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RIPPLE TANK STUDIES OF DEPTH DETERMINATION BY WAVE VELOCITY METHOD FROM TIMED VERTICAL PHOTOGRAPHS

BY

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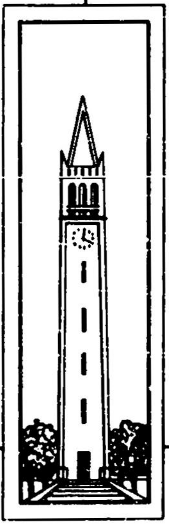

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INTRODUCTION

It is generally known, according to the wave theories, that the wave patterns in shallow water react to water depths and to the bottom configuration. This fact was recognized and employed with some success in World War II to evaluate water depths and bottom gradients of inaccessible beaches from timed aerial photographic records of sea surface behavior. Various methods have been developed; among them the wave velocity method seems to be the most promising one.

According to Airy, uniform periodic waves of small steepness in water of constant depth (roughly, wave height $< 1/200$ wave length) propagate at a velocity of

$$C = \sqrt{\frac{gL}{2\pi} \tanh \frac{2\pi d}{L}} \qquad (1)$$

where:

C = wave velocity
L = wave length
d = depth beneath still-water level
g = gravitational acceleration

The wave velocity can also be expressed as the wave length divided by the wave period, T

$$C = \frac{L}{T} \qquad (2)$$

In deep water where the water depth is larger than one-half of the wave length, $\tanh \frac{2\pi d}{L}$ is nearly unity. We thus have

$$C_o = \sqrt{\frac{gL_o}{2\pi}} = 5.12T \qquad (3)$$

and $$L_o = 5.12\ T^2 \qquad (4)$$

where the subscript o refers to the deep-water condition. In order to make use of these equations for depth determination, there must be no significant change in period as waves approach the shore; and this has been found to be true.

The fact that the wave velocity in shallow water is dependent on both the water depth and the wave length enables the depths of shoals to be deduced with some accuracy from the crest lines shown on timed aerial photographs. The wave velocity is determined by measuring the distance traveled by a crest in a known interval of time. The wave length is defined as being the distance between two crests, and is associated with the water depth midway between crests. The wave period is the time interval between the passage of two successive crests past fixed vertical line, and is associated with the water depth along this vertical and the mean time involved. Values of period and wavelength at other times and positions are to be determined by interpolation. Since linear interpolation appears to be quite accurate for our purposes, this method will be employed in the analysis as described later.

Ocean waves are never exactly like the ideal waves of which the wave velocity is given by Equation (1). Indeed, as Jeffrey stated so correctly, their irregularity is the conspicuous feature of the sea surface. Even in the laboratory, a wind of constant velocity blowing over the water surface generates waves with a spectrum of heights and periods.(1)* In addition, waves incident to a beach may be coming from more than one area, thus adding to the complexity. The apparent wave period, length, height and velocity differ from one wave to another, and such a wave pattern is called non-uniform. In shallow water, the formula for C_o, although derived for uniform periodic waves, approaches as $d \rightarrow 0$, the formula $C = \sqrt{gd}$, which has more general validity. This indicates that for non-uniform waves, even if there might be some doubt as what should be the proper wave length and period for a certain wave in a group, depth determination by the wave velocity method should still give satisfactory results in shallow water, since the velocity depends little on the wave length.

The principle of depth determination by the wave velocity method as outlined above is indeed simple. Yet to carry out a process of this nature involves numerous practical difficulties. The steepness of the waves may not be sharp enough to allow a clear-cut identification of wave positions in the photographs, tilting of the airplane used in the reconnaissance survey may largely distort the length scale, and difficulty in obtaining proper light conditions may deprive one of getting satisfactory photographic records in the first place, to cite only a few of the common practical limitations. Consequently, agreement between the measured beach profile and the calculated profile according to the wave velocity method seems to be highly erratic. On the one hand, good agreement between the two has been reported, notably the Bicsfored Survey(2) in World War II, the result of which greatly stimulated the research in this field. On the other hand, large discrepancies between the two have been found by some of the more recent investigations(3) (4). Part of the deviations may be attributed to the fact that the reconnaissance flights and direct surveys of beach profiles did not take place at the same time, and during the time interval the beach profile might have been modified to a certain extent by the action of the sea. A recent study by Moffitt(5) at Davenport, California indicated that agreement comes much closer when the time interval between photographic survey and beach profile sounding is relatively small, although discrepancies still exist.

It is the purpose of this study to make the comparison under controlled conditions in the laboratory, so that difficulties encountered in field surveys are generally eliminated. The result can thus be expected to indicate the reliability and limitations of the wave velocity method in depth determination.

EXPERIMENTAL APPARATUS

The experiment was conducted in a ripple tank(6) four feet wide, twenty feet long and five inches deep. A 45" x 72" x 3/8" glass plate was

*(1) Refers to reference number under REFERENCES at end of the report.

installed at the middle of the tank bottom for the passing of the light rays. On one end of the tank a water-supply line and drain were provided for the control of the water depth. A plunger type wave generator supplorted by a movable frame was located on the other end of the tank. The wave period was controlled by changing the speed of the motor. and the wave height adjustment was provided by a pair of eccentrics of equal eccentricities . The light source was located in an enclosed compartment underneath the tank. The light rays reflected from a 45° mirror, passed through in turn a wire grid, the glass section of the tank bottom, the transparent beach and the water in the tank and projected the images on a 42" x 42" screen supported directly over the glass section and three feet above the water surface. A system of strings and pulleys was provided to facilitate raising and lowering the screen. Light, while passing through a disturbed water surface, will not be uniform in intensity, due to the varying angle of refraction at the surface. According to the principle of light refraction, the images of the wave forms will be reproduced on the screen and thereby recorded by a 35 mm. movie camera with 25 mm. lens at a speed of 28 frames per second and at a focal distance of 5 feet.

The beach was made of 1/4 inch transparent lucite plate. It had a slope of 1:40 and was $1\frac{1}{2}$ feet wide onshore . and extended 2 feet 10 inches seaward. (This slope was chosen in order to facilitate comparison of model data with field data taken at Davenport, California on a beach having nearly the same slope(5).) The beach was oriented at an angle, α_1, with the wave generator, with two sides parallel to the side-walls of the ripple tank. A thin guide wall was installed along each side of the beach to eliminate shock waves generated by an abrupt change of water depth. The sharp-edged toe of the beach plate was taped down on the bottom of the ripple tank. Black tapes were also fastened on the back slide of the beach plate to indicate the base-line, range-line, and control points used in the measurements.

EXPERIMENTAL PROCEDURES AND METHOD OF ANALYSIS

The beach was first installed in the ripple tank and then water was run into the tank to cover the entire beach plate. The water surface and the beach profile along the range line were measured to 0.001 ft. by a movable point gage. Part of the water was then drained to maintain a still-water depth of 0.069 feet in the tank. The beach under still water and an electric clock in motion were then filmed separately, this allowed one to determine the length and time-scale of the films, respectively. The wave generator was put in action, starting at the lowest speed (or the longest wave period). After the wave pattern became steady, its images on the screen were recorded by the camera for a length of time of about six seconds. The motor speed of the wave generator was then changed to a shorter wave period, and movies of the wave forms over the beach were taken again. A total of four different wave periods, varying from 0.22 to 0.83 seconds were studied. The beach was constructed on a length scale ratio of 1:500, and this gave corresponding wave periods of 4.9 to 18.5 seconds in the prototype.

Next, non-uniform waves were generated by continuously and quickly increasing and reducing the speed of the wave generator motor. Photographs (about 50 feet of film) of the non-uniform wave patterns were taken, and this completed one run. The water was drained and the beach plate was re-oriented to another angle with respect to the wave machine. The whole process as described above was then repeated. In Table I are shown the characteristics of the waves and the beach for each run.

TABLE I

CHARACTERISTICS OF THE WAVES AND THE BEACH

Run No.	Beach			Waves				
	Still Water Depth Ft.	Orientation	Profile	Description	Wave period in sec.		Wave length in deep water in feet.	
					Model	Prototype*	Model	Prototype*
1-A	0.068	50°	slightly concave upward	uniform waves	0.83	18.5	3.495	1750
1-B					0.60	13.4	1.843	920
1-C					0.40	8.8	0.800	400
1-D					0.25	5.6	0.315	160
1-E				non-uniform waves	0.25- 0.35	5.6- 7.7	0.320- 0.610	160- 305
2-A	0.069	34°	essentially at 1:40 slope	uniform waves	0.81	18.1	3.360	1680
2-B					0.60	13.5	1.855	930
2-C					0.39	8.7	0.770	385
2-D					0.22	5.0	0.250	125
2-E				non-uniform	0.30- 0.53	6.6- 11.7	0.446- 1.420	225- 710
3-A	0.069	15°	essentially at 1:40 slope	uniform waves	0.77	17.2	3.030	1515
3-B					0.59	13.1	1.762	880
3-C					0.39	8.8	0.787	395

* Scale ratios of the model beach
length scale 1 to 500
time scale 1 to 22.4

Analysis of the photographic records involved the following steps:

1. The film taken of the beach under still-water was projected on the working table and the base-line, range-line and control points were drawn on a large sheet of paper. This determined the length scale of the projected film.

2. The time scale of the records was determined by examining the movies of the clock in motion.

3. The films of wave patterns over the beach were then projected on the table and the sheet of paper was oriented until the base-line, range-line and control points drawn on the paper in step (1) coincided with the corresponding ones on the images.

4. From the first few projections a line orthogonal to the wave crests was drawn near the range-line. All further measurements were made along this reference orthogonal-line. The locations of the base-line, the range-line and the reference orthogonal-line are shown in Plate 1, which is one of the photos indicating the pattern of 0.395-sec. waves over a 1:40 beach oriented at an angle of 50° with the wave generator.

5. Starting with one of the projections as time zero, each wave was identified with an appropriate number. In all subsequent projections these waves were identified by the same number.

6. The distance from base-line and along the reference orthogonal-line to each wave crest was measured.

7. In this manner every third frame was analyzed, and the time of each projection and distance along the orthogonal-line from the base-line to each wave was tabulated. The results were plotted in terms of distance from the base-line of each wave and the time of its advancement. Fig. 1 is a typical example of such a plot for non-uniform waves.

8. The wave period, T, was determined by averaging the time interval between successive waves in the time-distance plot for uniform waves. For non-uniform waves, the time intervals at either side of the wave, T_1 and T_2, were measured, and the period T of the given wave was determined by interpolation as follows:

$$T = \frac{T_1 \; T_2}{\frac{T_1+T_2}{2}} \qquad (5)$$

9. The slope of the time-distance curve gave the wave velocity C. For each wave, the wave velocity C was measured along the reference orthogonal-line at a spacing of 0.1 ft.

10. From the wave period, the wave characteristics in deep water were determined according to Equations (3) and (4).

11. For each wave the values of the ratio C/C_0 were computed from the data obtained above. Then the values of d/L_0, corresponding to the above values of C/C_0, were determined from published tables(6). Knowing the value of L_0, the water depths along the reference orthogonal-line were computed. Each wave in the distance-time curve gave a beach profile, and the average of these profiles was taken as the representative beach profile along the reference orthogonal-line, as determined by the wave velocity method.

12. The beach profile along the reference orthogonal-line was then converted into the beach profile perpendicular to the base line, and this calculated profile was compared with the measured beach profile along the range-line. The error of the wave-velocity method in depth determination was given by the equation:

$$\text{Percentage error} = \frac{d_c - d_m}{d_m} \times 100 \qquad (6)$$

where d_m and d_c were the measured- and calculated-depth, respectively.

EXPERIMENTAL RESULTS

The measured beach profile and the calculated beach profile, based on the wave velocity method, were plotted in Figs. 2 to 4 for the three different runs. The percentage errors for all the runs were plotted in Fig. 5 by using d/L_0 as the abscissa. Fig. 5 was constructed to detect any trend caused by the defects of wave theory. It should not be used to judge the reliability of depth determination by the wave velocity method, as a small absolute error in depth determination near the shore-line where the water is shallow may result in large relative error percentage-wise, yet it has little effect on the practical applicability of the method as a whole.

Discussion of the Experimental Results : The agreement between the measured beach profile and the calculated beach profile based on the wave velocity method was considered to be fairly good for all the runs, although errors in local depth determination of as much as 24 percent were noticed. These errors could have been caused either by the defects of the wave velocity method, or by errors in measurements.

The wave velocity method of depth determination is based solely on Equation (1) which indicates the relationship between wave velocity, wave length and water depth. Equation (1) was developed for wave motion over a horizontal bottom in water of constant depth and for very low waves, having a steepness of less than 1/200. Munk(8) found that the velocity at a specified depth on a sloping bottom was essentially the same as that over a horizontal bed of the corresponding constand depth, but there was some indication that the measured velocity over a sloping beach was about 5 percent less than that given by Equation (1). Because of this effect, there is a tendency to under-estimate the depth; although quantitatively it is believed to be negligible in the results presented in this report, as the change of depth is small with respect to the wave length over a 1: 40 slope beach.

When waves are traveling in shoaling water, the wave crests tend to be steeper and narrower while the troughs become flatter. Close to the breaking zone, waves begin to deform rapidly by increasing the front slope and reducing the rear slope, leading in the limiting case of a breaking wave, to a front slope which goes beyond the vertical. In the ripple tank this part of the wave diverges the light and results in a black band immediately at the front of the white crest line, as shown in Plate 1. After the breaking of a wave into a bore of foam, it still has energy, and this energy can be transmitted forward. Plate 1 indicates that over the flat beach there is ample room between the shore-line and the breaking zone for a wave to reform, and the bore actually reforms into several crests, each maintaining its identity until the depth is further reduced sufficiently to cause the wave to break again. Similar phenomenon have been observed on actual beaches.(9)

For a given wave length, a steep wave travels faster than a flat one. A small error is thus introduced in using Equation (1) to compute the velocity, since waves become progressively steeper when approaching shore-line. For the steepest waves, this may result in the computed depth being too great by about 10 percent(1). The wave steepness was not measured in this study, and its effect on the depth determination could only be studied qualitatively from the experimental results.

It is generally known that the increase in height of waves as they run into shallow water is a function of d/L_0. The wave height is also affected by the wave refraction phenomebon when waves approach the shore line at an angle. Refraction commonly tends to increase the length of the wave crest and thus reduce the wave height. In this respect, over the same beach, waves coming from a larger angle of approach tend to be flatter. This is also true from the wave reflection point of view. Waves are reflected whenever they move into a region where their speed of propagation changes. Against a shoaling bottom some of the wave energy is reflected and thereby augments the incident wave height. The effect is largest when waves move directly toward the beach instead of approaching it at an angle. However, Munk(10) has shown that the effect of wave reflection on a comparatively flat beach was probably small.

Considering the arguments given above (since the initial wave height was the same for all the runs) one would expect the depth to be over-estimated to a larger extent for:

1. the same run with smaller values of d/L_o ,
2. the same initial angle of approach, α_1, with larger wave periods (and therefore, larger wave lengths)and
3. the same wave period with smaller initial angles of approach,

providing the errors in depth determination by the wave velocity method are caused by wave steepness alone.

As can be seen in Figs.2 to 5 these trends, if they exist, are not recognizable with the possible exception of Item (3). In Run 3 (which had the smallest initial angle of approach) the wave velocity method tended to over-estimate the depth to a greater degree than in Runs 1 and 2.

It is not surprising that the above-mentioned trends were completely concealed by the errors in measurement, which had a much larger order of magnitude. In spite of caution in conducting the experiments, experimental error still existed and affected the results to a great extent. Among these experimental errors were:

1. Difficulty in identifying crest-lines. As the wave length shortened when the wave approached shore, the crests became more sharply defined. However, deeper water wave crests were difficult to identify. This was especially true for waves with longer periods. For instance, consider the measured beach profiles for Runs 2-A and 2-D in Fig. 3. Run 2-A had a period of 0.810 sec. The crest lines offshore were so faint that no waves could be identified seaward of 0.85 feet from the baseline. Within the interval of from 0.70 to 0.85 feet seaward of the base-line the positions of the waves were so uncertain that large errors resulted when determining the depth. On the other hand, Run 2-D had a wave period of 0.221 sec., and in that run waves could be identified as far as 1.5 feet from the base-line. Although large deviations were obtained when more than 1.0 feet seaward of the base-line, the agreement was fairly good in the interval of from 0.70 to 0.85 feet.

2. In most of the films (Plate 1 was one of the exceptions) the base-line was not clear and its allocation depended on the control points. However, the positions of the control points were affected, to a certain degree, by the relative positions of the waves. Light refraction due to the water surface undulation may have shifted the images of the control points on the screen from their true positions. In other words, the same wave in subsequent projections may not have been oriented with

respect to the same base line.

3. The actual beach profile and the still water surface were measured by a point gage read to 0.001 ft., i.e., there was a possible error of plus or minus 0.001 ft. in these measurements. Close to the shoreline where the water depth was small, the possible error of actual depth measurement became important and might have accounted for the local apparent large errors, percentagewise, in depth determination by the wave velocity method, as shown in Fig. 5. Furthermore, with the measured beach profile referred to the still-water surface, an error of plus or minus 0.001 ft. in the measurement of the latter might result in a beach profile 0.001 ft. too high or too low. Referring to Fig. 4, if it happened that the measured beach profile was 0.001 ft. too high; then the agreement between the actual beach profile and the calculated beach profile became much closer. If this was the case, the trend that depth tended to be more over-estimated for small initial angles of approach was somewhat upset; although this was to be expected from theoretical deductions.

The non-uniform waves seemed to give results comparable with those given by uniform waves. However, one must realize that the spectrum of the non-uniform waves generated in the ripple tank was narrower than that generally existing in the oceans. Or, in other words, the result might be different for waves with higher non-uniformity.

CONCLUSIONS

This study indicated that the determination of water depth in the laboratory by the wave velocity method from timed vertical photographs was satisfactory. This was true for both uniform and non-uniform waves. Any errors resulting from the application of this method were mainly due to errors in measurement, not to inherent defects of the theory upon which the method was based. The conclusion may need to be modified for extreme cases, such as beaches with very steep slopes.

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REFERENCES

1. Johnson, J.W. and Rice, E.K., "A Laboratory Investigation of Wind-Generated Waves", Trans. Amer. Geophys. Union, vol. 33, No. 6, pp.845-854, 1952.

2. Williams, W.W., "The Determination of Gradients on Enemy -Held Beaches", The Geographical Journal, vol. CIX, Nos. 1-3, July 1947, pp. 76-93.

3. Johnson, J.W., "Progress Report; Wave Velocity Method of Depth Determination by Aerial Photographs", Inst. of Engr. Res. University of Calif., Tech. Report No. 155-10 , October 7, 1949.

4. Crooke, R.C., Wiegel, R.L., Kolster, O.J. and Thomas, G.E., "Beach Profile Determination from Timed Vertical Aerial Photographs". IER. University of Calif. Tech. Report No. 29-48, June 1951.

5. Moffitt, F.H. "Analysis of Controlled Terrestrial Photographs taken at Davenport, California for Wave Velocity Method of Depth Determination", IER, University of Calif, Tech. Report Series 74, Issue 3, 1953.

6. Chinn, A.J., "The Effect of Surface Tension on Wave Velocities in Shallow Water", M.S. Thesis, University of Calif. 1949.

7. Wiegel, R.L., "Oscillatory Waves - Diagrams and Tables of Relationships Commonly used in Investigations of Surface Waves", The Bull. of the Beach Erosion Board, Special Issue No. 1, July 1948.

8. Munk, W.H., "The Solitary Wave Theory and its Application to Surf Problems," Annals of New York Ac. Sci., vol. 51, Art. 3.

9. Russell, R.C.H. and Macmillan, D.H., "Waves and Tides", Philosophical Library, New York, 1953, pp 85-86.

10. Munk, W.H., "Surf Beats", Amer. Geophys Union, Trans. vol 30, 1949, pp. 849-854.

REFERENCES

1. Johnson, J.W. and Rice, E.K., "A Laboratory Investigation of Wind-Generated Waves", Trans. Amer. Geophys. Union, vol. 33, No. 6, pp.845-854, 1952.

2. Williams, W.W., "The Determination of Gradients on Enemy -Held Beaches", The Geographical Journal, vol. CIX, Nos. 1-3, July 1947, pp. 76-93.

3. Johnson, J.W., "Progress Report: Wave Velocity Method of Depth Determination by Aerial Photographs", Inst. of Engr. Res. University of Calif., Tech. Report No. 155-10 , October 7, 1949.

4. Crooke, R.C., Wiegel, R.L., Kolster, O.J. and Thomas, G.E., "Beach Profile Determination from Timed Vertical Aerial Photographs". IER. University of Calif. Tech. Report No. 29-48, June 1951.

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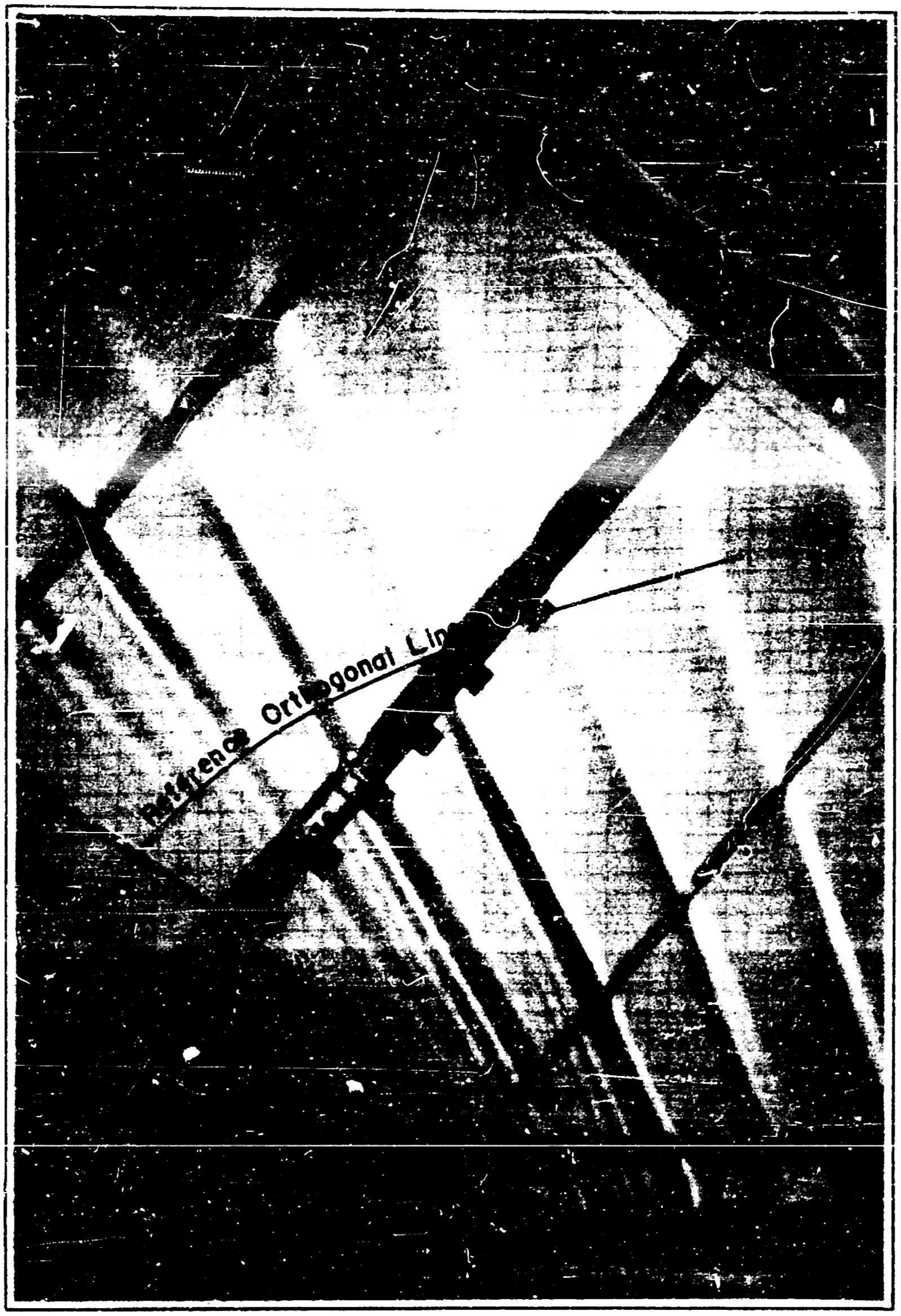
6. Chinn, A.J., "The Effect of Surface Tension on Wave Velocities in Shallow Water", M.S. Thesis, University of Calif. 1949.

7. Wiegel, R.L., "Oscillatory Waves - Diagrams and Tables of Relationships Commonly used in Investigations of Surface Waves", The Bull. of the Beach Erosion Board, Special Issue No. 1, July 1948.

8. Munk, W.H., "The Solitary Wave Theory and its Application to Surf Problems," Annals of New York Ac. Sci., vol. 51, Art. 3.

9. Russell, R.C.H. and Macmillan, D.H., "Waves and Tides", Philosophical Library, New York, 1953, pp 85-86.

10. Munk, W.H., "Surf Beats", Amer. Geophys Union, Trans. vol 30, 1949, pp. 849-854.



Reference Orthogonal Line

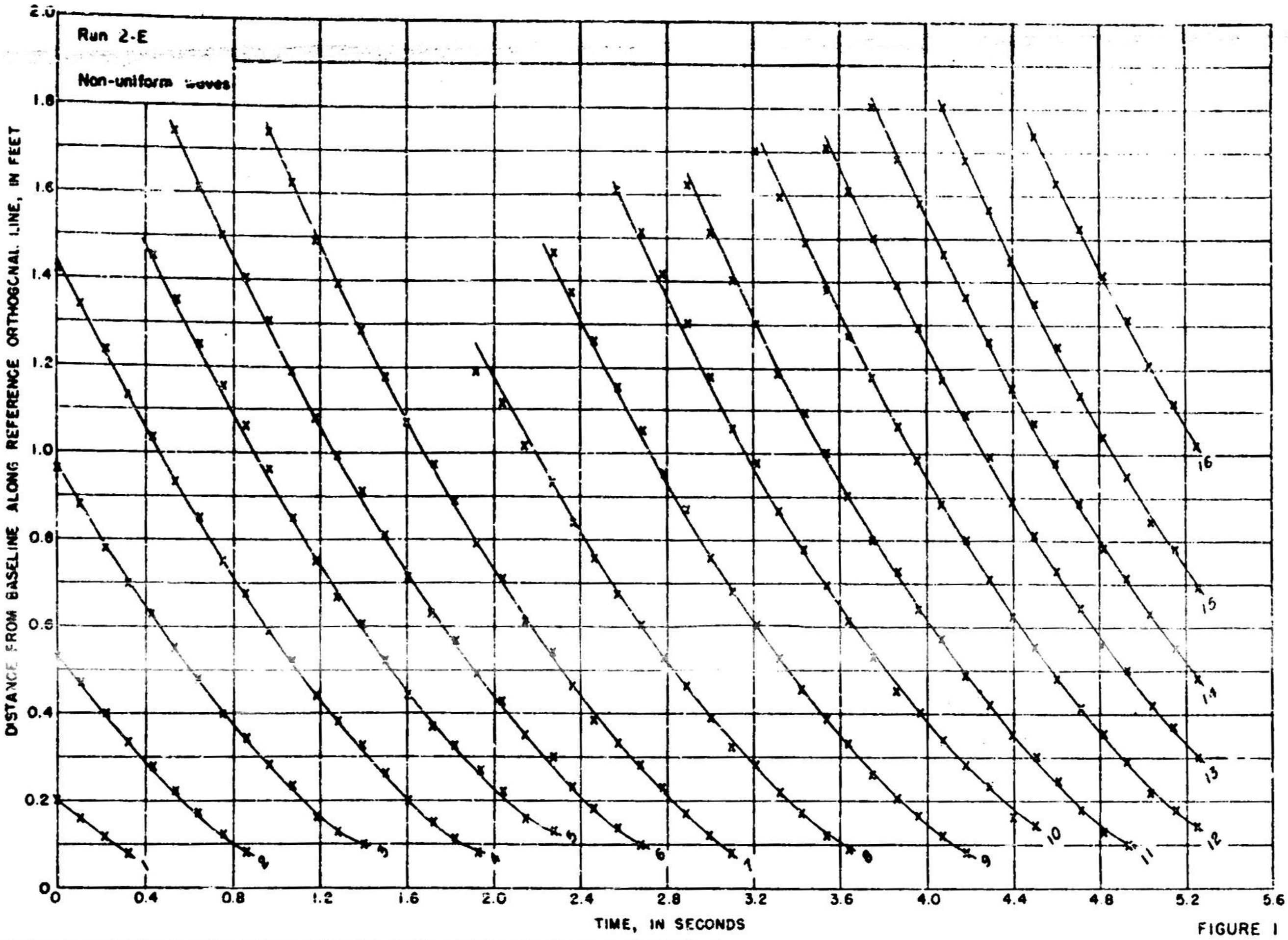


FIGURE 1

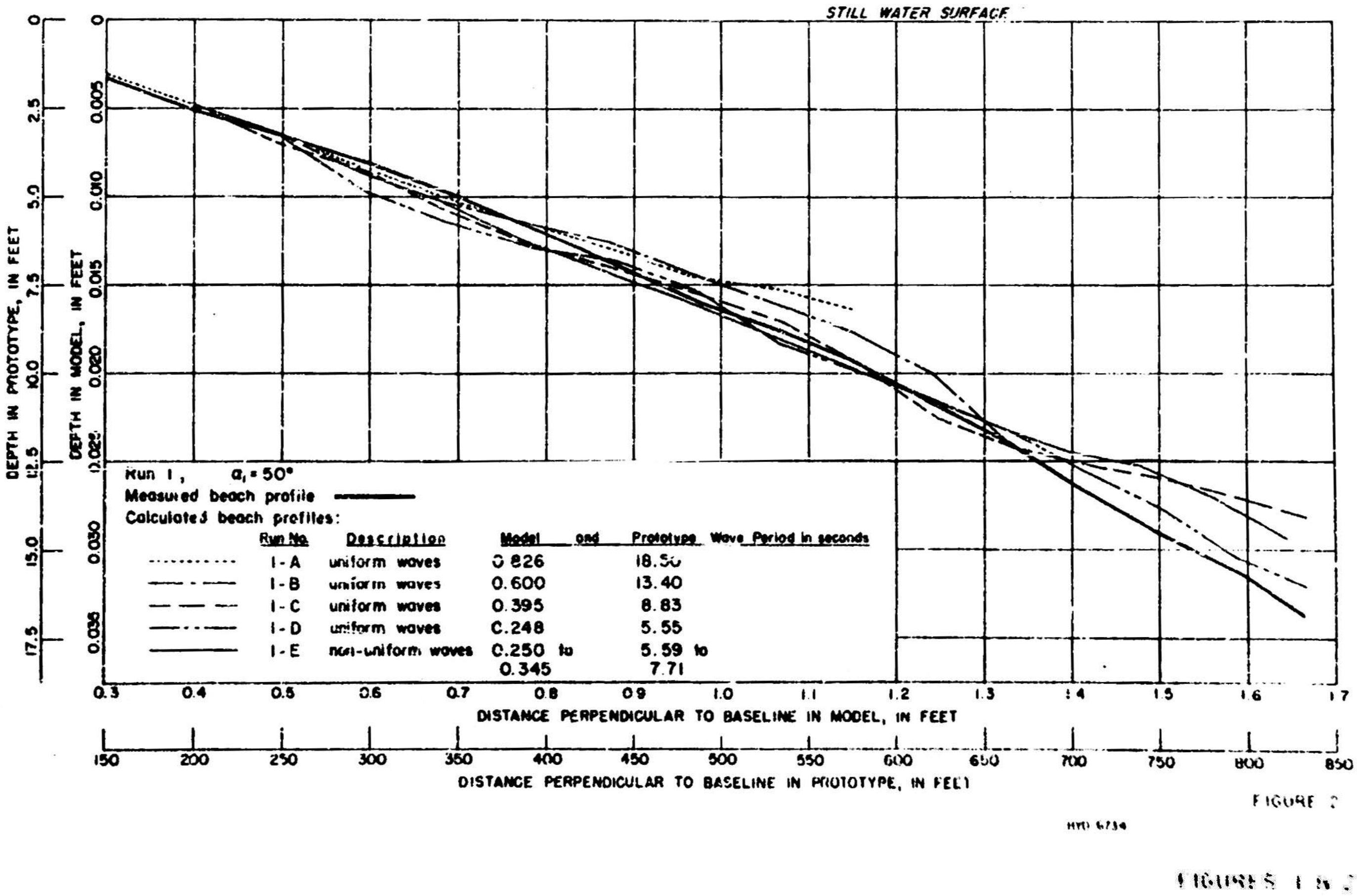


FIGURE 2

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FIGURES 1 & 2

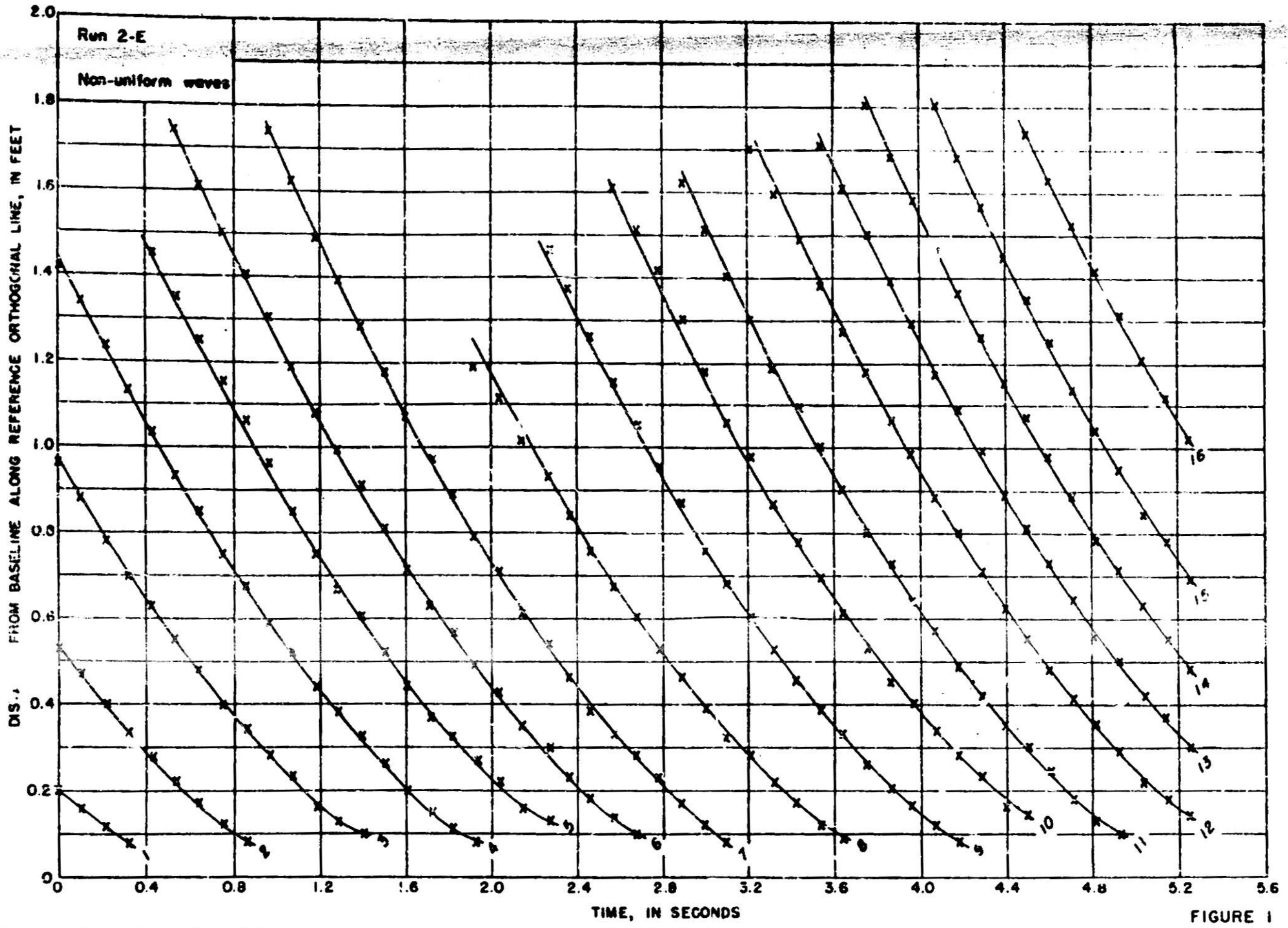

FIGURE 1

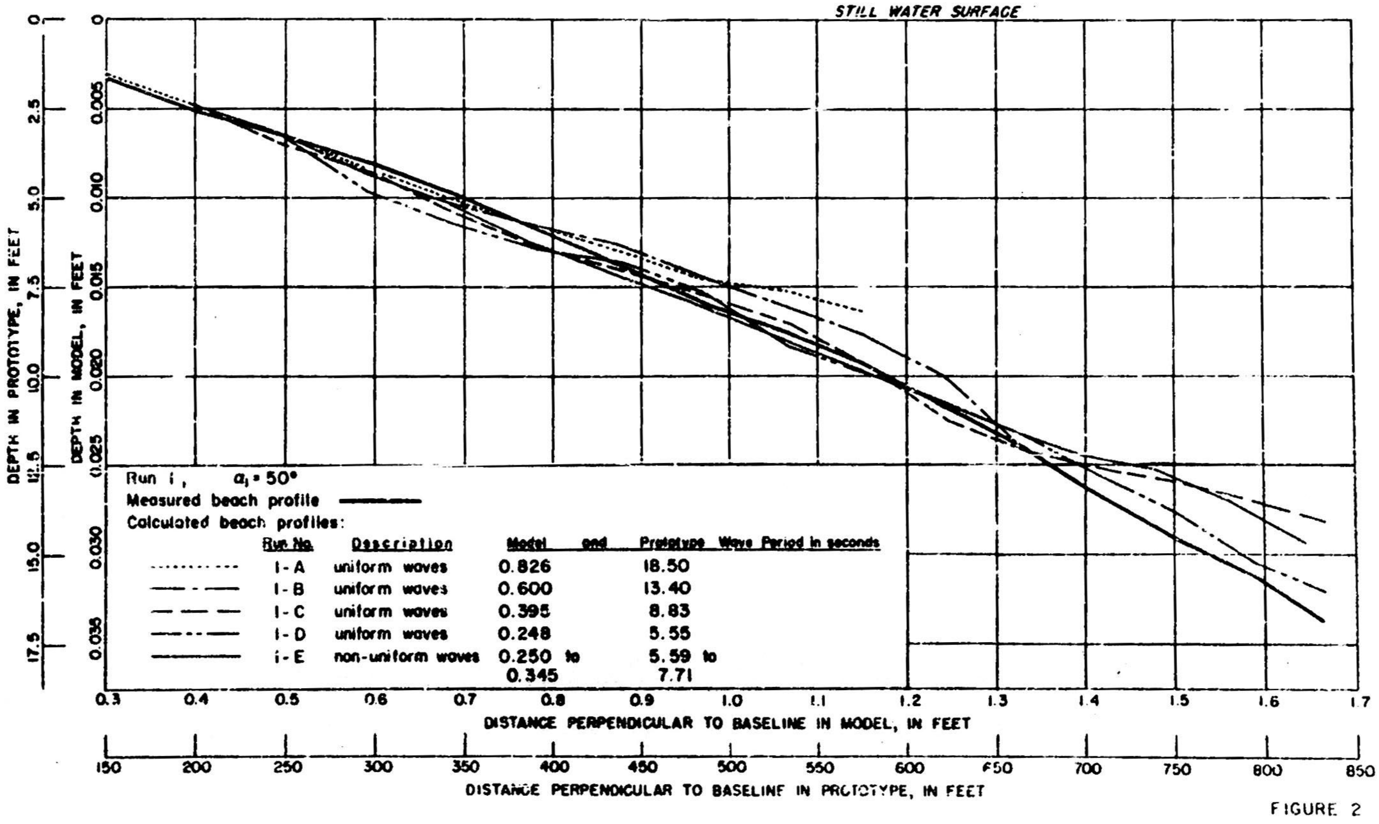

FIGURE 2

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FIGURES 1 & 2

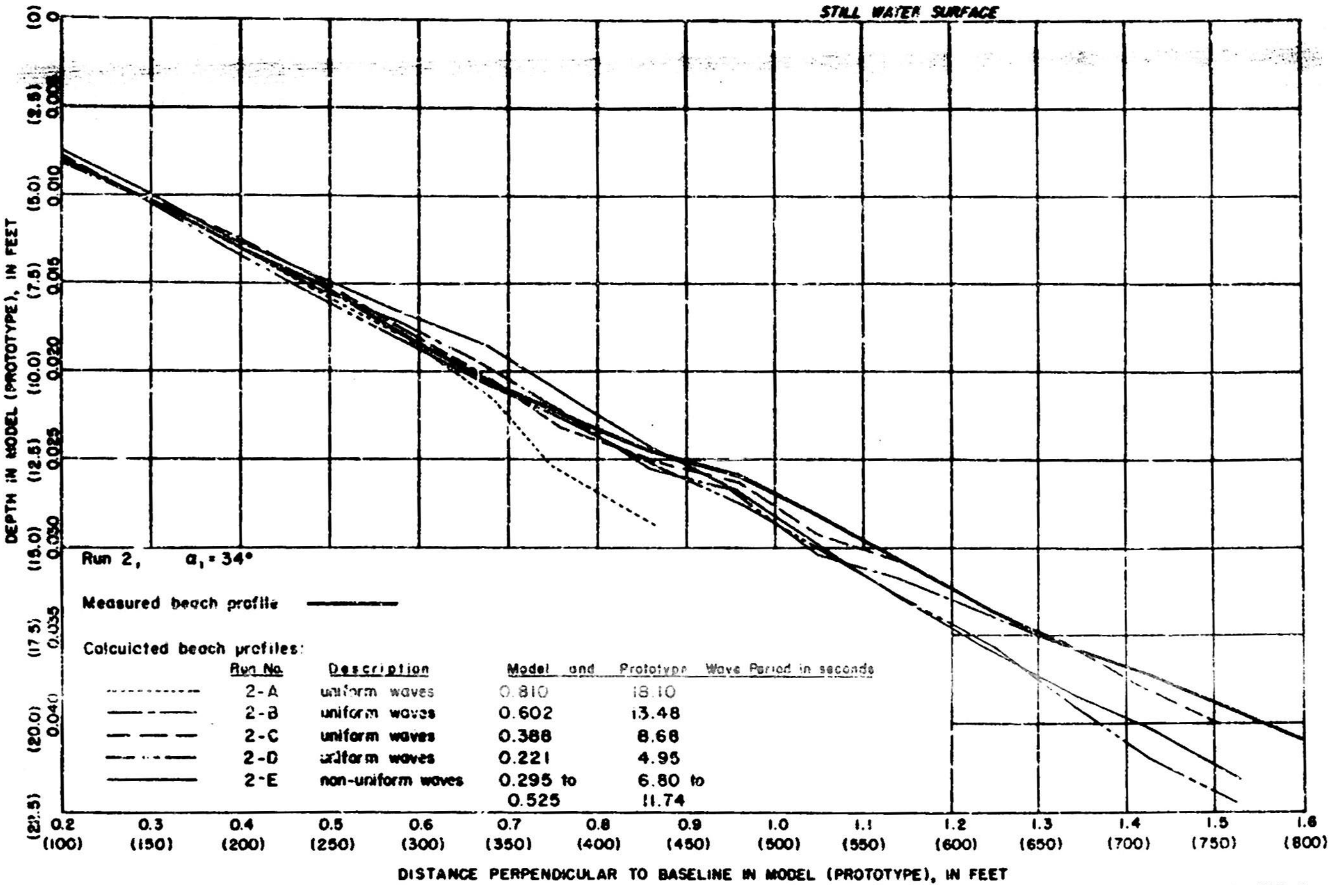

FIGURE 3

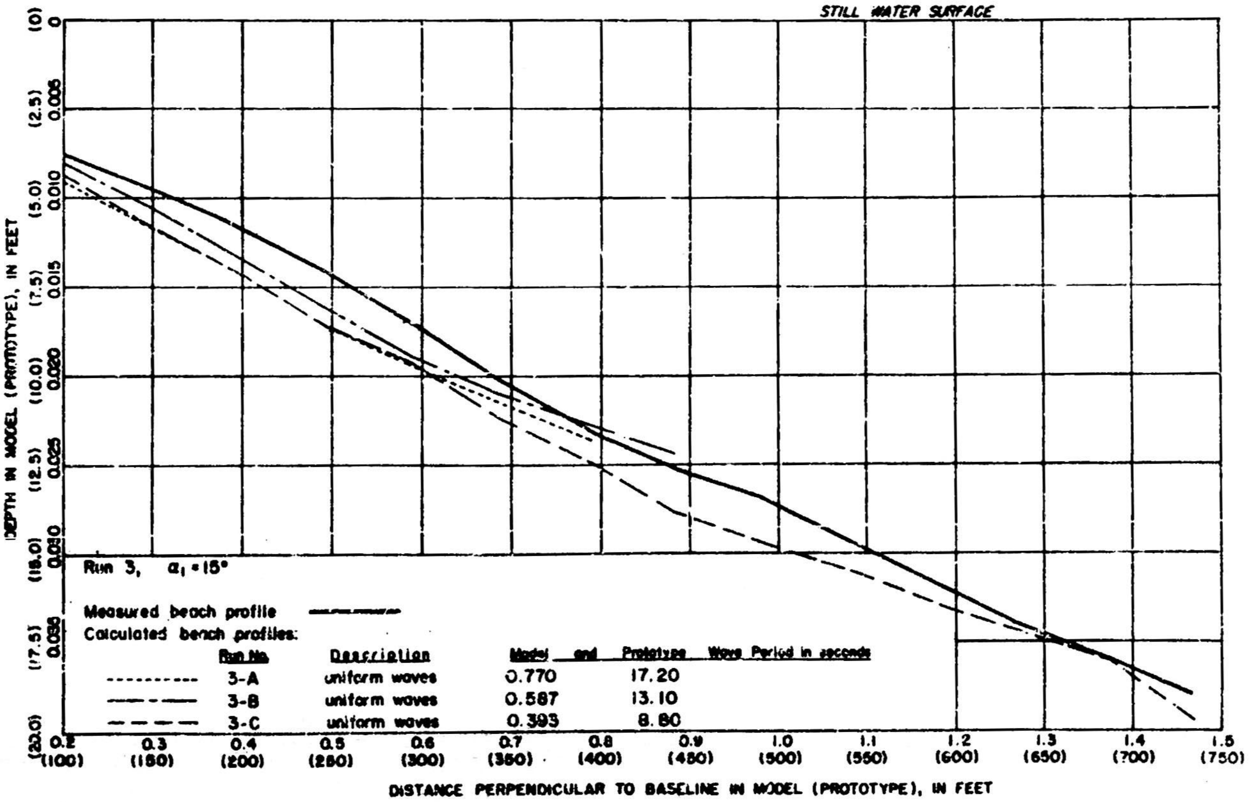

FIGURE 4

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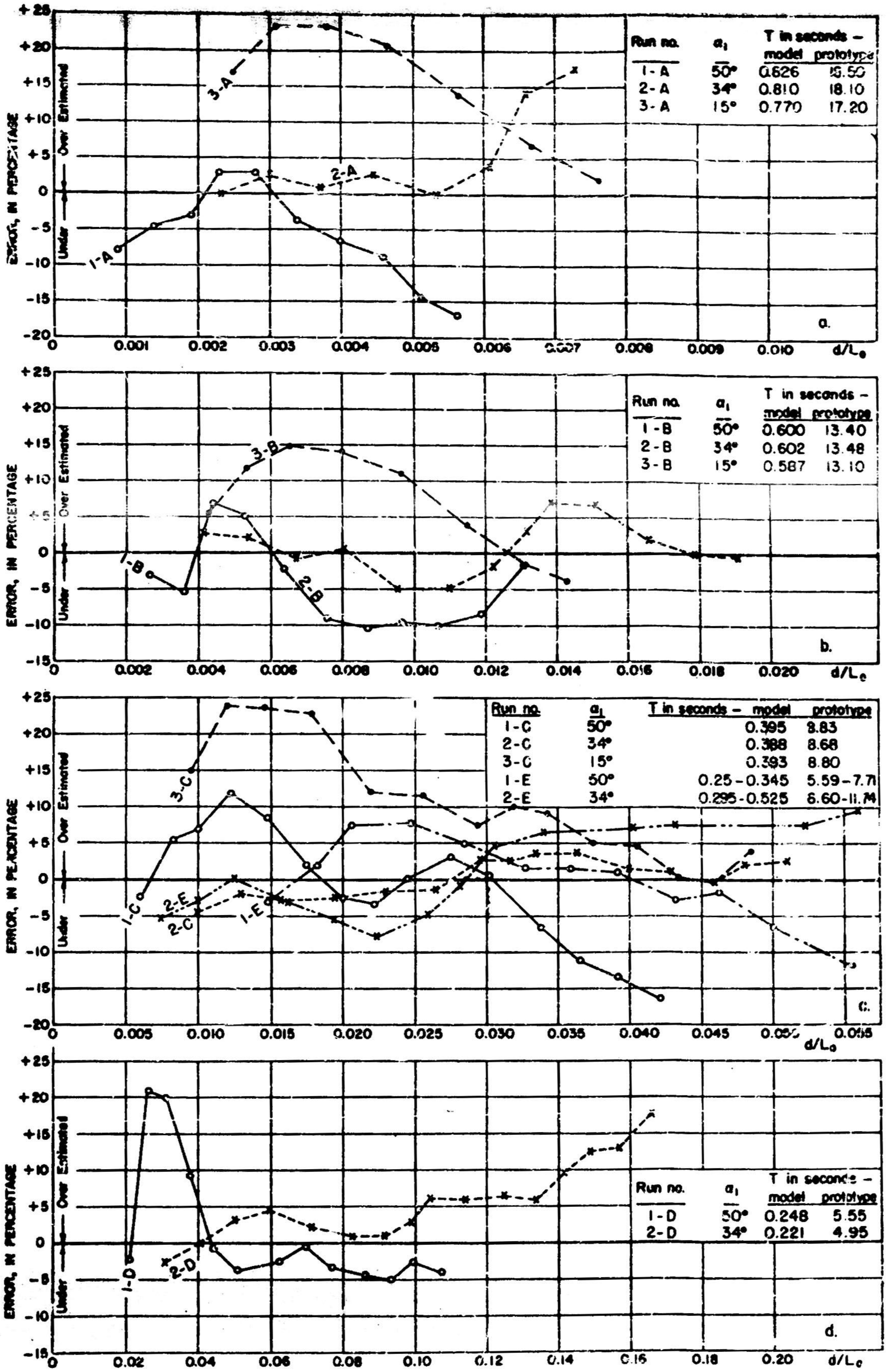

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FIGURE 5