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# A CONVENIENT AND ACCURATE SEMI-EMPIRICAL ENTROPIC EQUATION FOR USE IN INTERNAL BALLISTIC CALCULATIONS

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ABSTRACT: The solution of the internal ballistics problem requires a knowledge of the equation of state of the propellant gases. Usually the Noble-Abel equation of state is used for this purpose. However, this equation does not account for the intermolecular forces.

A semi-empirical equation of state has been developed which permits the solution of the internal ballistics problem and represents accurately the effects of the intermolecular forces over a wide range of pressures and densities. Since the semi-empirical equation is similar in form to the perfect gas equation, it is as convenient in application to interior ballistic calculations.

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This report presents some of the results of a theoretical and experimental study made at the University of Amsterdam and the Naval Ordnance Laboratory with an experimental gun, the "Expansion Rate Measuring Apparatus". This work was carried out under the sponsorship of the Office of Naval Research.

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# A CONVENIENT AND ACCURATE SEMI-EMPIRICAL ENTROPIC EQUATION FOR USE IN INTERNAL BALLISTIC CALCULATIONS

## I. INTRODUCTION

In order to calculate the muzzle velocity of a gun, the isentropic equation of state of the powder gases must be known. For this purpose the ballisticians universally employ the Noble-Abel isentropic relation,

$$p\,(v-b)^{\gamma} = \text{constant},$$

where $b$ and $\gamma$ are assumed constant during the expansion and are the co-volume and mean ratio of specific heats of the propellant, respectively. This equation is based upon the Abel thermal equation of state, $p(v-b) = RT$, and the assumption that the specific heat is constant. Although it is an approximate representation of the behavior of propellant gases, its use is justified by the fact that the approximation is as good as others introduced in the theory of internal ballistics and that it is mathematically extremely tractable. Nevertheless, it would be desirable to have an isentropic equation which more accurately describes the behavior of the propellant gases and which is convenient for ballistic calculations.

Such an entropic relation for a real gas (one in which the intermolecular forces are important, in contrast to a perfect or Abel gas in which the intermolecular forces are unimportant) was found by the present author in his study of the rapid expansion of compressed gases behind a piston. (This study was made at the van der Waals Laboratorium, Amsterdam, and is described fully in reference (a).) The entropic relation, which is semi-empirical, and the manner in which it was obtained are discussed in this paper.

## II. THE EXPERIMENTAL GUN APPARATUS

The experimental gun apparatus employed in this work was first used by Michels, Slawsky, and Jacobs (reference (b)); it is designated as ERMA (from the initial letters of the descriptive title "Expansion Rate Measuring Apparatus"). The instrument is essentially a hollow cylinder, closed at one end and open at the other end. Inside the constant cross-sectional area cylinder is a movable piston or projectile of known mass which is initially restrained from movement. Between the piston and the closed end is contained a highly compressed gas whose temperature and pressure are known. During the operation of the experiment the compressed gas propels the piston from rest out of the open end of the cylinder. From the

experiment the travel and time measurements of the piston could be determined. These travel-time data were used to study the dynamic expansion process in the experimental gun.

## III. THE BALLISTIC THEORY

The basic assumptions used to describe the motion in ERMA of the gas and the projectile are the following:

(1) The gas motion is one-dimensional in space.

(2) Each part of the gas expands isentropically.

(3) The projectile is propelled by the gas pressure and opposed by a frictional resistance between the projectile and the cylinder walls and by the air pressure in front of the projectile.

From these assumptions, hydrodynamic and thermodynamic laws lead to the "characteristic equations". It can be then shown that two types of expansions are possible. One type is the "non-reflection" case in which the behavior of the piston is the same as would occur if the back wall of the chamber were infinitely distant from the piston; the second type is the "reflection" case in which the behavior of the piston is influenced by the presence of the back end of the chamber. For both cases the entire behavior of the expanding gas and moving piston can be determined from the characteristic equations, the real gas equation of state (in the form of an entropic pressure-density relationship), and from Newton's force law applied to the projectile.

## IV. OBTAINING THE SEMI-EMPIRICAL ENTROPIC EQUATION

As a result of tests performed with the experimental gun on nitrogen gas, it was previously observed that the use of simplified equations of state, such as the perfect gas or Noble-Abel gas equations, yielded unsatisfactory agreement between the ballistic theory and the experiment; satisfactory agreement was only obtained when the isentropic nitrogen data as determined from experimental $p$-$v$-$T$ measurements was used (see references (a) and (b)).

An attempt was made to obtain an analytic expression relating the pressure behind the projectile with the projectile velocity for the non-reflection (i.e., infinite chamber) case. Therefore, from the $p-\rho$ data of nitrogen, isentropic curves of piston velocity, $u$, as a function of pressure, $p$, were obtained for the non-reflection case. From these curves derived only from nitrogen thermodynamic data, the position-time relation of the projectile could be obtained in the isentropic case of unopposed projectile from the expressions,

$$\int_0^x dx = \frac{M}{A}\int_0^u \frac{u\,du}{p} , \qquad \int_0^t dt = \frac{M}{A}\int_0^u \frac{du}{p} . \quad (1)$$

Various relations were attempted to approximate the p-u curves (power series, rational fractions, etc.) without success. Here the difficulty lay in obtaining an expression which fitted the curves and yet could be conveniently integrated to yield, for example, a relation between projectile velocity and barrel length.

It was recalled that the relation between p and u for a perfect gas in the non-reflection case is

$$p = p_o\left(1 - \frac{\gamma-1}{2}\,\frac{u}{a_o}\right)^{2\gamma/(\gamma-1)}$$

where $p_o$ is the initial pressure of the expansion, $a_o$ is the initial sound velocity, and $\gamma$ is the ratio of specific heats. In view of this a similar expression for the real gas was attempted.

$$p = p_o\left(1 - \frac{u}{\alpha_o}\right)^{\beta} \quad (2)$$

where $\beta$ is assumed a function of entropy alone and $\alpha_o$ is an initial parameter of the expansion. Then from the condition that equation (2) must hold with a constant $\beta$ along an isentropic p-u curve for all initial pressures $p_o$, follows the fact that $\alpha_o$ is equal to $\phi\, p_o$ where $\phi$ is a function of entropy alone.

This semi-empirical equation satisfies the requirements of a desired analytic expression very well. With the proper values of $\beta$ and $\phi$ it fits the isentropic p-u curves within the accuracy with which these curves were obtained (1 part in 50 to 1 part in 500), and it is extremely convenient, since it is readily integrated as necessary.

From equations (1) and (2) the expression for projectile travel as a function of time for an unopposed projectile propelled by an isentropically expanding gas in the non-reflection case can be obtained:

$$x = \alpha_o t - \frac{\alpha_o^2}{(\beta-2)P_o}\left\{\left[\frac{\beta-1}{\alpha_o}P_o t + 1\right]^{\frac{\beta-2}{\beta-1}} - 1\right\} \quad (3)$$

where $P_o = Ap_o/M$, and the initial conditions are $x = 0$, $u = 0$, $p = p_o$, $t = 0$ .

Equation (2) can be written as a relation between the Riemann function $\sigma$ (defined below) and the gas pressure along an isentrope.

$$p = p_o\left(1 + \frac{\sigma}{\alpha_o}\right)^{\beta} , \qquad (2a)$$

where the Riemann function $\sigma = \int_{p_o}^{p}\left(\frac{dp}{a\rho}\right)_s$. It is to be noted that equation (2a) is <u>independent</u> of the experimental gun; it expresses a relationship between gas properties.

By use of the definition of $\sigma$ the above expression can be transformed to obtain a relation between pressure and density for isentropes. The following result is obtained.

$$p^{(\beta-2)/\beta}\left(\frac{1}{\rho} - f\right) = \frac{\phi^2}{\beta(\beta-2)} \equiv \alpha , \qquad (4)$$

where $f$ is a function of entropy alone (as is $\alpha$). It is seen that $f$ is the volume occupied by a gas if compressed isentropically to an infinite pressure. Equation (4) is the semi-empirical entropic equation.

## V. METHOD OF OBTAINING THE CONSTANTS OF THE EQUATION

Each entropy has associated with it different values of $\beta$, $\phi$, and $f$; these are dependent only on the entropy and the gas. To obtain these constants for a given entropy, equation (4) was used with $p$-$\rho$ isentropic data. At three points on the isentropic $p$-$\rho$ curve corresponding values of $p$ and $\rho$ were obtained. Then by equation (4)

$$\frac{1}{\rho} - \frac{1}{\rho_o} = \frac{\alpha}{p_o^{(\beta-2)/\beta}}\left[\left(\frac{p_o}{p}\right)^{(\beta-2)/\beta} - 1\right] , \qquad (5)$$

and

$$\frac{1/\rho_1 - 1/\rho_o}{1/\rho_2 - 1/\rho_o} = \frac{(p_o/p_1)^{(\beta-2)/\beta} - 1}{(p_o/p_2)^{(\beta-2)/\beta} - 1} ,$$

which was solved for $\beta$; $\phi$ was then obtained from equation (5), and $f$ from equation (4). A plot is given in figure 1 of $\beta$, $\phi$, and $f$ as a function of entropy for nitrogen gas.

## VI. ACCURACY AND RANGE OF THE EQUATION

The range in which the experimental gun expansion occurs extends from states at about 50°C and high pressures (400 atmospheres to 3000

atmospheres) down to states at low temperatures, -75°C to -125°C, and low pressures (of about 100 atmospheres). The isentropes S = -12 through S = -17 cal/mole°C for nitrogen cover this range ( S is chosen to be equal to zero at 1 atmosphere and 0°C). Therefore, the constants for the semi-empirical equation were calculated for these entropy values.

The average difference between densities calculated from the semi-empirical equation (4) and those read from a plot of nitrogen isentropes* is 0.25% down to 200 atmospheres. Between 200 atmospheres and 100 atmospheres the deviation is greater, becoming 1% in some cases at 100 atmospheres. It is to be noted that in this low pressure region the accuracy of the data is also less (about 1/2%), and the co-existence region is beginning to influence the isentropes.

Although the constants of the semi-empirical equation for nitrogen were chosen to apply to regions below 50°C, the equation has been employed successfully (1% deviation in density) at 125°C and 6000 atmospheres with these same constants. Thus, it is probable that the equation can be applied to higher pressure and temperature regions.

For the only isentrope examined which extended into a perfect gas region, the semi-empirical equation seemed to apply. It fits the nitrogen isentrope, S = -4 cal/mole°C, in the range of -125°C and 1 atmosphere to +150°C and 34 atmospheres. The equation has been applied with success to argon in the region of 0°C to 150°C and pressures up to 2000 atmospheres.

It seems likely that the semi-empirical equation could be applied to powder gas in ballistic problems with success.

## VII. THERMODYNAMIC RESULTS OF THE EQUATION

From the entropic equation of state, equation (4), should follow all the thermodynamic quantities. Some of these are listed below:

$$p^{\frac{\beta-2}{\beta}}(v-f) = \frac{\phi^2}{\beta(\beta-2)} \equiv \mathcal{K} \qquad \text{entropic equation of state}$$

$$U = g + \alpha^2/2\beta \qquad \text{internal energy equation}$$

$$H = g + fp + \frac{\alpha^2}{2(\beta-2)} \qquad \text{enthalpy equation}$$

*These isentropes were taken from reference (c), where the thermodynamic properties of nitrogen for pressures up to 6000 atmospheres between -125°C and +150°C are given.

$$a = \frac{\beta p}{\alpha \rho} \quad , \qquad \text{velocity of sound}$$

$$T = p(v - f)\left\{\frac{\beta}{2}\frac{d \ln \kappa}{dS} + \frac{d\beta}{dS}\left[\frac{1}{2} - \ln p^{1/\beta}\right]\right\} + \frac{dg}{dS} + p\frac{df}{dS} \quad , \qquad \text{temperature equation}$$

where $g$, $f$, $\beta$, $\phi$, and $\kappa$ are functions only of entropy, and $V$ is the specific volume $= 1/\rho$, $\alpha \equiv \phi p^{1/\beta}$.

## VIII. THE USE OF THE SEMI-EMPIRICAL EQUATION TO OBTAIN THE DESCRIPTION OF THE EXPERIMENTAL GUN PERFORMANCE

If the semi-empirical equation is used, a relation between projectile velocity and travel can be obtained for an isentropic expansion when there are no reflections which reach the projectile.

$$\frac{p_o A}{M \alpha_o^2} \chi = \frac{1}{(\beta-2)(\beta-1)}\left[\frac{(\beta-2) - (\beta-1)\left(1 - \frac{u}{\alpha_o}\right)}{\left(1 - \frac{u}{\alpha_o}\right)^{\beta-1}} + 1\right] \qquad (6)$$

Thus, the dimensionless projectile velocity $u/\alpha_o$ can be plotted against the dimensionless projectile position $p_o A \chi / M \alpha_o^2$ to yield curves for each entropy of performance for the isentropic non-reflection unopposed projectile case. These curves are shown as solid lines in figure 2 for nitrogen gas.

If reflections reach the projectile, these curves of course are altered. By using the semi-empirical equation and the numerical characteristics method the dimensionless velocity can be obtained as a function of dimensionless projectile position for a given G/M (where G is the mass of gas behind the piston, M is the projectile mass) and entropy. In figure 2 these G/M lines are plotted dashed to indicate the reflected cases.

It was found from tests made with the experimental gun that the forces opposing the projectile could be conveniently and accurately accounted for by the use in the calculations of an "effective" projectile mass, $M/(1-\eta)$, in place of the actual projectile mass, M. (see references (a) and (d)). Thus, the curves of figure 2 can be applied to the opposed projectile case (if everywhere the projectile mass M is replaced by $M/(1-\eta)$ ) and are a complete description of the experimental gun performance.

## IX. APPLICATION OF THE SEMI-EMPIRICAL EQUATION TO ACTUAL BALLISTIC CALCULATIONS

The semi-empirical equation can be as conveniently applied to ballistic calculations as the Noble-Abel or perfect gas equations. The necessary gas constants for such calculation, $\beta$ and $\phi$, should be obtained by fitting the semi-empirical isentropic equation to the powder gas $p-\rho$ isentropic data. If this data is unavailable, $\beta$ and $\phi$ can be approximately found from the data of two gun firings made at different values of dimensionless length or velocity by the use of the non-reflection isentropic theory, i.e., from equations (3) and (6). (The application of the non-reflection isentropic theory to gun firings has been found to be a useful approximation, especially to high velocity gun firings. See reference (6).)

## LIST OF SYMBOLS

$a$ - Velocity of sound

$A$ - Cross-sectional area of projectile

$b$ - Co-volume in Abel thermal equation of state

$f$ - Entropy constant in semi-empirical entropic equation

$g$ - Entropy constant in internal energy and enthalpy equations

$G$ - Mass of gas behind the projectile

$H$ - Enthalpy

$M$ - Mass of projectile

$p$ - Gas pressure

$P_o$ - Defined as $Ap_o/M$

$S$ - Entropy of gas

$t$ - Time coordinate

$T$ - Gas temperature

$u$ - Gas velocity or projectile velocity

$U$ - Internal energy of gas

$v$ - Specific volume of gas

$x$ - Position coordinate

$\alpha$ - Defined as $\phi p^{1/\beta}$

$\beta$ - Entropy constant in semi-empirical entropic equation

$\gamma$ - Mean value of specific heat in Noble-Abel equation

$\eta$ - Defined as $1 - M/M_{EFFECTIVE}$ (see p.6)

$\kappa$ - Defined as $\phi^2/\beta(\beta-2)$

$\rho$ - Density of gas

$\sigma$ - Riemann function, defined on page 4

$\phi$ - Entropy constant in semi-empirical entropic equation

The subscript "o" refers to initial conditions of the gas.

ACKNOWLEDGEMENT

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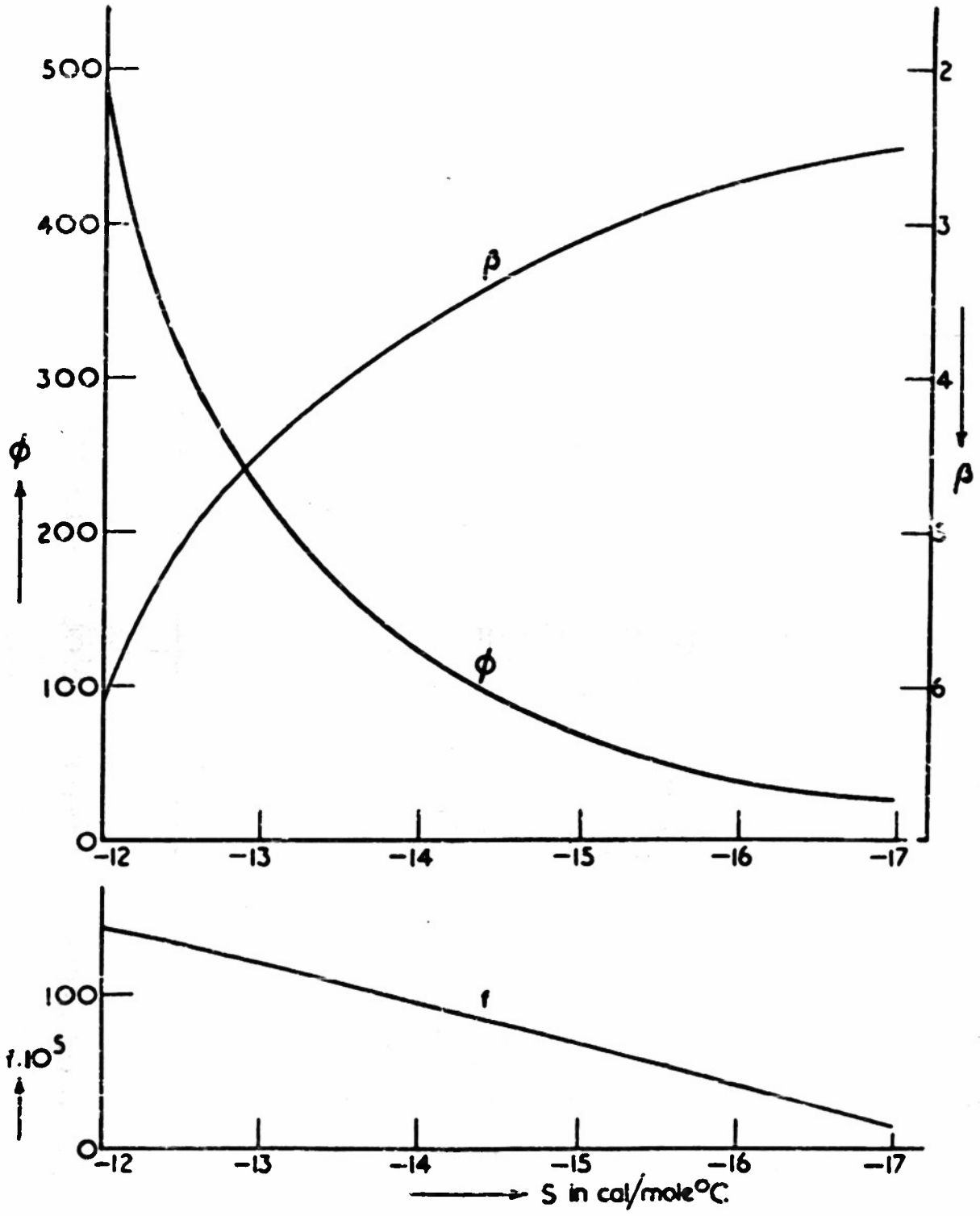


Fig. 1. Constants for nitrogen of the semi-empirical equation [$\varphi$ in m/sec (atm)$^{1/\beta}$, $f$ in Amagat$^{-1}$, $\beta$ dimensionless].

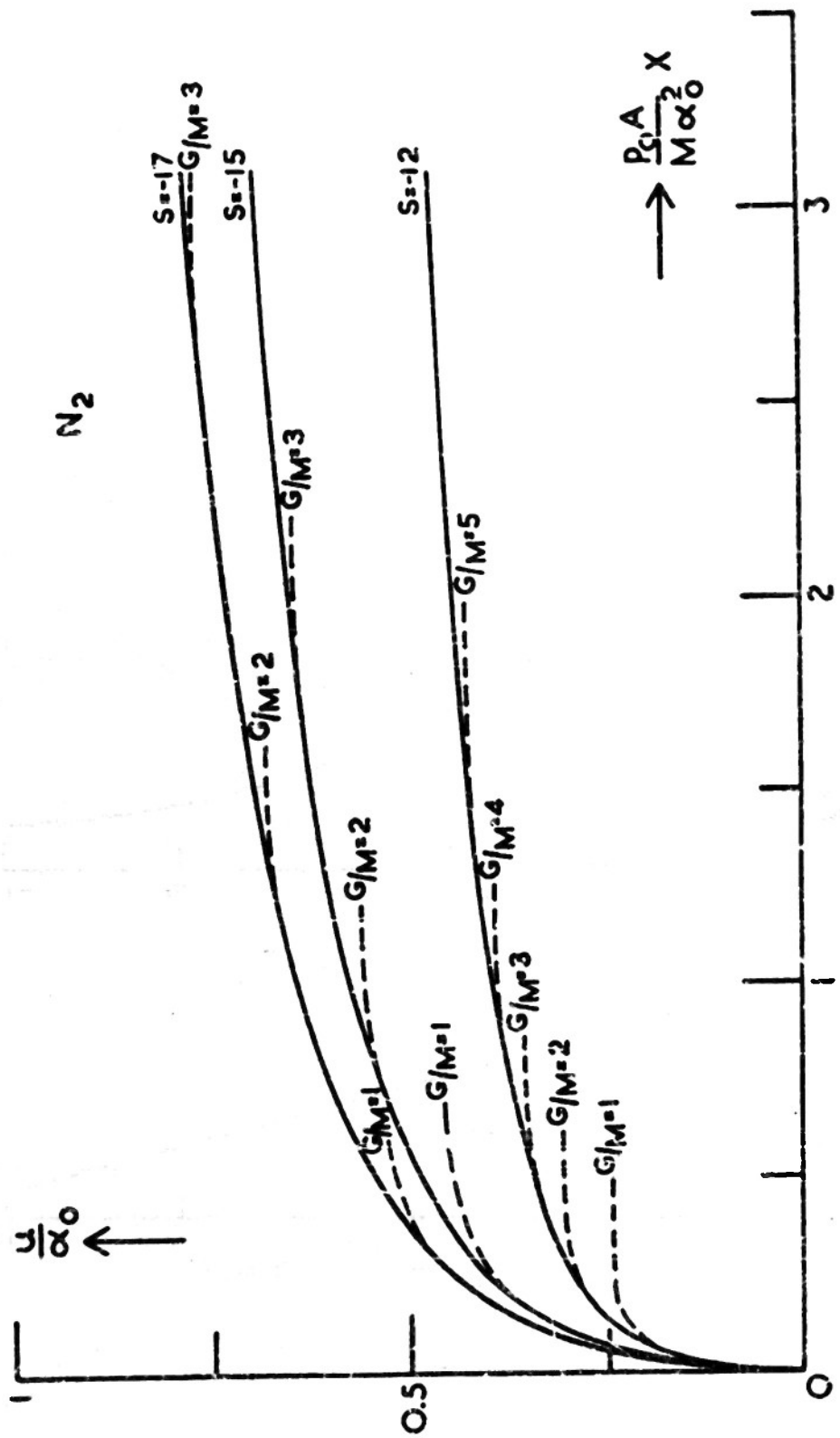


Fig. 2. Dimensionless piston velocity-travel plot for nitrogen.