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# U. S. Naval School of Aviation Medicine



U. S. NAVAL AIR STATION  
PENSACOLA, FLORIDA

## RESEARCH REPORT



PERCEPTION OF CONTOUR:  
II. EFFECT OF RATE OF CHANGE OF VERTICAL INTENSITY GRADIENT  
REPORT NO. NM 001 075.01.05

U. S. NAVAL SCHOOL OF AVIATION MEDICINE  
NAVAL AIR STATION  
PENSACOLA, FLORIDA

JOINT PROJECT REPORT NO. 5

The Kresge Eye Institute under Contract Nour-586(00)  
Office of Naval Research, Project Designation No. NR-142-023  
and

U. S. Naval School of Aviation Medicine

The Bureau of Medicine and Surgery  
Project No. NM 001 075.01.05

PERCEPTION OF CONTOUR:

II. EFFECT OF RATE OF CHANGE OF RETINAL INTENSITY GRADIENT

Report by

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17 August 1953

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## PERCEPTION OF CONTOUR

### II. Effect of Rate of Change of Retinal Intensity Gradient

#### SUMMARY

The present investigation continues the effort to ascertain the nature of the retinal stimulus effective in producing the perception of an edge under certain simple conditions of photopic foveal vision.

A method is briefly described by means of which a distal stimulus is computed which, under the conditions of observation existing, will result in the desired proximal or retinal stimulus. The observer can project a small fiducial dot of light onto the distal stimulus to indicate where an edge, break, contour or discontinuity appears.

It is shown that the subjective apparent intensity or brightness is not simply related to the intensity on the retina and, indeed, that the relationship may be inverse, namely, that intensely illuminated regions of the retina may appear dark while adjacent, less intensely illuminated regions may appear bright. It is shown that if one were to attempt to express this phenomenon in terms of the classical concept of the Weber-Fechner fraction,  $\Delta I/I$ , it would result in a positive intensity gradient producing a negative brightness increment.

It is further shown that the amount of energy decrement on the retina has little effect upon the appearance of an edge. It is then shown that the rate of change of energy with respect to retinal distance is an unimportant factor in the production of edges and that the lowest derivative of energy with respect to retinal distance of substantial significance for contour formation is the second derivative.

A fundamental principle is stated to be that a doublet edge may appear substantially symmetrically distributed on either side of a point where the rate of change of the rate of change of intensity on the retina is maximal. The separation of this doublet edge will be smaller the greater the value of the fourth derivative of energy with respect to retinal distance.

The phenomena of contour formation, simultaneous contrast, absolute photopic threshold or light sense, differential photopic threshold or light difference sense, minimum visible or visual acuity for a line and minimum resolvable or minimum separable or visual acuity may all be considered chiefly in terms of the second and higher derivatives of energy with respect to retinal distance.

#### METHOD

An artificial pupil, 1 mm. in diameter, and a source of light, the effective visual wave-length range of which is limited to the region 560 to 585 millimicrons, are employed to render defects of the human eye such as

chromatic aberration,<sup>1</sup> spherical aberration,<sup>2,3,4</sup> coma, refractive errors<sup>5</sup> and astigmatism by oblique incidence negligible compared to the overwhelming defect of diffraction and the smaller defect attributable to fixation tremor.<sup>6,7</sup> The Airy disc diffraction pattern may, for many practical purposes, be assumed to consist merely of a visually effective portion of the central disc<sup>8,9</sup> over the surface of which the energy is uniform. An external stimulus is presented such that derivatives of energy with respect to space in the horizontal dimension have definite values but in the vertical dimension are all substantially zero. Furthermore, the vertical extent of the energy distribution is relatively large and the resulting energy distribution on the retina is approximated by computing it as the integral of the energy distribution presented between the limits of plus and minus the radius of the visually effective blur disc as projected into the plane of the stimulus.

The procedure is first to decide what scalar field of energy on the retina is desired. For example, we may require that the third derivative of energy with respect to distance in the horizontal dimension on the retina pass through a maximum and that this maximum have a specific value. We may impose other conditions on the energy at this point and on the rate of change of energy with distance or the energy gradient at this point. The desired scalar field is computed expressing energy,  $E$ , as a function of  $x$  distance on the retina in the horizontal dimension. There is then computed that  $E$  as a function of  $x$  in external space which, when viewed by the eye at a given distance and through a given pupil and filter, will result in the desired energy distribution being obtained on the retina. That is, a distal stimulus is computed which, under the conditions of observation existing, will result in the desired proximal or retinal stimulus. The observer can project a small fiducial dot of light unto the distal stimulus to indicate where an edge, break, contour or discontinuity appears. The reading of the position of such dot on the distal stimulus constitutes one recorded observation. A more detailed description of the method and apparatus employed has been given in a previous paper.<sup>10</sup>

## RESULTS AND DISCUSSION

Figure 1 shows three different retinal energy distributions plotted as solid lines and the location of the experimentally observed breaks plotted as circles. On the  $y$  axis is plotted units of relative retinal energy. On the  $x$  axis is plotted distance on the distal stimulus in inches, each square being .2 inches. The distance of observation was 12 ft. and, therefore, each square corresponds to a retinal distance of about .023 mm., the exact figure depending upon the specific schematic or reduced eye used for calculation. A relative retinal energy of 1.0 corresponds to a brightness of 65 millilamberts on the distal stimulus. The results are the mean of 36 observations, 18 observations by one subject and 18 by another. The distance between the bars represents two standard errors of the mean. The retinal functions portrayed are: for curve A,  $y = .8957 \int .1207x - 1.00x^2 \int 26.04x^4 - 406.9x$ ; for curve B,  $y = .8787 \int .2698x - 1.00x^2 \int 5.208x^4 - 16.28x^6$  and for curve C,  $y = .7933 \int .6034x - 1.00x^2 \int 1.042x^4 - .6510x^6$ . All three curves have a maximum value of  $d^2y/dx^2$  at  $x = 0$ .  $d^3y/dx^3$  is, of course, zero at  $x = 0$ .  $d^4y/dx^4$  is maximal at  $x = 0$  and its maximum value is 625 for curve A, 125 for

curve B and 25 for curve C. All three curves have the characteristic that at some positive value,  $x = \frac{1}{2} s$ , of retinal distance,  $dy/dx$ ,  $d^2y/dx^2$  and  $d^3y/dx^3$  are all zero and  $y = .9$ . (For clarity, curves B and C have been lowered on the  $y$  axis.) At this point the curves are arbitrarily flattened out. All three curves have the characteristic that at some value,  $x = -s$ ,  $d^2y/dx^2$  and  $d^3y/dx^3$  are zero. The value of  $dy/dx$  at  $-s$  has been computed and the distal stimulus has been given the required slope at this point so that the first three derivatives show no discontinuity at either  $s$  or  $-s$ . Thus, the curves depicted are plots of the functions given between  $x = -s$  and  $x = \frac{1}{2} s$ . Outside of these limits they are arbitrarily made straight lines. Not illustrated in the figure is the fact that all three curves are prolonged beyond  $x = -.5$  until the  $y$  value drops to approximately .42, at which value of  $y$ ,  $dy/dx$  is arbitrarily made zero.

Figure 2 shows, on a reduced scale, the entire curve B plotted as a solid line. At the point indicated by the arrow the first and all higher derivatives are discontinuous. Two edges appear, one on either side of this point, but only four observations are taken on these frivolous edges.

The dashed line curve in Figure 2 is qualitatively illustrative of the subjective apparent intensity or brightness of the retinal energy distribution depicted as Curve B. To the right of  $E_1$  the figure appears as a uniform bright gray. At  $E_1$  there occurs an edge which is quite sharp in appearance as is indicated by the small standard error of the mean. Between  $E_1$  and  $E_2$  lies a strip of uniform brightness which is definitely brighter than the figure to the right despite the fact that the retinal energy at all points in the region  $E_1E_2$  is less than it is to the right of  $E_1$ . At  $E_2$  another sharp edge appears and to the left of  $E_2$  the figure is darker. In the immediate vicinity to the left of  $E_2$  the figure appears to be uniformly dark. There then occurs a gradual brightness gradient between  $E_2$  and the right of the pair of frivolous edges  $E_3E_4$ . On looking from region  $E_2$  to the nearer region  $E_3$ , it is subjectively apparent that a change in brightness has occurred but sharp edges are not formed despite the fact that the rate of change of energy with retinal distance is maximal in this region. Returning now to a consideration of figure 1, all three of the curves A, B and C produce a subjective appearance qualitatively similar to that just described for figure 2, namely, a bright strip with gray on either side, and eventually, to the left, the frivolous edges. The inverse relationship between brightness and intensity is most strikingly illustrated by the retinal energy distribution depicted by curve C. It is apparent that the retinal energy in the bright strip between the two observed edges at  $\frac{1}{2} .26$  and  $-.20$  is substantially less than the retinal energy to the right of  $\frac{1}{2} .26$ , yet the strip is markedly brighter than its background. The relative lack of brightness of the region to the right of  $x = \frac{1}{2} .26$  is not attributable to a deficiency in area since curve C continues flat out beyond the value of  $\frac{1}{2} 1.00$ , although, to conserve space this fact is not depicted on the graph. Curve C also illustrates the fact that no brightness variation is perceived within the region from  $-.20$  to  $\frac{1}{2} .26$  although the relative retinal energy has increased from .54 to .89. If one were to attempt to express this phenomenon in terms of the classical concept of the Weber-Fechner fraction  $\Delta I/I$ , it would mean that within the strip, a  $\Delta I/I$  value of .65 does not suffice to produce a perceptible brightness increment, whereas, a  $\Delta I/I$  of .015 has, in a certain sense, produced a perceptible, although negative, brightness increment at the right edge of the

strip. Furthermore, on looking from one edge of the strip to the other, no subjective gradient appears. At the distance of observation employed, the width of the strip stimulates a retinal region 13 to 55 foveal cone diameters wide, depending upon whether foveal cones are assumed to subtend 12 or 60 seconds of arc at the nodal point of the eye.

In the retinal energy distribution shown by curve A, an edge occurs at  $x = .13$  and the other edge occurs at  $x = -.13$  after an energy decrement of  $.03$  has occurred. In curve B, the left edge appears after an energy decrement of  $.08$  has occurred. In curve C, the energy decrement from one edge to the other is  $.40$ . It appears that the amount of energy decrement has little effect upon the appearance of an edge. Thus, curve A shows that an energy decrement of  $.03$  suffices to produce an edge and if an energy decrement of this magnitude were all that were required, curve C would produce thirteen edges in a region which, in fact, appears of uniform brightness.

Consider next the effect of  $dy/dx$ , the rate of change of energy with respect to retinal distance. On curve A, one edge occurs where  $dy/dx$  is zero whilst the other edge occurs where  $dy/dx$  is  $.24$ . The right edge of the strip of curve A, demonstrates that a sharp discontinuity in brightness may be seen in a region where the retinal energy is not changing with distance. In other words, an edge may appear where the retinal intensity gradient<sup>11</sup> is zero. In curve B, the left edge of the strip does not appear until  $dy/dx$  achieves a value of  $.51$ . While in curve C, the left edge does not appear until  $dy/dx$  has reached  $.96$ , a value four times as high as that at which the left edge occurs in curve A. The figures demonstrate that the value of  $dy/dx$  has little to do with the production of an edge.

Consider next the significance of  $d^2y/dx^2$ . All three curves of figure 1 are illustrative of the phenomenon which has been described in a previous paper by the author,<sup>10</sup> namely, that on either side of the maximum value of  $d^2y/dx^2$  there appears an edge. Thus, each maximum value of  $d^2y/dx^2$  is associated with a doublet edge and the separation of the doublet determines the thickness of the bright or dark line perceived. It is apparent that the two edges are substantially symmetrically distributed to the right and left of the point where  $d^2y/dx^2$  is maximal.

In curves A and B of figure 1, the edges occur in regions where  $d^2y/dx^2$  is practically zero but that this is not a necessary condition for the appearance of an edge is shown by curve C on which the right edge occurs in a region where  $d^2y/dx^2 = -1.24$  while the left edge occurs in a region where  $d^2y/dx^2 = -1.55$ .

It is, however, necessary for the formation of edges that the value of  $d^2y/dx^2$  at its maximum be some minimum absolute value. Thus, if  $d^2y/dx^2$  at  $x = 0$  is held at 525, the value it has in curve A, but the maximum absolute value of  $d^2y/dx^2$  is reduced from 2.0 to 1.2 the edges disappear. This does not mean that  $d^2y/dx^2$  at  $x = 0$  must, under all conditions, achieve a magnitude of 1.2. Thus, if the maximum value of  $d^2y/dx^2$  be reduced to 25, then  $d^2y/dx^2$  at  $x = 0$  may be  $-.8$  and clear edges will appear as is illustrated by curve B of figure 3.

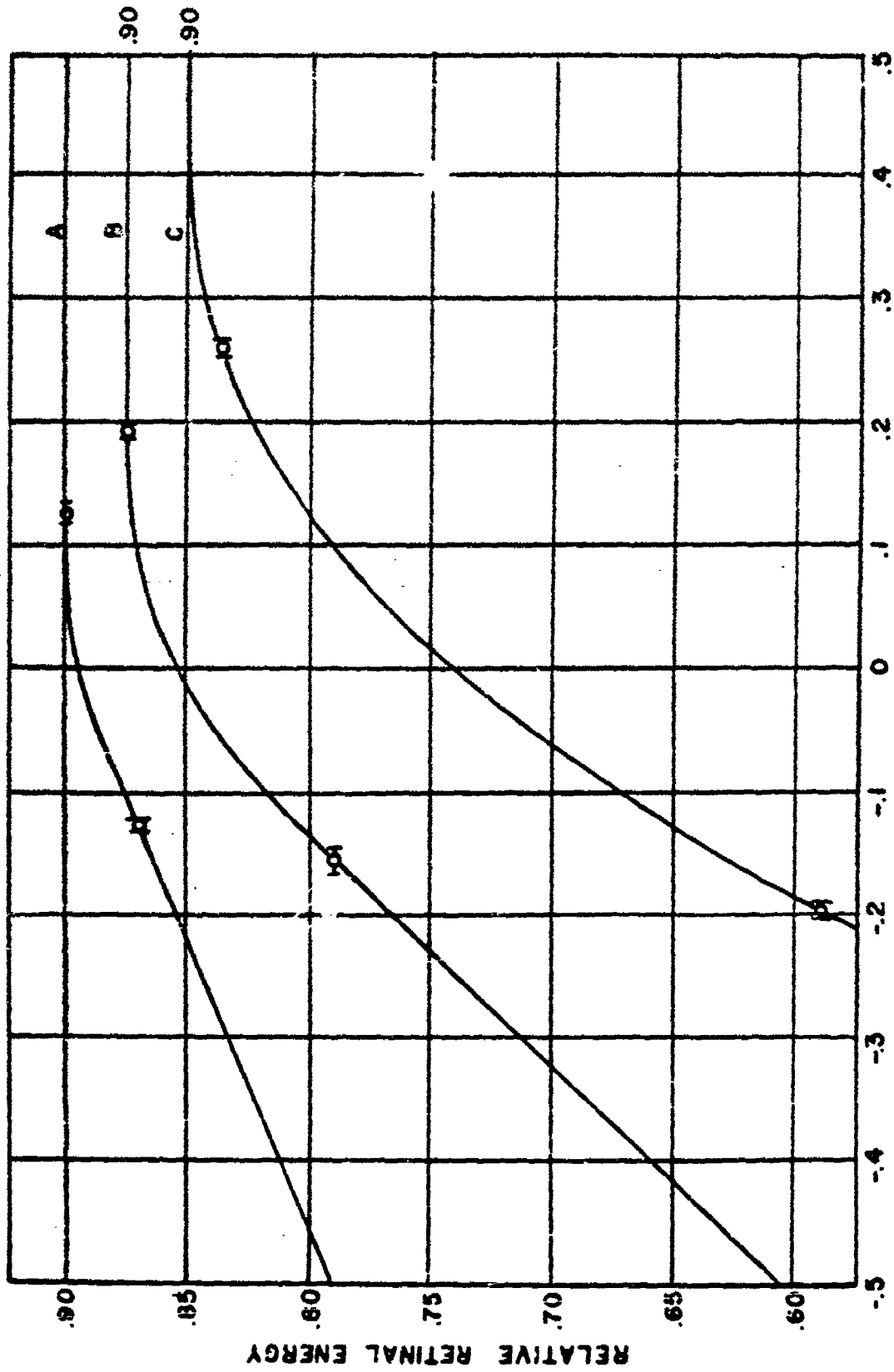
The foregoing observations and discussion indicate that the lowest derivative of substantial significance for contour formation is the second.  $d^2y/dx^2$  must achieve a certain maximum value whether positive or negative in order to produce a doublet edge. The separation of the doublet edge and hence, the width of the bright or dark strip decreases as the maximum value of  $d^4y/dx^4$  is increased as is shown by curves A, B and C. A comparison of curves C and E shows that in this general type of curve the zero, first and second derivatives may be altered substantially without serious change in doublet width if the maximum value of  $d^4y/dx^4$  is held constant.

The relative unimportance of the value of  $dy/dx$  has been mentioned. If the coefficient of  $x$  be set equal to zero in the equations for curves A, B, C and E these curves will become symmetrical about the  $y$  axis and a narrow or wide, black, gray or white line appears on a uniform background. The doublet edge is now associated with a test of (1) the absolute photopic threshold or light sense, (2) the differential photopic threshold or light difference sense or (3) the minimum visible or visual acuity for a line depending upon the magnitudes of  $y$ ,  $d^2y/dx^2$  and  $d^4y/dx^4$  which are employed. Thus, if  $d^4y/dx^4$  is greatly increased the line may be made too "small" to be seen; if  $d^2y/dx^2$  is greatly decreased the line will have too little "contrast" to be seen while if  $y$  is greatly reduced, all derivatives will decrease, and the line will become too "dim" to be seen.

If functions are employed which produce more than one doublet edge, adjacent edges from different doublets may fuse and the values of the various derivatives now affect the minimum resolvable as will be described in a subsequent report.

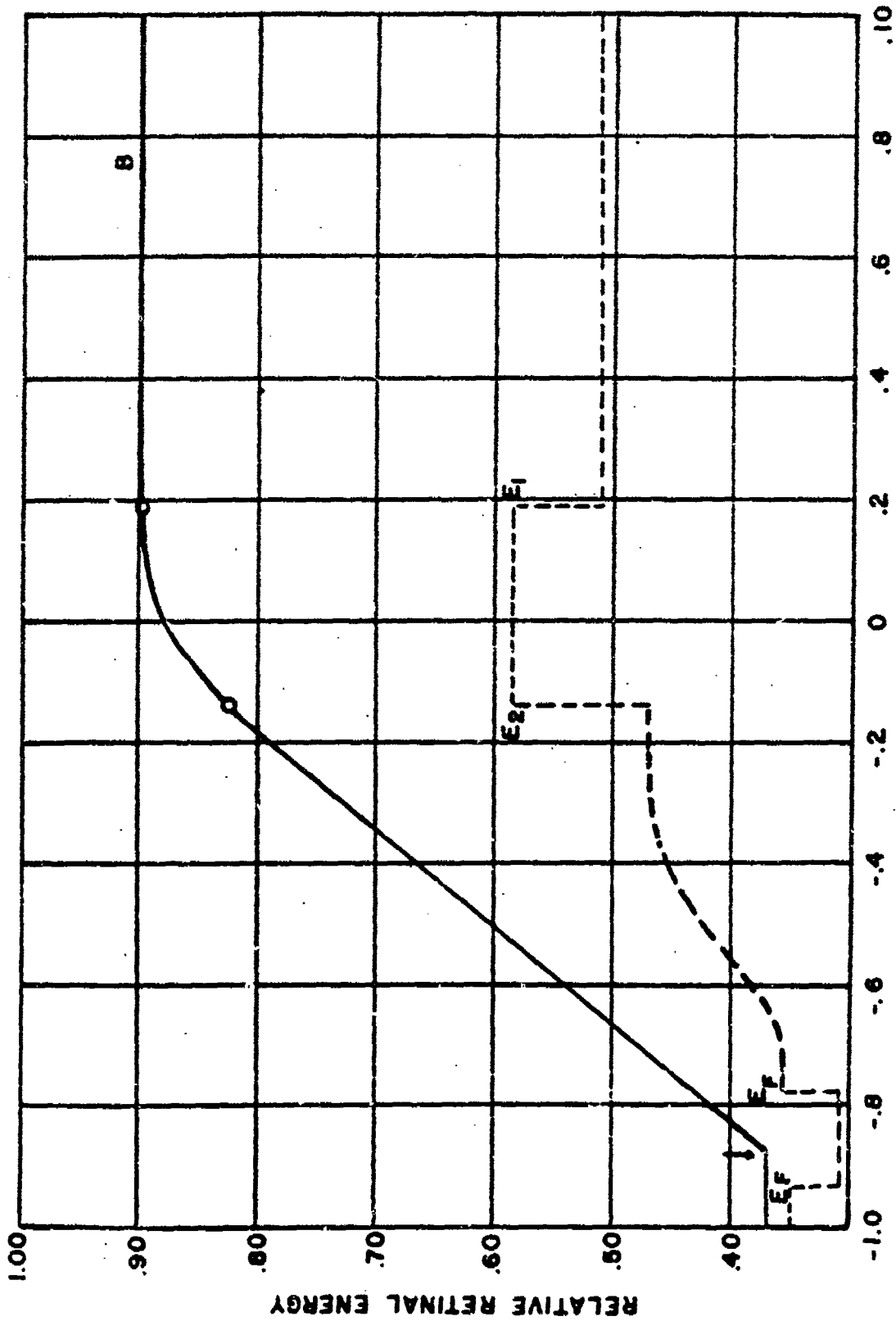
## BIBLIOGRAPHY

1. Sheard, C. S.: The Chromatic Aberration of the Eye, the Chromatic Variations in the Interval of Sturm, and Allied Phenomena as Determined by a Subjective Method of Skiascopy, Jr. Opt. Soc. Am., 12:82, 1925.
2. Jackson, E.: Symmetrical Aberration of the Eye, Trans. Amer. Ophth. Soc., 5:141, 1888.
3. Pi, H. T.: Total Peripheral Aberration of the Eye, Trans. Soc. U. K. 45, Part I:393, 1925.
4. Stine, G. H.: Variations in Refraction of the Visual and Extra-Visual Pupillary Zones, Am. Jr. Ophth., 13:101, 1930.
5. Ames, A. and Proctor, C. A.: Dioptrics of the Eye, Jr. Opt. Soc. Am., 5:22, 1921.
6. Adler, F. H. and Fliegelman, M.: Influence of Fixation on the Visual Acuity, Arch. Ophth., 12:475, 1934.
7. Riggs, L. A. and Ratliff, F.: The Steadiness of the Eye, Jr. Opt. Soc. Am., 39:630, 1949.
8. Jacobs, D. H.: Fundamentals of Optical Engineering, New York, McGraw-Hill Book Co., Inc., 1943, p. 178.
9. Hardy, A. C. and Perrin, F. H.: The Principles of Optics, New York, McGraw-Hill Book Co., Inc., 1932, p. 129.
10. Ludvigh, E. J.: Perception of Contour: I. Introduction, U. S. Naval School of Aviation Medicine and The Kreege Eye Institute, Project No. EM 001 067.01.01, Joint Project Report 4.
11. Ludvigh, E. J.: The Gradient of Retinal Illumination and Its Practical Significance, Am. Jr. Ophth., 20:260, 1937.



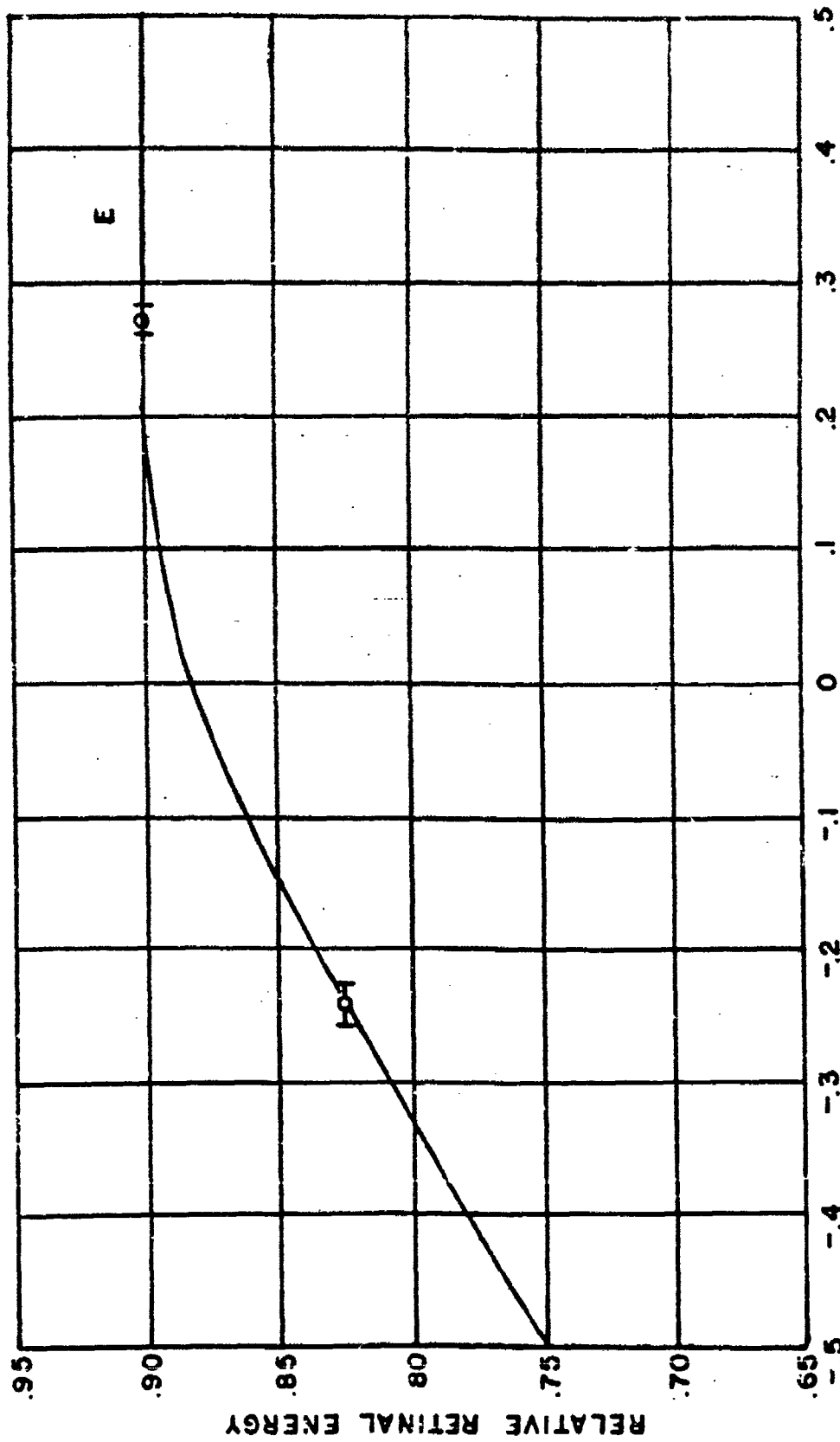
DISTAL STIMULUS DISTANCE IN UNITS OF INCHES  
 RETINAL DISTANCE IN UNITS OF .115 MM.

FIGURE 1



DISTAL STIMULUS DISTANCE IN UNITS OF INCHES  
RETINAL DISTANCE IN UNITS OF .115 MM.

FIGURE 2



DISTAL STIMULUS DISTANCE IN UNITS OF INCHES  
 RETINAL DISTANCE IN UNITS OF .115MM

FIGURE 3