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TECHNICAL REPORT NO. 6418-6

MEASUREMENTS OF FRICTION COEFFICIENTS
FOR SUPERSONIC FLOW OF AIR IN THE ENTRANCE
REGION OF A TUBE WITH AND WITHOUT
HEAT TRANSFER

BY

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FOR

OFFICE OF NAVAL RESEARCH

CONTRACT N5ori-07805

NR-061-028

D. I. C. PROJECT NUMBER 6418

SEPTEMBER 1, 1953

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING
AND
DIVISION OF INDUSTRIAL COOPERATION
CAMBRIDGE, MASSACHUSETTS

MEASUREMENTS OF FRICTION COEFFICIENTS FOR SUPERSONIC
FLOW OF AIR IN THE ENTRANCE REGION OF A TUBE WITH
AND WITHOUT HEAT TRANSFER*

By

Joseph Kaye¹ and Tau-Yi Toong²

SUMMARY

Experiments have been made for supersonic flow of air in the entrance region of round tubes for measurements of friction coefficients with and without heat transfer. The experimental data are presented and the results interpreted in terms of two simple flow models, the one-dimensional and the two-dimensional flow models.

In addition to the usual method of computing the local apparent and "true" friction coefficients on the basis of individual measurements over a short distance of flow, a new method has been proposed in order to eliminate the scattering due to small inherent errors in pressure measurement. The measured values of static pressure and rate of heat transfer are plotted versus the modified length Reynolds number, as suggested by a theoretical analysis for the laminar boundary layer of a compressible flow in the entrance region of a tube. By means of the method of least squares, mean curves have been computed to represent those points where a laminar boundary layer appears to exist. Both the local apparent and "true" friction coefficients are then computed on the basis of these mean curves.

The experimental results are compared with theoretical predictions for laminar flow over a flat plate and in the entrance region of a tube.

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Presented at the First Iowa Thermodynamics Symposium,
State University of Iowa, April 27-28, 1953.

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NOMENCLATURE

A	cross-sectional area
a	inside radius of tube
a_1, b_1, c_1, d_1	constants used in equations (8) and (11)
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
c_w	discharge coefficient of nozzle
D	inside diameter of tube
F_T, F_f	functions of k and M, defined by equations (25) and (26) respectively
f	local coefficient of friction, $2g\tau/\rho V^2$
g	acceleration given to unit mass by unit force
k	ratio of specific heats, c_p/c_v
L	distance from end of curved contour of nozzle
M	Mach number, $V/\sqrt{gkRT}$
n	exponent in equation (11)
p	static pressure
Q	heat transfer per unit time
q	heat transfer per unit time per unit area
R	perfect-gas constant
Re_D	diameter Reynolds number $\rho VD/\mu g$
Re_L	length Reynolds number $\rho VL/\mu g$
s	entropy per unit mass
T	absolute temperature
V	velocity
w	mass rate of flow
ρ	density
λ	thermal conductivity
μ	viscosity
τ	shear stress at wall
ϵ	modified length Reynolds number defined by equation (7).
Superscript *	refers to throat of supersonic nozzle where $M = 1$
Subscripts:	
b	refers to boundary layer
c	refers to isentropic core
j	refers to station numbers
o	refers to hypothetical entrance plane of the tube, where the boundary layer is of zero thickness
oi	refers to upstream stagnation conditions
oj	refers to stagnation conditions at station j
∞	refers to free stream conditions for flat-plate flow
w	refers to wall conditions

INTRODUCTION

Many engineering problems exist where the effects of development of the fluid flow in the entrance region of a duct or tube determine the nature of the flow for both subsonic and supersonic speeds. It is now well established that for smooth entrance into a tube, the flow of air at subsonic or supersonic speeds is attended by the growth of a boundary layer along the walls of the tube and a gradual filling of the tube cross section with this boundary layer. This simple picture is complicated by the existence of transition from a laminar to a turbulent boundary layer and for supersonic flow by the possible occurrence of interaction between shock waves and boundary layer. In addition, it is very difficult to maintain supersonic flow in the tube for a sufficiently long length in order to approach the well-known "fully developed" flow attained for subsonic speeds in long tubes. Hence the understanding of supersonic-flow phenomena in tubes must be sought almost wholly in terms of the flow behavior in the entrance region of the tube.

Up to the present, the phenomena occurring in supersonic flow in the entrance region of a tube have been studied mostly by experimental methods. The data obtained in these tests have usually been interpreted in terms of a simple one-dimensional flow model, which is based on the simplifying assumption of uniform fluid properties at any cross section of the tube. Recently a somewhat more complicated but arbitrarily simplified two-dimensional flow model has also been used to interpret the test data. However, within the last year, solutions have been obtained for the system of partial differential equations of energy, momentum, and continuity, together with the equation of state, for the case of supersonic flow of air in a tube with smooth entrance and with a developing laminar boundary layer originating at the entrance plane of the tube. Since this theoretical analysis is based on the equations for a laminar boundary layer, comparisons made with the test results will necessarily be limited to those portions of the supersonic flow where a laminar boundary layer appears to exist in the tube.

The objectives of the present paper are to give some recent data on supersonic flow of air in a round tube, with and without heat transfer to the air, and to compare the local friction coefficients computed from these data by means of the one-dimensional flow model (1-D.F.M.) and by the two-dimensional flow model (2-D.F.M.) with those computed from the theoretical solution.

The work described herein is a portion of a research program, sponsored by the Office of Naval Research, with the objective of obtaining reliable and accurate data on the rate of heat transfer to air moving at supersonic speeds.

THEORETICAL SOLUTIONS

A brief summary of theoretical analyses for flow in the entrance region of a tube is given here in order to comprehend better the phenomena occurring in the case of supersonic flow in the entrance region.

There are many analyses available in the literature which are based on the simple 1-D.F.M. for fully developed flow of incompressible and compressible fluids, with and without heat transfer. These analyses, however, should not be used to determine the behavior of fluid flow in the entrance region of tubes, where the effects of rapidly changing velocity and temperature profiles have a strong influence on the values of the friction coefficient, heat-transfer coefficient, and other variables.

Several theoretical analyses for tube flow are available based on a two-dimensional flow model wherein an approximation to an unknown changing velocity profile is used in the entrance region. The analyses of Hagenbach (1)*, Neumann (2), Couette (3), Boussinesq (4), Schiller (5), Atkinson and Goldstein (6), and Langhaar (7) have been restricted mainly to the adiabatic flow of an incompressible fluid and for a laminar boundary layer near the entrance. The analyses of Latzko (8) and Elser (9) are the only ones which were found for a developing turbulent boundary layer originating at the entrance plane of the tube. Similar analyses for flow of a compressible fluid in a tube have not been found.

A search of the literature revealed a need for an analysis of the flow of a compressible fluid in the entrance region of a tube. Furthermore, this analysis would help in the understanding of the experimental data for supersonic flow in a tube. It may be shown that flow in a tube is nearly the same as the flow over a flat plate with a laminar boundary layer starting at the leading edge, provided that the pressure gradient in the free stream for plate flow is the same as that for tube flow. However, most analyses for flow of a compressible fluid over a flat plate are restricted to zero pressure gradient; thus the results of these analyses can serve only as a first approximation towards a solution of the flow in a tube, in a region where the thickness of the laminar boundary layer is small. The values of local friction coefficient, shown in Fig. 1, are computed from one of the most recent analyses for supersonic flow of air

* Numbers in parentheses refer to items in Bibliography.

over a flat plate, with variable properties taken into account, given by Van Driest (10).

An analysis of the flow of a compressible fluid in the entrance region of a tube was undertaken by use of the basic partial differential equations of energy, momentum, and continuity for a developing laminar boundary layer which coexists with an isentropic core in the central portion of the tube. These equations, simplified by the usual assumptions of boundary-layer theory, were transformed into a series of simultaneous ordinary differential equations and these, in turn, were solved with the aid of the M.I.T. Differential Analyzer for various boundary conditions. The first attempt to obtain solutions was based on the simplifying assumption of constant fluid viscosity and thermal conductivity; this set of solutions has appeared in the doctoral dissertation of Toong (11).

The results of the above analysis for local friction coefficients are shown in Fig. 2 for Mach numbers of zero and 2.8 at the entrance plane of the tube. For tube flow a new variable, the modified length Reynolds number,

$2(\text{Re}_{\text{Lo}})^{1/2} / \text{Re}_{\text{Do}}$, is used as the abscissa on the basis of the

analysis, so that a simple chart corresponding to Fig. 1 for flow over a flat plate is no longer available. Fig. 2 shows also the comparison between the flat-plate and tube-flow solutions for the same boundary conditions; it is seen that the tube-flow solution approaches the flat-plate solution at a zero value of the abscissa, or at the entrance plane of the tube. The tube-flow solutions shown in Fig. 2 will be used as a basis of comparison with the friction coefficients computed from test data.

EXPERIMENTAL APPARATUS

In the course of this program, supersonic flow of air in round tubes was obtained in two different kinds of apparatus. For adiabatic flow, the supersonic stream was passed through a round tube made from a good thermal insulator, such as textolite or lucite, and the tube was insulated either with a 14-inch thickness of rock wool or by a region of high vacuum. For flow with heat transfer, the supersonic stream was passed through a round tube made from a good thermal conductor, brass, and the tube was surrounded by a large mass of condensing steam. The description of the different kinds of test apparatus,

the means of testing, the preparations of the air stream, and some results of the tests have been published in a series of papers (12), (13), (14), (15). For this reason only a brief summary is given in Table 1 of the different test combinations used in this general program.

TABLE 1
SUMMARY OF TEST COMBINATIONS

	Symbol	Nozzle	Test Section	L/D	M_o
<u>ADIABATIC</u>	A-1	Brass	Textolite	37.6	3.0
"	A-2	Brass	Lucite	41.6	3.0
"	B	Stainless steel	Lucite	41.5	2.8
<u>HEAT TRANSFER</u>	C	Brass	Brass	29.2	3.0

The test combinations A-1, A-2, and B and the data obtained for adiabatic supersonic flow have been described in detail in reference (14). The heat-transfer apparatus, test combination C, has been described in reference (12).

EXPERIMENTAL RESULTS

General

Table 2 presents a summary of the runs made with each test combination and of the ranges of Reynolds number covered. The diameter Reynolds number is based on the tube diameter and on the mean stream properties at the first station. The length Reynolds number is based on the distance from the end of the curved contour of the supersonic nozzle and on the mean stream properties at that distance.

TABLE 2
SUMMARY OF TESTS

Test combination	No. of runs	Minimum $Re_D \times 10^{-5}$	Maximum $Re_D \times 10^{-5}$	Maximum $Re_L \times 10^{-5}$
<u>ADIABATIC</u>	A-1	12	0.76	4.7
"	A-2	10	0.50	4.6
"	B	18	0.15	4.8
<u>HEAT TRANSFER</u>	C	11	0.52	2.1

In order to avoid repetition and to keep the size of this paper within reasonable bounds, neither the original data obtained in these tests will be given in detail nor will all the data be given in chart form. Rather a careful and objective selection of these data has been made so as to aid in the understanding of the phenomena occurring in the supersonic flow of air in the tube. This selection has been made to permit comparisons between results based on the 1-D.F.M., the 2-D.F.M., and the theoretical solutions for the tube flow.

Since the theoretical solutions are limited to that portion of tube flow where a laminar boundary layer coexists with an isentropic core of central fluid, the selection has been limited mainly to those test results where a laminar boundary layer appears to be present in the tube.

In references (14) and (15) describing the adiabatic flow measurements and results, six values of the diameter Reynolds number were selected to cover the range from laminar to turbulent flow. For reasons given above, the data for the larger values of the diameter Reynolds number will not be included here since they encompass the transition and turbulent zones. Thus for the comparison of friction coefficients with and without heat addition, only four values of the diameter Reynolds number are shown here.

Laminar Boundary Layer

It has been shown in references (14) and (15) that a simple interpretation of the data on friction coefficients is obtained on the basis of the growth of a laminar boundary layer near the entrance of the tube, with transition to a turbulent boundary layer occurring at some distance downstream from the tube entrance. Figs. 3, 4, 5 and 6 show the modified pressure ratios, the Mach numbers, the apparent friction coefficients and the "true" friction coefficients for four values of the diameter Reynolds number, in the range where all the evidence indicates that laminar boundary layer exists for some distance downstream from tube entrance.

The modified pressure ratio is used to compare pressure distributions with and without heat addition for different nozzle and test-section combinations. The Mach numbers shown are based on both the 1-D.F.M. and the 2-D.F.M. The apparent friction coefficients are based on the 1-D.F.M. while the "true" friction coefficients are based on the 2-D.F.M. The calculated quantities based on one flow

model are plotted on each chart without reference to the quantities calculated from the other flow model.

In Figs. 3, 4, 5 and 6, both the local apparent and the local true friction coefficients scatter considerably about the reference lines of the theoretical solutions. The reasons for this scattering are that each point represents the measured small pressure differential over a length of two or four diameters of flow and that any fluctuation in the pressure distribution or small error in pressure measurement produces a grossly exaggerated change in the calculated friction coefficient. In the region of the laminar boundary layer of the flow, near the entrance plane, the scattering is also caused by the over-expansion and the under-expansion of the air passing through the fixed-geometry supersonic nozzle as the stagnation pressure is changed to vary the diameter Reynolds number. It is evident that the scattering obscures any definite effect of heat addition on the apparent or true friction coefficients. An examination of the rest of the tests, not shown in Figs. 3, 4, 5 and 6, yielded the same conclusions.

In order to decrease the influence of the scattering, several different methods of examining the data were tried. Fig. 7 shows the result of plotting a group of tests on the same chart; here the individual values of friction coefficients on Figs. 3, 4, 5 and 6 are all plotted on a single chart with the theoretical solution for a flat plate as a reference line. The overall scattering in the apparent friction coefficients based on the 1-D.F.M. is quite large, and although this scattering is greatly reduced in the lower half of Fig. 7 for friction coefficients based on the 2-D.F.M., it is difficult, nevertheless, to reach any conclusion regarding the effect of heat addition on the friction coefficient.

NEW METHOD OF TREATMENT OF EXPERIMENTAL DATA

General

Since the customary method of interpreting the experimental data did not yield any conclusion with regard to the effect of heat addition on the apparent or on the true friction coefficients because of the large degree of scattering of the data, a new method of treating the data was tried. The general basis of this method is described first, then the details of the new analyses needed are presented, and finally the results obtained are shown.

Consider first the supersonic flow of air over a flat plate with zero pressure gradient. Van Driest (10) and others have shown that the true local friction coefficient is a function of the length Reynolds number, the ratio of wall temperature to free stream temperature, and of free stream Mach number, i.e.

$$f = f_1 (Re_L, M_\infty, T_w/T_\infty), \quad (1)$$

if dissociation of the air is ignored. Furthermore, it is also known that a particular combination of variables, namely the product of the friction coefficient and square root of the length Reynolds number, is a function of the remaining two variables, i.e.

$$f(Re_L)^{1/2} = f_2 (M_\infty, T_w/T_\infty) \quad (2)$$

This function is shown in Fig. 1.

Consider next the supersonic flow of air in a round tube with finite pressure gradient in the direction of flow. The theoretical solutions show that the true local friction coefficient is a function not only of the length Reynolds number, the ratio of wall temperature to core temperature, and the Mach number of the central core of fluid but also of the diameter Reynolds number at the entrance of the tube, i.e.,

$$f = f_3 (Re_{Lo}, M_o, T_w/T_o, Re_{Do}) \quad (3)$$

In addition, the analysis indicate that a particular combination of variables, namely the product of the friction coefficient and square root of the length Reynolds number, is a unique function of the Mach number, of the ratio of wall temperature to core temperature, and of another particular combination of variables, the ratio of square foot of the length Reynolds number to the diameter Reynolds number, i.e.

$$f(Re_{Lo})^{1/2} = f_4 \left[(Re_{Lo})^{1/2}/Re_{Do}, M_o, T_w/T_o \right] \quad (4)$$

Furthermore, the pressure ratio and rate of heat transfer can be written as

$$p/p_{oi} = f_5 \left[(Re_{Lo})^{1/2}/Re_{Do}, M_o, T_w/T_o \right] \quad (4a)$$

$$qa/\lambda_o T_o = f_6 \left[(Re_{Lo})^{1/2} / Re_{Do}, M_o, T_w/T_o \right] \quad (4b)$$

Since this second combination of variables appears quite often, it is called briefly the "modified length Reynolds number". For clarity the definitions of the two Reynolds numbers are repeated here:

$$Re_{Lo} \equiv L \rho_o V_o / \mu_o g \quad (5)$$

$$Re_{Do} \equiv D \rho_o V_o / \mu_o g \quad (6)$$

where subscript o refers to the origin of the boundary layer at a hypothetical entrance plane of the tube, where the fluid properties are assumed to be uniform, as shown in Fig. 8. Since the fluid properties at state o cannot be measured directly in any apparatus, they are evaluated from data taken near the entrance of the tube. The modified length Reynolds number will be abbreviated, hereafter, in the equations to follow by the definition

$$\mathcal{R} \equiv 2(Re_{Lo})^{1/2} / Re_{Do} \quad (7a)$$

$$\mathcal{R} \equiv 2(L/D)^{1/2} / (Re_{Do})^{1/2} \quad (7b)$$

The theoretical solutions for tube flow not only indicate the combination of variables to use but also lead to a method, based on equations (4a) and (4b), of utilizing all the experimental data on one test combination for flow either with or without heat addition. This result is significant since the previous calculations for individual tests, shown in Figs. 3, 4, 5 and 6, indicate the difficulties inherent in measuring pressure differences over small distances of flow. If the values of the ratio of static wall pressure to stagnation pressure, p/P_{oi} , are plotted against the modified length Reynolds number, as in Figs. 9 and 10, a single mean or faired curve could be drawn through the points of all the measurements. Likewise, according to equation (4b), a single mean or faired curve could be drawn through the points of a plot of the quantity, $qa/(\lambda T)_{c1}$, versus the modified length Reynolds number as shown in Fig. 11.

Two problems arise, however, which must be solved in order to utilize all the data for a given test combination. The first is how to obtain a mean curve through the experimental points in an objective

manner, and second, how to obtain the local friction coefficients from these mean curves.

Mean Curves by the Method of Least Squares

An objective and satisfactory method of placing a mean curve through the points on Figs. 9, 10, and 11 is to use the well-known device of least squares, although considerably more computation is required by this method than by use of a faired curve drawn visually. However, since the theoretical analysis is valid only for a laminar boundary layer, a careful examination of the data was made before using the method of least squares. First, all points corresponding to the start of the transition from a laminar to turbulent boundary layer were given a weight of one-half; these points were clearly discernible for the test combination C in Fig. 11. Second, all points after the start of transition were eliminated. Third, those points which corresponded to data taken at the first station downstream of the entrance were given a weight of one-half, since they were subjected most to the effect of over-expansion and under-expansion in the supersonic nozzle. About one-half of the points corresponding to a laminar boundary layer on Fig. 9 were thus reduced in weight to one-half, about one-third of the total points on Fig. 10, and about one-third of the total points on Fig. 11.

Examination of the data in Figs. 9 and 10 indicated that extrapolation from finite values of the modified length Reynolds number to zero would introduce considerable error in any mean curve. Hence an arbitrary extrapolation device was used. The value of the ratio of static pressure to inlet stagnation pressure, p/p_{oi} , was computed for isentropic flow for the actual area ratio of the supersonic nozzle; this isentropic value was assumed to be that existing at the entrance plane of the tube flow, or for state o in Fig. 8. Thus, a mean curve of the form

$$p/p_{oi} = a_1 + b_1 \zeta + c_1 \zeta^2 \quad (8)$$

was passed through the weighted points by the method of least squares, with an arbitrarily selected value of the constant a_1 .

For the points in Fig. 9, the resultant equation is

$$p/p_{oi} = 0.03698 + 0.8292 \zeta - 3.259 \zeta^2 \quad (9)$$

For the points in Fig. 10, the resultant equation is

$$p/p_{oi} = 0.02641 + 0.6584 \zeta + 3.097 \zeta^2 \quad (10)$$

For the points on Fig. 11, a simple equation of the form

$$qa/\lambda_o T_o = d_1 \zeta^n \quad (11)$$

was passed through the weighted points by the method of least squares; the result is

$$qa/(\lambda T)_{c1} = 0.1726 \zeta^{-1.027} \quad (12)$$

As explained previously, all fluid properties are evaluated from data taken at station 1.

Use of Mean Curves to Calculate Local Friction Coefficients

A. One-Dimensional Flow Model with Heat Transfer

The following assumptions are made in the analysis of the 1-D.F.M.:

1. All fluid properties and velocities are uniform at any cross-section of the flow.
2. Air is a perfect gas with a constant value of the ratio of specific heats ($k = 1.40$) over the range of temperature under consideration.
3. Adiabatic flow exists up to the entrance plane of the tube.

The following relations hold at each section of the flow in the tube

$$\text{Continuity: } w = \rho VA \quad (13)$$

$$\text{Equation of state: } p = \rho RT \quad (14)$$

$$\text{Energy: } T_{oj} = T + V^2/2gc_p \quad (15)$$

$$\text{Definition: } M^2 = V^2/gkRT \quad (16)$$

$$\text{Definition: } c_w = (w/A^*)/(w/A^*)_s \quad (17)$$

$$\text{Definition: } f \equiv 2g \tau / \rho v^2 \quad (18)$$

$$\text{Assumption: } p/p_{oi} = a_1 + b_1 s + c_1 s^2 \quad (8)$$

$$\text{Assumption: } q_a / \lambda_o T_o = d_1 s^n \quad (11)$$

$$\text{Identity: } T_{oj} / T_{oi} \equiv 1 + \sum_i^j \Delta T_{oj} / T_{oi} \quad (19)$$

For a control volume which encloses the fluid between two adjacent stations, separated by an infinitesimal distance, the energy and momentum equations are as follows:

$$\text{Energy: } dQ = q \pi D dL = w c_p dT_{oj} \quad (20)$$

$$\text{Momentum: } -Adp - \tau \pi D dL = d(wV/g) \quad (21)$$

From equations (13) through (17), it may be derived that

$$\frac{1}{c_w} \frac{p}{p_{oi}} \frac{A}{A^*} \sqrt{\frac{T_{oi}}{T_{oj}}} = \left(\frac{2}{k+1} \right)^{\frac{k+1}{2(k-1)}} \frac{1}{M \sqrt{1 + \frac{k-1}{2} M^2}} \equiv f_1(M, k) \quad (22)$$

It is to be noted that the right-hand side of equation (22) is a unique function of the Mach number if k is a constant. This function is tabulated under the heading $pA/p_o A^*$ in Table 30 of reference (16).

The symbol p_o of reference (16) is analogous to p_{oi} used in this paper.

Combining equations (13), (7), (11) and (20) and integrating between station j and station $j + 1$

$$\frac{\Delta T_{oj}}{T_{oi}} = \frac{4d_1}{(2+n)} \left(\frac{\lambda_o}{c_p \mu_o} \right) \left(\frac{T_o}{T_{oi}} \right) \left(s_{j+1}^{2+n} - s_j^{2+n} \right) \quad (23)$$

The friction coefficient may be computed by means of the following expression, derived by combining equations (13), (14), (15), (16), (18) and (21),

$$dM^2 = F_T \frac{dT_{oj}}{T_{oj}} + F_f 4f d\left(\frac{L}{D}\right) \quad (24)$$

$$\text{where } F_T \equiv \frac{M^2(1 + kM^2) \left(1 + \frac{k-1}{2} M^2\right)}{1 - M^2} \quad (25)$$

$$\text{and } F_f \equiv \frac{kM^4 \left(1 + \frac{k-1}{2} M^2\right)}{1 - M^2} \quad (26)$$

The quantities F_T and F_f are functions of k and M only and are tabulated as such in reference (17).

Equation (24) may be put in a finite-difference form, and rearranged, using equation (7)

$$\Delta M_j^2 \equiv M_{j+1}^2 - M_j^2 = \bar{F}_T \frac{T_{o(j+1)} - T_{oj}}{T_{oj}} + \bar{F}_f 4\bar{f}_{j+1/2} \overline{(Re_{Lo})}^{1/2} \Delta S \quad (27)$$

where $\bar{F}_T$ and $\bar{F}_f$ are to be evaluated at the mean value of the Mach numbers at the stations under consideration. The friction coefficient, $\bar{f}_{j+1/2}$, thus computed is assumed to be the mean value between these two stations.

Method of Computation Using 1-D.F.M.

Values of ΔS are assumed and from equations (8), (19) and (23), the left-hand side of equation (22) is evaluated, using the known value of the area ratio, A/A^* , for the particular test combination. The value of the nozzle discharge coefficient is assumed to be unity for this computation. The value T_o/T_{oj} is found from the isentropic area-ratio for the particular supersonic nozzle used. The values of λ_o and μ_o in equation (23) are found from T_o/T_{oi} and an assumed value for T_{oi} of 570°F abs . The values of T_{oi} used in the actual experimental work were very close to 110°F and thus this assumption is valid. Equation (22)

therefore gives values of the Mach number at the assumed values of the modified length Reynolds number.

The curves of local apparent friction coefficient versus length Reynolds number may be obtained from the previous computations for a particular value of the inlet diameter Reynolds number. For any consecutive values of S , the quantity $4 \bar{f}_{j+1/2} \Delta (L/D)$ has been calculated. This quantity is identical with the last term of equation (27). From equation (7b), L/D is known for each value of S and hence $\Delta (L/D)$ is known. The value of $\bar{f}_{j+1/2}$ may thus be computed.

The corresponding value of length Reynolds number for this value of local friction coefficient is found as follows: From equation (7a), Re_{L_0} is known. This is related to Re_L by

$$Re_L / Re_{L_0} = \mu_o / \mu \quad (28)$$

The Mach number is known for each value of S and hence the stream temperature and viscosity can be evaluated. The arithmetical mean value of Re_L for any two adjacent stations is used as the abscissa for the previously computed value of $\bar{f}_{j+1/2}$. The results of such a calculation are shown in Fig. 12 for an inlet diameter Reynolds number of 0.5×10^5 .

This one-dimensional analysis is valid for the computation of data with and without heat transfer, since adiabatic flow corresponds merely to a special case of equation (11), namely $d_1 = 0$.

B. Two-Dimensional Flow Model with Heat Transfer

The following assumptions are made in the analysis of the 2-D.F.M.

1. There exists in the entrance region of the tube a laminar boundary layer, where both the viscous and heat-transfer effects are assumed to be predominant.
2. An isentropic, one-dimensional flow exists in the central core of the tube.
3. The static pressure is uniform across each section of the tube.

4. Air is a perfect gas with a constant value of the ratio of specific heats.
5. The laminar boundary layer is assumed to have uniform velocity and temperature profiles.
6. Adiabatic flow exists up to the entrance plane of the tube.

At each cross-section of the flow inside the tube, the following relationships are valid:

$$\text{Continuity: } w = w_b + w_c = \rho_o V_o A \quad (29)$$

$$\text{Continuity: } w_c = \rho_c V_c A_c \quad (30)$$

$$\text{Continuity: } w_b = \rho_b V_b A_b = \rho_b V_b (A - A_c) \quad (31)$$

$$\text{Equation of State: } p = \rho_c R T_c \quad (32)$$

$$\text{Equation of State: } p = \rho_b R T_b \quad (33)$$

$$\begin{aligned} \text{Definition: } w c_p T_{oj} &\equiv w_c (c_p T_c + V_c^2 / 2g) \\ &\quad + w_b (c_p T_b + V_b^2 / 2g) \end{aligned} \quad (34)$$

$$\text{Definition: } M_c^2 \equiv V_c^2 / g k R T_c \quad (35)$$

$$\text{Assumption: } V_b = V_c / 2 \quad (36)$$

$$\text{Assumption: } p/p_{oi} = a_1 + b_1 \xi + c_1 \xi^2 \quad (8)$$

$$\text{Assumption: } q a / \lambda_o T_o = d_1 \xi^n \quad (11)$$

The following relationships hold between the upstream stagnation state and any state inside the one-dimensional isentropic core:

$$\text{Isentropic: } p/p_{oi} = (\rho_c / \rho_{oi})^k \quad (37)$$

$$\text{Isentropic: } p/p_{oi} = (T_c / T_{oi})^{k/(k-1)} \quad (38)$$

$$\text{Isentropic: } c_p T_c + V_c^2/2g = c_p T_{oi} \quad (39)$$

The following expressions may be derived for the control volume which encloses the fluid between two adjacent stations j and $j+1$:

$$\text{Energy: } Q_{j-(j+1)} = w_c (T_{o(j+1)} - T_{oj}) = w_c \Delta T_{oj} \quad (40)$$

$$\begin{aligned} \text{Momentum: } (p_j - p_{j+1})A - \tau \pi D(L_{j+1} - L_j) \\ = \left[\frac{(w_c V_c + w_b V_b)}{g} \right]_{j+1} - \left[\frac{(w_c V_c + w_b V_b)}{g} \right]_j \end{aligned} \quad (41)$$

The local friction coefficient is defined in terms of the wall shearing stress by

$$\tau = 1/2 f_{j+1/2} \left[\left(\frac{\rho_c V_c^2}{2g} \right)_j + \left(\frac{\rho_c V_c^2}{2g} \right)_{j+1} \right] \quad (42)$$

It is to be noted that equation (40) suggests that the value of T_{oj} defined by equation (34) is identical with that of the local stagnation temperature in the analysis of the 1-D.F.M. Hence, from part A, equations (19) and (23) may be used to calculate T_{oj}/T_{oi} at any station.

Equation (41) may be put into the following working form by means of equations (29), (30), (31), (32), (33), (35) and (42)

$$\begin{aligned} \frac{1}{kM_o^2} \left[\frac{p_j}{p_{oi}} - \frac{p_{j+1}}{p_{oi}} \right] - \bar{f}_{j+1/2} \left[\frac{L_{j+1} - L_j}{D} \right] \left[\left(\frac{p}{p_{oi}} \frac{M_c^2}{M_o^2} \right)_j + \left(\frac{p}{p_{oi}} \frac{M_c^2}{M_o^2} \right)_{j+1} \right] \\ = \left[\frac{w_c}{w} \frac{V_c}{V_o} \frac{p_o}{p_{oi}} + \frac{w_b}{w} \frac{V_b}{V_o} \frac{p_o}{p_{oi}} \right]_{j+1} - \left[\frac{w_c}{w} \frac{V_c}{V_o} \frac{p_o}{p_{oi}} + \frac{w_b}{w} \frac{V_b}{V_o} \frac{p_o}{p_{oi}} \right]_j \end{aligned} \quad (43)$$

$$\text{where } \frac{V_c}{V_o} = \frac{M_c}{M_o} \sqrt{\frac{T_c}{T_o}} \quad (44)$$

$$\text{and } \frac{w_c}{w} = \frac{1 - \frac{T_c}{T_{oj}} \frac{V_b}{V_c} \frac{\rho_c V_c}{w/A} - \frac{V_b^2}{2g_c p T_{oj}}}{\frac{T_{oi}}{T_{oj}} - \frac{T_c}{T_{oj}} \frac{V_b}{V_c} - \frac{V_b^2}{2g_c p T_{oj}}} \quad (45)$$

Method of Computation Using 2-D.F.M.

The curves of local true friction coefficient versus length Reynolds number can be computed from equations (43), (44) and (45) for any value of inlet diameter Reynolds number. An examination of the quantities in these three equations shows that they are functions of the modified length Reynolds number and the entrance Mach number only. The entrance Mach number may be determined from the isentropic area-ratio of the nozzle used in the particular test combination.

Thus, the quantity $\bar{f}_{j+1/2} \left[\frac{L_{j+1}}{D} - \frac{L_j}{D} \right]$ may be computed between any

two arbitrary values of the modified length Reynolds number. The necessity for assuming the inlet diameter Reynolds number in order to find the friction coefficient is obvious from equation (7b). The assumptions that the values of upstream stagnation temperature and the nozzle discharge coefficient are respectively 570° F abs and unity are made as in the 1-D.F.M. The value of the length Reynolds number corresponding to the friction coefficient is determined as in the 1-D.F.M. from an equation similar to equation (28), namely,

$$\frac{Re_L}{Re_{Lo}} = \frac{\mu_o}{\mu_{cj}} \quad (46)$$

The results of such a calculation are shown in Fig. 12 for a particular inlet diameter Reynolds number of 0.5×10^5 . This two-dimensional analysis is again valid for the computation of data with and without heat transfer.

Results

Fig. 12 shows the variation with length Reynolds number of the local apparent and true friction coefficients, computed by the use of the above analyses for a particular value of the inlet diameter Reynolds number of 0.5×10^5 . In the same figure are plotted also the theoretical

predictions for adiabatic laminar flow over a flat plate and in the entrance region of a tube. It is found that the values of local apparent friction coefficient increase with heat addition. However, contrary to what is predicted from theory, the values of local true friction coefficient are also found to increase with heat addition. This discrepancy may be explained by the fact that the arbitrary 2-D.F.M. used in computing the local true friction coefficient has not completely taken account of the actual change in the momentum flux. Thus, the "true" friction coefficient will be somewhat too high.

Good qualitative agreement is found to exist between the experimental results computed from the new method and the theoretical predictions for laminar flow over a flat plate and in the entrance region of a tube. Better quantitative agreement is expected if the theory for tube-flow takes into account the variation of viscosity and thermal conductivity of air inside the boundary layer.

CONCLUSIONS

By means of two simple flow models, values of local apparent and "true" friction coefficients have been computed from experimental data obtained for a supersonic flow of air in the entrance region of round tubes, with and without heat transfer. Four different values of the inlet diameter Reynolds number are selected to cover the region where a laminar boundary layer appears to exist for some distance downstream of the tube entrance. Due to small inherent errors in pressure measurement, considerable scattering has been observed for values of friction coefficients calculated on the basis of individual measurements over a short distance of flow. Although theoretical analyses for laminar flow both over a flat plate and in the entrance region of a tube indicate a decrease of true friction coefficient with heat addition, the experimental scattering mentioned above obscures any definite effect of heat addition on either the apparent or the true friction coefficients.

On the basis of a theoretical analysis for the laminar boundary layer of a compressible flow in the entrance region of a tube, a new method of calculating the friction coefficients has been proposed in order to eliminate this experimental scattering. The local apparent friction coefficient computed on the basis of the 1-D.F.M. is found to increase with heat addition. However, contrary to what is predicted from theory, the local "true" friction coefficient computed on the basis of the simplified 2-D.F.M. is found to increase also with heat addition. This discrepancy may be explained by the fact that this arbitrary 2-D.F.M. has

not completely taken account of the actual change in the momentum flux. Thus, the "true" friction coefficient computed on this basis will be somewhat too high.

For that portion of the flow where the boundary layer appears to be laminar, the values of both the local and the "true" friction coefficient computed on the basis of the mean curves obtained by the new method mentioned above are found to agree qualitatively with those predicted by theory for laminar flow both over a flat plate and in the entrance region of a tube. Similar qualitative agreement appears to exist also in the lower half of Fig. 7, where the local "true" friction coefficients are computed on the basis of individual measurements over a short distance of flow.

Better quantitative agreement between experimental and theoretical values of friction coefficient is expected, if the theory for the laminar boundary layer of a compressible flow in the entrance region of a tube takes into account the variation of viscosity and thermal conductivity of air inside the boundary layer.

ACKNOWLEDGMENTS

The assistance of G. A. Brown, W. S. Wu, E. A. Sziklas, J. J. Dieckmann, and Mrs. A. B. Walker in the preparation of this paper is gratefully acknowledged.

This investigation is sponsored as Contract Number N5-ori-07805 by the Office of Naval Research of the United States Navy.

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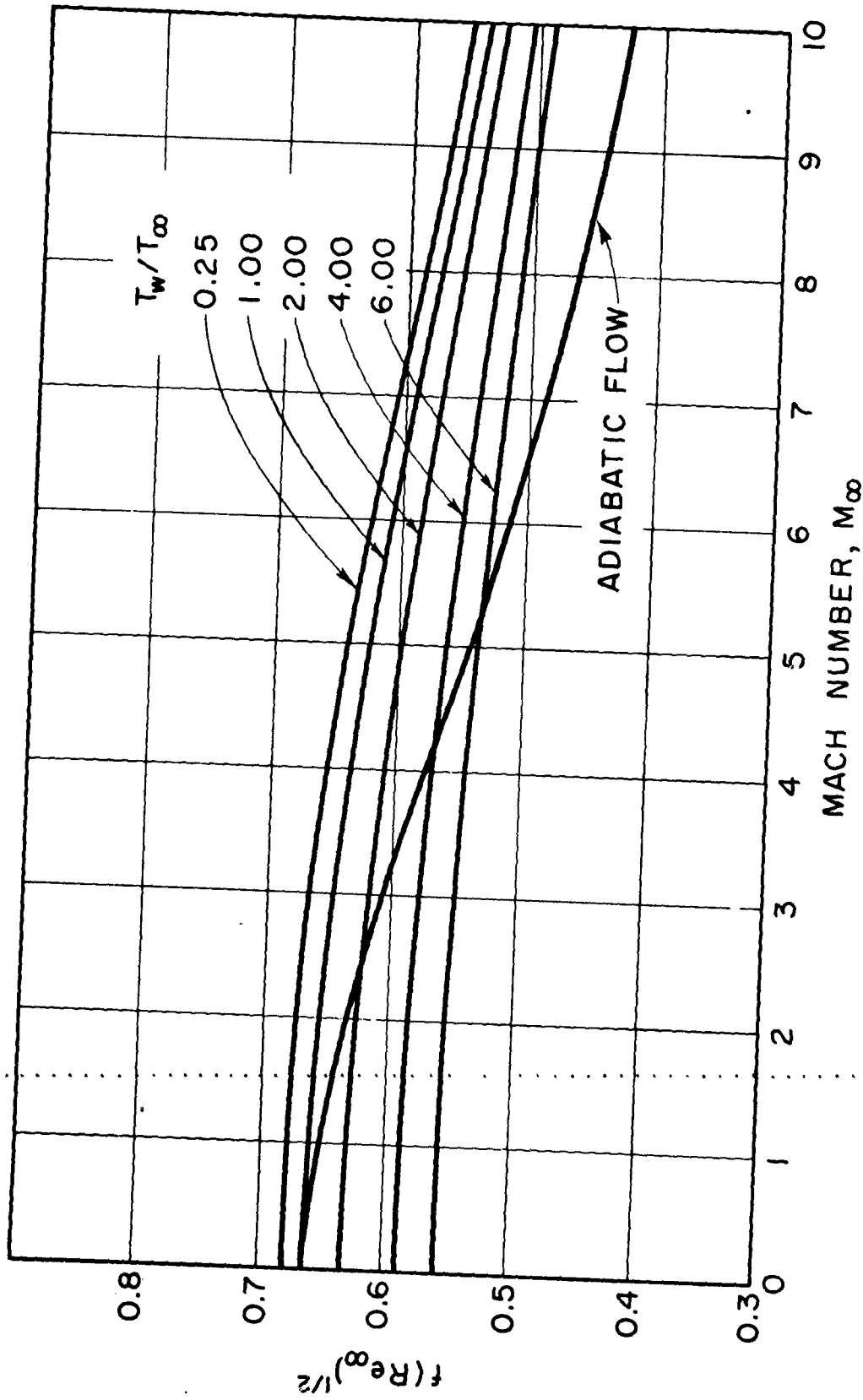


FIG. 1-LOCAL TRUE FRICTION COEFFICIENT FOR LAMINAR BOUNDARY LAYER OF A COMPRESSIBLE FLUID FLOWING ALONG A FLAT PLATE COMPUTED FROM VAN DRIEST (10)

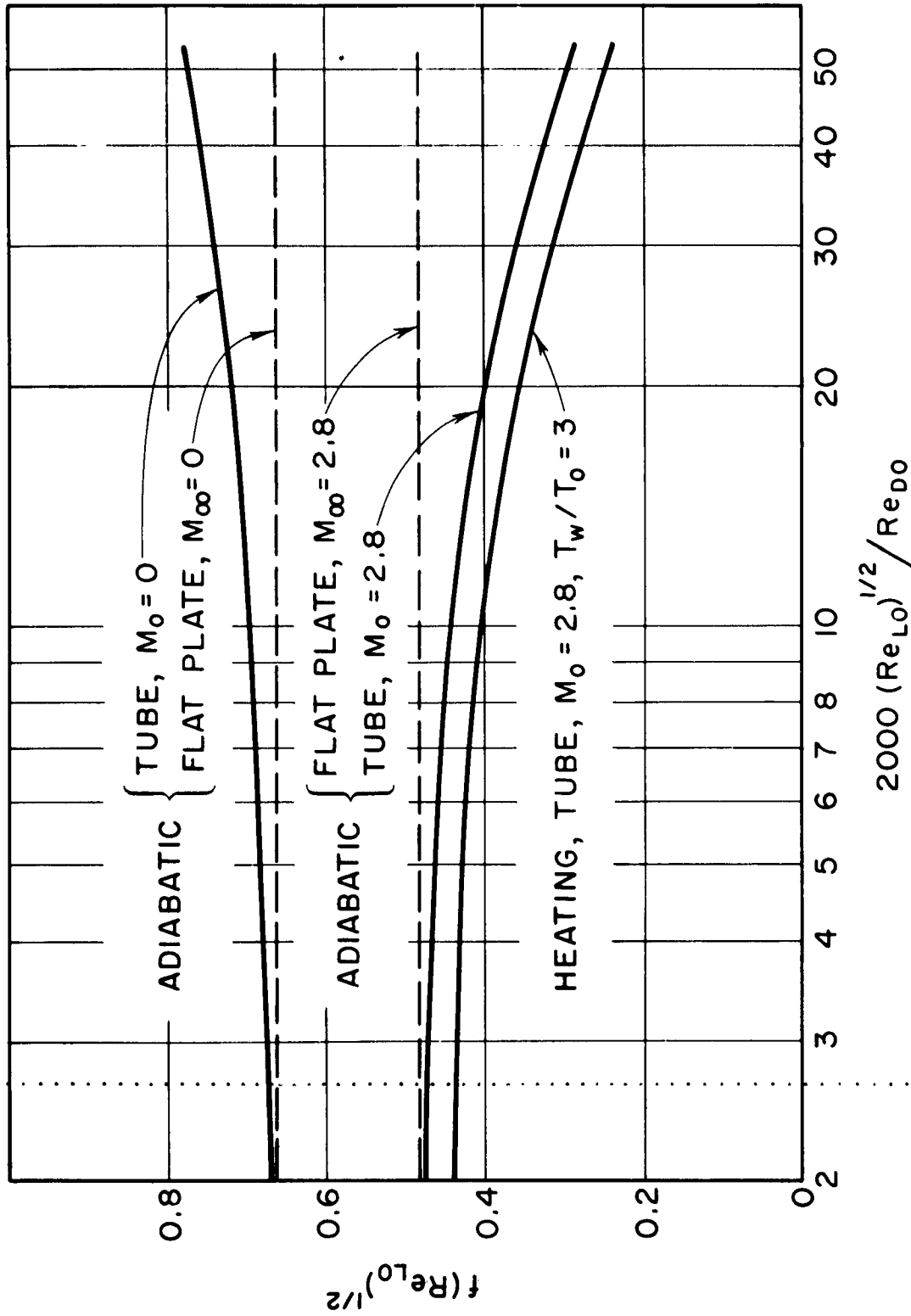


FIG.2 — COMPARISON OF LOCAL TRUE FRICTION COEFFICIENTS FOR LAMINAR BOUNDARY LAYER OF A COMPRESSIBLE FLUID FLOWING ALONG A FLAT PLATE AND IN ENTRANCE REGION OF A ROUND TUBE

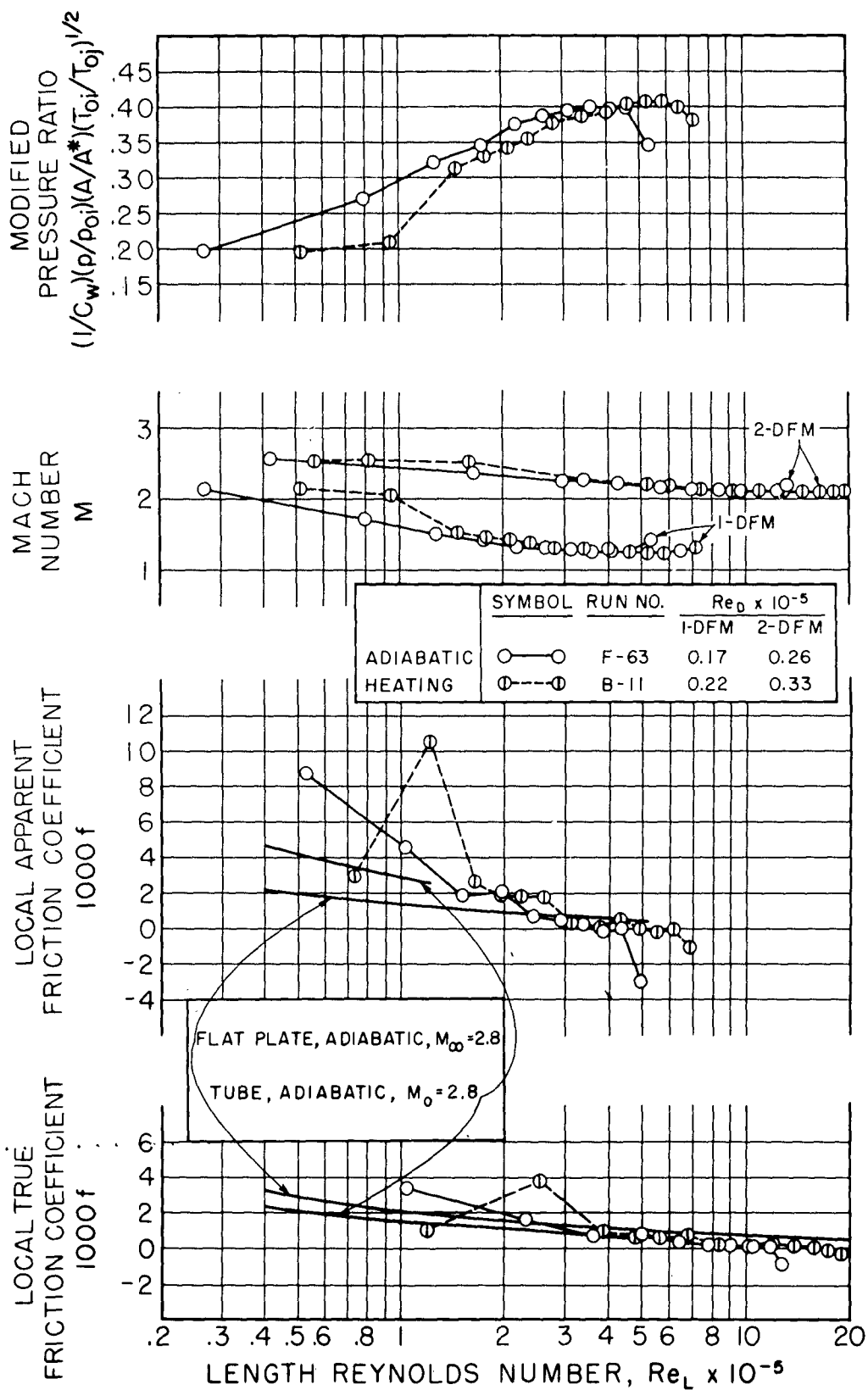


FIG. 3

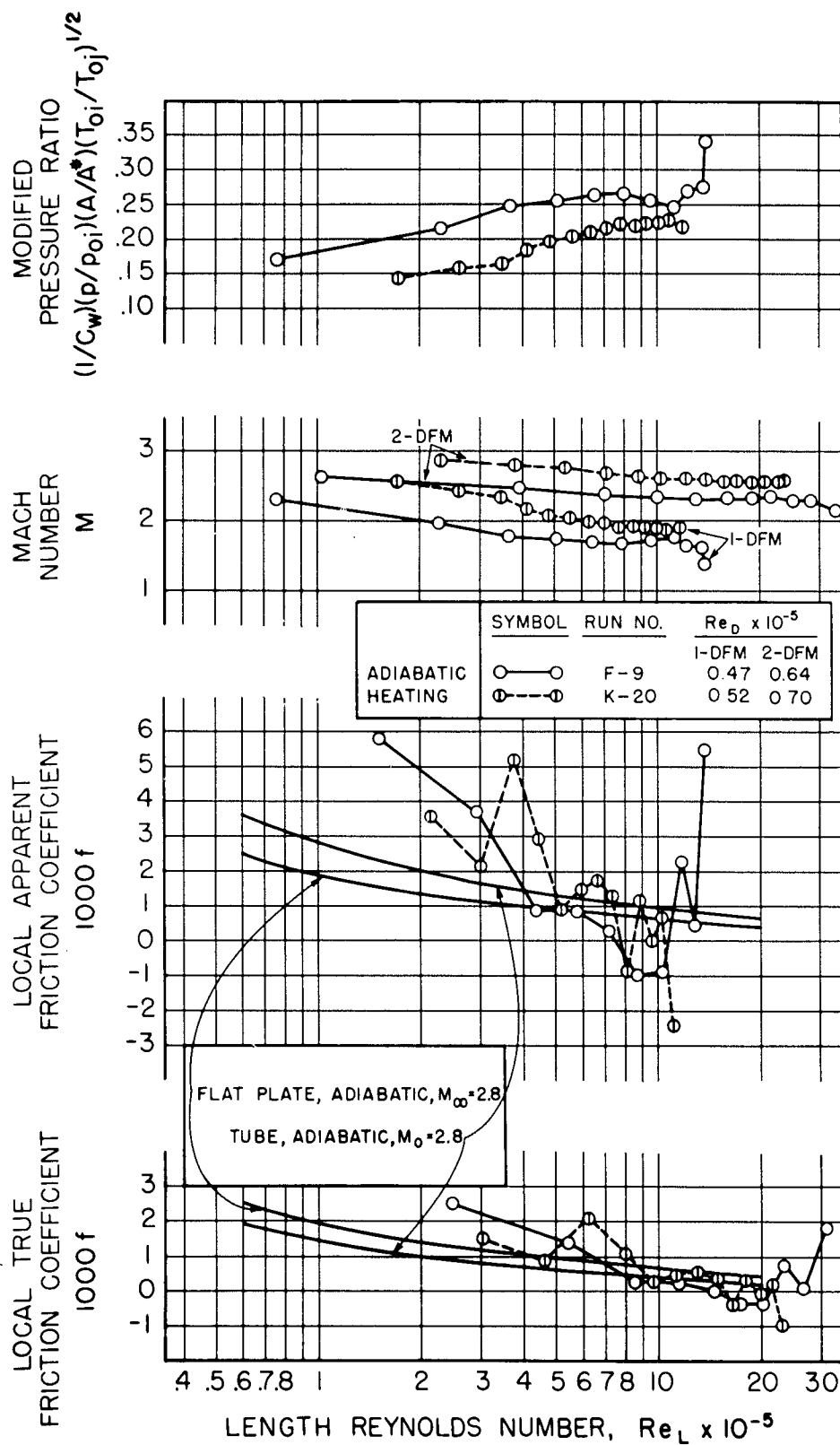


FIG. 4

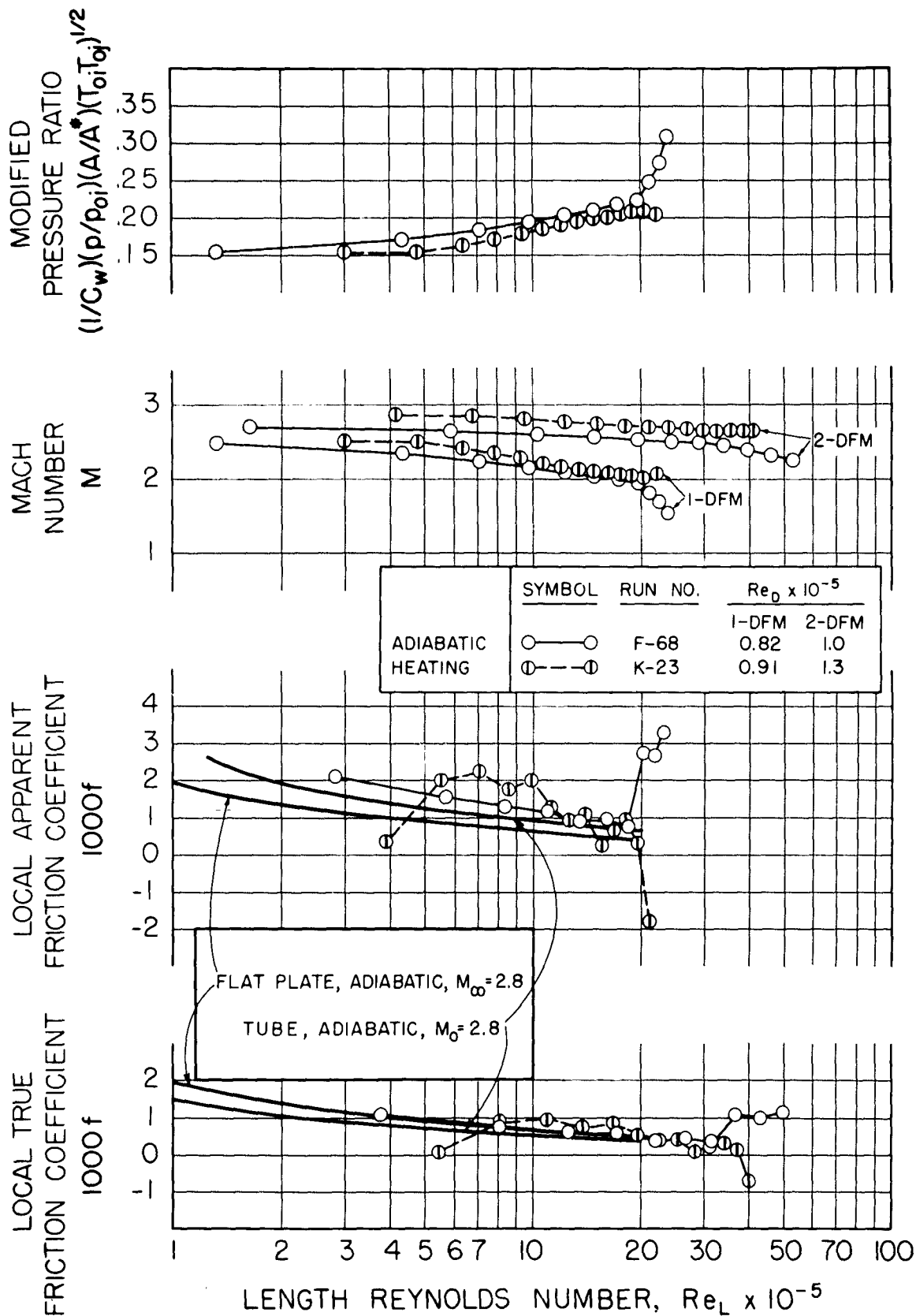


FIG. 5

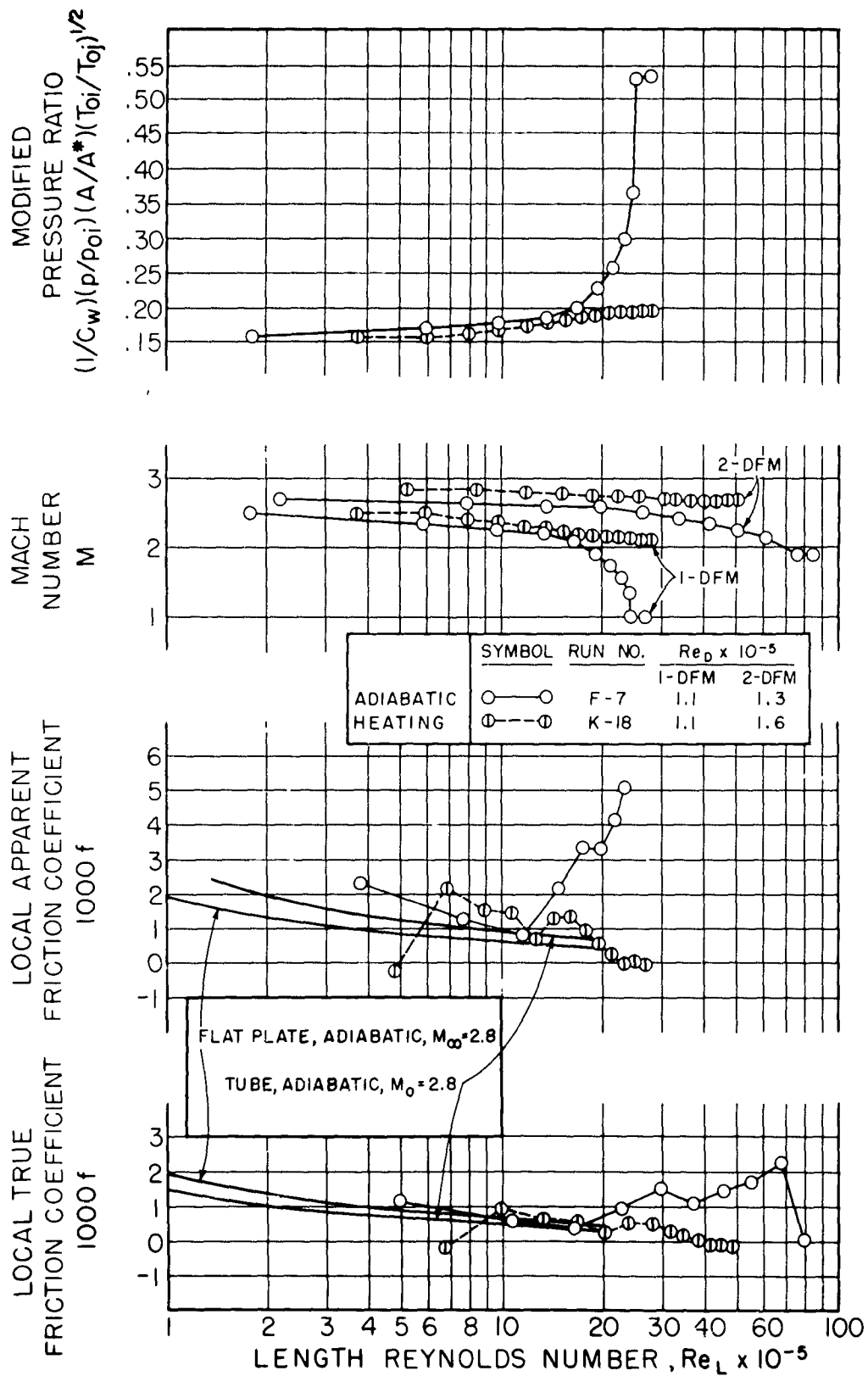


FIG. 6

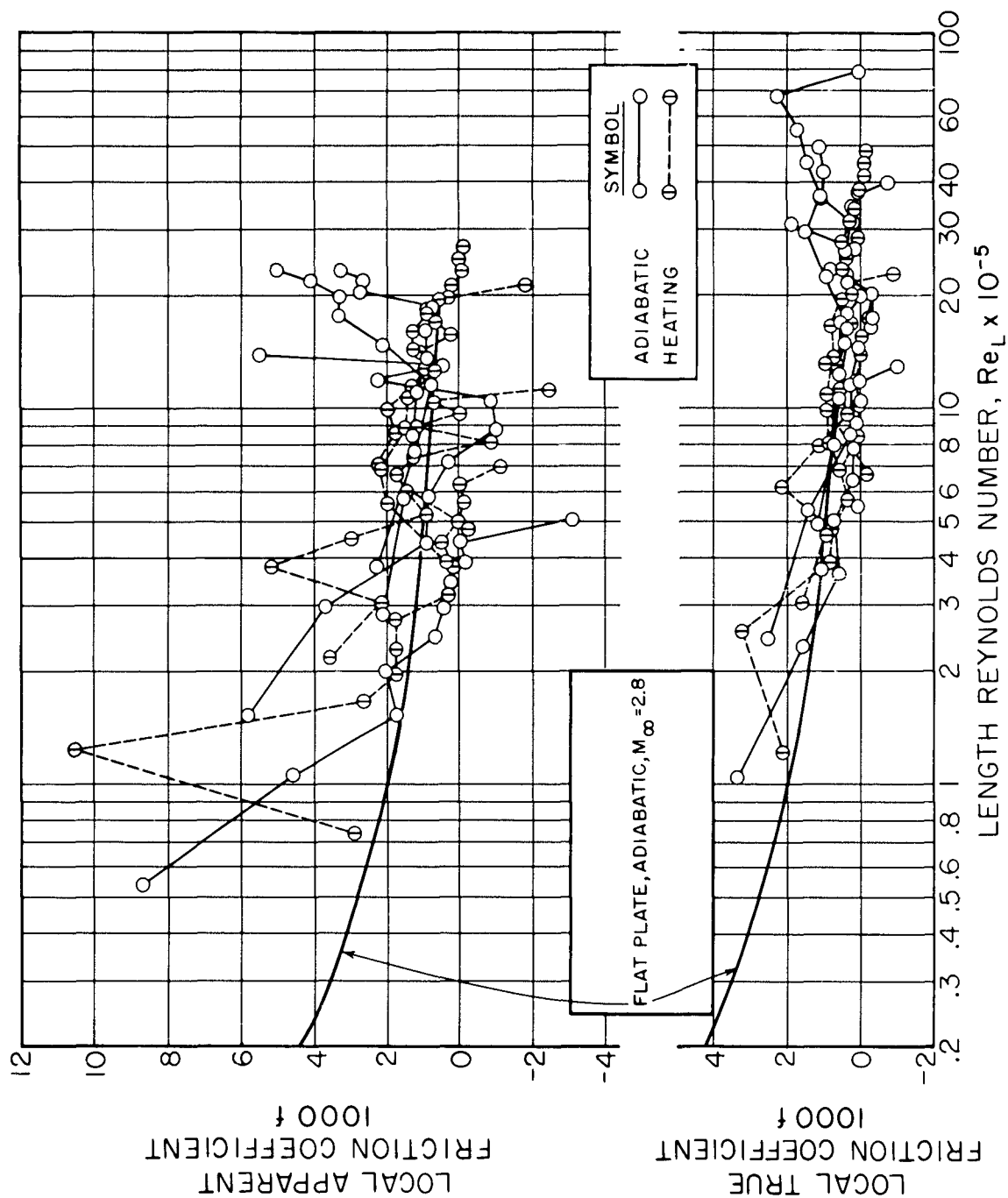
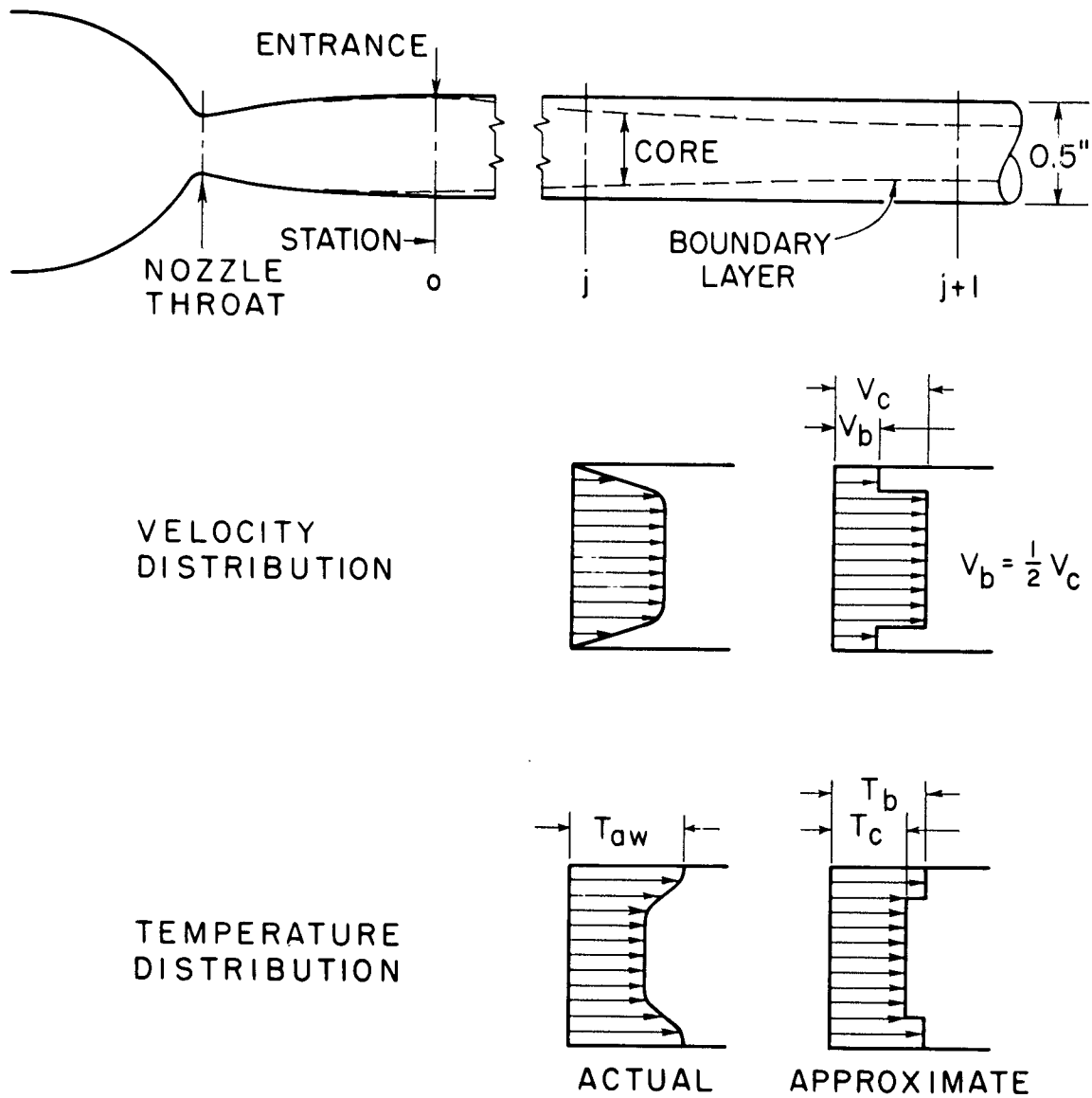


FIG. 7



SIMPLIFIED TWO - DIMENSIONAL FLOW MODEL

FIG. 8

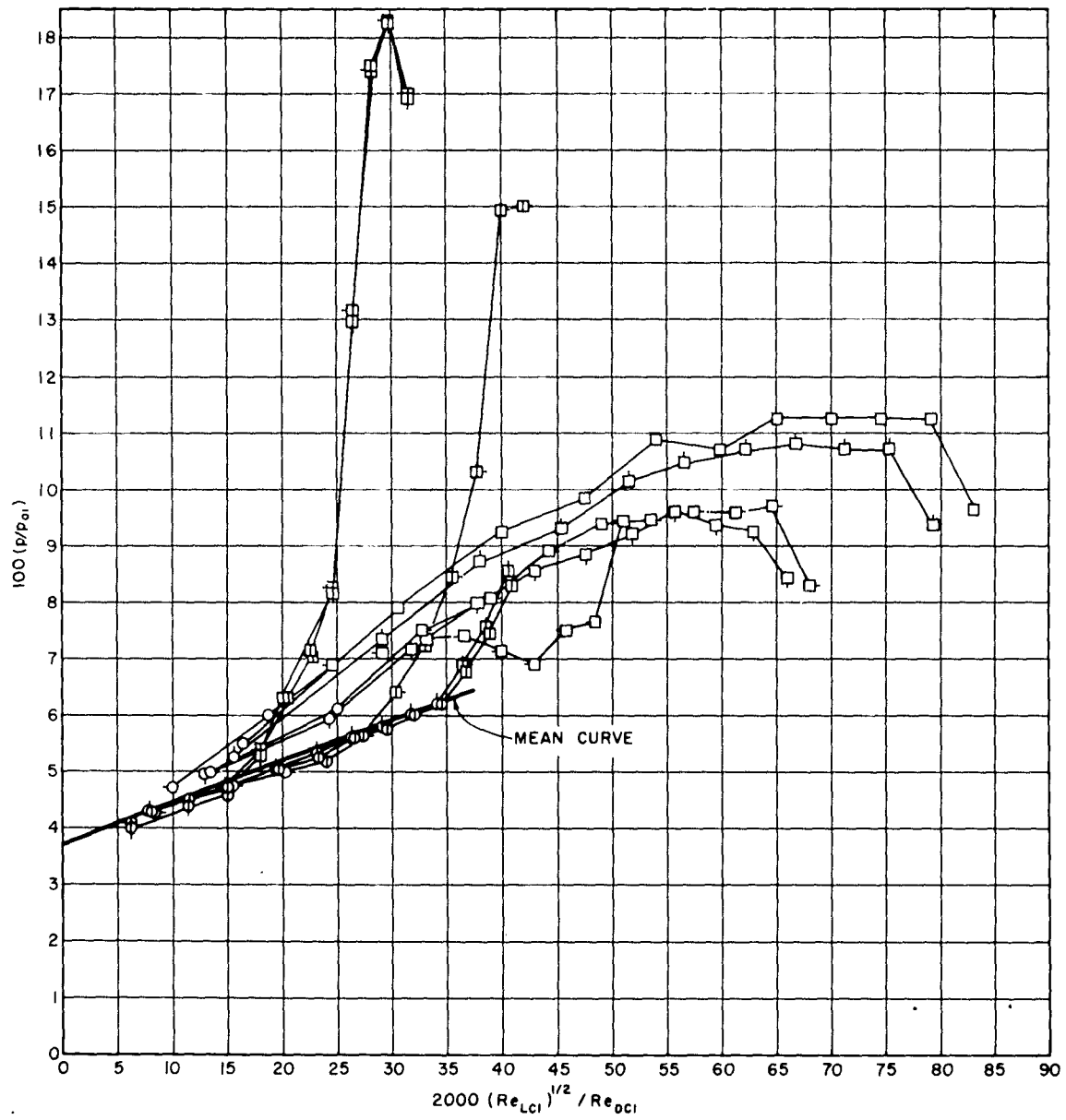


FIG. 9 - PRESSURE DISTRIBUTION FOR FLOW OF AIR IN THE ENTRANCE REGION OF A TUBE, APPARATUS B, ADIABATIC

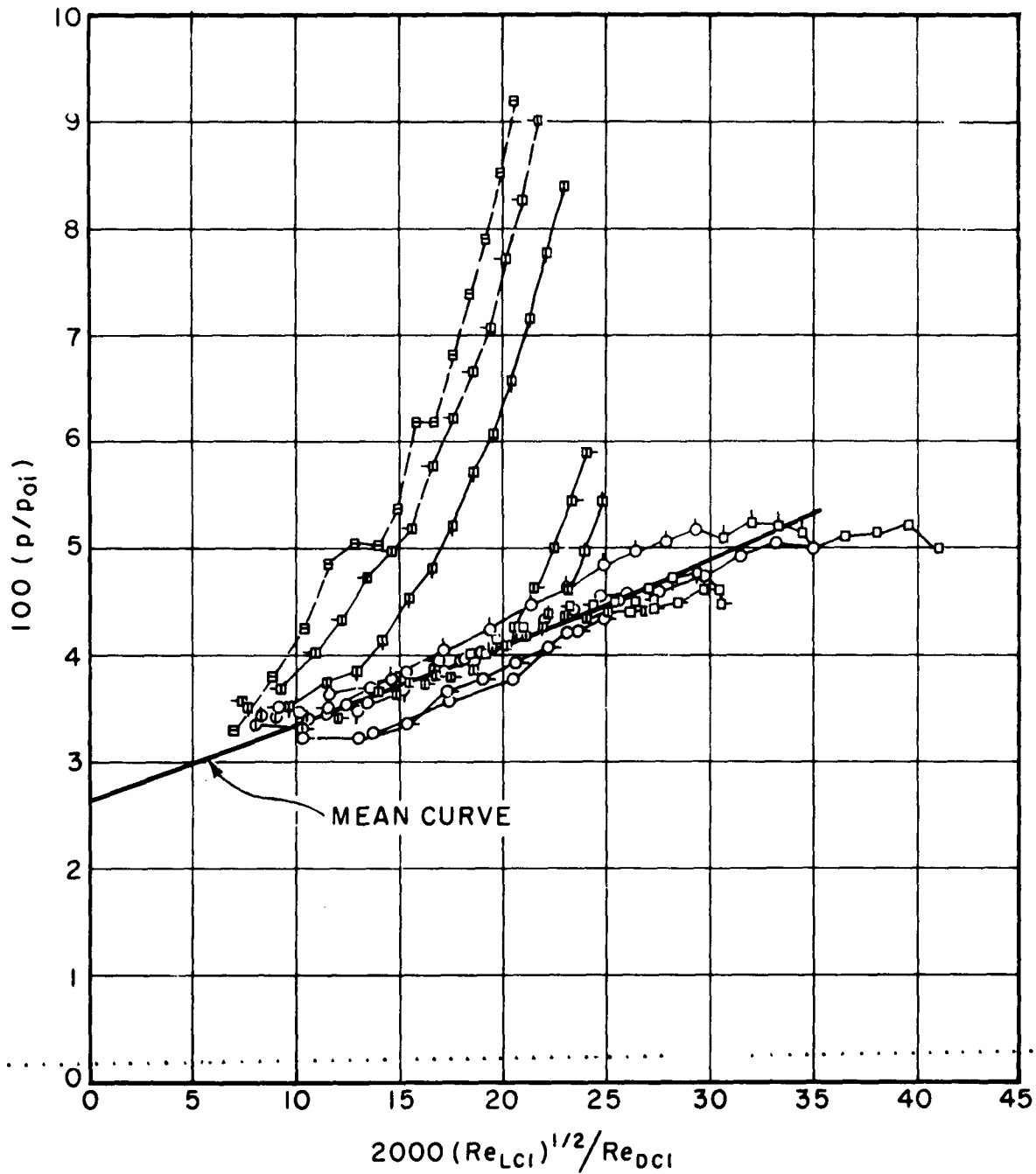


FIG. 10 - PRESSURE DISTRIBUTION FOR FLOW OF AIR IN THE ENTRANCE REGION OF A TUBE, APPARATUS C, HEATING

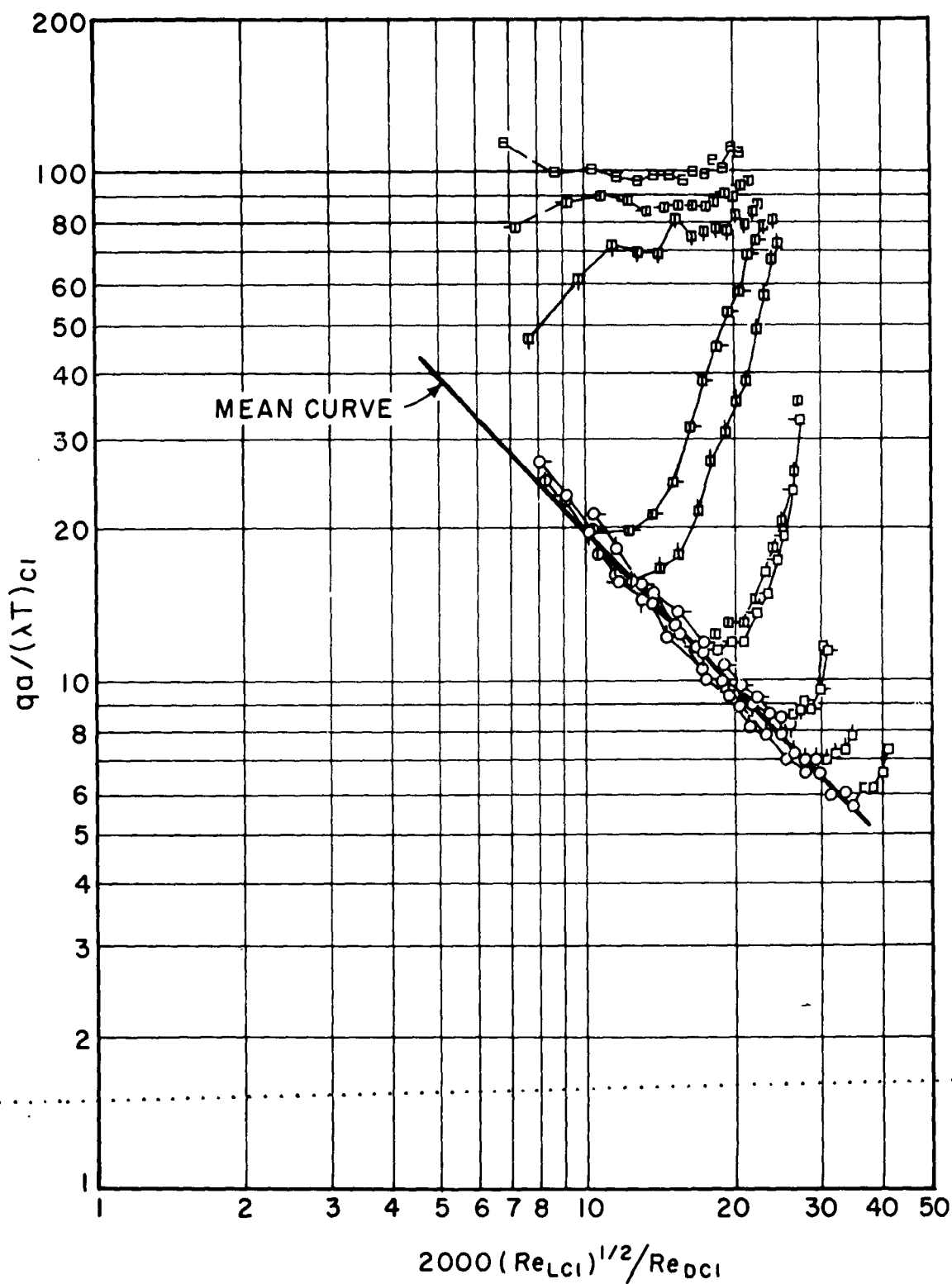


FIG. II - RATE OF HEAT TRANSFER TO FLOW OF AIR IN THE ENTRANCE REGION OF A TUBE, APPARATUS C, HEATING

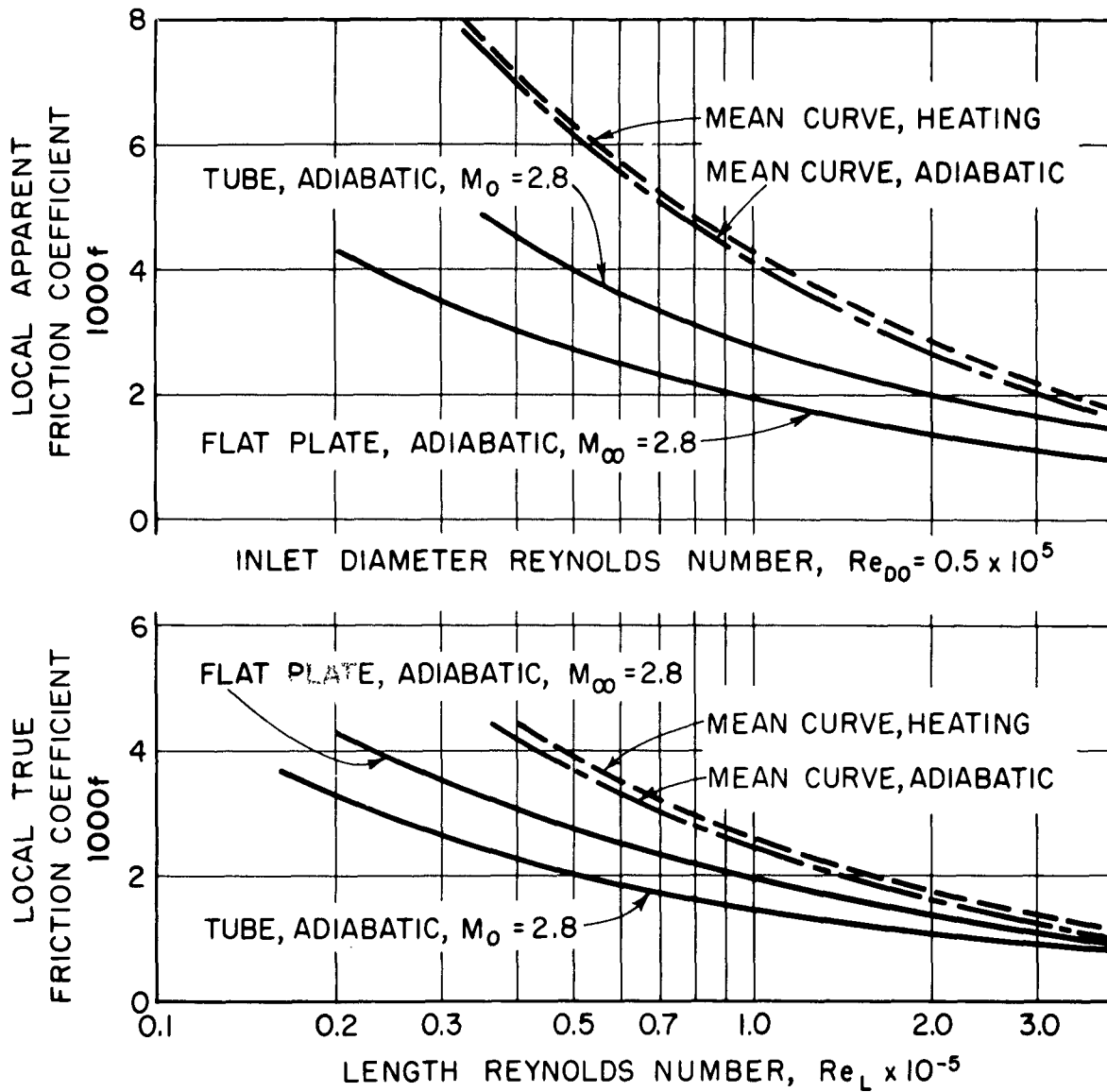


FIG. 12