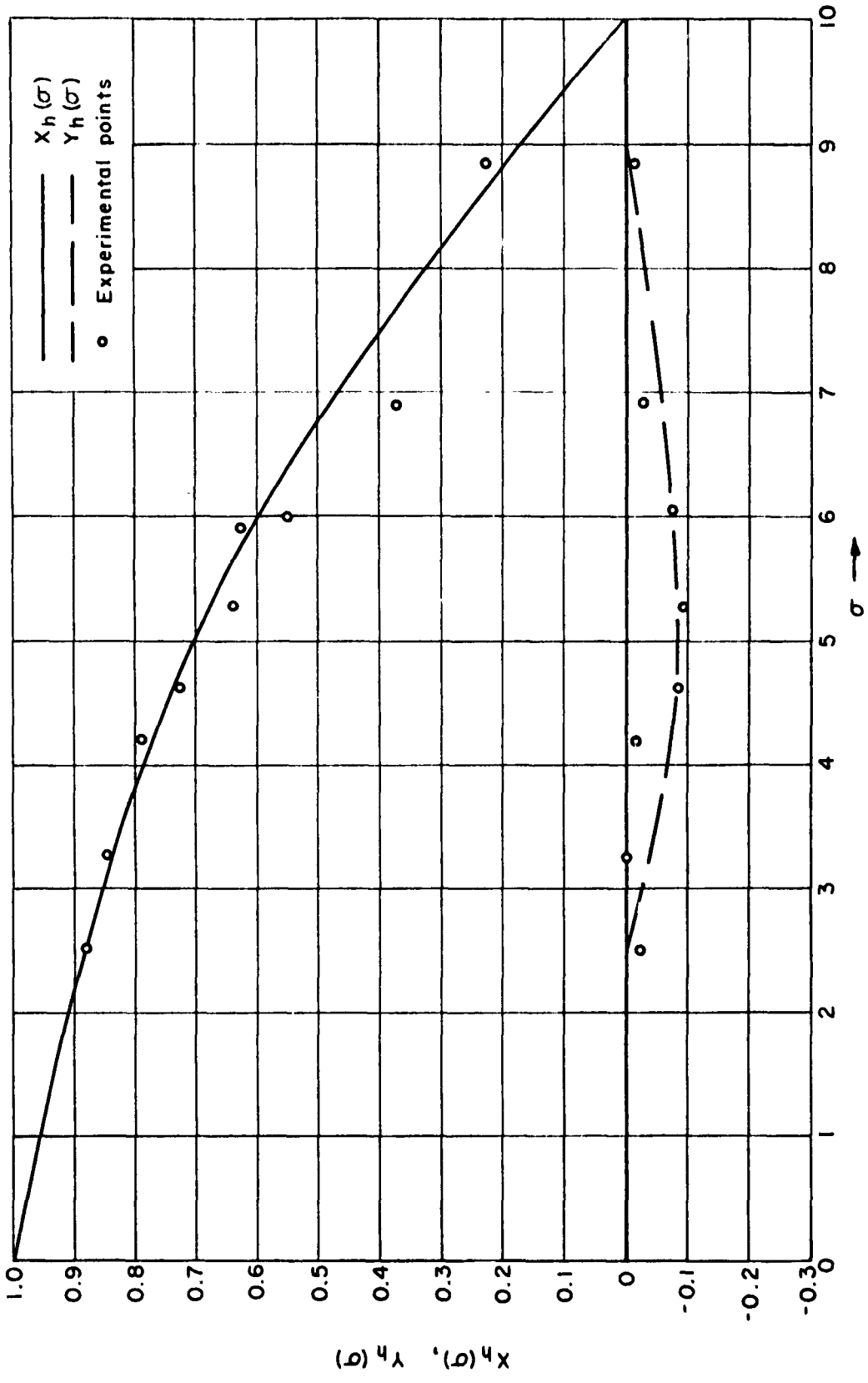
ANALYSIS OF PITCHING AND HEAVING OF A RECTANGULAR BLOCK IN IRREGULAR WAVES



HYD-684

ANALYSIS OF PITCHING AND HEAVING OF A RECTANGULAR BLOCK IN IRREGULAR WAVES

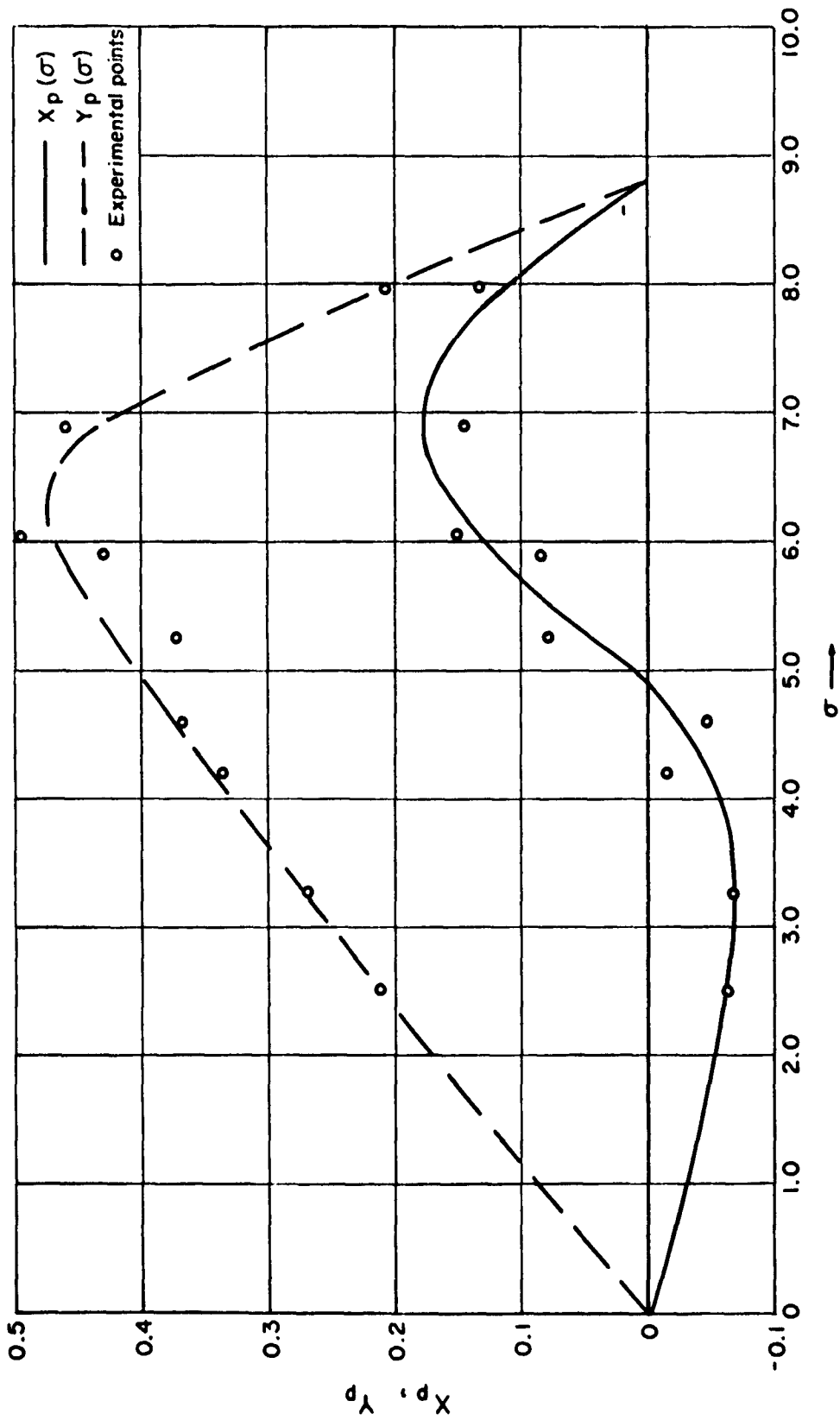
FIGURE 9



HYD-6615

FIGURE 10

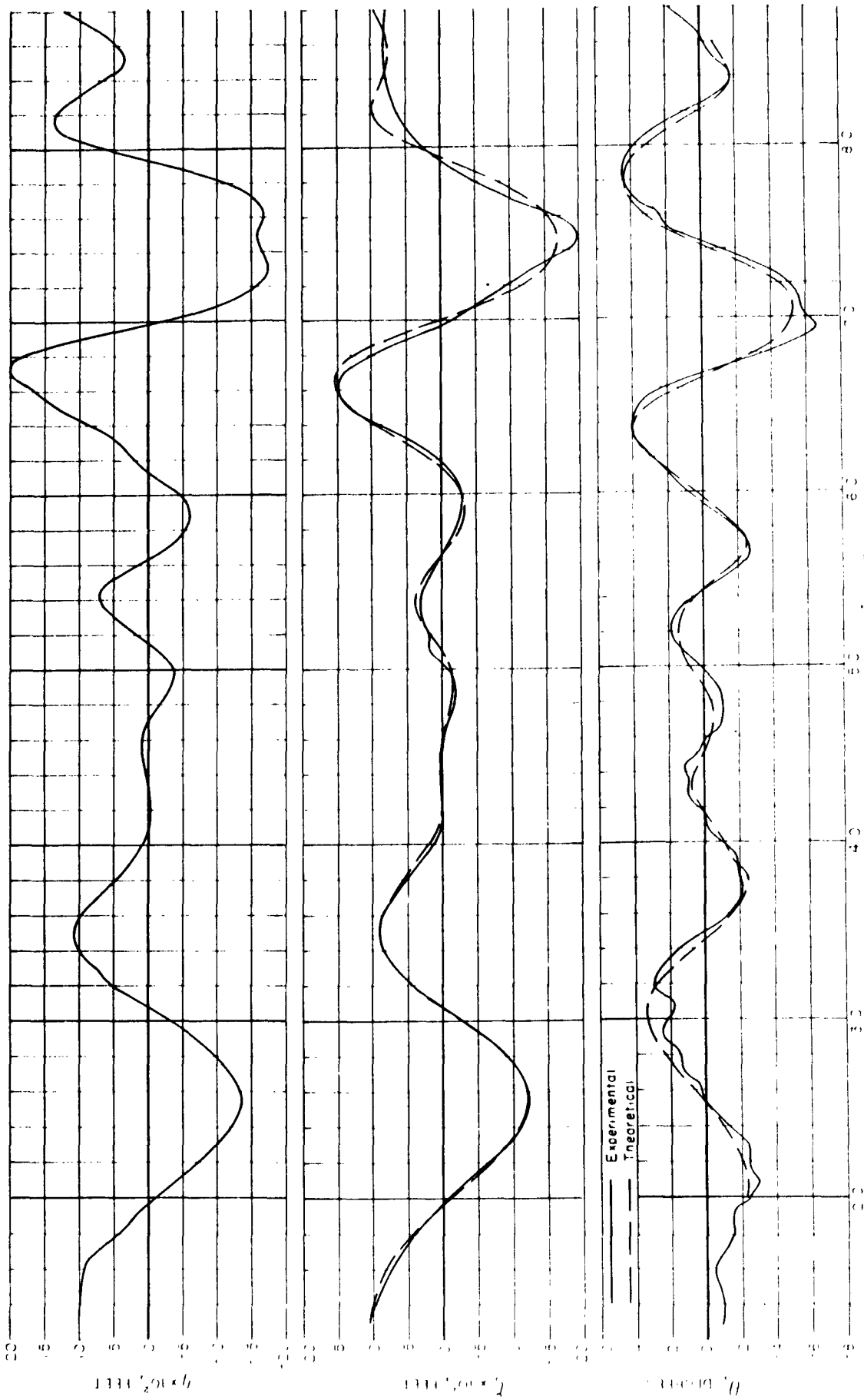
EMPIRICALLY DETERMINED RESPONSE FACTORS FOR HEAVING OF SHIP MODEL



HYD-6616

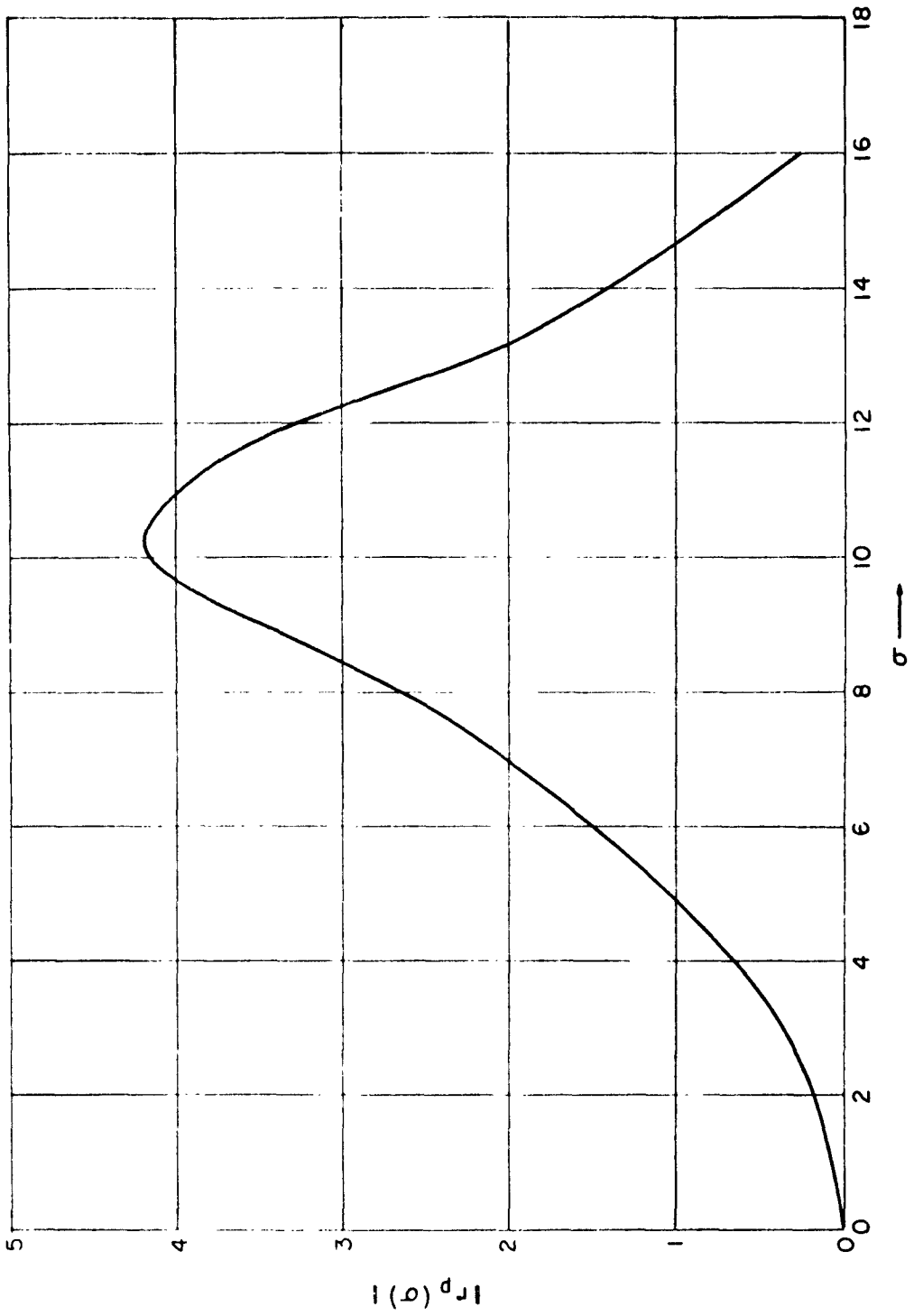
EMPIRICALLY DETERMINED RESPONSE FACTORS FOR PITCHING OF MODEL

FIGURE 11



NYO-667

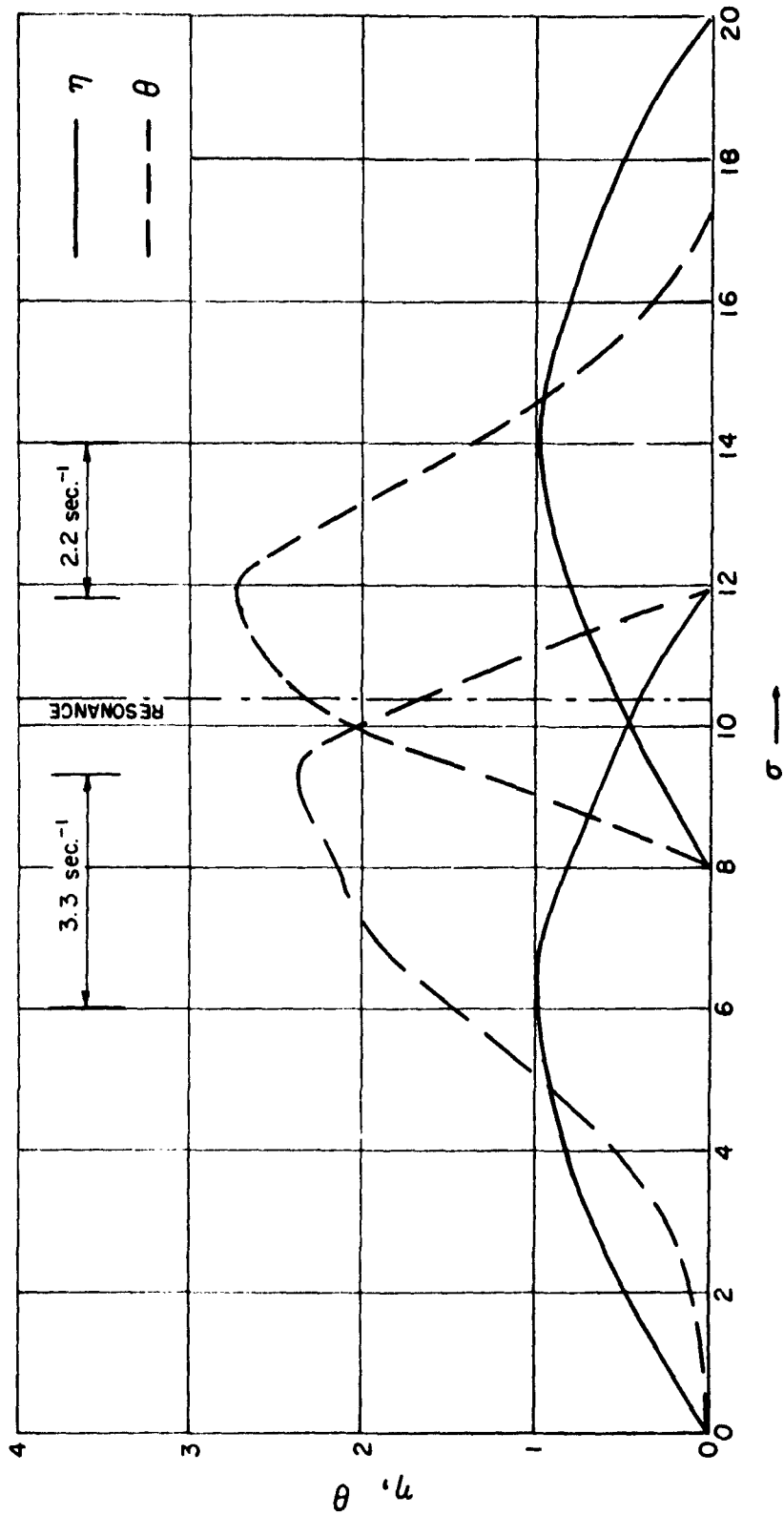
ANALYSIS OF PITCHING AND HEAVING OF A SHIP MODEL IN IRREGULAR WAVES



ABSOLUTE VALUE OF PITCHING RESPONSE

HYD-6618

FIGURE 13



HYD-6619

FIGURE 14

AMPLITUDE SPECTRA OF MOTION AND PITCHING