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# SOME RIPPLE TANK STUDIES OF WAVE REFRACTION

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## ABSTRACT

When the shore line and offshore contours are straight and parallel, wave refraction is governed by Snell's law, which states that the sine of the angle between the wave crest and the bottom contour is proportional to the velocity of wave propagation. Snell's law was verified in this study by analyzing the photographic records of wave refraction at a model beach with 1:40 slope and a straight shore line in a ripple tank. This study covers uniform waves with three different periods, non-uniform waves, and three initial angles of wave approach. For the longest period waves the experimental results seemed to give a refraction angle smaller than that indicated by Snell's law.

## INTRODUCTION

When a wave moves into shoaling water, its velocity of propagation decreases as the depth decreases, and continues to do so until the wave breaks. If waves approach the shore at an angle, the inshore portion of the wave front travels slowly while the portion in deeper water races ahead. Waves thus change direction and swing around in such a manner that the crests tend to conform to the bottom contour. This phenomenon is known as wave refraction.

Wave refraction plays an important role in coastal and harbor works, since it governs, together with the direct shoaling effect, the wave heights along the coast. The breaking of waves obliquely to the coastline also induces a longshore current, which, combined with the agitating action of breaking waves, is the primary factor in causing littoral sediment transport (JOHNSON, 1952). The fact that the bending of the waves conforms to the bottom configuration can also be used to estimate beach profiles of inaccessible beaches; and this has an important bearing on the planning of landing operations on enemy-held beaches in time of war. (WILLIAMS, 1947)

In the case of a straight shore line and parallel offshore contours, wave refraction can be determined by applying what is known as Snell's law (O'BRIEN, 1942).

$$\frac{\sin \alpha}{\sin \alpha_1} = \frac{C}{C_1} \qquad (1)$$

where $\alpha$, $\alpha_1$ equal angles between the wave crest and the bottom contour at any two points along the orthogonal to the wave crest, and $C$, $C_1$ equal the corresponding velocities of wave propagation at the points where $\alpha$ and $\alpha_1$ are measured.

In using Equation (1) to determine the position of wave crests at the surface, it is necessary to know the direction and velocity of the waves at a certain reference point 1. This reference point is generally taken in deep water where the direction and velocity are constant. In this study, the reference point was taken at the toe of the beach beyond which the depth of flow remained constant.

The wave velocity is given by the equation:

$$C = \sqrt{\frac{gL}{2\pi} \tanh \frac{2\pi d}{L}} \qquad (2)$$

where

$L$ = wave length
$g$ = gravitational constant
$d$ = depth beneath the still water surface

Equation 2 is true for waves with low steepness ( wave height < 1/200 wave length).

The wave velocity can also be expressed by the wave length divided by the wave period, T

$$C = \frac{L}{T} \qquad (3)$$

Snell's law has been used quite extensively in the past as the basis of wave refraction studies, and it is the purpose of this investigation to present experimental evidence of its general applicability and possible limitations.

## EXPERIMENTAL APPARATUS

The experiment was conducted in a ripple tank (CHINN, 1949) four feet wide, twenty feet long and five inches deep. A 45" x 72" x 3/8" glass plate was installed at the middle of the tank bottom. At one end of the tank a water supply line and drain were provided for control of the water depth. A plunger type wave generator, supported by a movable frame, was located on the opposite end of the tank. The wave period was controlled by changing the speed of the motor and wave height adjustment was provided by an eccentric. The light source was located in an enclosed compartment underneath the tank. The light rays reflected from a 45° mirror, passed through in turn a wire grid, the glass section of the tank bottom, the transparent beach and the water in the tank, and projected the images on a 42" x 42" screen supported directly over the glass section and three feet above the water surface. In order to facilitate raising and lowering the screen, a system of strings and pulleys was provided. Light, while passing through a disturbed water surface, will not be uniform in intensity, due to the varying angle of refraction at the surface. According to the principle of light refraction, the images of the wave forms were reproduced on the screen and thereby recorded by a 35 mm. movie camera with 25 mm. lens at a speed of 28 frames per second and at a focal distance of 5 feet.

The beach was made of 1/4 inch transparent lucite plate. It had a slope of 1:40 and a straight shore line $1\frac{1}{2}$ feet wide, and extended 2 feet 10 inches to the sea. The beach was oriented at an angle $\alpha_1$ with the wave generator with two sides parallel to the side-walls of the ripple tank. A thin guide wall was installed along each side of the beach to eliminate shock waves generated by an abrupt change of water depth. The sharp-edged toe of the beach plate was taped down on the bottom of the ripple tank. Black tapes were fastened on the back side of the beach plate to indicate the base-line, range-line and control points used in the measurements.

## EXPERIMENTAL PROCEDURE AND METHOD OF ANALYSIS

The beach was first installed in the ripple tank and water introduced into the tank to cover the entire beach plate. The water surface and the beach profile along the range-line were measured to 0.001 ft. by a movable point gage. Part of the water was then drained to maintain a still water depth of 0.069 ft. in the tank. The beach under still water, and an electric clock in motion were then filmed separately, this allowed one to determine the length and time-scale of the photographic records, respectively. The wave generator was put in action starting at the lowest speed (or the longest wave period). The wave period was determined by measuring the time, in seconds, for 100 waves to pass a fixed point and dividing by 100. After the wave pattern became steady, its images on the screen were recorded by the camera for a length of time of about six seconds. The motor speed of the wave generator was then changed to a shorter wave period, and movies of the wave forms over the beach were taken again. A total of three different wave periods, varying from 0.39 to 0.83 seconds were studied. The beach was constructed on a length scale ratio of 1:500, and this gave corresponding wave periods of 8.70 to 18.50 seconds in the prototype. After each run water was drained from the tank and the beach plate was re-oriented to another angle with respect to the wave machine. The whole process as described above was then repeated.

In Run 2 non-uniform waves were generated by continueusly and quickly increasing and reducing the speed of the wave generator motor, and the wave patterns were recorded. Table I gives the characteristics of the waves and the beach for each run.

TABLE I

CHARACTERISTICS OF THE WAVES AND THE BEACH

| Run No. | Beach | | | Waves | | | | |
|---|---|---|---|---|---|---|---|---|
| | Still water depth ft. | Orientation | Profile | Description | Wave period in sec. | | Wave length in deep water, in feet | |
| | | | | | Model | Prototype* | Model | Prototype* |
| 1-A | 0.068 | 50° | slightly concave upward | uniform waves | 0.83 | 18.5 | 3.50 | 1750 |
| 1-B | | | | | 0.60 | 13.4 | 1.84 | 920 |
| 1-C | | | | | 0.40 | 9.8 | 0.80 | 400 |
| 2-A | 0.069 | 34° | essentially at 1:40 slope | uniform waves | 0.81 | 18.1 | 3.36 | 1680 |
| 2-B | | | | | 0.60 | 13.5 | 1.86 | 930 |
| 2-C | | | | | 0.39 | 8.7 | 0.77 | 385 |
| 2-D | | | | non-uniform waves | 0.30-0.53 | 6.6-11.7 | 0.45-1.42 | 223-710 |
| 3-A | 0.069 | 15° | essentially at 1:40 slope | uniform waves | 0.77 | 17.2 | 3.03 | 1515 |
| 3-B | | | | | 0.59 | 13.1 | 1.76 | 880 |
| 3-C | | | | | 0.39 | 8.8 | 0.79 | 395 |

* Scale ratio of the model beach
length scale 1 to 500
time scale 1 to 22.4

Analysis of the photographic records involved the following steps.

1. The film of the beach under still water was projected on a working table and the base-line, range-line and control points were drawn on a large sheet of

paper. This determined the length scale of the projected film.

2. The time scale of the records was determined by examining the movies of the clock in motion.

3. The films of wave patterns over the beach were then projected on the table and the sheet of paper was oriented until the base-line, range-line and control points drawn on the paper in step (1) coincided with the corresponding ones on the images.

4. From the first few projections a line orthogonal to the wave crests was drawn near the range-line. All further measurements were made along this reference orthogonal-line. The locations of the base-line, the range-line, and the reference orthogonal-line are shown in Plate 1, which is one of the photos indicating the pattern of 0.395-sec. waves over a 1:40 beach oriented at an angle of 50° with the wave generator.

5. Starting with one of the projections as time zero, each wave was identified with an appropriate number. In all subsequent projections these waves were identified by the same number.

6. The distance from base-line to each wave crest across the reference orthogonal-line was measured. The angle between the wave crest and the base-line was also taken.

7. In this manner every third frame was analyzed. Knowing the wave period and the depth, the wave velocity could be calculated according to Equations (2) and (3). The direction sine of the wave crest was then calculated according to Equation (1) by taking the toe of the beach as the reference point where $\alpha_1$ was the orientation of the beach with respect to the wave generator, and $d_1$ equalled the still water depth.

8. For non-uniform waves, the distance travelled by the wave crests along the reference orthogonal-line was plotted against the time elapsed in Fig. 1. The time intervals at either side of the wave, $T_1$ and $T_2$, were measured, and the period T of the given wave was determined by linear interpolation as follows:

$$T = \frac{T_1 \quad T_2}{\frac{T_1 + T_2}{2}} \qquad (4)$$

## EXPERIMENTAL RESULTS AND DISCUSSION

The refraction angles, both measured, and calculated according to Snell's law, were plotted in Figs. 2 to 4 against $d/L_o$ for three different wave periods and for three different initial angles of wave approach. Among the non-uniform wave group (as indicated in Fig. 1) the refraction angles of waves No. 7 and No. 12 were plotted in Fig. 5.

A survey of Figs. 2 to 5 indicated that within the limits of accuracy of the experiment, Snell's law was verified, but there was some indication that the refraction angle was slightly less than that indicated by Snell's law for large wave periods.

The scattering of the experimental data could have been caused by the following experimental errors, or errors in measurement:

1. After considerable handling, the lucite beach plate tended to bend slightly, resulting in bottom contours which were not exactly straight and parallel. This effect can be seen in Plate 1, where the shore line is slightly curved, and is not parallel to the base line.

2. As the wave length shortened when the wave approached shore, the crests became more sharply defined. However, deeper water wave crests were difficult to identify. This was especially true for waves with longer periods. Close to the breaking zone the images of the wave crest lines assumed such a wide bend that it was difficult to judge their direction of motion.

3. In most of the films (Plate 1 was one of the exceptions) the base-line was not clear and its allocation depended on the control points. However, the positions of the control points were affected to a certain degree by the relative positions of the waves. Light refraction due to the water surface undulation might have shifted the images of the control points on the screen from their true positions. In other words, the same wave in subsequent projections might not have been oriented with respect to the same base-line.

The effect of wave steepness on wave velocity as waves approached the shoaling water was believed to be small. As wave lengths in all the runs were larger than 0.075 ft., according to CHINN (1949), the effect of surface tension on wave velocity could also be ignored.

Despite these errors, Fig. 2 indicated that for the longest wave period (and hence the longest wave length), the deviation of the experimental points from Snell's law was too consistent to be accounted for by experimental difficulties alone. The theory of wave refraction has been developed from purely theoretical considerations by LOWELL (1949). The characteristics of waves which approached obliquely into a gently shoaling straight beach was studied. He started with a linear shallow water theory (STOKER, 1947) in which the wave velocity was given by the approximate equation

$$C = \sqrt{gd} \qquad (5)$$

which was sufficiently accurate in the range of $d/L_0$ used in this study. By assuming that the wave amplitude remained periodic in the direction parallel to the beach and with time, the original partial differential equation could be reduced into an ordinary differential equation containing the wave number as a parameter. The asmyptotic solution of this equation in the limit of large wave number indicated that the wave crests were refracted in accordance with Snell's law. The analysis thus showed that Snell's law could be expected to hold good in shallow water only when the wave length was short. Lowell also attacked the problem by considering the bottom slope to be uniform, without making the assumption about the wave number being large. The solution of the differential equation could be expressed in terms of Whittaker functions. Unfortunately the solution is not well adapted for computation, since the Whittaker functions are poorly tabulated at the present time. No deduction could be made from the solution concerning the possible direction of deviation from Snell's law, although this study indicated that the refraction angle tended to be smaller than that indicated by Snell's law, at large wave lengths. Perhaps it is interesting to note here that the refracted wave pattern of a 16-sec. wave at Oceanside, California on June 16, 1944 as determined from aerial photographs also seems to give refraction angles slightly smaller than those given by Snell's law (SVERDRUP AND MUNK, 1944)

Ocean waves are seldom regular. Even in the laboratory a wind of constant velocity blowing over the water surface generates waves with a spectrum of heights and periods (JOHNSON and RICE, 1952). In addition, when wave trains of different velocities meet, they also superimpose on each other and result in a complicated wave pattern. For instance, the resultant waves will be short crested if the component trains are traveling in different directions. In consequence the inshore waves are often non-uniform in that the wave length, period and velocity differ from one wave to another. However, in shallow water the wave velocity can very well be represented by Equation (5) which indicates that the wave velocity is independent of wave length. From this we may conclude that non-uniform waves refract essentially the same as the uniform waves in shallow water, and the experimental evidence seems to bear this out.

In conclusion, this study essentially verified Snell's law on refraction of ocean waves. The experimental results, however, call for additional theoretical research in dealing with refraction of waves of long wave length.

## ACKNOWLEDGMENT

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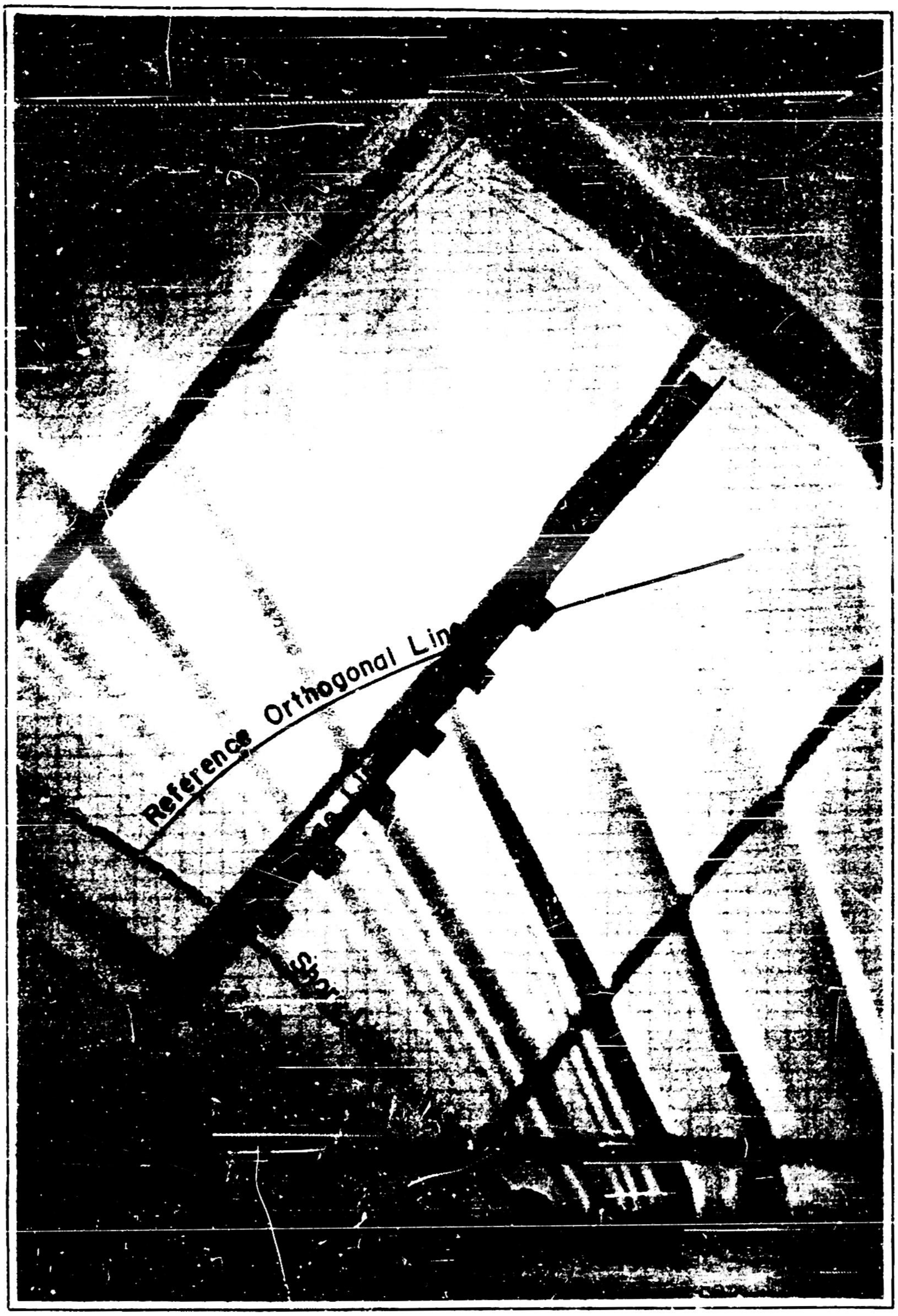

Reference Orthogonal Lin
Shor

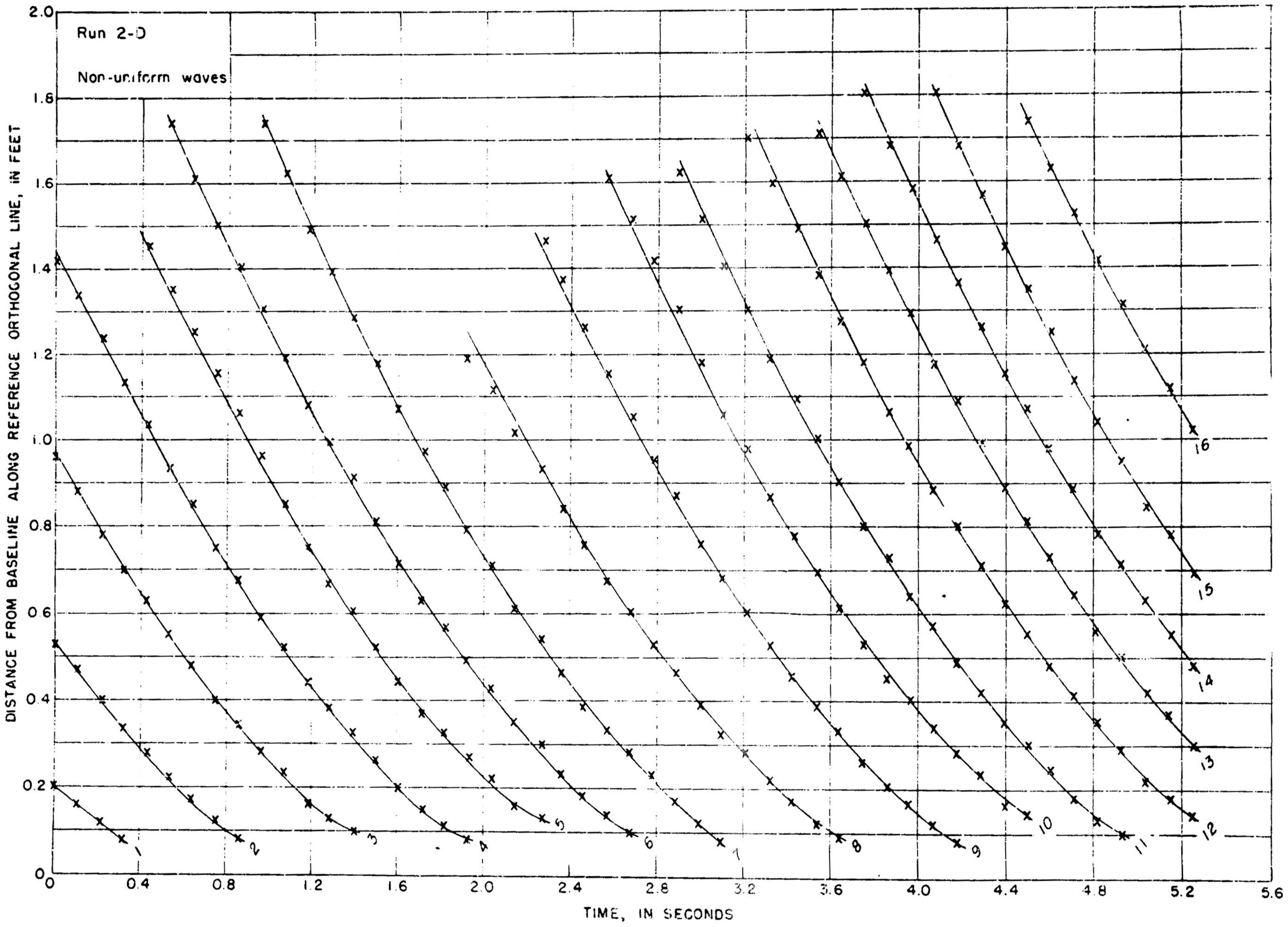
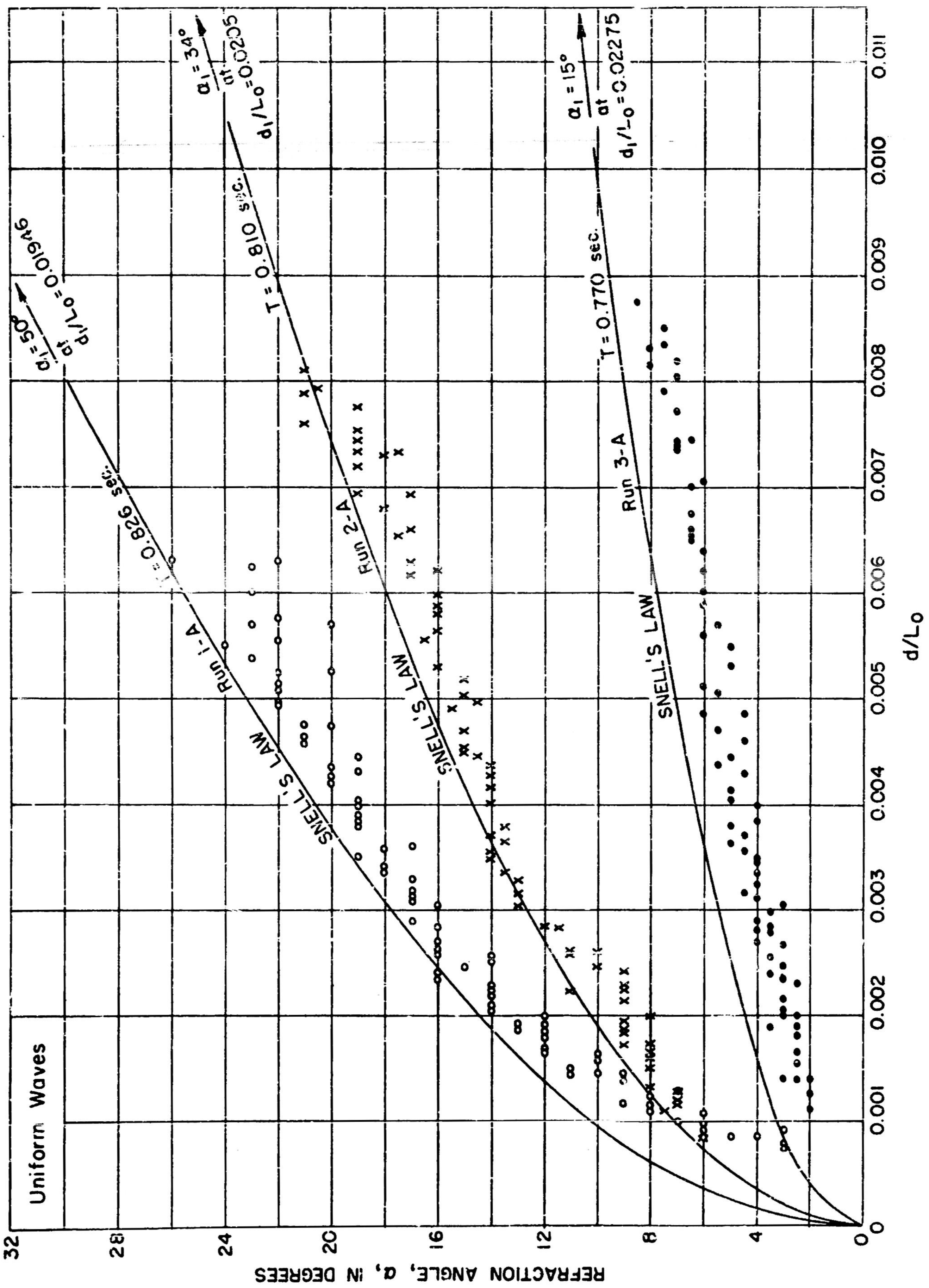
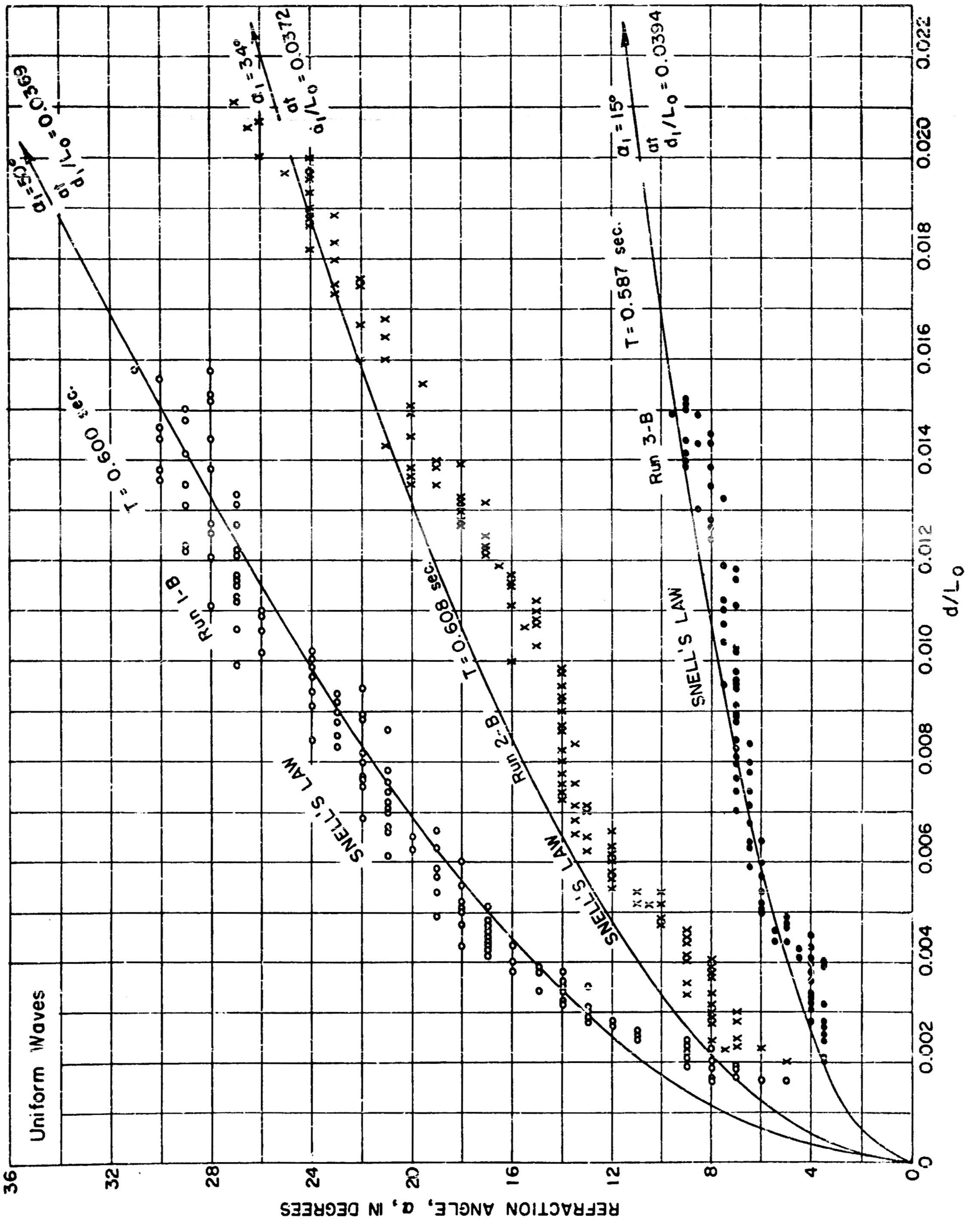

Run 2-D
Non-uniform waves
DISTANCE FROM BASELINE ALONG REFERENCE ORTHOGONAL LINE, IN FEET
TIME, IN SECONDS
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
0
0.4
0.8
1.2
1.6
2.0
2.4
2.8
3.2
3.6
4.0
4.4
4.8
5.2
5.6
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

FIGURE .



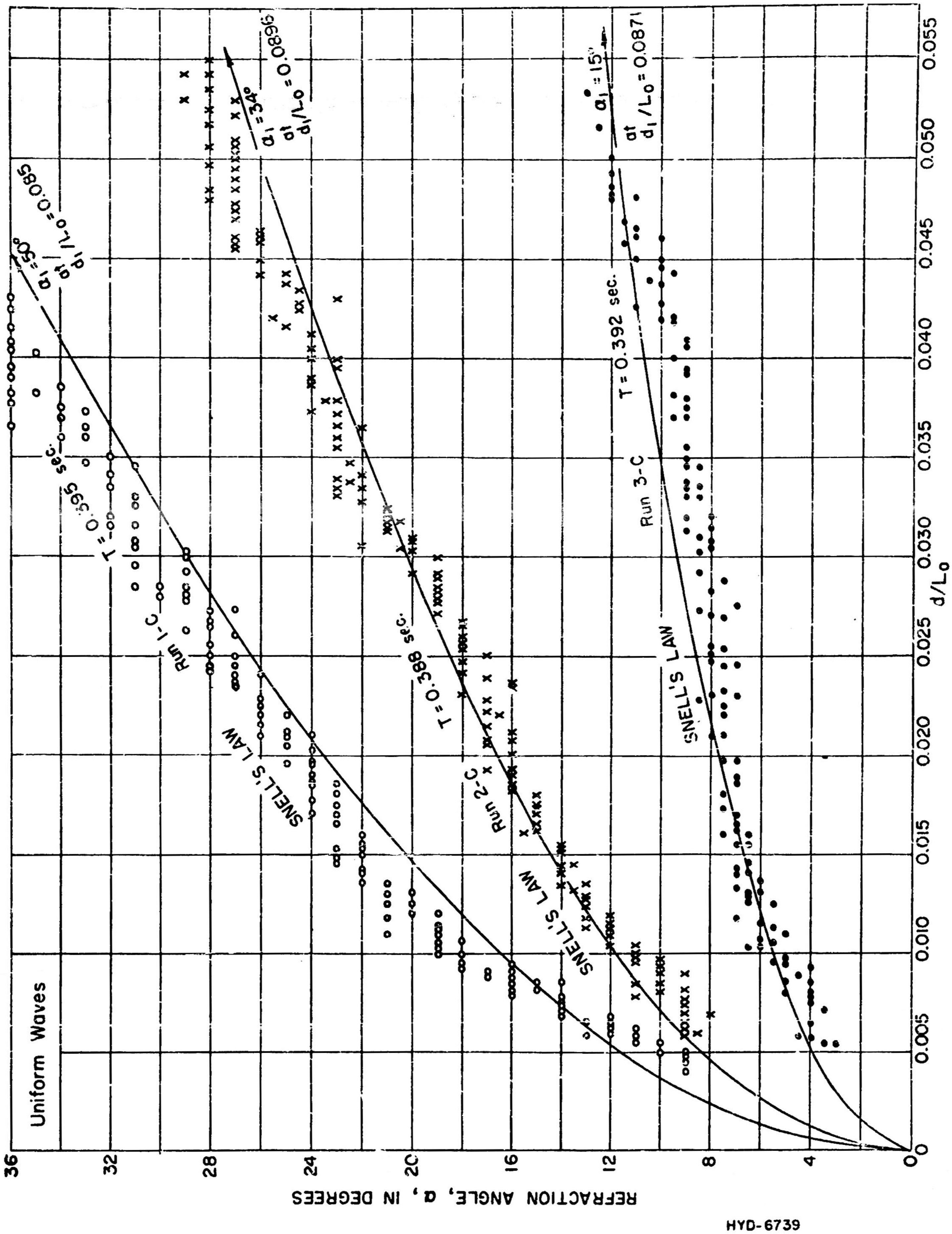
HYD-6737

FIGURE 2



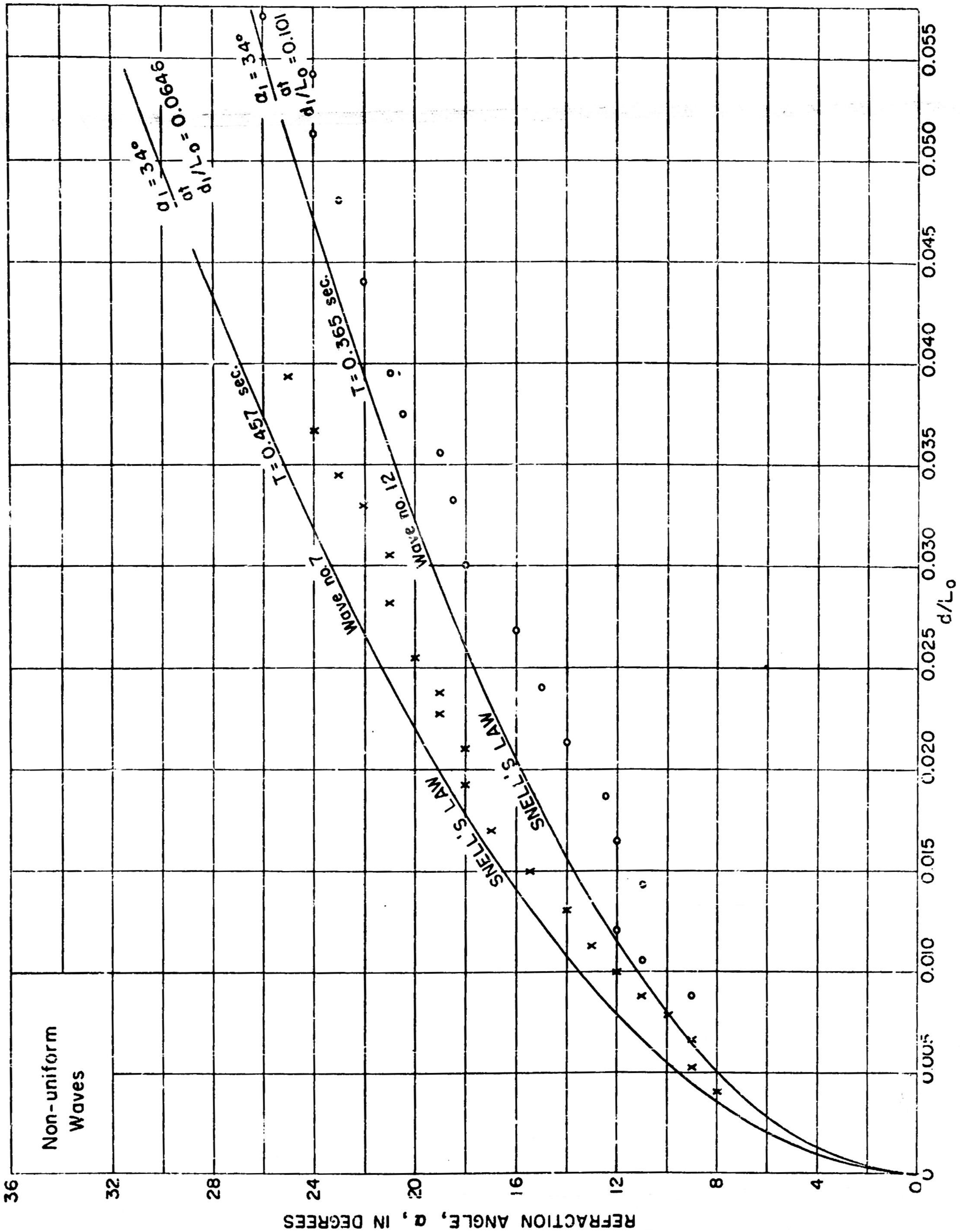
HYD-6738

FIGURE 3



HYD-6739

FIGURE 4



HYD-6740

FIGURE 5