PROCEEDINGS

Fourth Annual Logistics Conference

PART I - Unclassified Section

Jointly sponsored by

THE GEORGE WASHINGTON UNIVERSITY

Logistics Research Project

and

DEPARTMENT OF THE NAVY

Office of Naval Research

Held at the General Services Administration Auditorium
Washington, D. C.
16, 17, 18, 19 March 1953

Logistics Research Project

WASHINGTON, D. C.

1953

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Preface

It is with pleasure that we make available copies of the Proceedings of the Fourth Annual Logistics Conference. We hope that the four annual logistics conferences, beginning with a working conference of limited attendance in 1950, have served the purposes of stimulating fundamental and applied research in the field of logistics and disseminating useful information concerning the role of logistics in military planning and operations.

The Fourth Annual Logistics Conference contained papers of various security classifications. For the expedition of the distribution of the Proceedings, and for the convenience of the recipients of copies thereof, the Proceedings have been bound and distributed in four parts. It is hoped that this procedure will enhance the value of the Conference papers to the users.

We express our sincere appreciation of the active collaboration of extremely busy people in making the Fourth Annual Logistics Conference a success. We are indebted to the speakers for the interesting and important talks they prepared and delivered. We thank The George Washington University and the Office of Naval Research for their assistance in the Conference arrangements. And last, but in no sense least, is our expression of thanks to all those who attended the Conference and whose participation made it worthwhile.

E. W. Cannon
Principal Investigator
Logistics Research Project

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PROGRAM

General Chairman: Dr. Mina Rees
Director, Mathematical Sciences Division, Office of Naval Research

Chairman of "Practical" Sessions: Rear Admiral Henry E. Eccles, USN, Retired

Chairman of "Theoretical" Sessions: Professor Oskar Morgenstern
Princeton University

Monday, 16 March 1953
Planning Problems and Techniques
("Practical" Session, 1)

Welcoming Remarks
Dr. Mina Rees,
Director, Mathematical Sciences Division, Office of Naval Research

Rear Admiral C. M. Bolster, USN
Chief, Office of Naval Research

Dean Martin A. Mason
The George Washington University

Introduction to the "Practical" Sessions
Rear Admiral Henry E. Eccles, USN, Retired

1. Fleet Logistics
   Speaker: Vice Admiral F. C. Denebrink, USN
   Commander, Military Sea Transportation Service
   Formal Discussant: Rear Admiral J. E. Maher, USN
   Commander, Service Force Atlantic Fleet

2. Supply in the Pacific Theater
   Speaker: Capt. O. P. Lattu, SC, USN
   Commanding Officer, Naval Supply Depot, Newport, R.I.
   Formal Discussant: Capt. J. D. Parks, SC, USN
   Service Force, Atlantic Fleet

3. Maintenance and Repair in the Fleet
   Speaker: Rear Admiral W. D. Leggett, Jr., USN
   Deputy Chief, Bureau of Ships
   Formal Discussant: Rear Admiral W. M. Hague, USN
   Commandant, Industrial College of the Armed Forces

Tuesday, 17 March 1953
Logistics Data Processing
("Practical" Session, 2)

1. Survey of Modern Methods of Data Processing
   Dr. E. W. Cannon
   Principal Investigator, Logistics Research Project

2. Data Storage Devices and Techniques
   Jacob Rabinow
   National Bureau of Standards

3. Digital Input and Output Devices for Automatic Computers
   Dr. Nelson M. Blachman
   Office of Naval Research

4. A Description of the Logistics Computer
   Lt. R. J. Rossheim, USNR
   Office of Naval Research

5. Applications of UNIVAC to Air Force Programming Problems
   Emil Schell
   Headquarters, United States Air Force

6. Application of CRC Computer to Bureau of Aeronautics Problems
   D. O. Larson
   Bureau of Aeronautics

7. Application of the Logistics Computer to Naval Logistics Problems
   J. Jay Wolf
   Logistics Research Project

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Distribution Control
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   Professor Oskar Morgenstern
   Princeton University

1. Optimal Technology for Supply Management
   Speaker: Rear Admiral Frederick L. Hetter, SC, USN
   Bureau of Supplies and Accounts

   Formal Discussant: P. F. Hilbert
   Office of the Air Comptroller

2. The Limits of Centralization
   Speaker: Dr. T. M. Whitin
   Princeton University

   Formal Discussant: Prof. M. E. Salveson
   University of California

3. Estimating Shipping Requirements at Short Range
Dr. Harry M. Hughes
University of California

4. Optimal Scheduling in Transportation
Speaker: Dr. I. Heller
Logistics Research Project

Formal Discussant: Martin Shubic
(of Papers 3 & 4) *Princeton University*

Thursday, 19 March 1953
Theory of Games
("Theoretical" Session, 2)

1. The Solution of Games by Behavior Strategies
Speaker: Dr. H. W. Kuhn
Bryn Mawr College

Formal Discussant: Gerald Thompson
Princeton University

2. Reduction of Games in Extensive Form
Speaker: Dr. Norman Dalkey
The Rand Corporation

Formal Discussant: Lloyd Shapley
Princeton University

3. Blotto-Type Games
Speaker: Dr. D. W. Blackett
Princeton University

Formal Discussant: Dr. W. H. Marlow
Logistics Research Project

4. Machine Representation of a Symmetric Air War Game
BrigGen L. I. Davis, USAF
Air Research and Development Command

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WELCOMING REMARKS
by
The General Chairman, Dr. Mina Rees
Director, Mathematical Sciences Division
Office of Naval Research

This is the Fourth Annual Logistics Conference sponsored by the Office of Naval Research and The George Washington University. The first of these conferences was planned soon after ONR began its participation in the development of this relatively new art and science of logistics. We feel that one of the special needs is to provide for the exchange of the ideas and experiences of the diverse groups engaged in related work. It is in this way that scientific progress has been achieved across the ages and it is essential that the ideas that are generated in one group be subjected to the criticism, harsh or otherwise, of other groups. Thus they will be shaped into a form that may be good and useful. Those of us who are engaged on the civilian front in work related to logistics are eager that those of you who have large military experience should bring your experience and judgment and criticism, adverse or favorable, to play upon the ideas which we generate. In this particular field there are so many groups, military and civilian, working under the restriction of military classification, that the problem of interchange and criticism is particularly difficult. It is in this general area that we hope this conference will be effective. The George Washington University Project is only one facet of the ONR program in logistics. This program is concerned both with short range and with long range work and there are many other aspects of the program besides those which are represented at The George Washington University. It is particularly critical, however, to secure an interchange of judgment and criticism in the area of the short range program, and it is in this area that we hope to make progress during the first two days of the conference. The obverse of the picture is this: there is a need for some awareness of the rather long-haired work on the part of the practical workman. This is so partly because it can provide the research worker with an orientation in the selection of significant and fruitful directions for his research; partly because it can facilitate the translation into use at the very earliest possible time of any useful results which come out of the research. And so we hope that, although some of the papers on the last two days may seem a little remote, many of you will understand and participate in the discussions. It may be worth noting that the opening speaker of the so-called "theoretical" session will be Admiral Hetter who can hardly be accused of being too far removed from the scene of operations. The first two days are devoted to the so-called "practical" session and deal with planning problems and techniques and with logistics data processing. I am sure that you are all aware that the new Logistics Computer has just been delivered and will be on display during the conference. The second two days are devoted to the "theoretical" session and the topics will be distribution control and the theory of games. Before we proceed with our program, I am privileged to introduce representatives of the two organizations sponsoring this conference so that they may bring you greetings from their organizations. Rear Admiral Bolster, Chief of Naval Research, has been associated with the ONR Logistics Program since its inception and has given us support at every turn. I am particularly happy and privileged to present Admiral Bolster.

WELCOMING REMARKS
by
Rear Admiral C. M. Bolster, USN
Chief of Naval Research

Ladies and Gentlemen:

It is certainly an inspiration to all of us to see such a fine group here this morning and particularly to note that there is such a strong representation from the personnel of the Army, Navy and Air Force. As Dr. Rees has told you, we have been working very hard for a long time on this general problem of logistics, particularly that of bringing the many contributions of mathematics and science to bear in order to arrive at a more effective method of handling such problems. We have spent many hours both on the theoretical and practical side of this problem, since we all are extremely anxious to make a real contribution to the users of logistics services.

I want to welcome all of you and to thank you for coming here this morning, for your presence will help greatly in making this Symposium a success. It is through the efforts of you people, many of whom I know from personal knowledge are extremely busy with other things, that we achieve the understanding and progress so necessary for this effort. We appreciate the effort of those who have prepared papers which will be presented here. We also wish to express our thanks to those who have taken the time to prepare formal comments. For example, I have been working with Admiral Leggett on a special board, and I frankly do not see how he has had time to prepare his paper. On the other hand, I have been told that it is going to be a very fine paper which we are very grateful to have him present. As you look at the list of papers and speakers here today, you are struck by the wide variety of experience and background which they represent. For the first time, I believe we will be getting a lot of the flavor of real operating experience, and as Dr. Rees has said, the thing that makes such a project really successful is to be sure that it reflects the true needs of the services and not merely some theoretical problem.

As I said earlier, this project has been under way some time, and we have all looked forward to the period when we would have a computer available with which we could test some of the theories being developed. This computer is here in Washington now, and we are eagerly anticipating the demonstrations that will be possible on it, to see just how it will really solve the problems when you feed the right numbers into it. I, personally, have great confidence that it will do all the things claimed for it, simply because of my belief and great confidence in the people doing the work. Certainly, we couldn't have a finer group of people than those who have been working on this project and on the computer.

I want again to welcome you and to say that we in ONR are extremely proud to participate in this joint effort with you.

WELCOMING REMARKS
by
Dean Martin A. Mason
The George Washington University

Doctor Rees, ladies and gentlemen:

One of the pleasant things that a Dean has to do is from time to time to bring words of welcome from his University to those people whom the University serves. I was particularly happy to have this opportunity given to me to greet you people and make you welcome so far as I can, first because the University family has a deep and appreciative interest in the problems of supply or logistics, and secondly because I have had in my professional career some experience with logistics, insofar as it applies to military operations.

I have the feeling that there are three elements which characterize modern logistics. One might be called the trappings (those things that are luxuries in combat), and one might be called impedimenta (those things someone thinks are necessary); and the last is that body of necessary things vital to the success of a military operation. A few months ago I had occasion to talk to an underwater swimmer, a member of a Navy underwater demolition team, and I asked him what he really needed in order to do his job. It did not take him long to figure out the answer, and he did not take many words to tell me the answer. He just said, "Air, guts, and a gismo."

Now this probably reduces the logistics problem to its simplest terms, but these were terms which I, as an engineer, could understand. It appeared to me that if logistics problems could be solved with as much simplicity as this hardy character brought to his problem, perhaps there would not be quite so much need for computers, and money, and the large assembly of brains and talent that we have here.

So I like to think of logistics then, in terms of "air, guts and a gismo.' I don't know all that you are concerned with, but I am sure that you are going to put the problems and solutions that you have to work with in much more elegant language. I doubt if they can be any clearer than those of the underwater swimmer.

The University, as I have indicated, is happy, of course, to participate in the attack on the difficult problems of logistics. At the present time many people wonder what Universities are really for, whether they are havens for Communists, or a place where long-haired people can be given the necessary where-with-all to continue to have beans and bread at least once a week, or whether they really do develop new knowledge and try to disseminate that knowledge. We have a strong feeling, of course, in our University, as every other university has, that we are trying to develop in knowledge, and to disseminate it. The field of study of logistics appears to us to offer considerable opportunity for the development of new knowledge. Certainly, all of you people know that the knowledge that exists needs to be disseminated somewhat better. We feel this is truly an effort in which a university might well cooperate.

We are pleased indeed, therefore, for this opportunity to join with the Office of Naval Research in working on a relatively important problem.

Our interest, as one of your joint hosts, is to see that you are comfortable, and that you feel you are among friends. You probably won't after you have been here a few days; but that is simply the atmosphere in Washington.

We are grateful to the General Services Administration for making available to us this more comfortable auditorium than the austere classrooms and facilities that we would have had at the University.

Let me state again the pleasure of the University in welcoming you to this Conference, and our great appreciation to the Office of Naval Research for asking us to join in such a conference.

INTRODUCTION TO THE "PRACTICAL" SESSIONS

by

Rear Admiral Henry E. Eccles, USN Retired

1. The major problem facing the United States today is that of maintaining our National Security without resorting to rapid inflation. In order to do this we must develop strong, adequately supported combat forces, at a minimum cost. This overall problem resolves itself into several related lesser problems:

A. The selection of weapons and weapons systems most suitable for attaining National Security and our National Objectives.

B. The determination of how these weapons and weapons systems can best be employed to attain these objectives.

C. The manner in which these weapons systems should be organized; that is to say the command relations that should be established in the Combat Forces.

D. The determination of how the forces employing these weapons can be most effectively supported.

E. The determination of how best to provide for the overall command and departmental administration of these forces.

2. In considering these problems we find many strong differences of opinion and certain startling paradoxes.

A. The differences of opinion largely stem from differences in basic philosophy of war, strategy, and the employment of weapons and forces. However, these differences are greatly aggravated and made urgent by the problem of the Budget - "The Battle for the Dollar", - which is a Logistics Problem.

B. The most startling paradox is found in the fact that the slogan of "business efficiency" is being invoked by persons advocating administrative practices which are contrary to the trend in our major businesses. At a time when some authorities are emphasizing the evils of overcentralization in Government in general, and when others consider that many of our military deficiencies stem from overcentralization, there arises a demand for still greater centralization. All the while large companies are tending toward decentralization in their management.

3. The differences in military philosophy we should accept and work out in our traditional manner by patient study and education. The paradoxes and contradictions of the demand for more centralization stem from a superficial approach to the problem and from impatience.

4. The size of any enterprise can be roughly measured by the number of its employees and its gross income. In 1951 General Motors, General Electric, American Telephone and Telegraph and U. S. Steel employed a

total of about 1,630,000 persons. Their combined gross income was about $16,935,000,000.

In Fiscal year 1953 the U. S. Armed Forces were composed of a total of about 4,825,000 persons, both military and civilian, and they had a budget of about $48,600,000,000. That is to say they were about three times the size of the four industrial giants combined. In fact the Navy alone, with its 1,500,000 personnel and $13,170,000,000 budget, is about the same size as this hypothetical industrial combination.

Now, granted that statistics can be very misleading, nevertheless these figures do give us, in terms of well known industrial concerns, the order of magnitude of the problem of military management.

5. I presume that if we attempted a corporate consolidation of General Motors, General Electric, American Tel and Tel, and U. S. Steel, and then insisted that the budget for 1954 be submitted by each division of the combined company before its budget for 1953 had been established by the five hundred man Board of Directors, there might be some areas of imperfection, and the stockholders might become impatient. Some might even say that such a corporation is unmanageable in a democracy.

6. And yet the problem of creating, employing, and supporting our combat forces must be managed, and managed with efficiency. The application of sound principles of Logistics and of Logistics planning enters into every one of the problems mentioned. In some instances it is the vital element. And yet these vital principles have not yet been adequately formulated, let alone applied. Therefore, for the overall problem to be solved, there must be patient, continuing study and research. But first the problem must be seen in its whole immense size; and the relationships that exist among the various parts of the problem must be understood.

7. The logistics aspects are themselves so great that in this conference we can consider only certain portions. After stating several of them, we will discuss certain tools that we hope may be useful in their solution. Gradually, by clear statement, and by patient discussion, our understanding may be increased. Much of what will be said during these next four days will deal with new ideas, and with the development of theory. This is, of course, the purpose of our meeting and our only hope for continued progress. However, these ideas must be based on an understanding of the facts of life.

8. Because of our great preoccupation with what happens in Washington, there may be a tendency to forget that our Logistical Establishment has, as its sole purpose, the support of the Combat Commander. We can make many minor mistakes and readily absorb them; but if we ever forget the point of view of command in the field, we will make a major mistake that can be fatal.

9. And so today we will open our conference by a discussion of the problem of the support of the Naval Aspects of the Korean fighting. From 1945 - 1950, Vice Admiral F. C. Denebrink was engaged in various logistical tasks in the Pacific and in Washington. From 1950 to Nov. 1952 as Commander Service Force U. S. Pacific Fleet he was charged with the Logistic support of all Naval Forces in the Pacific Ocean. He is now Commander of the Military Sea Transportation Service. With this background he is particularly well suited to discuss "Fleet Logistics" from the working point of view--the point of view that we must never forget. I take great pleasure in introducing Vice Admiral Francis C. Denebrink, U. S. Navy.

SURVEY OF MODERN METHODS OF DATA PROCESSING
by
Dr. E. W. Cannon, Principal Investigator
Logistics Research Project

By modern methods we mean the application of the automatic, electronic digital computing devices developed during the last decade. Data processing is the manipulation of large quantities of data with the performance of a limited amount of computing on each item. A survey of the use of the new computers in data processing is quite revealing. The main result is the uncovering of the fact that this application has been given too little attention.

Suppose we dwell for a while on the development of the modern, electronic computer. This device has been popularized far beyond its capabilities. It has been widely heralded as a giant, automatic brain which will readily provide the answers to the knottiest problems in all fields of human endeavor. In truth, it is merely an extremely fast computing device, capable of doing the elementary operations of arithmetic thousands of times faster than the human beings who direct it. It will function properly, moreover, only in response to minutely detailed sequences of commands prepared for specific problems by the human operator.

The electronic computer is another step in the evolution of fast, mechanized computers, a concept which certainly did not originate with this generation. We are all acquainted with punched-card computing equipment, and its far-reaching application in accounting. This kind of digital computing equipment has also been used for some years in so-called scientific computing (or general purpose use), for solving problems in the fields of engineering, physics, chemistry, and the social sciences. Punched card equipment was invented about fifty years ago by Hollerith, then working at the Bureau of Census.

The idea of developing computers to solve problems automatically in the fields of pure and applied mathematics is much older, however, than the Hollerith machinery. At least as early as 1830, more than 120 years ago, Charles Babbage, an English mathematician, was struggling unsuccessfully to obtain satisfactory performance on a Government contract for the construction of his Difference Engine. Work on the Difference Engine was carried on for many years, but the device was never fully completed. A comment of Babbage, who had demonstrated the principles of his device by a small model was prophetic; to quote "But when I understood it to be the wish of the Government that a larger engine should be constructed, a very serious question presented itself for consideration, namely: Is the present state of the art of making machinery sufficiently advanced to enable me to execute the multiplied and highly complicated movements required for the Difference Engine?"

In 1834, Babbage had a brighter vision, the Analytical Machine, a device to do not only everything which the Difference Engine promised, but also to execute every kind of analytical operation indicated by formulas. Again, however, the inventor's ideas were too demanding of the tooling industry of his day, and his Government support was withdrawn long before the completion of the machine.

It is important to note (1) that the designs of Babbage were sound; (2) that in his day the capability of industry to construct mechanical devices was severely limited, and (3) that there was no electric circuit theory in his time. In fact, Faraday was only then performing his fundamental experiments on the induction of electric currents by motion of a conductor through a magnetic field -- the experiments which were the forerunners of the invention of the electric generator.

We mentioned the birth of punched card computing machinery. The next significant step in the evolution of digital computers came much later, in 1940, with the completion of the Bell Telephone Laboratories' Complex Computer. In 1938, the use of telephone relays and teletype apparatus for numerical computation had been suggested by Dr. George Stibitz, then a research mathematician with the laboratories. The resulting computer was operated from a keyboard to add, subtract, multiply and divide complex numbers, both the original numbers and the results being printed on a teletype printer.

The Complex Computer was demonstrated, by remote control, at a meeting of the American Mathematical Society in the Fall of 1940. The keyboard device and a teletypewriter for recording answers were installed in the auditorium of Dartmouth College, where the Society was meeting, and were connected to the relay computer in New York City by a telephone circuit. Those attending the demonstration placed their own test problems on the keyboard at Hanover, New Haven. The Computer in New York City made the computation and controlled the printing of the answer on the typewriter at Hanover. The complete operation required about one minute, from typing of the problem to receipt of the answer. This demonstration is of interest as the first practical demonstration of the feasibility of the remote control of automatic, digital computers.

The Complex Computer was followed by several successful relay computers, both special purpose and general purpose machines. The International Business Machines Corporation and Professor Howard H. Aiken, of Harvard University, placed the first of the large-scale, high-speed, digital calculators in operation in 1944. This machine, called the Harvard Mark I, is a relay computer, performing the basic operations of arithmetic and the logical operations essential for automatic sequencing by the use of relays, having a small number of relay storage registers, and having built in the additional operations of taking the logarithm, evaluating the sine of an angle and the exponential of x to base 10. The machine has IBM card input, and the results of a computation can be recorded by either or both of two output devices, IBM electromagnetic typewriters or an IBM punch.

Next, following a sequence of more or less special-purpose digital computers, designed essentially to handle fire-control computing, and several kinds of problems connected with the testing of fire control equipment, the Bell Telephone Laboratories delivered in 1946 and 1947 two large-scale relay computer systems, one to the National Advisory Committee on Aeronautics at Langley Field, Virginia, and the other to the Ballistics Research Laboratory at Aberdeen, Maryland. Each of these giant devices has 9,000 relays and 55 pieces of teletype equipment. Each embraces a system of computers which can be operated together or independently. There may be as many as six computers and ten problem positions. As one computer completes the problem on its associated problem position, it automatically picks up a waiting problem position.

Built into the computer are the functions of evaluating logarithms, sines, cosines, and anti-tangents. Included also is the floating decimal point (the computer automatically takes care of the scaling of numbers to keep them within the capacity of its registers), multiplication by short-cut addition, automatic rounding off of results, the basic arithmetic operations and rather elaborate discriminatory (or logical) controls. The system has elaborate self-checking features, and is capable because of this fact and the problem-selection characteristic, of effective unattended operation. Systemwise, the Bell Computer Systems were a marked advancement in the art of computer development.

The remaining of the giant, automatically-sequenced relay digital computers were the Harvard Mark II, designed and constructed by the Harvard Computation Laboratory, and a Bell Laboratories' model VI., which was a refinement of the System we have just described.

All these relay computers of the large-scale, automatic digital variety are still in use. Systemwise they are fully the equals of the newer electronic devices. Any problem which can be solved on the electronic computers can also be solved on the relay computers, although usually at a much slower rate. Moreover, the latter are fully automatic, as effectively so as the electronic counterparts. They represent a highly satisfactory state of the development of high-speed, large-scale computers using electromagnetic [redacted] apparatus. The only reason that they have not been dubbed giant brains is probably that the relay is more comprehensible to us than is the vacuum tube.

These relay computers have in common several interesting features. First, they are all automatically sequenced by decks of punched tapes, which may be prepared at the convenience, with respect to time and place, of the mathematicians translating the problem into suitable form for machine solution. Second, they all have so-called internal memory, that is, storage registers matched in reference or access time to the computational speed of the arithmetic speed of the machine. Third, they all incorporate sufficient control features, including the exercise of limited choice decisions, to make them fully automatic. And fourth, they lend themselves readily to the inclusion of self-checking features in their design.

By contrast, the first large-scale electronic digital computer, the ENIAC (Electronic Numerical Integrator and Calculator) was a step backward in many directions. This device, conceived by a theoretical physicist, Dr. John W. Mauchly, was designed and built in the Moore School of Electrical Engineering of the University of Pennsylvania for the Ordnance Department of the United States Army.

The ENIAC was essentially a special-purpose computer, designed for the purpose of calculating tabular data, such as firing tables. It was very effective in this kind of calculation. It performed numerical operations very fast, at the rate of 5,000 additions or subtractions of ten decimal-digit numbers in a second, or over three hundred multiplications of such numbers to give 20-digit products in the same time. It was a mammoth machine, weighing approximately twenty tons, and containing about eighteen thousand vacuum tubes. Yet it was unbalanced, combining extremely fast arithmetic operations with long and tedious set up time for problems, and extremely limited internal storage. The range of problems that could be solved on it was severely limited. Moreover, the necessity of handling hundreds of patch-cords and switches to set up the machine for a problem, and the fact that the nature of the computer precluded the intelligent onlooking of the scientist as his problem was being solved, tended to limit the expeditious use of the machine as a general-purpose computer.

The very engineers who designed and constructed the ENIAC were thinking longingly of more flexible computers even before its completion. Moreover, even the most rabid proponents of relay computers eventually were captivated by the potential of ultra-fast computing devices, and joined in the effort to develop more flexible, and more easily operated electronic digital computers than the ENIAC. The result of this concerted effort was the introduction of the era of the modern, large-scale, electronic digital computer.

This computer, in its ideal form, combines the computational speed of an ENIAC with the desirable features of the large-scale relay computers. Its functioning is based on the manipulation of electrical pulses, as in the ENIAC, and its arithmetic speed therefore borders on the fantastic. It is sequenced by coded instructions, which may enter it interspersed with numerical data, on the same input tape, or in the same deck of cards. It consists of four main parts, an input-output device, arithmetic unit, control or sequencing unit, and a storage unit. It is preponderantly an electronic machine, containing a minimum of relays and moving parts.

The first wave of these new computers has, as one would expect, an experimental nature. There have been almost as many ideas tried out with respect to the design of the arithmetic, control and storage units as there have been development groups in the field. To mention a few of the major design differences: some arithmetic units add numbers digit by digit, serially in time, and are called serial; others operate on all the digits in parallel, at the same instant, and are said to be parallel units. Some computers operate essentially cyclically, the time of each operation being an integral multiple of a basic cycle; others are purely acyclical, at the end of each operation a signal being produced to cause the next operation to begin. Some computers are capable of simultaneous computing and operation of the input-output device, while others are so designed that the arithmetic unit must stop when the input or the output unit is energized. Then again some internal storage units are electro-magnetic, some are electrostatic, and some are even acoustical devices. Electromagnetic storage is storage on a magnetic drum, electrostatic storage is storage on the face of a small television-type cathode ray tube, and acoustical storage is storage in the form of sound waves travelling along carefully controlled paths, with regeneration as long as needed.

As to the input-output devices in use, although they have not received the development emphasis that has been accorded the central electronic element, they again are not cast from the same mold. We find punched cards, punched paper tape, magnetic wire and magnetic tape in use as a medium for communication between operator and machine, with a corresponding variety of auxiliary input-output devices.

The input-output units have frequently been somewhat makeshift in nature, the reason being concentration on the utility of the amazing new computers on problems calling for a high ratio of computing per item of data. Many very important problems of this type have been solved on computers of very limited terminal equipment. In fact, during World War II, certain key problems of the Manhattan Project were successfully solved on the ENIAC in about three months (of which about three weeks were actually computing time) which could not have been solved in less than a year on any other existing computing equipment. The effect of such exploits has been a continuing emphasis on the development of powerful computing elements, with relegation to the background of serious interest in any terminal equipment involving mechanical design.

There has been, moreover, until recently, a blissful disregard of the existence of extremely important problems at the other end of the spectrum from the exciting scientific problems. The data processing, or data handling -- the mass performance of small bits of repetitive computing -- is at the same time less glamorous and more difficult in many respects than the more complicated applications of the large computers. Nevertheless, data pro essing is extremely important -- its value is not to be measured in terms of the demand it makes upon the arithmetic unit of an electronic computer.

The first commercial design aimed professedly at this application, quantity production of small bits of computing, was a product of the Eckert-Mauchly Computer Corporation. The Eckert-Mauchly machine, the UNIVAC, is essentially an all-purpose computer, intended, however, to compete with general-purpose scientific computers and also with accounting-type, data-processing devices. It is an acoustic storage (1000 item capacity), serial, tape-sequenced electronic computer, of 12 decimal digit precision. Its capacity for data processing, for computing demanding high performance of input-output unit, is arranged for by the use of up to 10 magnetic-tape handling devices, which can be used interchangeably for input or output purposes, and any one of which may be operated concurrently with the performance of computing.

While the UNIVAC, now manufactured by the Eckert-Mauchly Division of the Remington-Rand Corporation, is among the more reliable automatic, electronic computers, it is not the complete answer to the requirements of data processing. For one thing, the small internal storage, 1000-item capacity, must contain at the same time raw data, partial results, and coded instructions. The magnetic tape units receive a quite thorough work-out, especially in starting and stopping precisely, in data processing operation. Unfortunately, these units are probably the least reliable component of the UNIVAC system.

Then again, for predominantly data handling applications, the UNIVAC is unbalanced. Its all-purpose computing features and complicated control unit are not required, and they only add to the cost of keeping the computer in operation. What about the other large-scale electronic computers in use? Which of them are suitable for the modest kind of computing we are concerned with -- that occurring in accounting, in production planning, in inventory studies, and the like?

We have pointed out that the original stimulus of the great development in automatic, electronic, digital computers which has occurred during the last decade has been the desire to solve problems standing at the frontiers of scientific and engineering knowledge. One has no reason to expect that the computers developed would turn out to be economical and efficient for other uses. It is true that, in general, these computers are not economical, and in many instances not practicable, for use in data processing.

We may divide the requirements for data processing equipment into two groups: functional requirements and performance requirements. As functional requirements, we list (1) High-speed, controllable, input devices capable of handling data in huge volume expeditiously; (2) Large internal storage capacity (thousands of items); (3) Arithmetic capacity sufficient for, but not greatly in excess of the computing needs;

and (4) High-speed output devices, including printers.

The performance requirements are (1) accuracy; (2) freedom from break-down; (3) ease of repair and maintenance; (4) ease of operation; and (5) ease of problem preparation, or ease of programming, in computer jargon. These factors might well turn out to be the dominating figures of merit in the comparison of machines of comparable computational potential.

Of the functional requirements, (1) and (4), concerning input-output devices and high-speed printers, have to do with terminal equipment which is not an integral part of the electronic computer. Fortunately, almost all electronic computers have sufficient built-in flexibility to provide for conversion to improved forms of this terminal equipment, should they appear.

There is considerable development underway here, particularly with respect to high-speed printing devices, as Dr. Nelson Blachman will point out later.

Functional requirement (2) -- large internal storage capacity -- is realizable by means of the magnetic drum, which is a rugged, reliable storage medium. Moreover, this phase of computer design is attracting the interest of capable development groups, and there may be revolutionary advances here in the near future. Mr. Rabinow will tell us of some of the activity in the following talk. The remaining functional requirement, which I have stipulated, is easily taken care of -- it is merely a matter of design philosophy, and not a matter of technical difficulty, to manufacture an arithmetic unit tailored to the computational needs of the problems. All in all, the functional requirements appear to pose no obstacle to the development of electronic-digital computers suitable for a variety of data processing applications.

The situation with respect to the performance requirements is quite different, I think. It is sufficient to consider only items (1), (2), and (3) accuracy, freedom from breakdown, and ease of repair and maintenance.

For data processing does not lend itself to programmed, or mathematical checks of accuracy -- in this respect it is very unlike the scientific problem. It therefore calls for a much lower incidence of error of computing equipment than does the latter. Considering only electronic, digital computers which have been in actual use long enough for evaluation in this regard, the picture could be more favorable. There are several of these computers in use which would propagate errors at an intolerable rate if the computed results were not being constantly verified by programmed checks. These computers obviously could not be used for data processing.

There is, fortunately for us, a definite trend toward the development of more accurate computers, a trend which is exemplified, in two directions, in computers developed under military sponsorship. One computer, the RAYDAC, designed and constructed by the Raytheon Manufacturing Company for the

Bureau of Aeronautics, with the support of the Air Materiel Command, USAF, carries out in its design a rather complete checking system. Check computations, similar to the familiar casting out of 9's, parallels all computer operations, including transfers of information within the machine as well as the actual arithmetical or logical operations. Discrepancy between coded information within the computer at any point and the check performed for that point stops the computer and causes error indication. The RAYDAC has not been in operation long enough for a certain evaluation of its accuracy, but the checking system promises to reduce undetected errors to an insignificant number.

The other computer, the WHIRLWIND I, is an experimental computer developed at the Massachusetts Institute of Technology, under the support of the Special Devices Center, USN, and the Office of Naval Research. This computer does not have automatic checking, but rather, has been designed with careful attention to the installation of facilities for effective preventive maintenance. The various blocks of circuitry in the computer can be operated under conditions which would tend to accelerate failure, such as low voltages on the heaters of the vacuum tubes, wrong voltages to the grids of those tubes, and abnormal line voltages. If errors occur under the abnormal testing conditions, their causes are isolated and remedial action taken. It has been found that following a test period of this kind, called marginal testing, the computer can be relied upon for error-free operation for an extended period, usually of several hours.

We should mention also that the UNIVAC has built-in checking features. In it the steps of arithmetic are performed by two arithmetic units, and the results checked for agreement before the beginning of the next step. Lack of agreement indicates a failure of one or both arithmetic units, and causes stoppage of the computer and the turning on of an error-indicating light. Transfers of information from one part of the computer to another are checked by an odd-even check, which effectively reduces the undetected errors caused by faulty transfers.

The incorporation of automatic self-checking features in computers and the use of marginal checking techniques are a step in the direction of making computers perform more accurately. However, the really basic step which should always occur somewhere before the production of working models of complicated equipment, namely the exercise of prudence in the choice of design tolerances cannot be overemphasized. An outstanding example of the effect of conservatism in design and, possibly marginal checking, on the performance of an automatic, electronic computer is the Engineering Research Associates 1101.

In the 1101, all components are operated well within manufacturers' ratings. Maximum vacuum tube dissipation is limited to fifty per cent of rating. Resistor dissipation is limited to less than thirty-five per cent of rating. Crystal diodes, with few exceptions, are not subjected to inverse voltages greater than half the manufacturers' rating, nor are they required to conduct direct currents greater than fifty per cent of the ratings.

The reflection in the performance is the following: The computer was delivered in December, 1950, and only eight days later was installed, tested, and ready for operation. Maintenance personnel assigned to the machine were supplied by the customer, and were not under the manufacturer's supervision. Only two of them had seen the equipment before its delivery to the customer. For the first 4,500 hours the machine was turned on, it was available operationally for eighty-six per cent of the time. Of the remaining fourteen per cent, ten per cent was used in scheduled preventive maintenance and marginal checking. Only four per cent of the total time on, therefore, was spent in unscheduled maintenance in the remedying of the causes of machine errors. This is the kind of performance record every manufacturer of large-scale computing machinery, whether it be designed for general-purpose use or data processing, should aim for.

We return now to the requirements of reasonable freedom from breakdown and ease of repair. Because of accelerated design and construction schedules and a somewhat mad scramble for higher and higher computational speeds, these requirements have been sometimes disregarded in the manufacture of general-purpose computers. It has been the exception rather than the rule that one could count on a few hours of computation uninterrupted by errors. Moreover, computers have been constructed often in the small laboratory environment, and considered a laboratory tool, with no need for serious attention to designing so as to facilitate quick repair of breakdowns.

In such computers one finds very few plug-in, replaceable parts, so that the computer is out of operation for the duration of circuit repairs. In some computers gymnastics are required for the simple operation of removing for test, or replacing a single vacuum tube. Where it is desirable, even essential, to keep the computer in productive operation a maximum of the time, and where it is essential to have low operation and maintenance cost this prototype of design will not do.

The greatest impulse in the direction of reliable, easily serviced computers will probably come from the competitive market for the data-processing kind of device. In fact, this competition has already produced a noticeable emphasis on the design of computers for inventory control in chain mail-order, retail houses, like Sears Roebuck and Company, and there is in evidence a sincere interest in the potential use in production control of modest-scale computers, which are not too difficult to use and keep in service.

You will have observed, long before this time, that this survey is a consideration of the suitability for data-processing of electronic computers designed for other uses rather than a survey of existing data-processing systems. We had no choice in this matter. There simply are not yet in use such systems based on modern, electronic computers. In fact, the only specially-designed data-processing system I know of, intended to serve as a proving ground for the determination of the utility of modern electronic computing in the mass handling of data, is the Logistics Computer, which has recently been delivered to the Logistics Research Project. This computer was constructed by the Engineering Research Division of Remington Rand, Incorporated, under the sponsorship of the Office of Naval Research, and it is the computer which will be on display and demonstrated, according to your wishes, beginning tomorrow morning.

Since the Logistics Computer will be discussed in talks by Lieutenant Robert Rossheim, USNR, and Mr. J. Jay Wolf, which are to be given later today, I shall content myself with saying that the research performed in conjunction with it will serve a double purpose. We shall try to use it to assist in the devising and testing of new logistics procedures, and we are interested also in evaluating the design of the computer. In this evaluation we shall be testing the computer from both the functional and performance standpoints we have used before. The functional evaluation will test the foresight of those who collaborated on the logical design of the computer; the performance evaluation, on the other hand, will indicate, we hope, the practicability of electronic digital data processors, particularly in the field of naval logistics.

DATA STORAGE DEVICES AND TECHNIQUES

by

Jacob Rabinow
National Bureau of Standards

I would like to make some comments on Dr. Cannon's talk. As a working engineer I am sure that most of the troubles he ascribes to "unconservative" design were really due to the newness of the machines. Making machines "conservatively" has its difficulties too. If you make an airplane conservatively, it simply will not fly. This is true also in electronics. One cannot use vacuum tubes at a fraction of their rating. Tubes that are too large for the job have too much capacity and have other difficulties.

Engineers do not deliberately make flimsy machines, and I know the computer people well enough to know that they thought they were doing a conservative job. Experience is the only thing that can tell you what is and what isn't conservative.

Competition, of course, is very good in this business. There is no doubt that the Government spends millions of dollars for data processing machinery, and as the Government proves that its machines can work, more and more people of industry will enter this field. There is no doubt that many of these people will eventually say that the original machines were quite stupid, and we who work on them also think they are quite stupid. I have been in Ordnance for a long time, some twelve years, and always, as I look back on a design of a mechanism done some five years before, I wonder why I did it that way. Five years from now we will be looking back at the present designs of computing machinery and thinking this same thing. In any case, I hope so.

Now for the information storage devices. They can be classified in many ways. Perhaps the best way to classify the storage mechanism is by the type of things they store, and there are two types of information which one can discuss when one discusses recording memories. One is the type of information that must be preserved in facsimile. The signature on checks is such an item, as are maps, etc. These things must be photographed, and they are usually preserved in Microfile. Microfile, of course, is a trade name for a very fine process. The unfortunate thing about Microfile is that the original design of Microfile equipment did not anticipate mechanical handling. Thus most Microfile equipment, with the few exceptions I shall mention, requires hand-searching, and while Microfile gives you perhaps the greatest compactness of information, some poor clerk, usually a girl who is stuck with these jobs, has to sit and crank a knob or a dial until she gets the right bit of information out of storage.

The notable exception is the Rapid Selector, built by the Engineering Research Associates, now a subsidiary of Remington Rand. It is a machine re-invented by Dr. Vannevar Bush -- I say re-invented because someone of the Zeiss works in Germany invented it many years before, but apparently no model was ever built. In this Rapid Selector, information is stored in photographic form on 35 mm film. One-half of the width of the film is reserved for photographs of the documents one searches for. The other side of the film is reserved for a code of black and white dots. This code can be used to recognize the data in the documents by an optical coincidence device. The machine searches the data very rapidly. It searches at 300 frames per second, and in some four minutes one can search roughly 100,000 items of information. Whenever the particular code coincides with the card that one places into the machine, the machine says, "Bingo!" and copies the document. It does this "on the fly" without stopping the main film.

The photographs are taken in an exposure of two microseconds. The machine had some difficulties, particularly in the high speed copying camera, so that my group at the National Bureau of Standards made some modifications on it. That part works quite well. There are still parts that are giving trouble.

The Department of Agriculture, who has the machine now, does not have very much money for this sort of operation, not being a military agency. If one could get a military agency to support it, the work could be done at a much higher level.

My group has two kinds of programs. Most of the work is for the military, and a small amount is for the civilian agencies. The difference in cost for a comparable item, depending on whether it is being built for the military or civilian agency, is about 10 to 1. The difference in the rate of completion is about 2 to 1. I wish it weren't so, but this is the way it happens to work out. You can do the same job for a civilian agency, which gives you all the time in the world, and doesn't put any low temperature specifications on, in about twice the time, and for one-tenth the money. In the military development program the lowest priority we have is "crash" and the highest priority is "incandesence."

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Now to get back to facsimile data storage devices. Another machine is now being built by Yale University for one of the intelligence agencies. This will also scan Microfile mechanically. We are building at the Bureau, a machine that reads type. My little six year old daughter does it very much better, but the machine will read, and we can apply this machine to the reading of Microfile. The greatest advantage of such machines, of course, is the tremendous amount of information that one can photograph in two millionths of a second. In this time one can photograph several pages of newspaper. This is a great deal of information -- I mean quantitatively, not necessarily qualitatively. This type of data extracting and recording cannot today be duplicated by any other means.

The other type of data is the kind you are concerned with here today, and that is the type of data one can convert into some kind of a code. We can express this data as a number or as a series of dots and dashes, etc. This type of information has the particular virtue that it can be sent electrically over a wire. If one wants to get technical, one can say that this is also true of pictures. One can send them over a wire, also, by scanning and using television techniques.

This approach is not too practical today, but may be practical tomorrow. The information that can be coded easily has the other great advantage that one can make a duplicate of the information cleaner than the original. This, of course, sounds like double talk. One cannot get data that is lost -- that isn't there -- but you can at least preserve its cleanliness if it is there. In other words, every time you reproduce the data you can clean it up.

You can generate local pulses so that if the information you obtain is getting a little noisy, if the peaks are not even, one can regenerate them and make them even and clean. This is quite impossible in photographic reproduction. When one copies Microfile records, the second copy is worse than the first, the third is worse than the second, etc., and by the time one has copied it five or six times, the information is gone as far as machine reading is concerned. In any case, one can't trust it too much any more.

Digital information can be reproduced an infinite number of times, always checking to make sure that the copy is correct. As Dr. Cannon said, "There are techniques by which you can see that the information is not only clean, but that it is correct." This, of course, is the great advantage of digital machines.

Now for the machines that store this sort of information. The earliest practical machines were those using the punched cards with which you are familiar. It may be worthwhile at this point to remind you that this type of machinery was developed by the Government, and picked up by business. I would like to emphasize this because quite often we hear it said that we are competing with industry, and we would like to assure them that this is not so. We are just helping out. Maybe in individual cases, in a personal way, we are competing, but not as an overall program. The punched card is a wonderful method of storing information. It is of a binary type, that is, the information is either "yes" or "no," and this type of binary information is particularly good for electronic computers. This is because most electronic computers depend on vacuum tubes which are quite poor in their ability to distinguish multiple states and if one had less than a binary system, one would have no system at all.

Mathematicians tell us that binary arithmetic has some very great advantages. Frankly, I do not believe a word of this. I think the reason the binary system is being used is that the vacuum tube is such a poor device. If vacuum tubes were ever developed to a point where they have ten stable states, or sixteen, I believe the mathematicians would not use the binary system. Of course, these machines are so fast that it makes little difference, at the present time, as to what kind of arithmetic one uses. It is interesting to note that the glamour boys of this industry, the mathematical wizards, developed the computing parts much faster than the accessory organs. The result of that is that now everybody is desperately trying to devise input and output mechanisms to match the fantastic speeds of the electronic computing equipment.

There are several ways to store digital information: One is to punch holes in cards as I mentioned previously; another is to punch holes in magnetic material, except that instead of punching a real hole, one records a little plus or minus or a little negative or positive pole in the material. You can take small iron cores and wind small coils on them. By magnetizing the cores one can record a digit of information because the iron retains its magnetism. If one has to record a great deal of information, one takes a sheet of magnetic material and puts a little coil over a small area of this sheet, and records the material on a little magnetic spot. The spots may be only 1/100 of an inch by 1/100 of an inch. This magnetic material can be in form of wires, sheets, various other shapes, discs, drums, or anything you like, and the overall device can be quite elegant. Drums are very popular because they are very fast. They are cylinders perhaps ten inches or twelve inches in diameter, and perhaps a foot long. You can record on millions of these little areas and the magnetic field can then be read, later, by putting a small pickup coil near the drum. As the magnetic particle passes the coil, it induces a slight amount of current in it, and this is the output which can then be fed into a computer or into another recording device.

The difficulties with these drums are, first, that they are very expensive; secondly, one must not contact them because if one does, one rubs the surface away. The result of this is that one must have very high precision in the mechanical assembly. They are very difficult to use in a mobile unit. If one tries to fly such a drum you find that it is not only a magnetic recording device, but also a gyroscope, and it wants to fly the airplane in its own peculiar way. This gives rise to all sorts of difficulties. One can, of course, put drums in gimbals, so that if the carrier maneuvers, the drum remains relatively still. I am sure that any definite statement I make here will be wrong tomorrow. Someone who doesn't know the difficulties, or who knows them much better than I do will find an easy way out.

Magnetic recording on tape is very excellent, and gives very high compactness to the information, but there is a fundamental trouble with tape. The use of tape has rediscovered a lost art. In the old days, before books were invented, people had scrolls. The heralds read the scrolls by turning the two reels in their hands. The difficulty is that one has to unroll a great deal of length to find a particular bit of information, and if one happens to be looking for an item on Page 1,000, or the equivalent of Page 1,000, one has to go through a thousand pages to get there.

So, people invented books. These, of course, are scrolls chopped into short sections. Here the great advantage is that one can open the book at any place, if one has reasonable intelligence, and find the information one wants. The difficulty with books is that they do not lend themselves to mechanical handling. Such mechanical devices have been built, for instance, for pianists, and for people without hands, but the mechanisms are complicated and certainly not as reliable as one would wish for an automatic storage device.

We have recently developed at the Bureau, a machine which does read books without opening them. It is done this way: One takes a large number of disks and puts them together. I would like to have had the machine here, rather than explain it, but in fifteen minutes one does not have much time to do much exhibiting. If any of you wish to visit the National Bureau of Standards, we would be more than happy to show you the equipment.

Each of the stacked disks in our machine has a notch cut out of it, like a pie section. The notches are all aligned in one straight line. By turning any disk, one can read both sides of it by looking through the notches of the other disks. This is as if a book were made of round pages pivoted at the centers, so arranged that each page could be independently turned. The first machine is now being finished by one of our contractors. It will store about a billion digits and will have an access time of less than half a second. This type of machine is called a Random Access Memory.

Magnetic information is very elegant, not only because of its compactness, say 10,000 dots to a square inch, but also because one can erase the information very easily, and reinsert it at any time. This, of course, has its difficulties. Some types of information one does not wish to erase. I am sure if you were keeping permanent records, you would worry about this. We worry about it too. It is not so bad, however, if one takes special precautions, such as duplicating the records, having proper interlocks in the equipment, etc.

Magnetic information is quite permanent. If one does not put exactly the right type of demagnetizing magnetic fields on it, which is rather difficult to do, and if one does not heat it to a very high temperature, the records should last indefinitely. There are magnetic instruments in the world today that have not changed more than one or two per cent in the last fifty years. The safety from fire is no worse than one would expect from paper information, perhaps better. By the time the magnetism is destroyed, the base on which the magnetic material is deposited is usually destroyed also.

There are many other ways of storing information. The wire is excellent, permanent, but rather slow in access time.

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Another way of classifying these information storage devices, perhaps, is to differentiate between those where the information must be permanently stored and those where the device is only used as a temporary memory, that is, as a scratch pad, where the information is only required for a few moments of time. One of these devices is the acoustic delay line. It is a tube of mercury about two feet long, through which a series of sound pulses is transmitted. Actually the word "sound" is somewhat misleading. The pulses are of megacycle rates. An input device sends a pulse into the mercury and at the other end a microphone picks it up, then the signal is fed back through an amplifier, back to the input so that the device "chases its own tail" until you are ready to take the information out. If one shuts off the amplifier, the information dies in a single pass; if one wants to read it, one connects to the circuit and the information runs out on the connected wire. One can interrupt the circuit for a short time and thus erase a part of the information and then insert new information in its place.

The cathode ray tube can also be used for storage of information. The information is put on as small dots on the face of the tube and electrical capacity can be made use of to keep the information there for a short time. The material can be regenerated and kept on the face of the tube indefinitely; that is, as long as the power in the circuitry stays alive.

The beauty of this device is the very high speed with which the information can be placed on the face of the tube, or read out. Also, any bit of information can be reached directly without having to go through a long process as is done in the case of wire, tape, drums, etc. This type of storage is particularly good for temporary, working, type of memory, usually inside a computer. It is, of course, unsuitable for permanent storage.

An entirely new kind of data memory is now being developed. It makes use of the spin of electrons in a solid or liquid material. I am not a physicist, and do not know exactly how these devices work, but it seems that the electrons are spinning and really are gyroscopes, so that if one applies an external field, these gyroscopes can be made to precess. This precession, and the rates at which the precession is developed and decays, can be made use of to record information. The information is fed in as a series of pulses and at some time later the series of pulses come back, out of the device, and then can be reinserted and regenerated indefinitely. The beauty of the device is that it has no moving parts, in the usual sense of the word, and is extremely compact and fast.

There is no doubt that in the near future these will be working gadgets. Other memories are also being built such as electrostatic devices, where the information is recorded capacitatively on barium titanate and other materials. This development looks very promising and should result in inexpensive, very compact storage devices which can record information permanently.

Most of the high speed devices require very complicated electronic circuits to feed the information in and out. The magnetic devices are relatively simple, but require more mechanics.

I believe that the above gives you some sort of a picture of the field of recording of information. I am sure I have confused some of you. It goes without saying that all the remarks you have heard about computers not being too dependable, apply to memory devices as well. We hope that as experience is gained and the devices pass from the laboratory to the engineering and finally to the production engineering stages, these difficulties will be overcome and the gadgets will be as good as things like typewriters, for instance. It takes more than two or three years to get industrial experience and in perhaps ten to twenty years all of you will be able to go out and buy conventional, commercial equipment of this type. Certainly it appears that the dependability of the data will be as good as it is in typed or handwritten methods used today.

It is our experience in Ordnance, that the human factor is much more important in the question of dependability than the matter of machine design. I could tell you fantastic tales about the cases where intelligent people have made the most stupid errors. Machinery is quite dependable. It is our philosophy, at least in ordnance, that we would rather trust the machine than the operators. So that we believe that the computers with all their faults, if they are provided with some built-in checking equipments, will be far better from the point of view of dependability, than the operations that we are performing today.

I have discussed some of the operations of inventory control with people of industry, people from General Motors and Sears Roebuck, and it appears that a great deal of hand labor is now used in these operations and the results are far from satisfactory. The errors are quite frequent, and sometimes can be very large. We hope and believe that the machines now being built, because of their phenomenal speeds, which enable you to sacrifice some speed in exchange for checking time, will eventually store information much more dependably than anything we employ today.

DIGITAL INPUT AND OUTPUT DEVICES FOR AUTOMATIC COMPUTERS

by

Nelson M. Blachman

There seem to be about 15 different types of digital input and output devices for use with automatic computers. One of them, the keyboard, is good only for input. Four types can be used for either input or output, so that they can also be used for slow-access, large-capacity storage. These four are: Punched cards, punched paper tape, magnetic tape, and magnetic wire. The other 10 types, good only for output, are all printers.

A high-speed computer doing a problem involving any considerable amount of input or output is operating efficiently only if it is equipped with high-speed input and output equipment. PUNCHED CARDS are fairly fast; they are usually punched at the rate of about 2 a second, that is, about 150 alphabetic or numeric characters a second. They are read at the rate of $2\frac{1}{2}$ cards a second on standard equipment, that is, 200 alphameric characters a second, but there are special card readers that go 4 times that fast. Punched cards have the advantage of being widely used, easily sorted, easily altered by replacement with a new card, and they can be read visually if necessary.

PAPER TAPES are slow. Tapes from 5 to 7 holes wide, are read and punched on standard equipment at the rate of only 8 frames or characters a second, but special equipment can handle paper tape at 60 frames a second. Also, there are commercially available photoelectric tape readers able to read 200 frames a second which can start and stop the paper without missing a frame. Like punched cards, paper tapes can be read visually, but they are not easy to alter. Once information is put on a tape, it can't get shuffled around--which may be an advantage or disadvantage. Punched paper tape seems to be the most universal input and output medium for automatic digital computers, expecially for the simpler ones.

MAGNETIC TAPE is often divided up into 6 to 8 tracks, with pulses in each track recorded at 50 to 100 per inch. These tapes are usually run at 30 to 100 inches a second, so that something of the order of 2000 characters are handled per second. Magnetic tape is capable of much higher speeds and storage densities than punched cards or paper tapes. It can't be read visually, but any part of it can be erased and reused many times.

MAGNETIC WIRE is equivalent to single-track magnetic tape. At the Bureau of Standards, wire cartridges intended for office use are run at 8 feet a second with 50 pulses to the inch, that is, with 5000 pulses or about 1000 characters a second, but I'm told that speeds a dozen times that fast are practical. Since the wire is very thin, extremely high storage densities are obtained--up to 4 million bits per cubic inch. But a cartridge of magnetic wire will store only about 200,000 characters, while a reel of magnetic tape usually stores several times as much.

Having considered the 4 input-output devices, we may go on to the rapidly developing field of printers for computer output. PRINTERS may be classified as mechanical, electrical, or electronic. Among the MECHANICAL PRINTERS we have first the solenoid-operated typewriter, which is standard equipment on most computers, and the solenoid-operated adding machine, which can handle up to about 15 characters a second. Then there is the accounting machine, such as the IBM 407, which prints $2\frac{1}{2}$ lines a second of 120 characters each, that is, 300 characters a second. The accounting machine works like the adding machine except that it has more columns, and the type is on rotating wheels rather than on straight bars. Next there are "on-the-fly" printers, which come in two varieties.

In one variety, made by ANelex and Shepard, there is the equivalent of a set of wheels as in the 407 printer, but all of the wheels rotate continuously and in unison. Instead of the wheels being struck against the paper, hammers behind the paper--one for each column to be printed--push the paper against the ribbon and wheel at the instant when the desired character is passing by. Characters are printed in different columns at different instants instead of being printed simultaneously as in 407, and it is important that the paper should not move during the printing of a line of information. The problem of getting the paper to move quickly and stop quickly is one of the principal difficulties in developing high-speed printers. It is also one of the chief difficulties with magnetic tapes and wires.

The ANelex and Shepard printers, sometimes called "wheel printers", are able to print 15 to 20 lines a second, each having 40 to 56 characters, that is, about 800 characters per second. Another problem presented by printers handling a line more or less all at once is the necessity of storing a whole line's worth of information so that the printer can act on it simultaneously. These "wheel - type" printers are sold without any storage, and the user must supply one.

The other type of "on-the-fly" printer is represented by the Potter Flying Typewriter. Instead of using a separate wheel for each column, it uses a single, large wheel that rotates about a vertical axis, so that each of the up to 64 different characters on the edge of the wheel passes every position in the line to be printed in the course of one revolution. A hammer behind each of the 80 column positions strikes the paper against the ribbon and wheel as the desired character passes by. In its present form, the printer is operated at 5 lines a second, that is, 400 characters a second. The intricate timing problems are solved by the use of 80 binary electronic counters, which are also used as a shifting register to receive and store the information to be printed.

The fourth and last type of mechanical printer is the "matrix-type printer," which forms characters by a selection of dots from an array of, usually, 5 dots by 7. The dots are made by the pressure of thin wires or "styli" against a carbon ribbon. The styli can be actuated up to 2000 times a second, so that they are capable of extremely high printing speeds, but the Eastman Kodak printer prints only a fifth of a character at a time (one column of dots), so that the speed of one printing assembly is only 3 or 4 hundred characters a second. In printing addresses, 4 assemblies are used to print the 4 lines of an address at the same time, though, quadrupling the speed.

The Control Instrument Co. version of the matrix printer has 36 assemblies of 35 wires each, so that it can print a line of 36 characters all at once, but its speed is apparently limited by the paper-motion problem of 15 lines a second. These matrix-type printers require a lot of electronics to select the proper styli; and the crowding of all the styli into a small space has taken a good deal of ingenuity.

ELECTRICAL PRINTERS. The Atomic Instrument Co. has developed a printer that records 500 three-digit numbers a second on a continuously moving strip of Teledeltos paper. Each digit is formed by a selection of dots from a 3 x 5 array; a set of tungsten wires is made to spark through the gray Teledeltos paper, revealing the black inner layer. Improved models of this printer are supposed to use bigger arrays so that a greater variety of characters can be produced, and a larger number of columns--up to the width of a page.

The Hogan Laboratories and Austin Co. have proposed digital recorders using special types of paper that are marked by the passage of an electric current, but they have apparently not yet built these devices.

General Electric has been working on a system called "Ferromagnetography." This system resembles somewhat the wheel-type printer, but the hammers are replaced by electromagnets which put magnetic images of the characters on a wide magnetizable tape. The tape is passed through a suspension of magnetic particles, which adhere to the magnetized regions and are then transferred to the paper. Additional copies can be made by passing the still magnetized tape through the suspension again, or it can be demagnetized for the printing of new information. An experimental model of this printer is capable of 40 lines a second, and a line width of 120 characters is proposed, but the paper-motion problem has not been considered.

ELECTRONIC PRINTERS. Engineering Research Associates has built an experimental model of a "Magnetic Numeroscope Printer," which uses magnetic drums on which are permanently recorded the signals necessary to make the beam of a cathode-ray tube write the decimal digits on its screen. Additional deflection voltages place these digits on the proper part of the screen so that 100 characters can be written on a line, 5 times a second. A microfilming camera is used to record the images. Improvements contemplated in the Numeroscope Printer would increase its speed from 500 characters a second to 8000 and would allow 40 different characters.

At the Naval Research Laboratory, a display device has been devised using a monoscope to control the formation of sexadecimal digits on the display tube. The cathode-ray beams in the monoscope and display tubes are made to scan, respectively, the desired digit and the part of the screen where it is to appear. The monoscope electrode controls the beam current in the display tube.

Consolidated Vultee is developing a special type of cathode-ray tube called the "Charactron," which has two sets of deflection electrodes. Between the two sets is a plate stenciled with up to 64 different characters. The first set of deflection electrodes causes the beam to pass through the desired hole, giving it the shape of the character to be displayed; the second set deflects the beam to the position on the screen where this character is to appear. The Charactron is supposed to have a speed of about 10,000 characters a second. Recording may be accomplished by the use of "Xerography," which uses electrostatic forces in much the same way that magnetography uses magnetic. The light from the Charactron screen, focused on the surface of a charged xerographic drum, would discharge the areas where it fell. These areas would be able to pick up particles from a suspension and transfer them to paper, to which they would be fixed by momentary heating.

The Wang Laboratory and Laboratory for Electronics in Boston have developed a cathode-ray display with a speed of 12,000 characters a second. It uses a 7 x 8 array of magnetic cores through which is threaded one wire for each different character, in the shape of that character. One wire is selected every 80 micro-seconds to control the beam current in the display tube. The cores are pulsed sequentially as the beam scans the area where the character is to appear. A model of this device is working, with input from a 10-column keyboard. Since it is intended only for display, nothing is being done about recording its output. A similar device using a complicated diode switching network in place of the magnetic-core array is in use on a computer in Paris. Its speed is also 12,000 digits a second, and its output is recorded photographically on 35-mm film. These electronic printers are so fast that they raise the question of what to do with all the output.

This covers the 15 types of input and output devices I know of. Lt. Rossheim deserves credit for gathering much of the information I've presented. He and I have written a report on "High-Speed Printers for Digital-Computer Output" for the Office of Naval Research. There are a few copies left which are available to those of you who may need them.

SUMMARY OF GENERAL DISCUSSION ON
PAPERS BY CANNON, RABINOW, AND BLACHMAN

Dr. C. B. Tompkins commented that not all scientific research nor all engineering development is of a casual, experimental nature. In the development of computing machinery some people have applied their ingenuity to making very fast machines, others to building reliable ones with more or less predictable results. Business machine manufacturers in general--Bell Telephone Laboratories, International Business Machines, and National Cash Register, for example--have built reliable equipment, which they predicted would be reliable. Dr. Cannon's point was that in some problems, we have to pick reliability in lieu of speed, if we cannot have both features in the same device.

Mr. Rabinow stated that not a casual approach to design but rather the infancy of the art of computer development is the factor responsible for the limited reliability of the high-speed computer. Manufacturers of business computing devices, on the other hand, have had long years of experience and their new equipment is often excellent.

A DESCRIPTION OF THE LOGISTICS COMPUTER

BY

Lt. R. J. Rossheim, USNR
Office of Naval Research, Logistics Branch

On February 17, 1953, there was delivered to the George Washington University Logistics Research Project a special purpose electronic computing machine named the Logistics Computer. It was built under contract with the Logistics Branch of the Office of Naval Research by the Engineering Research Associates Division of Remington Rand. The primary reason for its construction is to assist research in the science of logistics by providing an instrument for rapid and effective data-handling and computation. This paper will trace the history and development of the computer, and describe the equipment and its operation in general terms. The description is intended to suggest some of the uses to which the computer may be put, and later today Mr. Wolf of the Logistics Research Project will describe in some detail a particular application of the computer.

In accordance with an assigned mission of the Logistics Research Project to explore the use of modern high-speed data-handling and computing techniques in logistic applications, the general specifications for a machine designed to perform appropriate operations were first formally set down at the end of 1950. A paper prepared by Dr. C. B. Tompkins, then Principal Investigator of the Logistics Research Project, suggested in some detail the possible design of a prototype logistics computer and the hardware components which might be used in it.

All of the Navy Bureaus, the Marine Corps, and other interested supply people were approached and were asked to express their opinions on the utility of the proposed equipment. They were also asked to describe any of their problems which required considerable data-handling and computation. Some of the problems were analyzed in sufficient detail by members of the Logistics Research Project to yield some quantitative figures describing the efficacy of the hypothetical computer. I should like to discuss very briefly the characteristics of these problems which led directly to the creation of the Logistics Computer.

An Inventory Control Problem of the Aviation Supply Office (Fig.1) involves the use of planning information, usage factors, and inventory reports to calculate net item requirements for each of 50 supply bases. The computation involved is basically one of accumulating gross requirements of items by multiplying planned activity levels by the item usage factors. For example, if 50 engine overhauls were scheduled, then the itemwise usage factor list indicating the requirements for parts to accomplish one overhaul would be multiplied, item by item, by 50. The requirements generated by this one activity would then be added to previously accumulated requirements, and this process continued until all planned activities had been considered. Gross requirements thus computed would be subtracted from inventory levels, item by item, to determine net requirements and to initiate purchases or, perhaps, to revise the activity levels.

An Advanced Base Critical Item Problem, (Fig. 2) based on the requirements for critical items, involves making a logistic feasibility test of a plan for advanced base development. Planning data, consisting of numbers of advanced bases of different types scheduled over a period of time, are applied against allowance lists of critical items (which may also involve time in the form of procurement lead times). The basic computation is again one of multiplying the appropriate allowance list by a planning number, accumulating gross requirements of specific items at specific times. With the total itemwise, time-phased demand thus stated, there remains for consideration the supply picture to test the logistic acceptability of the plan. If it is shown that the plan is not acceptable--that certain critical items can not be available at the proper time--the plan must be tailored or modified by eliminating or delaying one or more bases and recomputing the requirements.

Ordnance Supply Office Spare Parts Requirements Problem is to stock an overseas base with ordnance spares sufficient to support a given naval force. Starting with a specified group of ships, the population of major ordnance equipments is computed. Major items are then broken down into replenishable components or items, and the sums of these items in the entire force are computed. Usage factors are then applied, item by item, to these totals resulting in the numbers of replacements expected to be needed by the naval force over a specified period of time. This, then, is a three-level breakdown computation involving multiplication and addition to pass from one level to the next lower, and finally a multiplication of usage factor times total quantity of items, for each different item in the population, to find replacement requirements.

Each of the problems referred to was attacked by using the hypothetical logistics computer. Data preparation and computational procedures were described in precise detail and time estimates for solution, based upon a realistic quantitative analysis, were made.

I would like to emphasize the principal computational aspects of these problems so that you may understand the reasons for the special features of the computer. It must be immediately apparent that a major consideration is the mere handling of the tremendous quantities of data necessary to solve these problems on the required level of detail. Depending on the problem, thousands, tens of thousands, or hundreds of thousands of different items must be accounted for, and very often there are multiple entries to be made for a single item. Another clearly indicated aspect which is common to these problems is that in each case there are millions of data transfers and basic arithmetic operations, but on any single unit of data there are relatively few operations.

Figure 1

?

PLANNING — HISTORICAL INFORMATION — DEMAND | SUPPLY — SUPPLY ACTIVITIES — PRODUCTION

DATA

ACTIVITY LEVEL	PLANNING FACTORS	TOTAL DEMAND	TOTAL SUPPLY	STOCK LEVELS	PRODUCTION SCHEDULES
L_1	U_1 U_2 U_3 - - -	D_1 D_2	S_1 S_2	S_1 S_2 S_3 - - - A	P_1 P_2 P_3 - - - M_1
L_2	U_1 U_2 U_3 - - -	D_3	S_3	S_1 S_2 S_3 - - - B etc.	P_1 P_2 P_3 M_2

COMPUTATION

$L_1U_1+L_2U_1+L_3U_1+\ldots = D_1$ | $S_1=S_1(A)+S_1(B)+S_1(C)+P_1(M_1)+P_1(M_2)$ | $D_1-S_1=R_1$ (net requirements)

$L_1U_2+L_2U_2+L_3U_2+\ldots = D_2$ | $S_2=S_2(A)+S_2(B)+S_2(C)+P_2(M_1)+P_2(M_2)$ | $D_2-S_2=R_2$ (net requirements)

$L_1U_3+L_2U_3+L_3U_3+\ldots = D_3$ | $S_3=S_3(A)+S_3(B)+S_3(C)+P_3(M_1)+P_3(M_2)$ | $D_3-S_3=R_3$ (net requirements)

Figure 2

REQUIREMENTS FOR ADVANCED BASES

THE PLAN

TIME MONTHS

	1	2	3	4	-	-
LIONS	2		1	-	-	-
CUBS		1	1	-	-	-
ACORNS			2	1	-	-

Phased Gross Requirements

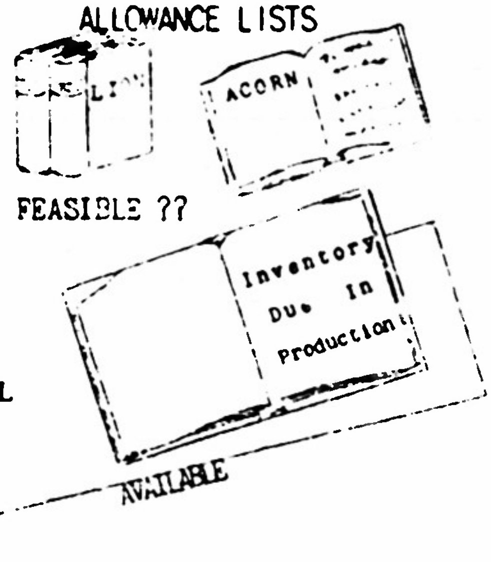
BASE TYPE

TIME MONTHS

	1	2	3	4	5	-
CRANE	4	2	3	0	-	-
PUMP	6	3	2	1	-	-

CRITICAL ITEMS

ALLOWANCE LISTS



Moreover, they are straightforward and simple. It is the difference between multiplying one pair of numbers and 100,000 pairs that is important.

The inventory operation points up the need for a large, easily accessible file or central information storage system which contains at all times the current stock levels of all the items in the system. In addition to pure size, since we can not normally forecast the order in which item transactions will occur, it would be most convenient to be able to search out quickly and at any time information about any item in the system. Stated in computer lingo, we would like a rapid arbitrary access storage device in which any selected unit of information may be modified without affecting any other information in the system.

What about the arithmetic properties of the machine? It is clear that addition, subtraction, and multiplication are the operations repeatedly called for, and division does not seem to be justified. It has been pointed out that the basic operations of the computer would normally be combined in relatively short, simple sequences, and that precisely the same sequence of operations would be performed on each successive unit of data. These considerations have largely determined the design of the arithmetic and control units of the computer.

After satisfactory completion of the initial phases of the analysis which I have described, a contract was let by the Office of Naval Research to perform research and to construct a prototype Logistics Computer. It was intended that the contract should run approximately 1½ years, starting on the first of March 1951. It consisted of four phases:

Phase I included investigation by the manufacturer of the logistics problems mentioned earlier. At the end of this phase the computer specifications were determined jointly by the Office of Naval Research, the Logistics Research Project, and Engineering Research Associates.

Phase II consisted of research and development of hardware which resulted in a breadboard model of the computer. It might be noted here that it was during this phase that continued problem analysis by the Logistics Research Project and logical design analysis by the manufacturer's engineers led to some changes resulting in additional flexibility in the operation of the computer. The flexibility, it was decided, was desirable in a prototype equipment to be used for research. Subsequent models intended for routine operational applications might not require all the features of the prototype machine and would, therefore, be scaled down in size and complexity.

Phase III covered the construction of a working model of the computer.

Phase IV of the contract provided a period for operational testing prior to acceptance. It was completed in February 1953 and the Logistics Computer is being operated by the Logistics Research Project. The computer has the following general features:

1. It is designed to read at high speed and to process large quantities of decimal data previously punched on standard teletype paper tape. The computer is unusual in that normal operation provides for simultaneous data input and computation.

2. The computer is capable of performing one or two sequences of simple data-handling and arithmetic operations on each unit of data as it is read into the computer.

3. An information storage unit internal to the computer will retain up to 160,000 decimal digits. These numbers may be initial data to be processed, intermediate results, or final results before they are taken out of the machine. The means of referring to any of these digits is easy and quick. Access to any number in the system is practical and efficient and there is no need to consider the order of reference.

4. Maximum reliability of operation has been consistently emphasized in the construction of the machine. It has been understood by all concerned that the usefulness of such a computing equipment is based upon complete reliability. Wherever possible, computer component types have been chosen which have been successfully tested and used on other equipments.

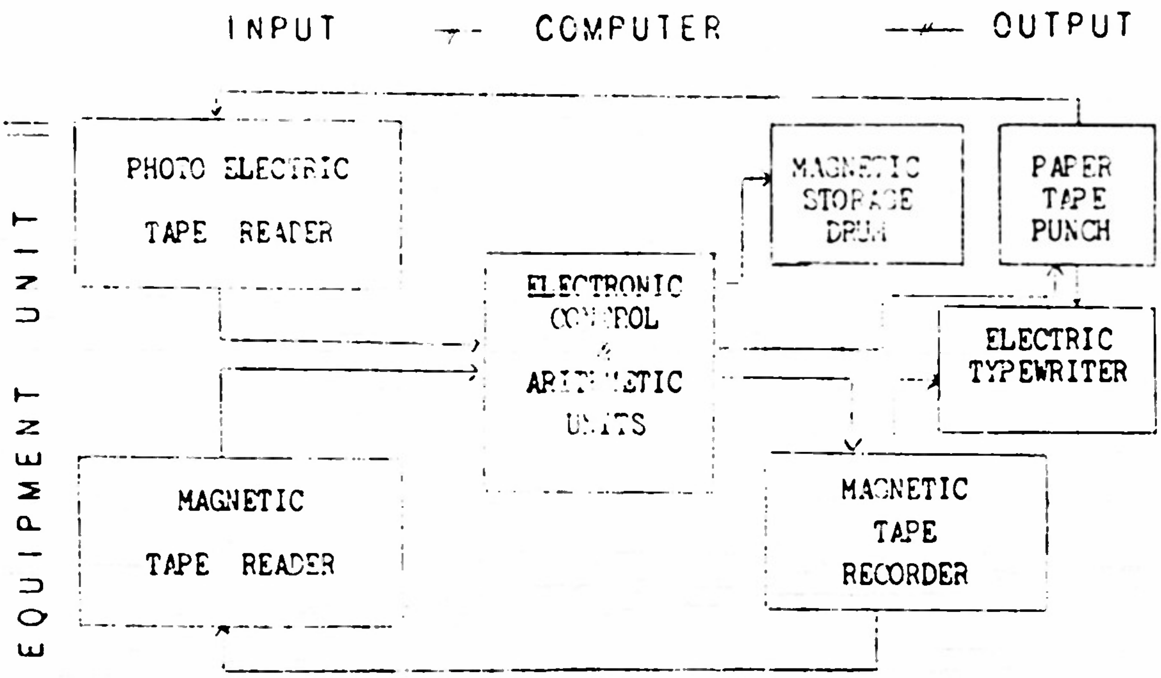
Both the setting up or programming of the computer, and its operation are intended to be simple and straightforward, so that a small amount of training will prepare people to use the machine. It is my belief that the people who now supervise the manual or punched card procedures in activities performing the operations for which the Logistics Computer is suited, could easily learn to use the computer.

In addition, from the start it has been visualized that the computer might be the heart of an extensive data-processing system. Recalling the Aviation Supply Office problem, earlier described, in which computations for 50 widespread activities were to be carried out at some central control point, we are reminded that it is necessary for the computer to have communication with all its data sources. With this in mind, punched paper tape compatible with standard teletype gear was selected as the medium of communication between the computer and the outside world.

The Logistics Computer, (Fig. 3) like all large-scale computers, may be divided into three basic sections--the input section, the computing section (including arithmetic, control, and storage units) and the output section. The computing units basically handle information in pulse form,

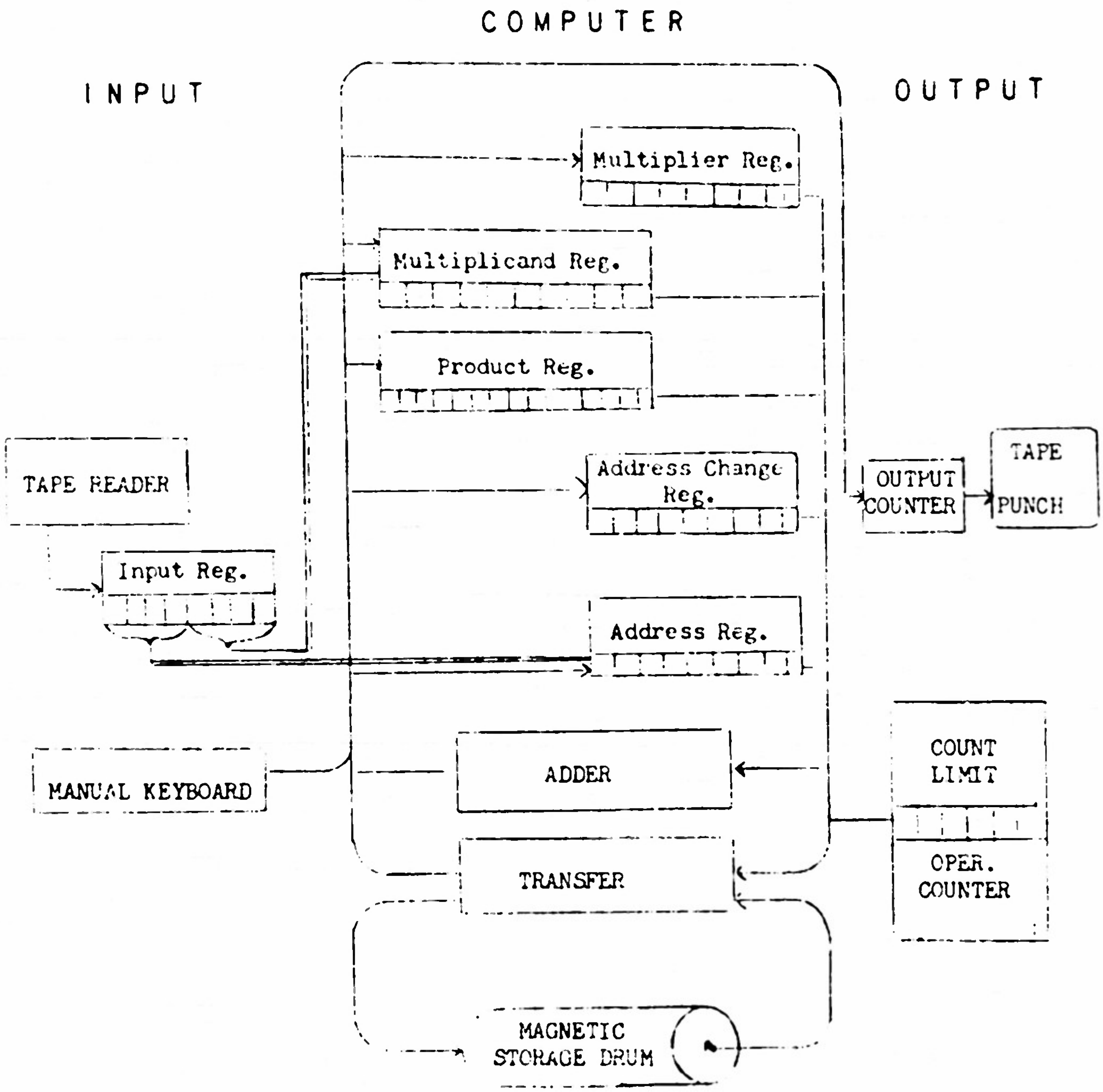
A DESCRIPTION OF THE LOGISTICS COMPUTER

Figure 3



FUNCTION	TRANSFER NUMERICAL INFORMATION INTO COMPUTER	DETERMINE AND CONTROL SEQUENCE OF OPER. ON DATA	STORE (with random access) approximately 176000 Digits	TRANSFER NUMERICAL INFORMATION OUT OF COMPUTER	PRINT COMPUTED RESULTS FROM TAPE
SPEED	300 DIGITS/SEC.	220,000 DIGIT TRANSFERS/SEC 40,000 DIGIT ADDITIONS/SEC	60 REFERENCES/SEC.	MECHANICAL PUNCH 10 DIGITS/SEC MAGNETIC RECORDER 600 DIGITS/SEC.	8 DIGITS/SEC

Figure 4


COMPUTER
INPUT
OUTPUT
Multiplier Reg.
Multiplicand Reg.
Product Reg.
TAPE READER
Address Change Reg.
OUTPUT COUNTER
TAPE PUNCH
Input Reg.
Address Reg.
COUNT LIMIT
MANUAL KEYBOARD
ADDER
OPER. COUNTER
TRANSFER
MAGNETIC STORAGE DRUM

permitting operations at electronic speeds. The input system must work with a medium capable of performing the conversion between human speeds and computer speeds. The output section must effect the opposite conversion, making the results intelligible to the user.

In this computer the medium presently being used to communicate data into and out of the machine is standard Teletype punched paper tape. A photo-electric tape reader is used to convert the information, punched in the tape, to pulse form in the computer. The reader is capable of sensing 300 lines of tape per second, and each line contains a decimal digit.

Numerical data and control signals are sent from the reader to the arithmetic and control units. As soon as a unit of data is transmitted to the central computer, a sequence of operations is initiated by the control unit. These operations will include additions, subtractions, multiplications, data transfers, and others -- all of which are performed on numbers temporarily stored in the arithmetic section, and under supervision of the control unit. The order of speed of these operations is indicated by the fact that single digit additions (i.e., $2 + 3 = 5$) can be performed at the rate of 30,000 to 40,000 per second. Multiplications of two six-digit numbers are performed at 100 per second. Closely associated with the arithmetic and control sections is the large internal storage unit, a magnetic drum capable of storing 160,000 decimal digits. During a sequence of operations on a unit of data, reference to this central file may be made to read out previous results, or to store intermediate and final results. Up to 60 of these references to storage may be made per second without regard to the order in the file in which the references are made. Once the selected position in the storage is located, decimal digits are transferred at the rate of 220,000 per second.

When the point in the processing is reached at which it is desired to convert answers stored in the computer to punched tape, this is performed by a standard electro-mechanical tape punch at 10 digits per second. The punched tape may then be interpreted by using it to control a decoding, standard electric typewriter at 4 to 8 characters per second.

The magnetic tape equipment, currently under development but not yet in use with the computer, will serve two major functions. In situations requiring the recording of thousands of answers, the speed advantage it has over mechanical punching (60 to 1) affords a considerable time saving. The second function is that of a semi-permanent, large capacity storage external to the computer. For instance, an inventory which has just been brought up-to-date and stored on the drum, may be read out onto magnetic tape at high speed, thus freeing the computer for other operations. We expect that a complete drum load of 25,000 numbers will be read out onto magnetic tape in 15 minutes.

Fig. 4 will indicate to a certain extent what is inside the blocks described above and how the units are tied together.

The photoelectric tape reader senses tape continuously, transmitting the data into the Input Register. There, each unit of data is assembled and when the computer signals that it is ready, the data is squirted out of the buffer storage Input Register at high speed into the electronic registers of the arithmetic section. Each unit of data may have two parts -- a five digit number which automatically is sent to the Address Register, and a six digit number which goes into the Multiplicand Register. The number in the Address Register may be identified with a particular item in the problem. It is assigned also as reference to a particular location in the storage, since the computer storage system breaks down the five digit number in the Address Register to select one of the fifty channels along the axis of the drum, and to select one location around the periphery in the chosen channel.

The Keyboard provides a means for manual insertion into the arithmetic registers of numbers which are used initially in a computation or which remain constant during it.

The flow of information through the computer is indicated by the arrows (Fig. 4). All operations are serial with respect to decimal digits. The electronic registers are of various lengths, one 5 digit, two 6 digit, and two 12 digit. The number in any one of them may be copied out onto the main bus (the original number remains in the register, shifting circularly). Numbers may be transferred normally, or transferred with change of sign; they may be added, subtracted, multiplied, or stored. Any register may also be made to receive the results of an operation.

Some specifications of the magnetic drum storage system may be of interest. It is a cast aluminum cylinder $8\frac{1}{2}$ inches in diameter and 14 inches long. Its surface is treated with a ferro-magnetic material capable of permanently recording the numbers handled by the computer. The effective density of the information around the drum is about 120 pulses per inch based on the non-return-to-zero system. Both recording and reading are performed by the same dual-purpose head which controls the information in the channel passing beneath it during rotation of the drum. A magnetization pattern, representing a number, will remain on the drum indefinitely, or until another pattern is recorded over it. A number may be copied off the drum time and time again.

Storage locations for decimal numbers may vary in size from 4 to 12 decimal digits inclusive, although only one number length may be specified on the drum at a time. Between 13,000 and 31,000 numbers may be stored simultaneously on the drum surface, depending on the number length selected.

A DESCRIPTION OF THE LOGISTICS COMPUTER

The Operations Counter is a decimal counter which may be stepped up one count at a time. Associated with the counter is a machine instruction which permits repetition of a sequence of operations a certain number of times, after which another different sequence may be performed.

The output gates are used to synchronize the crawling speed of the tape punch with the lightning speed of the computer.

The relationships between all of these units during a particular calculation is pre-determined by the wiring of a programming plugboard similar to those used in office accounting machines. Sequences of instructions to the computer are wired on the plugboard prior to running a problem. The plugboard may be inserted into the machine in a matter of seconds. A set of standard plugboards which instruct the computer to perform recurrent operations may be permanently wired so that they may be ready for use at any time. New plugboards to perform unusual operations can be wired in from 10 minutes to an hour depending on their complexity.

A typical instruction consist of 4 parts called commands, each **represented** on the plugboard by a single wire. For example, to transfer a 6 digit number from the Multiplier Register to the drum requires the following 4 command wires: One to Multiplier Register Read Out Transfer; one to Drum Read In; one to the operation Normal Transfer; and one to cycle length 6 indicating the number of decimal digits.

I would like now to demonstrate how these units are tied together under typical operating conditions:

Let us suppose, for example, that we have a simplified version of the critical item problem mentioned before, (Fig 2). We want to know the logistic acceptability of a certain plan involving numbers of various organizational units. With each organizational unit is associated an allowance list containing the quantities of critical items necessary to outfit the unit. Before we perform the computation, the data comprising each allowance list must be punched on a paper tape. Each unit of data on the tape describes the quantity of a specific critical item needed for the organizational unit. The tape contains an item identification number and the quantity needed. Where similar critical items appear on other allowance list tapes, they must be tagged with the same identification number.

These tapes, one for each organization, contain relatively fixed information and represent a basic file of allowance lists. Of course, they may be periodically brought up-to-date. The tapes may be punched manually using a tape punch with a keyboard similar to, but simpler than, an accounting machine card punch, or, if the allowance lists already exist on punched cards, tapes may be punched automatically by a card-to-tape converter which operates at the rate of 10 digits per second, punching decimal codes on paper tape.

Given these allowance list tapes, and a knowledge of the computation involved, the next step is to prepare a programming plugboard which will be plugged into the computer to select and to control the sequence of operations in this particular computation. Let us review the data-processing operations in terms of computer instructions.

Let us say that our plan calls for the outfitting of 5 Type A organizational units. What we will do is feed the allowance list tape for Type A into the computer, multiply a single unit of data by 5 and add the resulting product to the amount of that particular critical item already required for units previously considered. We will then store on the magnetic storage drum the current total requirement, and proceed to the next number from the tape.

As each unit of data is read from the tape into the computer, the item number is automatically transferred to the Address Register which controls the point of access to the storage drum. We have selected and assigned these item numbers with some thought so that the arrangement of the data on the drum is consistent with the operation of the storage system. The item number prepares the computer to reach into its memory and copy out the quantity of the desired critical item required by preceding tape runs. The first instruction on the programming plugboard is to transfer the previous requirement from the storage drum to one of the arithmetic registers where the computer is able to work on it. The other part of the unit of data from the tape representing the amount of the item in question in Type A unit is automatically placed in another arithmetic register.

Since we want to multiply by five every item on this allowance list tape, we may manually insert into a third register (using the keyboard) the number five, before the tape is run into the machine. The second step in the sequence of computer operations is to multiply 5 times the quantity from the tape and to add the product to the previous requirements. The third step is to transfer the newly computed total back to storage.

The identical 1, 2, 3 sequence of "read from storage, compute, and return to storage" is repeated for each item on the tape as it is read into the computer at the rate of 25 units of data per second.

When a complete tape has been read, the computer pauses while a new multiplier for organization Type B is manually inserted, and the B allowance list tape is placed in the photoelectric reader.

After all tapes have been run, the storage drum has recorded on its surface the gross requirements of all critical items demanded by the plan. An inventory tape may then be run into the computer to calculate net requirements. It might then be desirable to punch out a tape indicating all critical items which would be depleted by the plan under consideration. Tailoring of the original plan may then proceed, using the information on the drum and running in selected allowance list tapes with different multipliers than originally planned.

It is evident, I think, from the description of the computer, that we are and will continue to be faced with problems involving the preparation and input of data, and other problems involving the output, printing and dissemination of resulting data.

At present we have manual methods of punching tape limited by the dexterity of the human operator; and we have automatic means of converting IBM cards to input tape for the computer. This is a standard piece of IBM equipment limited in speed by the mechanical tape perforation process. Only through multiplicity of these tape preparation units can we expect to prepare tapes fast enough to keep the computer well fed.

The magnetic tape equipment will, if it proves reliable and satisfactory in other respects, enable us to relieve the computer of its highly uneconomical chore of punching tapes. Data may be dumped at high speeds on the magnetic tape and at any convenient time or place converted to printed copy or to punched tape if necessary. Magnetic tape will provide a medium which can feed any printer of reasonably high speed. It will also serve as a limitless external storage for data which we may use occasionally for short runs. It is quite economical and compact, a 1200 foot reel being capable of storing a complete drum load of numbers. Other newly conceived methods of storage may also be tied in with the computer.

When and if the magnetic tape equipment is in everyday use, the bottleneck will be the production of printed output. Present solutions of this problem involve paralleling a few of the slower speed electric typewriters. We are attempting to keep well informed about the development and production of printing equipment which will meet our requirements. I feel, after having seen some of the current high speed printers, that if we were faced with a critical requirement for a printer tomorrow, we could fill the need from existing equipment as long as our specifications for quality were not too rigid.

Another concept of which we are aware is one that was suggested by the original problems considered for the computer. In the Aviation Supply Office problem, some fifty widespread activities reported to the central inventory control point. We would like to investigate the possibility of transmitting source data from each of these points via standard Teletype communications equipment. The data would be received at the site of the computer in a form which could be fed directly into the machine. An experiment related to this problem has already been made with the cooperation of Western Union. Logistics Computer data was relayed to the St. Paul office of Western Union, where the computer manufacturer is located, and back to Washington on a standard line. The "message" consisted of 8,000 characters. During the entire round-trip transmission, which took about 25 minutes, only one error was detected.

Very little can be said at this time about the personnel and operating requirements of a Logistics Computer installation. Currently, 4 to 5 people with various degrees of mathematical ability are analyzing problems and preparing data for the machine. It is thought that a single maintenance man will suffice on a one-shift operating basis. It should be emphasized that this operation is a research application; problems are normally run on a pilot scale to minimize the routine work, and they are constantly being altered, or different problems are being considered. The personnel requirements for this type of operation are much greater than under a routine work load such as a supply activity is likely to involve.

SUMMARY OF GENERAL DISCUSSION ON

LT. ROSSHEIM'S PAPER

Captain Long inquired if the Logistics Computer were available for working out current problems that agencies of the Department of Defense might have. Dr. Rigby replied that the answer was yes, but with an important qualification. The intended primary use of the machine is as a research tool to find out how the resources of electronic computation can best be used in logistics work. Accordingly, it is expected to solve a wide variety of problems drawn from various agencies of the Defense Department (principally Navy, of course), on a pilot or demonstration basis. If the machine is to be used for routine service solution of problems whose status as research or pilot problems has passed, it should be on some kind of off-shift basis, preferably with the proprietors of the problems providing operators for the machine.

Captain Long asked if a problem would be entertained as a research program, for example, a problem in the Joint Chiefs of Staff that might be applicable to solution by machine methods. Such a problem might be recurring feasibility testing which might be readily adaptable to solution by such means. Dr. Rigby replied that such problems are welcome for trial on the Logistics Computer. In fact, it is expected that a good deal of time will be spent on the solution of such problems--even of problems that recur repeatedly--and also odd jobs, as long as they can be justified as research. It is believed, of course, that it would be improper for a research activity such as the Logistics Research Project to become an operating service agency for machine solution of logistics problems on a routine basis--this would certainly be a misuse of research talent. However, the Office of Naval Research is anxious to cooperate with all Defense Department activities having logistics problems potentially capable of machine solution in exploring the extent to which the computer's capacity can contribute to effective logistic planning and operation.

In response to questions by Commander Tynan, identification of items within the computer--the conversion of stock numbers to control numbers--and the trial of inventory control problems were discussed by Dr. Rigby and Captain Hunt. The conversion of stock numbers can be mechanized in some applications. The need for such conversion is not unique with the Logistics Computer; it is even a more pressing problem in the case of computers of smaller internal storage capacity.

With respect to the second question of Commander Tynan, Dr. Rigby pointed out that the inventory control problem, a problem of a type handled at a SDCP, was one of the problems used to determine the design characteristics of the Logistics Computer. "Dry runs" of actual inventory problems in data obtained from operating activities will be made to generate information as to the increase in inventory control efficiency which may be attainable through use of such machines as the Logistics Computer.

Admiral Eccles commented, in closing the discussion period, on the value of testing the Logistics Computer on typical problems actually occurring in Naval operations and planning, as compared to limiting its application to research use.

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LT. ROSSHEIM'S PAPER

Captain Long inquired if the Logistics Computer were available for working out current problems that agencies of the Department of Defense might have. Dr. Rigby replied that the answer was yes, but with an important qualification. The intended primary use of the machine is as a research tool to find out how the resources of electronic computation can best be used in logistics work. Accordingly, it is expected to solve a wide variety of problems drawn from various agencies of the Defense Department (principally Navy, of course), on a pilot or demonstration basis. If the machine is to be used for routine service solution of problems whose status as research or pilot problems has passed, it should be on some kind of off-shift basis, preferably with the proprietors of the problems providing operators for the machine.

Captain Long asked if a problem would be entertained as a research program, for example, a problem in the Joint Chiefs of Staff that might be applicable to solution by machine methods. Such a problem might be recurring feasibility testing which might be readily adaptable to solution by such means. Dr. Rigby replied that such problems are welcome for trial on the Logistics Computer. In fact, it is expected that a good deal of time will be spent on the solution of such problems--even of problems that recur repeatedly--and also odd jobs, as long as they can be justified as research. It is believed, of course, that it would be improper for a research activity such as the Logistics Research Project to become an operating service agency for machine solution of logistics problems on a routine basis--this would certainly be a misuse of research talent. However, the Office of Naval Research is anxious to cooperate with all Defense Department activities having logistics problems potentially capable of machine solution in exploring the extent to which the computer's capacity can contribute to effective logistic planning and operation.

In response to questions by Commander Tynan, identification of items within the computer--the conversion of stock numbers to control numbers--and the trial of inventory control problems were discussed by Dr. Rigby and Captain Hunt. The conversion of stock numbers can be mechanized in some applications. The need for such conversion is not unique with the Logistics Computer; it is even a more pressing problem in the case of computers of smaller internal storage capacity.

With respect to the second question of Commander Tynan, Dr. Rigby pointed out that the inventory control problem, a problem of a type handled at a SDCP, was one of the problems used to determine the design characteristics of the Logistics Computer. "Dry runs" of actual inventory problems in data obtained from operating activities will be made to generate information as to the increase in inventory control efficiency which may be attainable through use of such machines as the Logistics Computer.

Admiral Eccles commented, in closing the discussion period, on the value of testing the Logistics Computer on typical problems actually occurring in Naval operations and planning, as compared to limiting its application to research use.

APPLICATION OF THE UNIVAC TO AIR FORCE PROGRAMMING

by
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The work I shall describe has been conducted under Project SCOOP. This Air Force project was established in 1948 in the Planning Research Division, DCS/C, as an attempt to develop more suitable answers to the problem of programming Air Force requirements. Preparation of a detailed program from a specific strategic plan is at best a cumbersome endeavor. The expansion of defense efforts and the continuing increase in the technological complexity of military weapons has not made the programming process any less involved.

Requirements programming has traditionally been done on a piecemeal basis. This may, in part, be due to historical accident, but it undoubtedly reflects the usual circumstantial need for haste and for the technical knowledge of a host of subject-matter specialists. SCOOP has been inclined to treat the problem as an entity, thereby promoting consistency, but to do this without losing the essential services of the specialists.

This approach has necessitated extensive analysis, clarification, and mathematical formulation.[1] It has dictated a heavy initial burden of centralized digesting and editing of planning factor information. And in concentrating and mechanizing the inherent data-processing and computational aspects of programming, the approach has generated a heavy requirement for computational facilities.

In passing, I might say that Univac is the second electronic computer with which the project has been concerned. The first was SEAC, which was developed by the National Bureau of Standards under SCOOP support. SEAC was actually conceived of originally as a laboratory instrument for developing and testing logical designs and computer components. As the blueprints evolved, however, SEAC blossomed into a complete computer. In addition to functioning as a testing ground for future equipment, it has contributed much to the mathematical and computational research at Project SCOOP. Because of its design, however, it is not suited to full-scale day-to-day program computations.

Preliminary operations of this kind began with punch-card equipment. The introduction of IBM 604's as the main units in a chain of punched card machines, enabled the computation of programs to approach an operational basis. The chief contribution, however, lay in the opportunity provided for creating an organization capable of developing

1/ Cf. Dantzig, George B., "The Programming of Interdependent Activities," in Activity Analysis of Production and Allocation, Tjalling C. Koopmans, ed., New York: John Wiley & Sons, 1951, pp. 19-32.

and applying SCOOP methods while the Univac was being built. It had been clear from the start that problems would arise that demanded the adaptability and potential of general purpose electronic computer.

The Air Force Univac was accepted in February, 1952. Although the first Univac to be moved from the factory of the Eckert-Mauchly Division of Remington Rand, it was in partial operation at the Pentagon by April. On 25 June 1952, the installation was deemed complete, and the machine was formally turned over to the Air Force.[2] In the eight months since that date, during which the Univac has been continuously manned, over six thousand hours have elapsed. The categories used to account for these hours reveal the following percentage distribution:

Down time	15%
Modification, training and maintenance	26%
Testing of machine instructions	14%
Computation	45%

It would be difficult to present anything like a complete account of the diverse applications in program computation made possible by nearly three thousand hours on the Univac. I have selected one application, called Model 15, as a means of giving you an explicit picture. Although I shall not deal with the mathematics of the computation, some of you are undoubtedly familiar with the triangular model,[3] of which this is an important application.

Model 15 is designed to provide a programming document listing the amounts of selected material needed to support a war plan. Aviation fuel and lubricants, conventional ammunition and guided missiles, aircraft guns, assist take-off units, auxiliary fuel tanks, pylons, chaff, photographic film, boats, drop kits, and drop sondes are examples of classes of items for which detailed item requirements are computed.

Monthly schedules of requirements are obtained for each of the material items after an assumed D-Day. Allowance is made for the expenditure of material by combat aircraft and by aircraft engaged in other

2/ For an account of the efforts that precede this stage, see Johnson, Lyle R., "The Installation of a Large Electronic Computer," to appear in the Proceedings of the Toronto (September 1952) Meeting of the Association for Computing Machinery.

3/ Wood, Marshall K. and Geisler, Murray A., "Development of Dynamic Models for Program Planning," in Activity Analysis of Production and Allocation, ibid., pp. 189-215.

missions, such as training. Important kinds of losses, stock levels at installations and depots, and transit times enter into the determination of requirements.

In order to carry out a computation of this kind, a great deal of detailed information must be prepared for the computer. This information is placed upon reels of metal tape. Some appreciation of the important features of the computation can be obtained by considering the purpose of each of the six essential reels.

When data are prepared for the computer, it is efficient to arrange them in a manner which simplifies the handwork and writing of numbers, because operations performed by people have a greater error rate than those performed by machines. This arrangement is not intended to be most suitable for computation, and any rearrangement of the material to achieve a computable form is done by the machine. These processing and editing steps are followed by a sequence of computations which yield levels of material requirements by month. After the computed data are arranged in a form for printing of the answers, code numbers are replaced by descriptive headings, and printed listings are prepared for reproduction. The first tape reel, (called the TRIMOD tape), contains the machine instructions for all of these transformations.

A second reel contains the data from the war plan. In the Air Force these data naturally deal with the flying of aircraft, but the diverse kinds of flying to be considered in making accurate estimates of material is somewhat surprising. Schedules of deployed aircraft by type, model and series are shown for each month for the continental United States and for overseas. The number of aircraft for each month must also be listed for various kinds of schooling and practice; such as crew training, individual flying training, joint training with the Army, tactical support, administrative flying, testing, and special mission. Information on crews and on individuals completing training each month are also needed. Finally, the schedules of aircraft accepted by the Air Force from manufacturers for delivery, and their allocation to fighter-bomber interceptor units and other kinds of units are contained on this reel.

Planning factors and the structural information showing how the activities of the Air Force are related are placed on a third reel. Monthly hours of use for each kind of aircraft, gallons of fuel consumed per flying hour, material items expended per sortie, rounds of fire in life of weapons, materiel loss factors, and pipeline times for transit and for installation are typical of the detail required.

Three blank tapes are also mounted on the Univac for the inscription of the computed requirements. The Univac writes results on each of three tapes. As described above, these results are then rearranged, edited, and merged with descriptive alphabetic information before printing.

It is appropriate now to give some statistics on Model 15. The instruction tape contains a central computing routine which consists of 650 words of twelve characters each. On the punched card equipment used by Project SCOOP to test programming procedures, this computation routine took some sixteen plugboards containing 3818 wires. Preparing the Univac instructions took about as much in man hours as wiring the plugboards, but the instructions appear on about nine feet of metal tape which could be coiled in a sewing thimble. while the plugboards occupy several cabinets. The Univac computes six activities at a time, while only one was possible on punched cards. In using punched card equipment, the success of the computing process rested not only in the sixteen wired plugboards, but largely in the training, knowledge, and infallability of the operating personnel in selecting the plugboards and in properly routing cards from one machine to another. The automatic nature of comparable operations on the Univac is an attractive feature.

The program data used in the problem consisted of some 15,400 activity levels. The computed results contained 57,500 activity levels which were useful for analytical purposes and as supporting detail. Some 144,000 planning factors entered into the computation. Results of the computation are presented to the offices of primary responsibility which review and use them. A computation has been made to meet Munitions Board directives for the submission of war consumable requirements. It is now being computed under a different plan for budgetary evaluation.

Needless to say, a computation such as Model 15 is limited in value by the accuracy of the planning factors used. Improvement of these planning factors has been stimulated in the Air Force by a classified Wartime Planning Factors Manual initiated by the Planning Research Division. This publication, and another concerning peacetime factors, embraces all major areas of planning factors required in Air Force programming. The goal has been to bring these publications to as complete and comperhensive a stage as sound planning demands. The Planning Research Division does not originate the bulk of planning factors; they are prepared by staff offices and commands throughout the Air Force. It does, however, have responsibility for review and for publication of planning factor manuals.

In the description of the computational process, I indicated that the planning factors are on a reel of tape with the structure. We are presently completing a system that will relegate much of the work of preparing this tape to Univac. Planning factors will be kept on library tapes. From condensed structure information, the Univac will pick factor information off the library tapes and generate the structure tape. Many of these library tapes contain substantially the same information as the tables in the planning factor manuals. Periodic changes are made in the planning factors as change notices are received from the responsible offices. In the future, those changes will be made on the library tapes.

APPLICATION OF THE UNIVAC TO AIR FORCE PROGRAMMING

Combat consumables are but one major area of interest in the requirements field. In the past four years, about a dozen different types of models have been developed, which, taken together, make it possible to compute very comprehensive programs. Military personnel requirements by Air Force Specialty Code, technical training courses, aircraft engine procurement, overhaul and shipping requirements are illustrative of other areas. A number of these models were test computations to establish the adequacy of our methods and are not yet in routine use. From the viewpoint of comprehensive programs, further refinements are desirable in some of these models. In general, greatest difficulties are encountered at the beginning and at the end of the program computation process. At the start, we should logically begin with a schedule of anticipated aircraft acceptances. Allocations to training and to combat should then be accomplished through use of a computational system. On the other hand, this area has been handled in the past on a judgment basis, and complete delineation of the restrictions under which such judgments are made is an exceedingly difficult mathematical formulation task. Near the close of the computation, we encounter the problem of estimating requirements of hundreds of thousands of small items which the Air Force purchases from private contractors. We are just in the process, after several formative attempts, of constructing a computational procedure which we feel will approach the adequacy desired in this area.

Model 15 and the associated models represent Air Force activities and computation of these activities give information about the military aspects of the programming problem. If military appropriations are large, as in recent years, or as in wartime, the economic implications are of equal importance in appraising a plan. For the purpose of making such an appraisal, the Planning Research Division has aggregated the host of small items by industry according to the Standard Industrial Classification, which is widely used by governmental agencies in their statistical compilations. The value of these efforts depends to a considerable extent on parallel information from the other services, and the Division has therefore worked closely with the Munitions Board. Using modified Leontief methods for interindustry analysis, with some elaborations to take care of capital formation, procedures have been developed for assessing the economic impact of military programs. For analyses of this kind, we have since April, performed the inversion of four distinct matrices of order 200, as well as a large number of multiplication of matrices by vectors to show the industrial patterns under the purchases of various bills of goods.

Another aspect of the application of the Univac Air Force Programming involves the selection of activities for support of a program according to some criterion indicating an optimum selection. The Air Force has stimulated a great deal of mathematical research in this field, but the application of the techniques has barely begun. A set of instruction codes which will select and compute 250 non-negative unknowns from 500 has been prepared. These unknowns are subject to 250 linear restrictions and one linear optimizing condition. Explanatory work on the applications of this approach to Air Force problems is the next logical step.

I should like to suggest what now appears to be significant advantages to programming by the methods we are using:

1. It promotes consistency in the schedules.
2. Allows a self-checking electronic computer to perform a tiresome and time-consuming part of the work.
3. Permits a complete systematic presentation for review.
4. Provides for rapid recomputation when program assumptions lead to unacceptable results.
5. Makes possible the use of more manpower on planning factors and on areas which require judgment.

In addition to these primary advantages, straight thinking about programming is encouraged, inadequate data are highlighted, and an orderly sequence of program computational steps must be followed.

There are, of course, disadvantages. The method dictates a heavy load of centralized factor analysis, and the large quantities of data per person requires fairly inflexible rules of procedure. Systematic methods do not satisfy all or every special programming question. Finally, the methods contribute little to the problem of data preparation, which was present before.

The Univac itself has made these contributions to the program:

1. It has permitted the computation of larger models than could otherwise have been contemplated.
2. Total computational time has been substantially reduced.
3. Its automatic, self-checking mode of computation, practically eliminates the supervisory function that is characteristic of card computation.

The general purpose computer is well suited for processing masses of data, such as we meet in the programming and logistics work. The unparalleled adaptability of the machines permit the user to adjust his operations to fit nearly any change in procedure. This same facility poses a considerable test of the user's self-restraint; frequent changes in procedure are tempting, but they run counter to the stability required to achieve the production-type operation called for in handling a multitude of numbers. To obtain the greatest benefit from our machine instructions, we must use them more than once.

When tapes for a computation have been prepared, they are mounted on the Univac, and the starting of the machine should be the end of human intervention until the computer types out "end of problem". The fact that the computer occasionally halts with a cry of distress has provided the framework for much hilarity. The inevitable failures of the computer should not blind us, however, to the realization that today electronic computers are no longer research instruments, but production tools highly adaptable to the solution of management problems when we have the maturity to understand the advantages of the systematic approach and the forethought their use demands.

APPLICATION OF THE CRC-107 COMPUTER TO
PROBLEMS OF THE BUREAU OF AERONAUTICS
by

D. O. Larson
Bureau of Aeronautics

Today, both government and industry are faced with the problem of finding faster, less expensive, and more efficient means of processing information. The recent advances of input-output equipment has now made digital computers feasible for processing information in large volume. Many people are now beginning to study seriously the application of digital computers to the business type problem. This kind of problem, called "data processing" involves masses of input-output data and relatively simple computations, and frequently requires complex controls for sequencing operations.

In data processing we are confronted with two major questions:

(1) Is a particular problem adaptable to high speed machine processing?

(2) How can the problem be programmed to achieve the best results?

At the Bureau of Aeronautics, we are especially concerned with this business type of problem and the CRC-107 computer is scheduled to deal mainly with this problem. For this reason, I have chosen to describe the application of the CRC-107 computer to our version of a typical problem.

As I discuss the essential ideas in the application of the CRC-107 computer to a data processing problem, you will readily recognize how these ideas can be adapted to other problems of the business variety, principally in the fields of procurement, production control, inventory control and distribution, sales, and others.

Now, what are the principal characteristics of the CRC-107 computer? It is moderately low in cost, relatively small in size, simply designed to reduce maintenance and programming costs, and flexible in input-output equipment using magnetic tape, IBM cards, or flexowriter paper tape. It has a capacious memory, internal as well as external. The computer also has the advantage of special auxiliary equipment such as the tape-typewriter unit and the high speed printer.

The tape-typewriter unit is a novel piece of equipment, and is especially important on "data processing" problems because it permits fast preparation of the vital magnetic tapes in large quantities. This essential unit is completely independent of the main computer. Controls are provided on the tape-typewriter unit for searching out any section on a magnetic tape in either direction, so as to permit reading, filing, modifying, verifying or erasing the particular section of tape. The tape-typewriter unit is therefore a valuable time saver in making changes in data processing records and in keeping them up to date. These units are portable, and can be easily placed where needed. The tapes can be sent over to the computer when ready.

The high speed printer, another important auxiliary, makes possible a printed copy of computer results at a speed equal to the tape writing speed of the computer.

The main problem which I shall discuss now is one which concerns production control record keeping on aeronautical equipment ordered by the Production Division of the Bureau of Aeronautics. Tens of thousands of records must be handled to keep accurate accounts for this project. Due to limited manpower, it is impossible to accomplish this under manual methods without extensive overtime work. It takes 70,000 basic cards to maintain these records, and approximately 22 people plus the necessary overtime are required to process them.

The production problem which uses these cards is basically a scheduling problem, in which the necessary quantities and type of government-furnished equipment is ordered at the optimum month so that this equipment is made and distributed to meet production schedules of aircraft effectively. The scheduling involves the computation of monthly quantities of items required. It involves adjusting schedules to provide installation lead time to meet airframe manufacturers installation requirements. It involves adjustments to monthly requirements caused by changes in types and quantities of aircraft and equipment. Thus, we have the problem both of computing the data, and keeping the data up to date at all times.

To give you some idea of the magnitude of this problem, let us look at an illustration: One piece of equipment, say an airplane engine may have as many as 325 possible destinations; or as we call it, "end users". An end user may be one of the services, or one of the countries of the Mutual Security Pact, or it may be an aircraft, a spare, or requirement for the fleet, etc. This equipment--say the engine--must be furnished in quantities of 1 to 25 for each end user (such as an aircraft) and must be furnished with precise installation lead time. The installation lead time differs for each end user. These deliveries of engines must be furnished anywhere from 18 months prior to delivery, to 3 months after delivery of the specific aircraft. Conversely an aircraft will be furnished from 25 to 230 items of aeronautical equipment, each depending on the type of aircraft. Installation lead time on the aircraft will vary from 16 months to 2 months prior to shop completion of the aircraft and quantities of equipment would again vary from 1 to 25 per end user. Keeping all this source data up to date is a huge task. Changes occur daily and average about 50 changes a week in government furnished equipment alone. Other changes that would occur in the data might be a revision in aircraft schedules, quantities of aircraft, etc. Each change entails an insertion of one or more new cards, deletion of current cards, readjustment of total quantities, monthly quantities, etc. One example of a change on current operation occurred just a short time ago. Monthly schedules of aircraft were slowed down to spread out the program. This made a review of each card necessary and recomputation of each monthly requirement, a matter involving 50,000 cards.

To solve this scheduling problem, the programming for the CRC-107 computer has been done as follows: Source data, just described, is arranged on two master tapes. One tape is called the airframe tape and consists of 10,000 computer words of information. Each airframe is identified by a particular code stored in the block address word of a tape and followed by 50 words of information which pertains to this airframe. The block address word which identifies the aircraft is similar to any other word on the tape except it is a key word which can be numbered in sequence, or any other pattern, so that it identifies the 10 following words on the tape. Thus, each 11th word on a tape is a block address word. At any time, then, by programmed commands, any specific airframe can be found through the key word and the information pertaining to the airframe made available for computation.

The 50 words of information on an aircraft include manufacturer and plant location, fiscal year, end user, source type, contract number, priority, unit cost, total cost, total quantity of airframe for fiscal year, total to date, balance due, month in which schedule first starts and ends, along with a 3 year monthly schedule of airframes.

The other master tape is called the equipment tape and consists of approximately 60,000 computer words of information. The particular equipment is the code on this tape corresponding to the airframe as a code on the companion tape. Each piece of equipment is stored in the block address word of the tape, and is followed by as many two-word coded units of airframes that use this particular equipment. A two-word unit consists, therefore, of an airframe that uses the item of equipment, fiscal year, end user and source type (appearing again on this tape) quantity of equipment per airframe, installation lead time, and application. You start the ball rolling by selecting the first piece of equipment showing up on the equipment tape. Then the first airframe using this equipment is selected as a key to search the airframe tape. When a match is made between the corresponding airframes on the two tapes, a further search is made down the airframe tape, to match the corresponding end user and fiscal year. This search is necessary, since the same airframe is reproduced several times on the airframe tape for varying fiscal years and end users. Once the match desired has been achieved on all points, the computation consists essentially of determining monthly quantities of equipment required to meet specific aircraft manufacturers need. This computation is done by multiplying the quantity of equipment for the specific aircraft by the number of aircraft produced per month. The monthly requirements are then scheduled by the specific lead time established and located in the correct month of a 60 month spread covering a 3 year current production program. Intermediate results are transferred into 60 other summation cells required for accumulation of specific equipment requirements for the aircraft program. The last part of the program provides for editing the information computed, and producing a hard copy print. In this particular example, the editing is done as part of the main program, but in many problems it will be necessary to have special tape editing programs. In the case of special programs, another run would be made through the main computer using the output tape as the new input tape and the program would rearrange the computations to print out any desired type of report on the high speed printer.

In many of the cases of large data processing problems, much of the programming time may be spent in arranging information for a final output on a high speed printer, especially in the case where all kinds of reports are needed from some general set of computations.

To program the problem we just discussed for the CRC-107 computer, it required approximately 350 man hours. To prepare the data on the magnetic tapes, about 175 man hours more will be required, it is estimated. Of course, the programming time has now been completed once and for all; henceforth, only relatively minor additions or modifications will be necessary. Also, the data preparation time includes the preparation of all basic source data, and after the program is in operation, only changes in records will be required as each new run is made on the computer. Our estimates show that about 22 hours of computation time will be needed to run the program each time it is required, which should be about once a month. A small staff of people will therefore be able to run this specific problem at any time with 22 hours of computer time. These figures show quite a contrast as against the hand methods which require 22 persons working full time and overtime, all the time, processing 70,000 cards.

Another problem which has been prepared for the CRC-107 computer and will be run periodically is the mobilization scheduling problem. This is a much smaller project, in terms of machine time, than the one just described but I present it simply to illustrate the versatility of the computer. The mobilization scheduling problem is a planning one, concerned with scheduling aircraft and equipment items to meet a military objective of the future. The general plan of the problem is similar to the production problem but far less variations in parameters are employed. Briefly, the problem is this: A mobilization schedule projected for a 5 year span in the future is provided to the Bureau of Aeronautics by higher authority to meet a certain objective. This schedule has been examined and been set from estimates of what aircraft manufacturers can produce or can be expected to produce by expansion in a 5 year period. The quantity of a particular equipment to go on an individual aircraft is known, so the problem consists essentially of multiplying the number of aircraft scheduled times the quantity of equipment that go on the aircraft. This results in a total mobilization schedule for selected equipment to meet the mobilization plan. The computation here again is simple but it is time consuming because of the large numbers of different types of aircraft, and the many different types of equipment going on different aircraft. All equipment items must be kept up to date as to type, number, etc., and each change here reflects changes in equipment items or aircraft schedules.

Other problems that are being prepared for the CRC-107 computer include the following:

(1) Control of distribution of Navy Publications going to naval aeronautical establishments.

(2) Control of distribution of Navy Publications going to new naval aeronautical establishments.

(3) Stock accounting of equipment being overhauled at naval overhaul and repair stations.

(4) Work measurement problems which aim at the control of personnel and expenditures through the establishment of standards.

(5) Computational Problems. There are several of these, such as the addition to Tables of the Incomplete Beta Function; computations on stability parameters of selected aircraft; analysis of variance, etc.

So much for what we can do with the computer. Now for some of the "bugs". One of the first difficulties that we have experienced in programming a large data processing problem for the computer originated during the presentation of the problem. There was a lack of understanding between the person familiar with the problem and the person programming it. This of course resulted in a much longer study of the problem than would be ordinarily required. This difficulty most likely will be a common one for all concerned with programming large data processing problems in the future. One reason for this difficulty is that the individuals presenting a problem have been so misinformed as to what computers will do, that they do not present their problem in sequence, in simple terms, and in complete detail. As a result the programmer must go to several persons to gather the entire information, and so consumes more time rephrasing the problem. Of course, in many cases it is nearly impossible for any one person to present the problem in its entirety, because the problem is so far-reaching in scope and so many people are involved in the problem. This is especially true where several different divisions are all working on some phase of a common problem. A big job lies ahead in training personnel who will know the broad phases of a business and have a little better knowledge of what computers can do. These individuals are necessary, if mechanization is to be done efficiently, so that an entire office is converted to digital processing. This might be the case where purchasing, production, distribution, sales, inventory, payroll and other functions are included in one project.

Another problem which many people are concerned with at this time is the aptness of a particular problem for a digital computation. Certainly a lot of education will have to be given in this regard, until it is learned that these new computers cannot solve everything, and some problems would be a waste of time to solve. It is impossible to generalize and say what types of problems can or cannot be put on a digital computer since every problem has so many variations. Only a careful study by experienced programmers can determine a particular case. Too, the situation varies with each computer so that a problem that is well adapted to one computer may not be adaptable to another. Therefore, as each large scale project is considered for mechanization, where no particular computer has already been selected, only a careful study by programmers who are familiar with many types of computing machines, can determine the best way to handle it. The programmers should therefore be of the highest caliber.

Well, how to train the programmers? At first glance this seems like a minor problem, since the general impression is that after once programming a mathematical problem, a person should be able to program easily enough a data processing problem. This is very often not true on a problem of any size; in many cases several months of intensive training is required for a programmer to learn the overall problem. This brings up the question, then, whether it is more efficient to teach a person who is well acquainted already with a large problem to program it on a digital computer, or whether it is better to train the programmer on the problem. This is a very difficult question to answer, too, since often a person who knows the problem well, does not make a good programmer. A good programmer on the other hand, usually can learn the data processing problem given sufficient time, but this loss in time can not be tolerated. In the bureau, we are approaching this problem from two directions and will evaluate the results to determine future courses of action. In the case of the production problem first mentioned, a key person in the production division has been trained in programming who calls on regular programmers associated with the computer only for technical assistance.

In conclusion--we at the bureau are looking on the CRC-107 computer as an important new tool which can eliminate some of the drudgery of everyday routine and release personnel from tedious operations to other work. We are expecting to accomplish a larger quantity of work than was ever before realized by hand processing methods.

We know that one of our biggest jobs ahead will be in the transition period when many of our office records will be transferred to machine records.

A major problem confronting any agency will be training personnel who can both present a problem effectively and program it intelligently.

I think the statement is true that I have heard from several sources, that today we seem more advanced in solving problems from the technical point of view than we are from the viewpoint of preparing problems for computers.

More confidence will have to be gained by people presenting problems, since they have heard so many misstatements about computers, that they are confused as to what they can do. Everyone in the computer field will have to make an honest effort to give clear, accurate statements so that no more misleading interpretations can be taken.

For ways of improving computers for business type problems, let us look to the future for a closer link between the scientist and the business man. Let us dream a little, too, and visualize the age of the complete push button automatic data handling.

SUMMARY OF GENERAL DISCUSSION ON ELECTRONIC COMPUTERS

In response to questions by Admiral Eccles concerning the size and cost of the Bureau of Aeronautics computer, Mr. Larson, while he could not quote the cost, stated that the main electronic part of the machine and the magnetic drum occupied cabinets about six feet high, four feet wide and seven feet long. The computer system also would include tape editing units, a high-speed printer and two pieces of IBM equipment--a card input and a card output unit.

The need for a control or reference number in the Logistics Computer was questioned indirectly. The question as stated from the floor was the following: "Can seven-digit Federal identification numbers be adapted to use for identification purposes in the Logistics Computer without conversion to a control number?" Dr. C. B. Tompkins pointed out that direct use of the seven-decimal-digit numbers for identification within the computer would require expansion of the magnetic drum storage to 100 times its present size, an appalling idea. Dr. Tompkins stated that it is absolutely essential to face the issue--that to demand a piece of equipment be designed to accept stock numbers or Federal item identification numbers is simply out of reason because of the cost. The practical alternative, as pointed out by Mr. Wolf, in handling a problem requiring greater storage capacity than that of the Logistics Computer, consists of repeated runs of portions of the problem. Captain Hunt mentioned that engineers have stated that reliable magnetic drums, up to thirty-four inches in diameter and capable of storing 100,000 items, can be built. The use of such high-capacity storage devices, would, of course, tend to limit the number of computer-runs required for a large problem.

The development and use of large-scale, electronic, digital computers in foreign countries was discussed by Mr. Emerson, Cdr. Keene and Dr. Blachman in response to a question by Mr. Torrance. Such computers were said to be in use, or in various stages of development, in England, Germany, Sweden, Switzerland, Norway, Holland, Australia, Italy and Japan. "The Digital Computer News Letter", published quarterly by the Office of Naval Research, and the ONR report, "Large Digital Computer-Projects", were mentioned as a source of information on this subject.

Professor Morgenstern mentioned the existence of a Russian book on computing machines, which was published about two years ago. The text of the book describes the technical features of computing machinery just as any ordinary technical book in English does. It is interesting to note that the text contains not one word of identification of the origin of large-scale electronic devices, but that it is well illustrated with photographs of American computers.

In response to questioning by Captain Hunt, Mr. Schell discussed the programming effort required for major linear programming problems. The Air Comptroller UNIVAC, applied to problems of the SCOOP project, is manned by about 12 operators, 12 programmers and 20 people engaged in the preparation of data, and the operation of the auxiliary tape preparation and printing equipment.

Mr. Geisler pointed out differences between the Air Force computations and the personnel problem described in Mr. Wolf's talk. For example, flow of personnel into a theatre, increments of inventory, and rotation of personnel are a part of SCOOP computations of this type. These additional considerations add levels of complexity to the computation.

The built-in logical operations of the Logistics Computer, the comparison operations, were discussed briefly by Mr. Wolf in response to a question by Commander Stanley. Commander Stanley's interest stemmed from the fact that in matching bidders to determine the low bid, the computations, which are not trivial by any means if they are to serve the needs of his agency (ASTAPA), require repeated comparison of results of calculations and the tossing out of the low or the high one.

SUMMARY REMARKS
Tuesday, 17 March 1953
by
Rear Admiral Eccles, USN, (Ret.), The Chairman

I would like to make one or two comments on the computer situation. I would like to ask you to bear in mind the difference between the general purpose and the special purpose machine.

We need both, but there is no use in trying to drive a thumb-tack with a sledge hammer, nor a railroad spike with a tack hammer.

I believe this question of reliability and maintenance will be a matter of the greatest importance. I ask you to consider this: How do the requirements of defense against an atomic attack affect our thinking in regard to the greater mechanization of logistics planning? Is that greater mechanization an asset or a deterrent to the operation of a national logistics apparatus in time of an atomic war? I do not know the answers.

I think it is equally important for us to get better methods of logistics computation, and also to simplify our methods of logistics control. Efficiency will come from both ways. But it is as equally important for us to simplify our requirements as it is for us to find a simple means for satisfying requirements that are not simple.

Finally, I would like to have you leave with the thought--and this applies to other things than computation--no computer will ever ask the proper questions. Professional judgment determines whether or not the proper question is asked. If you ask questions within the capabilities of the computer, the computer will answer them quickly and accurately.

In the future we can be sure that one of the criteria we may use in estimating the ability of a logistics planner will be, "Does he know the capabilities and limitations of modern computers? Has he the knowledge and good judgment to ask the computer the proper question?"

And in closing today, I would like to express my great thanks to the gentlemen who so ably presented the situation this afternoon, and this morning, and to end on a note that Mr. Larson left, which is one I think we must never forget--computers, mechanization, or anything else--that regardless of how this develops, as usual we will come back to the eternal question of the training and education of personnel, which, in spite of everything, will always be our most difficult and most important problem.

Thank you very much.

INTRODUCTION TO THE "THEORETICAL" SESSIONS
by
Prof. Oskar Morgenstern

The general chairman, Dr. Mina Rees, said that on Wednesday a person named Oskar Morgenstern would take the chair for the so-called "Theoretical Session." I am this person and I want to say that I am very honored in having this privilege of introducing so many excellent speakers.

The overall program is broken into two parts: practical and theoretical. This sort of division easily irks somebody who works in theory, because it appears as if it were unpractical; but if you look carefully, you will find that the arrangers of the program have in their wisdom put these two words in quotation marks, thereby indicating that the common assumption that theory is not practical, might not be true. I hope that the speeches and the talks and other contributions and discussion will prove this.

Naturally, we will progress gradually more and more to abstract things in the sessions of today and tomorrow, but you may well be aware that, for example, if one can define an optimum operation of a system, one will really know something because one could discover whether a given system actually is near it or not and how one can approach the optimum.

Insofar as the theory of games is concerned, it appears of course, in many ways even more remote from such questions; yet on the other hand, some hardware has already been constructed with the aid of the theory. For example, certain types of planes have been selected over other types of planes precisely on the basis of this theory. Similarly, at present, for example, certain works are in progress which, if successful, will lead to the construction of a particular type of guided missile because it would be possible to build certain concepts of strategy into the hardware itself. So it is quite clear that there are very direct practical connections.

The talks of this morning begin with the talk on "Optimal Technology for Supply Management." The speaker is Rear Admiral Frederick L. Hetter of the U. S. Navy; he has a wide experience in the Atlantic and Pacific, and since January 1951 he has been Assistant Chief of the Bureau of Supplies and Accounts. It gives me great pleasure to introduce Rear Admiral Hetter.

THE LIMITS OF CENTRALIZATION

by
Dr. T. M. Whitin
Princeton University

Although the problem of centralization has received, and is receiving, much attention, an adequate definition of centralization has not yet been devised. People frequently have preconceptions concerning whether centralization is good or bad, although there is often a complete lack of reasons for these opinions.

Given any of the typical definitions, it is a simple matter to construct paradoxical examples that immediately demonstrate their inadequacy. For example, in the current controversy concerning unification of supply, much of the discussion implies that unification is equivalent to centralization. This is by no means necessarily the case. It is quite possible to have a unified but extremely decentralized supply system, and conversely, to have a separate supply system for each service with a high degree of centralized control of each service.

The problem of military supply is a general equilibrium problem. In wartime, more goods can be produced for the military only at the expense of giving up production for civilian sectors. Within the armed services, more tanks can be acquired only at the expense of ships, planes, etc. Unless some authority is placed over the services, grave difficulties are likely to arise concerning priorities, and it is not likely that an optimal program will be approximated. Nor is a system of priorities determined by the Munitions Board, whose members owe allegiance to separate services, satisfactory. A recent book by the War Records Section of the Bureau of the Budget contained the following passages:[1]

> "During the summer and fall of 1942 scores, if not hundreds of production lines were closed down for brief periods when the flow of materials ceased, and the conflict of high military priorities threatened to strangle the entire war production program, military and civilian."

A conversation with Archibald Alexander, former Under Secretary of the Army, stressed some of the difficulties that can arise because of different profit rates allowed manufacturers by the three services. Since manufacturers give preference to the service allowing the higher profit per cent, the working of the priority system is greatly impaired. Clearly, there is a great need for a

[1] The United States at War, p. 280

new priority system that gives more adequate consideration to the overall viewpoint.

Nevertheless, the conflict between the need for realism and detail on the one hand and for coordination on the other will persist. Ely Devons, wartime director of the Ministry of Air Production in England wrote that:[1]

> "Every attempt at planning reveals these two problems: first, the need to split up the field to be covered so that each administrative unit can deal efficiently with its own sector, and second, the need to secure that the actions of these separate units all fit into the general plan. But the implementation of these principles always leads to a conflict. For the first requires delegation and devolution, so that plans can be manageable and realistic; and the second requires centralization, so that plans can be coordinated."

Returning to the problem of defining centralization and decentralization, typical definitions state that under a centralized management the important decisions are concentrated at one central point. This type of definition clearly ignores a point brought out by Herbert Simon, namely that there are two very different aspects to centralization; --- "decision-making powers may be centralized by using general rules to limit the discretion of the subordinate. On the other hand, decision-making powers may be centralized by taking out of the hands of the subordinates the actual decision-making function."[2] In a sense the latter type of centralization is a special case of the former where the rules lead to a unique decision. An organization might be changed from completely "centralized" to completely "decentralized" by nominal changes in the decision-making process.

This illustration indicates some of the difficulties inherent in defining centralization. A somewhat different type of problem that may arise may be illustrated by the recent history of a large mail order company. The company was considering a decentralization of certain functions that had previously been highly centralized. Specifically, the company was considering discontinuing the keeping of sales records for style goods in a central office, the central office review of purchase orders, and the central office revision of demand estimates. At the same time the company was engaged in bringing together under one supervisor

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[1]Ely Devons, Planning in Practice, p. 14

[2]Herbert Simon, Administrative Behavior, New York, 1947, p. 234.

certain retail and mail order operations that had been handled by different supervisors. Purchasing and disposal of surplus are among the functions thus centralized. Clearly this example is centralization in one sense and decentralization in another. Is the net result more centralization, or less? The answer is not known. There is no single measurement that is satisfactory for measuring the degree of centralization. As soon as more than one characteristic must be measured, the problem of assessing the relative importance of the various characteristics arises.

If we assume for the moment that we can measure the degree of centralization, then the problem of ascertaining the optimum amount arises. Here again, many factors are relevant such as the size and nature of the organization, the capabilities of its members, and the costs of information. We could take the usual approach of writers in the field of Public Administration, and list various advantages and disadvantages of centralization. However, unless there are methods of evaluating the factors involved, this approach is not very fruitful. The problem of assigning values to the relevant variables is the most urgent and the most difficult task in the field.

As just indicated, much literature in the Public Administration field consists of listing advantages and disadvantages. Other literature in the field consists of a study of special cases. Herbert Simon wrote of the "steady shift of emphasis from the 'principles of administration' themselves to a study of the conditions under which competing principles are respectively applicable."[1] This is accomplished, of course, by choosing situations where the weights attached to some particular aspect or aspects is sufficiently high to dominate the other aspects. One leading authority in the field has written the following on integration:[2]

> "Public Administration has only a loose content of incomplete observations, speculative hypotheses, and unwarranted conclusions to offer the student and administrator who wishes to inform himself about the best settled experience."

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[1]Op.cit., p. 240.
[2]Earl Latham, The Federal Field Service, Public Administration Service, Chicago, 1947, p. 6.

Other quotations indicate that the problem of organization requires analysis, balancing, and setting values, but do not go on to carry out the analysis. In this paper, several aspects of the centralization problem that lend themselves to formalization will be presented. The models are partial equilibrium models and are at best only approximations to a complex reality involving all the various effects and their interactions. It is possible, however, in the state of present knowledge, that a systematic evaluation of these partial effects may be useful, for it may frequently be the case that indirect repercussions are small. The analysis is primarily intended as an example of the type of analysis that may be of use in many concrete situations involving a choice between centralization and decentralization, or rather, between a higher and a lower degree of centralization. The models discussed, as most economic models, involve a balancing of the advantages and disadvantages of centralization from various specific viewpoints.

Before discussing the aspects of the problem that can be formalized, I shall make a few general remarks on possible limits of centralization. It is generally agreed that increases in the amount of centralized control bring about substantial increases in the difficulty of the task of coordination. For example, Herbert Simon mentioned that "referral upward introduces new money and time costs into the decision-making process."[1] Also, "noise" is introduced that interferes with the transmission of information, thus necessitating the making of decisions on the basis of worse information than that available at the lower level. The resulting high-level decision may therefore not be better than the lower-level decision. Even in the event that better quality of decision prevails at the high level, the benefits must be weighed against the concomitant costs.

Urwick has written that there is no question that the functional method of distributing responsibilities does "very much increase the difficulties if coordination, and that, generally speaking, insufficient thought and attention have been given to the problems which it raises in this direction."[2]

Similarly, Russell Robb wrote, "The returns from increasing organization do not continue proportional to the effort and limits are reached - - - the loss on account of difficulties of coordination sometimes more than counterbalances

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[1] Op.cit., p. 236

[2] L. Urwick, The Elements of Administration, New York 1944, p. 52.

any advantage from economy that can come from functional division."[1] Difficulties of coordination either explicitly or implicitly play an important role in the "decreasing returns to management" that has received much emphasis in economic theory. Some of these difficulties were spelled out in detail in Sir Oliver Franks' Central Planning and Control in War and Peace,[2] as the following passages indicate:

> "All really large-scale organization has tendencies to inefficiency --- Successful determination of policy requires high intellectual ability to hold the many diverse factors clearly before the mind and grasp firmly their importance and their relationships. - - - Abilities sufficient to cope successfully with the planning of a small-scale organization are insufficient to deal with the same thing on a larger scale - - - Inertia has been described as the characteristic vice of the bureaucrat - - - Really large-scale organization compels men to take many decisions at a point remote from the places where their decisions will take effect. They work on paper. The in-tray and out-tray symbolize the situation. It is not easy to see beyond the paper and hard to preserve a lively awareness of the real issues - - - Large-scale organization tends to produce a steady flow of orders, instructions and memoranda from the centre. This has two results, one direct and the other indirect. The first is that initiative at the periphery is restricted progressively as the authority of the centre is asserted. - - - In the second place a habit of mind is engendered in the staff of the organization. They come increasingly, when confronted by the need to act, to look for rules and precedents in the light of which to determine action rather than to seize on the situation for what it is or attempt to devise techniques and methods of action which will yield a fully satisfactory outcome from it."

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[1] Russell Robb, "The Limits of Organization," reprinted in Papers on Organization and Management, edited by Catheryn Seckler-Hudson, American University Press, 1946, pp. 156, 171.

[2] Harvard University Press, 1947, pp. 27-30.

In a sense, a production line constitutes a highly centralized entity. There is need for extremely close coordination between all the components of the final product. Disturbances in one part of the line may necessitate stopping production. The operating costs of coordination in this extremely centralized case, however, are not high, once the production line has been set up. This type of extreme centralization can take place only in a special technological set-up. In fact, the degree of centralization should not be considered independent of technology. Consideration of the decision-making process during battles may illustrate the importance of technological considerations. In early days, the supreme commander accompanied his men to the battle field and made key decisions on the spot. In recent wars commanders have found it necessary to delegate important decision-making powers to the field. The latest developments in communications, television, etc., may make possible a recentralization of decision-making, for rapid access to good information may enable coordination to be brought about efficiently from the center.

Difficulties of coordination constitute one of the principal limiting factors to centralization of many types of organization. Ely Devons stressed this factor, stating that the effective limiting factor was "the limitation in the number of variables that could be comprehended by one brain - - - the decision had to be taken on the basis of rough-and-ready orders of magnitude that the co-ordinators carried in their minds."[1] It may be the case that, as a production line facilitated better coordination in the above case of centralization, modern techniques in large-scale computation may effect better coordination in overall planning problems. If the limiting factor is the number of interrelationships that can be handled in one brain, it is evident that an electronic brain can handle more of these than can a human brain. Modern programming techniques may thus be a tremendous aid in military planning. Programming estimates of feasible production plans may be far from accurate - - - and yet, if they are better than the estimates they replace, they constitute an improvement. The evaluation of these techniques depends on much more empirical testing than has been completed at the present time.

Some factors limiting the degree of centralization have been discussed. Current literature in economics often stresses these limits in discussions concerning decreasing returns to scale and their causes. At the same time, however,

[1] "The Problem of Co-ordination in Aircraft Production," in D. N. Chester, Lessons of the British War Economy, Ch. VII.

some factors operate to effect increasing returns to scale. The lack of emphasis on these factors constitutes an important deficiency in economic theory. The facts that large-scale industry makes possible a high degree of specialization and sometimes enables better adjustment of indivisible factors have received some attention, but certain other factors that bring about increasing returns have been discussed hardly at all. For example the "repairman problem" cited by W. Feller[1] is certainly relevant for economics. This problem is concerned with automatic machines which ordinarily require no human care. However, they are subject to breakdown and call for service. When the number of breakdowns and the repair time involved are assumed to be random variables, the optimum number of repairmen can be computed. The results of this analysis have been successfully applied in Swedish industry. The number of repairmen increases much less than proportionately with the number of machines if the degree of protection against stoppage is maintained constant.

Another example is that of group feeding, where the amount of food required per person to provide protection against random variations in demand increases much less than proportionately with the number fed in a mess hall[2] Still other examples occur in the field of inventory control, both with respect to purchase quantities and safety allowances. Economies of centralization will appear in the case of safety allowances wherever demand is subject to random variations, in the case of purchase quantities, economies of centralization may appear even in the cases of known demand. These cases will be discussed below, as they play an important role in the partial equilibrium analysis models constructed. In the field of inventory control empirical observations are in agreement with our theory. For example, W. F. Luttrell wrote of slower turnover rates caused by the existence of branches.[3] Other empirical evidence is not hard to find.[4] A final economy

[1]C. Palm "The Distribution of Repairmen in Servicing Automatic Machines," Indistritidningen Norden, Vol. 75, 1947, pp. 75-80, 90-94, cited in W. Feller, An Introduction to Probability Theory, New York 1951, p. 379 n.

[2]Edward U. Condon, "Food and the Theory of Probability," U. S. Naval Institute Proceedings, Vol. 60, Whole No. 371, January, 1934.

[3]W. F. Luttrell, The Cost of Industrial Movement, Cambridge 1952, p. 19.

[4]T. M. Whitin, "Inventory Control in Theory and Practice," Quarterly Journal of Economics, November 1952, pp. 502-521.

of scale is that lower cash reserves are required for liquidity purposes, relative to the volume of sales. In general, whenever there are random variations in economic variables it is quite likely that economies of scale exist. Since random variations permeate a great many aspects of economics, these economies are of considerable importance.

Now returning to the analysis mentioned above, four models will be discussed, named for convenience (1) the decision-making model, (2) the safety-allowance model, (3) the location model, and (4) the purchase quantity model.

(1) In the first model, assume the existence of N independent firms assume secondly that the nature of demand interdependence between firms is known, as are the costs of transmitting information concerning demand to an information center, the costs of operating on the information, and the costs of feeding back improved estimates of future demand to the firms. In the case of random demand variations, the demand feedback is subject to decreasing returns. Since the costs remain constant, it is profitable for only n of the N firms to transmit information to the information center which would feed back the results to all N units. The greater the demand variance, the larger is the number of firms that should submit sales results to the center. In the event that demand is certainly known, and there are no systematic interrelationships, then the need for an information center disappears and each firm or supply center makes its decisions independently.

Realistic examples that possess some of the essential characteristics of the model are not hard to find. For example, in the mail order business, it is often contended that there is more reason for centralization of control of style goods than for staple goods because of the volatility of demand. The centralized controller feeds back sales estimates to the individual mail order stores based on the sales history of all the stores.

(2) In the safety allowance model, assume the existence of various retail outlets. Assume that supplies can be distributed by several alternative methods among which are:

(a) directly from manufacturer to the outlets, with no inter-outlet transfer.

(b) from manufacturer to a central storage warehouse and from there to the retail outlets.

The following assumptions are made:

(1) The demand distribution for each outlet is known.

(2) The following delivery times are known:

(a) t_{mr}, the time from manufacturer to retail outlets

(b) t_{mw}, the time from manufacturer to warehouse.

(c) t_{wr}, the time from warehouse to retail outlets.

These times are the time elapsing between sending the order and receiving the merchandise.

(3) The costs of depletion are known.

(4) Carrying charges are known.

For different values of the above parameters one can construct some situations where goods should be shipped directly from manufacturer to retailer and other situations where goods should be shipped to retailers via a central warehouse. For example, the larger the demand variance the more likely, other things being equal, it is that the central storage function of the warehouse is desirable.[1] This results from the fact that safety allowances increase less than proportionately with demand --- hence central storage reduces carrying charges. Similarly other indications that central warehousing will lead to reduction in costs are a large difference between t_{mr} and t_{wr} and a large number of retail outlets, for each of these considerations makes reduction in safety allowances possible. The level of carrying charges and of the costs of depletion are also of extreme importance in determining the proper method of distribution. Increases in the cost of depletion bring about a lowering of the optimal probability of depletion leading to increased safety allowances and thus to increased probability that central storage is desirable

Changes in the level of carrying charges present a somewhat more complicated problem. An increase in carrying charges should have two effects, namely (1) to reduce the level of protection against depletion and (2) to increase the amount of saving per unit of reduced safety allowances. Effect (1) dominates (2) for high probabilities of depletion, while for low probabilities of depletion (2) dominates

[1] In the case of certainty, this function is not needed.

(1). Therefore, for high probabilities of depletion, increases in carrying charges decrease the likelihood of central storage being desirable. Conversely, for low probabilities of depletion, increases in carrying charges increase the likelihood of central storage.

Reductions in safety allowance constitute a field where tremendous savings may be possible. An example of the ridiculous level to which safety allowances can rise is described in the Bonner Committee Hearings where it is pointed out that 335 cans of paint in the pipeline from the United States to Germany were required to supply the using unit with one can of paint a day for maintenance purposes.[1] Rational analysis should illuminate the wastes involved in a system that allows such situations.

Several aspects of a realistic model have been omitted intentionally, for the sake of ease of presentation. For example, purchasing costs have not been included although methods of incorporating them into the analysis have been described elsewhere.[2] Quantity discounts and price anticipations are among the other factors that have been omitted. Nevertheless, the model does indicate that in some situations there is a need for various types of supply, some involving central storage, others not. This aspect of the model in a sense corresponds with reality.

(3) In locational models the conventional method of approach postulates the existence of a central office and then decentralization of functions in some time sequence. Perhaps a study of large mail order companies would shed light on this problem. The main advantage of increasing the number of mail order plants is that better customer service is provided and sales are increased. On the other side of the ledger, inventory levels and transportation costs must be increased. Attempts should be made to quantify these various aspects of the problem, and thus rationally arrive at the decision as to how many outlets are desirable

The problem of determining how many mess halls to have on a naval station possesses the essential characteristics of our locational model. From the point of view of the

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[1] Federal Supply Management Hearings, Feb. 22, 27, 28, March 5, 11, 1952, p. 42.

[2] T. M. Whitin, The Theory of Inventory Management, Princeton, 1953, pp. 56-62.

amount of food required the smaller the number of mess halls the more economical the feeding is.[1] On the other hand, the men must come to the mess hall from farther away if there are few mess halls. Thus time and expense are involved in getting the men to the food. A balancing of these advantages and disadvantages should help to determine the desirable number of mess halls.

The interrelationships between the various models considered here are sometimes of a fairly simple nature. For example, some aspects of the location model can be looked at as changes in the time factors in the safety allowance model. In the decision model, revised demand estimates fed back from the information center can be used for the safety allowance model's demand estimates.

(4) It was mentioned above that the determination of economic purchase quantities leads to economies of scale. More specifically it has been shown by many authors that the best balance between ordering costs and carrying charges may be achieved by ordering a purchase quantity equal to $\sqrt{\frac{2YS}{I}}$ where S represents the cost of placing an order, Y represents annual \$ sales (at cost prices) and I represents carrying charges (as a per cent of cost). However, such a formula is based upon the supposition that only one order is required for a purchase. In the event that the "economic purchase quantity" exceeds the capacity of the firm with which the order is placed, the simple formula is no longer applicable, although it can be modified to take this difficulty into account.

The much discussed Paint Study carried out for the Munitions Board by a Harvard Business School group was an attempt to evaluate the effects of centralization on purchasing. The results of this study have often been cited as indicating that single-department procurement does not lead to savings. However, the study is inconclusive in this respect, in that it merely indicated that single-department procurement does not lead to savings in one particular instance. The reasons for this failure may be eliminated. For example, consider the argument that a very large purchase quantity enables producers to charge a high price, for they know that several bidders will receive contracts.

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[1] See page 7, above. Of course, if this principle is carried too far dis-economies may set in due to type of equipment.

Surely, it is possible for the single department to space purchases in at least as efficient a manner as the individual services could. Furthermore, the single-department procurer could exert more powerful monopolistic influence. One of the reasons that this possibility did not materialize is that the relative urgencies of the orders of the departments was not evaluated. A revised theory of priorities and more standardization of requisitioning procedures should help to eliminate this difficulty.

Concerning the problem of unification of supply, no one is yet in a position to evaluate the merits and demerits of a unified system. The facts are not yet known. Nevertheless, to clarify some of the issues at stake, I should like to make the following remarks:

(1) A commonly expressed argument is (a) that unification of supply systems brings about centralization (b) that General Motors has found decentralized operations profitable; and therefore (c) unification is bad. As indicated in the first part of this paper, unification does not necessarily involve centralization. Therefore, this argument is clearly invalid.

(2) From the safety allowance standpoint, economies would be achieved by reducing the number of pipelines. Obvious though this statement may be a recent statement by an individual in an extremely high position specifically denied that inventories could be reduced in this way because stock levels depend on overall military requirements.

(3) The amount of material in military pipelines is tremendous, as the following statement before the Bonner Committee indicates:[1]

> "Those two pipelines (Army and Navy), at the end of the war in ETO, had about as much material in them as was consumed during the entire World War II. The Army alone reports $31,454,000,000 in supplies on hand at the end of World War II, exclusive of subsistence or petroleum products - - -."

The savings involved in reducing these pipelines would therefore be of a large order. For example, it has been stated that:[2]

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[1] Federal Supply Management, Sixth Intermediate Report of the Committee on Expenditures in the Executive Departments, June 27, 1951, p. 79.

[2] Federal Supply Management, Hearings, Feb. 22, 27, 28, March 5 and 11, 1952, pp. 23-24.

> "Cooperative action at the New York office demonstrated that under a unified operation it is possible to look for tremendous savings from intangible economic factors. These embrace the more efficient utilization of production facilities and materials, less competition for productive capacity, more favorable prices, improved planning and scheduling of supply requirements and better purchasing methods, as well as the reduced overhead of consolidated operations. If Army and Navy procurement of quartermaster common items, exclusive of subsistence, fuels and lubricants had been combined during the war, this ratio would have yielded savings amounting to $1,115,000,000."

(4) The Alameda Medical Supply Test indicated that there are possibilities of large savings by unified distribution of medical supplies. Alameda provides a representative cross-section of supply support for all three services in the Western States and in addition, supplies Korean operations.

> At the West Coast site selected for the medical test, the Army previously maintained a medical depot at Oakland, California, with 344,000 square feet of storage space. Across the street, the Navy had a similar depot with 535,000 square feet of space. The Alameda depot with 677,000 square feet of space replaces these two installations totaling 879,000 square feet of space. This represents a saving of some 200,000 square feet of valuable space.[1]

(5) The Report of the President's Committee on Administrative Management stated:[2]

> "The conspicuously well-managed administrative units in the government are almost without exception headed by single administrators."

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[1] Alameda Medical Supply Test, Seventeenth Intermediate Report of the Committee on Expenditures in the Executive Departments Government Printing Office, 1952, pp. 2,4. At $8 per square foot, this represents a saving of about 1 1/2 million dollars.

[2] p. 32

Although such evidence is not conclusive, it should be given serious consideration, particularly when the Bonner Committee reports:[1]

> "On the basis of field observations to date and other data available, supply management in the Department of Defense and the component military departments lacks adequate centralized direction and coordination."

(6) At the recent Supply Management Conference it was contended that the superficial thinking that leads to the false conclusion that there is an inherent virtue in centralization and standardization could be rejected. It cannot be denied that there are certain virtues inherent in centralization and standardization. On the other hand, there are many examples of the diseconomies involved. The question is one of balancing these economies and diseconomies. Concerning standardization, it is clear that where standardization can be effected without impairment of technical efficiency it leads to savings through reduced inventory levels.

(7) Concerning responsiveness to command as an argument against unification, the following can be said:

(a) That the various commands operating separate supply systems cannot act in the best interests of the nation, in wartime, for the overall allocation of the nation's scarce resources must be considered.

(b) The fact that the Air Force has been supplied by the Army without raising a large number of problems of this type is evidence that a unified system may work.

I wish to emphasize in closing that I am not contending here that unification is desirable. Good reasons for the maintenance of separate supply systems may be advanced. It is my contention, however, that the opponents of unification have not yet presented a good case, as the above remarks indicate.

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[1] Federal Supply Management, Sixth Intermediate Report of the Committee on Expenditures in the Executive Departments, June 27, 1951, p. 3.

FORMAL DISCUSSION ON
WHITIN'S PAPER
by

Prof. M. E. Salveson
University of California

Last year at the Logistics Conference I visited Admiral Watt, then head of the Production Division of ONM, together with Captain Dudley of his office. Both of these officers expressed the desire to have an "electronic machine" that would project onto a screen a "picture" of the status of production in the nation, at least as far as it affected Navy Procurement. This year I heard Admiral Hetter, from Bureau of Supplies and Accounts, say he would like to have a picture on the wall of how the world wide situation stands at any time in regards to distribution of Navy supplies. Both Admirals Watt and Hetter appreciated the potentialities of the electronic data processing, although the supply picture seems to be somewhat farther progressed toward high speed data processing -- probably because there is just one agency handling the function under fairly well "centralized" control.

In keeping with the views initiated by these officers, I will confine my remarks to the problems of data processing in relation to centralization. In particular reference to Dr. Whitin's talk, I would like to ask the question, "Centralization of What?" and "Why Centralize" My suggested answer is, "Centralization of decision-making, because it potentially leads to more efficient use of available resources."

The limits on centralization that I will consider then, are the limits which are inherent on centralization in decision-making. Incidentally, the class of decisions to which I will make particular reference is that concerned, of course, with logistical problems, i. e., production, procurement, and supply. These have to do with such questions as to what to make or buy, how many, when, and where, to whom to distribute, when, how, etc.

In order to determine the limits on centralization of decision-making it is necessary to look beneath the immediate question and determine what goes on under the broad name of decision-making. This, I suggest, includes the following aspects:

a. Information seeking activity
b. Information and data processing
c. Decision-making per se
d. Feed-back control.

I will explore briefly how these factors limit centralization, and, in turn, how the degree of centralization limits efficiency in logistics.

Firstly, it is obvious that gathering information can not be centralized. It must be done "on the spot" where it is generated, such as in the field, at the supply depots, in the factory, etc. But, there is an important limitation from this fact -- the amount of information which can be given to a centralized decision-maker is strictly limited. That is, regardless of the care with which information is collected, the mere act of reducing it to writing or of communicating it eliminates some of it. That is, some of the "richness" or content is eliminated by extracting only that which the collector considers important or relevant, and by placing it in some standard terminology, such as a language or code form. For example, it is seldom possible to report the "exact" shade of a color, or to give the "exact" conditions at the battle front. Hence, some of the relevant facts are immediately lost in conveying the information to the decision-maker.

The information and data processing step adds further losses. Firstly, the step of transmitting the information takes a further toll on its content, as well as on its "timeliness." There may be errors, further standardization of "shaded" information, etc. And finally, the information is transmitted and received usually much later than it was generated. The cost of transmission is, even in the most urgent case, non-trivial such that we shall not be able in the near future to have full content information transmission. Of course, some day when we have 3-D color television available at all strategic locations, we may be able to overcome some, but not all, of this loss. The consequence, in any case, is that the centralized decision-maker has less information than existed, and at a later time than it was created on which to make decisions.

Other steps in data processing make centralization more difficult, in particular, the amount of time required to process the data into a form which will permit a "well-informed" decision. The notion of a well informed decision requires explanation. By this term is meant a decision which considers the conflicting and mutually exclusive events which can or may arise from carrying out any plan which may have been decided upon. For example, if in a shop there are one thousand man hours per week available, and there are production orders from the Army for eight hundred man hours, the Air Force for eight hundred, and the Navy for eight hundred, it is necessary to consider all of these orders simultaneously in order to obtain the best compromise. For example, if a different shop "planner" handles each defense agency's orders (as often is the case) and there is no central authority at which there is common consideration of the effect of the loading decision of any one on plans or decisions of the others (also as often is the case), all or most of the procurement agencies must suffer in their procurement schedules.

Under this situation it is necessary to select some "optimum" compromise as to amount and timeliness to produce for each agency. This kind of situation is very common. There is almost always more than one "end" use to be made of limited means, such that it is necessary to have some method of finding the optimum compromise in satisfying the ends. But this requires a large amount of data processing, and is very often time consuming, at least until larger scale hi-speed computers are available in greater numbers. The inevitable consequence is either (a) there is no centralization of decision-making with the result that decisions are often conflicting and not attainable, or (b) there is centralization with inadequate data processing and hence, poor decisions.

A full discussion of this problem would require much more time than is alloted here, but the interested reader is referred to a paper from which much of this material was taken: Salveson and Canning, "Electronic Production Control." It contains, in particular, the way in which centralization with necessary data processing was conceived in at least one factory.

Decision-making per se, is often hopelessly lost in a morass of detail unless the data are processed in some precise manner. That is, the human "mind" can encompass only a small number of variables at any one time, such that a large decision must be broken down into many small ones, or must be aggregated into a few variables, either of which reduces the efficiency of the decision and the resulting plan. If the decisions are to be "good," therefore, we should have them made as often as possible by some automatic means which can consider large amounts of detail. This does not mean necessarily that an electronic computer will be the decision-maker, but that if the data reflect the subjective preferences as well the objective relations and values in any situation, it can materially aid in making the decision. The process then can show the decision-maker the consequences and conflicts of various alternatives which may be considered before any of them are carried out, so that he may select the "best" one.

Finally, there is the need for feedback control. This need is reflected in the many common proverbs, such as the "slip twixt cup and lip," or "best laid plans of mice and men go astray," etc. The fact that decisions may be poorly made, that plans must change with changes in circumstances, that additional information alters the situation, etc., is well known by all men of affairs. The consequential need is for feeding back information from those who execute plans to those who have prepared and decided upon them. This information is used both for making new plans, as well as for revising old.

This means, of course, "reports," the bane of all operating personnel. There is little cure for them, as long as we are to take advantage of the increased efficiency of centralization. Our decision center must be informed. Of course, much can be done to make the reports automatic "by-products" of regular operating controls and to make them more efficient in terms of data that are actually required.

In any case, it strongly appears, therefore, that increases in efficiency from greater centralization -- at least of decision making -- must come from more efficient methods of information and data processing. In fact, the "Logistics Computer," which is available for inspection during this conference, is just such a device. In essence, it is an electronic machine especially designed to speed up the processing of large amounts of logistics data and thereby aid in the more efficient planning and decision-making processes by logistics planners. It is understood that this computer is still in the "research" stage, but the eventual adoption of machines of this type will surely make for more efficient planning and decision-making.

A word of caution is added, however. That is, although these machines will handle many problems in a very small fraction of the time required by conventional manual or punched card methods, they are only hi-speed clerks that need very careful instructions. Hence, it will be necessary to define the steps taken in planning and decision-making with far more care than has heretofore been necessary. While this involves interesting mathematical problems, this additional care of itself also will increase the efficiency of the planning and decision-making processes. And, finally, this general approach to centralization should be devoid of the many pitfalls formerly found in plans for centralization. That is, it would be possible to give each agency more adequate consideration in the central decision process, and avoid the "shortchanging" any of them unnecessarily in the broad decisions and plans.

SUMMARY OF GENERAL DISCUSSION ON
DR. WHITIN'S PAPER

Admiral Eccles emphasized the magnitude of the problem of unification, with respect to the logistics implications. The objective in logistics is not economy; rather, economy is an essential means to attain the objectives of the national welfare and security of the United States.

Questions considered pertinent by Admiral Eccles, each of which, he and Dr. Whitin agreed, requires a tremendous degree of scientific, objective study, are the following:

(1) What will be the possible effect of atomic-biological warfare and sabotage on a very highly centralized logistic support system?

(2) What is the meaning of the word "unify"?

(3) What is the importance of good faith in the establishment of common objectives as opposed to improvement in mechanical techniques in the operation of a military defense system for a nation?

(4) In the field of supply, does the law of returns operate? If so, at what size in a military supply system does the law of diminishing returns begin to operate and how does it operate?

(5) What were the true causes of massive failures in the military pipe lines? Were those caused by the fact that there was an imperfect knowledge of the basic principles of theatre logistics in our operating forces in World War II, or were they caused by the fact that we had separate supply systems?

(6) Which supply systems that operated in the war were actually the most efficient and produced the greatest effectiveness of combat support in relation to the energy and effort expended in their establishment?

(7) What is the effect of increasing size of a military operation or establishment on the quality of the product of that establishment?

The discussion, between Admiral Eccles and Dr. Whitin, on economy as a primary desideratum was interesting and enlightening. Dr. Whitin held the view that minimization of cost must be the foremost objective in operation at given production levels, under given military requirements or under a fixed budget. Admiral Eccles expressed concern over the use of criteria in judging the effectiveness of operations that are not based on a thorough analysis of the objective to be desired, analysis taking into account the difference between being given an operation and the determination of the effectiveness of the operation.

Dr. Salveson commented during the discussion on the pitfalls in drawing analogies between the military establishment and large industrial firms. The latter are economic entities, he stated, subject to competition and normal attrition, whereas the military establishment provides one service--defense in war.

The discussion ended with some brief remarks by Professor Morgenstern on the statement, often quoted as a maxim in certain large-scale operations, that it is necessary to obtain a maximum effect with minimum effort. He pointed out that the elimination of such statements, which serve only to generate misunderstandings, is traditional in science, and could well be applied in the field of logistics.

ESTIMATING SHIPPING REQUIREMENTS AT SHORT RANGE

by
Dr. Harry M. Hughes
University of California

During the first day of this conference various general problems in fleet logistics were presented. My purpose this afternoon will be to take a particular set of such problems which have a similar formulation and to discuss some of the methods of solving one of them. This particular set all ask the question "How many measurement tons of a certain type of material arrive at tidewater for a given destination during a designated time period?" To make it more concrete, a typical problem would be "How many measurement tons of dry cargo, excepting aircraft, will arrive at Pacific Coast tidewater for the Far East Theatre during May?" This is known as the shipping-estimate or cargo-generation-estimate problem.

Generally speaking, there are three different groups of such questions corresponding to the length of time in advance that an estimate is made. Long range estimates are needed at staff levels for budget planning, for shipbuilding plans, for establishment or disestablishment of port bases, and for judging feasibility of strategic plans. Intermediate estimates for a period of several months ahead are needed at general and subordinate staff levels for allocations of shipping among the services and among the theatres of operation in time of short supply of ships, for overall scheduling of shipping at any time, for major diversions of supplies to provide maximum utilization of loading and unloading ports, and for judging feasibility of tactical plans. Short-range estimates are useful at the operating level in scheduling individual ships and in preventing temporary congestion at ports. For research purposes and occasionally for control purposes the question is even asked after the designated time period has passed -- that is, after May in the foregoing concrete question.

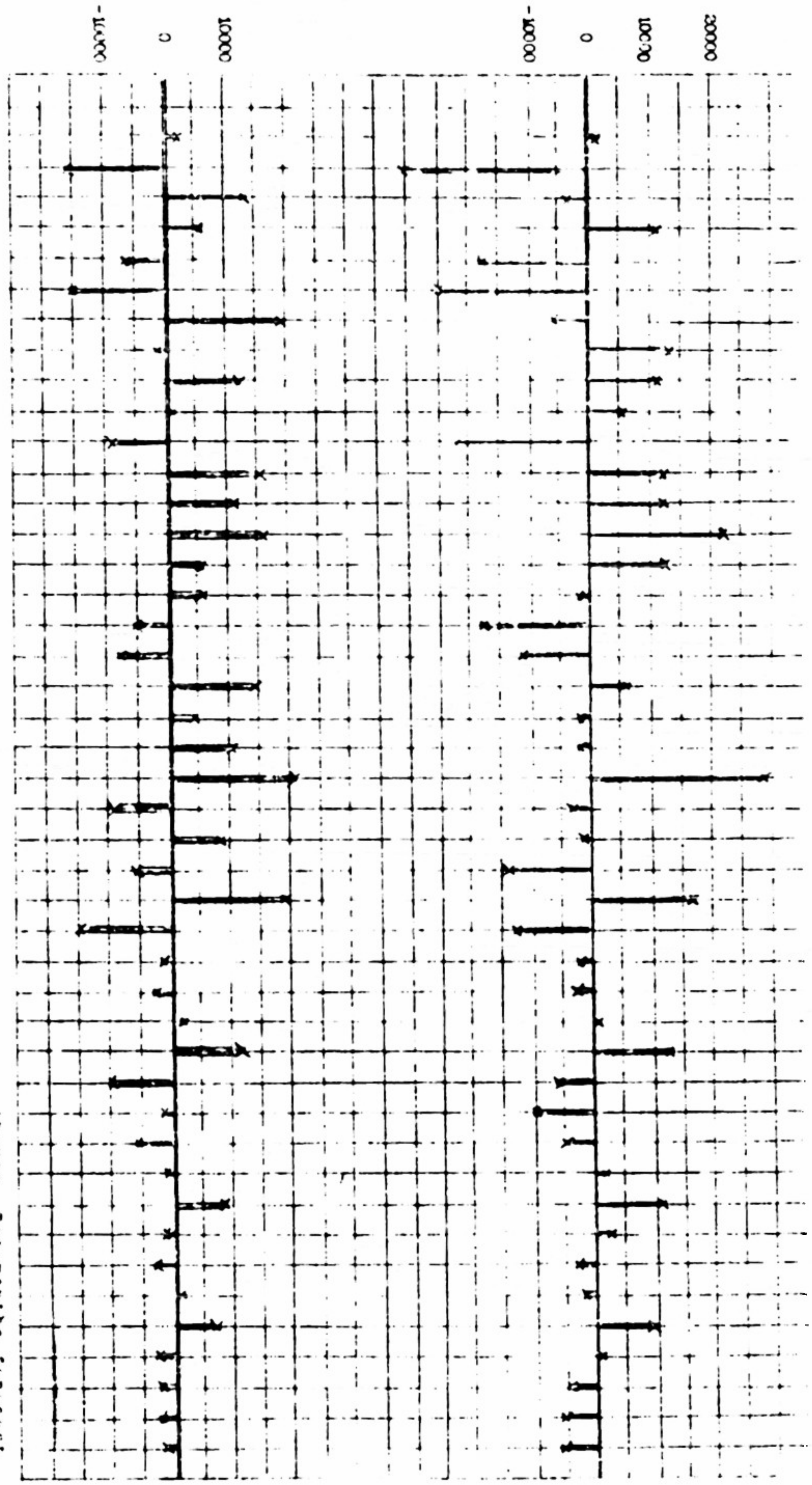
The method of answering the question -- that is, of making the estimate -- will be essentially different for each of the foregoing groups. This is primarily due to the fact that there are different amounts of information available at different times preceding the designated period. A secondary difference in the groups is that different accuracies are required. However it is this availability of information which calls for different methods of attack in answering the question when asked a year in advance from the methods to be used when the question is asked only two months in advance. Even so, there are certain elements in common to both situations so that progress toward an effective method of forecasting in one situation becomes an advance in the solution of the other.

Probably the most common approach to a solution in any of the cases has been that of using an average cube-per-man-per-month factor along with some fairly accurate estimate of the number of men involved. This is of course more successful with certain types of material than with others -- with food for example but not with repair parts.

Probably the second most common method for estimating up to four months ahead is that of observing the jagged curve of measurement tons generated in previous months and extrapolating, at the same time adjusting subjectively for any presumed changes in the previous level or trend of activity. In periods of abrupt change this obviously gives little to go on. Even during periods which one would otherwise consider quite stable the cargo generated from month to month fluctuates in a manner very difficult to sharpshoot by this method. To evaluate the errors, one many consider a chart which presents these errors above and below a central line representing the actual tonnage. "One-month" and "two-month" forecasts have been displayed in Figure 1. You may note that the one-month forecast is not as much more accurate as might be believed without such illustration. The average error for the two-months forecast was over 5,000 measurement tons before Korea and about 12,000 measurement tons since Korea. The three months of transition are not included in these computations. This gives some idea of the size of the error in terms of ship space involved. On the theory that a large-scale operation could expect or permit a larger measurement-ton error we might consider what the percentage error was for this period. Figure 2 is a chart, which shows, month by month, the percentage deviations above and below the horizontal line at true generation. Note that the percentage error varies from 1 percent in several places up to 50 percent, still omitting the three months of transition. The average percentage error is between 15 and 18 percent for both the before Korea and after Korea periods. These are for the two-months forecast, which actually was made about the tenth to fifteenth of the month preceding the month forecasted.

(Hereby hangs the tale of reporting information, because the accumulation and assimilation of the information regarding the actual generation in a given month was not accomplished until the fifteenth of the following month, thus delaying the forecast. This is a difficulty which plagues most of the methods -- accurate and timely information accumulation plus assimilation. But let me return to the methods.)

What information then is available besides past performance which might reasonably give a clue to future cargo generation? Before material moves there must be issued a demand document for it; hence, it appears reasonable to assume that there is information in demand documents about cargo generation. Consider for a while a control center through which pass all the documents (or copies thereof) concerning the movements of the type of material concerned from original demand to arrival at tidewater. What information on the demand documents would constitute perfect information? Three things are required: Method of shipment, time, and cube. First (method of shipment) it is necessary to know which items on the document, if any, will eventually be shipped by surface (as opposed to air or parcel post shipments and cancellations). If forecasts for parcel post generation are desired, then further detail would be necessary. Second (time) it would be necessary to know when each item on the document (to be shipped by surface) will arrive at tidewater. And third (cube) it would be necessary to know what number of cubic feet of packed material will result from each item to be shipped by surface. Consider each of these three separately.



Above: Two-months forecast
Below: One-month forecast

FIGURE 1. Typical Errors in Forecasting by Trend Method
Cargo Generation

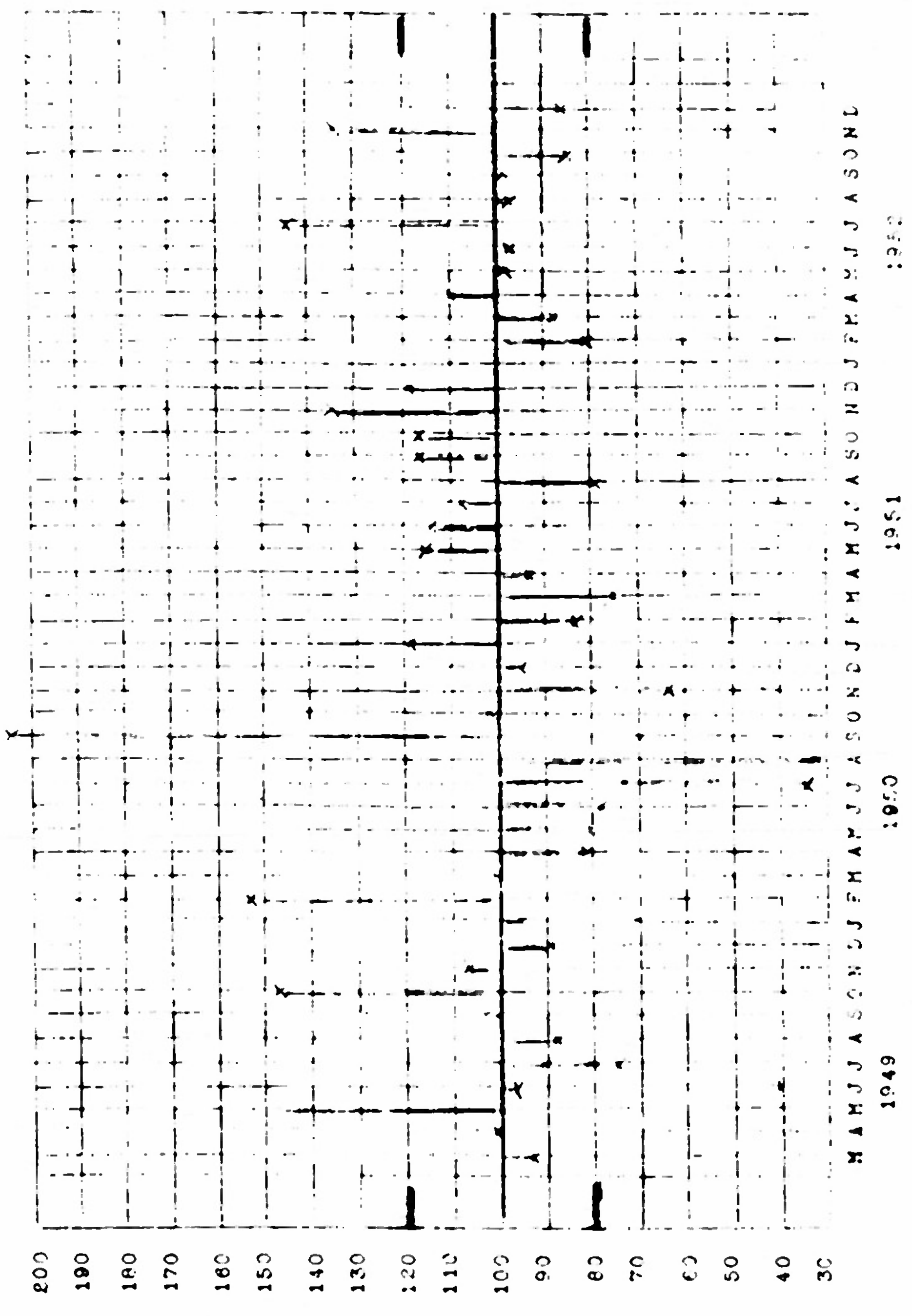


FIGURE 2. Percent Error in Forecasting by Trend Method

Cargo Generation to POA (plotted as Forecast over Actual)

ESTIMATING SHIPPING REQUIREMENTS AT SHORT RANGE

Method of shipment -- It is not in general known on a demand document just what the method of shipment will be. There are of course general inferences -- emergency requisitions will most often result in air shipments; very small items may go parcel post depending on the quantity of other material generated for the same addressee at the same time and depending on the current policy with respect to parcel post shipments -- that is, whether all packages below a specified size and/or weight are to be shipped parcel post or whether such packages are to be held for a few days for the purpose of consolidation if possible and subsequent surface shipment. But with a few exceptions of shipment by air it is not known from the demand document what the method of shipment will be for the particular item "demanded".

Yet it is possible to make statistical statements about the method of shipment; that is, the proportion of items requested which are shipped by surface remains fairly constant over a period of time. This situation was investigated for the material controlled by the Pacific Requisition Control Office for a period before and after the Korean impulse. After subtracting out the cancellations, the proportion going by any particular method of shipment remained roughly constant. There was one noticeable change in the proportions which was found to coincide with a change in parcel post policy at NSC Oakland looking toward more movement by parcel post. Thus while it may not be feasible to place a method of shipment on each demand document, it may be feasible to assume that out of each thousand line items so many hundred will be shipped by surface. The precise factor to be used may be determined from the current completion data arriving at the control center. While this involves an extrapolation just as does the prevalent method of forecasting from past shipments, the variability of the factor from month to month is much less. If a sudden change or initial movement is to be included in the forecast, the method of shipment is likely to be completely determined.

The real difficulty in this first point (method of shipment) is the determination of those items which will be cancelled. In the Pacific Requisition Control Office data the volume of such cancelled items was extremely high. A major source of such cancellations were the items ordered by ships in the forward area which returned to tidewater before the items were shipped. Such items, delivered to the requisitioning ships at tidewater, were cancelled insofar as PRCO was concerned. Since ship schedules are fairly well known in advance, combination of such information with the time information to be discussed as point two may well resolve most of the problems of predicting cancellations. So much for the various aspects of the first piece of information, method of shipment.

Consider now the second piece of information -- time. One way to determine from a demand document the time at which the items will arrive at tidewater is to place a scheduled arrival period on the document. This system has been in use by several agencies. The success of such a procedure of course depends on the knowledge and realistic attitude of the scheduler as well as the cooperation of all the sections which are involved in issuing and transporting the material. Such a scheduling system obviously runs afoul of the difficulty of allowing for local conditions and temporary deviations as well as the other attributes of scheduling treated recently by Professor Salveson. If the scheduler sits at the position of the control center, where he is better able to obtain current information on the capabilities of the supply system, many of the present difficulties of the DDD (deadline delivery date) system can be eliminated. However there still remain many hazards, not the least of which is the scheduling of obligated material, which entails some idea of the time a contractor will actually deliver.

So again we may turn to the statistical approach. Is there a constancy in the time of supplying an item to tidewater which will enable us to assign a time of arrival to a certain percentage of a stack of line items even though it is not known just which particular ones will arrive in that period? Again the answer is yes, there is an essentially constant distribution of supply times, provided the supplying depot and stock status are known. With a good flow of information, these two bits of information may well be available promptly at the control center for use in the forecasting process. Specifically, material that is in stock is furnished according to a relatively stable skewed curve from month to month with percentages quite closely,forecastable from the immediately preceding median times. The shape of this distribution was found to remain stable through the Korean impulse.

Material obligated -- that is, placed on standby status until the material actually arrives at the depot concerned from action already taken by the depot -- has practically a uniform distribution of supply time; that is, of the line items obligated in one month, roughly a constant number will arrive at tidewater in the third month, fourth month, fifth month, etc., up to a distant month depending on the type of material concerned. For different depots the in-stock and not-in-stock curves will center at different time points but will have the same shape. This is again a constancy which changes gradually from month to month due to improved efficiency or reduction of workload or improved availability in the open market, etc. Three samplings recently performed on PRCO records indicate a high reliability of time forecasting by this method.

Possibly the most variable of the three components--method of shipment, time, and cube--is the last, the cube of each item on the demand document. One way of obtaining this information would be to place in the various catalogues a cube figure for each item and require that it be copied onto the requisition or other demand document. This is not an impossible task, although there is a question as to its efficiency in the end for several reasons. One reason is the extra burden placed on the requisitioning agency. If the requisitioning agency be a ship or if it be any activity in a forward area, it should not be expected to do more than a minimum of such detail work. Another reason is that quite a few items are ordered for which a Navy number -- and hence presumably a cube figure -- is not assigned. Such items are numerous in the Yards and Docks cognizance material. Allowance lists and automatic resupply lists in some of the cognizance categories are such lists of cube item by item, particularly for the faster-moving material. These may eventually find their place in the control process as well as the planning processes -- both in forecasting and in other applications. It must always be remembered in any of the foregoing, however, that some arrangement must be made to allow for the cube introduced by packing the item for overseas shipment.

One compromise to reduce the load of assigning a cube to each and every item or line item is that of ignoring the small items and giving particularly close attention to the large items, with routine treatment of the in-betweens. For example, all items less than one cubic foot might be completely ignored in setting up the forecast. In Navy practice there is a sufficient number of such small items to add up to a quite significant proportion of the total cube shipped. Hence they cannot be ignored. Fortunately, it is quite possible and relatively dependable to identify each item as being in a certain cube range, counting the number of items in each range and using an average cube within each range. An incomplete study of material arriving at NSC Oakland tidewater during one month, analyzed class by class, indicates the possibilities in this direction with some suggestion for useful cube ranges.

Four charts will indicate what I mean. For class 51 material measured in gallon units the distribution of cube per item is very closely grouped around the two-tenths point (Fig. 3). Thus the assigning of the figure two-tenths to all class 51 material measured in gallon units would not lead to a very high error, either in absolute value or percentagewise. Quite a few class-unit combinations were found to have this general type of distribution, though of course with different values of cube per unit at the centering point of the distribution. Another quite prevalent type of distribution is typified by class 15 material measured in feet (Fig. 4). Here is a great preponderance of very small material with a few items relatively larger but still nothing over two-tenths of a cubic foot per item. While the dispersion is greater for this type, the use of an average value is still justified with any reasonable number of items. A third type of distribution is represented by class 33 material measured in square yards (Fig. 5). Here it is obvious that different kinds of material are involved, each quite homogeneous within itself. However the dispersion of the groups is not so great but what one could use a single average for a reasonable number of items although it would probably not be too hard to identify the different kinds and assign an average to each Finally there is a fourth type of distribution which absolutely demands the isolation and individual cubing of particular items. This type is represented here by class 12 material in "each" units (Fig. 6). The preponderance of the items is down in the same cube range as before, even more so than is indicated on the chart, but there exist significant quantities of items having around 20 cubic feet per item. These are sufficient to make any average figure quite unrealistic so that they must necessarily be identified before a cube figure is assigned.

Since line items are relatively easy to count (and are counted anyway for other purposes) in the present system, and since an increase or decrease in number of line items should in some way be reflected in a corresponding subsequent increase or decrease in measurement tons shipped, attempts have been made to find a cube-per-line-item factor which might be used for various planning purposes. Over the period previously mentioned, a study of line items completed versus measurement tons shipped for material under Pacific Requisition Control Office cognizance shows a lack of steadiness which precludes the use of such a general factor (Fig. 7). Note the changes of more than 100 percent even during a period which was considered relatively stable. However it may be still possible to use the cube-per-line-item approach if a continual cubing of sample line items properly selected is carried on to determine the current factor. Thus the factor would be based on current rather than past data. This would of course require that practically 100 percent knowledge be available at the sample cubing point regarding cube-per-item information.

Hence we are brought back to the statistical evaluation of cube-per-item for each of a set of material subdivisions such as classes. The encouraging fact about determining cube-per-item for each class, estimating cube by class, and then combining for the total estimate is that in this manner the error can be greatly reduced. The principle involved is that of additivity of variance for sums of independent variables. Thus, since the deviation of the cube of the particular items requisitioned from their class mean is quite reasonably assumed to be independent of the deviation of the cube of any other particular items in a stack from their class mean, it follows that the ability to forecast the cube of any particular item need not be exceptionally accurate. For example, suppose the cubing process were broken down into one hundred factors -- that is, one hundred class-unit groups or some other such recognizable breakdown. If then the accuracy of the cube-per-item factor in each group were only on the order of 30 percent, the accuracy of the total tonnage computed from these figures would be on the order of 3 percent, which is obtained by dividing 30 percent by the square root of 100, the number of groups involved. This is a very generalized statement but indicates the order of improvement which might be obtained. The approach has been well proven in populational sampling under the heading of stratified sampling. More specific analysis of variances within groups for the previously mentioned sample has not been completed, though the graphical portrayals of the distributions, as already indicated, are hopeful.

With several alternative procedures available in each of the components -- shipment method, time, and cube -- there is a variety of combinations from which to choose for the forecasting method. The combination chosen will depend on various record-keeping policies for other purposes than forecasting and on the prevalent methods of transmitting and handling information (paper work). A criterion for optimum system must take into account the cost in time and money of the data-gathering and data-manipulation as well as the accuracy of the forecast in terms of variance, M/T, or percentage error of some sort.

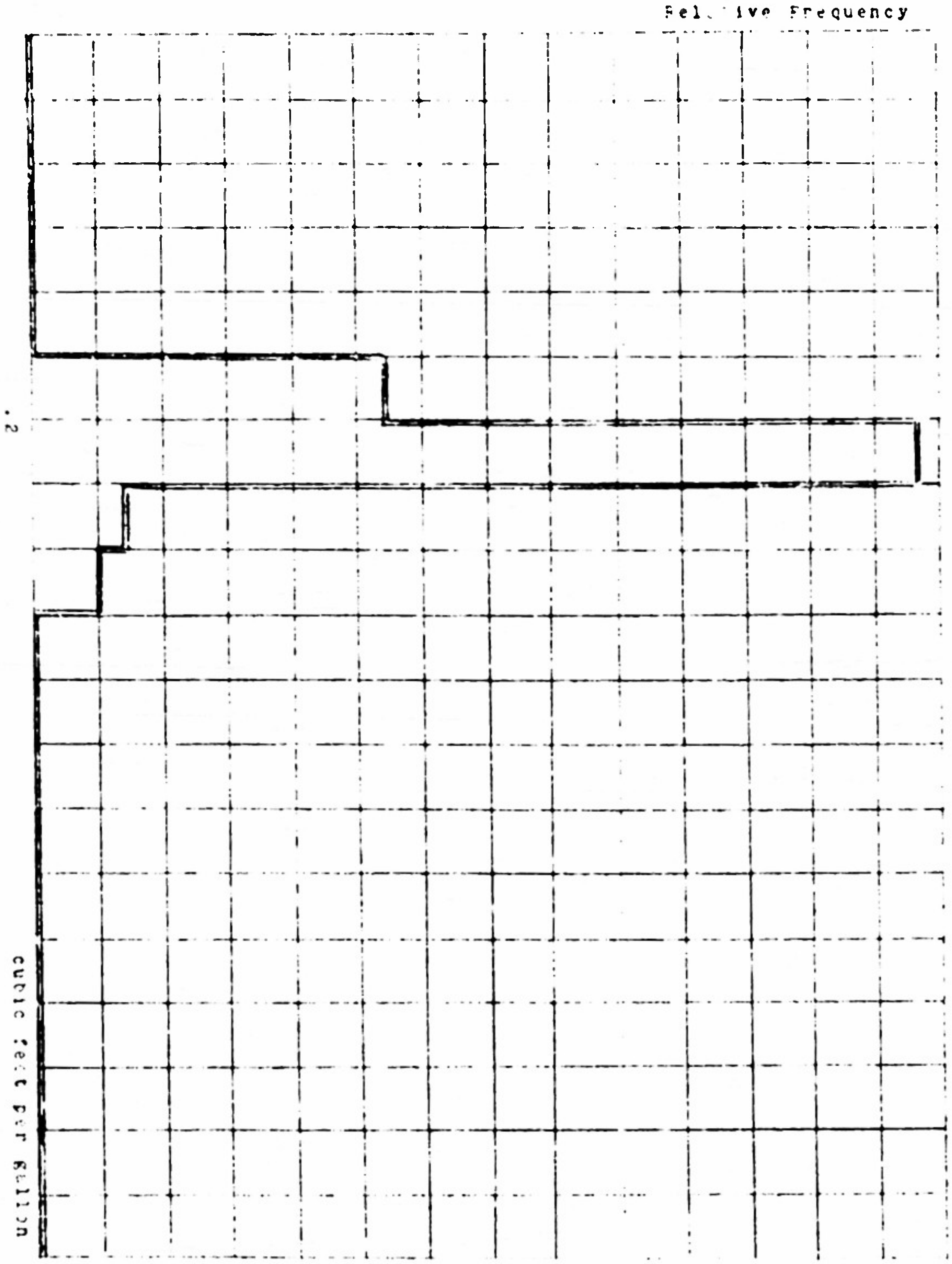


FIGURE 3. Distribution of cube per gallon for Class 51 material

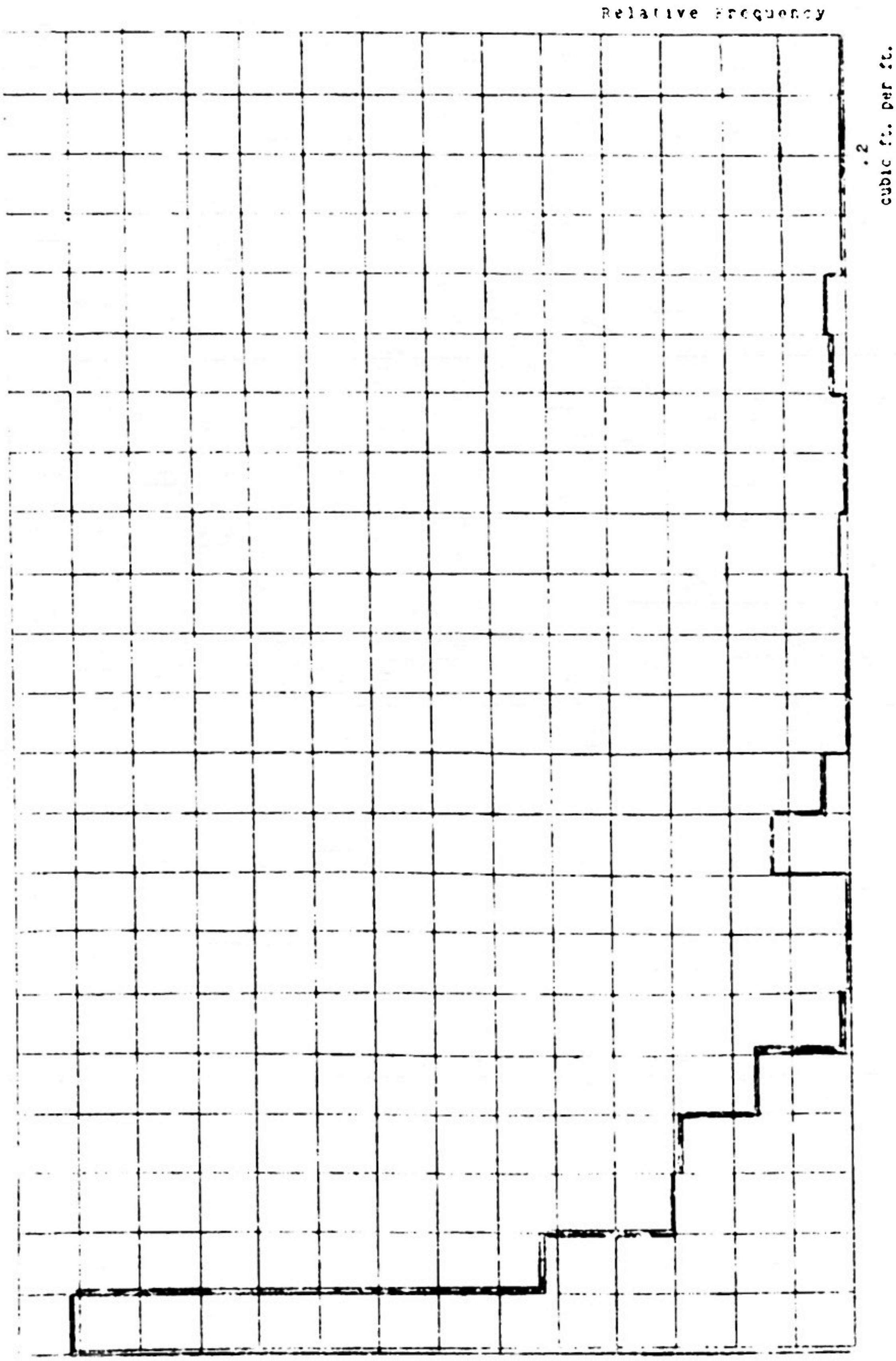


FIGURE 4. Distribution of cube per foot for Class 15 material

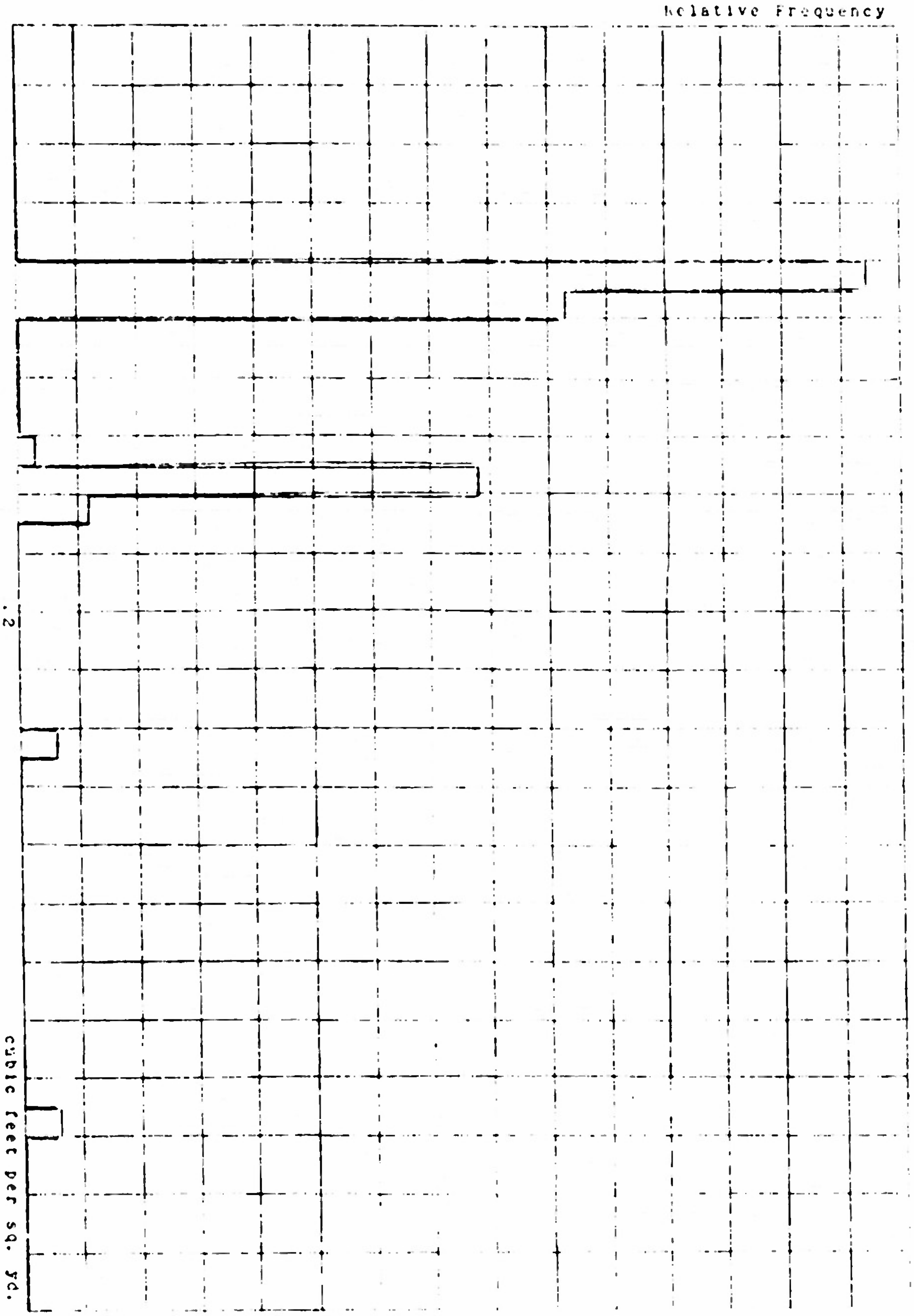


FIGURE 5. Distribution of cube per square yard for Class 33 materiel.

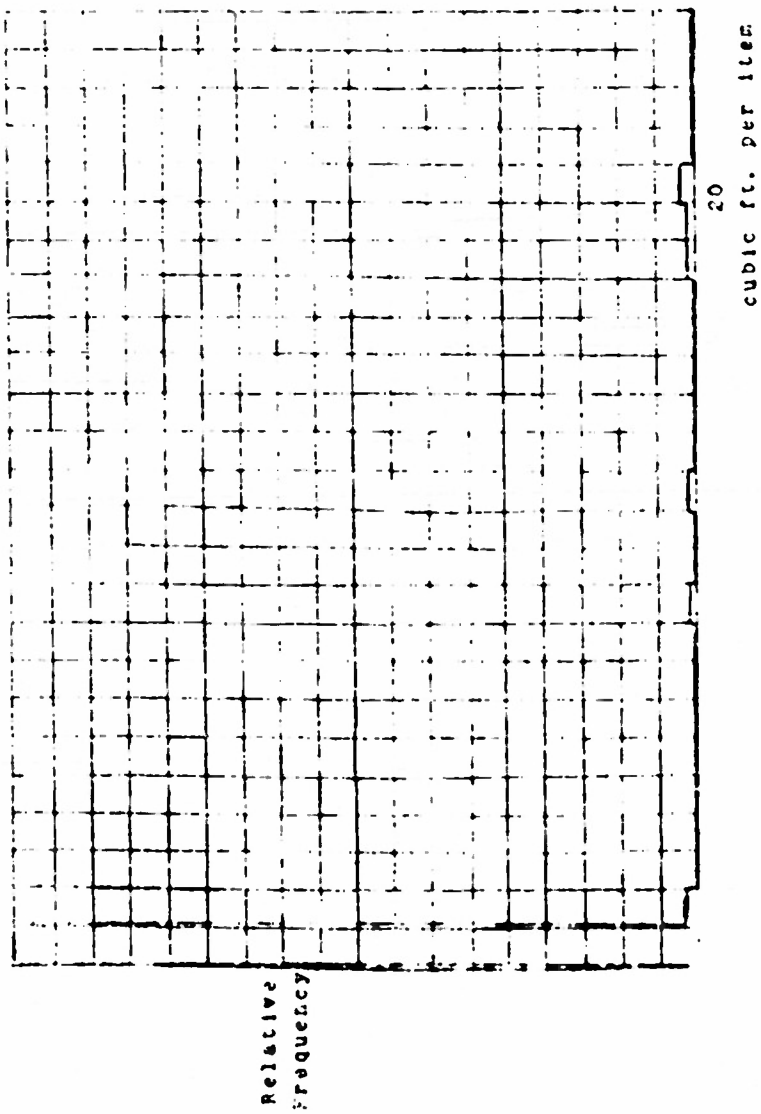


FIGURE 6. Distribution of cubic feet per item for class 12 material

ESTIMATING SHIPPING REQUIREMENTS AT SHORT RANGE

Basically any system of the foregoing type will require keeping track of thousands of line items with information for each including cognizance, supplying depot, stock status, or perhaps directly method of shipment, time of shipment and cube. As well as a running file of such active items, there will be involved various counting procedures, elementary computations of adding and multiplying, and perhaps some sampling. It is at once evident that if the running file could be placed in the storage of an electronic computer, that very few routines would be needed to provide the necessary forecast at stated intervals. Furthermore sharp changes or impulses such as the Korean activity could easily and quickly be handled.

Now forecasting alone may not be sufficient justification for the complete utilization of an electronic computer -- or of a particular storage capacity of one. However the storage envisioned above would have many other control uses. It would provide a very convenient file for movement control functions and for follow-up functions such as are performed now by PRCO. The most formidable difficulty is probably the matter of getting the proper information to the machine, particularly with respect to progress and completion information.

Either with or without the use of computing machines, there are many places in control and planning where properly designed samples can reduce time and manpower involved yet produce realistic and dependable numerical values for the purpose desired. Consider as an example the matter of the time lapse from transmittal of demand document by PRCO to arrival of material at tidewater. The determination of this time for each of thousands of items is a burdensome task either by hand or by machine. Having once established the stability of the form of the distribution, as previously mentioned, and having decided what accuracy is desirable, it is a simple computation to determine the size of sample required. Because of the shape of the distribution, it was felt that the median would be a more meaningful statistic and would in many cases require a smaller amount of sampling. For each combination of cognizance, stock status, and method shipment a sample size was determined which would yield an estimate within 10 percent of a true median at least 95 percent of the time. This is given by the formula:

$$n = (yc)^{-2}$$

where y is the ordinate of the frequency distribution at the median and c is 10 percent of the true median. Since the true values were not known, the observed values of y and c were used from several different months to make the estimate of n. The results were quite consistent from month to month. In general the number of computations per month for each combination was reduced from over a thousand to one or two hundred.

Thus by further development of information procedures, by introduction of well-designed sampling procedures, and perhaps by the use of machine aids, control and planning responsibilities such as that of estimating shipping requirements may have much of the subjective element removed.

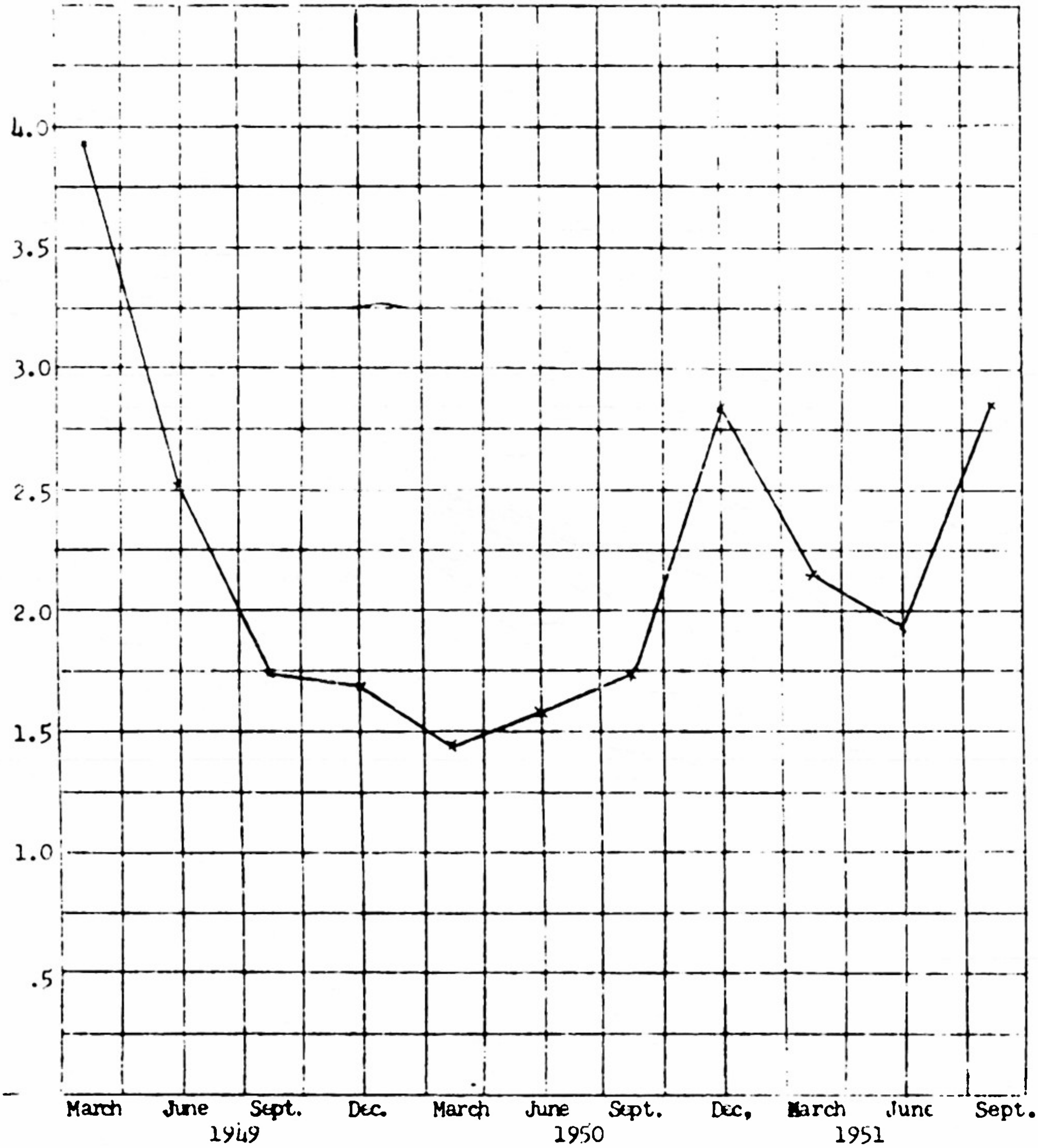


FIGURE 7. Ratio of Measurement tons shipped to line items completed, by 3 month increments.

OPTIMAL SCHEDULING IN TRANSPORTATION

by

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Logistics Research Project

Summary

Given the available supply of a commodity in m shipping points and given the demand at n receiving points, the problem is to schedule the shipments of cargo, or the return of empty ships, or both, in an optimal way. This means that routes and times of shipping have to be so determined that the schedule

1° is feasible with regard to time-phasing

2° minimizes travel cost (cargo- or ballast travel or both)

3° uses minimal ship requirements.

The problem is described by means of a mathematical model leading to linear programming as a method of solution. It amounts to minimizing a linear function under constraints represented by linear inequalities, and solutions yield feasible schedules which minimize total cost of transportation.

Problems postulating only one of the three mentioned properties at a time have been investigated by several authors. In particular, consideration of 2° has led Hitchcock [5] and Koopmans [6] to their classical transportation model.

Properties 1° and 2° taken jointly have been considered by G. B. Dantzig, J. Fennell, C. B. Tompkins and the author.

Computational methods for linear programming have been developed by Dantzig [1] and others.

1. Notation*

To fix the ideas, we speak of ships and cargoes, although the considerations are not restricted to sea transportation. The ships are assumed to be all of the same type and to carry a single commodity, which is measured in shiploads as unit.

Shipping points are labelled 1, 2, .. i, .. m; receiving points are labelled 1, 2, .. j, ... n, so that generally $\underline{i}$ will always refer to a shipping-, $\underline{j}$ to a receiving point.

The cargo shipping is described by the following (m x n) cumulative functions of time

$F_{ij}(t)$ the total number of cargoes which, up to time $\underline{t}$ (inclusive), leave the shipping point $\underline{i}$ for the j-th receiving point as destination.

Similarly the ballast shipping is described by the (m x n) cumulative functions

$X_{ij}(t)$ the total number of ships which, up to time $\underline{t}$ (inclusive), leave the receiving point $\underline{j}$ in ballast travel towards the shipping point $\underline{i}$.

Some of the routes may have no cargo- or ballast travel; then the corresponding functions will be identically zero.

Evidently, total shipping during a specified interval of time,

$a < t \leq b$

*See diagrams at end of this paper.

is given by

$F_{ij}(b) - F_{ij}(a)$ for cargo from i to j, and

$X_{ij}(b) - X_{ij}(a)$ for ballast from j to i.

We now consider a time period of length T, say from $t = 0$ to $t = T$. In accordance with practice in discrete scheduling, the period T (say a month) is divided in intervals of unit length (say 1 day) each, which we number $1, 2, \ldots k, \ldots T$. Shipping during one of these intervals is then briefly denoted by:

$f_{ijk} = F_{ij}(k) - F_{ij}(k-1)$ cargo shipping from i to j during k-th interval

$x_{ijk} = X\ (k) - X\ (k-1)$ ballast shipping from j to i during k-th interval

Sailing times (including time for loading and unloading) are approximated by integers and denoted by:

a_{ij} cargo sailing time from i to j

b_{ij} ballast sailing time from j to i.

Eventually we may have $a_{ij} = b_{ij}$.

Per ship such cost of travel as is in excess over pure maintenance cost at anchor, is denoted by:

d_{ij} cost of ballast travel from j to i.

d^{*}_{ij} cost of cargo travel from i to j.

2. The basic problem

The shipments during the considered time period T will in general be part of a larger shipping program, which has been under operation before—and is to continue after that period. However it may be assumed, without loss of generality, that the whole operation is restricted to the period T, starting at $t = 0$ and ending at $t = T$. Also all essential features are preserved if it is first assumed that the schedule for cargo shipping is given, the problem then being to optimally schedule the ballast shipping.

The cargo schedule is represented by the system of numbers f_{ijk}. This means we have a set of tables, one for each time interval k; each table has two entries, thus stating, for every shipping point i and every receiving point j, the number of cargoes scheduled to leave i for j. To find a ballast schedule, means to construct a similar set of tables containing numbers x_{ijk} of ships scheduled to leave j in ballast towards i during k. Such schedule will be a solution to the problem if it fulfills the 3 conditions mentioned in the summary.

By the condition of feasibility for a ballast schedule we mean, first, that, at any time interval k, only those ships that are present at a receiving point j can be scheduled to leave this point, and second, that, at each shipping point i, empty ships arrive at times and in numbers as needed to meet the given cargo schedule.

A ballast schedule is understood to fulfill the condition of minimal travel cost if the schedule is feasible and has least travel cost among all possible feasible schedules.

The last of the 3 conditions, demanding that an optimal schedule should use the smallest possible number of ships, is closely related to t[illegible] the first 2 conditions and depends moreover on the initial geographical positions of the ships. To assure the existence of a feasible schedule, a given number of ships may be sufficient if favorably located, and not sufficient if too far away from the points where needed. Concerning relations between ship requirements and ballast travel, it may seem attractive to assume that a schedule with least ballast travel will require the smallest number of ships, and conversely. This however is not true, since in increasing the number of ships the number of feasible schedules increases, and among the additional schedules so obtained there may be some with less ballast travel. In such case the operating agency will have to decide which of the two possibilities it prefers: less ballast travel with more ships in use, or less ships with larger ballast travel cost.

It has become customary to deal with ship requirements and travel cost separately. The illustrated interdependence however shows the need for a more consistent formulation of the problem. This is obtained by going back to the general definition:

A schedule is optimal if it minimizes total cost. The problem then is to find an optimal schedule.

Cost of operating a ship can be broken down into cost of procurement, maintenance, devaluation, cargo- and ballast travel (in excess over maintenance).

Cost of procurement may mean the difference of the price one has to pay when buying a ship and the price he would obtain on immediately reselling it; it may also mean a constant fee one has to pay, in addition to the time rate, when chartering a ship; or it may represent the disutility of having to withdraw that ship from another task in order to make it available to the claiming agency. Such disutility, measured in the same units as the other cost factors, will then depend on how badly the ship is needed elsewhere (and may at times be zero).

Cost of maintenance and devaluation are assumed linear in time and therefore proportional to the number of ships and the length of time under operation.

Cost of cargo- and ballast travel are determined by the respective schedules. Since at present the cargo schedule is assumed to be given, the corresponding cost is fixed and need not be considered when minimizing total cost.

To make the model simple, it can be assumed, without loss of generality, that the transportation agency has no ships available at the beginning of the operation. It is further assumed that the agency can not only procure the ships necessary for the operation but can also dispose of ships when no longer needed. Evidently the agency will not procure and dispose of ships more often than necessary, since such a transaction involves loss. Even if redisposal is not permissible (for instance, an agency operating its own ships, buying but never selling them), we still <u>formally</u> provide such possibility in the model, for bookkeeping reasons; disposal followed later by procurement can be prevented by simply presetting the mentioned loss sufficiently high; disposal without subsequent procurement will then mean: no longer needed.

3. The model

We refer to notations in section 1. In addition to the m shipping- and n receiving points, numbered 1 to m and 1 to n respectively, we introduce two more points: a source (for procurement) and a sink (for disposal) of ships. Since ships needed at shipping points generally come in ballast from receiving points, the source is formally included in the system of receiving points and labelled zero. Similarly the sink will appear as shipping point with label 0. In this way procurement and disposal of ships are formally included in the ballast schedule, represented by the system of numbers x_{ijk}:

If i and j are both different from zero, x_{ijk} still denotes the number of ships leaving (during time interval k) the actual receiving point j, in ballast travel towards the actual shipping point i.

For $j = 0$, x_{i0k} is the number of ships acquired (during k) at the source, for first use at the shipping point i.

For $i = 0$, x_{0jk} is the number of ships leaving j, bound for the sink, i.e. no longer needed after delivery of their last cargo at receiving point j.

The case that i and j are both zero is excluded, since no ships are sent directly from the source to the sink (unless the operating agency also engages in dealer's activities). Henceforth by (ballast) schedule we will understand this general type of schedule $\{x_{ijk}\}$ that includes procurement and disposal of ships, in admitting nonsimultaneous zeros for subscripts i and j.

If no limitations are put on the number of ships available at the source at any time, then feasible schedules will certainly exist.

Given a feasible schedule, what is the total cost?

The cost of actual ballast travel of a ship from j to i has been denoted by $d_{ij} (i \neq 0, j \neq 0)$.

The cost of procurement maintenance and devaluation will be denoted by d_{i0k}, if the ship is acquired during time interval k and bound for first use in shipping point i. Formally this cost then appears as "cost of ballast travel from 0 (the source) to i". That part of this cost which, in some later time interval k, is recovered through disposal of the ship at the point j, will be denoted by d_{0jk}. A possible form of these functions is:

$$d_{i0k} = p_i + c(T - k)$$
$$d_{0jk} = -[s_j + c(T - k)],$$

where

p_i is "purchase price fob i",

s_j is "selling price at j", and

c is cost of maintenance and devaluation (amortization) per ship and unit time (for other possible meanings see section 2).

The total cost then is obtained by multiplying each of the x's with the corresponding d's and algebraically adding up the products.

A feasible schedule that has minimal total cost is a solution to the problem.

The total number of ships used in the schedule is the sum of all the x_{i0k}'s.

4. Technical formulation

The time variable k will be written as subscript or as argument, whichever is more convenient.

The feasibility conditions are:

$$(1) \quad x_{ijk} \geq 0 \qquad \begin{Bmatrix} i = 0,1,\ldots m \\ j = 0,1,\ldots n \\ k = 1,2,\ldots T \end{Bmatrix}$$

$$(2) \quad \sum_{j=1}^{n} f_{ijk} = \sum_{j=0}^{n} x_{ij}\,(k - b_{ij}) \qquad \begin{Bmatrix} i = 1,2,\ldots m \\ k = 1,2,\ldots T \end{Bmatrix}$$

$$(3) \quad \sum_{i=1}^{m} \sum_{r=1}^{k} f_{ij}\,(r - a_{ij}) \geq \sum_{i=0}^{m} \sum_{r=1}^{k} x_{ijr} \qquad \begin{Bmatrix} j = 1,2,\ldots n \\ k = 1,2,\ldots T \end{Bmatrix}$$

(2) expresses the condition that the total number of cargoes leaving i during k should equal the total of arrivals at i during the same k. Arrivals consist of ships coming in ballast from receiving points $(j \neq 0)$, or coming new from the source $(j = 0)$.

(3) states the simple fact that at any time no more ships can leave a receiving point j than are present at that point, which means that, for any k, the total arrivals during $(0,k]$ must be $\geq$ the total departures during $(0,k]$, including disposals $(i=0)$. It has been assumed that no ships are present at the beginning; initial values different from zero would merely introduce constant terms in (3).

The decision on equality in (2) and inequality in (3) means that ships will spend their eventual waiting times at receiving points j (and not at shipping points i). The converse assumption would yield optimal schedules which are less economical (since the assumption would decrease the number of feasible schedules). Inequality in both constraints is of course possible, and somewhat more general, in the sense that it may furnish additional schedules, although without improving on the minimal cost.

The loss function, representing total cost of operation (disregarding the fixed cost of the fixed cargo schedule), is

$$L = \sum_{i,j,k} d_{ijk}\, x_{ijk} \,,$$

with the agreement that $d_{ijk} = d_{ij}$ when both i and $j \neq 0$.

The feasible schedules that minimize total cost are solutions of the linear programming problem of minimizing the linear function L under the constraints (1)-(3).

The existence of solutions can be confirmed in proving for instance that

(a) the constraints are satisfied by some schedule $\{x_{ijk}\}$

(b) the homogeneous form of the problem (i.e. left-hand sides of (2) and (3) replaced by zeros) has a null-solution.

Since ships are assumed to be available at the source at any time in any number, (a) is proved by choosing

$$x_{ijk} = 0 \text{ for } j = 0\,, \text{ and } x_{i0}(k - b_{i0}) = \sum_{j=1}^{n} f_{ijk}$$

where b_{i0} is procurement lead time. This schedule satisfies the constraints (it simply uses a new ship for each cargo).

To prove (b), we remark that only the null-schedule satisfies the homogeneous forms of (2) and (3) simultaneously; it is therefore the only feasible schedule, and consequently also a solution, of the homogeneous problem.*

5. Extensions

In the basic problem treated so far, the cargo schedule, i.e. the system of numbers $\{f_{ijk}\}$, had been assumed as given, and only the ballast schedule $\{x_{ijk}\}$ was sought. Without changing essential features several other assumptions are possible, a few of which will be indicated in the sequel.

5.1. Total cargo shippings from i to j during entire time period T are given; optimal ballast and cargo schedules are sought.

Let c_{ij} denote the given totals. The only changes in the technical set up of the model are, that

(a) not only the x_{ijk} but also the f_{ijk} have to be considered as variables

(b) in addition to the constraints (1)-(3) we have the constraints

(1a) $f_{ijk} \geq 0$

(4) $\sum_{k=1}^{T} f_{ijk} = c_{ij}$ $(i = 1,2,...m;\ j = 1,2,...n)$

*The question of existence of integral solutions will be discussed in a forthcoming paper.

The loss function $\underline{L}$ remains the same, since, because of the fixed totals c_{ij}, the cost of cargo shipping does not vary with the f_{ijk}.

Similar treatment can cover the cases where totals for specified subperiods are given.

5.2. For the entire period T, the total supply in each $\underline{i}$ and the total cargo shipping to each $\underline{j}$ are given; optimal cargo and ballast schedules are sought. This is the general case, where total cost of ballast and cargo shipping has to be minimized.

Let c_i be the total supply in $\underline{i}$ and q_j the total shipment to $\underline{j}$. In addition to the constraints (1)-(3) of the basic problem, we have the following two systems of constraints:

(1a) $f_{ijk} \geq 0$

(4*) $$\sum_{k=1}^{T} \sum_{j=1}^{n} f_{ijk} \leq c_i \qquad (i = 1,2,\ldots m)$$

(5) $$\sum_{k=1}^{T} \sum_{i=1}^{m} f_{ijk} = q_j \qquad (j = 1,2,\ldots n)$$

The loss function changes to:

$$L^* = \sum_{i,j,k} (d_{ijk}\, x_{ijk} + d^*_{ijk}\, f_{ijk}),$$

where $d^*_{ijk} = d^*_{ij}$ = travel cost per cargo from $\underline{i}$ to $\underline{j}$.

5.3. Tolerance in Timing. Here again, as in the basic problem, the cargo schedule is considered to be given, however with the understanding that slight deviations in shipping times are permitted. To fix the ideas, say that cargoes may leave up to 3 days earlier and up to 2 days later than marked in the given cargo schedule. Such assumption brings the model closer to reality, since in most cases the given cargo schedule will have been constructed on the basis of estimates and averages of data, and therefore is itself to be considered as an approximative rather than an exact frame of requirements.

Making use of the admissible tolerance, the domain of feasible schedules becomes larger, with the possible effect of savings in total cost, or of making a plan feasible, which, because of limitations in ships, would be unfeasible under the rigid schedule.

When using the admissible tolerance, the given f_{ijk} can no longer be considered as representing the actual cargo schedule. Hence, in addition to the variables x_{ijk} of the ballast schedule, we introduce variables y_{ijk} for the cargo schedule. The problem is again one of linear programming. As constraints we have:

The relations (1)-(3) of the basic problem with the f_{ijk} replaced by y_{ijk}, and in addition

$$(1a) \quad y_{ijk} \geq 0$$

$$(4^{**}) \quad \sum_{r=1}^{k-d} f_{ijr} \leq \sum_{r=1}^{k} y_{ijr} \leq \sum_{a=1}^{k+a} f_{ijr} \qquad \left\{\begin{array}{l} i = 1,2,\ldots m \\ j = 1,2,\ldots n \\ k = 1,2,\ldots T \end{array}\right\}$$

if the tolerance of advance and delay in shipments is of $\underline{a}$ and $\underline{d}$ time intervals respectively.

The loss function remains the same as in the basic problem.

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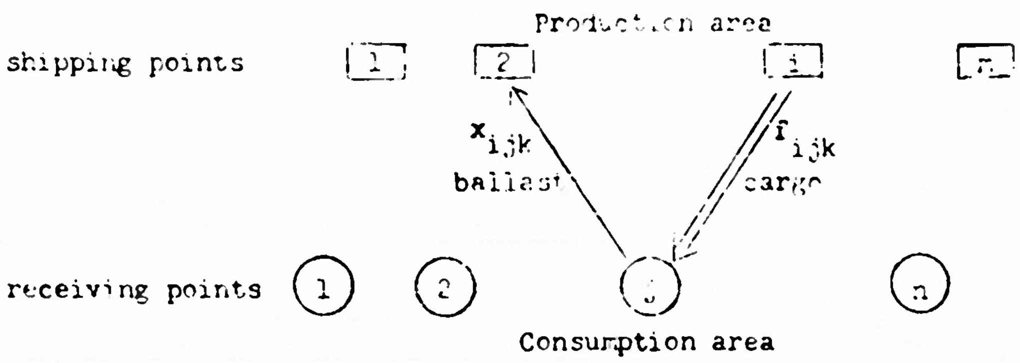
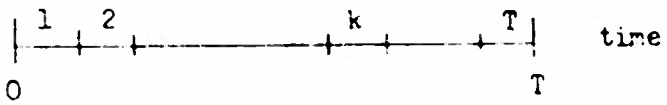
The problem





given cargo schedule f_{ijk} sought ballast schedule x_{ijk}

March 19

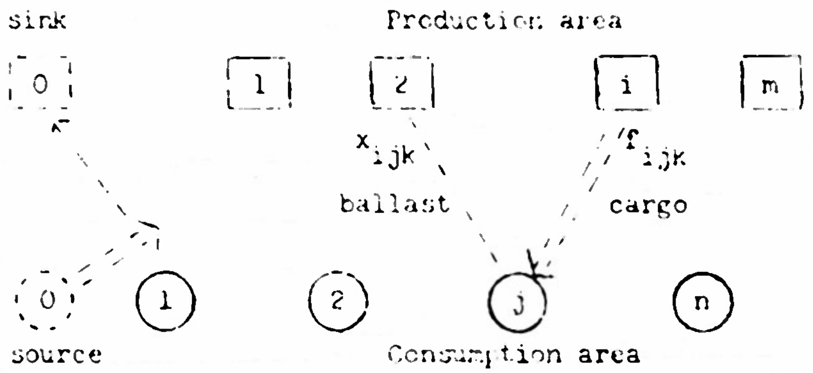
	1	2		i		m
1						
2						
j				f_{ijk}		
n						

1) feasible

2) minimum ballast cost

3) minimum ship requirements

The Model



Notation

	cargo $i \to j$	ballast $j \to i$
shipping during k	f_{ijk}	x_{ijk}
sailing time	a_{ij}	b_{ij}
travel cost	d^*_{ij}	d_{ij}

Cost

1) procurement $\quad d_{i0k} = p_i + c(T - k)$

2) maintenance

3) amortization $\quad d_{0jk} = -[s_j + c(T - k)]$

Constraints

(1) $x_{ijk} \geq 0$

(2) $$\sum_{j=1}^{n} f_{ijk} = \sum_{j=0}^{n} x_{ij}(k - b_{ij}) \qquad \left\{\begin{matrix} i = 1,2,..m \\ k = 1,2,..T \end{matrix}\right\}$$

(3) $$\sum_{i=1}^{m} \sum_{r=1}^{k} f_{ij}(r - a_{ij}) \geq \sum_{i=0}^{m} \sum_{r=1}^{k} x_{ijr} \qquad \left\{\begin{matrix} j = 1,2,..n \\ k = 1,2,..T \end{matrix}\right\}$$

Loss function

$$L = \sum_{i,j,k} d_{ijk} x_{ijk} \qquad d_{ijk} = d_{ij} \quad (i,j \neq 9)$$

Extensions

Sought: optimal cargo and ballast schedules.

I Given total cargo for each route

$$(4) \quad \sum_{k=1}^{T} f_{ijk} = c_{ij} \qquad \left\{ \begin{array}{l} i = 1,2,..m \\ j = 1,2,..n \end{array} \right\}$$

II Given total supply and demand in each $\underline{i}$ and $\underline{j}$ respectively

$$(4^*) \quad \sum_{k=1}^{T} \sum_{j=1}^{n} f_{ijk} \leq c_i \qquad (i = 1,..m)$$

$$(5) \quad \sum_{k=1}^{T} \sum_{j=1}^{m} f_{ijk} = q_j \qquad (j = 1,..n)$$

$$L^* = \sum_{i,j,k} (d_{ijk}\, x_{ijk} + d^*_{ijk}\, f_{ijk})$$

III Tolerance in shipping times: admissible advance $\underline{a}$, delay $\underline{d}$

In (1)-(3) replace f_{ijk} by variables y_{ijk}.

$$(4^{**}) \quad \sum_{r=1}^{k-d} f_{ijr} \leq \sum_{r=1}^{k} y_{ijr} \leq \sum_{r=1}^{k+a} f_{ijr} \qquad \left\{ \begin{array}{l} i = 1,2,..m \\ j = 1,2,..n \\ k = 1,2,..T \end{array} \right\}$$

FORMAL DISCUSSION ON

HELLER'S PAPER
and
HUGHES' PAPER
by

Martin Shubik
Princeton University

I will discuss Dr. Heller's paper first. Dr. Heller told me just a while ago that this paper is envisaged as one of a series, and he trusts that as he can develop this topic, some of the extra conditions needed for models of transportation will be considered. Hence, my remarks, which will point out some features that would be desirable to have in models, should not be construed as a negative criticism of the previous paper.

As many of you here know, this study on transportation has been founded on the work of Koopmans, Reiter, Dantzig, and others. It is in an eminently practical field, because the problems of foreign aid supply, and military mission supply, fit very well into the conceptual framework given.

We can discuss four aspects of the paper. They are the mathematical formulation and proofs, the economic relevance of the work, the logistic and operational applications, and the problems of computation and the possibilit of obtaining data.

The transportation problem, as presented, has been a skillful formulation of various aspects of transportation as a minimization of a linear form, subject to various constraints.

It is of interest to note that although the work has been presented from the point of view of programming or activity analysis, much work has been done on a very closel allied problem, looked at from the point of view of geomet: In fact, there is a fairly classical problem, I believe, first formulated by Gauss many years ago, which is now known as the Traveling Salesman Problem. I hope that after my comments, Professor Menger may mention some of the work done on this problem.

As far as the economic relevance of this programming work is concerned, Koopmans has already pointed out the analogy between the shadow pricing system that we can obtain from this sort of model, and the pricing system that would exist in a competitive industry. He has even pointed out that he thinks that the tramp steamer industry of the Twenties actually provided a realistic example. I do not wish to pursue this aspect any further.

The third point I believe is of the most interest to us here -- that is, logistics and operational applications.

The last war, and the present situation in Korea, and for that matter, all over the world, have brought up problems of coordination and centralized shipping. These problems have been associated with a centralized body dealing with a mission. Hence the models presented, which deal with supplying points, or shipping points, and receiving points, provide a far better model of military or over-seas aid missions than they do of international trade, because with missions, much of the shipping is unidirectional. With international trade there is the problem of trying to get a pay-load going in both directions at the same time.

We have seen that experience has emphasized the importance of shipping limitations, and that maximal routings are not intuitively obvious. This is, I believe, why we have to go to the level of abstraction, pointed out in the programming work. We can examine the assumptions and conditions on the models put forward by Professor Heller. He has uniform shipping and one commodity. He introduces a consideration of time periods. He allows the purchase and sales of ships, then considers a lee-way in scheduling periods. He does not put in any limits on on port facilities -- as Koopmans incidentally does.

It is hard to say how reasonable a condition uniform shipping is, at this time, for an applied model for some actual situation. Possibly in studying a gasoline or petroleum program, where we have only two or three major ships, it would not be unreasonable to enlarge the model to take care of these. However, it would be difficult.

The assumption of one commodity is made. One commodity can be interpreted in practical terms as the old apples and nuts aggregation proposition, and could possibly be looked at as a mission ton, if we had a mission. Then if we agreed not to look at the results, we might be able to talk about a mission ton, and report our deliveries in one aggregated figure, which we hope is homogeneous.

So it is possible that the conception of one commodity in Dr. Heller's model is not a bad limitation at the present time. He does introduce time periods explicitly, and he points out the cost of introducing time periods in the way it affects computation.

His basic model has the nice property of introducing ship purchasing and the possibility of getting rid of ships.

In a purely abstract model of programming, this certainly is a very nice generalization of the transportation problem, but we must ask ourselves where our major interest lies, in eventual application. If we are interested in long term planning, or in relatively short term planning, I feel that many problems faced are such that the purchase and getting rid of ships happens to be a very, very heavy bounding condition. Very often you have just got the number of ships, and you can't get any more ships, and the question is, "What can you do with them at this time?"

Therefore, although from the point of view of formulation, I find the model very useful and very stimulating in advancing the technique to take care of something that was not done before, for application purposes, I do not think that this is too vital.

Dr. Heller's last point I found very interesting, and believe to be very important. That was the introduction of lee-ways into the program, because, as we all know, if everything ran according to the book, we could all pack up and go home. It really is the foul-up in any system that forces us to think hard. As such, the introduction of flexibility into a programming model, I believe, is very desirable, both from the shipping and from the mission point of view. I would have liked to have seen Dr. Heller go on and develop a probabilistic model, which I think has much to merit it, both from the viewpoint of shipping and mission.

I say this because when you have the problem of getting shipping across the North Atlantic or Pacific in wartime, you have to add in the question of probable loss or alternate routes vs the costs. In the war we had an example of this, in deciding whether or not to send convoys via the South Atlantic route or the North Atlantic route -- and this was also an economic problem that could have been phrased, as I believe the operations research people did know a fair amount about the probabilities of a ship being sunk in one area or another. Hence, this could have been introduced explicitly.

In conclusion, I feel that Dr. Heller's model has done a great service in introducing the lee-way, and I do hope he will develop the probabilistic model. I do not know how the computation of a useful problem would be. I think that the availability of data should not be too bad, because it has been my impression that the transportation problem offers one of the easier data gathering problems that we are faced with in Naval Supply.

If we are to regard activities analysis as a tool of executive control, I think that it will eventually be necessary to be able to compute these various programs within hours, or at most, within days. I believe that situations change so much that although we may be able to formulate problems, unless we can compute immediately and use the results for very short term planning, much of the effect will be swept away by our having a model that tells us how we could have been right on this plan or that plan six months after the plan has become obsolete.

So, all in all, Dr. Heller has offered us a formulation whose theoretical framework and whose importance I believe merit to be teste. empirically as soon as we can get around to handling such si:eable quantities of data as will be needed.

I hav: a feeling that in all of this work the mathematical formulation can only be tested against operations. The mathematical formulation may be correct, but the proof of the pudding will be in the eating later.

I would now like to turn to the paper by Dr. Hughes. Dr. Hughes brings up many of the practical problems involved in estimating and gathering data for the kind of transportation program envisaged in the paper of Dr. Heller. The problem looked at by Dr. Hughes is that of finding what we need in the way of tonnage estimates, the mission aspect of Dr. Heller's problem.

In Dr. Hughes' presentation, the old three-in-one problem of naval statistics comes up. That is, information, accuracy, and aggregates. The problem faced by all people working in programming is to start with present information -- to put it in the words of Old Bill, for want of a better "ole" to go to, they have to examine it and see what useful conclusions can be drawn, and then attempt gradually to improve it and set up adequate statistical gathering procedures.

Dr. Hughes examines the average cube per man, per month. As we saw, he tested the two month extrapolation to see how well we can guess next month's tonnage from the previous two months. He then asked, "What else do we have available at the present moment that we can use to improve our methods?" And he talked about the information on Demand documents, number of lines, priority dates, cube measures. He also mentioned other information, such as cancellations.

The problem of line item per cube measure, which was mentioned, is an old aggregation chestnut.

Dr. Hughes' analysis, and work done by others, including Mr. Young, of Princeton, and Isenberg, of the Logistics Research Laboratory at Bayonne, all indicate that we just must have finer breakdowns. The solution to obtaining a finer breakdown, as is the case with most of these data problems, involves much hard leg work, combined with fairly careful design of statistical procedures to cut down the mountains of work that would otherwise be faced.

To be frank, I think that most people do not know whether or not one should disaggregate the overall line item per cube, per man measure.

Possibly there are some people in this audience who have looked at a sufficiency of different methods of trying to break up the apples and nuts problem, and do have some very definite suggestions based on a decent enough collection of data that has been analyzed in a careful manner.

Dr. Hughes has actually presented us with at least some analysis of the relationship between items and cube measure. He has pointed out that we can use estimations of cancellation, distribution or priority data on demand documents, to help tidy up some of our purely numerical guesses as to what the tonnage is going to look like in the next period. This brings up the question of looking for aggregative numbers -- in other words, finding a few numbers with which we can represent a complex situation.

What we are really looking for is to be able to add another set of "variable constants" to the planner's bag of tricks. We, of course, know that in any of the figure-gathering activities we indulge in, the figures are only as good as the bounding conditions placed on them -- and the bounding conditions change.

My own experience seems to indicate that great care to keep an accurate verbal history of changes in the statement of missions, and the status of various supply points and some of the major order trickeries that are taking place, should go along with any numerical work that is being done, because I do feel that a verbal report to go along with figure processing can let you throw out great quantities of figures that are no longer relevant, and can also let you know whether the establishment you are talking about is still the same sort of animal.

When we look, for instance, at a supply depot at two different periods in its history, we may be able to say, by gathering all the figures that we have on it, that it seems to have become vastly more efficient, or vastly more inefficient than it was two or three years previously. In the meantime, if we had cared to have gotten a statement of the change of mission which may still be leaving the same volume of measurement tons, or something else, pouring through, at both times, we would have found out that the nature of the operation has changed. Things such as technological change in transportation, for instance, the difference in the trucking industry at the moment and the trucking industry several years ago, may have had great effect. Such like bounding conditions have to be tied in somewhere with any numerical analysis that we do.

It is very easy to make cynical and facile statements concerning lies, damn lies, and statistics. The fault in statistical analysis usually lies with the people who try to read into figures things which are simply not there, because the reporting method and the models involved do not account for the many qualifications that have to be made before using. I feel that Dr. Hughes stresses the magnitude and the delicacy involved in the statistical processing methods.

I would like to conclude with a few observations on Navy statistics. I think that in previous times we lacked people with the prerequisite scientific training. By scientific training, I mean more an attitude and a general approach than ability to turn diagrams upside down.

The desire for a single number to be the answer to every question has been another bugbear that has just got to go by the board. Admittedly, at a high enough Command level you have to have a single number, because there is not time to operate with great arrays, but I still believe that it would be a very desirable thing to have pages on the shelf that maybe never get looked at, that contain the justifications for that single number, and the disaggregations of the hundred or the thousand numbers that went into getting the one monster fudge factor.

Another factor tied in with this problem, of course, is the sheer lack in most cases of anybody quoting a variance on any figure. If you give someone a figure, a figure is a figure, and that is it. It is a good figure. Say it is ten. This does not mean nine and one-half. It does not mean ten and one-half. It usually means somewhere between three and twenty-five.

It has been my sad observation that there is a tremendous lack of continuity in the gathering of Naval statistics. It is hard to get decent time series, and sometimes we need decent time series. I believe it pays to have continuous programs going, rather than let everything go to pieces for a number of years, then put in a tremendous gathering process whereby everybody's files get crammed for a couple of years after which nobody can remember why all the work is being done, and everything gets thrown away until the next spasmodic drive.

I, for instance, find that I can, say, get the inter-depot tonnage figures from Clearfield for part of 1943, all of 1948, and some of 1952. I just mention that as being rather an interesting example of the lack of continuity.

I am possibly wrong, but it seems there exists three types of statistics in the Services. They are what I would call the financial statistics, the Courts martial statistics, and the operational statistics.

The financial statistics are the ones that the accountants and the various other financial people thrust upon us -- and they have a long and an honorable history. You can usually still get information about an extra ten cents that was spent in the time of John Paul Jones. The statistics certainly are good, and certainly do provide material whereby one can get gross aggregates such as costs per ton of something, over a period. These are essentially dead figures, and frankly, I do not know how useful they are for good planning. However, they certainly are better than nothing.

The Courts martial statistics, as I call them, are not kept officially, but if one goes around a depot one usually finds that any person who has been in a position of any influence for some time is keeping his own dossier either mentally or in a small black book. I know there are certain laws on keeping statistics, however, an operator knows that some day, if something goes wrong, he may have to give the real reason why it went wrong, even though his measurement ton for line item figures were favorable on that particular month. So he often keeps good sets of figures.

The other, and these are the figures that we are essentially interested in, happen to be operational figures. Operational figures are the figures that people in various sub-sections have to have if they are going to run their operation. These are the figures which occur in great disaggregation in every issue control, or in any other section you want to mention, held by the operating personnel who have to do a job tomorrow. These are the figures that we try to gather into bigger reports, then put them into even higher reports, and hope to filter through to the top where the planners hope to get aggregated figures in order to be able to work.

Of course it has only been recently that we have tried to go down to the operating levels to see how far we can proceed with aggregating until we have but few figures at the top reporting levels, but those few figures are meaningful.

I feel that cooperation between operators, mathematicians, statisticians and economists may eventually make a permanent, continuing statistical processing organization possible for the Navy. I do not think that it can be built over night, and I think it will take very many years, but I feel that it has to be done. I feel that we have more linear programming theory at the present moment than we can apply, although it is certainly an obviously good thing to see the theory program go even further.

At the present time our major needs are statistics and statistical processing techniques, to be able to do justice to the activities analysis theories at our disposal.

The intangibles in Navy planning will be sufficiently great for many years to come, that the most that we can hope for is to be able to establish a satisfactory machinery which is capable of taking advantage of those aspects of the modern supply system which can, we hope, be reduced to objective computation.

SUMMARY OF GENERAL DISCUSSION ON

HELLER'S PAPER
AND
HUGHES' PAPER

Professor Menger explained that he had not treated Dr. Heller's problem geometrically, as Mr. Shubik had stated in his discussion. Professor Menger discussed the manner in which we might attempt to geometrize the tanker scheduling problem, and compared the problem with the travelling salesman problem.

Professor Tucker mentioned a similar problem, the so-called optimal assignment problem. This problem had recently been treated by Professor von Neumann, and shown by him to be equivalent to a matrix game problem. Professor Tucker stated that problems of this general type were of interest from the computational standpoint, and asked Dr. Hoffman if he, as the authority on comparison of the various well known computation methods, would care to make any remarks concerning them. And, finally, as a rejoinder to Mr. Shubik, Professor Tucker asked if we can have too much theory.

Mr. Shubik agreed to the value of theory. Dr. Hoffman then mentioned that the size of the problem (for example, number of variables and accuracy demanded) would dictate the answer to Mr. Shubik's question if Dr. Heller's problem could be attacked effectively by modern computing machinery. In response to questions of Dr. Tompkins and Dr. Hughes, Dr. Hoffman did state that the general computational methods available did apply to Dr. Heller's problem.

Dr. Hoffman stated that two trends of computing in the field of linear programming exist: one, the development of methods which are applicable to general problems; the other, the development of special methods tailored to the efficient solution of special, important problems. In this connection, he pointed out the difference between the Simplex and the Double-description methods, the former having proven reasonably outstanding for pure linear programming computations; the latter being superior if one wishes to obtain the set of all solutions to a set of linear equalities.

The remaining discussion of Dr. Heller's paper, by Dr. Heller, Mr. Shubik, Professor Morgenstern and Dr. Tompkins, centered about mathematic details of the paper, for example, the artifice of "source" and "sinks" for ships, the realism of the model and the existence of statistical data in the Navy's records. With regard to the last point, Dr. Tompkins felt he could paint a rosier picture than that envisaged by Mr. Shubik by mentioning the Navy's comprehensive fuel consumption data, the data which had been available to Admiral Denebrink, as COMSERVPAC, as illustrated by his charts and the current material consumption data of the Logistics Research Project. With respect to the realism of the model, Dr. Heller stated that it had resulted from about $1\frac{1}{2}$ years experience with an actual problem, the number of tankers involved and the general condition being such that the model appeared to be applicable.

The discussion was turned to Dr. Hughes' paper by questions of Commander Bennett. Commander Bennett inquired if anyone would care to comment on the utility of the measurement ton per man per month type of long range planning, and the effect of varying lead times, as applicable to material not-in-stock, or the short-range planning required for effective immediate operations.

Dr. Hughes explained that the key purpose of his paper was to treat the question raised by Commander Bennett concerning the short-range forecasts. While he had not suggested a specific, complete method to be followed, he hoped he had at least pointed to the highway to follow. The combined use of past trends, exact or statistical information as to method of shipment, date of arrival at tidewater and weight and cube data--all this information being assembled and interpreted for each item--would seem to provide a basis of forecasting shipping requirements with a good chance of being accurate.

Admiral Eccles discussed the assumption that movement of Naval vessels in wartime is predictable, and he told about the problems associated with fleet freight. There is a school of thought amoung Naval Officers that in time of war shipment of fleet freight must be stopped and individual requisitioning from ships outside the continental limits be therefore banned. The disposition of the fleet-freight concept may have a profound effect on certain shipping plans and may affect the question of the unit load.

Dr. Hughes stated that discontinuance of fleet freight, and individual requisitinning by ships outside the continental limits, would simplify the problem of estimation and forecasting of tidewater shipping requirements, because then the demand documents would originate at Bureau or Staff level, where presumably, there would exist complete knowledge. Furthermore, use of unit loads would also tend to facilitate forecasting of requirements. In fact, Dr. Hughes thought that such changes could only improve the situation considered in his paper, and, therefore, tend to make more accurate the estimations or forecasting he had discussed.

Dr. Hughes closed his remarks during the discussion period with a statement of the need for an overall evaluation of record-keeping systems of the Navy, taking into account the value of records not only for operational purposes, but also for control, which he felt, implied research purposes.

INTRODUCTION TO THURSDAY SESSION ON
THEORY OF GAMES
by
Chairman Professor Oskar Morgenstern
Princeton University

Ladies and gentlemen, this opens the last day of the conference, and we are turning to a new topic entitled, "Theory of Games". Of course, by this is meant "games of strategy". I might mention the first person who had a clear notion of what games of strategy are, and that they exist, and that a theory could be made of them, and such a theory would be most interesting, and furthermore such a theory would be an excellent model for social, economic, and military situations and conditions. That was Leibnitz, the German philosopher and mathematician, who wrote about games in 1710. He even had a clear idea that there is such a thing as pure strategy, and he actually contributed something to the solution of a particular problem with this idea of a pure strategy.

Furthermore, Leibnitz even went so far as to see that games of strategy might be used for the construction of a maneuver board of a fleet. That is rather interesting. And another notion he had, all in the same paper--so you see it is quite packed with information--was that the situation of a statistician vis à vis nature corresponds to that of a game of strategy, an idea which has been developed by A. Wald. The real step in game theory was taken by the establishment of the min max theorem by John von Neumann, who is the originator of the theory of games of strategy, and who has developed it. However, since the main work--his first paper was in 1928, and then his book, "The Theory of Games and Economic Behavior"--appeared in 1944 much other work has been done. The whole spectrum of the works can be seen in the two volumes entitled, "Contributions to the Theory of Games", Volume One, in 1950, and Volume II in 1953, edited by Professors Kuhn and Tucker.

Today we will hear first from Professor Kuhn, who has made a personal sacrifice by leaving his home at 4:00 o'clock this morning in order to get here from Bryn Mawr College. We are very pleased to have him talk on "The Solution of Games by Behavior Strategies".

Dr. Kuhn.

THE SOLUTION OF GAMES BY BEHAVIOR STRATEGIES*

by

H. W. Kuhn
Bryn Mawr College

(NOTE: In addition to the computational problems discussed in this paper, an attempt has been made to provide an informal introduction to the concepts and technical terminology of games in extensive form. Accordingly, an expositional appendix is provided for those readers new to this subject.)

INTRODUCTION

The process of normalization, in the sense of von Neumann and Morgenstern, replaces an arbitrary finite zero-sum two-person game by a matrix game with the following rules:

(1) Player A chooses an integer i from the set $1, \ldots, s$.

(2) Player B, in ignorance of A's choice, chooses an integer j from the set $1, \ldots, t$.

(3) Player B pays player A the amount a_{ij} .

In this normalization, each row i of the game matrix corresponds to a pure strategy for A, each column j corresponds to a pure strategy for B, and the payoff a_{ij} is defined to be the expected payoff to A if the players use these strategies in the original game as defined by its rules.

*The preparation of this paper was sponsored by the O. N. R.

I would like to propose a new matrix form, called a <u>split matrix game</u>, for an arbitrary finite zero-sum two-person game that represents such a game by a prototype with the following rules:

(1) A chance device C chooses an ordered pair of integers (k, χ) where $k = 1, \ldots, S$, and $\chi = 1, \ldots, T$. The ST possible pairs are equally likely.

(2) Player A, informed of k only, chooses an integer i from the set $1, \ldots, s_k$.

(3) Player B, informed of χ only, chooses an integer j from the set $1, \ldots, t_\chi$.

(4) Player B pays player A the amount $STa_{ij}^{k\chi}$.

The advantages of the split matrix form, as will be developed in the following, derive from the possibility of a direct application of behavior strategies to the matrix to yield a bilinear form to which the customary computational procedures can be applied. The particular computational method examined is that of "fictitous play" as developed by George Brown.

To show how the matrix entries of the split matrix game are defined in terms of the given game in extensive form, it is necessary to use some of the technical terminology of extensive games. This is done in what seems to be the most painless manner in Section 1 by reducing an example of an extensive game to the split matrix form. This is done with full recognition of the risk of exposing the essential simplicity of the whole idea that might be concealed in a more general context and with the full mathematical symbolism. In Section 2 the application of behavior strategies and the method of fictitous play is made to split matrix games with a computational example, while in Section 3 the possibilities of the general application of the split matrix form are discussed.

1. AN EXAMPLE

The game to be reduced to split matrix form in this section is called Simplified Poker; it is Example 3 in the Appendix and the notation in the following refers to the figure to be found there. Every instance in which this game is played from beginning to end, a play in game parlance, starts with a chance event (labeled C_1) at which one of six equally likely alternatives is chosen and continues out along a path to its end. The possible occasions of something happening in this game are called positions and are represented by vertices. They are of three possible types with the chance event just mentioned comprising the first type. Secondly, there are the personal moves in which the rules require a player to choose one from a number of alternatives. We label moves for A by A_1, ..., A_6 and moves for B by B_1, ..., B_4. Finally, there are the play-ends where the rules of the game terminate the play and specify the payoff to the players. In accordance with the usual convention for zero-sum two-person games, where what one person wins the other loses, we label each play-end with the amount that B pays to A. Finally, this game does not have perfect information in that a player is not always completel informed of the precise course of play preceding each of his moves. This is a common enough phenomenon in card games where a player is seldom informed of the exact outcome of the deal. We can add these data to our diagram through the medium of information sets containing moves for a player that seem identical to him according to the information at his disposal, while moves in different information sets can be distinguished. We denote information sets for A by $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3$ and set them off by enclosing their moves in dotted curves. In like manner, information sets for B are denoted by $\mathcal{B}_1$ and $\mathcal{B}_2$. Notice that all of the moves in an information set must have the same number of alternatives, since otherwise a player could distinguish between moves with a different number of alternatives. Moreover, it is important to realize that he is presented with one set of alternatives at an information set although the effect of the choice of an alternative will differ according to the move that is actually occuring. A useful convention is to index the alternatives at a move in counter-clockwise order around that move. This completes the description of the game.

Since the representation of a game by such a game tree may be unfamiliar, we will state explicitly how it is played in this form. Each player is assumed to know the rules of the game, that is, to have a copy of the game tree. All plays start at C_1. At this chance event, an alternative is chosen in accordance with the given probabilities (say by throwing a single die, then choosing the alternative that corresponds to the face that appears). If the play thus progresses to the move B_1, then player B is informed that he is information set $\mathcal{B}_1$ and asked to choose alternative 1 or 2. If the alternative chosen is 1 then the play progresses to A_4 and player A is given a choice, told only that he is information set $\mathcal{A}_2$. If he chooses alternative 2, then we reach the play-end of this play which is labeled with -1 which means that A is to pay one unit to B.

We are now in a position to describe the von Neumann and Morgenstern normalization which is accomplished through the introduction of <u>pure strategies</u> for the players. A pure strategy for A consists of a combined choice of one alternative for each situation that might confront him, that is, of one alternative for each of his information sets. Thus, a possible pure strategy for A is the choice of 1 on $\mathcal{A}_1$, of 2 on $\mathcal{A}_2$, and of 1 on $\mathcal{A}_3$. We would denote this by $\alpha = [1,2,1]$. In like manner, a possible pure strategy for B is the choice of 2 on $\mathcal{B}_1$ and of 1 on $\mathcal{B}_2$, denoted by $\beta = [2,1]$. With these pure strategies, it is a simple matter to calculate the probability that each play-end will occur and hence the expected payoff to A. This achieves the von Neumann and Morgenstein normalized form of this game; we merely list the pure strategies for A and B as the rows and columns of a matrix, then calculate the expected payoff for each pair to obtain the game matrix, which is given below.

GAME MATRIX FOR SIMPLIFIED POKER

	[1,1]	[1,2]	[2,1]	[2,2]
[1,1,1]	-1/6	1/6	-1/3	0
[1,1,2]	-1/6	1/3	-1/2	0
[1,2,1]	-1/2	-1/6	-1/6	1/6
[1,2,2]	-1/2	0	-1/3	1/6
[2,1,1]	1/6	0	0	-1/6
[2,1,2]	1/6	1/6	-1/6	-1/6
[2,2,1]	-1/6	-1/3	1/6	0
[2,2,2]	-1/6	-1/6	0	0

To obtain the split matrix form of this game we must introduce mixed strategies. A <u>mixed strategy</u> for player A is an assignment of a probability to each of his pure strategies, and thus is a vector

$$X = (x[1,1,1],\ x[1,1,2],\ \ldots,\ x[2,2,2])$$

with eight non-negative components $x[\alpha]$ that sum to one. In like manner, a mixed strategy for B is a vector

$$Y = (y[1,1],\ y[1,2],\ y[2,1],\ y[2,2])$$

with four non-negative components $y[\beta]$ that sum to one. In each play of the game a player will choose his pure strategy according to the probability distribution thus defined. The expected payoff is then easily calculated from the game matrix as

$$\sum_{\alpha,\beta} x[\alpha]y[\beta]a_{\alpha\beta} \quad .$$

However, if we go back to the game tree to calculate this payoff rather than taking it from the game matrix, an interesting fact emerges. The payoff is just the sum

$$\sum_{E}\ \sum_{\alpha,\beta} x[\alpha]y[\beta]\,p_{\alpha\beta}(E)h(E)$$

where $p_{\alpha\beta}(E)$ is the probability that the play-end E occurs if the players use the pure strategies α and β and $h(E)$ is the amount that B pays A if the play-end E occurs. Directing our attention to the terms of this sum corresponding to a given play-end, we find that the occurrence or non-occurrence of that play-end depends upon the choices of A and B at no more than one information set. That is, exactly those pure strategies for A that choose the unique alternative for A on the play and those pure strategies for B that choose the unique alternative for B on the play (if any) appear in the sum. Thus, if we make the substitutions

$$q[k,i] = \sum_{i_k = i} x[i_1,i_2,i_3] \qquad (k = 1,\ 2,\ 3;\ i = 1,\ 2)$$

$$r[\gamma,j] = \sum_{j_\gamma = j} y[j_1,j_2] \qquad (\gamma = 1,\ 2;\ j = 1,\ 2)$$

we find that the payoff is bilinear in these variables. Notice that $q[k,i]$ measures the probability that A will use a pure strategy that chooses alternative i on his information set $\mathcal{A}_k$ and $r[\chi,j]$ measures the probability that B will use a pure strategy that chooses j on his information set $\mathcal{B}_\chi$. In terms of these variables the expected payoff is

$$-1/3q[1,1] - 1/3q[1,2] - 1/3q[2,1] - 1/6q[2,2] + 1/6q[3,1] + 1/6r[1,2]$$
$$-1/3q[1,1]r[2,1] + 1/6q[1,1]r[2,2] + 1/3q[2,1]r[1,1] - 1/6q[2,2]r[1,1]$$
$$+1/3q[3,2]r[1,1] + 1/6q[3,2]r[2,2] + 1/3q[3,1]r[2,1] + 1/6q[3,1]r[2,2].$$

We can fill out the "short" terms that are missing a variable $q[k,i]$ or $r[\chi,j]$ by using the fact that

$$\sum_i q[k,i] = 1 \text{ and } \sum_j r[\chi,j] = 1 .$$

By using these devices consistently, we obtain as the expected payoff

$$-1/3q[1,1]r[2,1] + 1/6q[1,1]r[2,2] - 1/6q[2,1]r[1,2]$$
$$-1/3q[2,2]r[1,1] + 1/6q[3,1]r[2,1] + 1/6q[3,2]r[1,1] .$$

This information can be given by the following matrix:

SPLIT GAME MATRIX FOR SIMPLIFIED POKER

	[1,1]	[1,2]	[2,1]	[2,2]
[1,1]	0	0	-1/3	1/6
[1,2]	0	0	0	0
[2,1]	0	-1/6	0	0
[2,2]	-1/3	0	0	0
[3,1]	0	0	1/6	0
[3,2]	1/6	0	0	0

This is the split matrix form announced in the introduction; the rules of this normalized form are:

(1) A chance device C chooses an ordered pair (k,χ) where $k = 1, 2, 3$ and $\chi = 1, 2$. Each pair occurs with probability $1/6$.

(2) Player A, informed of k only, chooses $i = 1$ or 2.

(3) Player B, informed of ℓ only, chooses $j = 1$ or 2.

(4) Player B pays player A the amount $6a^{k\ell}_{ij}$, where $a^{k\ell}_{ij}$ is the matrix entry in the row (k,i) and the column (ℓ,j) above.

Summarizing the results of this section, we have obtained two normalized forms for the given game in extensive form. They are equivalent in the strong sense that they have the same sets of pure strategies and the same payoffs. (The reader should apply the definition of pure strategies to the above rules of the split matrix game to see that this is indeed the case.) Comparing the two matrices one remarks immediately the simpler data of the split game matrix. In a later section we shall see how this comes about and give quantitative estimates of the simplification.

2. BEHAVIOR STRATEGIES AND FICTITOUS PLAY

A player in a game can randomize his actions in one of two ways: by mixed strategies, already described, or by behavior strategies. When using behavior strategies, rather than using a chance device to choose a complete plan before each play of the game, a player uses a chance device at each information set to decide which alternative to play at that information set. It is my conviction that behavior strategies are much closer to the way people actually play games than are mixed strategies. This is clearly the case for games such as Poker or Gin Rummy, where players do not formulate complete plans, much less assign probabilities to complete plans, but rather play by intuitive rules that randomize at specific situations. No Poker player can tell

With this payoff defined, we consider behavior strategies in the split matrix game

$$(3)\quad \begin{aligned} q[k,i] &= \sum_{i_k=i} x[i_1, \ldots, i_S] \quad (k = 1, \ldots, S;\ i = 1, \ldots, s_k) \\ r[\chi,j] &= \sum_{j_\chi=j} y[j_1, \ldots, j_T] \quad (\chi = 1, \ldots, T;\ j = 1, \ldots, t_\chi) \end{aligned}$$

If the players use the behavior strategies thus defined, the expected payoff to A is

$$\sum_{i,j,k,l} a_{ij}^{k\chi}\, q[k,l]\, r[\chi,j]$$

$$= \sum_{i,j,k,l} a_{ij}^{k\chi} \left(\sum_{i_k=i} x[i_1, \ldots, i_S] \right) \left(\sum_{j_\chi=j} y[j_1, \ldots, j_T] \right)$$

$$= \sum_{\alpha,\beta} a_{\alpha\beta}\, x[\alpha]\, y[\beta]$$

and thus the substitutions (3) provide a mapping of the mixed strategies in the matrix game into the behavior strategies in the split matrix game that preserves the payoff. The simplest way to see that every behavior strategy is obtained by this mapping is to exhibit an "inverse" mapping:

$$(4)\quad \begin{aligned} x[i_1,\ldots,i_S] &= q[1,i_1] \ldots q[S,i_S] \quad \text{for all } \alpha = [i_1,\ldots,i_S] \\ y[j_1,\ldots,j_T] &= r[1,j_1] \ldots r[T,j_T] \quad \text{for all } \beta = [j_1,\ldots,j_T]. \end{aligned}$$

In summary, to every split matrix game there is a corresponding matrix game defined by (2) such that (3) provides a many-one mapping of the mixed strategies in the matrix game <u>onto</u> the behavior strategies in the split matrix game that preserves the payoff. (This mapping appeared with different motivation in Section 1.) This assures us that the game will have a solution in behavior strategies since we know that there is a solution in mixed strategies.

THE SOLUTION OF GAMES BY BEHAVIOR STRATEGIES

Considering possible computational procedures to use to solve a split matrix game with payoff given by (1), a method that comes naturally to mind is that of fictituous play devised for matrix games by George Brown and shown to converge by Julia Robinson. It is motivated by the observation that people improve their strategies (if so they may be called) in actual play by adjusting to the replies of their opponents. This is translated into a computational technique as follows:

FICTITIOUS PLAY IN A MATRIX GAME

The procedure is iterative and can be started with any mixed strategy with components $x_1[\alpha]$ for player A. If the strategy for A at the nth step has components $x_n[\alpha]$ we let $\bar{\beta}$ be any pure strategy for B such that

$$(5)\quad \sum_\alpha a_{\alpha\bar{\beta}} x_n[\alpha] = \min_\beta \sum_\alpha a_{\alpha\beta} x_n[\alpha]$$

and define

$$(6)\quad \begin{aligned} y_n[\beta] &= (n-1)y_{n-1}[\beta]/n \quad \text{for } \beta \neq \bar{\beta}, \\ y_n[\bar{\beta}] &= ((n-1)y_{n-1}[\bar{\beta}] + 1)/n . \end{aligned}$$

Then, if $\bar{\alpha}$ is any pure strategy for A such that

$$(7)\quad \sum_\beta a_{\bar{\alpha}\beta} y_n[\beta] = \max_\alpha \sum_\beta a_{\alpha\beta} y_n[\beta] ,$$

we define

$$(8)\quad \begin{aligned} x_{n+1}[\alpha] &= nx_n[\alpha]/(n+1) \quad \text{for } \alpha \neq \bar{\alpha}, \\ x_{n+1}[\bar{\alpha}] &= (nx_n[\bar{\alpha}] + 1)/(n+1) . \end{aligned}$$

The remarkable fact is that this procedure can be carried out equally well on the split matrix form of the game. Namely, a pure strategy for a player in a split matrix game is nothing more or less than the choice of one index for each block. If we are searching for the pure strategy that is the best counter strategy to a given mixed strategy for our opponent, we can hunt for the best counter index in each block since the blocks are independent of each other in the payoff. Moreover, we can record our cumulative strategy as a behavior strategy due to the equivalence established by (3) and (4) above. To fix this procedure precisely, we give its general formulation and then apply it to the example of Section 1.

FICTITOUS PLAY IN A SPLIT MATRIX GAME

The procedure is iterative and can be started with any behavior strategy for player A with components $q_1[k,i]$. If the strategy for A at the nth step has components $q_n[k,i]$ we choose $j_\lambda(n)$ such that:

$$(9) \quad \sum_{i,k} a^{k\lambda}_{ij_\lambda(n)}\, q_n[k,i] = \min_j \sum_{i,k} a^{k\lambda}_{ij}\, q_n[k,i] \quad \text{for } k = 1,\ldots,S,$$

and define

$$(10) \quad \begin{aligned} r_n[\lambda,j] &= (n-1)r_{n-1}[\lambda,j]/n \quad \text{for } j \neq j_\lambda(n) \\ r_n[\lambda,j_\lambda(n)] &= ((n-1)r_{n-1}[\lambda,j_\lambda(n)] + 1)/n\ . \end{aligned}$$

Then, if $i_k(n)$ is such that

$$(11) \quad \sum_{j,\lambda} a^{k\lambda}_{i_k(n)j}\, r_n[\lambda,j] = \max_i \sum_{j,\lambda} a^{k\lambda}_{ij}\, r_n[\lambda,j] \quad \text{for } \lambda = 1,\ldots,T,$$

we define

$$\begin{aligned} q_{n+1}[k,i] &= nq_n[k,i]/(n+1) \quad \text{for } i \neq i_k(n) \\ q_{n+1}[k,i_k(n)] &= (nq_n[k,i_k(n)] + 1)/(n+1). \end{aligned}$$

To make the procedure unambiguous once $q_1[k,i]$ is chosen, we choose $i_k(n)$ and $j_\lambda(n)$ to be the smallest indices satisfying (11) and (9) respectively.

A convenient arrangement of the computation is provided by the tabular form:

n	$i_k(n)$	$[\chi, j]$	$\underline{v}_n$	$\bar{v}_n$	$j_\chi(n)$	$[k, i]$

In this arrangement, the columns labeled $[\chi, j]$ contain the cumulative sums $n \sum_{i,k} a_{ij}^{k\chi} q_n[k,i]$ while the columns labeled $[k,i]$ contain the cumulative sums $n \sum_{j,\chi} a_{in}^{k\chi} r_n[\chi,j]$. The columns labeled $\underline{v}_n$ and $\bar{v}_n$ contain the amounts that A and B assure themselves when using the behavior strategies at the nth step. On page 14 we give the solution of Simplified Poker as an example of this scheme.

COMPUTATION SCHEME FOR SIMPLIFIED POKER

n	$i_1(n)$	$i_2(n)$	$i_3(n)$	[1,1]	[1,2]	[2,1]	[2,2]	$\underline{v}_n$	$\bar{v}_n$	$j_1(n)$	$j_2(n)$	[1,1]	[1,2]	[2,1]	[2,2]	[3,1]	[3,2]
1	2	1	1	0	$\underline{-1/6}$	1/6	$\underline{0}$	-1/6	1/6	2	2	$\underline{1/6}$	0	-1/6	$\underline{0}$	$\underline{0}$	0
2	1	2	1	$\underline{-1/3}$	-1/6	$\underline{0}$	1/6	-1/6	0	1	1	-1/6	$\underline{0}$	$\underline{-1/6}$	-1/3	$\underline{1/6}$	1/6
3	2	1	1	$\underline{-1/3}$	-1/3	$\underline{1/6}$	1/6	-1/18	1/18	1	1	-1/2	$\underline{0}$	$\underline{-1/6}$	-2/3	$\underline{1/3}$	1/3
4	2	1	1	-1/3	$\underline{-1/2}$	1/3	$\underline{1/6}$	-1/12	0	2	2	-1/3	$\underline{0}$	$\underline{-1/3}$	-2/3	$\underline{1/3}$	1/3
5	2	1	1	-1/3	$\underline{-2/3}$	1/2	$\underline{1/6}$	-1/10	-1/30	2	2	-1/6	$\underline{0}$	$\underline{-1/2}$	-2/3	$\underline{1/3}$	1/3
6	2	1	1	-1/3	$\underline{-5/6}$	2/3	$\underline{1/3}$	-1/12	-1/18	2	2	$\underline{0}$	0	$\underline{-2/3}$	-2/3	$\underline{1/3}$	1/3

In this table the cumulative sums that determine $i_k(n)$ and $j_\ell(n)$ have been underlined. The entries $\underline{v}_n$ and $\bar{v}_n$ which equal the value of the game, -1/18 have been circled. The corresponding optimal behavior strategies are:

$q[1,1] = 1/3$ $\qquad r[1,1] = 1/3$

$q[1,2] = 2/3$ $\qquad r[1,2] = 2/3$

$q[2,1] = 2/3$ $\qquad r[2,1] = 1/3$

$q[2,2] = 1/3$ $\qquad r[2,2] = 2/3$

$q[3,1] = 1$

$q[3,2] = 0$

3. CONCLUSIONS

The motivation of this paper was a suggestive remark by Dr. Rigby of the Office of Naval Research (he takes no responsibility for the paraphrase): "Can't one solve games by behavior strategies directly on the game tree? Can't one apply some such method as fictitious play, that seems close to the extensive form of a game, to the direct solution by behavior strategies?". The answer is complicated by the fact that certain games without perfect recall do not have a solution in behavior strategies. Indeed, it is a simple matter to give an intrinsic characterization of the payoff function of an arbitrary n-person game in terms of behavior strategies. By using the trick of filling out "short" terms as applied in Section 1, it turns out to be an arbitrary multilinear function defined on a cartesian product of simplices, where each player controls the variables ranging over one <u>or more</u> of the simplices. The result of applying fictitious play to such a payoff function is not known.

But what of the split matrix form? It was claimed in the Introduction that every finite zero-sum two-person game has such a representation. However, it is obvious that an arbitrary game will not behave as nicely as Simplified Poker, with no play intersecting more than one information set for A or B. However, in general, we look for a decomposition of the game tree into sub-trees, each containing the initial position and such that if it contains one move in an information set for A then it contains the entire information set. It is clear that the choices for A in two subtrees are independent and that the subtrees can correspond to the row blocks $k = 1, \ldots, S$ of the split matrix form.

A decomposition for player B will give the column blocks. Of course, there are many games without such a decomposition and in which the von Neumann and Morgenstern normalization coincides with the split matrix form with $S = T = 1$. However, for games with a chance move as initial position (a deal, say) and perfect recall, the split matrix form is the natural form and presents the following advantages:

(I) The number of parameters involved is a sum rather than a product (for player A, $s_1 + s_2 + \dots + s_S$ rather than $s_1 s_2 \dots s_S$) and hence, in general, is much smaller. The fictitious play scheme proposed takes advantage of this fact.

(II) The split matrix form admits a wider class of solution-preserving transformations than the von Neumann and Morgenstern normal form. In addition to the usual transformation of changing the unit of payoff throughout the matrix, one can add or subtract a constant amount from all of the entries in an individual block. Also one may add a constant vector to a given row block in a fixed column provided the same constant is subtracted from the entries of another row block in the same column. The consistent use of these devices reduced the number of non-zero entries in Simplified Poker considerably and seems to give promise of doing the same in other games.

APPENDIX

1. An Informal Glossary of Terms and Notation.

(The list of terms and notation presented here is an informal introduction to the concepts and technical terminology of games in extensive form. In accord with the purpose and mode of this presentation, the definitions are given in the common parlance of games; however, an attempt has been made to phrase them precisely enough to permit an easy translation into mathematical terms. For the sake of notational simplicity, only zero-sum two-person games have been included in the notation; however, the extension to arbitrary games is obvious and without difficulty. The list breaks naturally into three groups of terms corresponding to the definition and description of a game, the introduction of the concept of strategy, and the study of the effect of information on the solution of a game.)

GAME: A "game" is the object defined by a set of rules and conventions of play.

PLAY: A "play" of a game is a particular instance in which the game is played from beginning to end.

POSITION: A "position" is an occasion in a play; it is defined by (and may be identified with) the sequence of actions preceding that occasion. An arbitrary position will be denoted by Z. Three types of positions may be distinguished by the rules of a game.

> PERSONAL MOVE: A "personal move" is a position in which the rules of the game require a player to choose one from a number of alternatives. Personal moves for players A and B will be denoted by $A_1, A_2, \ldots$ and $B_1, B_2, \ldots$ respectively.

CHANCE EVENT: A "chance event" is a position in which the rules of the game specify the probabilities that a number of alternatives will be chosen by a chance device. Chance events will be denoted by $C_1, C_2, \ldots$.

PLAY-END: A "play-end" is a position in which the rules of the game terminate the play and determine the payoffs to the players. Play-ends will be denoted by $E_1, E_2, \ldots$. At the play-end E, player A pays player B the amount $h(E)$.

INFORMATION SET: An "information set" for player A (B) is a set of personal moves for player A (B) which seem identical according to the information provided A (B) by the rules of the game, while moves in different information sets can be distinguished. All of the moves in an information set have the same number of alternatives. No information set contains two moves on the same play. Information sets for player A will be denoted by $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_m$; the information set $\mathcal{A}_\mu$ will have a_μ alternatives indexed by $\alpha_\mu = 1, 2, \ldots, a_\mu$ where $\mu = 1, 2, \ldots, m$. Information sets for player B will be denoted by $\mathcal{B}_1, \mathcal{B}_2, \ldots, \mathcal{B}_n$; the information set $\mathcal{B}_\nu$ will have b_ν alternatives indexed by $\mathcal{B}_\nu = 1, 2, \ldots, b_\nu$ where $\nu = 1, 2, \ldots, n$.

GAME TREE: A "game tree" is a graphical representation of the possible plays of a game. The positions are represented by vertices; at a personal move or a chance event, edges representing alternatives branch off to new positions. There is a distinguished position where all plays begin and information sets are set off by enclosing their moves in dotted lines.

DESCENDENTS: The "descendents" of a position Z consist of Z and all positions following Z in some play; the set of all descendents of Z is denoted by $D(Z)$ and those which follow Z by the λth alternative at Z by $D(Z; \lambda)$.

RANK: The "rank" of a position Z is the number of choices preceding that position and is denoted by $g(Z)$.

PURE STRATEGY: A "pure strategy" for player A (B) consists of a combined choice of one alternative for each situation that might confront him, that is, of one alternative for each information set. Pure strategies for A will be denoted by $\alpha = [\alpha_1, \ldots, \alpha_m]$; pure strategies for B will be denoted by $\beta = [\beta_1, \ldots, \beta_n]$. The number of pure strategies for A and B is $N_A = a_1 a_2 \ldots a_m$ and $N_B = b_1 b_2 \ldots b_n$ respectively.

MIXED STRATEGY: A "mixed strategy" for player A (B) is an assignment of a probability to each of his pure strategies. Thus, a mixed strategy for A is a set of N_A non-negative numbers $x[\alpha_1, \ldots, \alpha_m]$ which sum to one while a mixed strategy for B is a set of N_B non-negative numbers $y[\beta_1, \ldots, \beta_n]$ which sum to one.

BEHAVIOR STRATEGY: A "behavior strategy" for player A (B) is an assignment of a <u>set</u> of probabilities to the alternatives in <u>each of his information sets</u>. Thus, a behavior strategy for player A is a set of $a_1 + \ldots + a_m$ non-negative numbers $q[\mu, \alpha_\mu]$ where $\mu = 1, 2, \ldots, m$ $\alpha_\mu = 1, 2, \ldots, a_\mu$, and $q[\mu, 1] + q[\mu, 2] + \ldots + q[\mu, a_\mu] = 1$ for all information sets $\mathcal{A}_\mu$. A behavior strategy for B is a set of $b_1 + \ldots + b_n$ non-negative numbers $r[\nu, \beta_\nu]$ where $\nu = 1, 2, \ldots, n$, $\beta_\nu = 1, 2, \ldots, b_\nu$, and $r[\nu, 1] + r[\nu, 2] + \ldots + r[\nu, b_\nu] = 1$ for all information sets $\mathcal{B}_\nu$.

PERFECT INFORMATION: A game has "perfect information" if a player is always informed by the rules of the game of the precise course of play preceding each of his information sets, that is, if each information set contains exactly one move.

EFFECTIVELY PERFECT INFORMATION: A game has "effectively perfect information" if a player always knows all the previous choices of his opponent and knows at least as much as his opponent knew when they made those choices.

SUBGAME: The descendents of a position Z form a "subgame" if all of the players with moves following Z are informed when Z occurs in the play.

POSSIBLE: A position Z is "possible when playing the pure strategy α (β)" if α (β) chooses all the alternatives for A(B) in the course of play preceding Z. This is denoted by $Z \epsilon$ Poss α and $Z \epsilon$ Poss β, respectively.

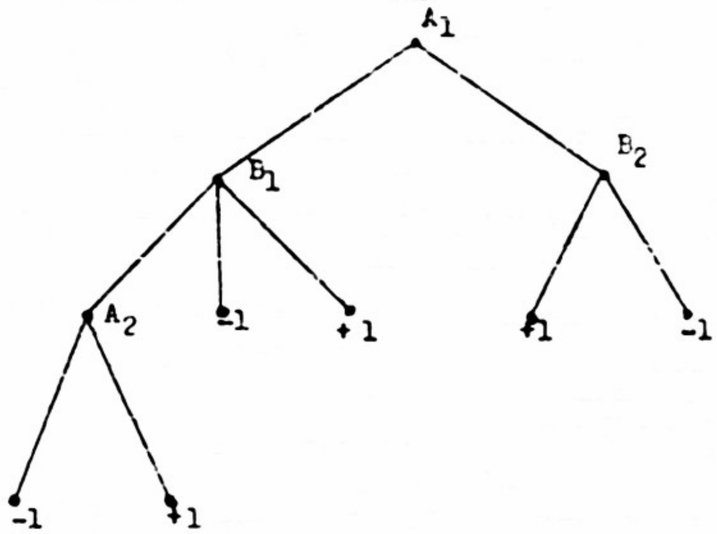
RELEVANT: An information set $\mathcal{A}_\mu$ ($\mathcal{B}_\nu$) is "relevant when playing the pure strategy α (β)" if $\mathcal{A}_\mu$ ($\mathcal{B}_\nu$) contains a move which is possible when playing α (β). This is denoted by $\mathcal{A}_\mu \epsilon$ Rel α and $\mathcal{B}_\nu \epsilon$ Rel β, respectively.

DEFLATION: A game is a "deflation" of another game if, (1) they differ only in the amount of information given to the players by the rules, (2) the players in the inflation are given more information than in the original game, and (3) they are given no more information than they could have deduced through the use of pure strategies.

PERFECT RECALL: A game has "perfect recall" if a player always knows all of his previous choices and knows at least as much as he knew when he made those choices.

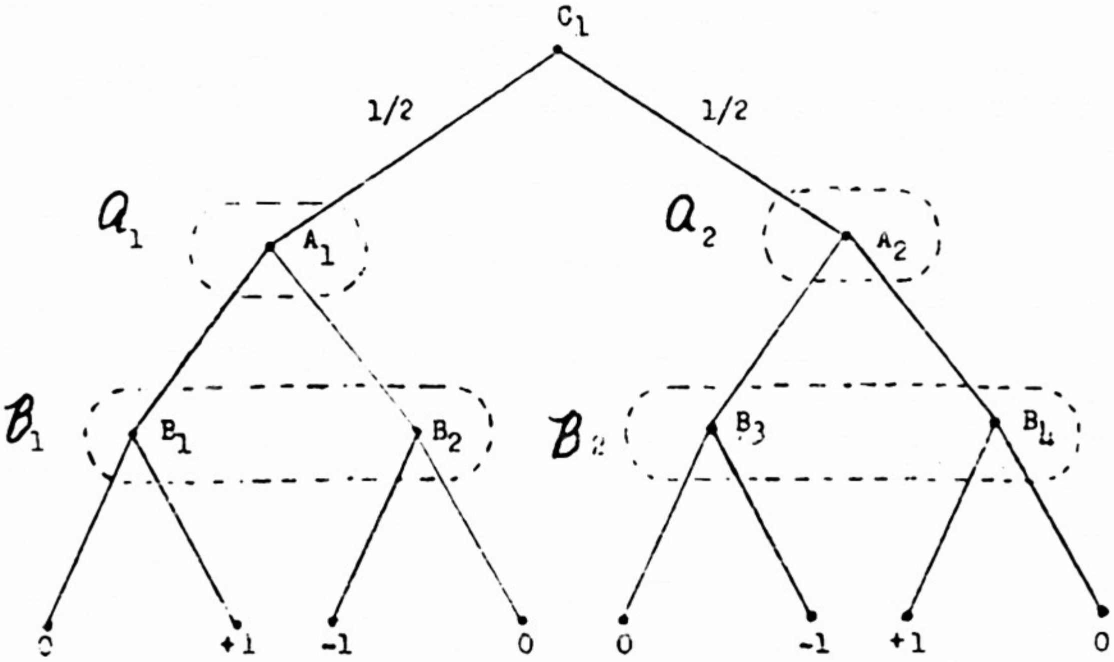
2. Illustrations of Games in Extensive Form.

EXAMPLE 1. NIM



This is a game with perfect information; the rules and solutions of the general game can be found in Hardy and Wright, An Introduction to the Theory of Numbers, Oxford (1938). The game tree above shows the game played with two piles of two matches each with A to move first, with a win for A or B assigned payoff ± 1.

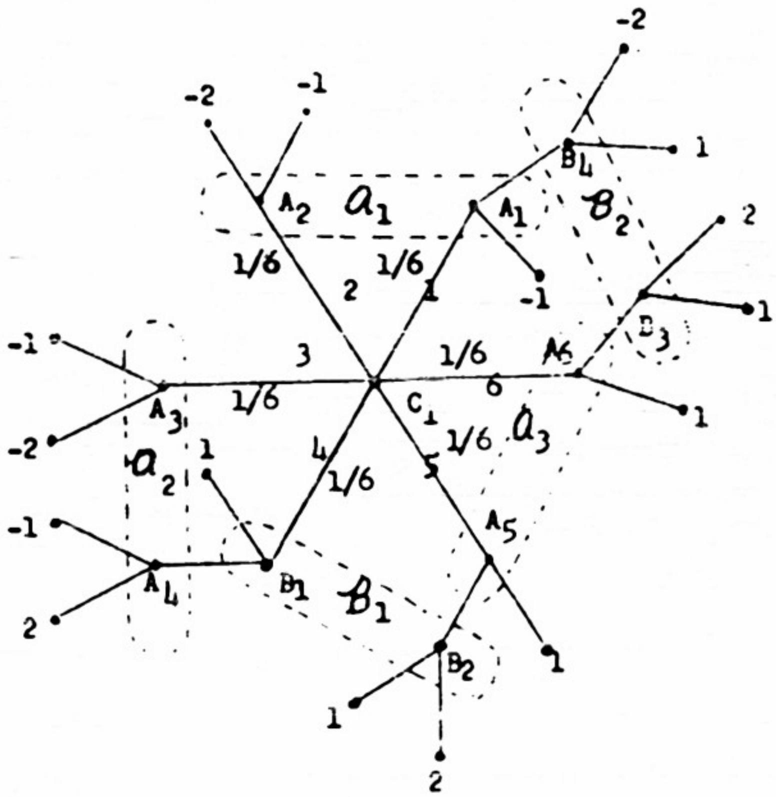
EXAMPLE 2. N-CARD GOOFSPIEL



In this game there are three decks of n-cards, each set labeled with the numbers 1, 2, ..., n. Players A and B hold one set each, and the third is shuffled and placed face down. There are n-stages to a play of this game; in each stage the top card of the hidden deck is turned up, the players bid simultaneously by each playing one of their cards, and the player bidding the higher card wins from his opponent the face value of the card turned up (if identical cards are bid there is no payoff).

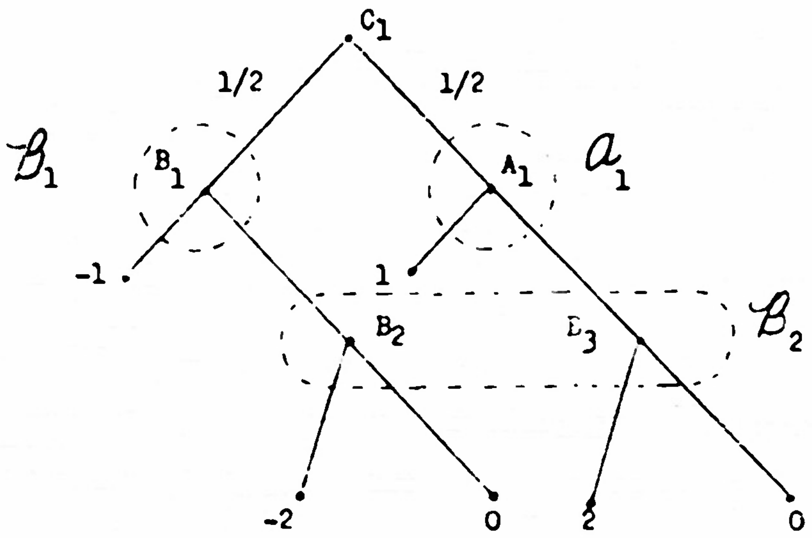
The game tree shown above represents 2-card Goofspiel. It is a game with perfect recall and contains two proper subgames starting at A_1 and A_2. It is itself a subgame of n-card Goofspiel for $n > 2$.

EXAMPLE 3. SIMPLIFIED POKER



The above game tree represents Simplified Poker (see: "Simplified two-person poker," by H. W. Kuhn, Annals of Math. Study No. 24, Princeton, 1950) after the dominations have been accounted for. The game starts at C_1 which is the deal. This is a game with perfect recall but has no proper subgames.

EXAMPLE 4. A PARTNER GAME



In this game, player B consists of two agents, called the Dealer and Partner, respectively. Two cards, one marked "High," the other "Low," are dealt to the Dealer and player A; the two possible deals occuring with equal probabilities. The agent with the High card then receives one dollar from the agent with the Low card and has the alternatives of terminating or continuing the play. If the play continues, the Partner, not knowing the nature of the deal, must instruct the Dealer to change cards with A or to hold his card. Again, the holder of the High card receives a dollar from the holder of the Low card.

This game does not have perfect recall since the Partner "forgets" information made available to the Dealer. As all games without perfect recall, this game requires the use of more than one agent for a player.

3. A Bibliography for Extensive Games.

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FORMAL DISCUSSION ON

Dr. Kuhn's Paper
by

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I would like to begin by pointing out that in a game without perfect recall, if one or more of the players does have perfect recall then that player or those players can achieve the value of the game by using behavior strategies. The players who do not have perfect recall must use either mixed or composite strategies (c.f. "Signaling strategies in n-person games," by G. L. Thompson, Annals of Math. Study No. 28). Thus the computational advantages of behavior strategies are available for those players who have perfect recall. This remark may be useful in studying the new way of normalizing a game which was discussed by Professor Kuhn this morning.

Next I would like to discuss Professor Kuhn's "Problem A" as stated in his abstract. This is the problem of finding the best behavior strategies for players in a game. This problem is particularly interesting if the game has perfect recall, for then it is equivalent to the problem of finding optimal strategies in a game. However, in a game in which at least one of the players does not have perfect recall these problems may not be equivalent. In such a game there exists assignments of the payoffs such that the players without perfect recall cannot achieve their maximum payoff in the game. In this case, signaling (ibid.) is possible and desirable, so that from a game-theoretic point of view, it is the problem of signaling that is important.

On the other hand, problem A would regain its game-theoretical significance if one were studying a game in which the players were actually restricted to behavior strategies. I think that the question of determining the allowable strategy space for players in actual games is one that has been largely neglected so far, but is one which deserves serious consideration. It can happen that the rules of the game (written and unwritten) restrict a player to the use of behavior strategies, non-convex subsets of pure strategies, etc.

Next I would like to discuss the Poker Game example which Professor Kuhn mentioned in his talk. You will recall that he had a tree on the board which he said represented the Poker Game but which he also said was not really the Poker Game because he had eliminated a great deal of the original tree of Poker. The tree that he had on the board was what might be called the "essential core" of the game tree of Poker.

Now this is a perfectly typical situation in game theory. When a game is given by its rules, that is when it is given in extensive form, there are numerous redundancies and symmetries present in the game. In many cases the trees of the game are so large that it is virtually impossible to write them down, and yet the game can be understood and played. In solving such a game one always begins, as Professor Kuhn did, by removing as much of the redundancy as one can, and then normalizing the game.

The problem of formalizing this procedure will occupy the rest of this discussion. The work which I shall describe has been done in collaboration with Dr. H. D. Mills of Princeton University.

The most widely used simplifying operation is to break the game into subgames whenever possible. By this one can either mean subgame in the sense which Professor Kuhn discussed this morning or in the sense of Dr. Dalkey. (Both of these concepts are defined in papers by the respective authors in the Annals of Mathematics Study No. 28.) In what follows we shall always assume that we are dealing with games that have no proper subgames.

The first two types of simplifying operations are what might be called "local decisions". These are decisions which are made at terminal information sets (that is, information sets that are followed only by plays). The decisions are local in the sense that they depend only on the information set and the payoffs of the plays following the information set. The two types of local situations which I shall discuss today are illustrated in Figure 1.

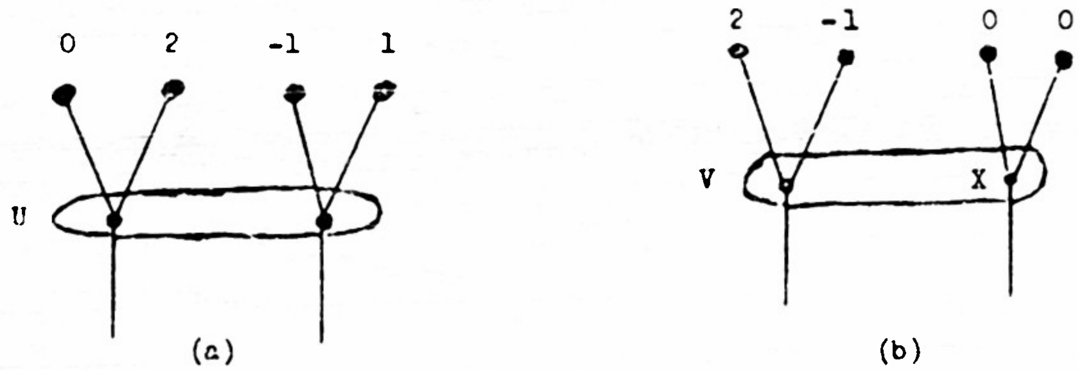


Figure 1

In Figure 1 (a) is illustrated what I shall call an <u>immaterial information set</u>. It is a terminal information set U which has the property that the choice of one of the alternatives is uniformly better than the other one for the player in question. In the figure it is clear (assuming that the player who controls the information set is the maximizing player) that the right-hand alternative is always better than the left-hand one. From the definition it is clear that every one element terminal information set is immaterial.

In Figure 1 (b) is illustrated what I shall call an immaterial move. It is a move X in a terminal information set V which has the property that all plays following it have identical payoffs. It is clear from the figure that the move X will not influence the strategic decision of the player who controls the information set.

The simplifying operation connected with immaterial moves and information sets is to eliminate the plays following them and change the moves in the information set, or the immaterial move, into plays. The examples in Figure 1 after each of these operations has been performed are shown in Figure 2.

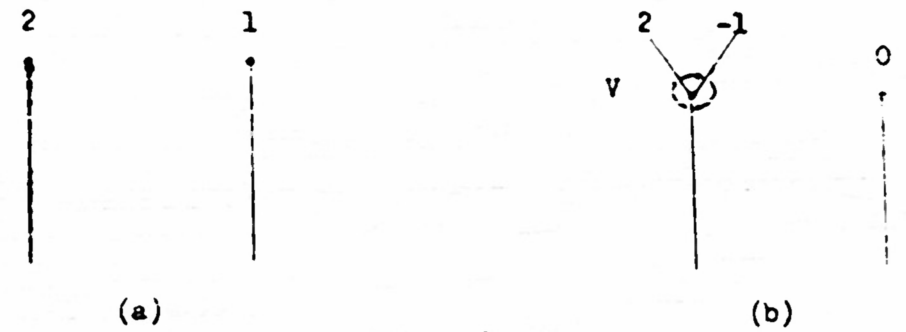


Figure 2

It will be noticed that in Figure 2 (a) we have reduced the number of information sets for a player by one and hence have decreased the number of his pure strategies. In Figure 2 (b) we have, by eliminating the plays following the immaterial move, made the information set V immaterial and hence susceptible to further simplification. The latter event happens only when, as in our example, the number of moves in the information set is two. But it does bring out the fact that these operations can be repeated several times.

We have, of course, changed the game tree and, in a strictly logical sense, now have a different game. It is clear how to obtain optimal strategies in the original game once they have been found for the reduced game. Although these operations are obvious, we have found that in some games they permit an enormous reduction in the size of the tree.

The third type of game tree reduction that I shall describe is one that takes advantage of symmetries in the game. I shall call it <u>telescoping</u>, and illustrate it by the example in Figure 3.

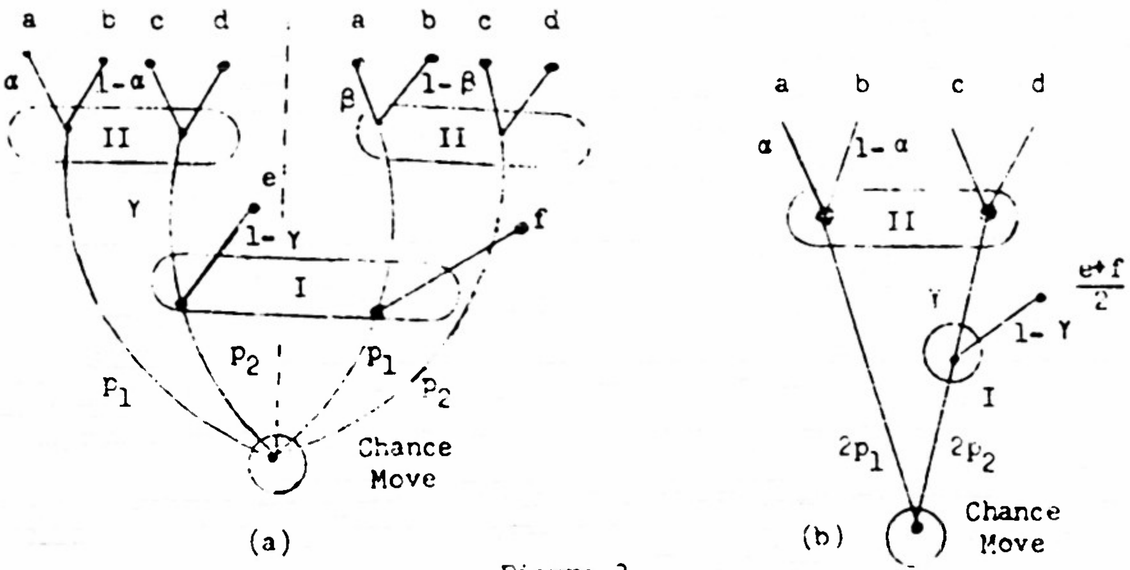


Figure 3

This is a two-person zero-sum game with perfect recall. The letters a, b, c, d, e, f stand for real numbers, and the symbols p_i for probabilities. It is apparent that the game tree exhibits bilateral symmetry, the part of the tree on the left of the dotted line being isomorphic to the part on the right. If we normalize the game, using the behavior coefficients as indicated we find that the expectation of player I is

$$(\alpha + \beta)\,[p_1(a - b) + p_2\gamma(c - d)] + 2p_1 b + p_2(e + f)$$
$$+ \gamma\,[2p_2 d - e - f]\,.$$

The important thing is that the behavior coefficients α and β always appear together in a sum. It is clear that any payoff which player II can get by choosing α and β, he can also get if he first sets $\alpha = \beta$ and then chooses α. In other words, the extra parameter of freedom is not useful in increasing his payoff. If one computes the expectation to player I in the game shown in Figure 3 (b) one sees that it is identical to the one obtained above with $\alpha = \beta$. From a game-theoretic point of view, then, one can replace the game of Figure 3 (a) by the one in Figure 3 (b).

This kind of symmetry occurs, for example, in card games in which the location of the cards of small denomination does not influence the outcome of a hand.

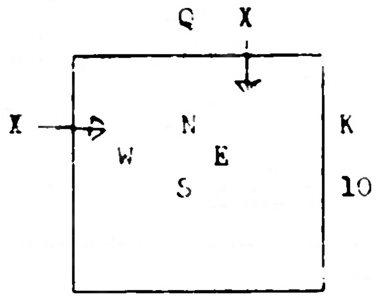
These operations by no means exhaust the number of simplifying operations that can be performed on trees, but they are sufficient to solve some games which are quite complicated. I would like to devote the remainder of the discussion to such a game.

Let us consider an eight card bridge game with one suit, four players, North, East, South and West, paired into partners as usual. We assume that the shuffling is done by a chance device, that there is no bidding but otherwise that the rules of play are as in ordinary bridge. The payoff to East-West is the number of tricks they get; thus, there are three payoffs, 0, 1 or 2. The payoff to North-South is the negative of this amount. We shall assume that South is declarer, and therefore North is dummy, and West leads.

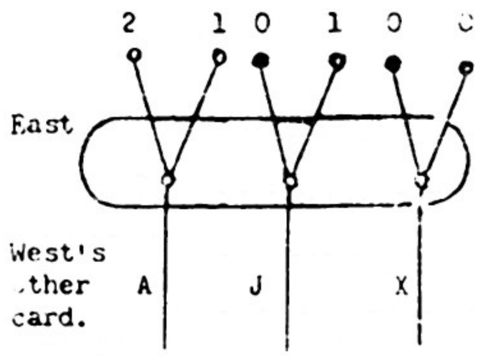
In this game there are 2,520 deals, and if you look at the game tree of this game you will find that it is too complicated to write down. There are an astronomical number of pure strategies for each player, but it is clear that it is very much simpler than the ordinary game of contract bridge, and that a good bridge player could easily play it very well.

Since I have assumed that South is declarer, he will make the last play of the first trick and, effectively, the last play of the game, since the payoff is determined after the first trick. It is not hard to see that all of South's information sets are immaterial since the outcome of the second trick depends only on which pair has the highest card and not on the location of the highest card. Thus all of South's information sets can be removed from the tree and replaced by plays with suitable payoffs. But the tree is still too large to write down.

It is also true that almost all of East's information sets are immaterial. However, there are situations in which it is not true, and one such is shown in Figure 4 (a).



(a)



(b)

Figure 4

In Figure 4 (a) the symbol x stands for a card smaller than 10 and the other symbols have their usual meaning. The diagram shows the information available to East; the arrows indicate the cards that were played by West and North; the queen in North's hand is visible to East since North is dummy. Figure 4 (b) illustrates East's information set. (It is assumed that all of South's information sets have been removed and replaced by plays). The lack of East's information can be characterized by the lack of knowledge of the other card in West's hand, since the other cards are necessarily in South's hand. From the diagram it is clear that the third move in East's information set is immaterial and can be removed. However, the modified information set so obtained is not immaterial and cannot be removed. This can be called a quandary situation for East.

After the rest of this analysis is carried out for this game it is found that there are other quandary situations for East and some for North. It is also found that the game tree breaks up into four subgames, which, after telescoping, become simple enough to draw on one sheet of paper. After all of these reductions are carried out it is easily seen that the game is strictly determined and the optimal pure strategies for each player can be obtained.

It seems clear then that, before one tries to solve the game either by normalizing in the sense of Kuhn or in the sense of von Neumann and Morgenstern, one should first carry out as many simplifying operations on the tree as possible and thus reduce the labor of solving the game.

SUMMARY OF GENERAL DISCUSSION ON DR. KUHN'S PAPER

The general discussion was largely concerned with the role of information in game theory. In connection with the signaling strategies of Mr. Thompson, Professor Tucker pointed out that (disregarding Hoyle), in order to receive the value of such a two-person zero-sum bridge game, the players have to use a chance device (concealed from their opponents) which they each observe and follow in order to achieve joint mixed strategies. The Chairman referred to a forthcoming McGraw-Hill volume by John Williams of Rand entitled "The Compleat Strategyst", which he described as being a most satisfactory background source for persons interested in game theory.

REDUCTION OF GAMES IN EXTENSIVE FORM

by

Dr. Norman Dalkey
The RAND Corporation

Abstract

A two-person game will be said to be essentially reducible when its matrix of outcomes is singular, independently of the specific payoff or specific probability distributions at chance moves. A general, n-person game is essentially reducible when some outcome matrix obtained by specifying strategies for all but two players is singular independently of payoff or chance-move probabilities. A game will be said to be essentially reduced if it is not essentially reducible.

A theorem is proved that any game in which one or more players has some information about previous moves by opponents is essentially reducible. In general, it is not possible to construct a game which is essentially reduced from one which is essentially reducible solely by equivalence operations on the information pattern when the game is expressed as a tree in the manner of Kuhn. In many cases of practical importance, however, this can be done. The major devices for reduction are deflation and resolving the game into partial subgames.

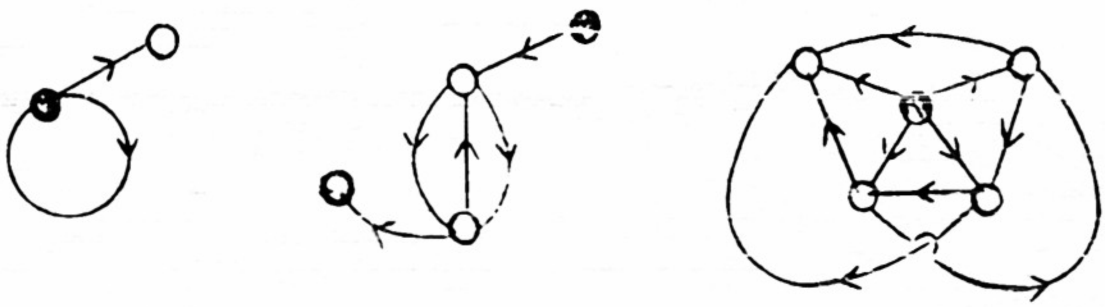
The notion of reduction involved here is closely related to behavior strategies. It can be proved that a necessary condition for a game to be solvable in behavior strategies is that the complete inflation of the game have perfect recall. All such games are essentially reducible; in fact, the number of pure strategies for a given player in the essentially reduced form of the game is precisely one greater than the minimum number of parameters defining the behavior strategy space for that player. Thus, the number of independent parameters involved in solving the game by mixed strategies is precisely the same as the number involved in solving it by behavior strategies. This negates the principal advantage of behavior strategies, namely a reduction in the size of the game. On the other hand, behavior strategies often have the advantage that they can be constructed directly from the information pattern.

There is the possibility, as yet undemonstrated, that a more general description of games will permit complete reduction of any game directly by equivalence operations on the game structure.

FORMAL DISCUSSION ON
DALKEY'S PAPER
by
L. S. Shapley

My remarks will deal with a generalization of the games in extensive form whose informational structure has just been discussed by Dr. Dalkey. I am not at present able to give an equally complete account for the wider class of games, which I shall here call "positional games", but I feel it would be useful and illuminating for such an analysis to be carried out.

A positional game differs from an extensive game in one main respect: the graph of its representation need not be a tree--i.e., free from closed circuits. In fact, the end-points of a single edge may be the same, and there may be several edges running between the same pair of vertices. Also, it is not required that the graph be embeddable in a plane (see the figures), and the condition that the

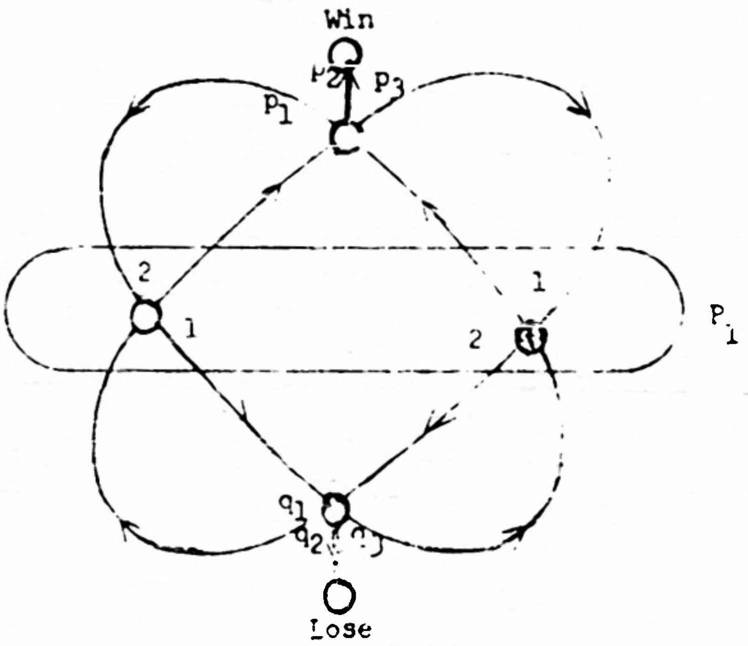


graph be finite is no longer important. To avoid ambiguity, it becomes necessary to orient each edge. The outgoing edges from a vertex represent the alternatives at that position. They must be numbered for identification purposes in some definite way. Vertices having no outgoing edges are terminal positions, and have the payoffs attached to them. The unique initial position, however, may have incoming as well as outgoing edges. The possibility of infinite plays arises even in a finite graph; it can be handled either by assigning a definite payoff (say, 0 to all players) for nonterminating plays, or by insisting on enough chance moves distributed through the graph to ensure with probability 1 that a terminal position will eventually be reached. The information sets are defined as in extensive games, except that the proviso that no play can pass through the same information set twice is dropped. It is not essential that the initial position be the only point in its information set; the players are of course informed of where the play begins, but they do not have to be able to find it out whenever the initial position is restored.

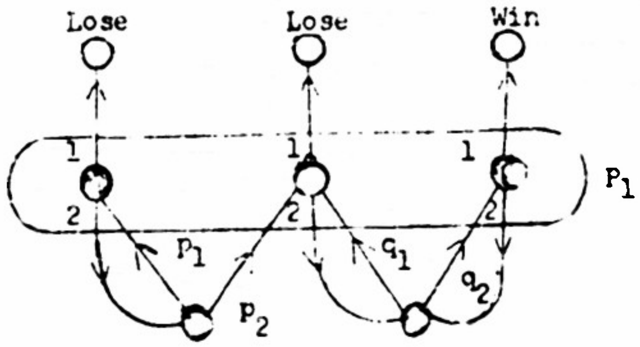
A pure strategy will consist of a sequence of choices on each information set of the player in question, instead of a single choice.

In playing the game, the first choice of the sequence is used when the play arrives at an information set for the first time; if the play returns to that set, the second choice is used, and so on. Thus, it is assumed that a player's agents in the field can count, but not communicate.

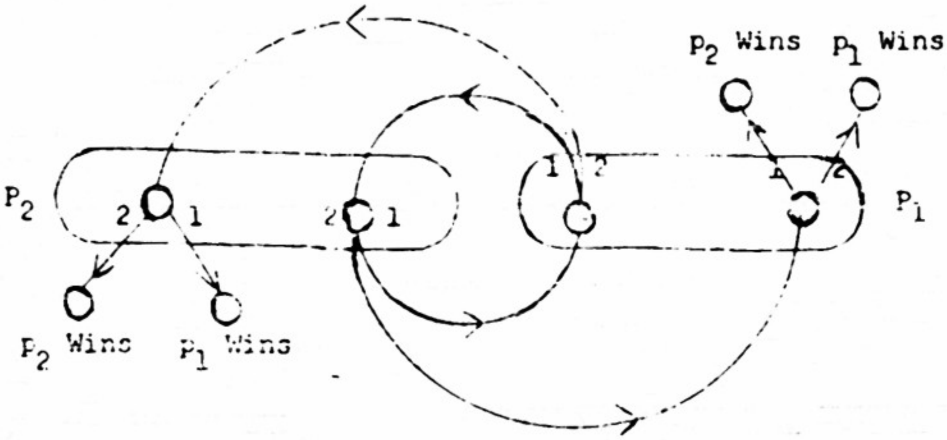
In the one-person game illustrated, there are two (indistinguishable) personal-move positions, two chance-move positions, and two terminal positions; the game begins at the black dot on the right.



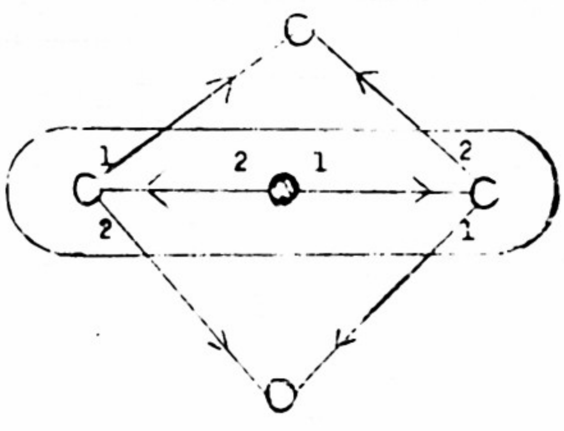
In this case the optimal pure strategy can be determined once the probabilities p_1, p_2, p_3, q_1, q_2, q_3 are given: the player should make "1" his first choice, and choose "1" or "2" thereafter depending on whether it is more likely that the play is at the right-hand or left-hand vertex, respectively. In the example below, however, there is no optimum strategy if all the probabilities are positive, since the longer the player waits before choosing "1", the more chance he has of winning.



In the two-person example below, there are no chance moves, and an infinite play is possible; nevertheless the strategies which produce it are so dangerous that the value ascribed to the infinite play is immaterial to the solution. A 50-50 probability of the two sequences (1, 2, 2, 2, ...) and (2, 2, 2, 2, ...) is an optimal mixed strategy for either player; it is essentially unique for the second player, but not for the first.



A useful concept in many applications is that of stationary strategy. A stationary pure strategy is one where all the choices at each information set are the same—so that the player's agents do not have to count their decisions after all. A mixed strategy is stationa if it has the effect of making each agent make his choice according to the same probability distribution whenever the play returns to his information set. A stationary mixed strategy is almost never a mixtur of stationary pure strategies, and conversely a mixture of stationary pure strategies is almost never stationary itself. The distinction m: be illustrated by the following game:



Here no stationary pure strategy, or mixture of stationary pure strategies, can ever win, while any stationary mixed strategy will win with probability $2p(1 - p)$, where $(p, 1 - p)$ is the associated probability distribution.

In a positional game with perfect information, it can be shown that if there is any solution, there is a solution in stationary pure strategies. Under a somewhat weaker condition than perfect information, which in effect guarantees that information is not too long delayed, and is never forgotten, it can be shown that a positional game can be solved in stationary mixed strategies, if it can be solved at all.

A number of investigators--among them, J. R. Isbell, R. P. Isaacs, D. L. Yarmush, and R. Bellman--are currently involved in research on infinite games in extensive form, more or less similar to the type that I have been describing. I make no claim of originality or superiority for my particular formulation and choice of nomenclature. So far as I am aware, there has been no thoroughgoing analysis of equivalence of information patterns in this field.

SUMMARY OF GENERAL DISCUSSION ON DR. DALKEY'S PAPER

The question of solution of games by the use of electrical analogue machines was raised. Professor Tucker stated that the originator of the theory of games, Professor John von Neumann, had given considerable attention to the solution of games in matrix form by such a machine. Games in matrix form have a great deal of stability and have unique values. There is never any question in them about the instability of the denominator vanishing. Professor von Neumann, a proponent of digital computing machines, has found the matrix game one place where good reasons for using analogue machines are indicated.

Dr. Dalkey pointed out that, except for the rather intriguing feature in the particular game under discussion--the possible circular groups--such games are represented by redundant trees. In other words, one can take any position where two or more alternatives meet and divide that into several positions, but that will give him a tree where parts will be repetitive, will be isomorphic to the other parts.

Conversely, in many cases one finds in the tree representation isomorphic elements of the game, and such games, then again, are representable in these forms where the isomorphic parts are represented by single positions.

Chess is a game of that kind. In the case of chess, there are many different ways in which you can get to the same position on the board, and you can represent them either as a tree, where each of the different parts will arrive at the same position, or representation can be made by a single point as general data.

Chairman Morgenstern closed the discussion with some remarks on games of pursuit. These games are of enormous interest and should be studied intensively. Apparently not enough people are interested in these games at present because the simultaneous application of various branches of mathematics is required.

The situation in a game of pursuit would be this, for example that there are two ships at sea, and that one ship is trying to capture the other. Capture would be defined as getting within gun range or within range of airplane attack, or as bringing the hunted ship near the coast.

Of course, one has to introduce various speeds for the ships participating, and various kinds of information. He may have to introduce communications in various manners (for example, signals passing, which are perhaps to be identified as being true signals or fake signals etc.).

In short, it is easy to set up very complicated--in fact, too complicated--schemes. Even the simplest schemes of this nature are of enormous complexity, and games theory applies to them because they are games--however, with other aspects, because one has to introduce such elements as speeds (for example, speeds of maneuver). All military services, but particularly the Navy and the Air Force, should be especially interested in promoting investigations along those lines.

SUMMARY REMARKS

by

Chairman Morgenstern

It is 3:24 and the time table here says that at 3:29 the chairman has to make summary remarks. I think they would be somewhat anticlimactic if they were long, as they would have to be. So all I will do is to thank the various speakers, the main speakers and the formal discussants, and all those who have sat through these two days of so-called "theoretical" sessions.

I am turning the meeting over to Dr. Rees.

CONCLUDING REMARKS
by
The General Chairman, Dr. Mina Rees

If Dr. Morgenstern's remarks would be anticlimactic, I cannot think of any that would be more anticlimactic than mine.

I wish to express, not only to the speakers, but to the Chairmen of the two sessions, the gratitude of all the participants for their excellent planning and presiding and participation.

I should like also, on behalf of the Office of Naval Research, to thank particularly The George Washington University people who have been largely responsible for the planning and carrying out of this symposium.

I mentioned in my opening remarks that one of the great needs in this new logistics game is to provide a forum for interchanging of opinion and experiences. Although I myself have been rather heavily engaged in some practical games over in the Pentagon for the last couple of days, I hear there has been a lively exchange of opinion, and some disagreement expressed on the floor, and sometimes outside. I think this is all to the good.

I want particularly to thank our military participants. I wonder if the rest of the civilians feel as I do, that we have learned a tremendous lot from what they said. Not many are here, but I would particularly like to thank General Davis, whose talk was so stimulating and worth while.

On the general front of providing a forum for interchange of opinion, I would like to take this chance to announce that the Office of Naval Research plans to initiate a journal, probably some time in the Fall, to be called the Naval Research Logistics Quarterly. It is intended that this journal should contain both theoretical and practical articles, that the writers should be drawn both from the scientific community and from the military community. You are all invited and urged to submit articles for consideration. The Journal will be carefully edited. We do not guarantee publication of anything. The objective is to provide a really adequate interchange of good work in logistics. Articles should be sent to Code 436, the Logistics Branch of the Office of Naval Research.

With these remarks I would like to declare this session of the Logistics Conference at an end, and thank all the participants, both on the floor and on the stage.