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NOL HYPERSONIC TUNNEL NO. 4 RESULTS V
EXPERIMENTAL AND THEORETICAL INVESTIGATION
OF A COOLED HYPERSONIC WEDGE NOZZLE

13 APRIL 1953



U. S. NAVAL ORDNANCE LABORATORY
WHITE OAK, MARYLAND

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Aeroballistic Research Report 93

NOL HYPERSONIC TUNNEL NO. 4 RESULTS V.
EXPERIMENTAL AND THEORETICAL INVESTIGATION OF
A COOLED HYPERSONIC WEDGE NOZZLE

By

P. Wegener, R. K. Lobb, E. M. Winkler,
M. Sibulkin and H. Staab

ABSTRACT: A new water cooled, wedge-type nozzle has been in operation in the NOL 12 x 12 cm Hypersonic Tunnel No. 4 since January 1952. This report presents results of theoretical and experimental investigations covering the performance of this nozzle. It was found from measurements of pitot and static pressure that the nozzle produced a shock-free, almost isentropic expansion having a test-section Mach number distribution comparable in quality to that obtained in existing conventional nozzles at lower Mach numbers. The cooling system used was adequate to maintain the nozzle block temperatures at constant low values. The temperature distribution in a nozzle block was also measured and it showed a rise near the throat. This rise agreed qualitatively with theoretical calculations. A comparison of the calculated and measured rate of heat transfer to a nozzle block suggested that boundary layer transition occurs after the throat. A pressure and temperature survey at the end of a nozzle block where the boundary layer is turbulent showed that the velocity distribution is similar to that found at low speed. This survey was also used to calculate the overall heat transfer rate to the nozzle block. This result agreed with that obtained from the direct method of determining the heat transfer by measuring the cooling water rate and temperature increase.

In conclusion it is found that a hypersonic wedge nozzle produces shock free expansions and acceptable flow in the test section and serves well as a research tool.

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1
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13 April 1953

This is the fifth NAVORD Report on an investigation carried out in the continuous NOL 12 x 12 cm Hypersonic Tunnel No. 4. This facility was first put into operation in May 1950. The titles of the previous NAVORDS discussing results from the tunnel are:

- I Air Liquefaction. NAVORD 1742, 4 Jan 1951
- II Diffuser Investigation. NAVORD 2376, 5 May 1952
- III Diffuser Investigation with Models and Supports, NAVORD 2435, 1 July 1952
- IV High Supply Temperature Measurement and Control, NAVORD 2574, 8 Oct 1952

The present NAVORD (Results V) presents an experimental and theoretical discussion of a cooled high Mach number nozzle. The work was jointly sponsored by the U. S. Naval Bureau of Ordnance and the U. S. Air Force, Flight Research Laboratory.

The authors are indebted to Messrs. Charles E. White and E. J. Stollenwerk for the mechanical design of the nozzle. Prof. N. Hall's advice on the cooling problem is gratefully acknowledged. Messrs. L. Liccini and R. Garren participated during the tests and Messrs. J. Kendall, Jr. and H. Maxwell assisted in making the boundary layer calculations.

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NAVCOR Report 2701

OUTLINE

	Page
I. Introduction P. Wegener	1
II. Nozzle design and performance. . . R. K. Lobb	3
III. Turbulent boundary layer characteristics at end of nozzle P. Wegener	8
IV. Measurement of surface temperature and total heat transfer E. Winkler	13
V. Theory of heat transfer to nozzle and comparison with experiment M. Sibulkin	18
VI. Notation	30
VII. References	33
VIII. List of Manufacturers	37
IX. Appendix: Construction of nozzle and cooling system H. Staab	38

77

UNCLASSIFIED
NAVORD Report 2701

ILLUSTRATIONS

- Figure 1. The NOL 12 x 12 cm Hypersonic Tunnel No. 4.
Figure 2. Cooled Wedge Nozzle Block with Thermocouple Plugs Removed.
Figure 3. Cooled Wedge Nozzle for Tunnel No. 4.
Figure 4. Schlieren Photograph of the Flow Near the Throat of a Minimum Length, $M = 5.18$, Nozzle, $p_0 = 4.5$ atm, $T_0 = 333^\circ\text{K}$.
Figure 5. Shadowgraph and Schlieren Photographs of Flow in the Wedge Nozzle.
Figure 6. Mach Number Variation in Nozzle Exit vs Time for Changing Coolant Rate $p_0 = 10$ atm, $T_0 = 593^\circ\text{K}$.
Figure 7. Static Pressure Distribution along Centerline of Nozzle Sidewall, $M = 7.0$, $p_0 = 10$ atm, $T_0 = 593^\circ\text{K}$.
Figure 8. Mach Number Distributions along Surveys in a Plane One Inch Upstream from the Nozzle Exit, $p_0 = 10$ atm, $T_0 = 593^\circ\text{K}$.
Figure 9. Comparison of Measured and Calculated Boundary-Layer Growth along Nozzle Wall.
Figure 10. Measured Turbulent Boundary-Layer Profiles with Heat Transfer at a Free Stream Mach Number of 7.
Figure 11. The u^+ , y^+ Representation of the Measured Velocity Profiles at $M = 7$ and Comparison with Theory.
Figure 12. Temperature Distribution in Nozzle Inlet at Three Different Levels of Supply Temperature (Temperature in $^\circ\text{K}$).
Figure 13. Nozzle Wall-Temperatures Near Throat (Station No. 13) at Various Supply Temperatures, $p_0 = 10$ atm, $M_s = 7.6$. Cooling Rate: 6.5 Gallons/Min.
Figure 14. Nozzle Wall-Temperatures Near Throat (Station No. 13) at Various Supply Pressures, $T_0 = 610^\circ\text{K}$, $M_s = 7.6$. Cooling Rate: 6.5 Gallons/Min.
Figure 15. Wall Temperatures at 9 Stations, Temperature Gradient in Wall, and Extrapolated Surface Temperatures of Nozzle $M_s = 7.6$, $p_0 = 10$ atm, $T_0 = 593^\circ\text{K}$. Cooling Rate: 6.5 Gallons/Min.
Figure 16. Wall Temperatures at 9 Stations, Temperature Gradient in Wall, and Extrapolated Surface Temperatures of Nozzle, $M_s = 7.6$, $p_0 = 20$ atm, $T_0 = 593^\circ\text{K}$. Cooling Rate: 6.5 Gallons/Min.
Figure 17. Isotherms in Nozzle for $M_s = 7.6$, $T_0 = 593^\circ\text{K}$, $p_0 = 10$ and 20 atm, Cooling Rate: 6.5 Gallons/Min.

7 7

UNCLASSIFIED
NAVORD Report 2701

- Figure 18. Nozzle Wall-Temperatures as Function of Running Time at Boundary-Layer Survey Station, $M_g = 7.6$, $p_o = 21.4$ atm, $T_o = 593^\circ\text{K}$. Cooling Rate: 6.5 Gallons/Min.
- Figure 19. Temperature Drop in Nozzle Wall, $M_g = 7.6$, $p_o = 21.4$ atm, $T_o = 593^\circ\text{K}$. Cooling Rate: 6.5 Gallons/Min.
- Figure 20. Temperature Increase of Cooling Water as Function of the Cooling Rate for Tunnel Operation, $M_g = 8.0$, $p_o = 20$ atm, $T_o = 600^\circ\text{K}$.
- Figure 21. Heat Flow Rate to Cooling Water as Function of Cooling Rate for Tunnel Operation, $M_g = 8.0$, $p_o = 20$ atm, $T_o = 600^\circ\text{K}$.
- Figure 22. Coolant Tube Positions and Dimensions Used in Heat-Transfer Analysis.
- Figure 23. Variation of k/k_e with Coolant Tube Size and Position.
- Figure 24. Determining Nozzle Surface and Coolant Tube Surface Temperatures.
- Figure 25. Velocity Distribution along Nozzle, $M_g = 8.0$, $T_o = 600^\circ\text{K}$.
- Figure 26. Acceleration Distribution along Nozzle, $M_g = 8.0$, $T_o = 600^\circ\text{K}$.
- Figure 27. Heat-Transfer Coefficient Distribution Assuming Laminar Flow and Neglecting the Effects of Pressure Gradients, $M_g = 8.0$, $T_o = 600^\circ\text{K}$, $p_o = 20$ atm.
- Figure 28. Heat-Transfer Coefficient Distribution Assuming Turbulent Flow and Neglecting the Effects of Pressure Gradients, $M_g = 8.0$, $T_o = 600^\circ\text{K}$, $p_o = 20$ atm.
- Figure 29. Growth of Momentum Thickness Assuming Laminar Flow and Including the Effects of Pressure Gradients, $M_g = 8.0$, $T_o = 600^\circ\text{K}$, $p_o = 20$ atm.
- Figure 30. Heat-Transfer Coefficient Distribution Assuming Laminar Flow and Including the Effects of Pressure Gradients, $M_g = 8.0$, $T_o = 600^\circ\text{K}$, $p_o = 20$ atm.
- Figure 31. Growth of Momentum Thickness Assuming Turbulent Flow and Including the Effects of Pressure Gradients, $M_g = 8.0$, $T_o = 600^\circ\text{K}$, $p_o = 20$ atm.
- Figure 32. Heat-Transfer Coefficient Distribution Assuming Turbulent Flow and Including the Effects of Pressure Gradients, $M_g = 8.0$, $T_o = 600^\circ\text{K}$, $p_o = 20$ atm.
- Figure 33. Comparison of Heat-Transfer Coefficients Showing Effect of Type of Boundary-Layer and Pressure Gradients, $M_g = 8.0$, $T_o = 600^\circ\text{K}$, $p_o = 20$ atm.
- Figure 34. Installation of Thermocouple Plug in Nozzle Block.

v
UNCLASSIFIED

UNCLASSIFIED
NAVORD Report 2701

NOL HYPERSONIC TUNNEL NO. 4 RESULTS V:
EXPERIMENTAL AND THEORETICAL INVESTIGATION OF
A COOLED HYPERSONIC WEDGE NOZZLE

INTRODUCTION

1. During the initial planning phase (reference 1) of the NOL 12 x 12 cm Hypersonic Wind Tunnel (Figure 1), it was decided to use two plane blocks as a nozzle (Figures 2 and 3) producing a continuous expansion of the air. The Mach number in any section of such a "wedge nozzle", and in particular, the maximum Mach number in the exit section can then be easily varied by varying the nozzle area ratio only. This feature appeared to be advantageous for the study of such basic phenomena as air condensation (reference 4), needed pressure ratios to operate at high Mach numbers (reference 39), etc. It was planned to later attempt the design of a nozzle producing uniform flow in a greater region for testing larger models. Another reason for the initial selection of such a simple nozzle was the fact that boundary layer corrections of theoretical potential flow nozzle walls designed to produce such uniform flows are insufficiently known at high Mach numbers.

2. Experimentation with two types of wedge nozzles during the last 2-1/2 years has shown that these nozzles produce satisfactory flow, and a theoretical and experimental discussion of the nozzle performance with some comparison with uniform flow nozzles will be given in section II of this report.

3. To prevent air condensation in the wind tunnel at high Mach numbers, the supply air must be preheated. Even highly purified air does not supercool appreciably (reference 2) in contrast to the well known case of rapid expansions of water vapor in nozzles (reference 3). It is therefore advisable that the expansion of the air in the tunnel does not enter the condensation region. Wind tunnel supply temperatures and pressures for such operation are given on Figure 8 of reference 4. Although these temperatures are much lower than those which would simulate actual free-flight stagnation temperatures (compare Figure 1, reference 5), they are high enough to pose a severe

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NAVORD Report 2701

technical problem at Mach numbers above say $M = 8$. At the peak Mach numbers for practicable tunnel operation, one may therefore well resort to extreme purification to obtain some super-cooling and resultant saving in heating as pointed out by Nagamatsu (reference 2). In this report we will deal, however, mainly with Mach numbers of the order 7 to 8 where a moderate supply pressure range from 5 to 30 atmospheres at temperatures of the order 300 to 400°C makes continuous tunnel operation quite practical.

4. During operation at elevated supply temperatures, all components of the tunnel are being heated by conduction, forced convection and radiation. The nozzle and test section walls, for example, will tend to reach a temperature close to the recovery temperature after some time of operation. This recovery temperature is in turn of the order of the supply temperature. Depending on the local rate of heat transfer from the flow, the locally different heat capacity of the tunnel components, etc., the equipment will become non-uniformly hot. An example of such uneven heating of the nozzle surface and other components is given on Figures 4 and 6 of reference 5. It can be seen that the throat of the nozzle heats up much more rapidly than the nozzle end (compare section V). If supply pressure and temperature are held constant for a fixed Mach number, one expects that finally all tunnel components will reach some constant temperature. Such a constant temperature could not be attained in our tunnel after several hours of operation. On the other hand, the slowly increasing temperatures of all tunnel components cause warpage of metal parts, breakdown of all rubber based seals, glass breakage, etc. Warpage is particularly effective at the narrow nozzle throat where small area changes result in major Mach number changes in the test section, (see section II). In order to avoid these difficulties, retain the flexibility of Mach number change inherent in a two-dimensional wedge nozzle, and rapidly attain a constant temperature and geometry, a nozzle water-cooling system was designed and built, (Figures 1, 2 and 3). It was further found that the top of the test section frame had to be cooled (Figure 1) to obtain fixed base points for the nozzle bed. With such an arrangement, the following is found (see sections II and IV):

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- (a) Most of the nozzle surface remains near room temperature at all times. (In our tunnel the cooling water discharge is of the order 10 gallons per minute.)
- (b) Constant Mach number readings in the test section are attained after a few minutes of operation.
- (c) The tunnel can be operated continuously for hours.
- (d) Materials, gaskets, etc. do not undergo permanent changes and all operating conditions can be repeated accurately.

5. The following sections of this NAVORD report give an account of some of the theoretical and experimental work done to date on this nozzle.

II. NOZZLE DESIGN AND PERFORMANCE

6. Design Problems of Hypersonic Nozzles. The aerodynamic design of a potential flow nozzle for Mach numbers above 5.0 is the same as that for conventional supersonic nozzles. There are several methods (all based on theory of characteristics) for determining the contours of both two and three-dimensional nozzles. A two-dimensional nozzle can be designed by the graphical method of Prandtl and Busemann (reference 6) or by the semi-graphical method given by Puckett (reference 7 and 8). These methods are tedious and are subject to graphical error especially in the hypersonic range for which the characteristic net becomes greatly elongated. The analytical methods of Foelsch (reference 9) and Atkin (reference 10) are usually preferred for practical reasons because they are more accurate and can be used on high speed computing machines. In the three-dimensional case the design is complicated by the fact that the characteristic net in the hodograph plane has to be determined step by step along with the net in the physical plane. However, for axially symmetric flows there are available several graphical, numerical and analytical methods (see references 11, 12 and 13). A three-dimensional nozzle of arbitrary cross-section can be obtained by tracing the stream lines that outline the area throughout a known axially symmetric flow (see references 14 and 15). This method has the disadvantage that the cross-sectional shape so obtained

varies along the nozzle thus making it difficult to fabricate. It may be possible to correct the surfaces to give a constant cross-sectional shape by a linearized method of characteristics such as given by Ferri in reference 16. Another type of three-dimensional nozzle employing a square throat and expanding in two stages, first in one plane then in the plane at right angles, is described in reference 17.

7. One of the main practical problems in hypersonic nozzle design is the achievement of a smooth flow through the nozzle throat. For high Mach numbers the ratio of the test section area to throat area becomes very large. As a result for medium sized tunnels the throat must be a narrow slit or some configuration of equivalent area. This is illustrated in the table below. The throat must therefore be machined very accurately. The mechanical design is further complicated by the possible warpage of the throat due to the high surface temperatures

TABLE II-I

Throat and Exit Areas for Two and Three-Dimensional
(M = 8) Nozzles.

Nozzle	Test Area	Throat Area
Two-dimensional	 12 x 12 cm	 .063 x 12 cm
Axially Symmetric	 12 cm dia.	 .87 cm dia.
Square throat and exit plane	 12 x 12 cm	 .87 x .87 cm

that can occur in this region during "hot" tunnel operation (see section IV and V). Also the subsonic section must usually withstand high pressures while in the supersonic portion the pressure is very low. (The NOL hypersonic tunnel no. 4 operates at supply pressures up to 30 atm). High supply pressures are required in order to have continuum flow in the test section at high Mach numbers.

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NAVORD Report 2701

8. A two-dimensional nozzle was selected for the NOL hypersonic tunnel for the following reasons. (1) Nozzle boundary-layer corrections are not accurate at high Mach numbers (primarily due to the lack of skin-friction data, see reference 15). In the two-dimensional case the boundary layer can be allowed for approximately, by adjusting the nozzle blocks. For an axially symmetric or square nozzle, on the other hand, errors in the boundary-layer correction resulting in poor Mach number distribution, would require remachining the surfaces or discarding the nozzle. (2) Disturbances in an axially symmetric nozzle tend to focus along the axis. (3) A two-step, three-dimensional nozzle is unsatisfactory (see reference 17) because of boundary-layer effects. (4) The cost of fabricating a three-dimensional nozzle is considerably higher than for a two-dimensional nozzle.

9. For the initial tests in the hypersonic tunnel a two-dimensional wedge nozzle (straight diverging walls) was chosen in preference to a uniform flow nozzle. With this wedge nozzle a wide range of Mach numbers can be achieved by simply changing the throat opening without introducing disturbances due to incorrect nozzle contour. The flow in a hypersonic wedge nozzle, for small wall angles, closely approximates two-dimensional source flow (constant properties along circular arcs with centers roughly at the throat). In a uniform flow nozzle, on the other hand, the flow expands slowly along the nozzle walls as compared to that along the axis. This produces relatively strong "cross-flow" pressure gradients which cause the boundary layer on the sidewalls to build up along the centerline (references 19 and 20). Also a wedge nozzle of the same length and Mach number, as a uniform flow nozzle, has a smaller maximum wall angle. This reduces the possibility of flow separation.

10. Previous investigations in a minimum length uniform flow nozzle designed for $M = 5.18$ have shown that a small radius at the throat can lead to such flow separation and shock waves (see Figure 4). Here the throat radius is about 1/16" and the wall angle slightly downstream of the throat is 36.2° . In order to minimize this effect, the throat of the wedge nozzle was formed as a cylinder of 3/8" radius and the wall angle was kept to approximately $5-1/2^\circ$. Shadowgraph and schlieren photographs of the flow near the throat of the wedge nozzle for a Mach number setting of 5.0 are given in Figure 5. (This setting corresponds to the

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NAVORD Report 2701

largest throat opening possible with the present nozzle). From these photographs we can see that there is no separation or large disturbances in the flow field. The triangular pattern appearing in the schlieren photograph agrees roughly with that found graphically by the method of characteristics. The distortion of the throat in the shadowgram is optical and is due to the large density gradients in this region of flow.

11. When the tunnel is operated at high supply temperatures the nozzle blocks become heated and tend to expand, particularly in the throat region where the rate of heat transfer is large. The expansion is inward because the tunnel casing, to which the blocks are attached, remains relatively cold. As a result the throat area decreases and the nozzle exit Mach number increases with time. By introducing a cooling system (see Figures 1, 2 and 3) the nozzle dimensions can be kept constant after a short period of operation (For $p_0 = 10$ atm, $T_0 = 593^\circ\text{K}$ and $M = 7.0$ the nozzle blocks reach equilibrium after about ten minutes of operation). The sensitivity of Mach number to coolant rate is illustrated in Figure 6. It can be seen that without any nozzle cooling the Mach number increases with time. Nozzle cooling also tends to keep the boundary-layer characteristics constant with time.

12. Flow Calibration. An example of the flow calibration will be given for a Mach number of 7.0 only. However, the results are generally true for other Mach number settings. This particular setting corresponds to that chosen for the boundary-layer survey and the heat-transfer measurements given in sections III and IV.

13. The flow in a plane 1" upstream from the nozzle exit was surveyed with pitot and static pressure probes. Static pressures along the centerline of the nozzle sidewall were also measured. The pitot probes were made from .095" hypodermic tubing with a wall thickness of .01". The static pressure probes were made from the same tubing with an 80° cone attached to the tip. Four .025" orifices were located 20 probe diameters aft of the shoulder. The static orifices in the nozzle sidewall had an opening of .025". In order to measure the low static pressures near the nozzle exit (of the order of 2mm Hg) it was necessary to use a silicone oil micro-manometer equipped with an optical reading system. This instrument has an accuracy of better than 1/2% in the

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NAVORD Report 2701

range under consideration. A precision mercury manometer or calibrated aneroid type gauges were used for measuring the higher pitot pressures.

14. A plot of the static pressure distribution along the centerline of the nozzle sidewall in terms of the supply pressure, for an exit Mach number of 7.0, is given in Figure 7. There appears to be a smooth expansion along the entire length of the nozzle. The pressure gradient at the nozzle exit corresponds to a drop in Mach number of about .02 M/cm. This compares with the value of .03 M/cm derived from one-dimensional theory assuming inviscid flow throughout the nozzle in which case the growing boundary layer on the nozzle wall is neglected.

15. The Mach number distribution for various traverses in a plane 1" upstream from the nozzle exit as determined from pitot and static pressure measurements at the identical location using the Rayleigh formula are given in Figure 8. The maximum variation in Mach number, outside of the boundary layer, is 2%. In earlier surveys it was found that there were disturbances in the flow origination near the junctions of the nozzle throat with the sidewalls. By using metal to metal seals between the nozzle and sidewalls these disturbances could be practically eliminated as indicated by the smooth Mach number distributions. From these measurements we find the stagnation pressure loss throughout the nozzle to be about 4%. A schlieren photograph of the flow in the nozzle exit for a Mach number of 7.0 is given in Figure 5.

16. Boundary-Layer Growth. If at some station along the nozzle the Mach number in the free stream is known then we can estimate the boundary-layer displacement thickness on the nozzle walls by comparing the flow area obtained from isentropic flow tables with the geometrical area enclosed by the walls. (A one-dimensional analysis is sufficient here since it closely approximates the two-dimensional analysis for a wall angle of only $5-1/2^\circ$.) In the calculations we assume that the boundary layer is of equal thickness on all four walls and zero at the throat (in section V the displacement thickness is shown to be about .005 mm at the throat). Also since the stagnation pressure loss in this nozzle is small, we can determine the flow area with good accuracy from simply the ratio of the static pressure to the supply pressure using isentropic flow tables. Therefore, for each static pressure measurement on the nozzle sidewall we can obtain

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NAVORD Report 2701

a displacement thickness. The growth of the boundary layer displacement thickness along the nozzle obtained in this manner is compared to that calculated by Tucker's method (reference 18) in Figure 9. In the theory the walls are assumed to be thermally insulated, whereas the actual nozzle walls are cooled. This probably accounts for the experimental values being thinner (i.e. increased density near the wall) than those calculated by the theory. The displacement thickness determined directly from the boundary-layer survey (see section III) fits well with the other experimental points. It is interesting to note that the total thickness of the boundary layer at the nozzle exit obtained by Tucker's method is 24 mm which is close to the 26 mm measured in the boundary layer survey.

III. TURBULENT BOUNDARY-LAYER CHARACTERISTICS
AT END OF NOZZLE

17. In addition to the indirect determination of growth of the displacement thickness along the nozzle wall discussed in the last section, a detailed survey of the boundary-layer profiles at one point was undertaken. A survey was made perpendicular to the west nozzle wall one inch from the nozzle exit. The data were unaffected by the juncture of nozzle end and first diffuser plates. Since the position of the transition point along the nozzle is still unknown, it is interesting to note that at this survey station a fully developed turbulent boundary layer was found. The pertinent results are summarized in the following table.

TABLE III: Boundary-Layer Survey

Station: Center of west nozzle wall, 1" upstream of
nozzle exit.
Supply pressure: $p_s = 10$ atmospheres
Supply temperature: $T_0 = 593^\circ\text{K}$ (50° above that temperature
needed to avoid air condensation
throughout.)
Wall temperature at survey station: $T_w = 316^\circ\text{K}$ (cooling
system in operation)
Free stream temperature: $T_\infty = 56^\circ\text{K}$ ($T_w/T_\infty = 5.7$)
Free stream Mach number: $M_\infty = 7.0 \pm 0.01$
Free stream velocity: $u_\infty = 1040$ m/sec
Free stream Mach number gradient: Appx. 0.02 M/cm (compare
section II)

77

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NAVORD Report 2701

Reynolds number based on displacement thickness:

$$Re(\delta^*) = 43,000$$

Re based on equivalent flat plate length (calculated):

$$Re(x) \approx 10^7$$

Boundary-layer thickness at point where $u = 0.995 u_\infty$,

$$\delta = 26 \text{ mm}$$

Displacement thickness: $\delta^* = 10.5 \text{ mm}$

Momentum thickness: $\Theta = 1.24 \text{ mm}$

Energy thickness (see section V): $\Phi = 2.78 \text{ mm}$.

18. The above data were taken in the following manner:

Static pressure was measured by a hole 1/2 mm diameter in the wall and by a 5° cone-cylinder static probe in the free stream just outside the edge of the boundary layer. (Silicone oil manometers of about ± 2 microns measuring accuracy during tunnel operation were used.) Since agreement between these two static pressure was found to be of the order of 1 per cent, a constant value of static pressure was assumed to exist through the boundary layer.

19. Pitot pressure was surveyed from wall to free stream with a flattened hypodermic tube of about 0.18 mm opening. This opening is large enough to avoid errors due to slip flow effects on the impact pressure measurement in the region of low Reynolds numbers in the boundary layer near the wall (reference 21).

20. From measured pitot pressure and static pressure, the local Mach number could be determined from the Rayleigh formula (reference 8 p. 95). The Rayleigh Mach number is shown on Figure 10a. (The assumption of the validity of the perfect gas law implicit in this evaluation and the use of $\gamma = 1.4$ is warranted in our range of thermodynamic data (reference 22, Figure 19). The distance from the wall y , is made dimensionless by the boundary-layer thickness $\delta = 26 \text{ mm}$. At the measured point closest to the wall ($y = 0.1 \text{ mm}$), the flow is still supersonic ($M \approx 1.3$).

21. Total temperature was surveyed with a shielded, total temperature probe. This probe was calibrated in the free stream in approximately the range of Mach numbers, densities and temperatures encountered in the boundary-layer measurements. The probe recovery factor

$$r = \frac{T_t - T_\infty}{T_o - T_\infty} \quad (\text{III}, 1)$$

9
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is smaller than one because of instrumental errors due to radiation and conduction from the thermocouple junction to the surroundings. Such errors are particularly hard to overcome at the high Mach numbers (low test section density) and high temperatures (large radiation losses) in question. Using the measured free stream recovery factor to correct the boundary-layer total temperature measurement, we obtain Figure 10b. The measurement closest to the wall was at $y = 2$ mm. A linear total temperature distribution was assumed between this point and the measured wall temperature. From the measured Mach number and total temperature, the static temperature can be computed using the energy equation in the form

$$T = \frac{T_0'}{\frac{\gamma-1}{2} M^2 + 1} \quad (\text{III},2)$$

22. Finally the velocity profile may be calculated employing the definition of Mach number $M = u/a = u/\sqrt{\gamma RT}$ and obtaining

$$\frac{u}{u_\infty} = \frac{M}{M_\infty} \sqrt{\frac{T}{T_\infty}} \quad (\text{III},3)$$

This velocity profile is shown in Figure 10c which includes the results of four independent runs. A $1/7$ - power profile is indicated for comparison.

23. Since all flow properties in the boundary layer are now known, the integrands in the equations defining displacement, momentum and energy thickness, (see section VI), could now be calculated and plotted. The integrals themselves were determined using a planimeter. The results are given in Figure 10 and Table III,2. Comparing these ratios to those given by the estimates of either Tucker (references 18 and 35) or Eckert (reference 24) for the insulated flat plate case at the same M_∞ , we find the following qualitative effects of heat transfer to the wall:

TABLE III,2

$M_\infty = 7$

	Measured Case: $T_w/T_\infty = 5.7$	Estimate (reference 18 or 24), assuming insulated flat plate; $Pr = 1$, $1/7$ - power profile and $T_w/T_\infty = 10.8$.
δ^*/δ	.4	.61
θ/δ	.05	.027

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NAVORD Report 2701

The direction of deviation of the measured values from those derived theoretically is expected from the increased density near the wall in the experimental case with heat transfer. (Cooling of the wall has an effect qualitatively somewhat similar to suction.)

24. The Reynolds number based on free stream conditions and displacement thickness is 43,000. On the other hand an estimate of an equivalent flat plate Reynolds number was made assuming that free stream conditions of the survey station would have prevailed outside the boundary layer of an equivalent flat plate. Using the incompressible formula for boundary-layer thickness as function of Reynolds number or that obtained by Eckert (reference 24, equation 37) for a compressible case with unaltered velocity profile and no heat transfer, we obtain a flat plate Reynolds number of the order 10^7 . Conversely, our survey station would correspond to that taken on a plate with a turbulent boundary layer of about 1.5 m length and our free stream condition. Our small tunnel may then be taken to simulate conditions in a large tunnel capable of operating with a flat plate model of such length successfully, if the small pressure gradient and previous boundary-layer history do not affect a fully developed turbulent boundary layer.

25. In section IV it will be seen that the temperature in the steel nozzle blocks drops linearly with distance from the surface after a steady state of supply conditions and cooling system operation has been reached. The time history of such a measurement is shown on Figure 18. A cut parallel to the T - axis through this figure at some time produces plots of temperature distribution like that given on Figure 19. Since the heat conductivity of the nozzle material is known, the rate of heat transfer per unit area at the boundary-layer station could be determined from

$$q = h \frac{dT}{dy} \quad (\text{III}, 4)$$

On the other hand the heat transfer coefficient is defined by

$$q = h(T_e - T_w) \quad (\text{III}, 5)$$

UNCLASSIFIED
NAVORD Report 2701

26. We could now determine the Stanton number (sometimes called dimensionless heat transfer coefficient Ch_∞) from

$$St_\infty = \frac{h}{C_p \rho_\infty u_\infty} = \frac{k(dT/dy)}{C_p \rho_\infty u_\infty (T_e - T_w)}, \quad (III, 6)$$

if we knew the insulated flat plate temperature, T_e . However, since $T_e \approx T_0 \approx 600^\circ K$, we were not able to measure its value at present. We therefore calculated St_∞ in two ways. First we take the known approximate expression for the turbulent recovery factor (reference 29) $r = \sqrt[3]{Pr}$. We then choose $Pr = 0.73$ based upon thermodynamic properties near the wall and obtain $T_e = 539^\circ K$ and $St_\infty = 0.00075$. Secondly by using $Pr = 1$ (and thus $T_e = T_0 = 593^\circ K$), we obtain $St_\infty = 0.00061$.

27. To obtain the local friction coefficient we apply the relation (given in reference 26, equation 20).

$$Nu_\infty = \frac{C_f \sqrt{Re_\infty}}{2} \sqrt{Re_\infty} \sqrt[3]{Pr}, \quad (III, 7)$$

which may be transformed into

$$\frac{C_f}{2} = St_\infty Pr^{2/3} \quad (III, 8)$$

Using (III, 8) we obtain numerically $C_f = 0.0012$ for both combinations of Pr and T_e discussed above. We now calculate the shear stress at the wall

$$\tau_w = C_f \frac{\rho_\infty}{2} u_\infty^2 \quad (III, 9)$$

28. Finally, it is possible to use this value for the representation of the velocity profile in the parameters

$$u^+ = \frac{u}{\sqrt{\frac{\tau_w}{\rho_w}}}, \quad \text{and} \quad y^+ = \frac{y \sqrt{\frac{\tau_w}{\rho_w}}}{\nu_w} \quad (III, 10)$$

(reference 27). These parameters are based on wall density and kinematic viscosity since this plot is of

greatest interest near the wall. Figure 3 shows our measurement with the region based upon interpolated total temperature data given as a dashed line. For comparison, von Karman's incompressible flow semi-logarithmic law (reference 27) and assumed linear velocity distribution ($u^+ = y^+$) in the laminar sublayer are shown. Finally a compressible flow profile calculated from equation 72 of van Driest's analysis (reference 25) for our M_∞ and T_w/T_∞ is given. Aside from other assumptions inherent in van Driest's analysis, the calculated profile has the boundary condition $u = u_\infty$ at $y = \delta$, matching the measured profile at this point.

29. Summarizing, it may be stated that the boundary-layer characteristics near the nozzle exit are quite similar to those encountered in incompressible turbulent boundary layers. Although the heat transfer to the wall does not affect the velocity distribution materially, it alters the relative magnitudes of displacement and momentum thickness with respect to the insulated flat plate case. Finally, one cannot discern a laminar sublayer from our data possibly because we were unable to take total temperature measurements in that range which is roughly coincident with the estimated extent of the incompressible laminar sublayer.

IV. MEASUREMENT OF SURFACE TEMPERATURES AND TOTAL HEAT TRANSFER

30. Any experiments to determine recovery temperatures, heat transfer data, and many other types of investigations rely upon the accuracy with which the wind-tunnel supply temperature, T_0 , is known. The correct reference T_0 is assured if the supply temperature is uniform and constant over practically the entire nozzle inlet cross-section. The achievement of such conditions, Figure 12, was therefore a prerequisite to the experiments described in the following (reference 28).

31. Due to design (cooling system) and operational range (high M , high T_0) of the hypersonic tunnel, experimental arrangements to obtain wall surface temperatures or overall heat transfer data have to be different from those used at other places (reference 29). There is some

UNCLASSIFIED
NAVORD Report 2701

question about the precision that can be obtained if surface temperatures are measured with a temperature gradient in the nozzle wall induced by the nozzle cooling system. However, the results obtained show that practically steady-state conditions are reached within a few minutes after starting a tunnel run, and that errors due to conduction and radiation are probably of the same order of magnitude as the experimental scatter of the data.

32. Measurement of Surface Temperature. With the nozzle cooled by a measured rate of water flow through the cooling system (see section IX), a steady-state temperature distribution will be established in the nozzle wall. This distribution is a function of the temperature, pressure, Mach number, coolant flow, etc. of the tunnel operation. Depending on these variables and the amount of preheating done through the by-pass, it will take some time until practically steady-state temperatures are attained. If there are no serious non-uniformities in the wall material, no strong temperature gradients in the nozzle in the z-direction, and if the nozzle wall can be practically treated as a slab of fixed dimensions, then the temperature in the nozzle wall will drop linearly with distance from the nozzle surface. By measuring the temperatures at various depths, it should then be possible to obtain surface temperatures by linear extrapolation. In this fashion one avoids the insertion of measuring elements that may disturb the flow on the nozzle surface.

33. Temperatures of the nozzle wall can be measured at any of 13 points along the center line of each nozzle block with four thermocouples imbedded into the nozzle wall at various depths from the surface, (see Figure 2). To make installation and exchange of this arrangement more convenient, the thermocouples were inserted in separate plugs (see Figure 34). These plugs were ground to fit appropriate holes in the nozzle wall. With the plugs made of the same material as the nozzle wall, it is assumed that they do not cause a greater disturbance of the temperature field than does the presence of the thermocouples itself. To reduce conduction errors along the thermocouple wires in the plugs, they are led for a certain length behind the junction parallel to the nozzle surface, in the direction of the z-axis.

7 7

UNCLASSIFIED
NAVORD Report 2701

34. The e.m.f. output of the thermocouples is measured with Brown temperature recorders (model 153X11P28A1)¹ (Superscripts refer to the list in section VIII) or a K-2 type Leeds and Northrup² potentiometer, depending upon the temperature difference across the nozzle wall and the accuracy required. With the Brown recorders the temperature of the individual points can be read to $\pm 0.30^\circ\text{C}$ (reference 28). Using the K-2 type potentiometer wall temperatures and/or wall temperature differences can be read consistently to within 0.01°C during actual operation of the tunnel.*

35. Most of the tests described here were made with the cooling rate adjusted to 6.5 gallons per minute per nozzle block. With this rate practically steady-state readings are achieved in a few minutes. (A change of this cooling rate just gives a displacement of the temperature level.) In this case the cooling water passed through two passages near the throat (Figure 2). One passage at the nozzle exit was partially open and all the remaining ones were closed. Measurements of surface temperature at $M_3 = 7.6$ given here as example cover the temperature range $340^\circ\text{K} \leq T_0 \leq 615^\circ\text{K}$ (Figure 13) and the pressure range $9 \text{ atm} \leq p_0 \leq 15 \text{ atm}$, (Figure 14). These measurements were taken near the nozzle throat and it can be seen that in all cases the nozzle surface temperature (distance = 0 cm) is considerably below the tunnel supply temperature and equal to or slightly higher than room temperature. An increase in supply pressure (Figure 14) and in supply temperature (Figure 13) causes an increase of the steady-state nozzle surface temperature due to the increased rate of heat transfer for a fixed rate of coolant flow. Measurements at given p_0 or T_0 at nine stations along the center line of the

*This accuracy requires the use of selected, calibrated thermocouple wire, same wire material from hot junction to ice bath, Cu-wire from ice bath to potentiometer and galvanometer (i.e. the same as used within these instruments), careful and repeated balancing of the potentiometer circuit, achieved with Leeds and Northrup standard cell and a Willard constant potential storage battery), and finally a stable reference ice bath.

UNCLASSIFIED
NAVORD Report 2701

nozzle surface (Figures 15 and 16) were evaluated to extrapolate nozzle surface temperatures, wall temperature gradients (Figures 15 and 16) and to construct isotherms through the nozzle wall, (Figure 17). Finally measurements of wall temperatures at the boundary-layer survey station (see Figure 2) were repeated with higher precision using the K-2 type potentiometer. The results indicate a slow increase of the temperature at each thermocouple location with time, (Figure 18). This is possibly due to the fact that exact steady-state conditions cannot be reached, as long as radiative heat transfer from the uncooled sidewalls whose temperature increases slowly during the test is present. However, the fact that this temperature change is only about 0.7°K in 3/4 hours permits us to consider operation as practically stationary.

36 Assuming no longitudinal heat conduction in the nozzle wall, complete insulation of the nozzle block from sidewall and test section frame (a fact that is very nearly true), absence of radiation effects, etc., temperature-depth curves such as shown may be used to determine the local rate of heat transfer. (The rate of local heat transfer per unit area is given by the product of heat conductivity of the material times the temperature gradient) (see equation III,4). Figure 19 is a cross-plot of Figure 18 averaged for 10 different abscissa values to eliminate the small time effect. With $k_{\text{steel}} = 50.4 \text{ kcal/h m } ^\circ\text{K}$ and $\Delta T/\Delta y = 48.1 \text{ } ^\circ\text{K/m}$ from Figure 19, we find a local rate of heat transfer per unit area $q = 2424 \pm 73 \text{ kcal/m}^2 \text{ hr}$ at the boundary-layer survey station ($M_\infty = 7$). This rate of heat transfer at the test section is very low in contrast to the throat region, mainly due to the low test section density. (Compare e.g. the heat output of $675 \text{ kcal/m}^2 \text{ h}$ from a steam radiator under standard conditions, i.e. room temperature 24°C , steam temperature 110°C , reference 30, p. 91).

37. Tests to determine the total rate of heat transfer require again a constant p_0 , a constant and uniform T_0 and a steady-state temperature distribution across the nozzle walls. The quantities to be measured are the coolant rate, the temperature of the incoming or outgoing coolant, and the temperature increase of the coolant. The cooling rate is measured with a Flowrator, (see section IX), after the coolant pressure is equalized in the lines to both nozzle blocks. The temperature of the incoming

UNCLASSIFIED
NAVORD Report 2701

COOLING RATE (GALLONS/MIN)	WATER TEMPERATURE INCREASE (°K)	HEAT FLOW RATE ($6.27 \times 10^{-2} \text{ AT} \times \text{COOLING RATE}$) (10^3 KCAL/HR)
$M_s = 8.0 \quad p_o = 20 \text{ ATM} \quad T_o = 600^\circ\text{K}$		
$6.6 \pm 0.8\%$	$1.50 \pm 0.5\%$	2.25 ± 0.054
$6.5 \pm 0.9\%$	$1.50 \pm 0.5\%$	2.23 ± 0.054
$5.3 \pm 0.9\%$	$1.84 \pm 0.6\%$	2.20 ± 0.036
$5.2 \pm 1.0\%$	$1.89 \pm 0.8\%$	2.22 ± 0.040
$3.9 \pm 0.8\%$	$2.47 \pm 0.8\%$	2.17 ± 0.036
$3.8 \pm 0.9\%$	$2.55 \pm 0.7\%$	2.19 ± 0.036
$2.6 \pm 1.9\%$	$3.50 \pm 0.6\%$	2.07 ± 0.050
$2.5 \pm 0.8\%$	$3.68 \pm 0.9\%$	2.04 ± 0.036
$1.4 \pm 1.5\%$	$6.30 \pm 0.6\%$	1.92 ± 0.040
$M_s = 7.6 \quad p_o = 10 \text{ ATM} \quad T_o = 593^\circ\text{K}$		
$6.3 \pm 2.3\%$	$0.92 \pm 2.2\%$	1.32 ± 0.06

TABLE I-IV SUMMARY OF RESULTS OF TOTAL HEAT
TRANSFER MEASUREMENTS

UNCLASSIFIED
NAVORD Report 2701

and outgoing water is measured in the center of the inlet duct and in the center of both outlet ducts, respectively, with 30 gauge I.C. thermocouples. The thermocouples are immersed into the water for about 2.5 cm, with the unsupported portions coated with Glyptal and sealed into the support. Because of the range of temperature differences involved (order of 1° to 10°C), the thermocouple e.m.f. is measured with the K-2 type potentiometer. Measurements at various cooling rates, 1.5 gallons/min to 6.5 gallons/min per nozzle block, were made for $M_2 = 8.0$ and $p_c = 20$ atm, $T_c = 600^{\circ}\text{K}$ (Figure 20) the conditions for which the heat transfer will be calculated in the next chapter. The results are given in Figure 21 and Table I.

38. The errors given in Table I are those resulting from the scatter of the experimental data. Estimates show that in comparison to them errors due to conductive and radiative heat transfer to and from the nozzle surface and surrounding boundaries can probably be neglected. The nozzle blocks are well insulated from most of the surrounding metal parts by an air space, silicone rubber seals, and only a small band of metal-to-metal contact. (The temperature gradient across the metal-to-metal contact is comparable to that existing in the subsonic part of the nozzle, see Figure 17). Conduction errors are therefore believed to be negligibly small. Radiative heat transfer from the uncooled tunnel walls can also be neglected if the walls are kept smooth and unoxidized. For an order of magnitude estimate of the error we assume a pessimistic case where the sidewalls of the tunnel over their entire lengths are at supply air temperature while the nozzle walls are kept at room temperature. We assume further that the total energy radiated from the sidewalls is transferred to the nozzle neglecting the geometry. Using $Q_{\text{rad}} = A \sigma \epsilon (T_1^4 - T_2^4)$ (reference 31, p. 215), with $A = 360 \text{ cm}^2$ for the sidewall area, the Stefan-Boltzmann constant $\sigma = 1.37 \times 10^{-12} \text{ cal/sec cm}^2 \text{ }^{\circ}\text{K}^4$, $T_{\text{sidewall}} = 600^{\circ}\text{K}$, $T_{\text{nozzle}} = 300^{\circ}\text{K}$, we obtain Q_{rad} of 22 kcal/hr if the emittance of the sidewalls is 0.1. This upper bound of the radiative heat transfer from the sidewall to the nozzle surface amounts to only about 1 per cent of the total measured heat transfer across the nozzle wall, (Table I), and may therefore be neglected.

V. THEORETICAL HEAT TRANSFER CALCULATIONS AND
COMPARISON WITH MEASUREMENTS

39. Since the air flowing through the tunnel is heated and the nozzle walls are cooled, there is a continual flow of heat from the air, through the nozzle wall, into the cooling water. A steady-state heat transfer analysis can be applied to this problem (see section IV). This analysis can be subdivided into the following parts.

- (a) The heat transfer by forced convection from a high velocity, compressible fluid to a plane wall.
- (b) The flow of heat by conduction in a homogeneous solid of a particular geometry.
- (c) Forced convection to water flowing through tubes.
- (d) The transfer of heat by radiation from the uncooled sidewalls to the nozzle walls.

The amount of heat transferred by (d) may be neglected (see section IV).

A. Overall Heat Transfer

40. If we assume that there are effective adiabatic planes midway between the transverse cooling passages (Figure 22) and that all the heat which enters the nozzle wall in the length ΔL is removed by the cooling water, the overall heat transfer can be defined by the equation

$$q = UA_w(T_e - T_c) \quad (V,1)$$

Furthermore, we have

$$\frac{1}{UA_w} = \frac{1}{h_c A_c} + \frac{b}{k_e A_w} + \frac{1}{h_a A_w} \quad (V,2)$$

from, for example, reference 32, page 136. The quantities on the right-hand side of equation (V,1) represent respectively the resistance to heat flow of

UNCLASSIFIED
NAVORD Report 2701

the cooling water, the nozzle wall, and the air stream; they are discussed in detail below.

41. Heat transfer to cooling water. The heat transferred to the cooling water per unit area of nozzle surface is given by

$$\frac{q}{A_w} = \frac{A_c}{A_w} h_c (T_b - T_c) \quad (V,3)$$

In calculating h_c we assume

- (a) that the entire surface of the cooling passage is at the same temperature T_b ,
- (b) that the temperature rise of the water is small enough to neglect the change of u , μ , and Pr with temperature, and
- (c) that the water flow is turbulent (true for our Reynolds numbers). We select the heat-transfer correlation based upon experimental results for turbulent flow through smooth pipes suggested in reference 32, page 167.

$$Nu = 0.24 R^{0.8} Pr^{0.4} \quad (V,4)$$

Substituting the following values of fluid properties taken at 14°C (the temperature of our cooling water)

$$\begin{aligned} \mu &= 1.25 \times 10^{-3} \text{ kg/sec m,} \\ k &= 1.40 \times 10^{-4} \text{ kcal/sec m } ^\circ \text{K, and} \\ D &= .00762 \text{ m} \end{aligned}$$

in equation (V,4) gives

$$h_c = 7.60 W_c^{0.8} \quad (V,5)$$

It should be noted that in equation (V,5) W_c refers to the weight of water entering the center of the nozzle block (see section X-X, Figure 3). Half of this water flows through each side of the cooling passage.

Substituting equation (V,5) into equation (V,3) with $\Delta L = 0.0190$ m gives

$$\frac{q}{A_w} = 10.0 W_c^{0.8} (T_b - T_c) \quad (V,6)$$

42. Heat conduction in nozzle wall. The heat transferred through a homogeneous wall of constant thickness is given by

$$q = \frac{k A \Delta T}{x} \quad (V,7)$$

By analogy the heat transferred through the nozzle wall can be formally defined as

$$q = b \frac{k_e}{k} \frac{A_w}{b} (T_w - T_b) \quad (V,7a)$$

where the effects of the coolant tube size and position on the heat flow pattern in the wall are put into an "effective conductivity" k_e . Assuming the cooling passage to act as a heat sink and using the method of images, N. Hall found that for this particular case

$$\frac{k_e}{k} = \frac{\Delta L}{2\pi b} \ln \left[\frac{\sinh \sqrt{(\pi D/2\Delta L)^2 + (2\pi b/\Delta L)^2}}{\sinh(\pi D/2\Delta L)} \right] \quad (V,8)$$

Equation (V,8) is shown graphically in Figure 23. In our nozzle $b = 0.0159$ m and from Figure 23, $k/k_e = 0.915$. It is sufficiently accurate to use a mean value of k . Selecting 440°K as a mean wall temperature gives $k = 0.012$ kcal/sec m $^\circ\text{K}$, and substituting these values into equation (V,7a) gives

$$\frac{q}{A_w} = 0.825 (T_w - T_b) \quad (V,9)$$

43. Heat transfer from air to nozzle wall. The heat transferred to the nozzle wall is defined as

$$\frac{q}{A_w} = h_a (T_c - T_w) \quad (V,10)$$

The calculation of h_a is a major problem in itself and will be considered in detail later on.

7

UNCLASSIFIED
NAVORD Report 2701

44. Equations (V,6), (V,9), and (V,10) giving alternative expressions for q can be considered a system of three equations in the three unknowns q , T_w , T_b and could be solved analytically for particular values of T_e , T_c , W , and h . Alternatively, a graphical form of trial-and-error solution using Figure 24 may be more useful since it is usually quicker than an analytical solution and gives a better feeling for the effects of changes in the values of the parameters. To use this graph a value of q is assumed and, for the prescribed values of W and h , this fixes $T_e - T_w$, $T_w - T_b$, and $T_b - T_c$. The sum of these three temperature differences must equal the prescribed $T_e - T_c$. If they do not, a new value of q is chosen and the procedure repeated until agreement is reached.

B. Calculation of h_a

45. The calculation of h_a must be based upon the use of the boundary-layer concept, i.e. the effects of heat transfer to the walls are confined to the portion of air flowing near the walls leaving a core of air in the center of the tunnel having the pressure and temperature corresponding to an isentropic expansion from supply conditions. Unfortunately, it has not been possible to experimentally determine the point of transition from a laminar to a turbulent boundary layer in our wedge nozzle. It is known that the boundary layer at the exit of the nozzle is turbulent (see section III); however, the strong favorable pressure gradient and the extraction of heat from the boundary layer are likely to delay transition (reference 33), and it is quite possible that the boundary layer is laminar at the throat. For these reasons, calculations will be made assuming either a laminar or a turbulent boundary layer throughout for a typical case $M_s = 8.0$, $p_0 = 20$ atm., and $T_0 = 600^\circ\text{K}$.

46. The velocity distribution along the edge of the boundary layer is given with sufficient accuracy by assuming one-dimensional flow based upon the geometric area (see section II). This velocity distribution is shown in Figure 25 together with an outline of the nozzle walls. It can be seen that most of the velocity change takes place near the throat. This is shown again by plotting the local value of air acceleration along the nozzle (Figure 26); the peak value is approximately 10^7 g's.

47. The simplest approach to the boundary-layer problem is to neglect the pressure gradient (and any temperature gradient) along the nozzle. The nozzle is then hypothetically divided into sections, each of which has a different but constant velocity to which the flat plate (i.e., zero pressure gradient) boundary-layer laws are applied. This was done by calculating local values of Reynolds number along the nozzle. The length used in computing the Reynolds number was measured from the nozzle inlet screen (the point shown as 0 meters in Figure 25). The local skin friction coefficients were found from

$$\begin{aligned} c_f &= 0.664 Re^{-1/2} \text{ (laminar)} \\ \text{or} \quad c_f &= 0.059 Re^{-1/5} \text{ (turbulent)} \end{aligned}$$

depending upon the type of flow assumed. The heat transfer was then assumed to be given by the modified Reynolds' analogy (see paragraph 27)

$$St Pr^{2/3} = \frac{c_f}{2} \quad (V,11)$$

where St is based upon free-stream conditions. The heat transfer coefficient can now be found from

$$h_a = St c_p \rho u = St Pr^{2/3} \frac{c_p (\rho u)^*}{Pr^{2/3}} \frac{A^*}{A} \quad (V,12)$$

Using equation (V,11), $c_p = 0.247 \text{ kcal/kg } ^\circ\text{C}$ and $Pr = 0.68$ (for $t^* = 500^\circ\text{K}$),

$$h_a = 2130 c_f \frac{A^*}{A} \quad (V,13)$$

is obtained. Since A^*/A varies much more rapidly with distance along the nozzle than c_f , the heat transfer coefficient will have its maximum value near the nozzle throat (since A^*/A is greatest at $M = 1$). This is shown in the graphs of h_a vs. x , assuming either turbulent or laminar flow, given in Figures 27 and 28. Since the value of h_a given by equation (V,13) approaches infinity as x approaches 0, it was felt that as our initial point was chosen somewhat arbitrarily it would be undesirable to use equation (V,13) too near it. Instead the value of h_a from

UNCLASSIFIED
NAVORD Report 2701

$x = 0$ to $x = 0.016$ m ($M = 0.01$) was arbitrarily taken as the value at the latter point. This is also done on all subsequent graphs of h_a vs. x .

48. Because the analysis given above does not consider the effects of longitudinal pressure gradients on the growth of the boundary layer and on h_a , its validity near the nozzle throat (where there is a strong pressure gradient) is doubtful. For this reason further calculations were made by methods which include some of the effects of pressure gradients. Again separate calculations were made assuming either a completely laminar or completely turbulent boundary layer.

49. In both cases the methods used were based upon the momentum - integral equation (reference 34).

$$\frac{d\theta}{dx} + \left[\frac{H+2}{u_\infty} \frac{du_\infty}{dx} + \frac{1}{\rho_\infty} \frac{d\rho_\infty}{dx} \right] \theta = \frac{\tau_w}{\rho_\infty u_\infty^2} \quad (V,14)$$

For the laminar boundary layer the method of Howarth (reference 35) as modified by Low (reference 36) was used. This method is based upon the assumption of a one parameter family of boundary-layer velocity profiles using the Pohlhausen parameter λ which is a function of the local boundary layer thickness and pressure gradient. Since in our case, values of λ larger than the maximum given in reference 36 were obtained, the method was extended using some of the results given by Thwaites (reference 37). Low's analysis also includes the following assumptions: (a) constant T'_0 across the boundary layer (the result of assuming zero heat transfer and $Pr = 1$), (b) $\mu/\mu_\infty = C (T/T_\infty)$, and (c) constant T_w along the nozzle surface (implied by assumption (a)). Since this method involved a changing velocity profile, both the growth of θ with x and the value of τ_w for a given θ are affected by the value of the local pressure gradient. Equations (V,11) and (V,12) were used to compute h_a since no better relation exists between heat transfer and skin friction for compressible flow. The variation of θ and h_a with x assuming laminar flow is shown in Figures 29 and 30.

50. For the assumption of turbulent flow, the method of Tucker (reference 38) was used. In this analysis a

constant boundary-layer profile given by the power law

$$\frac{u}{u_{\infty}} = \left(\frac{y}{\delta}\right)^{\frac{1}{N}} \quad (V,17)$$

is used, and the skin friction is assumed as

$$\frac{\tau_w}{\rho u^2} = \frac{0.0131}{Re^{1/7}} \quad (V,18)$$

The length Reynolds number along the nozzle surface is of the order of 10^6 , and consequently a value of $N = 7$ was used. In equation (V,18), ρ and Re were based upon stagnation temperature. Since reference 38 assumes zero heat transfer and a "turbulent Prandtl number" equal to 1, T_0 is constant across the boundary layer and equal to T_{∞} . The tabulated values given in reference 38 begin at $M = 0.1$. For the initial growth of the boundary layer where $M < 0.1$ an incompressible form of the momentum integral equation was used. Assuming

$$\frac{dp_{\infty}}{dx} = 0, \quad H = \frac{9}{7} \quad \text{and} \quad \frac{\tau_w}{\rho_{\infty} u_{\infty}^2} = \frac{0.0252}{\left(\frac{u_{\infty} \theta}{\nu_{\infty}}\right)^{1/4}}$$

and integrating equation (V,14) gives

$$\theta^{5/4} = \frac{0.0157 \nu_{\infty}^{1/4}}{u_{\infty}^{4.11}} \int_0^x u_{\infty}^{3.86} dx + \left(\frac{u_{\infty 0}}{u_{\infty}}\right)^{4.11} \theta_0^{5/4} \quad (V,19)$$

from which the growth of the boundary layer can be computed (the subscript 0 refers to conditions at the chosen initial point). Since θ_0 appears only in the last term of equation (V,19), it is evident that since $u_{\infty 0}/u_{\infty}$ decreases rapidly downstream of the starting point (see Figure 25), the effect of the initial value of θ becomes negligible. This means that the calculated thickness of the boundary layer at the nozzle throat is practically independent of the assumed distance from the throat to the boundary-layer starting point. The values of h_a were again computed using equations (V,11)

UNCLASSIFIED
NAVORD Report 2701

and (V,12). The variation of θ and h_a with x assuming turbulent flow is shown in Figures 31^a and 32.

51. Boundary-layer thickness at nozzle throat. It should be noted that despite our small nozzle throat opening (equal to 0.025" at $M_g = 8.0$) the boundary layer at this point is still relatively thin. Assuming turbulent flow, the calculation of paragraph 50 gives

$$\frac{2 \times \delta^* (\text{throat})}{\text{throat opening}} = \frac{2 \times 0.00016}{0.025} = 0.013$$

52. All the heat-transfer coefficient distributions which have been calculated are shown in Figure 22 where they can be more easily compared. The average heat-transfer coefficient $\bar{h}_a$ has been calculated for each case, and the values are tabulated in Table V-I together with the maximum value of h_a (in units of kcal/sec m² °K). The value of $\bar{h}_a$ is important in determining the overall

TABLE V-I

Case	$\bar{h}_a$	h_a (max.)
Laminar with pressure gradient	0.033	2.0
Laminar without pressure gradient	0.0091	0.18
Turbulent with pressure gradient	0.067	3.3
Turbulent without pressure gradient	0.060	1.5

cooling requirements for the nozzle, while the maximum value of $\bar{h}_a$ is important in determining the maximum temperature on the nozzle surface. In some cases the maximum surface temperature (which occurs very near the throat) may be more of a design limitation than the overall cooling load. It can be seen from Table V-I that including the effect of pressure gradients in the laminar boundary-layer case triples $\bar{h}_a$ and multiplies h_a (max.) by ten.

UNCLASSIFIED
NAVORD Report 2701

In the turbulent boundary-layer case, including the effect of pressure gradients doubles h_a (max.) but only increases $\bar{h}_a$ slightly.

C. Estimation of T_w at Nozzle Throat

53. The nozzle surface temperature T_w could now be calculated using Figure 24 and the values of h_a found in the preceding section. Near the throat, however, h_a varies rapidly while Figure 24 is based upon having a constant value of h_a over the length ΔL . Nevertheless, conservative estimates of T_w at the throat (in the sense of giving too high a value of T_w) have been made using the four values of h_a (max.) previously calculated. The calculation is also conservative because it neglects the conduction of heat in the nozzle wall parallel to the surface away from the maximum temperature point. The calculation was made for $T_e = 567^\circ\text{K}$ (based upon a recovery factor of 0.88), $T_c = 287^\circ\text{K}$, and $W_c = 3$ gal/min (the estimated coolant flow rate through the cooling passages on each side of the throat; see section IV). The computed values of T_w and T_b at the nozzle throat are shown in Table V-II.

TABLE V-II

Case	T_w ($^\circ\text{K}$)	T_b ($^\circ\text{K}$)
Laminar with pressure gradient	516	341
Laminar without pressure gradient	354	303
Turbulent with pressure gradient	538	346
Turbulent without pressure gradient	497	337

Except for the assumption of laminar flow without pressure gradient, Table V-II shows that most of the temperature drop takes place in the steel nozzle block.

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NAVORD Report 2701

D. Comparison of Calculated Values of Heat Transfer with Measurements.

54. Using the measured nozzle surface temperature (see section IV) and the estimated values of T_w at the throat, the rate of heat transfer to the nozzle block can be calculated from

$$Q = \bar{h}_a A (T_e - T_w) \quad (V, 20)$$

where $T_e - T_w$ is an average temperature difference along the nozzle surface. For a coolant flow rate of 6.5 gal/min a value of $T_e - T_w$ equal to 225°K was used (where T_e is based upon a recovery factor of 0.88). The values of Q calculated from equation (V,20) and the measured value of Q (Figure 21) to one nozzle block for a coolant rate of 6.5 gal/min per nozzle block are shown in Table V-III.

TABLE V-III

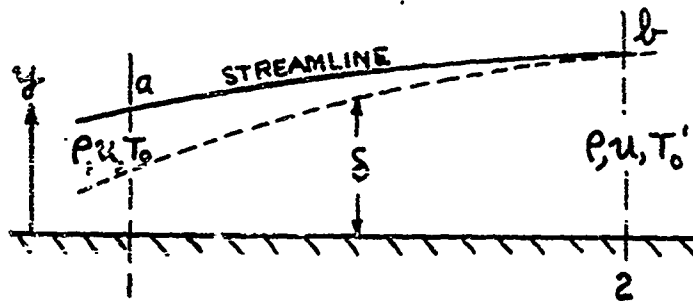
Case	Q (kcal/sec)
Laminar with pressure gradient	0.58
Laminar without pressure gradient	0.16
Turbulent with pressure gradient	1.2
Turbulent without pressure gradient	1.0
Experimental value	.62

It can be seen that the calculated value of Q based upon the assumption of a completely laminar boundary layer and including the effects of pressure gradients is closest to and lower than the measured value. This is the condition which should prevail if transition from a laminar to a turbulent boundary layer occurred

several centimeters downstream of the nozzle throat since the difference in heat transfer between laminar and turbulent flow downstream of such a transition point would be small compared to the heat transfer from the laminar boundary-layer upstream of the point.

E. Estimation of Heat Transfer Rate from Boundary-Layer Measurements.

55. It is interesting to note that an estimate of the total heat transfer rate to the nozzle walls for $M_s = 7.6$, $p_0 = 10$ atm., and $T_0 = 593^\circ\text{K}$ can be obtained from the boundary-layer measurements given in section III. This heat transfer rate can then be compared with the value (given in section IV) obtained from the cooling water temperature rise for the same conditions. Consider the growth of a boundary layer from stations 1 to 2 as shown in the sketch below.



It is assumed that the streamline $\bar{ab}$ is always outside of the velocity and temperature boundary layers. It is also assumed that T_0' is constant and equal to T_0 across the boundary layer at station 1, and that c_p is constant. From the conservation of energy law, the heat transfer Q to the wall between stations 1 and 2 must equal the difference in the energy flowing past station 1 and station 2,

$$Q = w c_p T_0 \int_0^a \rho_1 u_1 dy - w c_p \int_0^b \rho_2 u_2 (T_0') dy \quad (V, 21)$$

but from the continuity law

$$\int_0^a \rho_1 u_1 dy = \int_0^b \rho_2 u_2 dy \quad (V, 22)$$

combining equations (V,21) and (V,22) gives

$$Q = w c_p \int_0^b \rho_2 u_2 [T_0 - (T_0')_2] dy \quad (V,23)$$

Equation (V,23) can be used to define an energy thickness ϕ such that

$$Q = w c_p (\rho_\infty u_\infty)_2 (T_0 - T_{w_2}) \phi_2 \quad (V,24)$$

where

$$\phi_2 = \int_0^b \left(\frac{\rho u}{\rho_\infty u_\infty} \right)_2 \left[\frac{T_0 - (T_0')_2}{T_0 - T_{w_2}} \right] dy$$

In applying equation (V,24) to our nozzle, station 1 was taken at the nozzle inlet and station 2 at the boundary-layer measurement station 1 inch upstream of the nozzle exit. The heat transferred to the 1 inch portion of the nozzle block downstream of this station is negligible compared to the total. A more serious error of undetermined magnitude results from neglecting the variation of T_0' across the nozzle inlet station. Consequently, the value of Q obtained must be considered approximate. A comparison of Q (for one nozzle block) obtained from equation (V,24) and the value of ϕ measured in section III, and from the cooling water temperature rise (see section IV) is shown in table V-IV.

TABLE V-IV

Type of Measurement of Heat Transfer Rate	Q(kcal/sec)
Boundary-layer energy deficiency	0.33
Cooling water temperature rise	0.36

The agreement between these two measurements is probably within the experimental accuracy.

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VI. Notation

A	flow area
A_c	surface area of coolant tube ($=\pi D_w$)
A_w	nozzle surface area per coolant tube ($=\Delta L \times w$)
b	perpendicular distance from nozzle surface to center of coolant tube
c_f	local skin-friction coefficient
c_p	specific heat at constant pressure
D	diameter of coolant tube
h_a	local heat-transfer coefficient for the air
h_c	heat-transfer coefficient for the coolant
$\bar{h}_a$	average air heat-transfer coefficient ($= \frac{1}{L} \int_0^L h_a dx$)
$\bar{h}$	equals δ^*/ϵ
k	thermal conductivity
k_e	"effective thermal conductivity" defined by Eq. (V,8)
L	length
ΔL	distance along nozzle surface between "effective adiabatic planes" (See Fig. 22)
M	Mach number
M_g	nozzle Mach number setting
Nu	Nusselt number ($= hL/k$)
p	static pressure
p_o	supply pressure
p'_o	pitot pressure (in supersonic flow the stagnation pressure behind a normal shock)

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NAVORD Report 2701

Pr	Prandtl number ($= c_p \mu / k$)
q	rate of heat transfer per unit area
Q	rate of heat transfer
r	recovery factor ($= [T_w - T_\infty] / [T_o - T_\infty]$)
R	gas constant
Re	Reynolds number ($= \rho u x / \mu$)
St	Stanton number ($= h / \rho u c_p$)
T	static temperature
T _o	supply temperature ($\approx$ free stream stagnation temperature)
T' _o	local stagnation temperature
T _i	temperature indicated by stagnation temperature probe (uncorrected)
T _e	equilibrium temperature (also called recovery temperature)
T _b	coolant tube surface temperature
T _c	coolant temperature
u	velocity
w	nozzle width
W _c	weight flow rate of coolant
x	distance in flow direction
y	distance perpendicular to flow direction
γ	ratio of specific heats
δ	boundary-layer thickness
δ^*	boundary-layer displacement thickness ($= \frac{1}{\rho_\infty u_\infty} \int_0^\infty (\rho u_\infty - \rho u) dy$)
Θ	boundary-layer momentum thickness ($= \frac{1}{\rho_\infty u_\infty^2} \int_0^\infty \rho u (u_\infty - u) dy$)

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μ viscosity
 ν kinematic viscosity
 ρ density
 τ shear stress
 ϕ boundary-layer energy thickness (defined
by Eq. (V, 24))

Subscripts

w at the nozzle surface
 ∞ in the free stream

Superscripts

* at $M = 1$

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UNCLASSIFIED
NAVORD Report 2701

VIII. LIST OF MANUFACTURERS

1. Brown Instrument Co.,
4539 Wayne Avenue
Philadelphia 44, Pennsylvania
2. Leeds & Northrup
4901 Stenton Avenue
Philadelphia 44, Pennsylvania
3. Potomac Rubber Co.,
1421 H Street, N. W.
Washington, D. C. Manufactured by:
Connecticut Hard Rubber Co.,
407 E Street
New Haven 9, Connecticut
4. Johns Manville Co.,
22 E. 40th Street
New York 16, N. Y.
5. Fisher & Porter Co.,
Hatboro,
Pennsylvania
6. General Electric Corporation
Schenectady 5,
New York
7. V-B Ames C.,
Fremont,
Ohio
8. Bausch and Lomb Optical Co.,
Rochester,
New York

IX. APPENDIX

Construction of Nozzle and Cooling System

1. The nozzle blocks shown in Figures 1 and 2 were machined from a solid block of steel purchased under the Navy Department Specification 46 SiJ-Grade M. This material has the following chemical composition: 0.31% carbon, 0.25% Mn, 0.045% P, 0.055% Si, 0.35% Cu and 0.25% Ni. The finish machining of the surfaces of the nozzle in contact with the flow was accomplished by surface grinding parallel to the centerline of this surface. The surfaces were made flat to within 0.00025 inches, and smooth within an average of 16 RMS (root-mean-square) micro inches measured perpendicular to the nozzle centerline or flow direction and 5 RMS micro inches parallel to nozzle centerline. The inlet or subsonic portion and the .375 inch throat radius portion were ground perpendicular to the nozzle using a contour-shaped wheel. During this operation the nozzle blocks were set up side-by-side so that each cut would traverse both blocks; thus producing identical throat radii. During this set-up the nozzle alignment surface was ground, thus providing means for accurate alignment of the nozzles with respect to each other when installed in the tunnel. After grinding, final polishing of the throat was done by hand to produce a 5 RMS micro inch finish. Particular care was taken so that there would be no irregularity at the line of tangency of the supersonic plane surface and the throat radius.

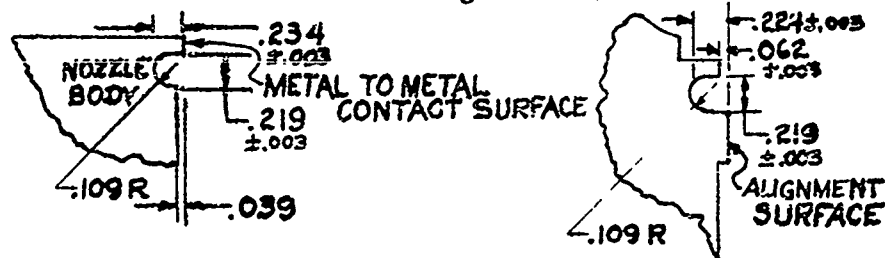
2. The method of aligning and supporting the nozzle is shown in Figure 1. The nozzle clamping linkage, which is actuated by a jackscrew mechanism at its lower end, is geometrically arranged such that it exerts one major force component on the nozzle in the vertical direction to seat the nozzle against its upper alignment surface, and another component to set it firmly on its 4 adjustable legs. Alignment of the nozzle crosswise of the case is controlled by two alignment pins which protrude from the bottom surface of the nozzle and engage in slots in the case.

3. The change of Mach number setting is accomplished by adjusting the two screw type legs nearest the nozzle throat using feeler gage stock in either side of the

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nozzle throat. The final throat is measured with vernier calipers graduated to one thousandth of an inch. The accuracy of the throat setting is within 0.0005 inches by checking with a series of feeler gages using a "go and no go" system. The accuracy of the final opening measurement of the nozzle is no better than 0.003 inches because of the wedge angle of the nozzle and the human error of locating repeatably the final opening.

4. The dimension of the gasket grooves, as shown below were determined to both retain the gasket material and to allow the door to close to a metal-to-metal seat while affecting a seal.



NOZZLE GASKET GROOVE
PARALLEL TO FLOW

GASKET GROOVE BETWEEN
NOZZLE BODY & TUNNEL CASE

The gasket material used throughout the tunnel is "Cohrlastic Silicone Rubber R-8925"³ which was purchased in 1/4 inch diameter extruded cord form. The best tolerance available on the diameter is $\pm .015$ inches, to this date.

5. A portion of the tunnel that is intimately associated with the cooling wedge nozzle is the air supply inlet system, (reference 28), Figure 12. This arrangement consists of a baffle system followed by a honeycomb in an insulated chamber for mixing the air to obtain a nearly uniform temperature distribution ahead of the nozzle throat. A stainless steel screen in a "transite"⁴ frame was found desirable to prevent clogging of the throat and probes by foreign particles, chips of the ceramic insulators and oxide flakes, from the exposed elements of the electric heaters. This system produces a nearly uniform supply temperature at the nozzle throat.

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Cooling System

1. The internal cooling passage arrangement of the nozzle block, as shown in Figure 3, consists of a central inlet-water-distribution passage with 34 holes of 0.312 inch diameter located $5/8$ inches from the air-flow surface. These holes connect this inlet passage to each of the two side outlet passages and can be easily closed (e.g. by removal of the side plates and stopping the holes with rubber cord), or restricted by insertion of an orifice plate.

2. The exterior piping arrangement, as shown in Figure 1, is made up of standard 1 inch copper water tubes. The water supply main is located beneath the floor on the lower left. It is controlled by a valve attached to a removable "T" handle (not shown). The water is fed to the right side of the tunnel through the tube near the floor on the far side of the tunnel. The inlet water pressure was equalized by the right-angle valves and the rate of flow is measured, in the nozzle under study by a variable area type flow Meter⁵ on the intake pipe shown at left. The two exhaust water pipes coming from each nozzle converge and join immediately outside of the tunnel. The flow from each nozzle is throttled separately by valves located at the level of the circular window on the far side of the tunnel.

3. The coolant used is tap water. For the usual flow rate of 6.5 gallons per minute the inlet water pressure is 6.5 psi and the outlet pressure is throttled to maintain a pressure of 1 psi.

Thermocouple installation

1. The nozzle temperatures are measured by iron-constantan thermocouples made of #30 gage wire² and electric welded without flux using a Variac and Pyrotip burner⁶ with a carbon electrode. The thermocouples were secured in place using technical "B" copper cement⁷ in a modified taper pin, shown in Figure 34. The exact location of each of the four thermocouple junctions in

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each plug is determined to 0.01 millimeters after assembly by use of a tool maker's microscope⁸ (Bausch and Lomb - Type UK 352). Special care is taken in the manufacture of the plugs so that each plug would bottom in its respective hole in the nozzle block.

Plug #	1	2	3	4	5	6
Location from Final Nozzle Opening (Inches)	.656	1.406	4.406	7.406	9.656	11.906
	7	8	9	10	11	12
	13	14.156	16.406	18.656	20.156	21.656
					23.156	24.156

TUNNEL SIDEWALLS

1. Several door arrangements are available for the tunnel. There is one set of blank steel doors, one steel door with 42 centerline pressure taps located one inch apart, a set of doors having a rectangular plate glass window, and a set of doors having 3 circular windows (see window in Figure 1). The pressure tap door and the circular window doors are provided with two sets of hinge arms so that when the door is inverted the tap holes and windows bisect their former location. The circular windows now in use are cut from selected commercial plate glass. Experience indicates, that while running at supply temperatures of 900°F these windows can be used anywhere from the nozzle exit through the diffuser for a maximum period of 20 minutes without fracture. A fused quartz window of similar arrangement is now being tested to increase the time of exposure.

2. When the doors are closed they are forced by bolting against the raised nozzle lips, which forms the edges of flow surface and against the metal-to-metal bearing surface, of the tunnel case (see Figure 1 outside of gasket). In addition to bolting around the periphery,

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the doors are reinforced in the high pressure region,
by a retractable bar and jackscrew arrangement (shown
in Figure 1).

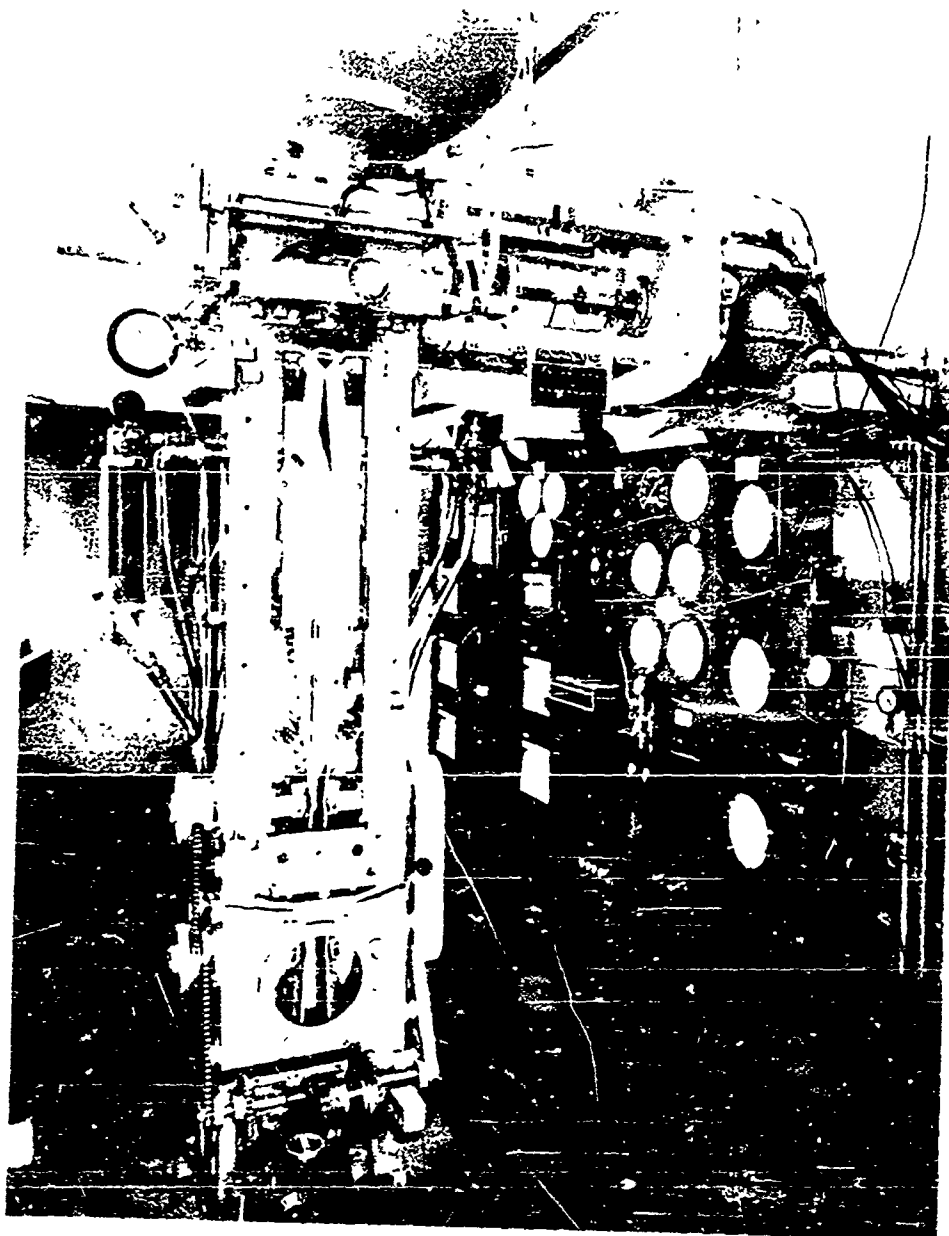


FIG.1 THE NOL 12X12 CM HYPERSONIC TUNNEL NO.4

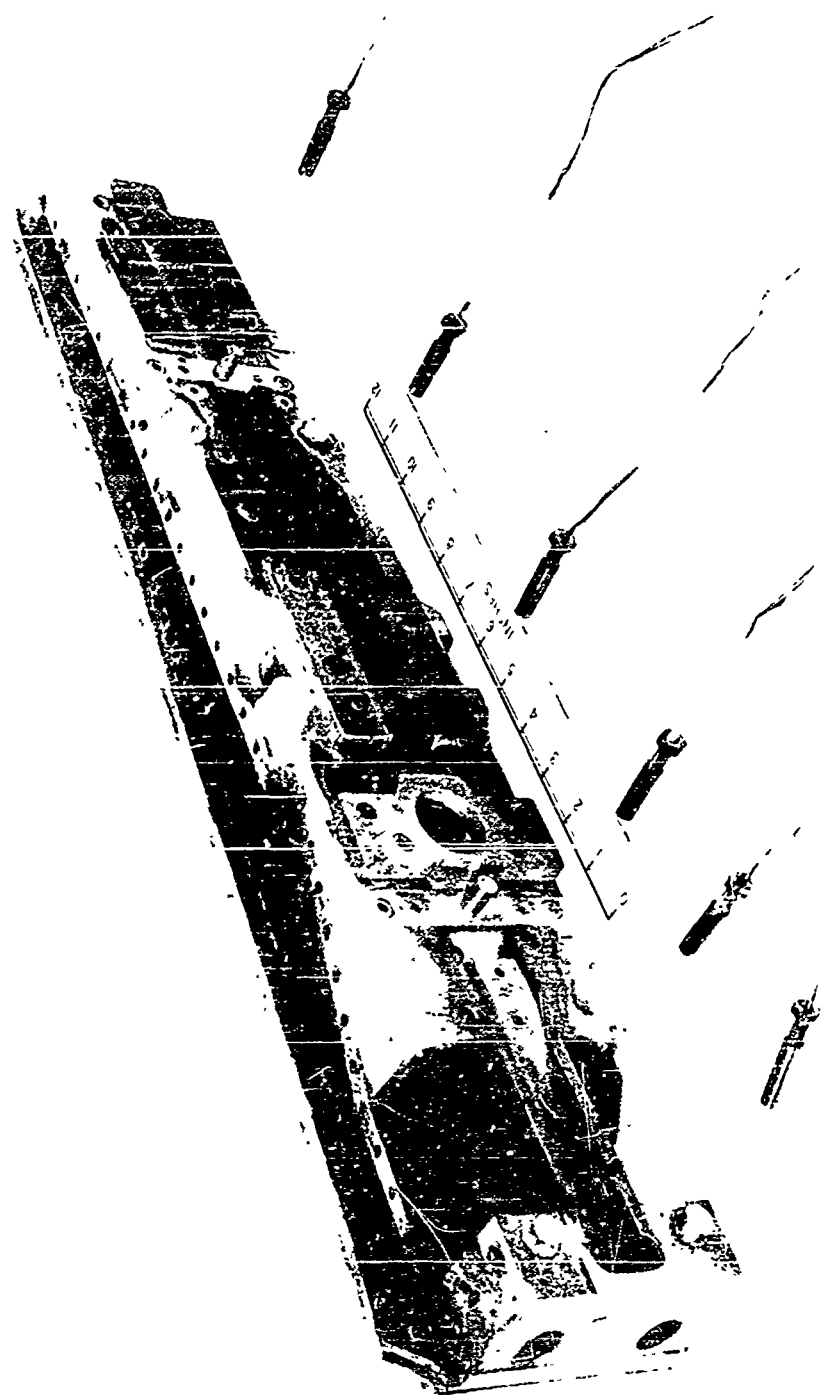
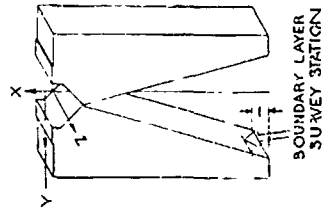
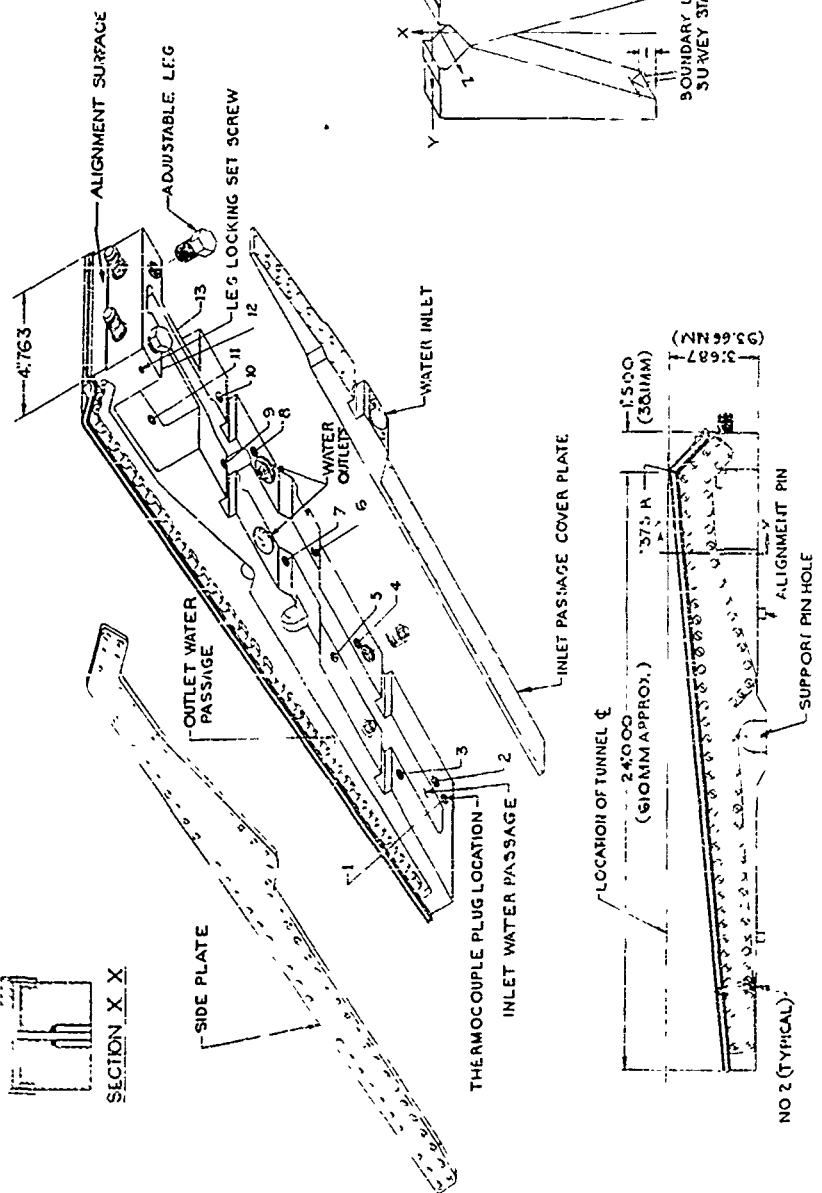
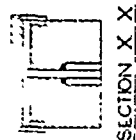
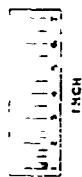


FIG. 2
COOLED WEDGE NOZZLE BLOCK WITH THERMOCOUPLE PLUGS REMOVED

FIG-3
COOLED WEDGE
NOZZLE
FOR
TUNNEL NO.4



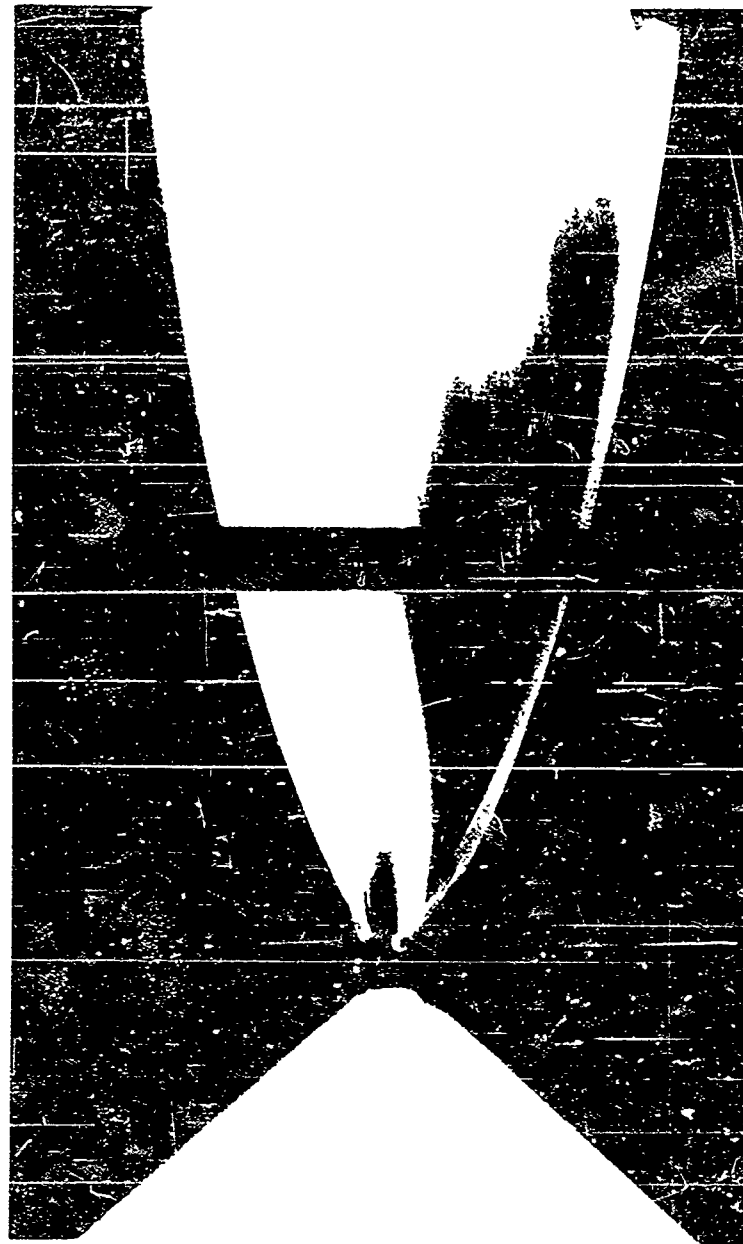
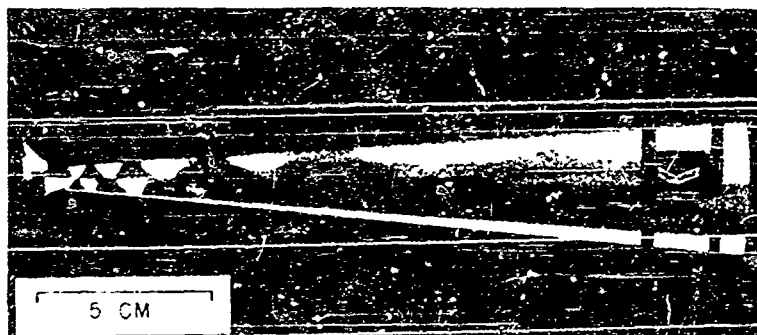


FIG. 4 SCHLIERON PHOTOGRAPH OF THE FLOW NEAR
THE THROAT OF A MINIMUM LENGTH; $M = 5.18$, NOZZLE
 $P_0 = 4.5 \text{ atm}$
 $T_0 = 333^\circ \text{K}$

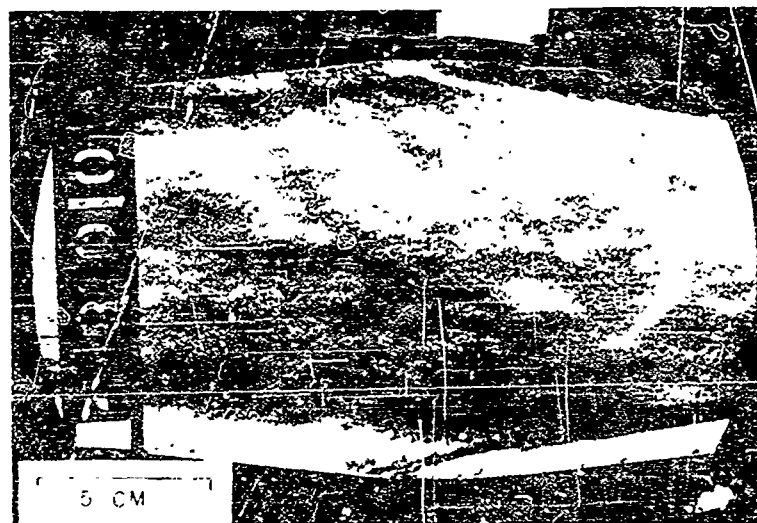


(SHADOWGRAPH)



(SCHLIERN)

(a) THROAT REGION, $M_5 = 5.0$, $P_0 = 4 \text{ atm}$, $T_0 = 293^\circ \text{K}$



(SCHLIERN)

(b) NOZZLE EXIT, $M = 7.0$, $P_0 = 20 \text{ atm}$, $T_0 = 293^\circ \text{K}$

FIG. 5 SHADOWGRAPH AND SCHLIERN PHOTOGRAPHS OF FLOW IN THE WEDGE NOZZLE

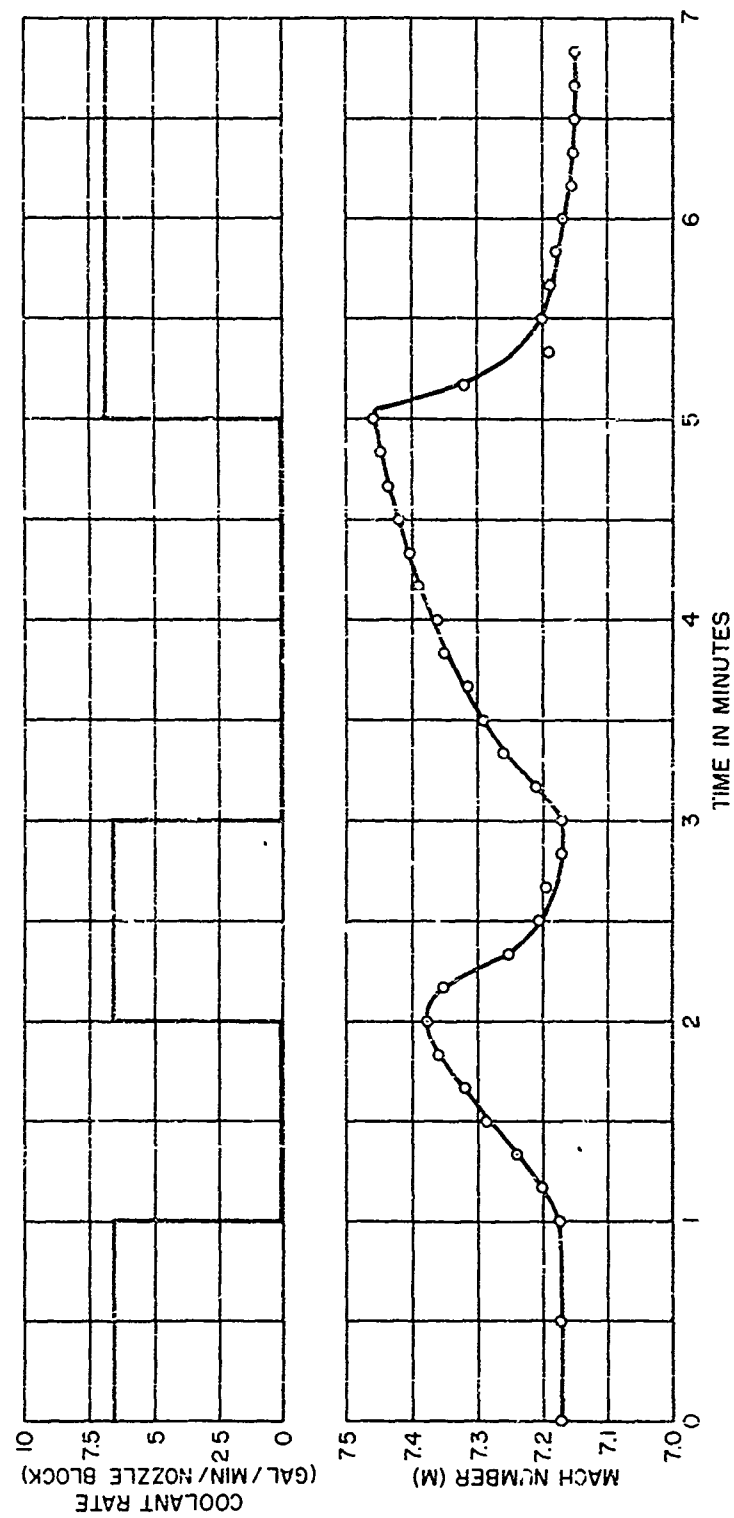


FIG. 6 MACH NUMBER VARIATION IN NOZZLE EXIT vs TIME
FOR CHANGING COOLANT RATE
 $P_0 = 10 \text{ atm}$, $T_0 = 593^\circ \text{K}$

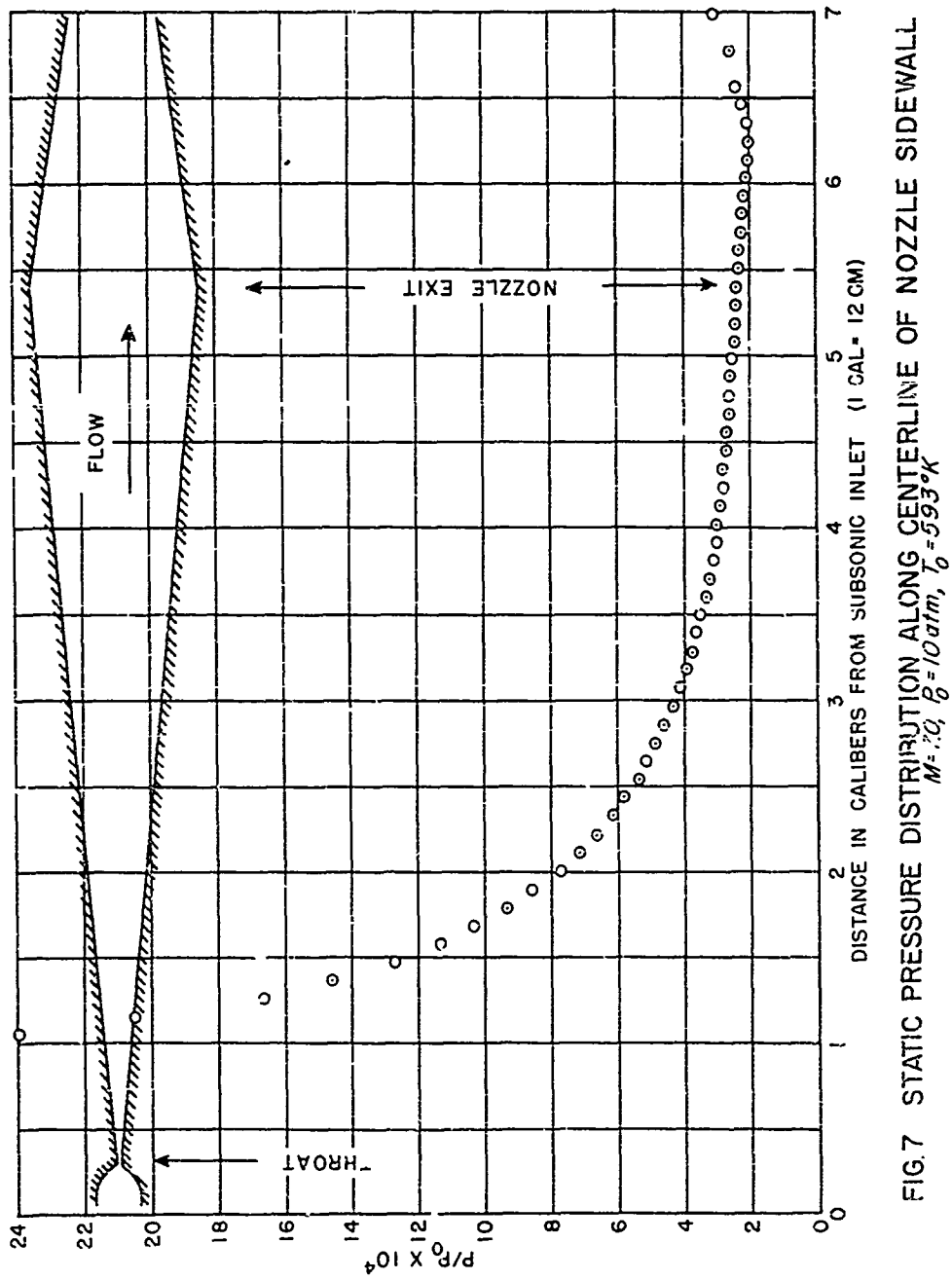


FIG. 7 STATIC PRESSURE DISTRIBUTION ALONG CENTERLINE OF NOZZLE SIDEWALL
 $M = 2.0, P_0 = 10 \text{ atm}, T_0 = 593^\circ \text{K}$

NAVORD REPORT 2701

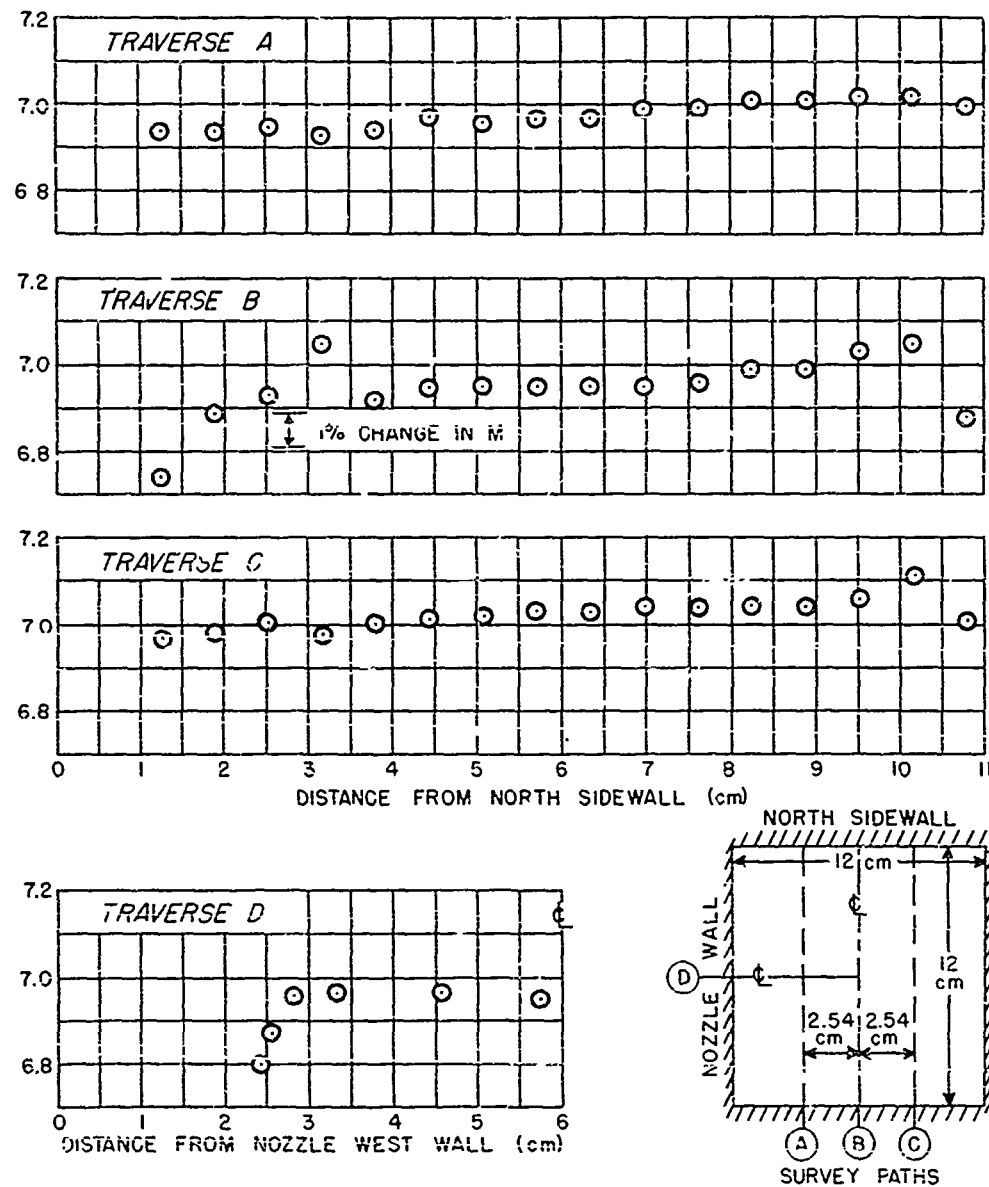


FIG. 8. MACH NUMBER DISTRIBUTIONS ALONG SURVEYS IN A PLANE ONE INCH UPSTREAM FROM THE NOZZLE EXIT
 $P_0 = 10 \text{ atm}$, $T_0 = 593^\circ \text{K}$

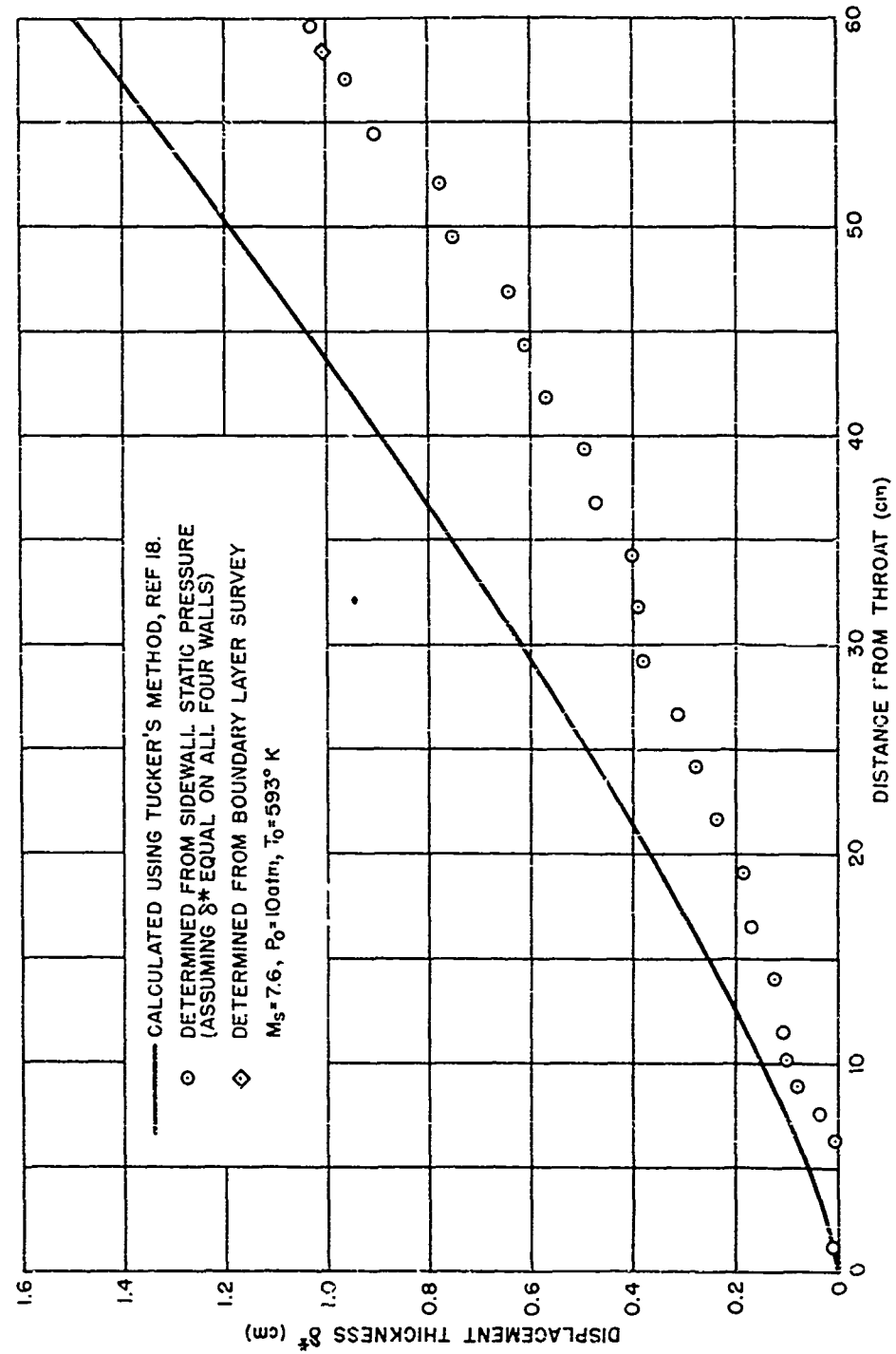


FIG. 9 COMPARISON OF MEASURED AND CALCULATED
BOUNDARY LAYER GROWTH ALONG NOZZLE WALL

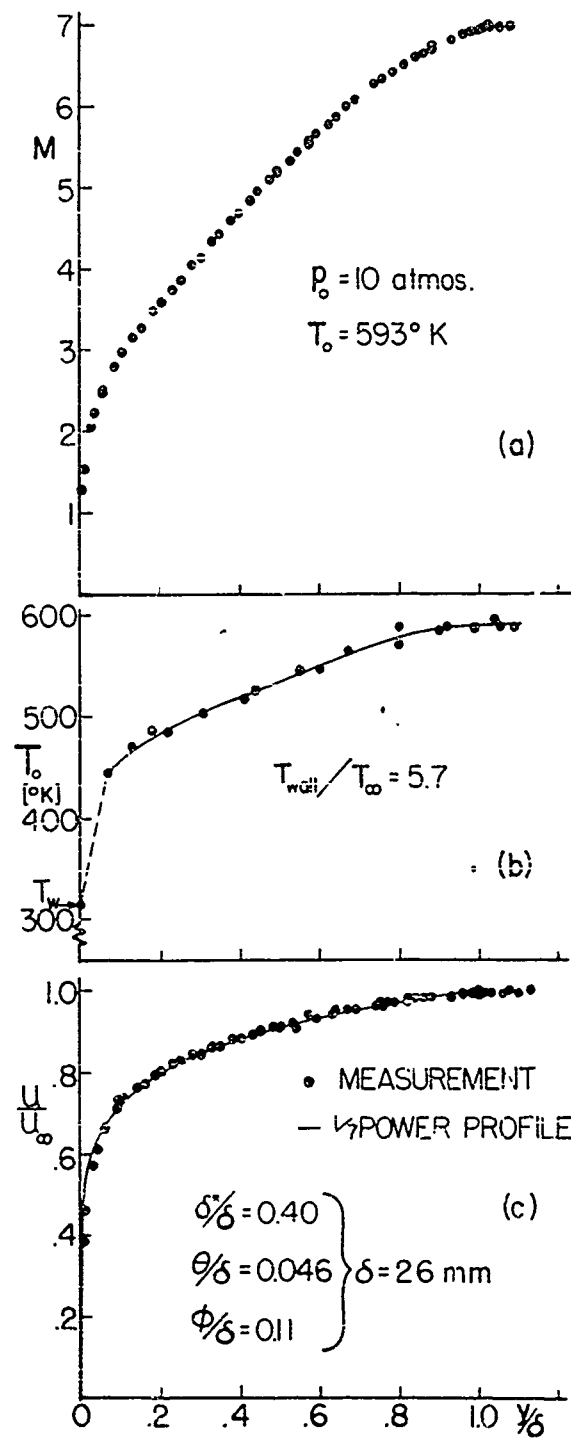


FIG. 10 MEASURED TURBULENT BOUNDARY LAYER PROFILES
 WITH HEAT TRANSFER AT A FREE STREAM MACH NUMBER OF 7

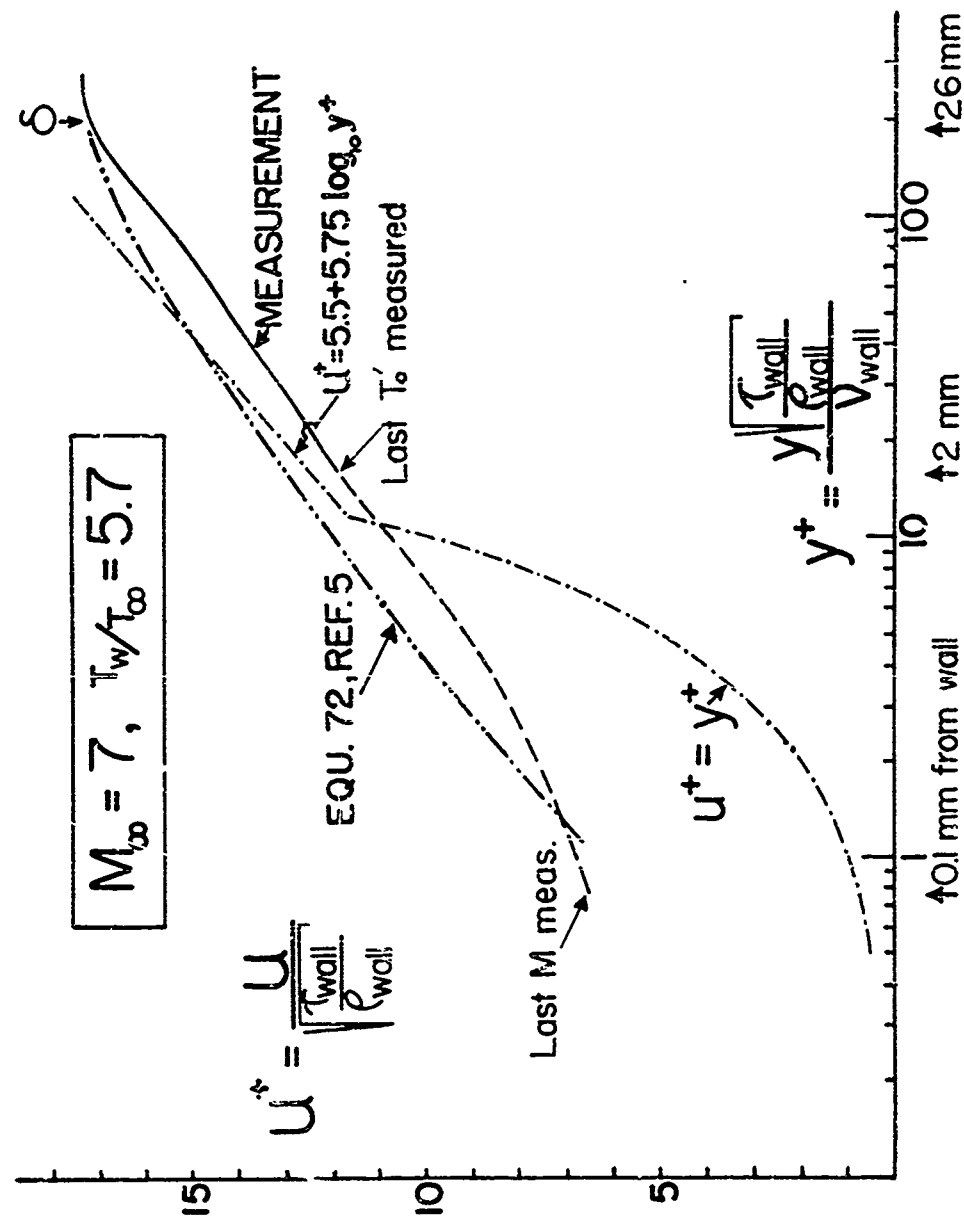


FIG. 11 THE u^+ , y^+ REPRESENTATION OF THE MEASURED VELOCITY PROFILES AT $M=7$ AND COMPARISON WITH THEORY

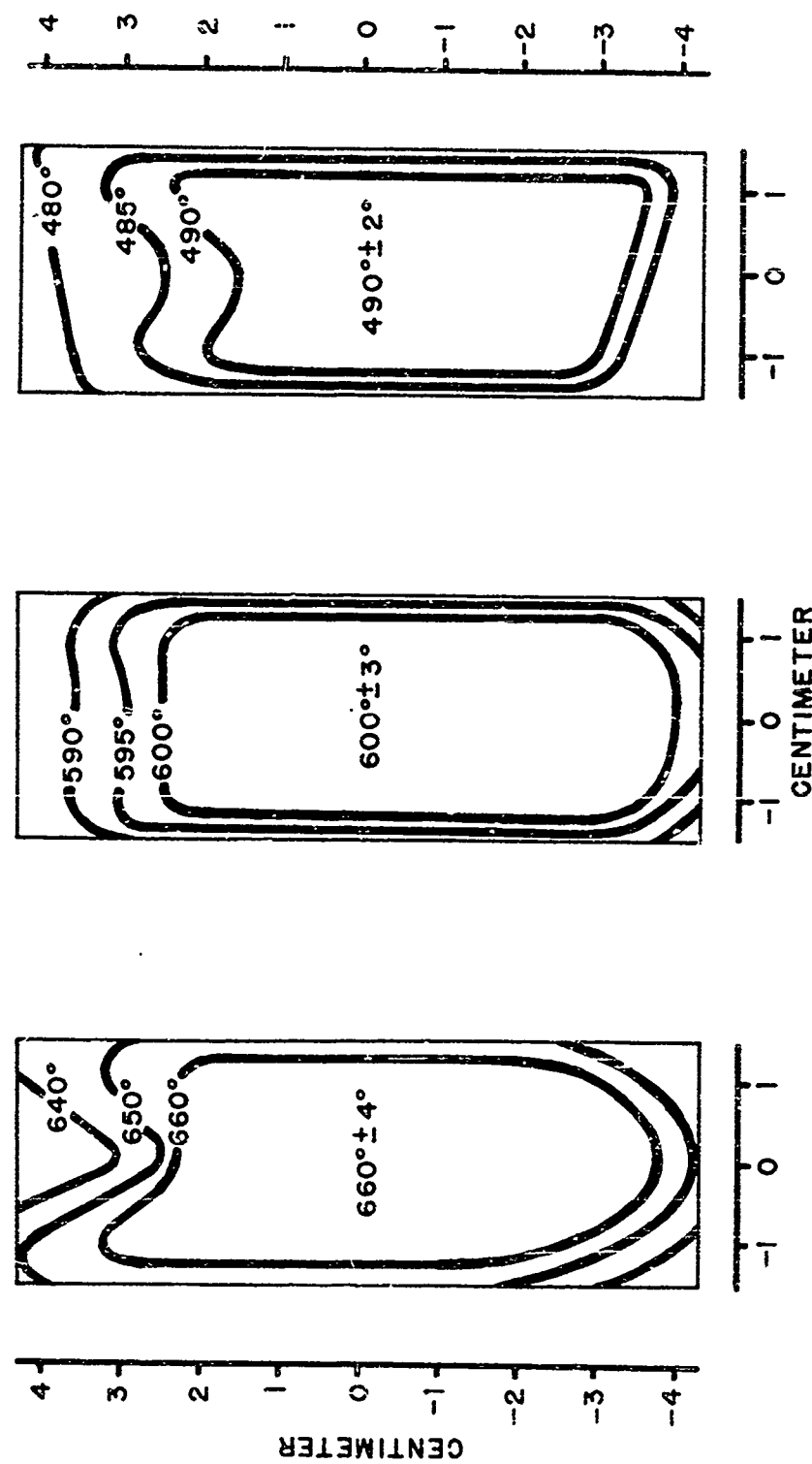


FIG. 12 TEMPERATURE DISTRIBUTION IN NOZZLE INLET
AT THREE DIFFERENT LEVELS OF SUPPLY TEMPERATURE
(TEMPERATURES IN °K)

NAVORD REPORT 2701

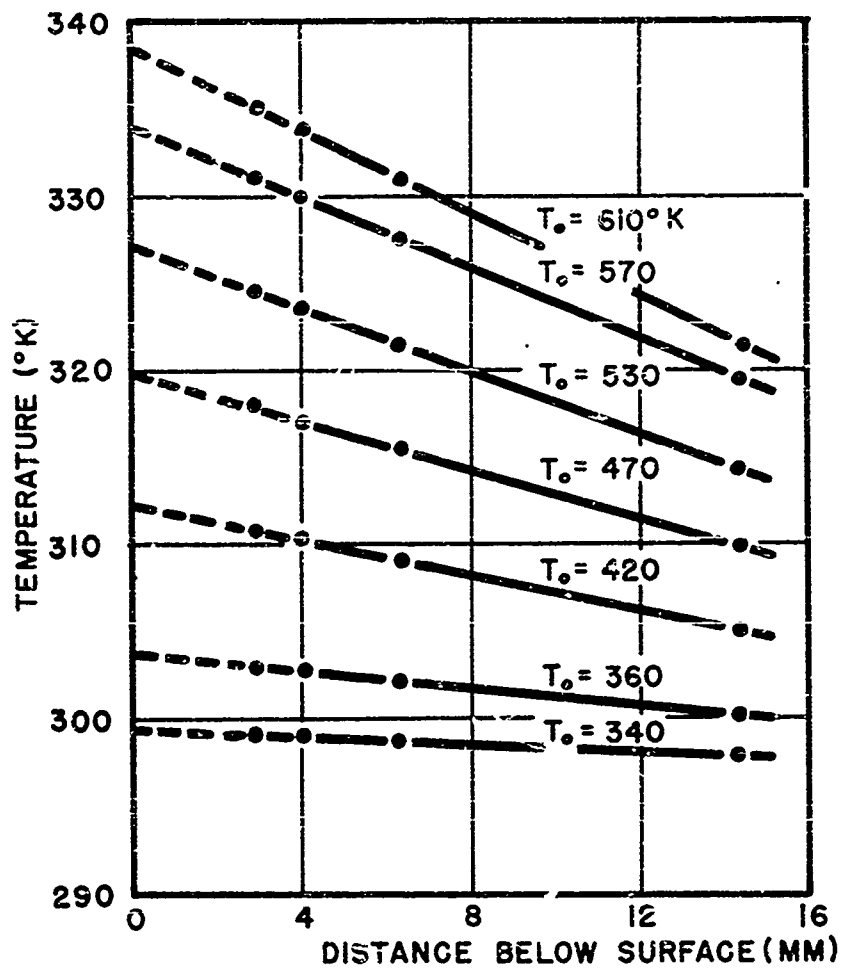


FIG. 13 NOZZLE WALL-TEMPERATURES NEAR THROAT
(STATION NO. 13) AT VARIOUS SUPPLY TEMPERATURES

$p_o = 10 \text{ ATM}$, $M_s = 7.6$
COOLING RATE: 6.5 GALLONS/MIN

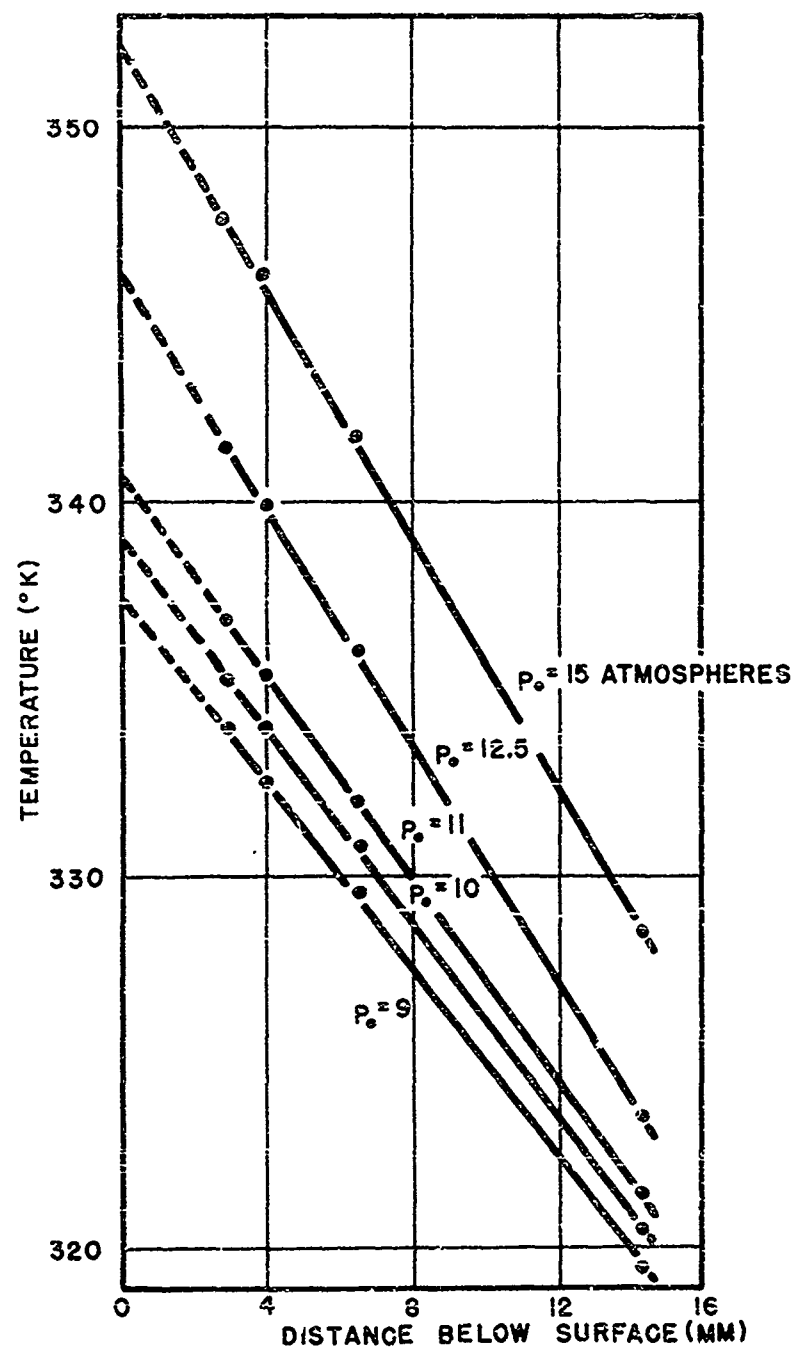


FIG. 14 NOZZLE WALL-TEMPERATURES NEAR THROAT
(STATION NO. 13) AT VARIOUS SUPPLY PRESSURES

$T_0 = 610^\circ \text{K}$, $M_5 = 7.6$

COOLING RATE: 6.5 GALLONS / MIN

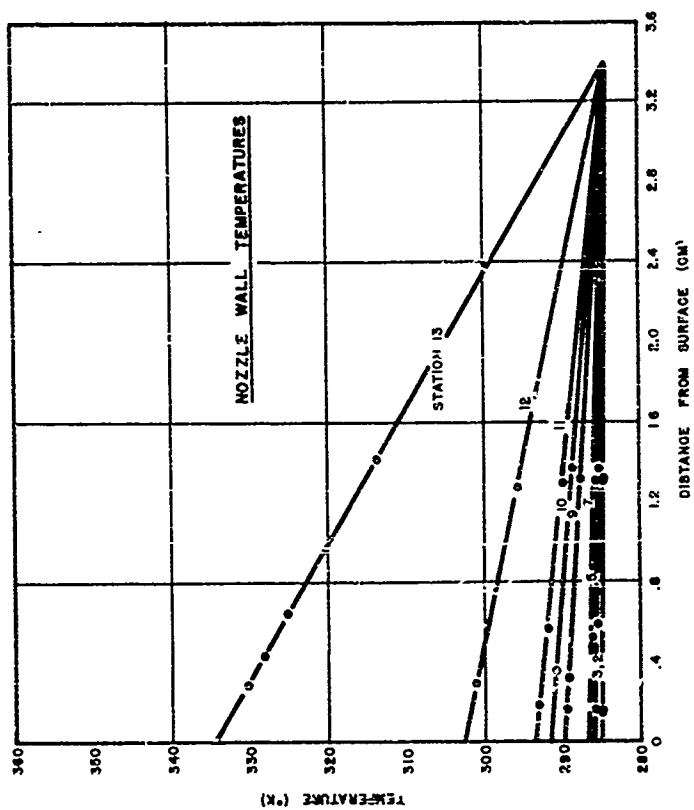
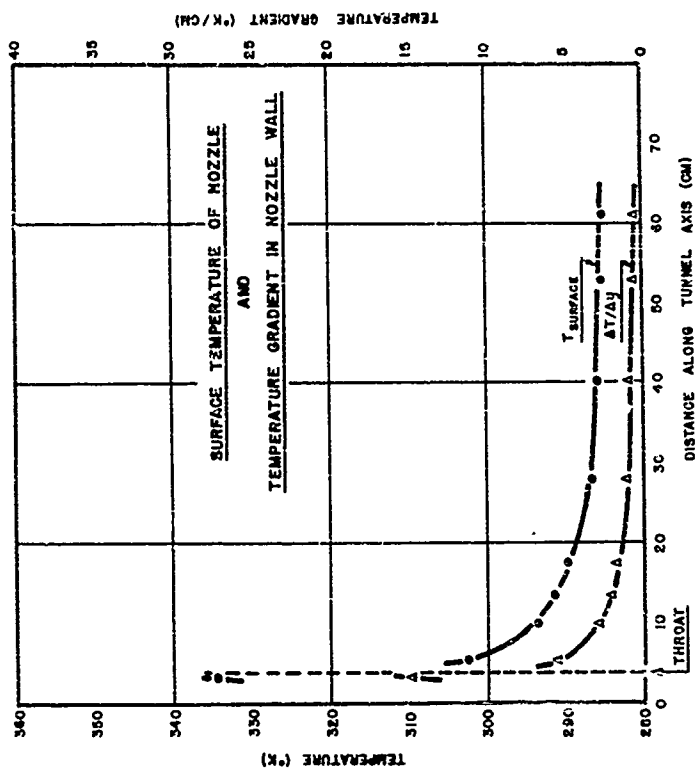


FIG. 15 WALL TEMPERATURES AT 9 STATIONS, TEMPERATURE GRADIENT IN WALL, AND EXTRAPOLATED SURFACE TEMPERATURES OF NOZZLE

$M_0 = 7.6$ $P_0 = 10$ ATM $T_0 = 993^\circ\text{K}$
COOLING RATE: 6.5 GALLONS PER MINUTE

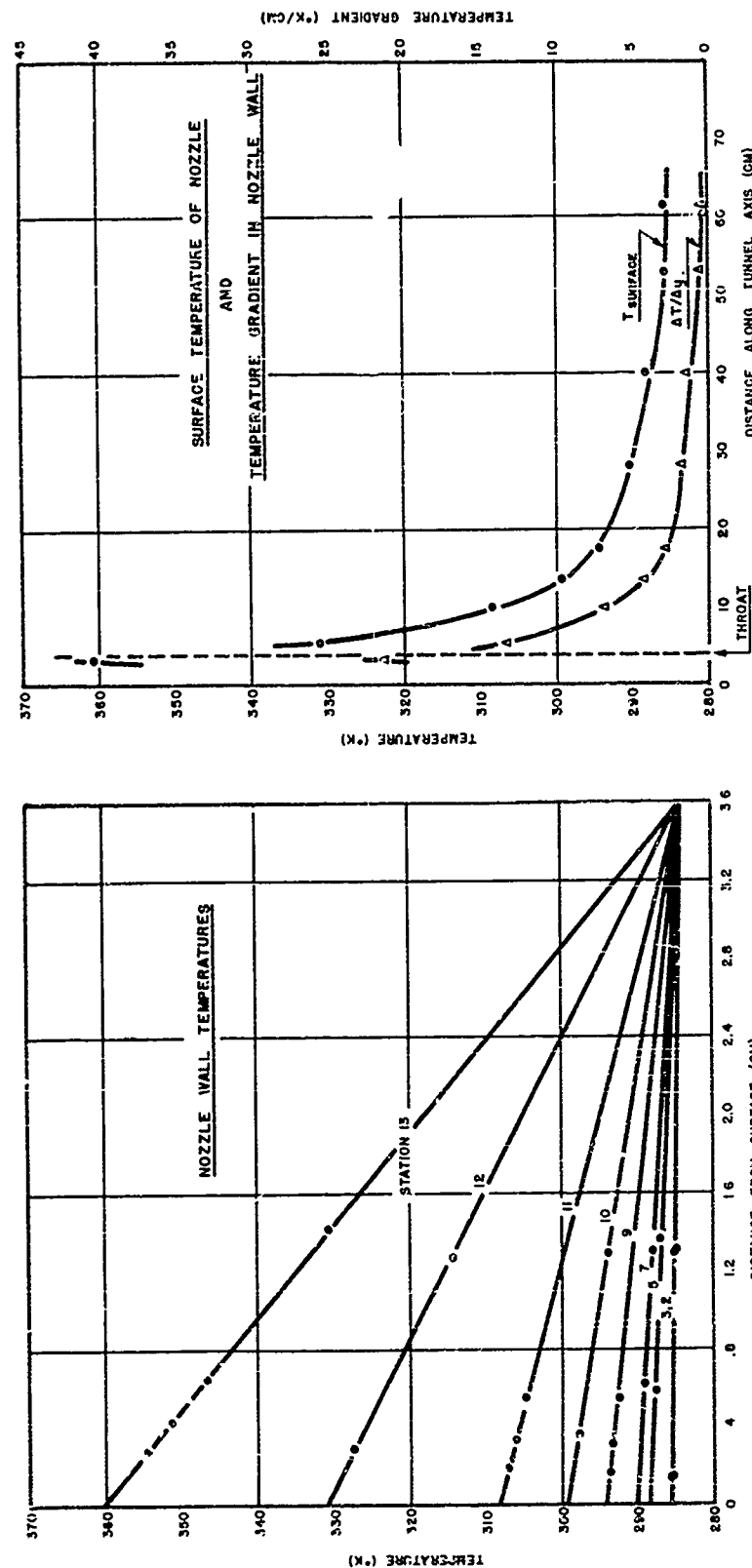


FIG. 16 WALL TEMPERATURES AT 9 STATIONS, TEMPERATURE GRADIENT IN WALL, AND EXTRAPOLATED SURFACE TEMPERATURES OF NOZZLE

$M_0 = 7.6$ $P_0 = 20 \text{ ATM}$ $T_0 = 593^\circ \text{K}$
COOLING RATE: 6.5 GALLONS PER MINUTE

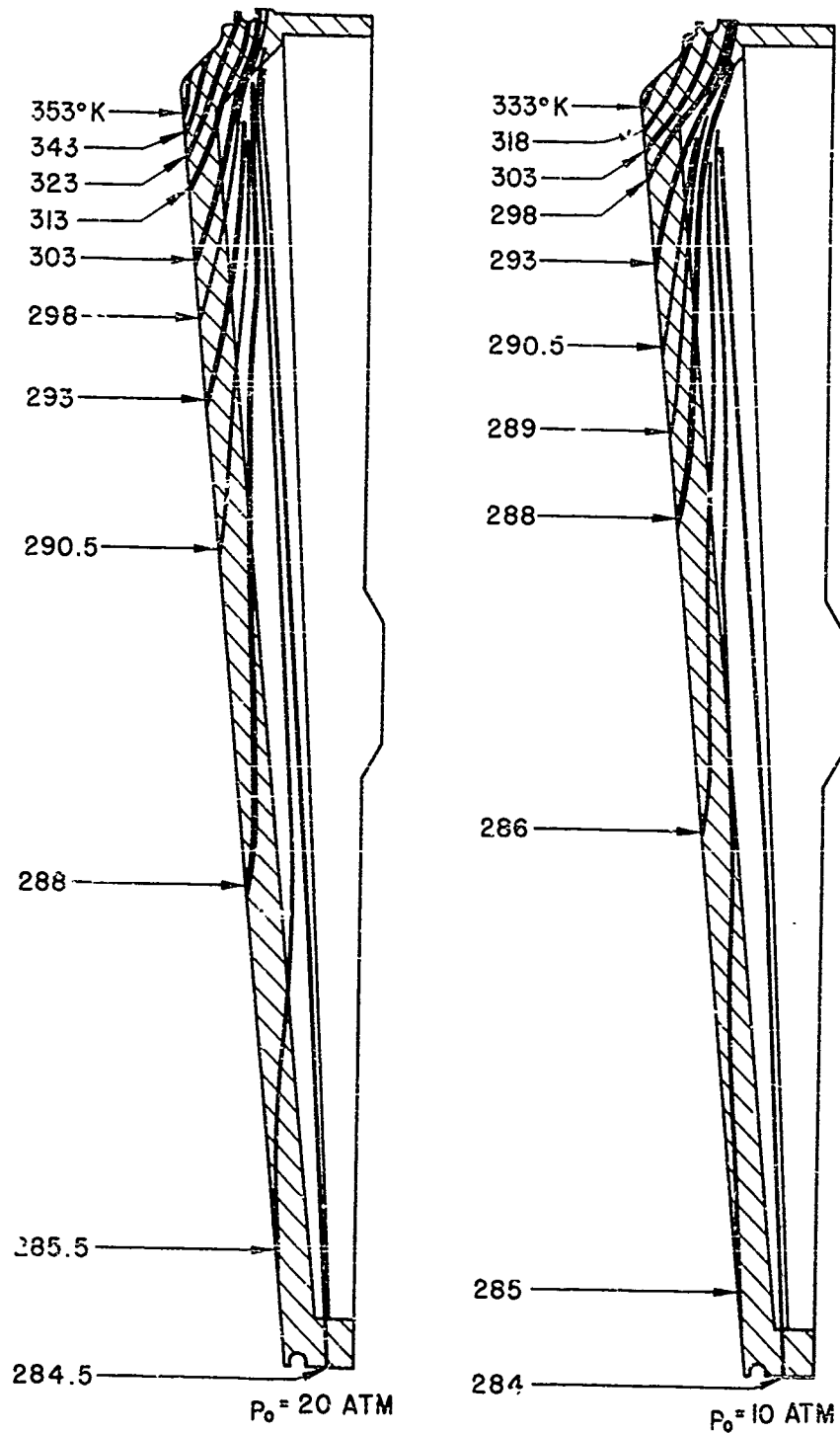


FIG. 17 ISOTHERMS IN NOZZLE
FOR

$M_s = 7.6$ $T_0 = 593^\circ\text{K}$ $P_0 = 10$ AND 20 ATM
COOLING RATE: 6.5 GALLONS PER MINUTE

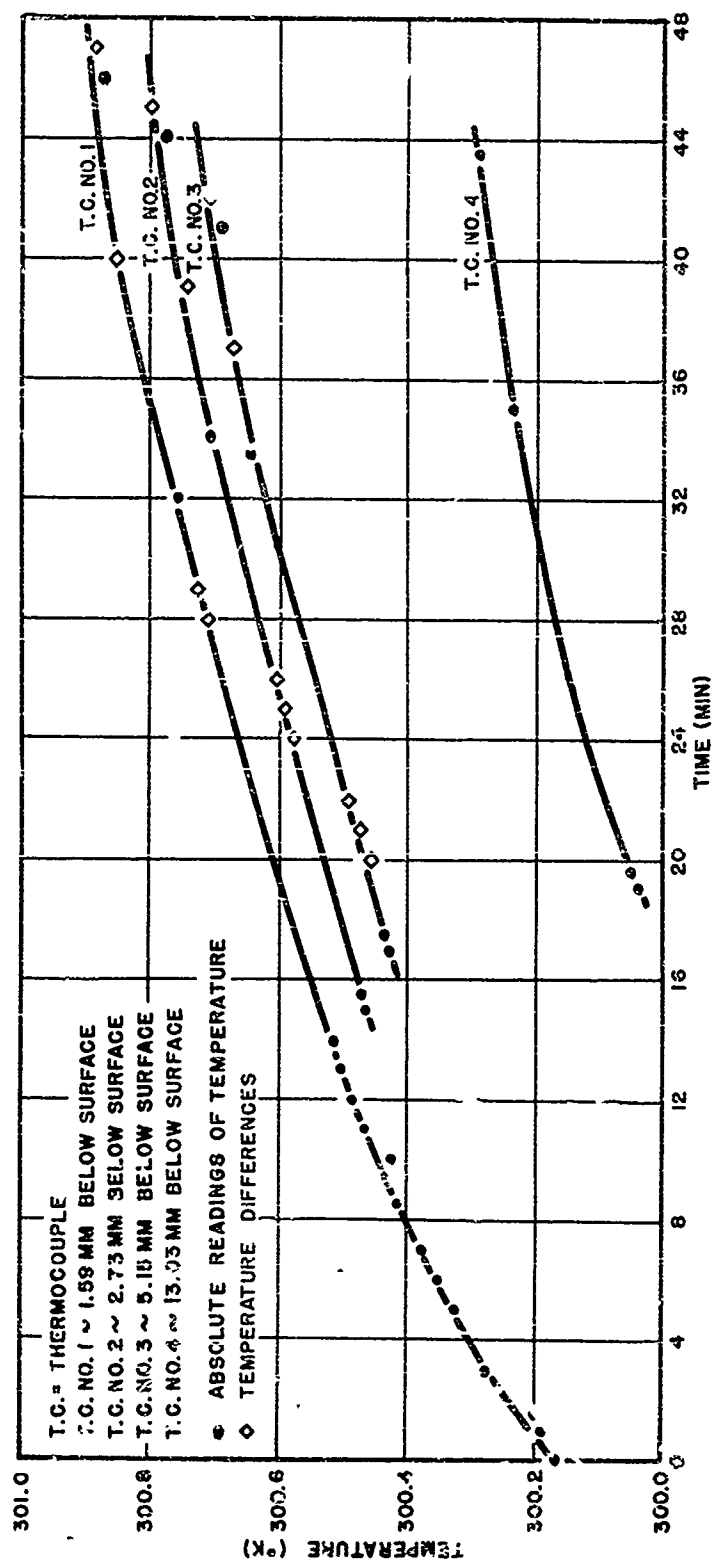
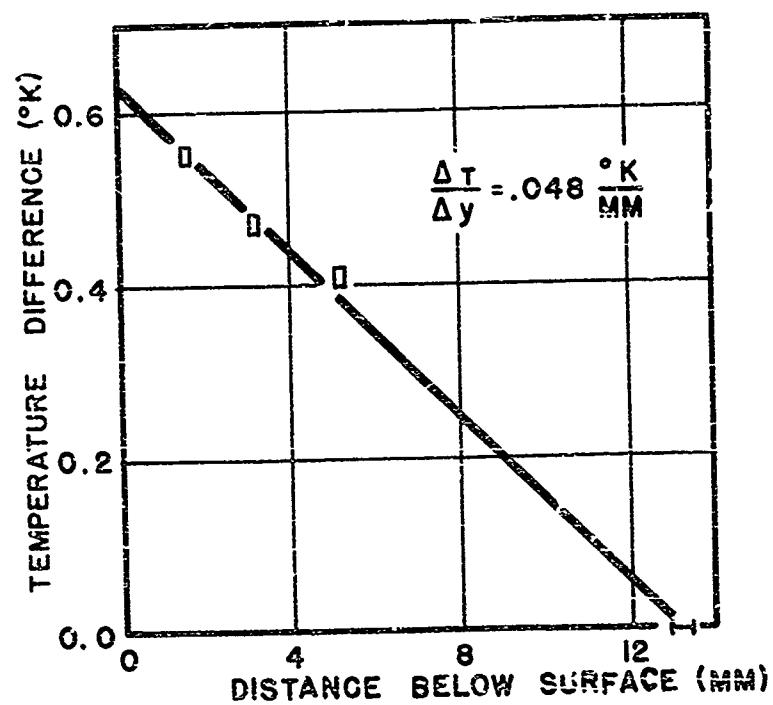


FIG. 18 NOZZLE WALL-TEMPERATURES AS FUNCTION OF RUNNING TIME
AT BOUNDARY LAYER SURVEY STATION

$M_0 = 7.6$, $P_0 = 21.4$ ATM, $T_0 = 593^\circ K$
 COOLING RATE: 6.5 GALLONS/MIN

NAVORD REPORT 2701



ERROR { DISTANCE: ± 0.1 mm
 ΔT : $\pm 0.01^{\circ}\text{K}$

FIG. 19 TEMPERATURE DROP IN NOZZLE WALL

$M_s = 7.6$, $p_o = 21.4$ ATM, $T_o = 593^{\circ}\text{K}$
 COOLING RATE: 6.5 GALLONS/MIN

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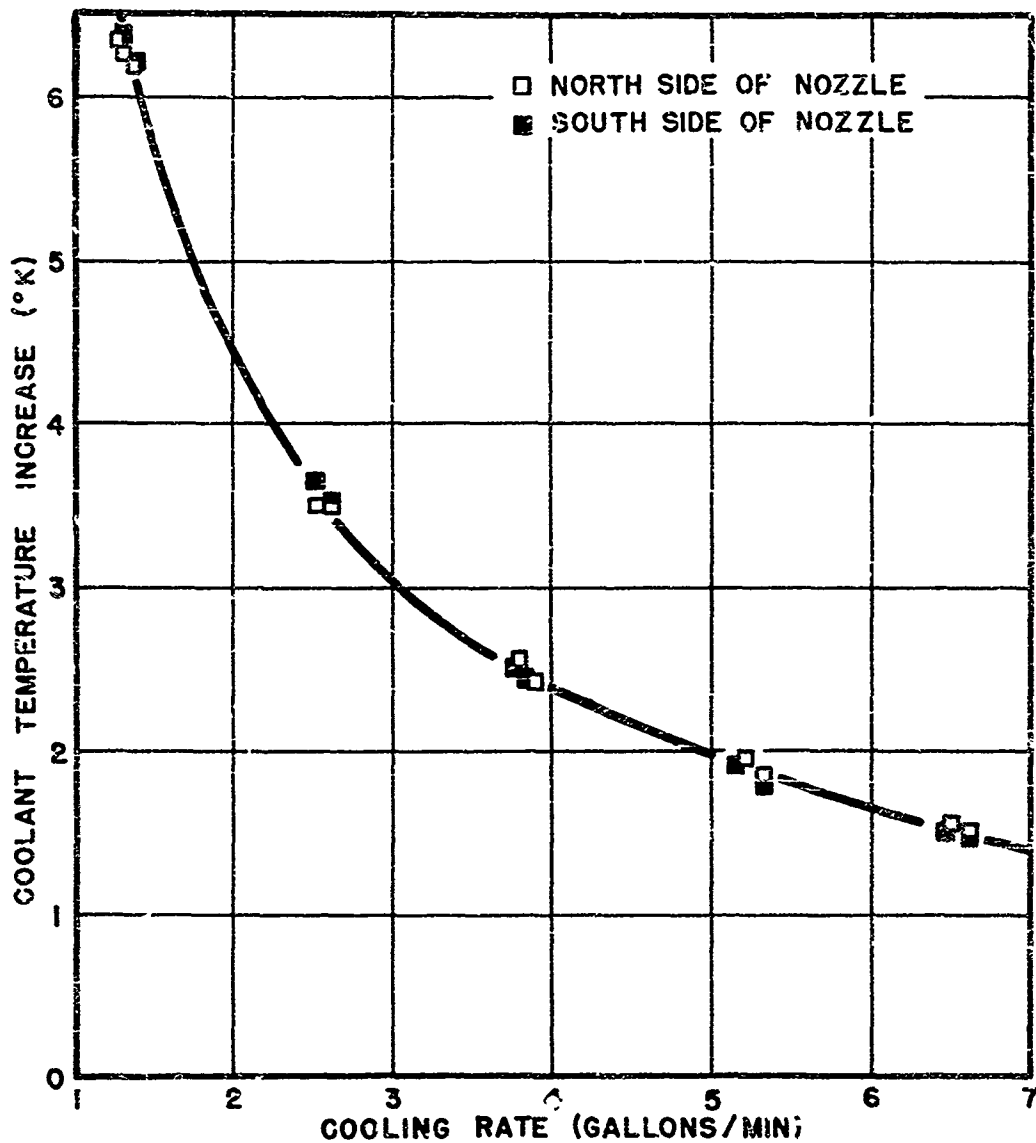


FIG. 20 TEMPERATURE INCREASE OF COOLING WATER
AS FUNCTION OF THE COOLING RATE FOR
TUNNEL OPERATION

$M_5 = 8.0$, $p_0 = 20 \text{ ATM}$, $T_a = 600^\circ \text{K}$

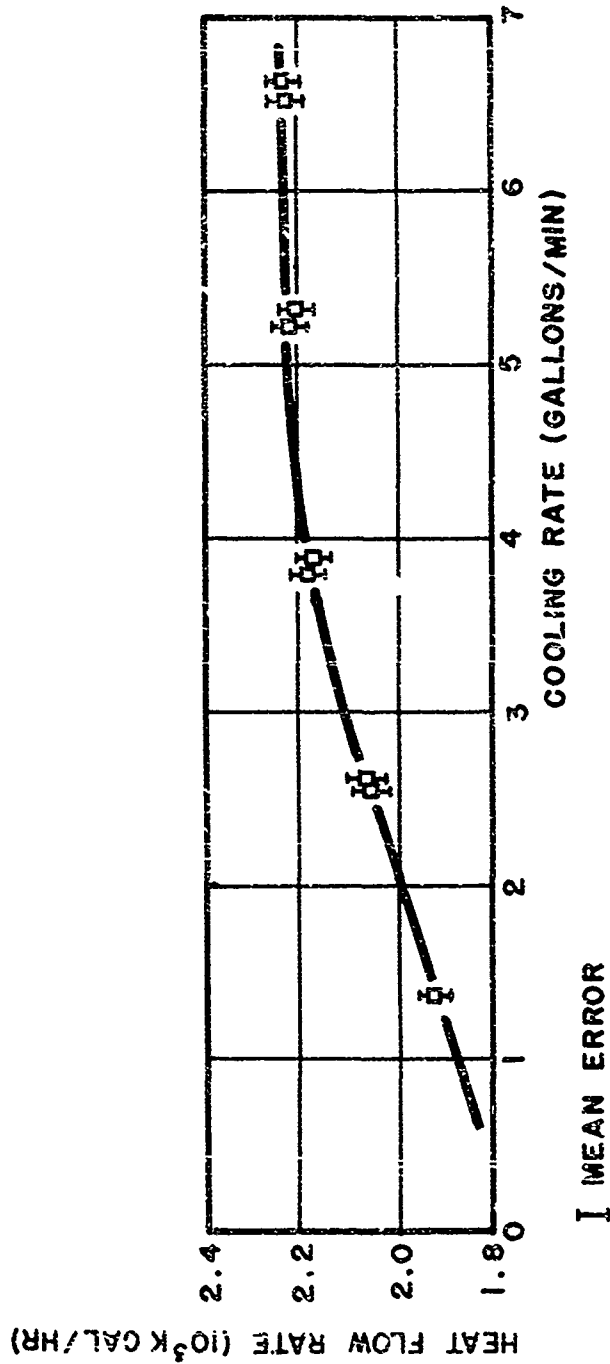


FIG. 21 HEAT FLOW RATE TO COOLING WATER AS FUNCTION OF COOLING RATE FOR TUNNEL OPERATION

$M_S = 8.0$, $P_c = 20 \text{ ATM}$, $T_c = 600^\circ \text{ K}$

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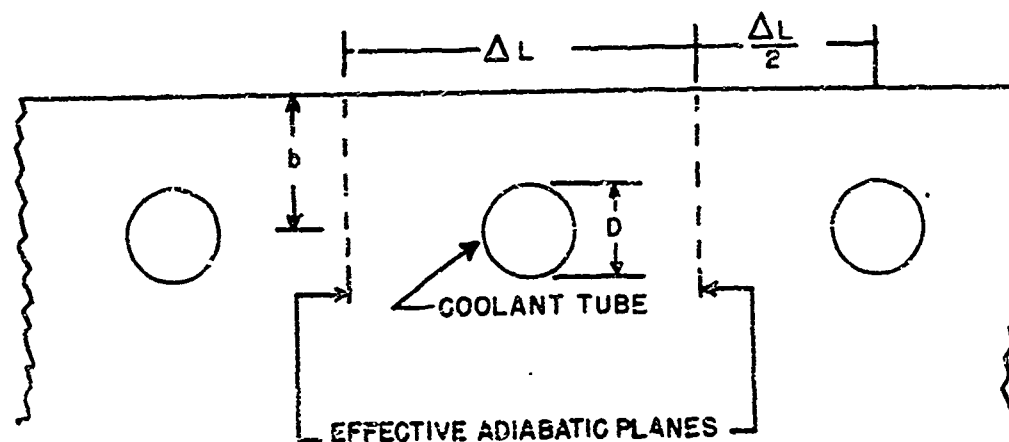
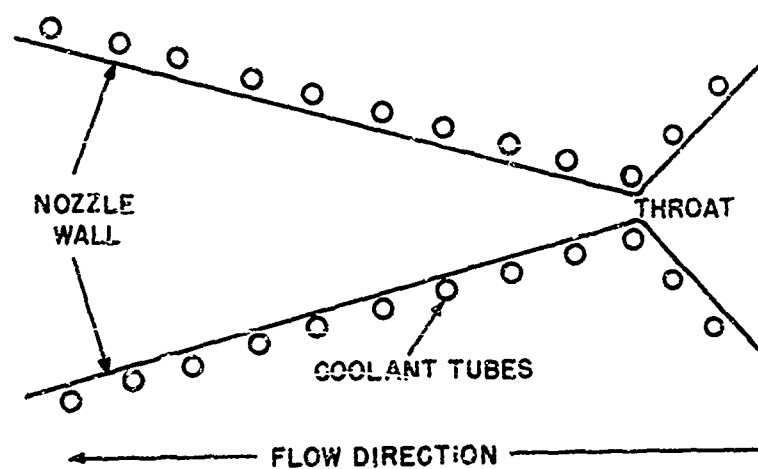


FIG. 22 COOLANT TUBE POSITIONS AND DIMENSIONS
USED IN HEAT TRANSFER ANALYSIS

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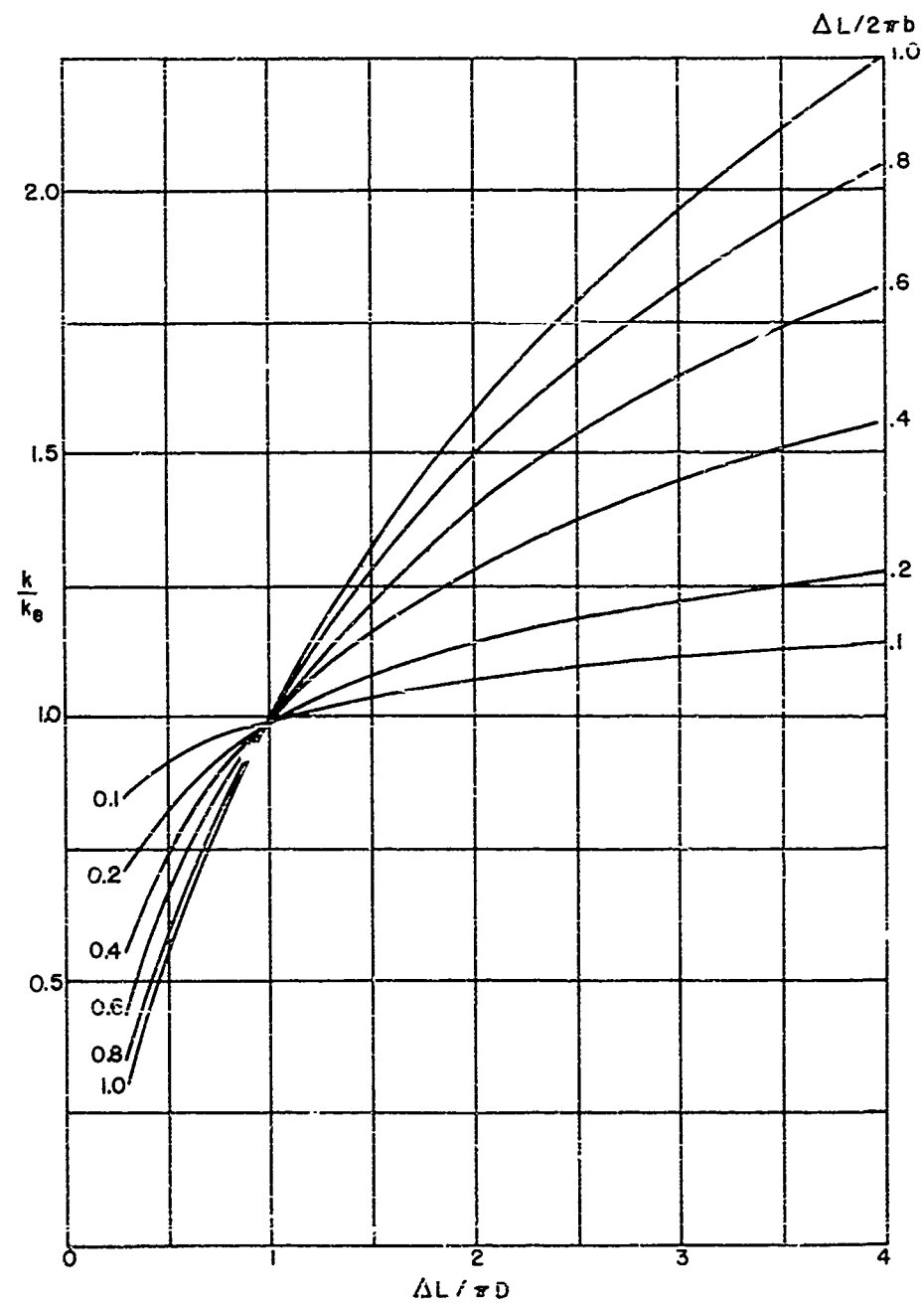


FIG. 23 VARIATION OF k/k_e WITH COOLANT TUBE SIZE AND POSITION

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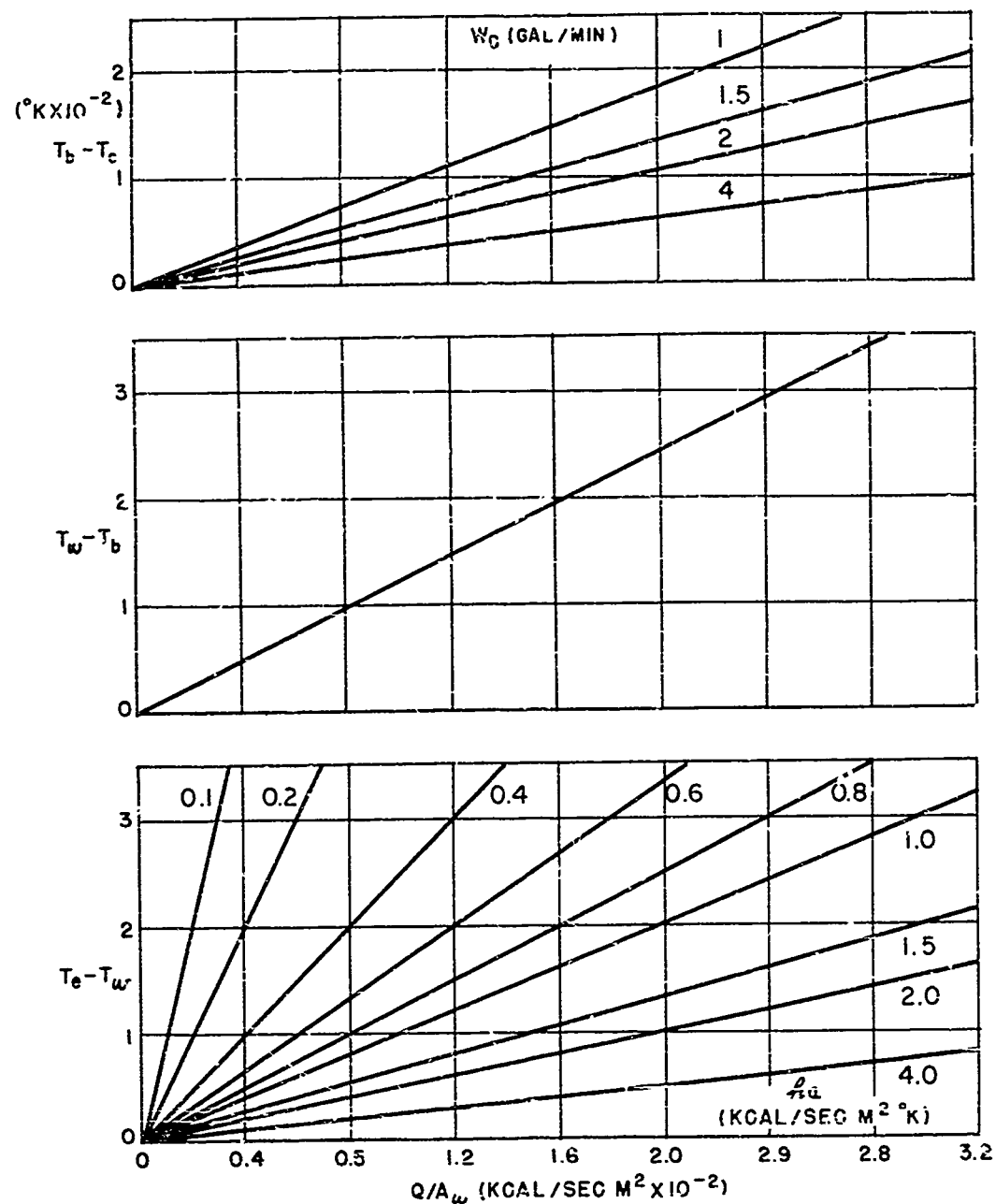


FIG. 24
DETERMINING NOZZLE SURFACE AND COOLANT TUBE SURFACE
TEMPERATURES

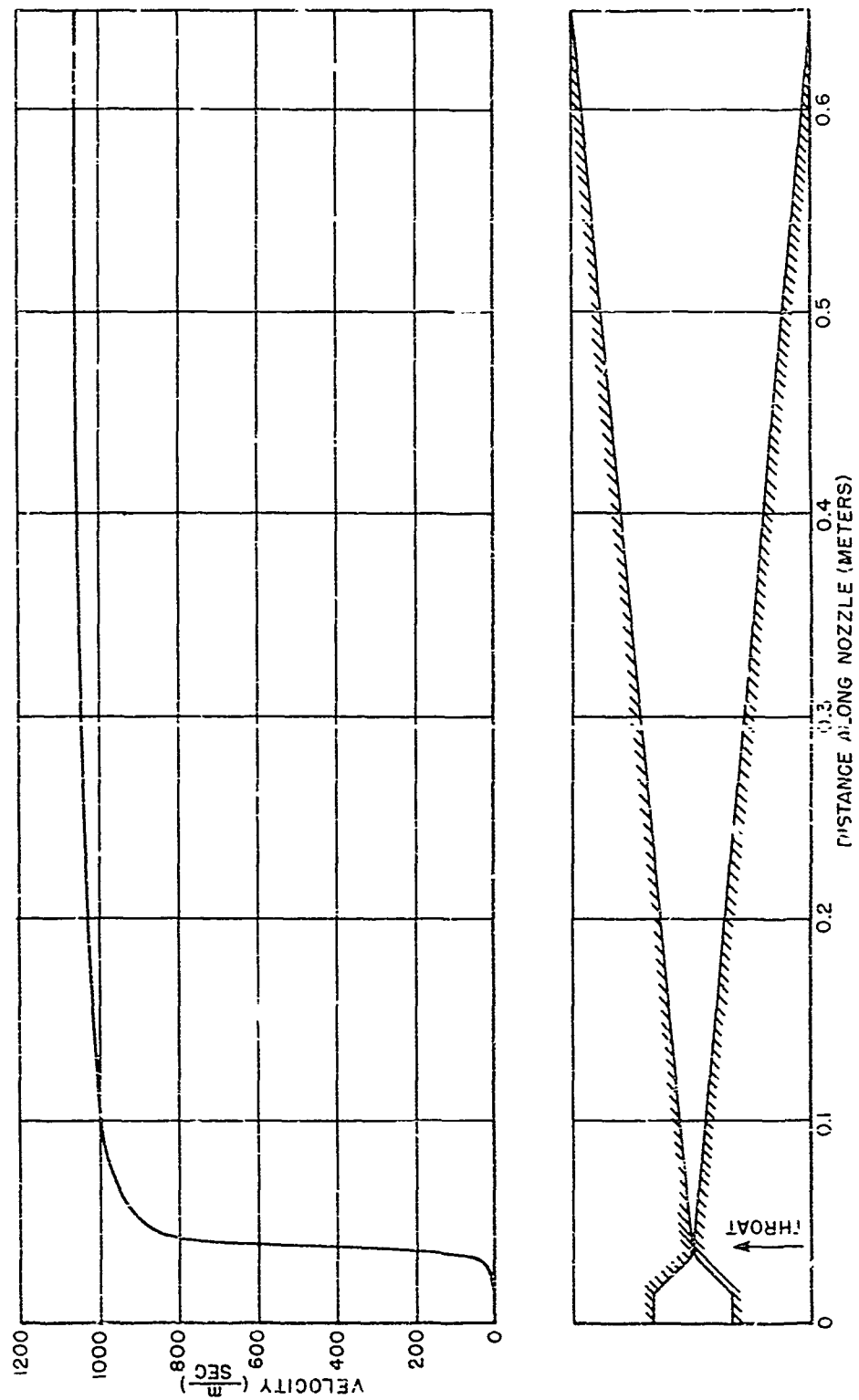


FIG 25 VELOCITY DISTRIBUTION ALONG NOZZLE
 $M_s = 8.0, T_0 = 600^\circ K$

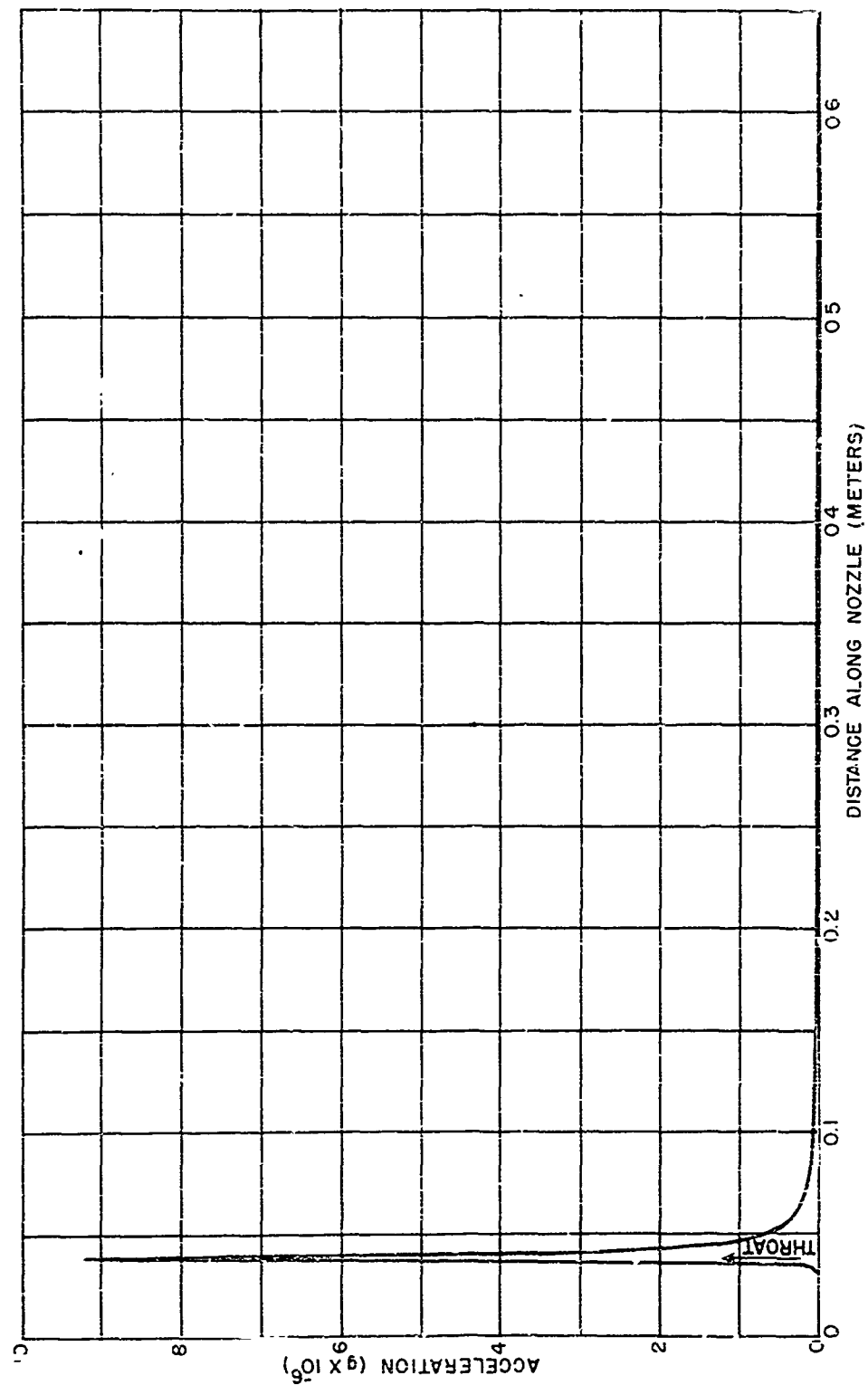


FIG 26 ACCELERATION DISTRIBUTION ALONG NOZZLE
 $M_s = 8.0, T_0 = 600^\circ K$

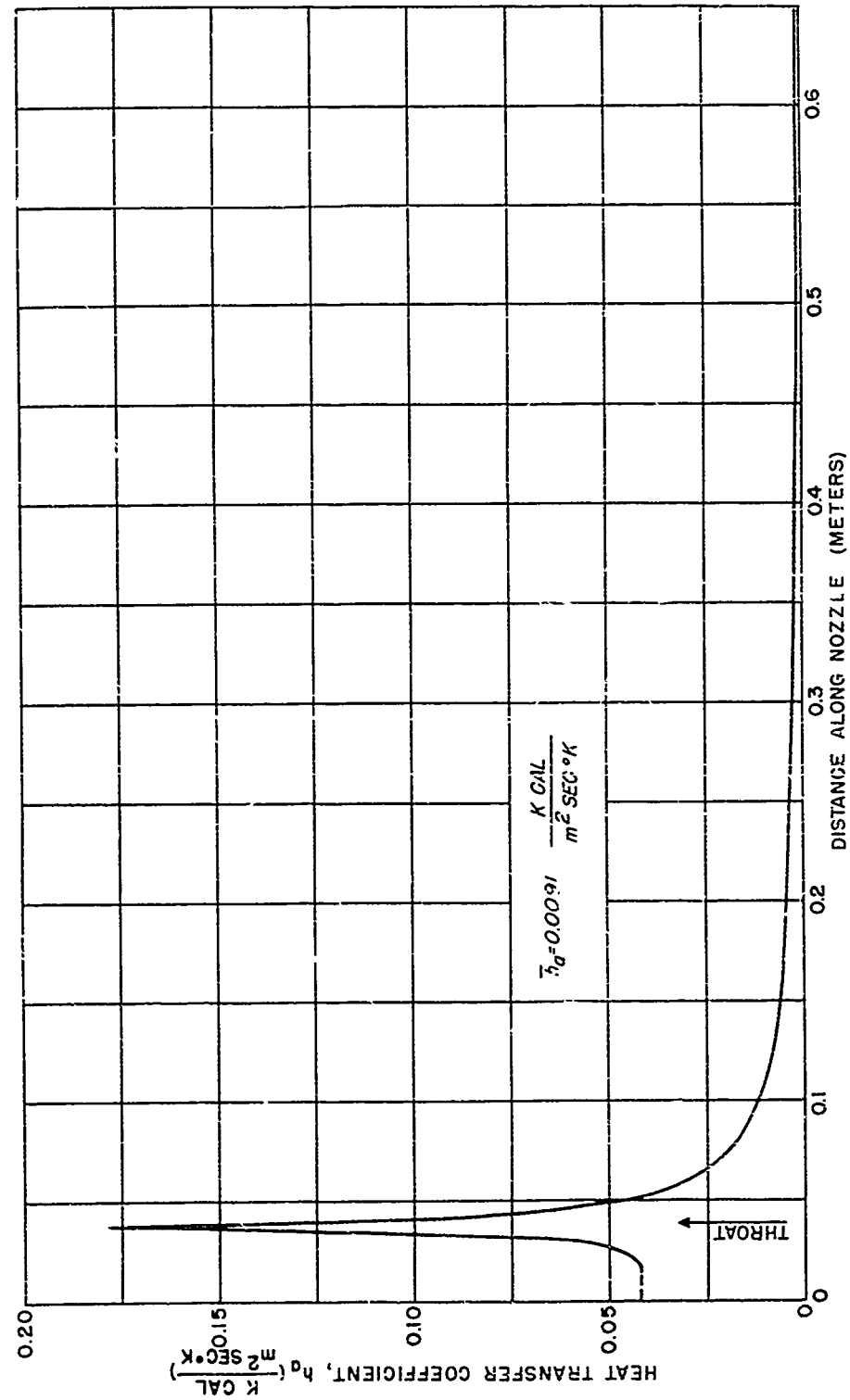


FIG. 27 HEAT TRANSFER COEFFICIENT DISTRIBUTION ASSUMING LAMINAR FLOW AND NEGLECTING THE EFFECTS OF PRESSURE GRADIENTS
 $M_s = 8.0, T_0 = 600^\circ \text{K}, P_0 = 20 \text{ atm}$

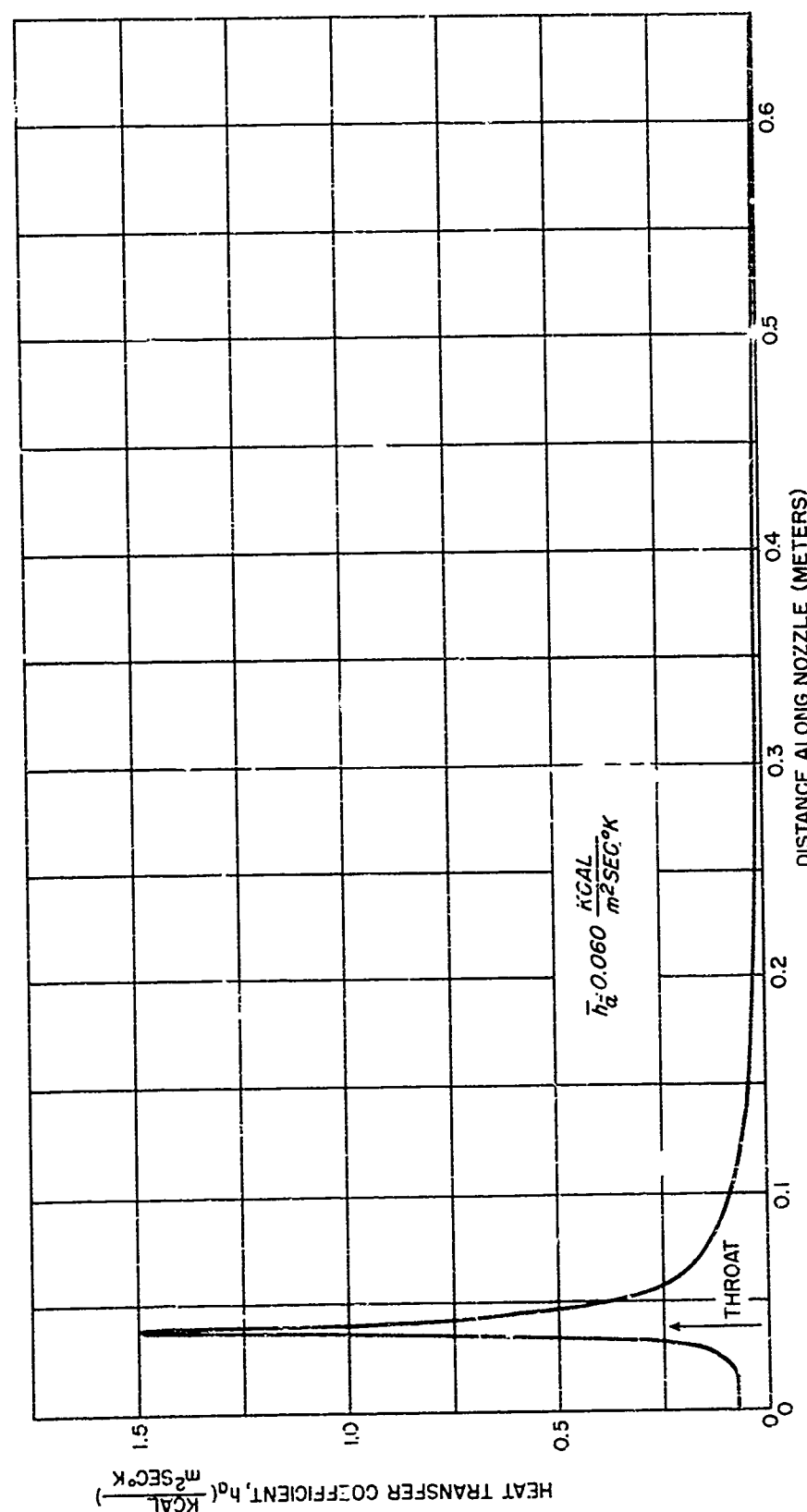


FIG. 28 HEAT TRANSFER COEFFICIENT DISTRIBUTION ASSUMING TURBULENT FLOW
AND NEGLECTING THE EFFECTS OF PRESSURE GRADIENTS

$$M_5 = 8.0, T_0 = 500^\circ \text{K}, P_0 = 20 \text{ atm}$$

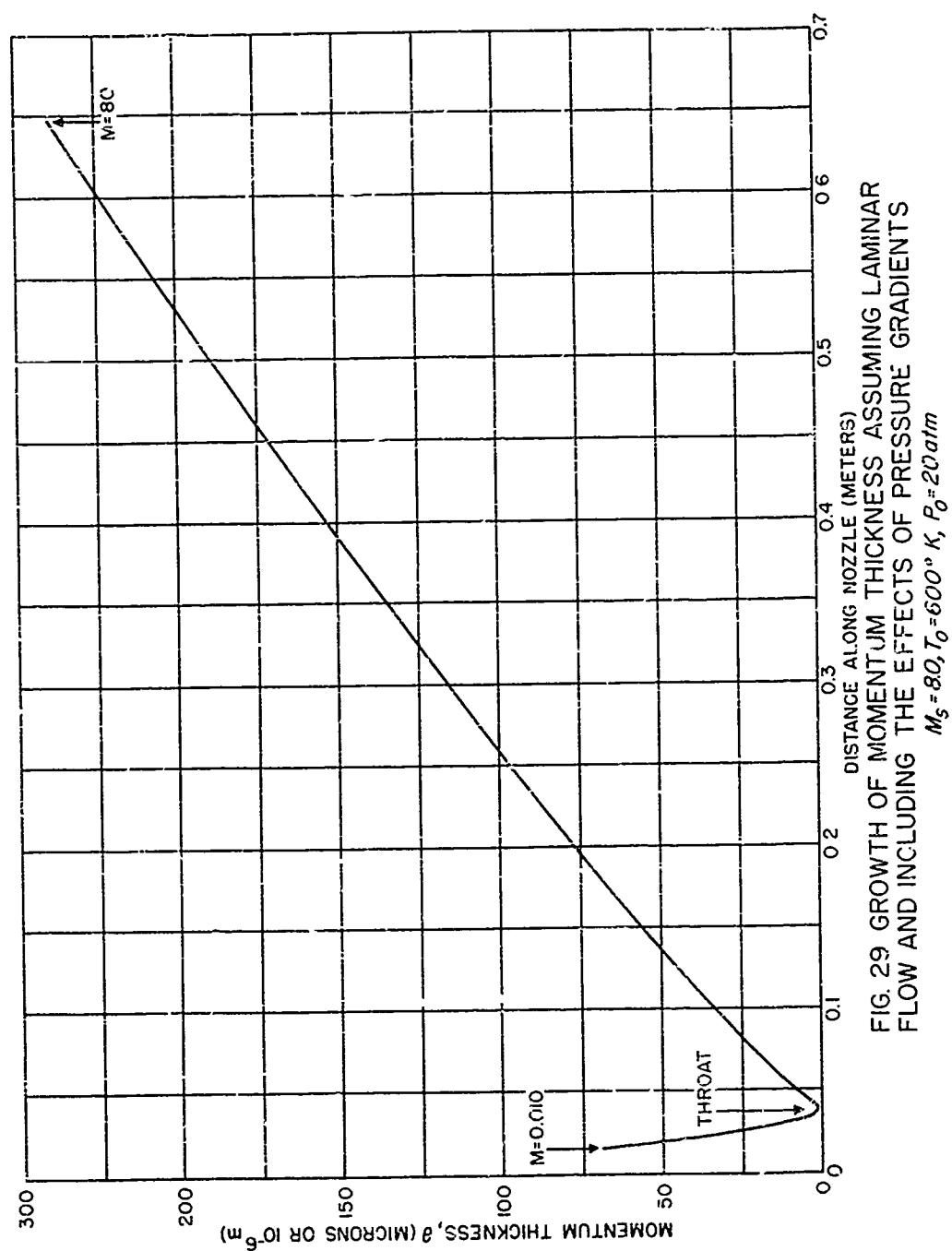


FIG. 29 GROWTH OF MOMENTUM THICKNESS ASSUMING LAMINAR FLOW AND INCLUDING THE EFFECTS OF PRESSURE GRADIENTS

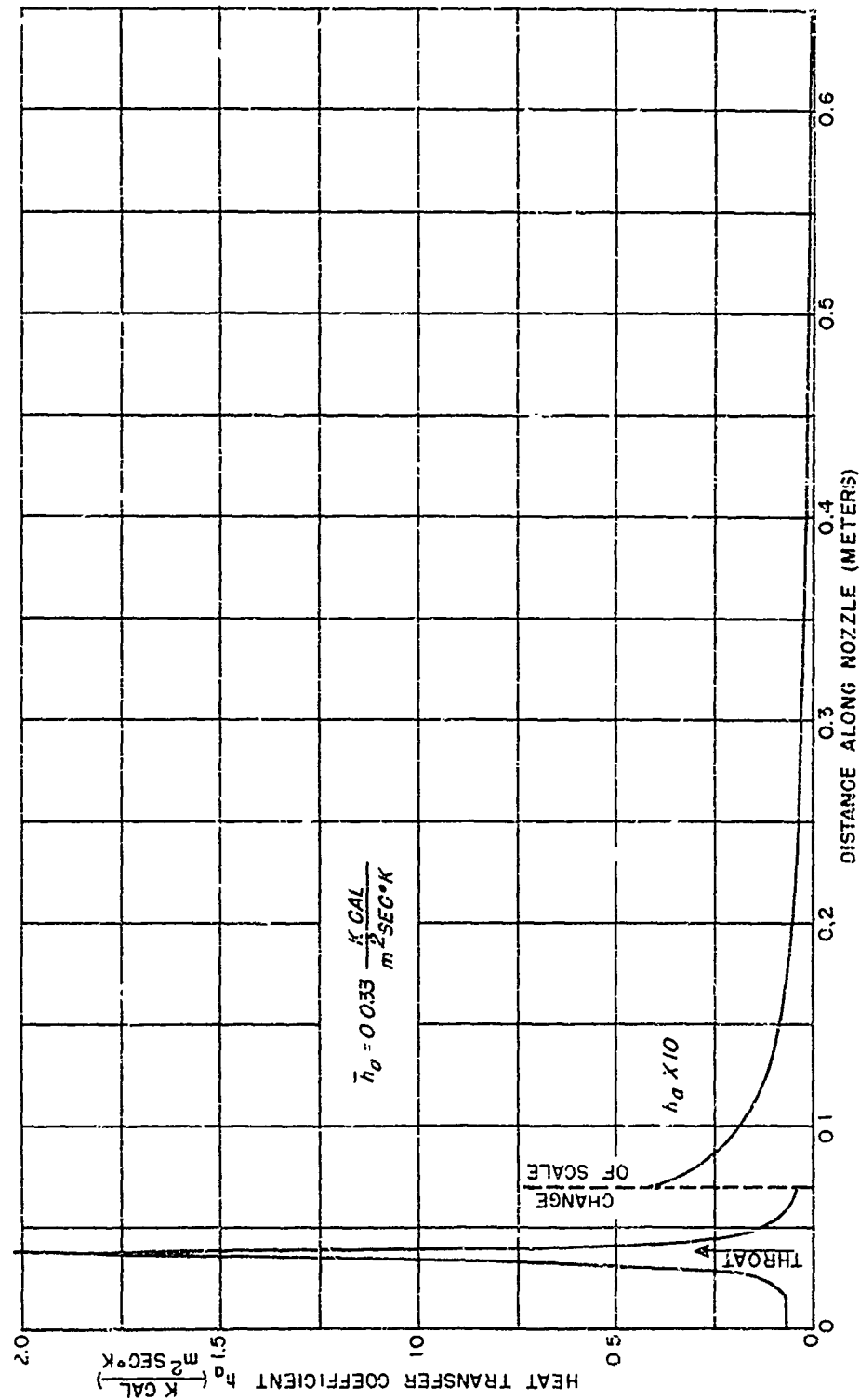


FIG. 30 HEAT TRANSFER COEFFICIENT DISTRIBUTION ASSUMING LAMINAR FLOW AND INCLUDING THE EFFECTS OF PRESSURE GRADIENTS $M_s = 8.0$, $T_w = 600^\circ \text{K}$, $P_0 = 20 \text{ atm}$

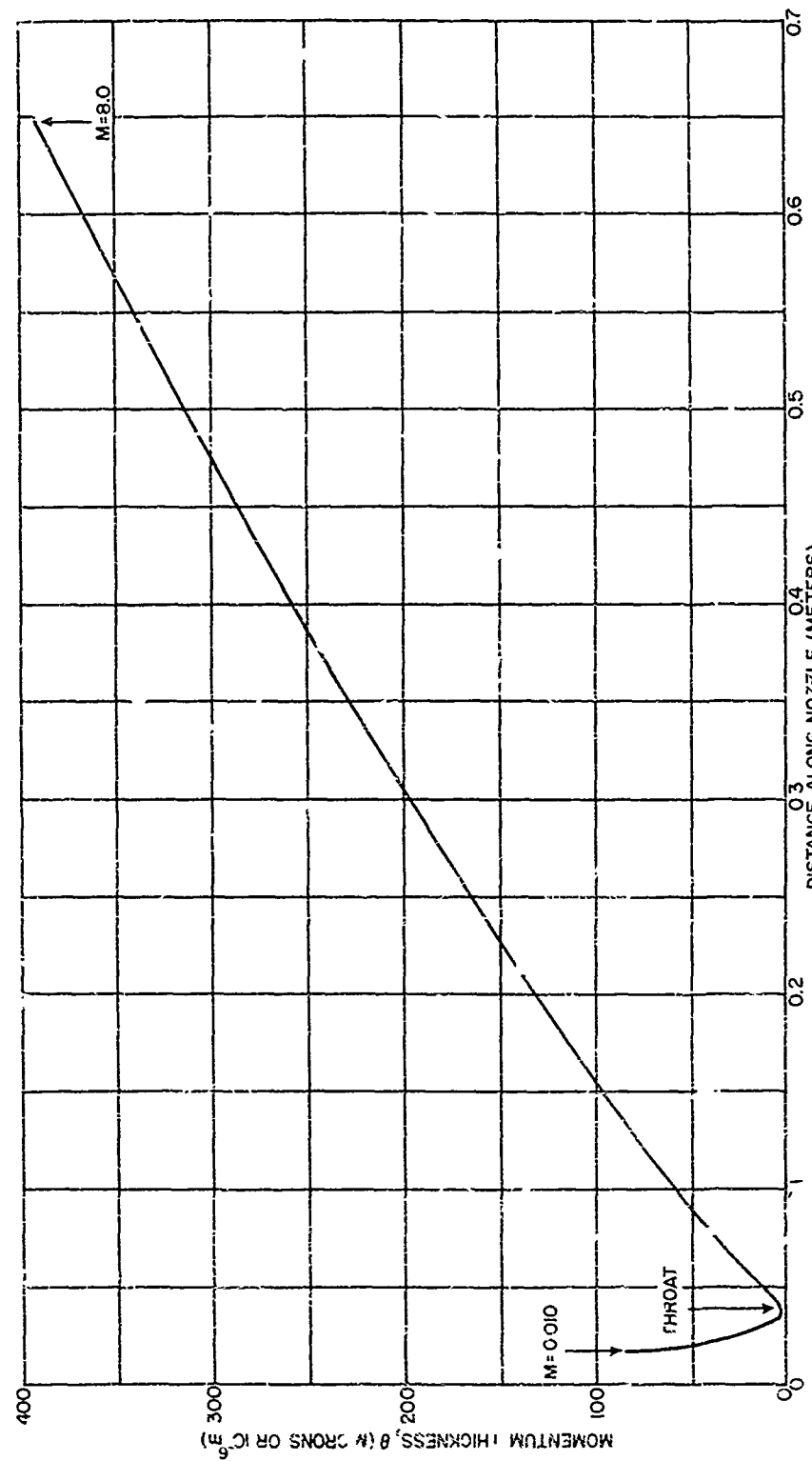


FIG. 31 GROWTH OF MOMENTUM THICKNESS ASSUMING TURBULENT FLOW AND INCLUDING THE EFFECTS OF PRESSURE GRADIENTS
 $M_s = 8.0, T_o = 600^\circ K, P_o = 20 \text{ atm}$

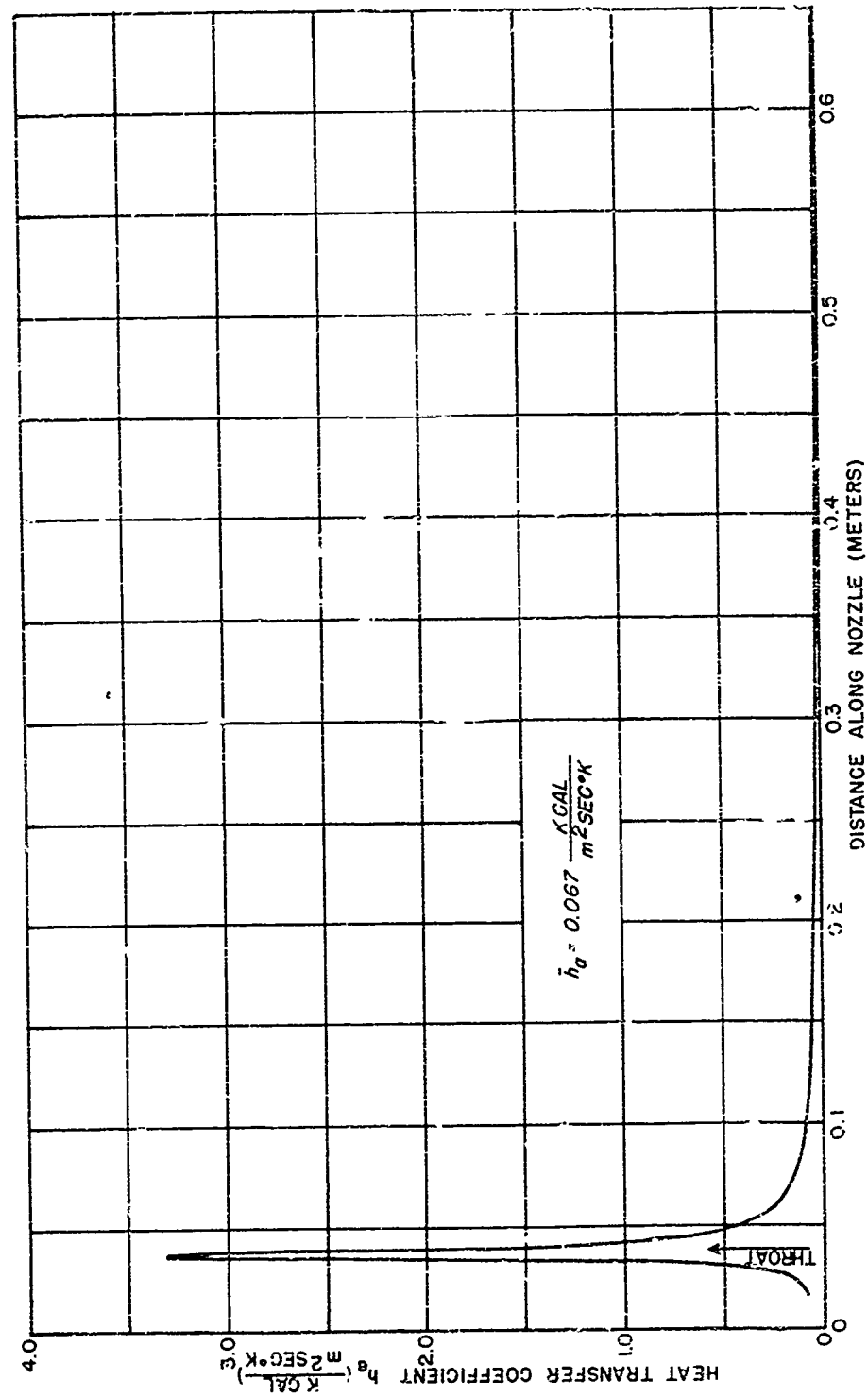
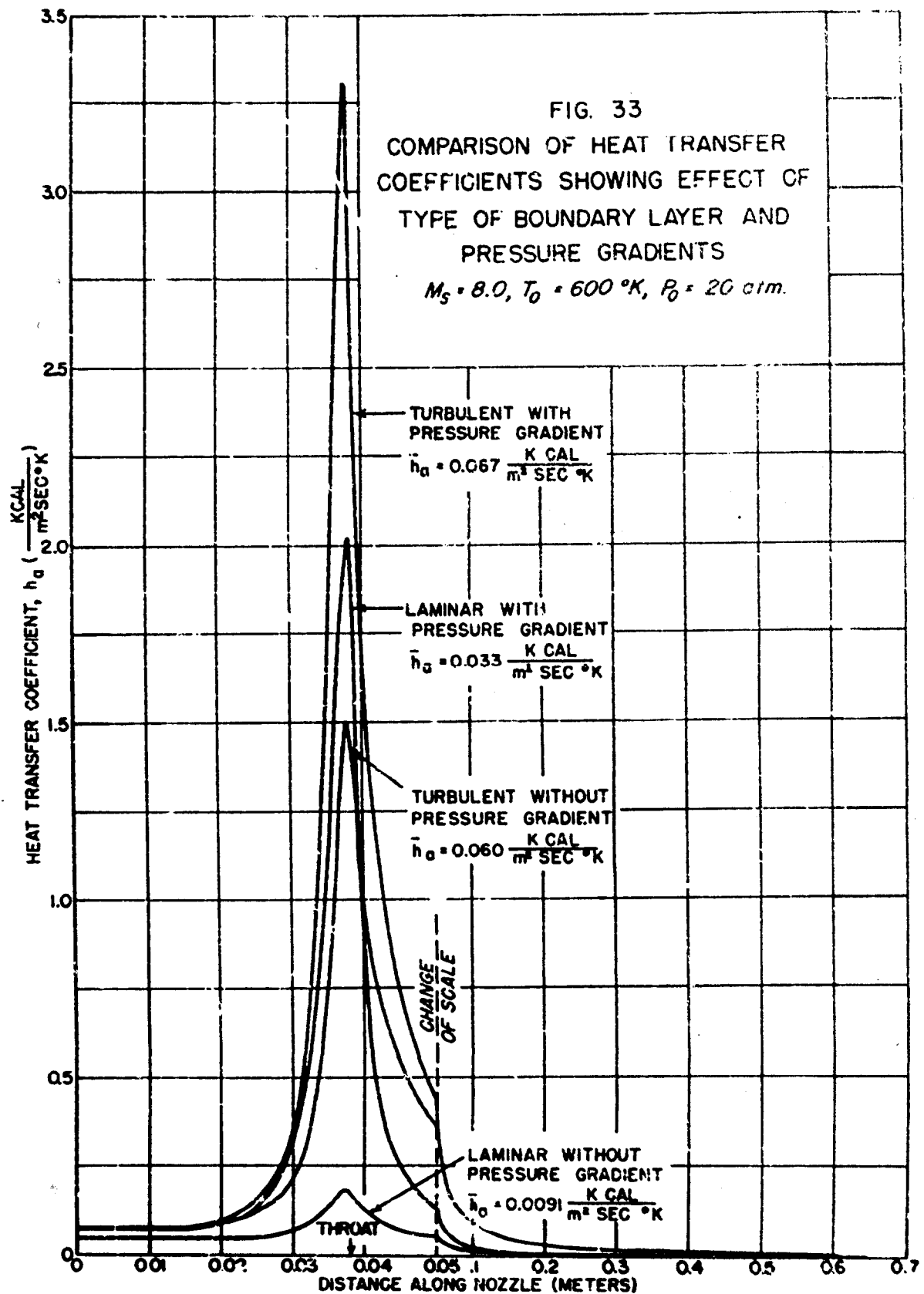


FIG. 32 HEAT TRANSFER COEFFICIENT DISTRIBUTION ASSUMING TURBULENT FLOW AND INCLUDING THE EFFECTS OF PRESSURE GRADIENTS
 $M_s = 8.0$, $T_0 = 600^\circ \text{K}$, $P_0 = 20 \text{ atm}$



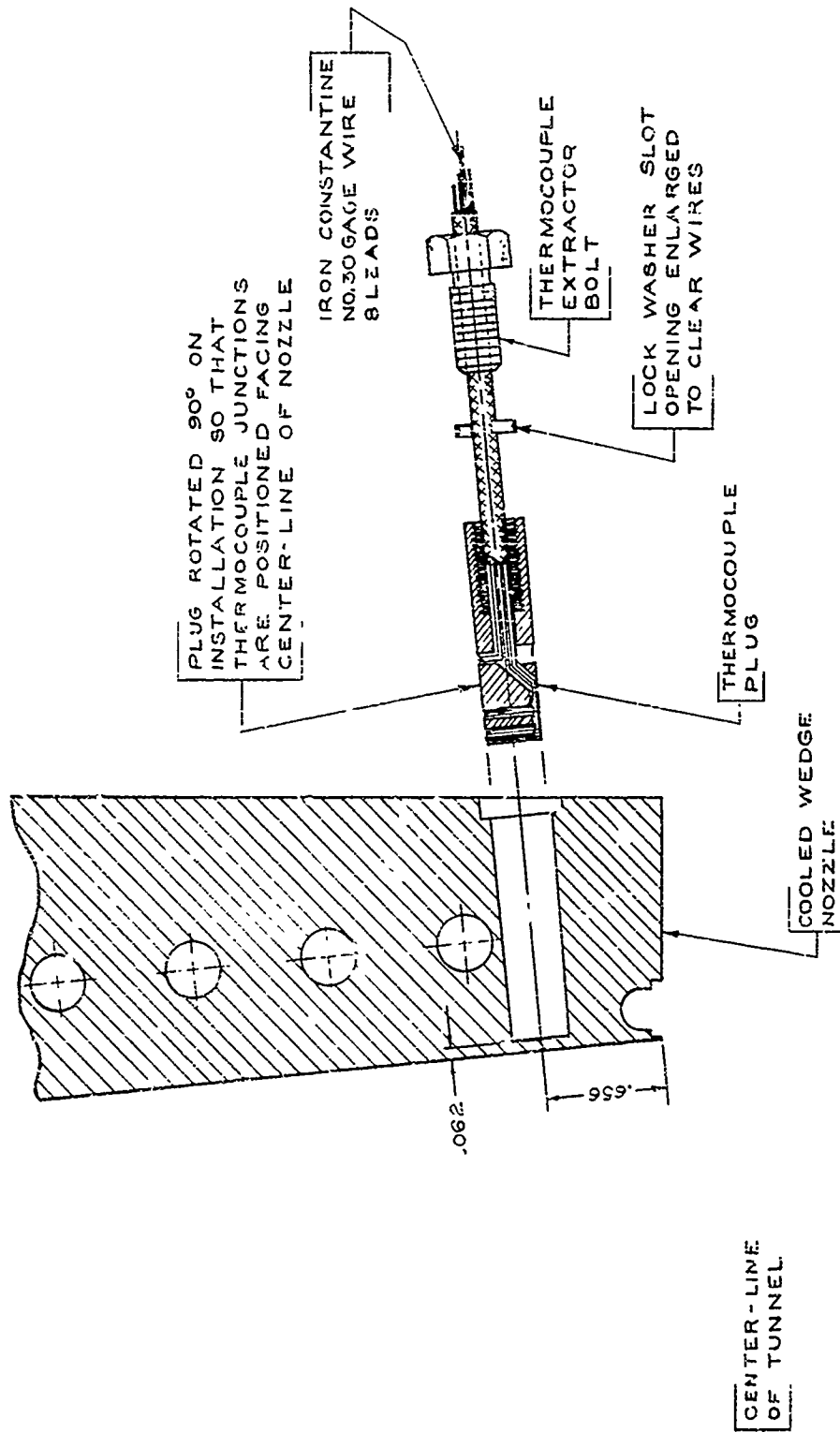


FIG.-34 INSTALLATION OF THERMOCOUPLE PLUG IN NOZZLE BLOCK