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## SOME OBSERVATIONS ON PHOTOELASTIC MATERIALS STRESSED BEYOND THE ELASTIC LIMIT

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### INTRODUCTION

Much of the recent literature on the plastic state of materials has been concerned with the development and application of various theories of plasticity while very little has appeared with regard to experimental methods of stress analysis beyond the elastic range.

The brilliant success of mathematical methods is certainly generally recognized and acknowledged. But one cannot limit a problem by reason of a method of attack. There is abundant evidence that experimental stress analysis can become as useful and necessary a tool for the solution of problems in the plastic state as it has been for the determination of elastic states of stress when the mathematical methods have become too cumbersome or complex.

An experimental method which has given some - at least qualitative - data of interest in connection with plastic flow is the photoelastic method. The isochromatic lines appearing in the photoelastic stress pattern are lines of constant principal stress difference and hence, for materials which follow Tresca's criterion for plastic flow, represent the boundaries between the elastic and plastically deformed regions of a stressed body. The work of Coker<sup>1\*</sup> and Mesmer<sup>2</sup> in this connection are of particular interest. They have also shown that the shear stress trajectories, which are readily obtained from the isoclinics in a photoelastic test, correspond very closely to the slip lines produced in a geometrically similar mild steel member under similar load conditions.

Another possible approach to the problems of the plastic state suggested again by the photoelastic method - is that of investigating flow of a purely viscous nature through analysis Presented at the SESA Annual Meeting, Dec. 1 1949

\*Superscripts refer to bibliography at end of paper

of the birefringence accompanying such flow. Recent work in this field (cf. (3) (4) (5) and (6)) has shown that in some media the amount of birefringence produced by the flow varies linearly with the shear stress developed by that flow.

Finally, it is well known that the optical effects produced by stress (and/or strain) in photoelastic materials persist well beyond the range of perfect elasticity and are, in fact, observable up to fracture. Thus it appears that the most direct approach to the problem of evaluating plastic stresses would be the simple extension of the usual photoelastic method to the plastic range. Before this can be done, if indeed such a simple extension can be made at all, it is necessary to establish the stress-optical relationships which are opera-

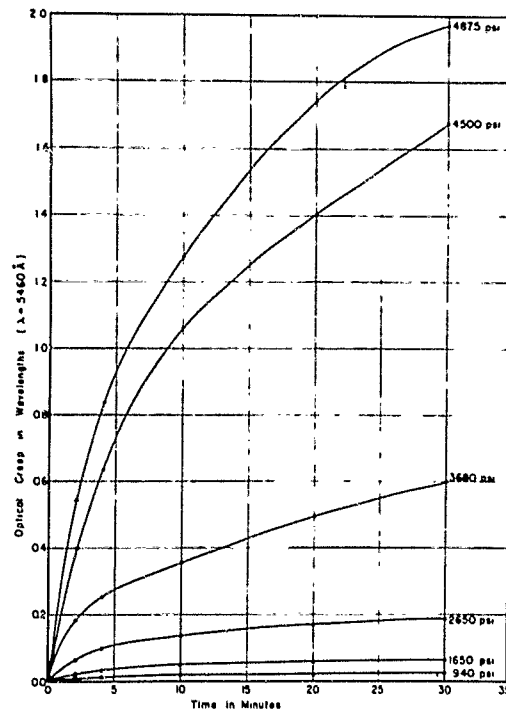


Fig. 1. Optical creep vs. time curves for cellulose nitrate at six levels of stress.

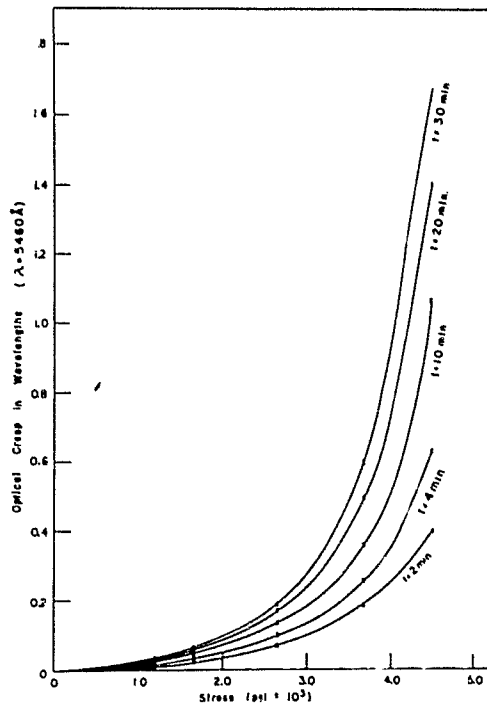


Fig. 2. Optical creep vs. stress for cellulose nitrate

tive beyond the elastic limit of the photoelastic material used.

#### THE STRESS-OPTICAL EFFECT IN SOME PHOTOELASTIC MATERIALS

Published results on the stress optical properties of photoelastic materials have, with but two or three notable exceptions, been concerned with behavior in the elastic range only. Within this range, the functional relationship between optical retardation and stress must, of course, be the same as that between optical retardation and strain. However, beyond this range these relationships will, in general, differ. It should be emphasized at this point that we are not here concerned with the question of whether the optical effects are caused by the stress or by the strain in the material under load, but only with their functional relationships.

The materials studied thus far have been: polystyrene, lucite, plexiglass, nylon, cellulose acetate, silver chloride, and cellulose nitrate. Of these, the first three were judged unsuitable for further study because of their low optical sensitivity.

The nylon (type FM 3001 supplied by DuPont) although possessing good optical sensitivity was only available in rather thin sheets (0.0083" thick) and consequently could sustain no compressive stresses. Further, in this thickness, the material became opaque shortly after its yield point in tension was reached.

Cellulose acetate was found to have good optical sensitivity and a linear relationship between optical retardation and stress well above its elastic limit. However, under high stress the structure of the material became rather badly broken up and the isochromatics appeared somewhat fuzzy and distorted.

Some photoelastic tests were made with rolled polycrystalline silver chloride strips which had been annealed prior to testing. This transparent material has a cubic crystal structure and hence is optically isotropic in the unstressed state. It is also very ductile and, in contrast with the usual photoelastic materials, has the property of strain hardenability. Under a uniform tensile load in the elastic range, the birefringence appears "spotty", i.e. adjacent grains show different amounts of birefringence and, indeed, parts of the same grain show different colors under white light illumination. This, of course, is due to differences in crystallographic orientation of adjacent

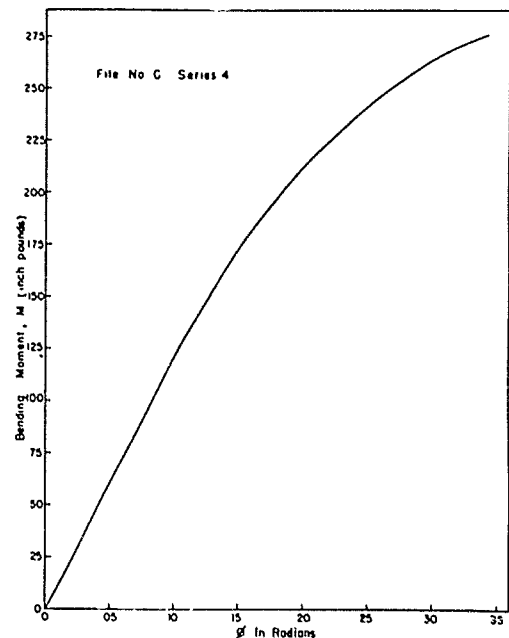


Fig. 3. Bending moment vs. slope of elastic line.

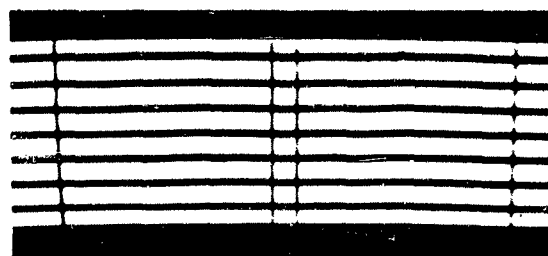
grains. Under plastic deformation, birefringent bands are produced in individual grains and investigation of states of stress on other than a microscopic scale, with this material, appears quite remote. However, optical studies with silver chloride may throw considerable light on the mechanism of slip<sup>7</sup>, and perhaps on other fundamental microstructural changes (8) which have been observed in metals. Further experiments on the stress-optical properties of this material are being continued in our laboratories and will be reported at a later date.

The most suitable material tested, from the point of view of the application of the proposed method to the solution of problems in technical mechanics, was cellulose nitrate. The optical retardation increased linearly with the stress up to a practical limit of approximately twice the elastic limit stress. This result is in agreement with reports of other investigators (9,10, and 11).

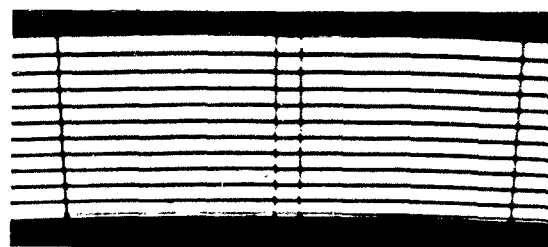
Cellulose nitrate exhibits certain mechanical and optical properties, characteristic of viscoelastic materials, which one must recognize and consider in the planning of experiments and in the valid interpretation of the results of such experiments. The most important of these are the following:

- 1) Its optical and mechanical properties vary with the age of the material.
- 2) It exhibits both optical and mechanical creep under sustained loads.
- 3) Its properties vary with rate of loading and with temperature at time of test.
- 4) Its properties depend, to a variable extent, on its strain history or previous mechanical treatment. (This dependence is, however, of a transient nature and does not persist over a long period of time as in the case of a work hardenable material).

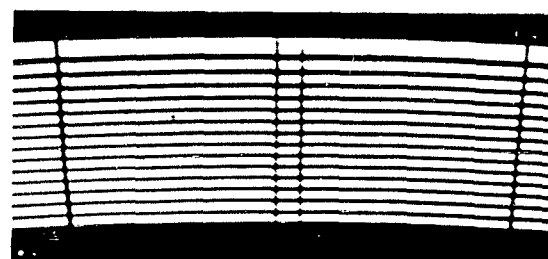
In the work reported here, the calibration specimens and the model specimens were cut from the same sheet. All tests were run in a constant temperature room at 70°F. and were made on previously unstrained material. Calibration and model tests were performed within a few hours of each other. With the exception of the creep tests, all measurements of birefringence and strain were made 15 seconds after full load application. Birefringence in calibration and creep tests was measured with a Babinet-Soleil Compensator and monochro-



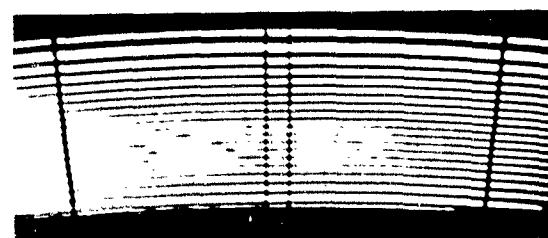
M = 50 inch-pounds



M = 100 inch-pounds



M = 150 inch-pounds



M = 200 inch-pounds

Fig. 4. Photoelastic stress patterns for uniform bending in cellulose nitrate beams.

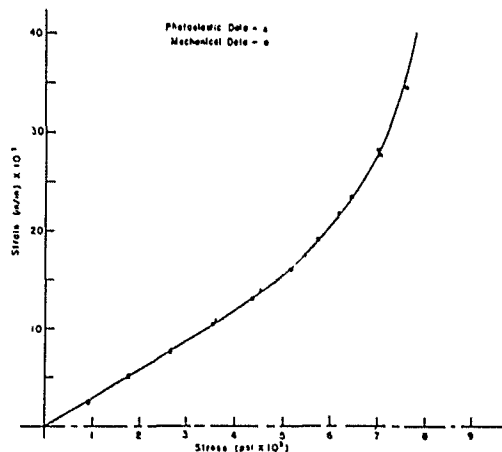


Fig. 5. Stress distribution in cellulose nitrate beam in plastic bending. (Tension side)

matic light ( $\lambda = 5460 \text{ \AA}$ ) was used throughout.

Since we were interested in applying the method to the problem of plastic bending under a uniform bending moment, the calibration specimens were loaded at neither a constant strain rate nor at a constant stress rate. (cf. Mindlin 13). Instead, each was loaded in such a way that the full tensile load was developed 5 seconds after start of load application. Thus, specimen no. 1 was loaded at a rate of approximately 100 psi/sec. and measurements were made at the maximum stress of 500 psi; specimen no. 2 was loaded at a rate of approximately 200 psi/sec and measurements were made at the maximum stress of 1000 psi; etc. Care was taken to avoid dynamic load effects.

Properties of two grades of cellulose nitrate used in these experiments are tabulated below

Material	A	B
Elastic limit	4400 psi	3300 psi
Ultimate strength	8600 psi	6800 psi
Modulus of Elasticity	$3.3 \times 10^5$ psi	$2.4 \times 10^5$ psi
Material fringe value	110 psi shear	128 psi shear

Both sets of values fall within the range of variation in physical properties of cellulose nitrate listed by Frocht<sup>14</sup>. The material fringe values were constant up to a tensile stress of 8000 psi for material A, and up to 6000 psi for material B.

Optical creep tests were made by maintaining the loads on the tensile calibration specimens and measuring the change in birefringence at various intervals of time. Results of creep tests are illustrated in Fig. 1 and 2.

#### APPLICATION TO PLASTIC BENDING OF RECTANGULAR BEAMS

One of the few problems in plasticity for which a solution is known, is that of pure flexure (cf. Nadai<sup>15</sup>). The theory is based on the assumptions usually made in the theory of elastic bending that cross-sections remain plane during bending. Experimental evidence of the validity of this assumption for non-elastic materials has been given by Bach<sup>16</sup> and others.

The results of the calculation for pure bending of a bar with rectangular cross-section may be summarized in the following two formulae:

$$M = \frac{bd^2}{(\epsilon_1 + \epsilon_2)} 2 \int_{-\epsilon_1}^{\epsilon_2} f(\epsilon) \epsilon d\epsilon \quad (1)$$

$$\phi = \frac{(\epsilon_1 + \epsilon_2)L}{2d} \quad (2)$$

Where:

b = width of beam

d = depth of beam

$\epsilon_1$  = strain in extreme compression fiber

$\epsilon_2$  = strain in extreme tension fiber

$f(\epsilon) = \sigma$  = stress corresponding to strain  $\epsilon$ .

L = distance between two initially parallel cross-sections of the beam which, under the bending moment M, make an angle  $2\phi$  with each other.

These equations give the moment M as a function of the slope  $\phi$  of the elastic line if the law of deformation of the material is known.

We have elected here to solve the inverse problem, i. e. determine the stress distribu-

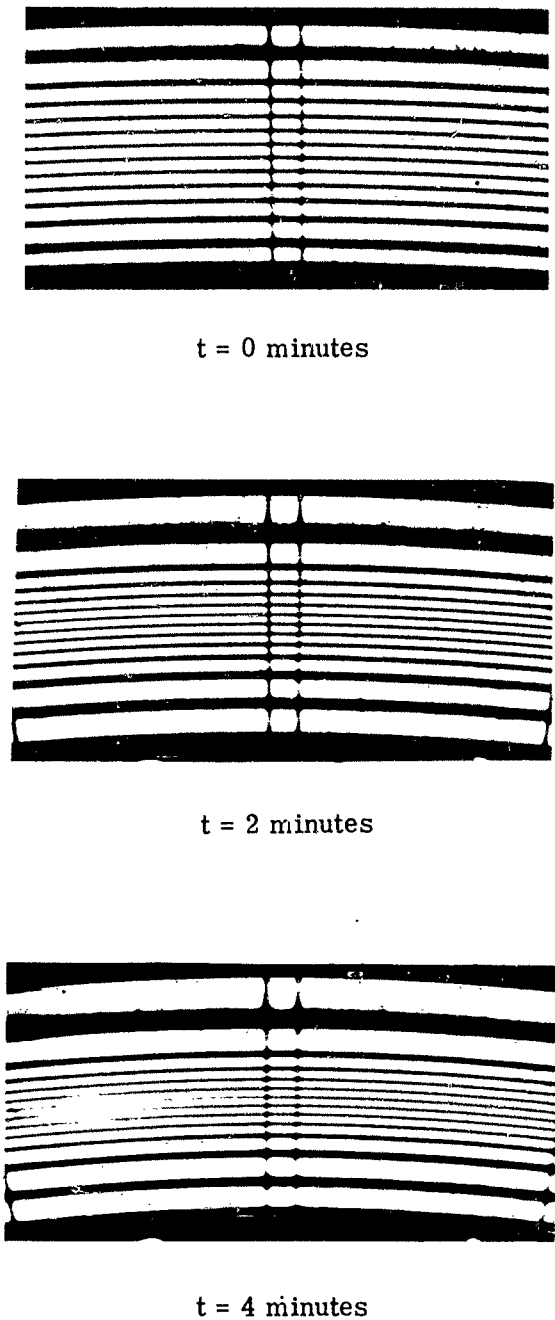


Fig. 6. Photoelastic stress patterns for uniform bending with creep. Bending moment = 150 inch-pounds.

tion in a beam under a bending moment,  $M_{max}$ , from the measurement of  $\epsilon_1$  and  $\epsilon_2$  for various values of  $M$ , and compare that stress distribution with the one obtained from simultaneous photoelastic measurements. In effect, the problem becomes one of determining the shape of the stress-strain curve  $[\sigma = f(\epsilon)]$  from measurements of  $M$  and  $\epsilon_1$  and  $\epsilon_2$ .

The cellulose nitrate beams used in these tests were 1/4 inch thick and 3/4 inch deep and had an effective length of 40 inches. Loads were applied at the ends and the beams were supported at points 10 inches in from each end.

Again, loads were applied at such a rate that the full bending moment was developed in 5 seconds and measurements and photographs were taken 15 seconds later. The slope  $\phi$  of the tangent to the elastic line was determined for a number of loads or bending moments  $M$ . The variation of  $M$  with  $\phi$  for the example treated here is shown by the curve in Fig. 3.

Photoelastic patterns were simultaneously obtained for each value of bending moment and a number of these are illustrated in Fig. 4. From the photoelastic patterns, the distances,  $h_1$  and  $h_2$ , of the extreme fibers from the neutral axis were measured and the extreme fiber strains determined from the relationship

$$\epsilon = 2\phi h/L \tag{3}$$

The corresponding fiber stresses were determined from the equations (cf. 15)

$$\sigma_1 d\epsilon_1 = \sigma_2 d\epsilon_2 \tag{4}$$

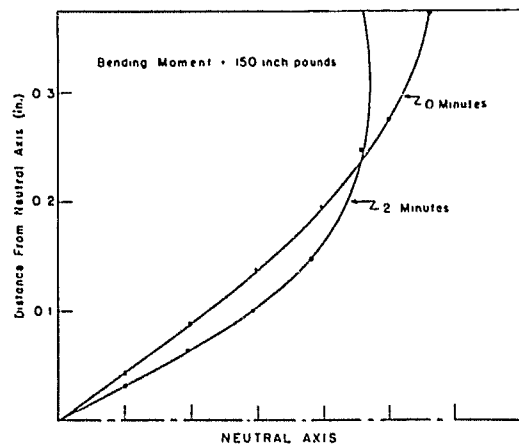


Fig. 7. Approximate stress distribution in beam at times  $t = 0$  and  $t = 2$  minutes. (tension side)

and

$$bd^2 \sigma_1 \sigma_2 / (\sigma_1 + \sigma_2) = \frac{1}{\phi} \frac{d}{d\phi} (M\phi^2) \quad (5)$$

We thus obtain coordinates on the curve  $\sigma = f(\epsilon)$ .

Since, in uniform bending, the strain varies linearly from the neutral axis to the extreme fibers, we can determine for a particular bending moment (say  $M_{\max}$ ) the resultant stress distribution in the beam. This has been represented by the circles in Fig. 5. In the same figure, results of the photoelastically determined stress distribution at the same bending moment are represented by the crosses.

The agreement between results obtained from theoretical considerations and from the "photoelasticity" data is excellent. The further requirements of static equilibrium (i. e.  $\Sigma F = 0$  and  $\Sigma M = M$ ) were, in all cases, satisfied by the photoelasticity data to within 8 percent.

#### CREEP IN BEAMS SUBJECTED TO SUSTAINED BENDING MOMENTS

This problem, and the closely related one of creep in eccentrically loaded columns, is of considerable interest in technical mechanics.

The change in the photoelastic stress pattern as a function of time for a beam in uniform flexure is illustrated in Fig. 6.

The optical creep data available (Figs. 1 and 2) is inadequate for the calculation of the change in stress distribution due to creep since in the one case optical creep was determined under constant stress, while in bending the stress in each fiber changed with time. However, an approximated correction for optical creep gives a fair qualitative picture of the change in stress distribution due to mechanical creep, Fig. 7.

The dependence of optical retardation on stress path and on time, as indicated here, is analogous in many respects to the dependence of strain on stress path and on time in the non-elastic range. The difficulties arising from this do not seem insurmountable. (cf. Popov<sup>17</sup>).

#### ACKNOWLEDGEMENT

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