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The Flow Field Surrounding a Moving Vessel
by H. Margenau, T.R. VanZandt, and L. Hellerman



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CONFIDENTIAL

Table of Contents

	<u>Page</u>
Introduction	2
Mathematical Preliminaries	3
Point Source Model	9
Linear Source Model	16
A. Velocity along the direction of the ship's motion	18
B. Velocity transverse to the direction of the ship's motion	24
Appendix I: Proof that the Computed and Observed Velocities are Equivalent	27
Appendix II: Closed Streamsurfaces of Point Source Distributions	29
Appendix III: Pressure Beneath a Moving Ship	32
Notes	35
Figures 15 thru 31	following page 36

CONFIDENTIAL

SECURITY INFORMATION

CONFIDENTIAL

- 2 -

THE FLOW FIELD SURROUNDING A MOVING VESSEL

Introduction

The calculation of a flow pattern is the first step in every theoretical explanation of pressure fields. Theories of pressure fields are numerous; in a sense, therefore, the substance of the present report is not new. Yet, in most of the work that has come to the author's attention, the emphasis is so preponderantly on matters other than flow, and discussion of the water motions is so sketchy, that more work with singular stress on flow seemed greatly needed. Furthermore, in one of the best reports on pressure fields ^{(a)(*)}, which uses the model of a line source and a line sink, transverse velocities of the water are omitted from consideration. Extension of the work to fill that gap is required.

Hence, the purpose of the present account is twofold: (1) It is to review the hydrodynamics of the problem, outlining the methods that have been found fruitful and worth knowing. In attempting this we shall not be bound by the usual inhibitions of the writers of technical reports and present all the material necessary for a coherent picture--even though most of it is available in the literature. For, while the other procedure saves the writer's time, it often wastes the readers'. (2) We desire to provide the numerical material concerning the theory of flow that seems to be still missing.

The disturbance created by a moving ship can be simulated by a variety of mathematical models, best perhaps by combinations of sources

(*) Small letters above the line refer to Notes, which are on page 35.

CONFIDENTIAL

SECURITY INFORMATION

CONFIDENTIAL

- 3 -

(b)
and sinks. Two such models are here investigated. The first consists of a point source and a point sink placed slightly below the surface of the water. The second model is composed of a line source of constant strength near the bow of the ship and a similar line sink near its stern. A point source discharges water in radially symmetric fashion and at a uniform rate about a point, a point sink absorbs it in similar fashion. Henceforth, where it will be unambiguous, we will say "sources" to mean "sources and sinks", etc. The line source may be thought of as a long pipe uniformly perforated on its lateral surface and exuding water at a uniform rate. A point source is said to have strength m if it emits a volume of water equal to $4\pi m \text{ ft}^3$ per second, i.e. if it causes a flow of $m \text{ ft}^3$ per second into every solid angle of unit magnitude. The strength of a line source, $f(\xi)$, is in general a function of the coordinate of a point in the source. We shall define it in such a way that:

$$\int f(\xi) d\xi = m, \quad (1)$$

the total strength. In this case, $4\pi m$ is again the total volume of water emitted per second. In our application $f(\xi)$ is constant and equal to m/ℓ provided ℓ is the length of the line source. (c)

To adjust our models to actual conditions, the device of images is employed. These will be so disposed that the flow is parallel to the surface and to the bottom of the water, as explained below.

Viscosity effects are not considered.

Mathematical Preliminaries.

Since the flow outside the sources and sinks is everywhere irrotational, velocities can be derived from a velocity potential ϕ with the

CONFIDENTIAL

SECURITY INFORMATION

CONFIDENTIAL

- 4 -

use of the relation.

$$\vec{v} = -\nabla \phi \quad (2)$$

For a point source

$$\phi = \frac{m}{r}, \quad (3)$$

For a line source

$$\phi = \int \frac{f(\xi) d\xi}{r} \quad (4)$$

r being the distance from the source element $d\xi$ at ξ to the point where ϕ is required.

In addition to the velocity potential we shall need the stream function, ψ . The ordinary stream function is useful in connection with problems of two-dimensional flow. It is defined as the rate of flow, from right to left, across the curve OP (see Fig. 1). In the absence of sources and sinks, this rate of flow is independent of the curve and depends only on the coordinates of the point P. Thus,

$$\psi = \psi(x, y).$$

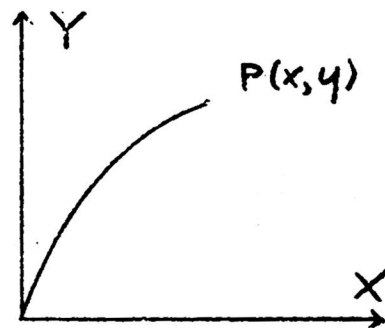


Figure 1

The water displaced by a moving ship does not flow in two dimensions, even approximately. A more reasonable first approxima-

tion would consider axisymmetric motion, such as results from translation through the water of a cigar-shaped body along its axis. A modified stream function, designed to describe axisymmetric motions, was introduced by Stokes. Let the axis of symmetry be X and use the set of cylindrical coordinates presented in Fig. 2. Select a point A on the axis, connect it by a curve to P, the point where the stream function is to be evaluated. Finally, construct a surface of

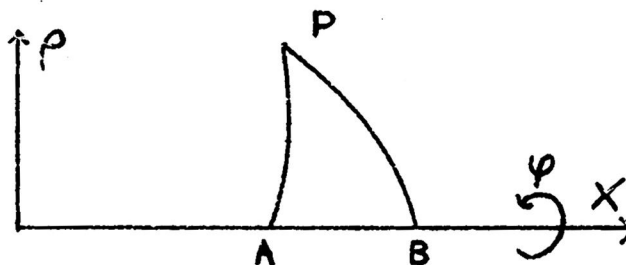


Figure 2

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CONFIDENTIAL

- 5 -

revolution by rotating that curve about $\underline{x}$. The flow to the left across this surface, in terms of Stokes' stream function ψ , is $2\pi\psi$. From this definition it is clear that ψ will in general depend on the coordinates of P and A, as well as on the curve connecting these points. In the absence of sources and sinks, however, the curve is irrelevant and $\psi = \psi(P, A)$. If, furthermore, there is no flow across the X-axis, the fluid crossing PA will also cross PB, and the position of the point A does not affect ψ provided A is on the axis. Both of these conditions are satisfied in the problem under study. Hence we conclude that ψ is a function of the coordinates of P alone. In the remainder of this report ψ denotes Stokes' stream function. It follows from the definition just given that $v_x = -\frac{1}{\rho} \frac{\partial \psi}{\partial \rho}$, $v_\rho = \frac{1}{\rho} \frac{\partial \psi}{\partial x}$.

Note that the physical dimensions of Stokes' stream function are $[L^3T^{-1}]$, whereas those of the ordinary ψ are $[L^2T^{-1}]$. Since there is no flow across a curve of constant ψ , the equation $\psi = \text{constant}$ defines a streamline. Herein lies the principal importance and the usefulness of the stream function.

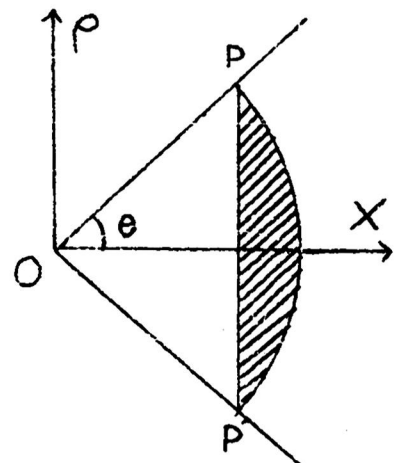
The fluid emitted by a point source at O through the spherical cap PP (of Fig. 3), from right to left, is $-m\Omega$.

Fig. 3

Ω being the solid angle subtended by the cap at O. But $\Omega = 2\pi(1 - \cos\theta)$. Hence, by definition, $\psi(P) = m(\cos\theta - 1)$.

If the source is located not at O but at some point x_0 on the X-axis,

$$\psi = m \left(\frac{x - x_0}{[(x - x_0)^2 + \rho^2]^{\frac{1}{2}}} - 1 \right).$$



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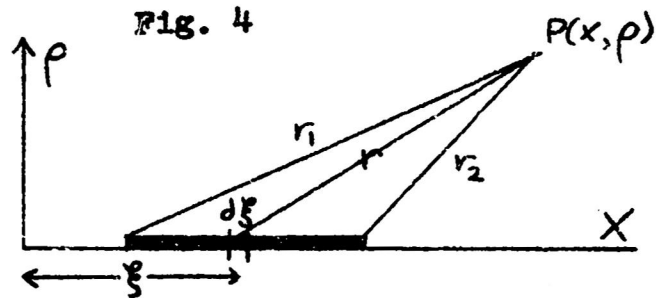
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CONFIDENTIAL

- 6 -

We shall now find the stream function generated by a line source extending along X. Consulting Fig. 4 we find that

$$\begin{aligned}\psi &= \int f(\xi) \frac{(x-\xi) d\xi}{[(x-\xi)^2 + \rho^2]^{\frac{3}{2}}} - m \\ &= \rho \int f(\xi) \frac{\cos \theta d\theta}{\sin^2 \theta} - m \\ &= \rho \int f(\xi) \frac{du}{u^2} - m\end{aligned}$$



if we take $u = \sin \theta = \rho/r$. In our special case, where $f(\xi) = m/l$,

$$\psi = \rho \frac{m}{l} \left[\frac{1}{u_1} - \frac{1}{u_2} \right] - m$$

Here u_1 corresponds to the left, u_2 to the right end point of the line source. If we denote the distances of these end points from P by r_1 and r_2 we have finally

$$\psi = \frac{m}{l} (r_1 - r_2) - m \quad (5)$$

There remains the simple task of determining the stream function for uniform flow of an infinite body of fluid. If its velocity to the right is V , then clearly the flow to the left through a circle of radius ρ perpendicular to X, which by definition is $2\pi\psi$, equals $-\pi\rho^2 V$. Therefore $\psi = -\frac{1}{2}\rho^2 V$.

The flow pattern produced by a single source is a radial outward current. A source and a sink together generate streamlines identical with the magnetic force lines surrounding a bar magnet with two poles. Neither model bears any relation to the flow pattern around a ship. But if a source and a sink are embedded in a stream of constant velocity V , the resulting flow simulates the situation in which water flows parallel to the keel of a stationary boat. There will be two points

CONFIDENTIAL

SECURITY INFORMATION

CONFIDENTIAL

- 7 -

where the velocity imparted to the water by the source sink system is reduced to zero by the velocity of the stream: These are stagnation points and correspond to the bow and the stern of the boat.

It is indicated, therefore, that a useful ψ for present purposes is a superposition of a stream function of some source-sink model with one representing uniform flow: $\psi = \psi_s - \frac{1}{2} \rho^2 V$. (6)

For ψ_s we have chosen different forms, and these will be discussed in due course.

The models described thus far simulate the motion of cigar or egg-shaped objects through an infinite medium in the direction of their axis of (highest) symmetry. They ignore the presence of boundaries, which destroy axial symmetry. One of the effects of surface and bottom is to enforce a flow which is wholly parallel to these surfaces in their immediate neighborhood. A formal method for guaranteeing this in our theory is available through the use of images, as follows:

Assume the water to have a depth h . Theory takes the water to be infinite in all directions, but singles out a hypothetical "slab" of thickness h , the horizontal surfaces of which correspond to the real surface and the real bottom. A source anywhere within the "slab" will cause a vertical flow across the surface, since the source knows no boundary conditions. But if another source of equal magnitude is placed at the image point of the first, beyond the surface, its vertical flow will cancel that of the primary source. Both of these sources, however, send water across the sea floor. To remedy this fault, both must be neutralized by a construction of images beyond the bottom surface.

CONFIDENTIAL

SECURITY INFORMATION

CONFIDENTIAL

- 8 -

These must then be reflected again in the top surface, and so on, until an infinite set of sources in a vertical line is produced, which serves to satisfy the boundary conditions with respect to vertical flow.

If the primary source lies a distance f below the surface and we take the origin within the surface, images must be constructed at

$$z = 2jh - f, 2jh + f, -(2ih - f), -(2ih + f); \quad \begin{matrix} i = 1, 2, 3 \dots \infty \\ j = 0, 1, 2 \dots \infty \end{matrix} \quad (7)$$

In particular, if the primary source is situated in the surface itself images occur at $-2jh$.

The presence of images makes the treatment of certain aspects of the problem difficult. This is especially true with respect to the details of flow near the hull of the vessel; for instance, to find the exact streamline which defines the boat contour in the presence of all images is a tedious task. Fortunately, it need not be undertaken, for the flow near the vessel is controlled predominantly by the primary source (or sources). To obtain it we ignore the images and treat the problem as though we had axial symmetry. But the computation of the water motions near the bottom, which are the result of the source and many images, must take account of the infinite array of images.

We now present the special features of the two models chosen for careful study.

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CONFIDENTIAL

- 9 -

Point Source Model

Our method for both models is to compute the velocity on the sea floor caused by a stationary source distribution lying in water which is stagnant at great distances. The ship speed enters the computations only thru the strengths of the sources which are proportional to V . This steady velocity is the same as that which would be observed instantaneously when the distribution moved by with the speed of the ship, provided of course that the vector distance between the distribution and the point of observation were the same in both cases. (d)

The present model approximates the ship by a point source near the bow and a point sink near the stern. The water surface and the sea floor are approximated by rigid horizontal planes. The boundary conditions - that there be no flux across these planes - are satisfied by constructing image sources and sinks as described on pages 7 and 8. Let the water surface be the XY plane; the sea floor, the plane $z = -h$, and let the source and sink be a distance f below the surface and a distance 2ℓ apart along the X axis. (e)

First we calculate v_x , caused only by the source and its images, on the sea floor along the keel line, $(y, z) = (0, -h)$, from $x=0$ to 500 feet. To do this we must sum the individual contributions from the source and its images. The general z -coordinates of the images have been given in Eqs. (7):

$$\begin{aligned}
 2jh - f &= 2(i-1)h - f \equiv C_1^{(i)} \\
 2jh + f &= 2(i-1)h + f \equiv C_2^{(i)} \\
 -(2ih - f) &\equiv C_3^{(i)} \\
 -(2ih + f) &\equiv C_4^{(i)}
 \end{aligned}
 \quad
 \begin{aligned}
 i &= 1, 2, 3 \dots \infty \\
 j &= 0, 1, 2 \dots \infty
 \end{aligned}
 \quad (8)$$

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CONFIDENTIAL

- 10 -

The four infinite sequences of images are represented in Fig. (5).

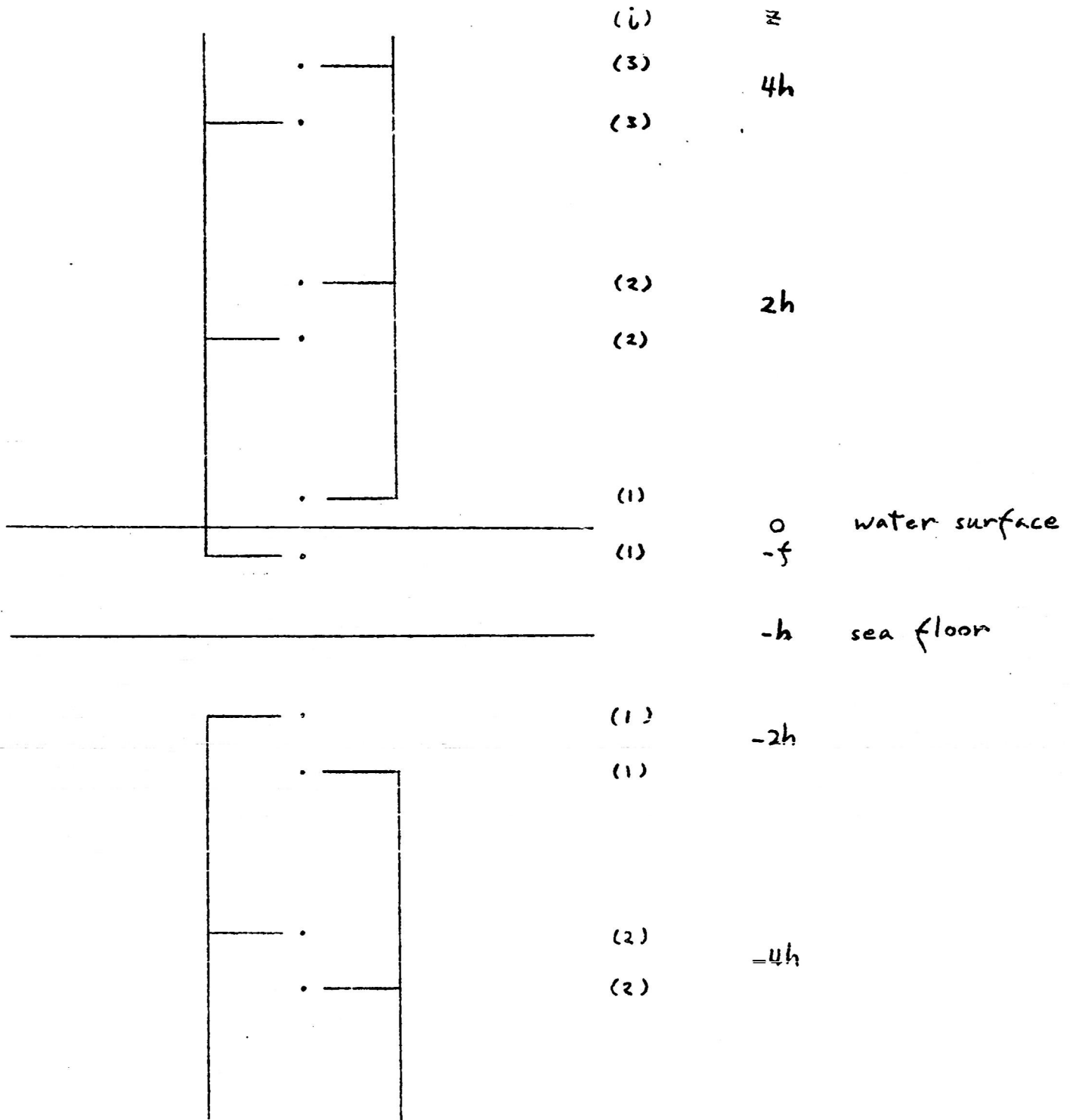


Figure 5

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From Eqs. (2), (3), and (7)

$$v_x = m(x-a) \sum_{i=1}^{\infty} \left[\frac{1}{(r_1^{(i)})^3} + \frac{1}{(r_2^{(i)})^3} + \frac{1}{(r_3^{(i)})^3} + \frac{1}{(r_4^{(i)})^3} \right] \quad (9)$$

where $(r_s^{(i)})^2 = (x-a)^2 + (y-b)^2 + (z-c_s^{(i)})^2$, where $(a, b, c_s^{(i)})$ are the coordinates of the source and its images; $s = 1, 2, 3, 4$.

$$\text{Also, } v_y = \frac{y-b}{x-a} v_x, \quad v_z = m \sum_{i=1}^{\infty} \left[\frac{z-c_1^{(i)}}{(r_1^{(i)})^3} + \dots \right].$$

We must now compute the sum.

Under certain conditions the sum of a series may be approximated (and its convergence simultaneously proved) by integrals. If we denote by $u^{(i)}$ the general term of the series and by $u(i)$ the function formed by allowing i to take all values, we have

$$\left. \begin{array}{l} u^{(1)} + \int_2^{\infty} u(i) di \\ \text{or} \\ u^{(1)} + u^{(2)} \end{array} \right\} < \sum_1^{\infty} u^{(i)} < u^{(1)} + u^{(2)} + \int_2^{\infty} u(i) di$$

whichever limits the sum more narrowly. The conditions on these inequalities are: (1) The integrals must exist; and (2) beyond $i = 2$, $u(i)$ must be monotonically decreasing. This can be seen from Fig. 6. The area represented by the integral $\int_2^{\infty} u(i) di$ on the left side of the inequality is indicated by shading thus: ; on the right by: .

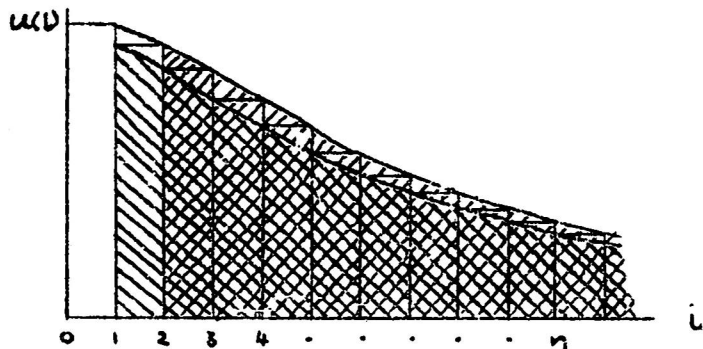


Figure 6

Then as an approximation

$$\sum_i u^{(i)} = u^{(1)} + \begin{cases} \frac{1}{2} u^{(2)} + \int_2^{\infty} u(i) di \\ \text{or} \\ u^{(2)} + \frac{1}{2} \int_2^{\infty} u(i) di \end{cases} \quad (10)$$

One of the terms in the summations constituting v_x (Eq. 9) is

$$v_{xs}^{(i)} = \frac{m(x-a)}{[\rho^2 + (z - c_s^{(i)})^2]^{\frac{3}{2}}}$$

where $\rho^2 \equiv (x-a)^2 + (y-b)^2$,

so the integral we need in Eqs. (10) is

$$\int \frac{dy}{(\rho^2 + y^2)^{\frac{3}{2}}} = \frac{1}{\rho^2(\rho^2 + y^2)^{\frac{1}{2}}} + \text{constant},$$

where $y \equiv z - c_s^{(i)}$. For sequences No. 1 and 2, $dy = -2h di$; for sequences Nos. 3 and 4, $dy = 2h di$. For 1 and 2 the limits become $-\infty$ and $z - c_s^{(2)}$. For 3 and 4 the limits become ∞ and $z - c_s^{(2)}$.

Since the point of evaluation must lie within the water all the conditions on the integrals in Eqs. (10) are satisfied; this can be seen most readily from Fig. 5. As ρ increases the distant images have a proportionately greater effect on the velocity, so that the series converge less rapidly.

Then for:

$$\begin{aligned}
 s=1,2 \quad m(x-a) \int_2^{\infty} \frac{di}{[\rho^2 + (z - c_s^{(i)})^2]^{\frac{3}{2}}} &= \frac{m(x-a)}{-2h\rho^2} \left[-1 - \frac{z - c_s^{(i)}}{r_s^{(i)}} \right] \\
 s=3,4 \quad d, \pi o &= \frac{m(x-a)}{2h\rho^2} \left[+1 - \frac{z - c_s^{(i)}}{r_s^{(i)}} \right] \quad (11)
 \end{aligned}$$

To compute $\tilde{V}_x$ using Eqs. (11) we must select values of $\underline{h}$, $\underline{m}$, and $\underline{f}$. To be able to compare our result with some of those in NOL Report No. 504, we have taken $h = 54$ ft., the shallowest depth treated. According to Lunde, op. cit., p. 38, $m = \pm \frac{AV}{4\pi}$, where A is the transverse midsection area and V is the ship speed; and $\underline{f}$ is the depth of the centroid of A . The average specifications of the four ships for which the experimental pressure signature is given in Reference (2) (plates 82, 87, 88, 93) are:

beam	B	60 feet
draft	D	24.5 "
length	L	441-7/12 feet
tonnage		6838 tons
displacement	Δ	214,000 ft ³
speed	V	10.6 knots

If the transverse midsection were rectangular, A would equal BD ; and if it were semi-elliptical, A would be $(\pi/4) BD$. Its shape is probably between these two: hence we take $A = (0.9) BD$. Then

$A = (0.9) (60) (24.5) = 1320$ ft². Take $m = 2000$ ft³/sec. For the ships under consideration this strength corresponds to a speed of 11.5 knots. The velocity induced by a ship of different cross-section or moving with a different speed, can be obtained from our results by one

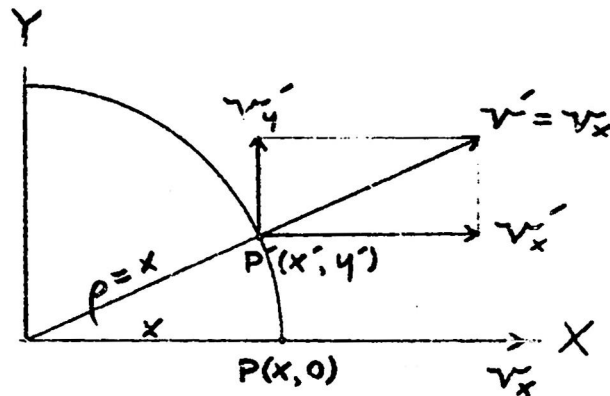
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- 14 -

multiplication. Again, if the midsection is rectangular $f = 12.25$ feet. Since it is somewhat elliptical, we take $f = 10$ feet.

v_x and v_y at any other point on the sea floor may be derived from v_x along the keel-line by noting from Fig. 7 that for points on a circle of constant ρ ,

$$v_x' = v_x \frac{x'}{x}, \quad v_y' = v_x \frac{y'}{x}. \quad (12)$$



With the use of these relations, v_x and v_y were obtained as functions of x for $y = 0, 5, 15, 25, 40, 50, 75, 100$ feet. All these data are presented in Figs. 15 and 16.

The quantity ℓ was determined by requiring that the points on the sea floor beneath the keel at which $v_x = 0$ be the same distance apart for both the point and the line model. This gave $\ell = 177$ feet.

Having found ℓ and knowing by inspection the signs of the velocities contributed by the source and the sink, the total v_x and v_y along each line $y = \text{constant}$ was obtained from the graphs by appropriate additions and subtractions. The contours of constant v_x and v_y on the sea floor beneath the ship are shown in Figs. 18 and 19.

CONFIDENTIAL

SECURITY INFORMATION

CONFIDENTIAL

- 15 -

To get a rough estimate of the size of the closed stream-surface the dimensions of a simple source-sink Rankine solid were computed for $m = 4000 \text{ ft}^3/\text{sec}$ (to include the first surface images). The appropriate theory is given in Milne-Thomson, pp 411-412. From this calculation: $B = 57$, $D = 28.5$, $L = 382$ feet. Since the solid is blunt and the ship sharp, making the equivalent solid shorter, comparison with the average dimensions of the real ships treated (p 5) shows that the m and l we have used are substantially correct.

The streamlines and stagnation points in the XY plane near the sources are shown in Fig. 20, and the longitudinal and transverse sections of the closed streamsurfaces of our source distribution are sketched in Fig. 21. Note that the solid is divided into two separate parts, symmetrical about the XY plane. The length of these solids is about 374 feet; the beam, less than 57 feet; the draft, greater than 28.5 feet. Thus, the actual solid does not approximate the dimensions or the shape of a ship as well as the simple source-sink solid. If one were able to fit a streamsurface exactly to a ship hull the velocities derived should be exact within the limitations of the other approximations made. But since the values we find for the velocities do not differ appreciably from those obtained with the use of a line source model whose streamsurface fit a ship unusually well, we may conclude that this "goodness-of-fit" criterion, based upon an examination of the dividing streamline, does not provide on a priori test of the validity of velocity calculations.

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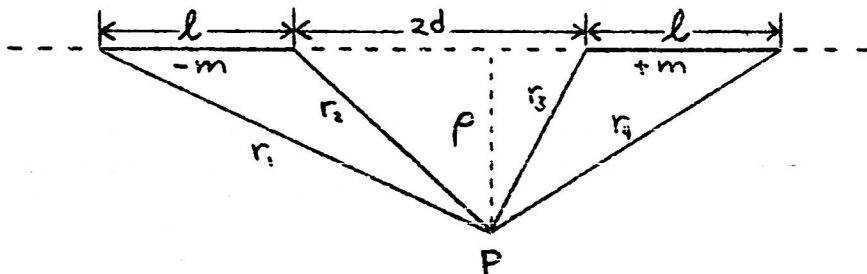
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Since the results of the present section, which are based upon the point source model, are not very different from, and indeed tend to confirm, the results of the next section, no separate discussion of the present data will be given. It is interesting to note, however, that the maxima and minima are more closely spaced for the point model, and that the velocity gradients are somewhat steeper near these maxima and minima than in the line model. The present model was used to obtain evidence about the trustworthiness of the different ways of simulating ships, and it is gratifying to note that computed velocities are so remarkably independent of the choice of model.

Linear Source Model

A line sink and a line source of equal strength $\underline{m}$ and of equal length $\underline{l}$ are placed along the X-axis, as shown in Fig. 8.

Figure 8



The X-axis is in the surface, the ρ -axis extends at right angles to it. Our analysis here follows, and in many respects repeats the work of NOL Report No. 504. The stream function for this system is given by Eq. (5) which must now be applied to each of the two line sources, the first with negative $\underline{m}$. Hence,

$$\psi_s = \frac{m}{l} (r_2 - r_1) + \frac{m}{l} (r_3 - r_4)$$

CONFIDENTIAL

- 17 -

To simulate fluid motion past a boat, we superimpose the constant velocity - $\underline{V}$. Eq. (5) then leads to the expression for the stream lines:

$$\frac{m}{l} (r_2 - r_1 + r_3 - r_4) + \frac{1}{2} \rho^2 V = \text{constant} \quad (15)$$

In this approximation images have been ignored and boundary conditions are not satisfied. It is fairly certain, however, that the lower half of the axially symmetric streamline pattern represented by Eq. (15) will portray reasonably well the flow in the neighborhood of the primary source even though the flow is perturbed by the images.

Eq. (15) may, therefore, be used to determine the streamline which contains the two stagnation points. Often it is called the "dividing streamline" because of its branching at the stagnation points. By proper choice of parameters, this dividing streamline can be made to fit a boat very well (see Fig. 22), so we may expect our velocity results to be satisfactory.

Far ahead of, and far behind the boat, this streamline must clearly lie on the X-axis. That condition is met if in Eq. (15) we put the constant equal to zero, for the left side is identically zero at points beyond the source-sink system. Hence the dividing streamline must be given everywhere by the condition

$$\frac{m}{l} (r_2 - r_1 + r_3 - r_4) + \frac{1}{2} \rho^2 V = 0 \quad (16)$$

To represent a real boat, we adopt the recommendation of NOL Report #504 and take $l = 135 \text{ ft.}$, $ld = 170 \text{ ft.}$, $\frac{m}{V} = 315 \text{ ft.}^2$ Eq. (16) is plotted for these values in Fig. 22. The fit of the present model is excellent. It corresponds to a vessel 470 ft. long. Perhaps it is also of interest to recognize that our choice of constants

CONFIDENTIAL

SECURITY INFORMATION

CONFIDENTIAL

- 18 -

implies a source strength m of 5360 ft³/sec for a vessel moving with a speed of 10 knots, that is, an exchange of about 67,500 cubic feet of water per second between source and sink. We now proceed to the calculation of flow velocities.

A. Velocity along the direction of the ship's motion.

Since the images remove the axial symmetry of the problem, the advantage of cylindrical coordinates disappears and it now becomes preferable to use Cartesian coordinates. The Z-axis is taken vertical, and x is in the direction of V within the water surface. The present task is to find v_x .

According to Eq. (4)

$$\phi_s = \int \frac{f(\xi) d\xi}{r}, \quad r = \sqrt{(x-\xi)^2 + y^2 + z^2}$$

and our model is specified by

$$f(\xi) = \begin{cases} -\frac{m}{2} & \text{if } x_1 \leq \xi \leq x_2 \\ \frac{m}{2} & \text{if } x_3 \leq \xi \leq x_4 \\ 0 & \text{elsewhere} \end{cases}$$

$$v_x = -\frac{\partial \phi}{\partial x} = - \int f(\xi) d\xi \frac{\partial}{\partial x} \left(\frac{1}{r} \right)$$

But $\frac{\partial}{\partial x} \left(\frac{1}{r} \right) = -\frac{\partial}{\partial \xi} \left(\frac{1}{r} \right)$, and a partial integration leads to

$$v_x = \frac{f(\xi)}{r} \Big|_{\text{limits}} - \int \frac{f'(\xi) d\xi}{r}$$

In the present case $f(\xi)$ vanishes at the limits of integration.

Furthermore, $f'(\xi) = \frac{m}{2} [-\delta(\xi, x_1) + \delta(\xi, x_2) + \delta(\xi, x_3) - \delta(\xi, x_4)]$

δ being Dirac's singular function. Consequently

$$v_x(P) = \frac{m}{2} \left(\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_4} \right),$$

CONFIDENTIAL

SECURITY INFORMATION

CONFIDENTIAL

- 19 -

where the r 's are the (positive) distances of the point P from the four ends of the two line sources. Interestingly, v_x due to our line-source distribution is identical with the potential resulting from four point sources of magnitude $\frac{m}{l}, -\frac{m}{l}, -\frac{m}{l}, \frac{m}{l}$, situated at the four end points of our lines.

At this stage all images are introduced, and v_x will be computed for a point P on the sea floor. The situation is depicted in Fig. (9). It is necessary to consider the effects of four infinite vertical sequences of point sources, occurring in the figure below the numerals 1, 2, 3 and 4. Thus ($r_j^{(1)}$ is defined in the figure)

$$v_x = \frac{m}{l} \sum_i \left(\frac{1}{r_1^{(i)}} - \frac{1}{r_2^{(i)}} - \frac{1}{r_3^{(i)}} + \frac{1}{r_4^{(i)}} \right) \quad (17)$$

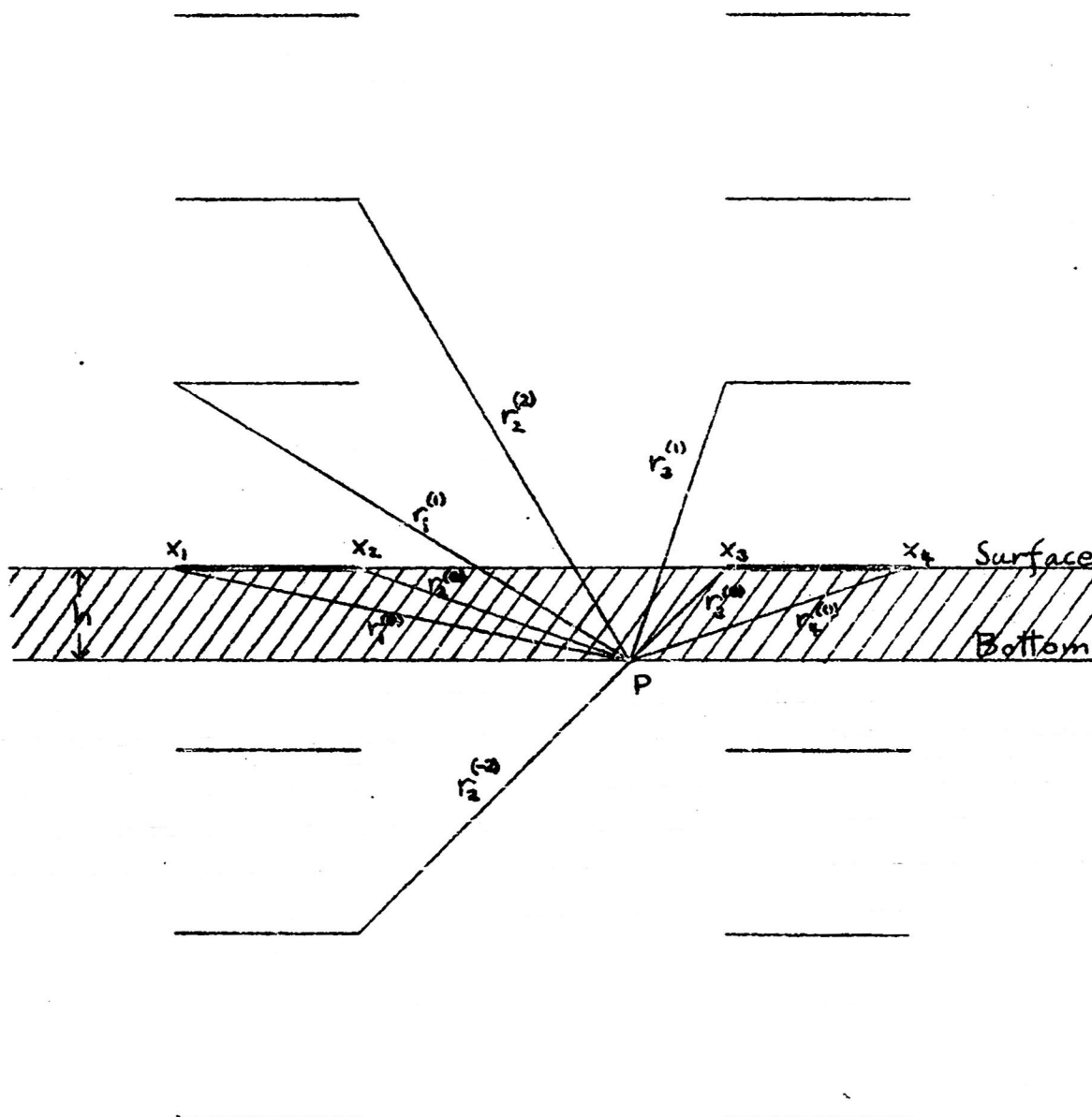
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- 20 -

Figure 9



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CONFIDENTIAL

- 21 -

These sums converge too slowly for direct evaluation. However, each can be converted into another rapidly converging series by a device of Madelung^(g) whose interest was in an electrical analog of the present problem. He has shown that

$$\sum_{j=1}^{\infty} \frac{1}{r_j} = \frac{1}{h} \left[\ln \frac{4h}{d_j} + i\pi \sum_{n=1}^{\infty} (-1)^n H_0^{(1)} \left(\frac{n\pi i d_j}{h} \right) \right] = \frac{1}{h} L \left(\frac{d_j}{h} \right) \quad (18)$$

Here d is the horizontal distance of P from the vertical row of points under consideration: $d_j^2 = (x - x_j)^2 + y^2$, $j = 1, 2, 3, 4$. H is the Hankel function. The result as stated is valid for a point P whose vertical position is midway between two sources, and not in the line of sources. Constant terms which cancel when Eq. (17) is formed have been omitted. In a line of sources, i.e. when $d_j = 0$, L takes on the limiting value $L(0) = 1.963$.

When the point P lies in the surface, we are also led to Eq. (17), but with an altered meaning of the $r_j^{(1)}$. The summations then yield a result similar to Eq. (18); however, all terms in $\sum_n$ are now positive. This modified function will be called L_0 ; it differs from L only for small values of the argument. Both L and L_0 have been computed and graphs are shown in Fig. 23. The function L_0 becomes ∞ for zero argument.

In conclusion, then,

$$v_x = \frac{m}{h\ell} \left[L \left(\frac{d_1}{h} \right) - L \left(\frac{d_2}{h} \right) - L \left(\frac{d_3}{h} \right) + L \left(\frac{d_4}{h} \right) \right] \quad (19)$$

As it stands, this formula is true for a point on the bottom; in the surface the same formula holds but L must be replaced by L_0 everywhere.

With the data adopted in the preceding section, $m/h\ell = 2.33 \text{ ft V/h}$.

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CONFIDENTIAL

- 22 -

The ratio $\frac{v_x}{V}$ is therefore directly given by Eq. (19). This ratio was computed in NOL Report #504 and plotted in the form of contour graphs, which we have checked. As supplementary material we present here some calculations of the variation of $\frac{v_x}{V}$ in plane vertical sections parallel to X.

Fig. 24 shows this ratio in the keel section of the boat, where it takes its largest values. The line source and the schematic form of the boat are sketched in. At the bottom of a 40 ft. channel the vessel is preceded by a positive current, i.e. a flow in the direction of the vessel, which is 2% of V at a distance of 200 ft. in front of the bow. As the bows passes, this rises to 8% and then falls rapidly to negative values. Under the left end of the line source, when about 1/4 of the ship's length has passed, $\frac{v_x}{V}$ reaches its maximum negative value of nearly 0.14, corresponding to 2.3 ft/sec for a ship moving at 10 knots. This flow diminishes slightly but continues at more than the ratio 0.1 until about 3/4 of the ship's length has passed, when the flow returns to its positive sign, the water filling in behind the vessel. The pattern drawn in Fig. 24 is symmetric; it repeats itself at negative values of x. The rise of $\frac{v_x}{V}$ near x = 0 appearing on this diagram occurs also in the point source model treated in previous section. It is absent at greater depths as is shown in graph 25 which gives $\frac{v_x}{V}$ at 60 ft. Here the maximum value of the ratio is .065; this indicates that, below the keel, flow velocities decrease more rapidly than $\frac{1}{h}$ but not quite as rapidly as $1/h^2$.

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- 23 -

Figures 24 and 25 also show $\frac{v_x}{V}$ on the bottom of 40 ft. and 60 ft. channels, a distance of 50 ft away from the keel section. The beam of the ship model used is about 70 ft, hence the flow here considered is no longer under the ship. It is evident that the lateral diminution of v_x is not very rapid, particularly at the greater channel depth. This conclusion can also be read from the contours of NOL Report #504. In interpreting the last two graphs it should be remembered that v_x has the same value for $+y$ and $-y$.

The behavior of v_x in the surface is not particularly interesting, although it illustrates some features of our model. In Fig. 26 a graph of $\frac{v_x}{V}$ in the surface is given. The ratio is infinite at the exact end points of the line source. Beyond the exterior end it descends very fast to small values, crossing the value 1 at a point just outside the line source (about 5 ft. outside in this instance). This point is the bow of the boat, a stagnation point where the water moves with the velocity of the ship. The part of Fig. 26 to the left of the stagnation point has no physical significance since it is excluded by the body of the vessel.

At large distances ahead of the boat the velocity is given by a simple formula. Let the distance, measured from the bow, be f and suppose that $f \gg h$, and $f \gg l$. One may then approximate

$$\begin{aligned} \text{and } \frac{v_z}{V} \text{ (surface)} &\approx \frac{L \cdot \left(\frac{f}{h}\right) \approx \ln \frac{4h}{f}}{2.33 \text{ ft.}} \left[\ln \left(1 + \frac{l}{f}\right) - \ln \left(1 + \frac{l}{f+2d+l}\right) \right] \\ &\approx \frac{2.33 \text{ ft.}}{h} \left(\frac{l}{f} - \frac{l}{f+2d+l} \right) \end{aligned}$$

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CONFIDENTIAL

- 24 -

The dependence on $1/h$ is interesting.

B. Velocity transverse to the direction of the ship's motion.

We have

$$v_y = -\frac{\partial \phi}{\partial y} = -\int f(\xi) d\xi \frac{\partial}{\partial y} \left(\frac{1}{r} \right) = \gamma \int \frac{f(\xi) d\xi}{(\xi^2 - 2x\xi + R^2)^{3/2}}$$

for a single source of strength density $f(\xi)$. Here $R^2 = x^2 + y^2 + z^2$.

Thus our primary source yields

$$\begin{aligned} v_y &= -\gamma \frac{m}{\ell} \left(\int_{x_1}^{x_2} - \int_{x_3}^{x_4} \right) \frac{d\xi}{(\xi^2 - 2x\xi + R^2)^{3/2}} \\ &= \frac{\gamma}{y^2 + z^2} \frac{m}{\ell} \left(\frac{x_1 - x}{r_1} - \frac{x_2 - x}{r_2} - \frac{x_3 - x}{r_3} + \frac{x_4 - x}{r_4} \right) \end{aligned} \quad (19)$$

by an elementary integration. Here again, the X-axis is in the surface and extends in the direction of the boat; z is vertical; x_2 is the coordinate of an end point of the primary source system; r_2 is the distance from one of these end points to (x, y, z) , where v_y is to be evaluated.

We now introduce the images depicted in Fig. 9. Each makes a contribution like Eq. (19) but with its own $z^{(i)}$ and $r^{(i)}$. Hence we find

$$v_y = \frac{m}{\ell} \gamma \left[\sum_i \frac{1}{y^2 + (z^{(i)})^2} \frac{x_i - x}{r_i^{(i)}} - \dots \right] \quad (20)$$

where only the summation for end points has been written, and $z_i = \pm (2n+1)h$

$$r_i^{(i)} = [(x-x_i)^2 + y^2 + (2n+1)^2 h^2]^{1/2}, \quad n=0,1,2,\dots$$

The summations encountered in Eq. (20) converge rapidly and can be

CONFIDENTIAL

SECURITY INFORMATION

CONFIDENTIAL

- 25 -

computed directly.

We define the function

$$T(a, b) = a \sum_{n=0}^{\infty} [a^2 + b^2 + (2n+1)^2]^{-1/2} [b^2 + (2n+1)^2]^{-1} \quad (21)$$

T is a pure number. In terms of it

$$v_y = \frac{2m}{\rho} \frac{y}{h^2} \left[T\left(\frac{x_1-x}{h}, \frac{y}{h}\right) - T\left(\frac{x_2-x}{h}, \frac{y}{h}\right) - T\left(\frac{x_3-x}{h}, \frac{y}{h}\right) + T\left(\frac{x_4-x}{h}, \frac{y}{h}\right) \right] \quad (22)$$

Evaluation of T proceeds by direct summation up to $n = t$, such that

$$(t+1)^2 \gg \frac{a^2 + b^2}{4} \quad (23)$$

The remainder is computed by converting the sum into an integral:

$$T_s = a \int_{t+1/2}^{\infty} [b^2 + (2n+1)^2]^{-1} [a^2 + b^2 + (2n+1)^2]^{-1/2} dn$$

With the use of elementary formulas this becomes

$$T_s = \frac{1}{2b} \left\{ \tan^{-1} \frac{a}{b} - \tan^{-1} \left[\frac{a}{b} \frac{2(t+1)}{\{a^2 + b^2 + 4(t+1)^2\}^{1/2}} \right] \right\}$$

and under condition (23)

$$T_s = \frac{1}{16(t+1)^2}$$

The foregoing considerations permit the computation of T without much labor. We present the results in Figs. 27 and 28. Values for intermediate b 's are easily found by interpolation between the three curves plotted.

Eq. (22) shows that $v_y = 0$ below the keel of the boat ($y = 0$). In every transverse section it reaches a maximum (on the bottom) some distance from the ship and then goes gradually to zero. Our numerical work shows that this maximum comes at about $y = h$. Also, v_y differs in sign on the two sides of the keel line, as expected. The quantity in the

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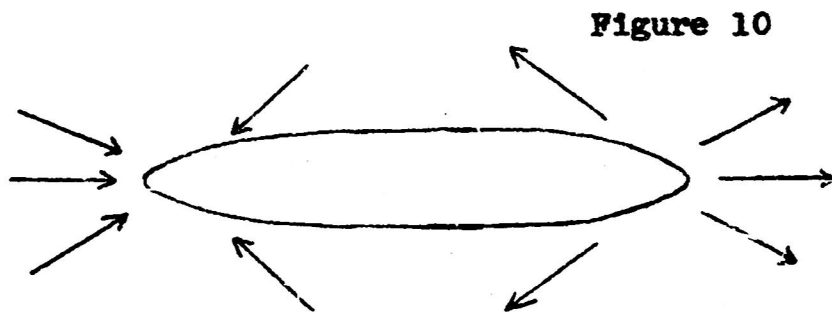
SECURITY INFORMATION

CONFIDENTIAL

- 26 -

bracket of Eq. (22) changes sign when x is reflected in the point midway between the sources. Hence the transverse flow pattern under the aft half of the boat is opposite to that under the fore half.

Details are given in Figs. 29 and 30. In a channel of depth 40 ft, the transverse velocity reaches a maximum of about $0.1 V$. At 60 ft. this drops to about $0.05 V$. The maximum always appears under the center of the source line. For $y = 2h$ the flow is less than its maximum by 20 to 30%. The total flow is, of course, in accord with the source-sink model here employed and is represented schematically in Fig. 10.



To complete the picture we have plotted in Fig. 31 curves of constant $\frac{v_y}{V}$ on the bottom of a 40 ft. channel.

Measurements of flow velocities beneath moving vessels have not come to the authors' attention. The theory can, therefore, be tested only in an indirect way by computing pressures and comparing them with observed data. This is done in Appendix III.

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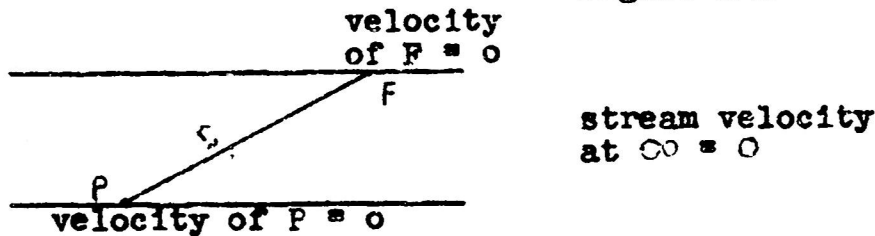
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APPENDIX I

Proof that the Computed and Observed Velocities are Equivalent.

The proof of the theorem stated on page 2 is illustrated step by step below, from the hypothetical situation with which the theory deals to the actual situation. F is a point fixed relative to the source distribution and P, the point at which the computations (or measurements) are made. First, the fictitious case:

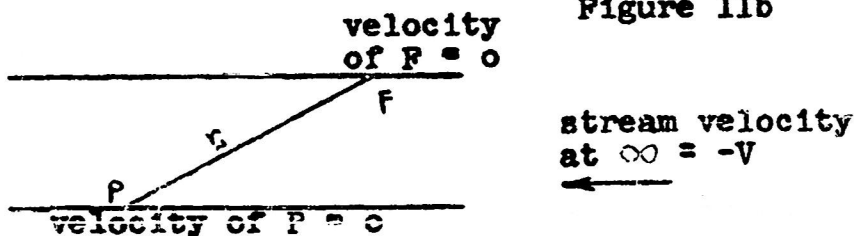
Figure 11a



Call the velocity of water at P v_x . This is the velocity we compute.

Next consider the source-sink distribution in a stream of velocity $-V$.

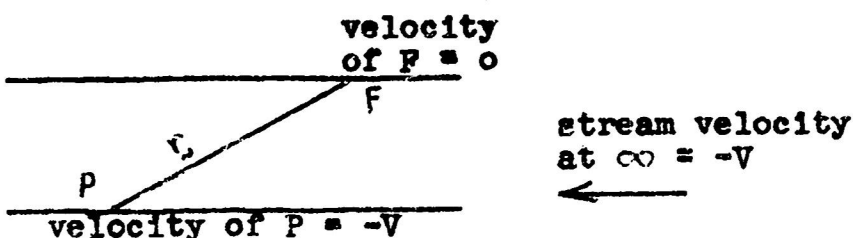
Figure 11b



The water velocity at P is now $v_x - V$.

If the point P (that is, the sea floor) also moves with the velocity $-V$:

Figure 11c



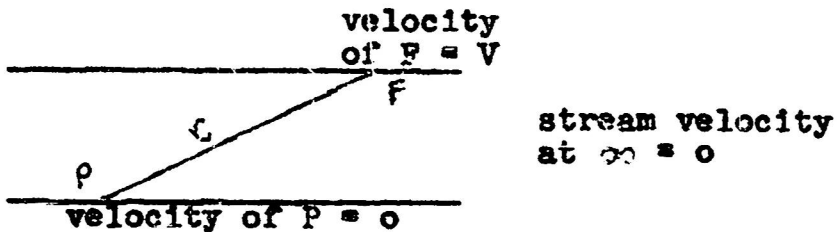
The water velocity at P (measured relative to P) is $(v_x - V) - (-V) = v_x$.

CONFIDENTIAL

- 28 -

This situation is equivalent to the actual situation; for if we refer it to coordinate axes moving with a velocity $-V$ we have:

Figure 11d



The velocity of water at P is still v_x , the computed velocity. Thus the theorem is proved. It should be emphasized that the velocity computed is steady, but that the velocity observed is transient. These two velocities can be compared according to the above theorem only when the position of the distribution relative to the point P is the same as the position of the ship relative to P. Since the ship is moving this occurs only instantaneously.

CONFIDENTIAL

SECURITY INFORMATION

CONFIDENTIAL

- 29 -

Appendix II

Closed Streamsurfaces of Point Source Distributions

Rankine's method for finding the limiting streamlines is not applicable to a problem like that treated here; for our use of a double source and a double sink, one set above and one below the water surface, destroys axial symmetry, and a Stokes stream function does not exist. Nor is it proper to neglect one set, since both are equally far from the water surface.

Nevertheless, it is desirable to know the enclosing streamlines, and we have computed them. The method used is explained in this Appendix.

The first aim is to find the curves on which $v_x = 0$ and $v_z = 0$ in the plane of the sources. The intersections of these curves are either sources or stagnation points. At a stagnation point streamlines meet at right angles ⁽¹⁾. For a stagnation point in front of a source, one of the two incoming streamlines is from the source, the other from the stream. The two outgoing streamlines form the longitudinal midsection of the Rankine solid (see Figs. 14, 20 and 21). Also, streamlines must cross the $v_x = 0$ curves vertically and the $v_z = 0$ curves horizontally. Thus, once the v_x and $v_z = 0$ contours are obtained the streamlines may be sketched with considerable accuracy.

The equation which the v_z contours satisfy is in general homogeneous, so it is easier to solve than the equation for $v_x = 0$, which is not. For our problem, the $v_x = 0$ curve may be obtained by finding the locus of points for which

$$V_z^{(1)} + V_z^{(2)} = \frac{z+f}{(r_1^{(1)})^3} + \frac{z-f}{(r_2^{(1)})^3} = 0$$

CONFIDENTIAL

SECURITY INFORMATION

CONFIDENTIAL

- 30 -

where $(r_1^{(n)})^2 = x^2 + (z + f)^2$, $(r_2^{(n)})^2 = x^2 + (z - f)^2$.

We thus ignore the sinks, but their effect is negligible near the sources.

For an arbitrary planar distribution of n sources, the contours of v_x satisfy exactly the equation:

$$\sum_{i=1}^n \frac{m_i x_i}{r_i^3} - V = -v_x \quad (24)$$

The labor of solving this equation can be greatly reduced by considering instead the n equations of the form

$$\frac{m_i x_i}{r_i^3} - \frac{m_i}{M} V = \left(\frac{m_i}{M}\right) v_x \equiv v_{xi}, \quad i = 1, 2, \dots, n, \quad (25)$$

where, $M = \sum_{i=1}^n m_i$. The sum of these eqs. is Eq. (24), so that if we solve each of them for v_{xi} at the same point P and add we have the total v_x . But since the equations are all of the same form we need solve only one of them, say the j th, for the contours of v_{xj} . Then, using the fact that for points P_k and P_j with the same coordinates relative to sources m_k and m_j respectively, $v_{xk} = \frac{m_k}{m_j} v_{xj}$ (see Fig. 12a), we can evaluate v_x at a particular point by a simple process of multiplication and addition (see Fig. 12b).

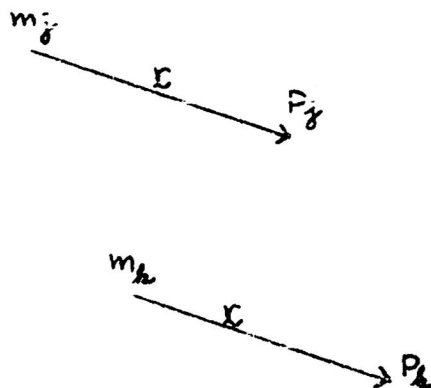


Figure 12a

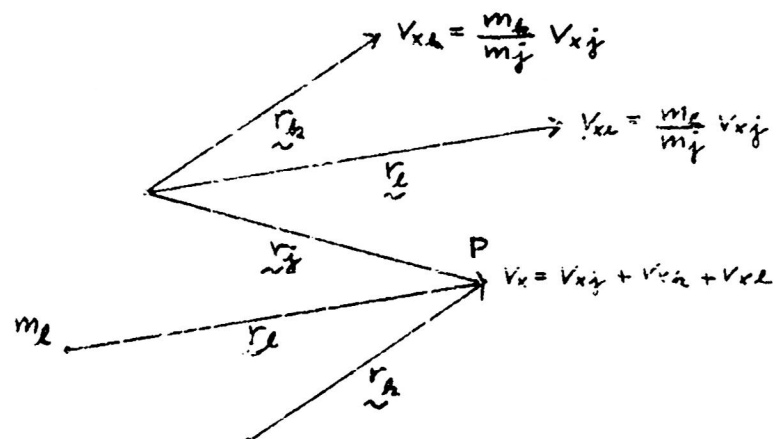


Figure 12b

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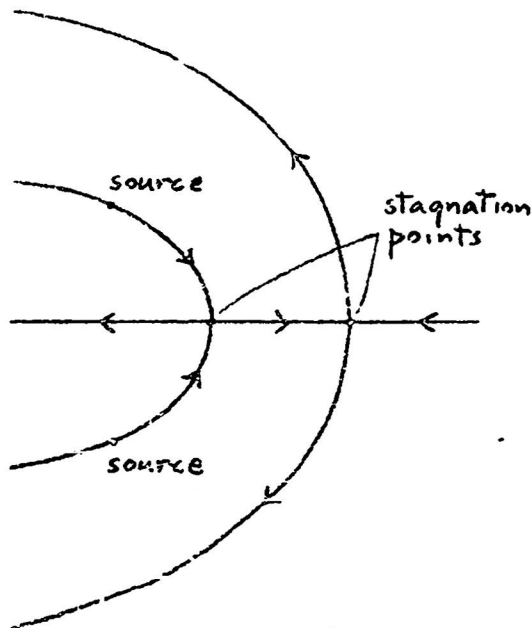
- 31 -

It is always helpful to know what to expect. Stagnation points on a plane of symmetry can be found more easily. In our case, the equation for stagnation points on the X-axis is:

$$\frac{2mx}{r^3} - V = 0,$$

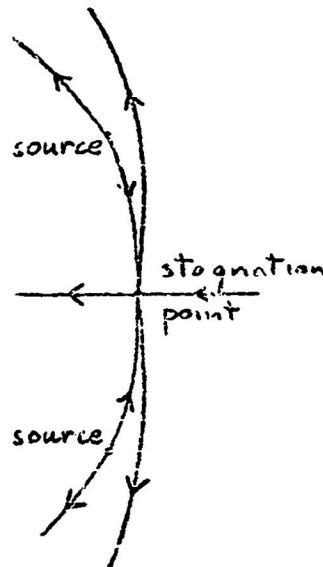
and it was found that there are 2, 1, or 0 stagnation points for f less than, equal to, or greater than some critical f . This critical f is $0.6204 \left(\frac{2m}{V} \right)^{\frac{1}{2}}$. For $\frac{m}{V} = 100 \text{ ft}^2$ it is approximately 8.8 feet.

The streamlines for the different cases are illustrated:



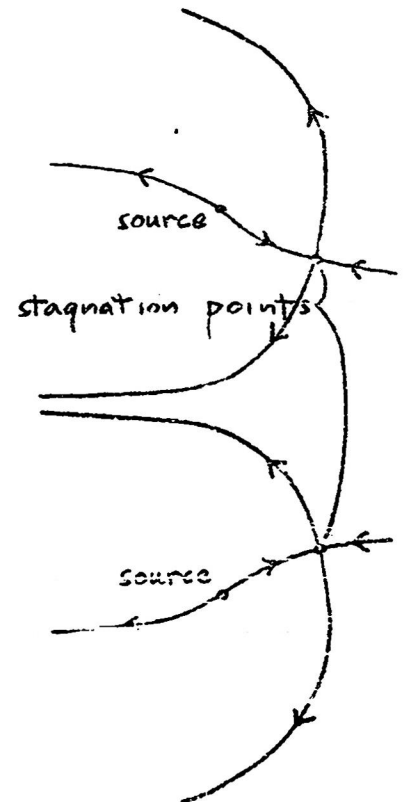
$f < \text{critical } f$

Figure 13a



$f = \text{critical } f$

Figure 13b



$f > \text{critical } f$

Figure 13c

Since $f = 10$ in our problem, this tells us our streamlines are like those in Fig. 13c.

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SECURITY INFORMATION

Appendix III

Pressure Beneath a Moving Ship

Thoughtless application of Bernoulli's equation leads to an instructive paradox, which we will discuss briefly. The pressure will be calculated by two methods, with apparently different results: A. Consider the actual situation, depicted in Fig. 14. The ship moves with a velocity V , and the water far from the ship is stationary. At the points a and b the water velocity is V .



Figure 14

Bernoulli's formula
(with the obvious ρgh
omitted) asserts that

$$p_{\infty} + \frac{1}{2} \rho v_{\infty}^2 = p + \frac{1}{2} \rho v^2$$

Since

$$v_{\infty} = 0, \quad p = p_0 = \frac{1}{2} \rho v^2 \quad (26)$$

B. Consider now the fictitious situation in which the ship is at rest and the water moves past it with velocity $-V$. The points a and b are now stagnation points, and the water velocity at ∞ is $-V$. Indeed, if we denote velocities in this situation by v' , in the former by v , we have

$$v'_x = v_x - V \quad v'_y = v_y$$

Bernoulli's formula is

$$p_{\infty} + \frac{1}{2} \rho V^2 = p + \frac{1}{2} \rho [(v_x - V)^2 + v_y^2]$$

CONFIDENTIAL

- 33 -

whence

$$p = p_{\infty} + \rho V v_x - \frac{1}{2} \rho v^2 \quad (27)$$

The discrepancy between (26) and (27) is apparent.

To resolve it we note that the use of Bernoulli's formula in A is improper, since the pressure at a stationary point under a moving ship changes in time: hence the situation is not a steady state. B does represent a steady condition, but not the actual state of affairs. Nevertheless, it leads to the correct pressure, as will now be shown.

For a non-steady state, Bernoulli's equation must be replaced by Euler's which, for irrotational flow of an incompressible fluid, takes the form

$$\frac{p}{\rho} + \frac{1}{2} v^2 - \dot{\phi} = \text{const.}$$

where ϕ is the velocity potential. Now

$$\phi = \phi(u, y, z) \text{ where } u = x - Vt;$$

hence

$$\dot{\phi} = \frac{\partial \phi}{\partial u} \dot{u} = -v_x (-V)$$

On inserting this in Euler's equation, we have

$$(p + \frac{1}{2} \rho v^2 - v_x V)_{\infty} = p + \frac{1}{2} \rho v^2 - \rho v_x V$$

or, since $v(\infty) = 0$,

$$p = p_{\infty} + \rho v_x V - \frac{1}{2} \rho v^2 \quad (28)$$

which is Eq. 27.

The term $-\frac{1}{2} \rho v^2$ makes a relatively small contribution to $p - p_{\infty}$. Hence a knowledge of v_x is generally sufficient to understand the pressure field of a moving vessel.

CONFIDENTIAL

SECURITY INFORMATION

CONFIDENTIAL

- 34 -

In the absence of significant flow data, we can test our theory only by computing pressures via formula (24), which reads in practical units

$$\Delta p \text{ (in inches of water)} = 1.06 V^2 \left(\frac{v_x}{V} - \frac{1}{2} \frac{v_x^2 + v_y^2}{V^2} \right) \quad (29)$$

The ship's velocity (relative to the water at ∞), V , is here expressed in knots, and Δp is the increase in pressure at the bottom over its normal value, ρgh . Except for the correction term proportional to v^2 , Δp has the sign of v_x . This agrees with observations.

For numerical comparison we choose some data on pressure distribution contained in NOL Report No. 504. S.S. Robin Tuxford, having a length of 480 ft. and a beam of 66 ft, is closest to the dimensions of the theoretical model among the four boats for which measurements are listed. It developed a maximum pressure reduction of 17 inches in the keel section under the fore part of the boat when proceeding at 10.2 knots, and the depth of the channel was 40 ft. Under these conditions Fig. 24 shows that $\frac{v_x}{V} = 0.135$, $v_y = 0$. Hence the linear term in Eq. (25) contributes $\Delta p = -14.9$ in, the other -1.0 in. The total of 16 in. compares well with the measured value. The agreement is not much worse for the other vessels discussed in NOL Report No. 504, even though their dimensions differ somewhat from those of our model.

CONFIDENTIAL

SECURITY INFORMATION

CONFIDENTIAL

- 35 -

Notes

(a) Page 2. W.F. Whitmore, NOL Report No. 504, "Representation of ships' pressure fields by source-sink models"; 14 pp., 11 plates; 7 April 1942.

(b) Page 3. For references to papers developing and applying various mathematical ship models see the bibliography on pages 35-37 of R. Guilloton, "Potential Theory of Wave Resistance of Ships with Tables for its Calculation", Society of Naval Architects and Marine Engineers, New York, advance paper No. 2 for the 6 and 7 September 1951 meeting.

(c) Page 3. See pp. 407-413 of L.M. Milne-Thomson, Theoretical Hydrodynamics, 2nd ed.; 600 pp., 1949. This book is the primary hydrodynamics reference for this paper.

(d) Page 9. This equivalence is usually left implicit. A proof is given Appendix I, p. 27.

(e) Page 9. Placing the source and sink below the surface rather than in it was suggested by J.K. Lunde, "On the linearized theory of wave resistance of displacement ships in steady and accelerated motion", Society of Naval Architects and Marine Engineers, New York, advance paper No. 1 for the 6 and 7 September 1951 meeting. This may not be the best placement. Its interesting effect on the Rankine solid is discussed in Appendix II, p. 29. Its effect on bottom velocity cannot be great for the situation we analyze.

(f) Page 15. The method by which these streamlines were found is presented in Appendix II, p. 29.

CONFIDENTIAL

SECURITY INFORMATION

CONFIDENTIAL

- 36 -

(g) Page 21. E. Madelung, "Das elektrische Feld in Systemen von regelmaessig angeordneten Punktladungen", Physikalische Zeitschrift, 19, 524-533 (1918).

(h) Page 21. H. Margenau and G.M. Murphy, The Mathematics of Physics and Chemistry, 1943, p. 115.

(i) Page 29. Milne-Thompson, op.cit., pp.110-111.

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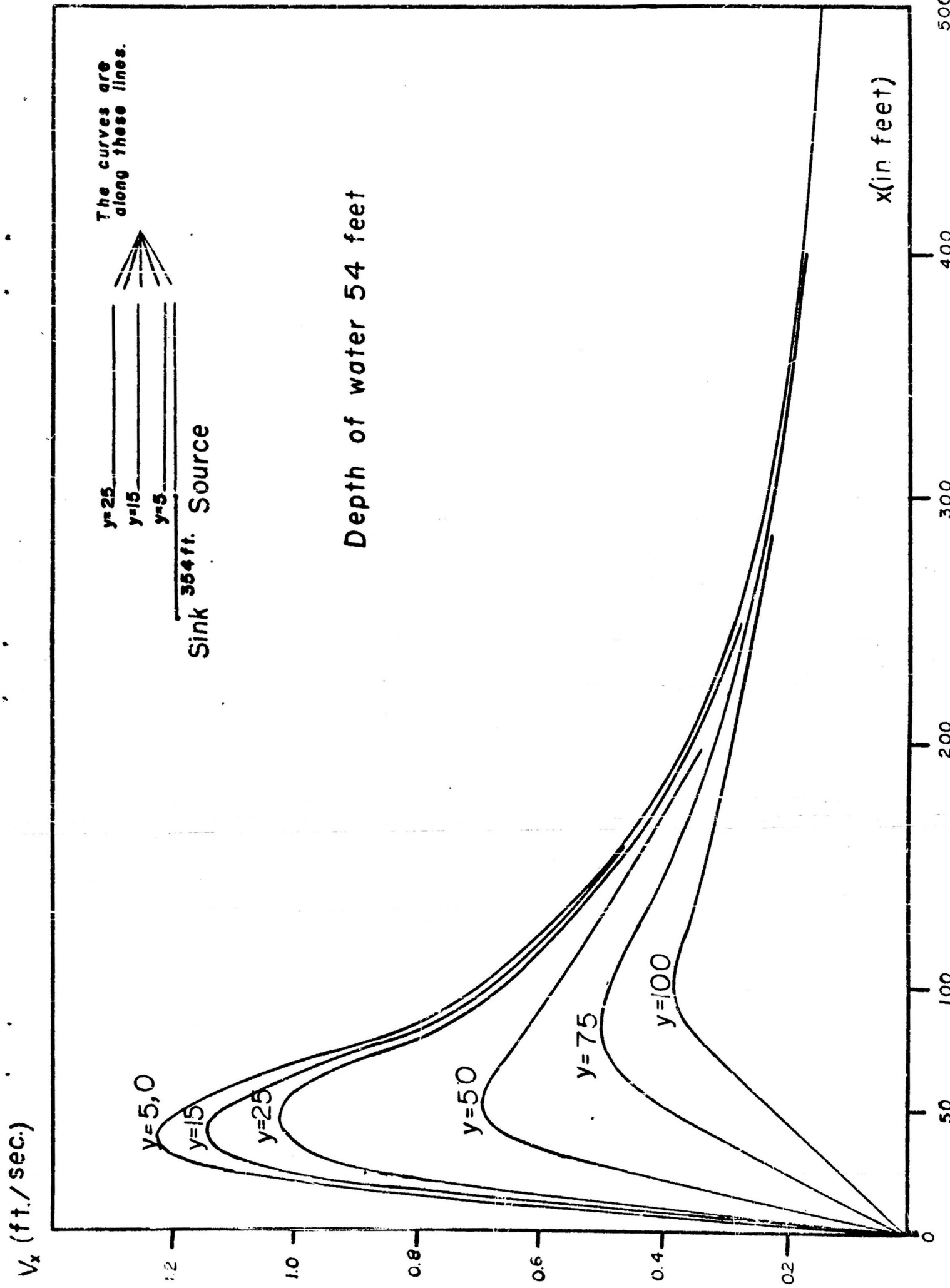
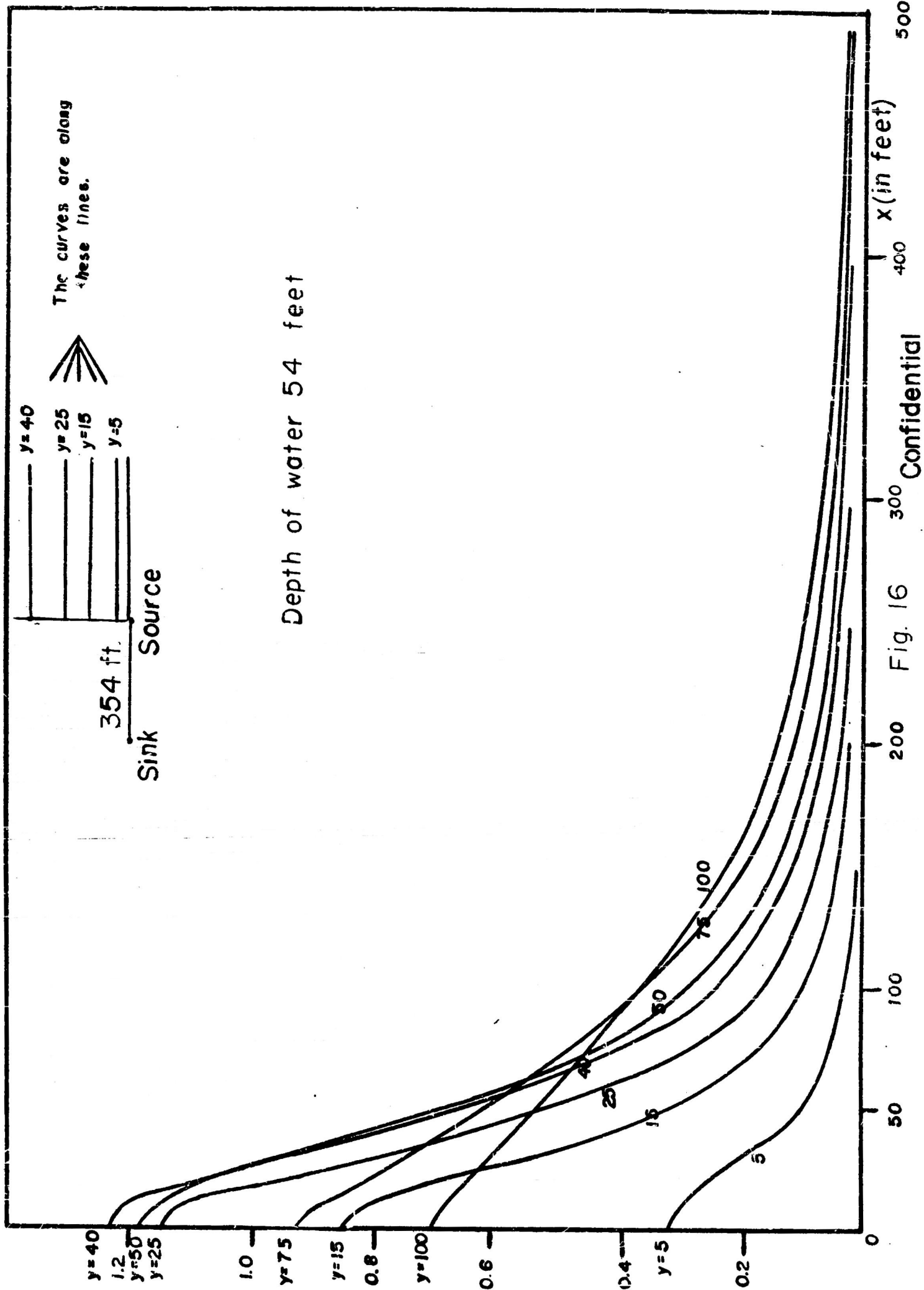


Fig. 15

V_y (ft./sec.)



V_x (ft./sec.)

V_x vs. x for two sources m and $-m$

Distance between sources 354 ft.

Distance between zeros 360 ft.

Depth of water 54 ft.

Velocity 20 ft./sec.

m at $(x, y, z) = (0, 0, -10)$

x (in feet)

Sink - m

Source + m

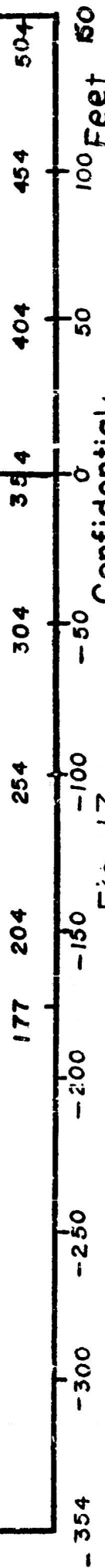


Fig. 17

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Lines of constant V_x in ft./sec.

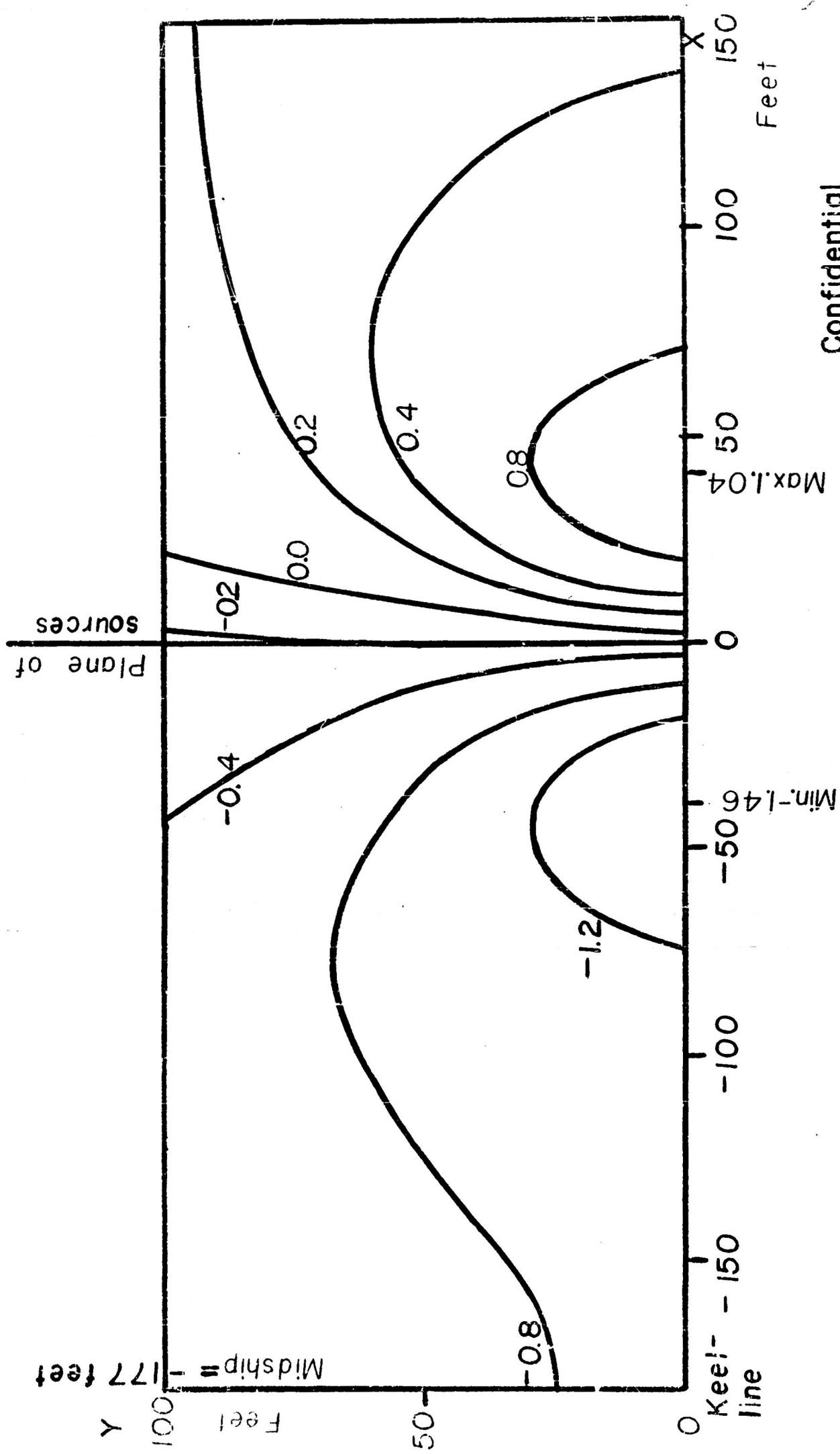
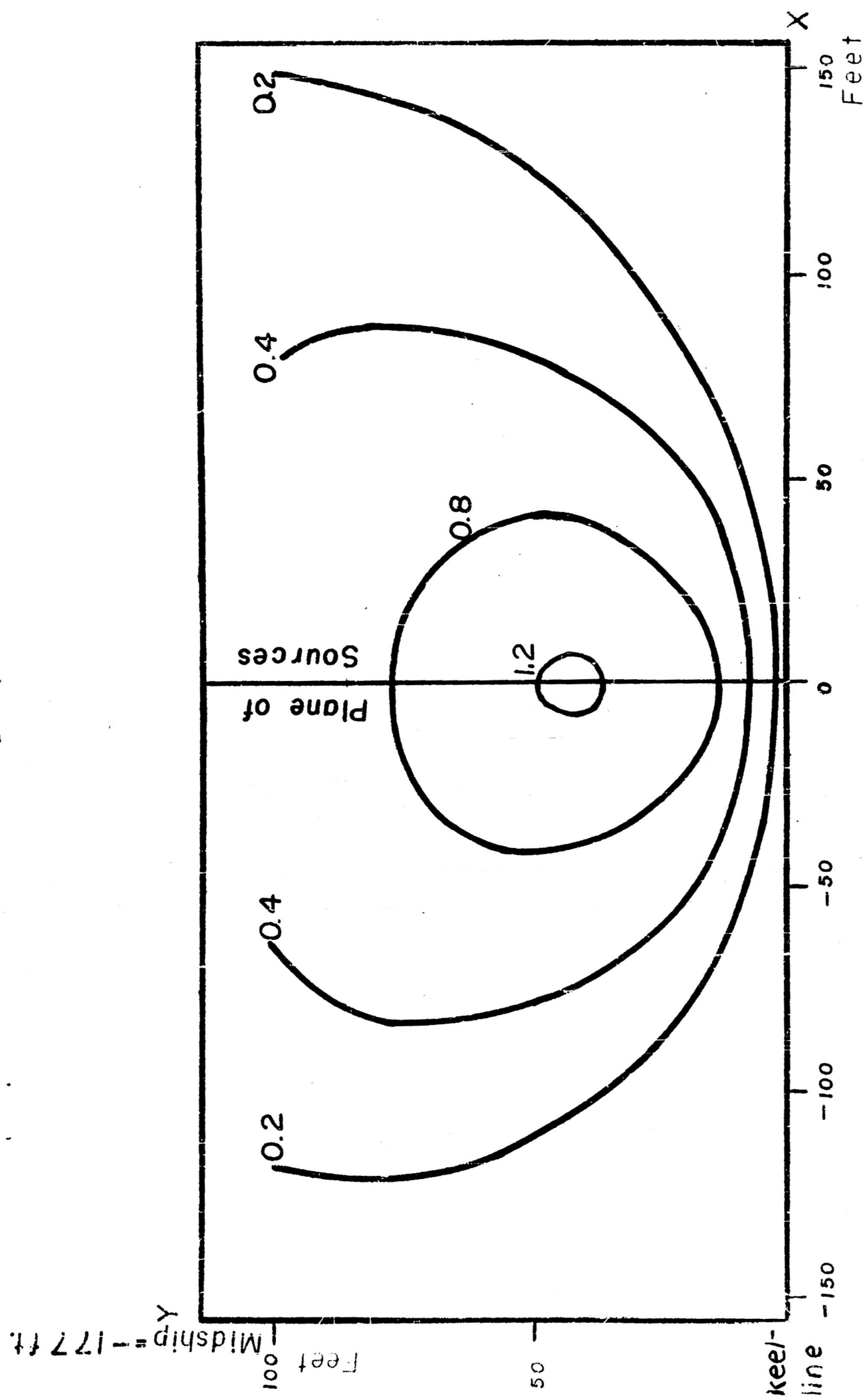


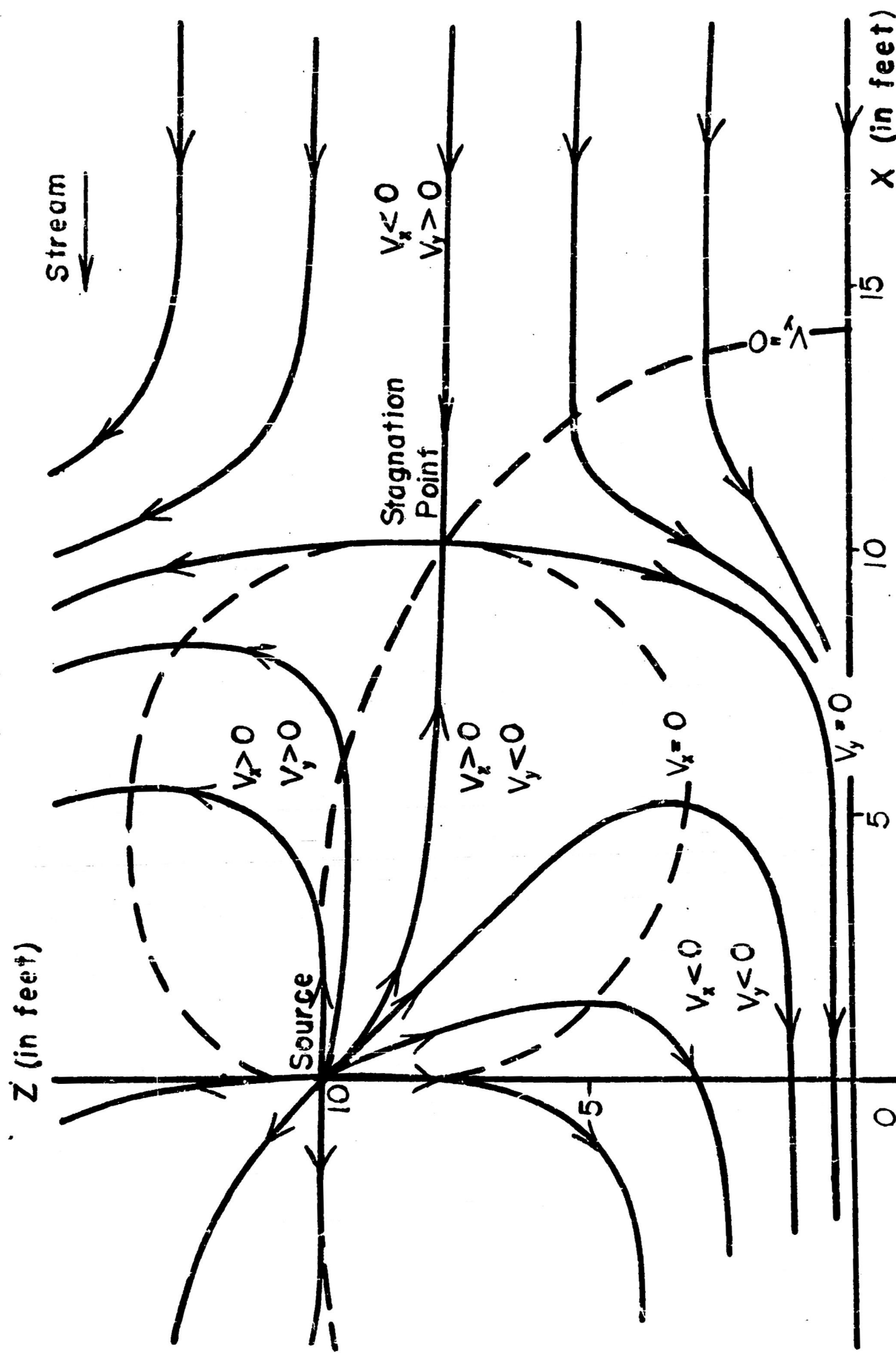
Fig. 18



Lines of constant V_y in ft./sec.

Fig. 19

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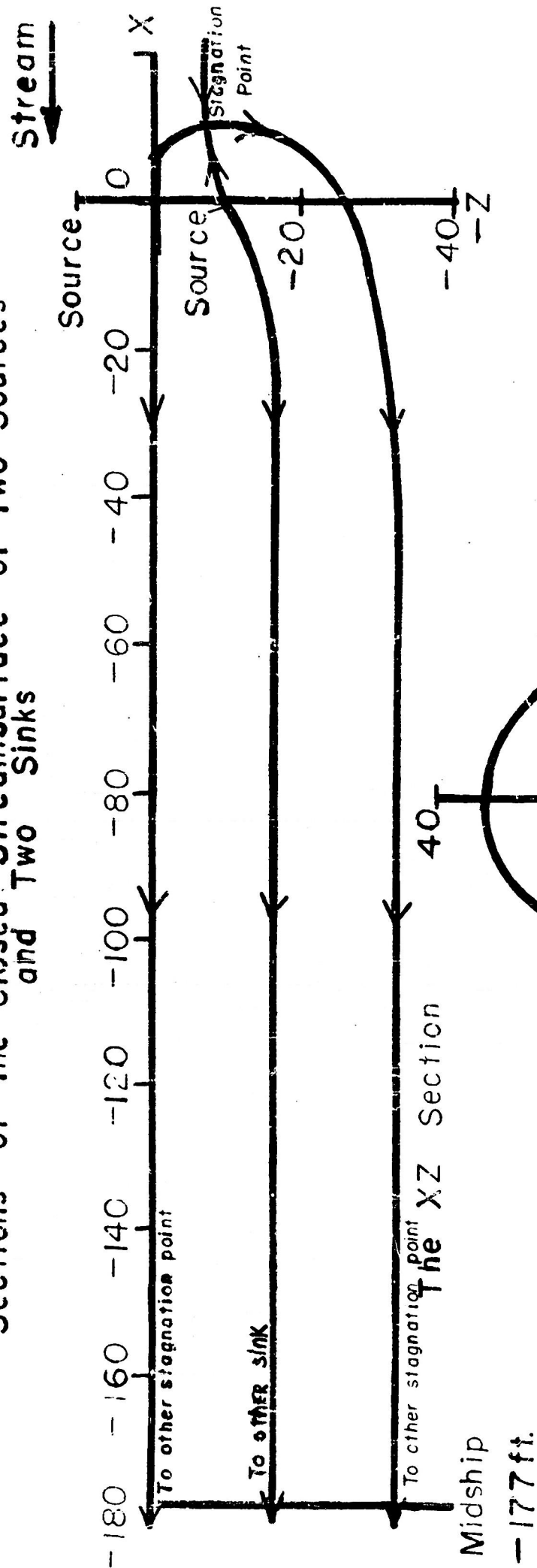
Streamlines Near Two Sources

Other Source
at -10 Ft.

Fig. 20

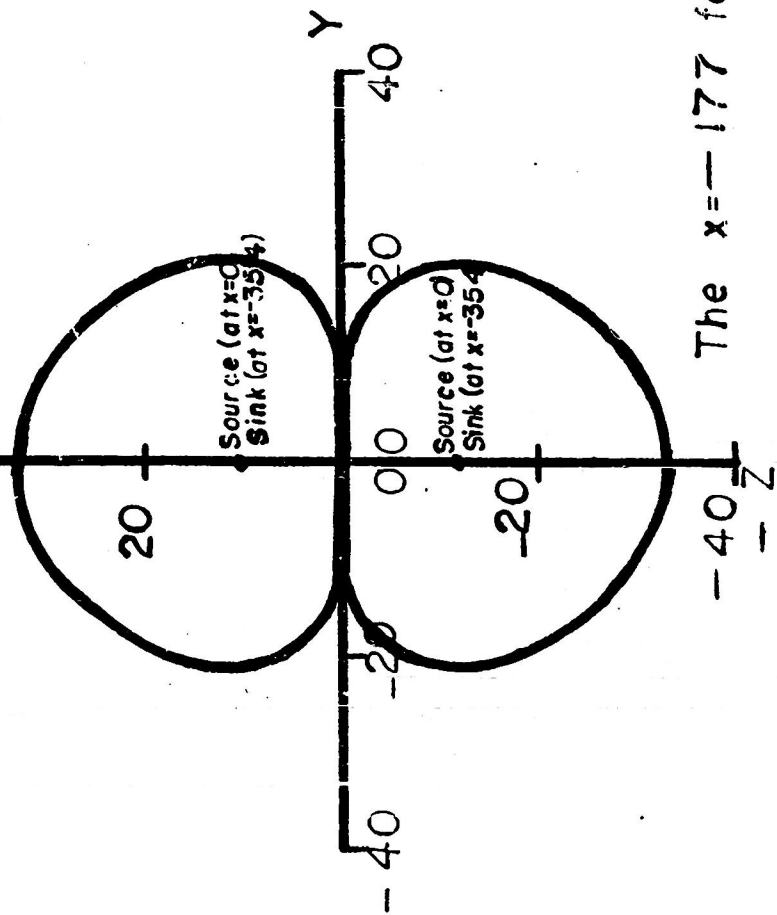
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Sections of the Closed Streamsurface of Two Sources and Two Sinks



Sink at -354 feet

$$\frac{V}{V_\infty} = 100 \text{ ft.}^2$$



The $x = -177$ feet Section (Midship)

Fig. 21

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Dividing Stream Line for Model Consisting of Line Source

and Line Sink

Length of sink: 135 ft.

Length of source: 135 ft.

Clear distance between source and sink: 190 ft.

Total strength of source = m of report = $315V \text{ ft}^3/\text{sec}$. if V is in ft./sec.

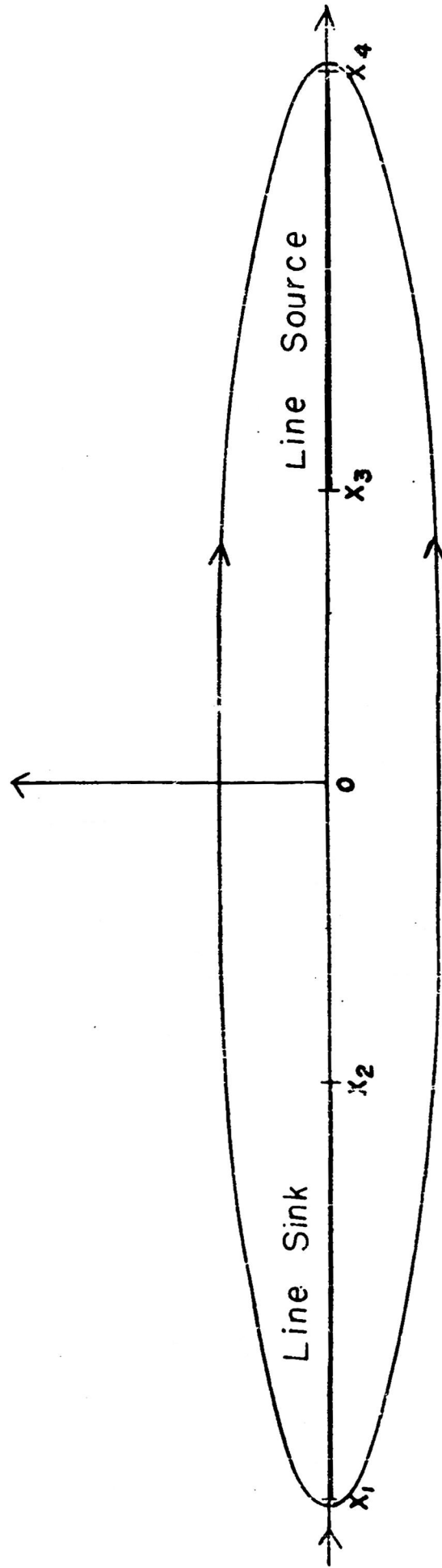


Fig. 2 2

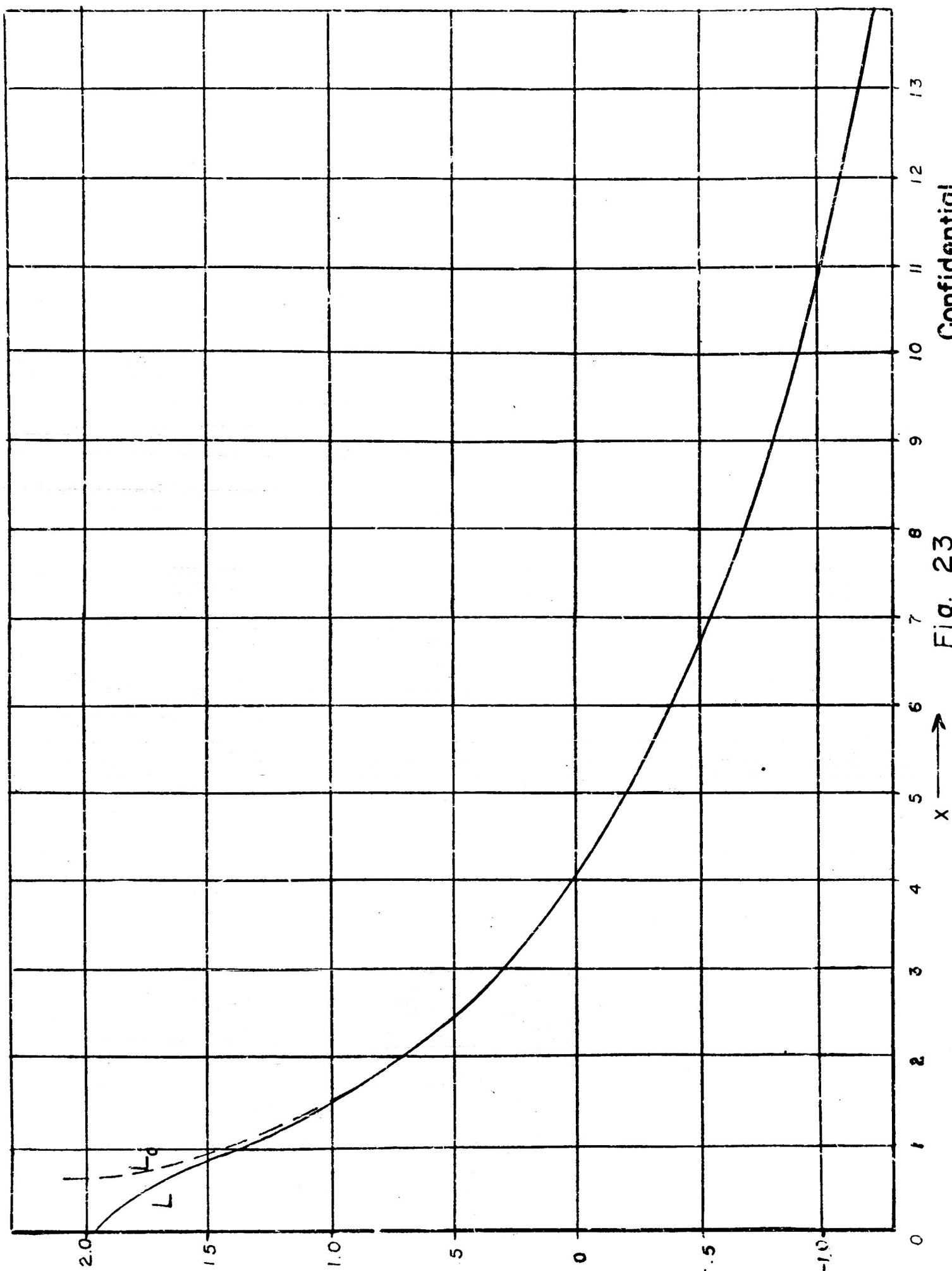


Fig. 23

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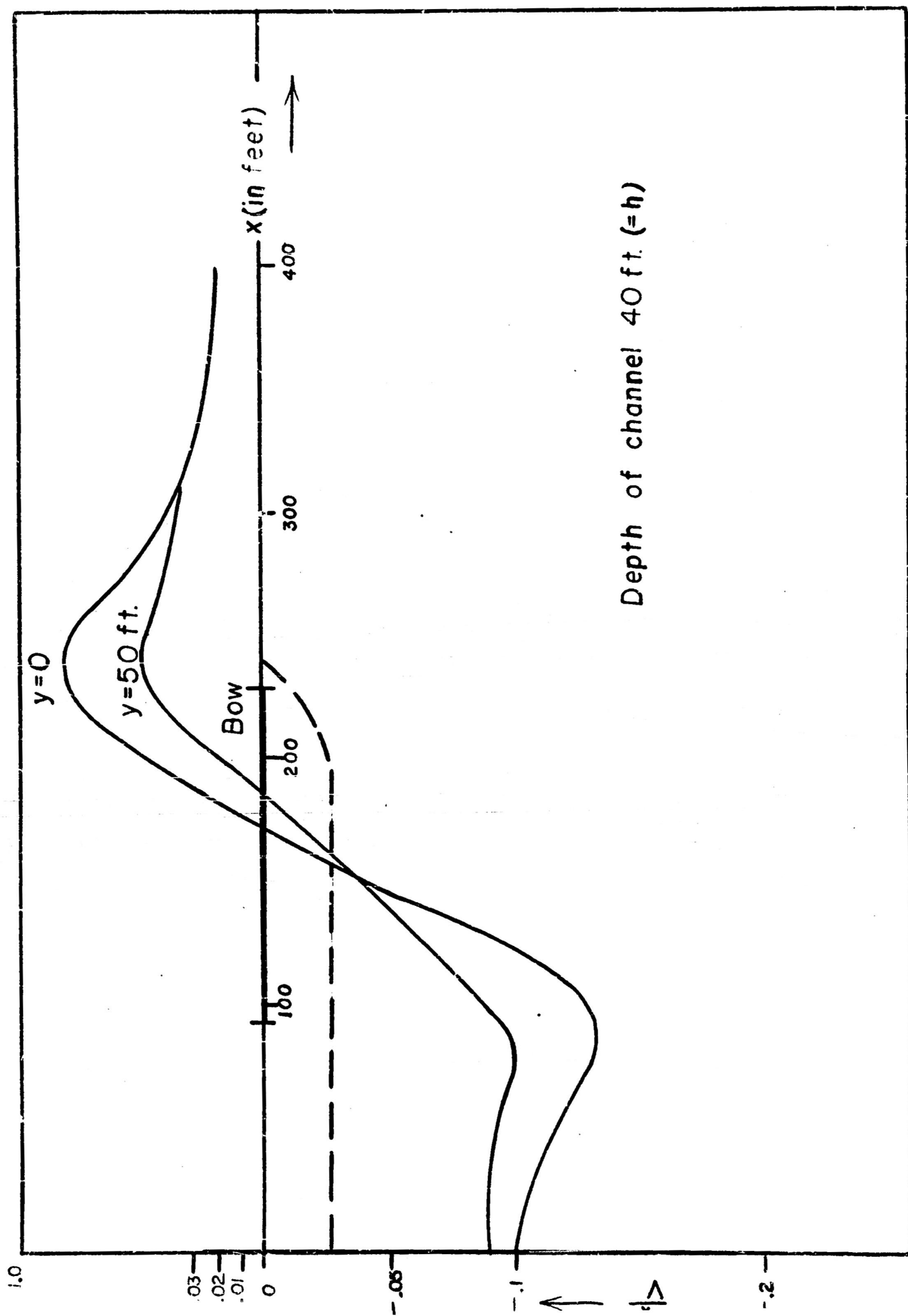


Fig. 24

Confidential

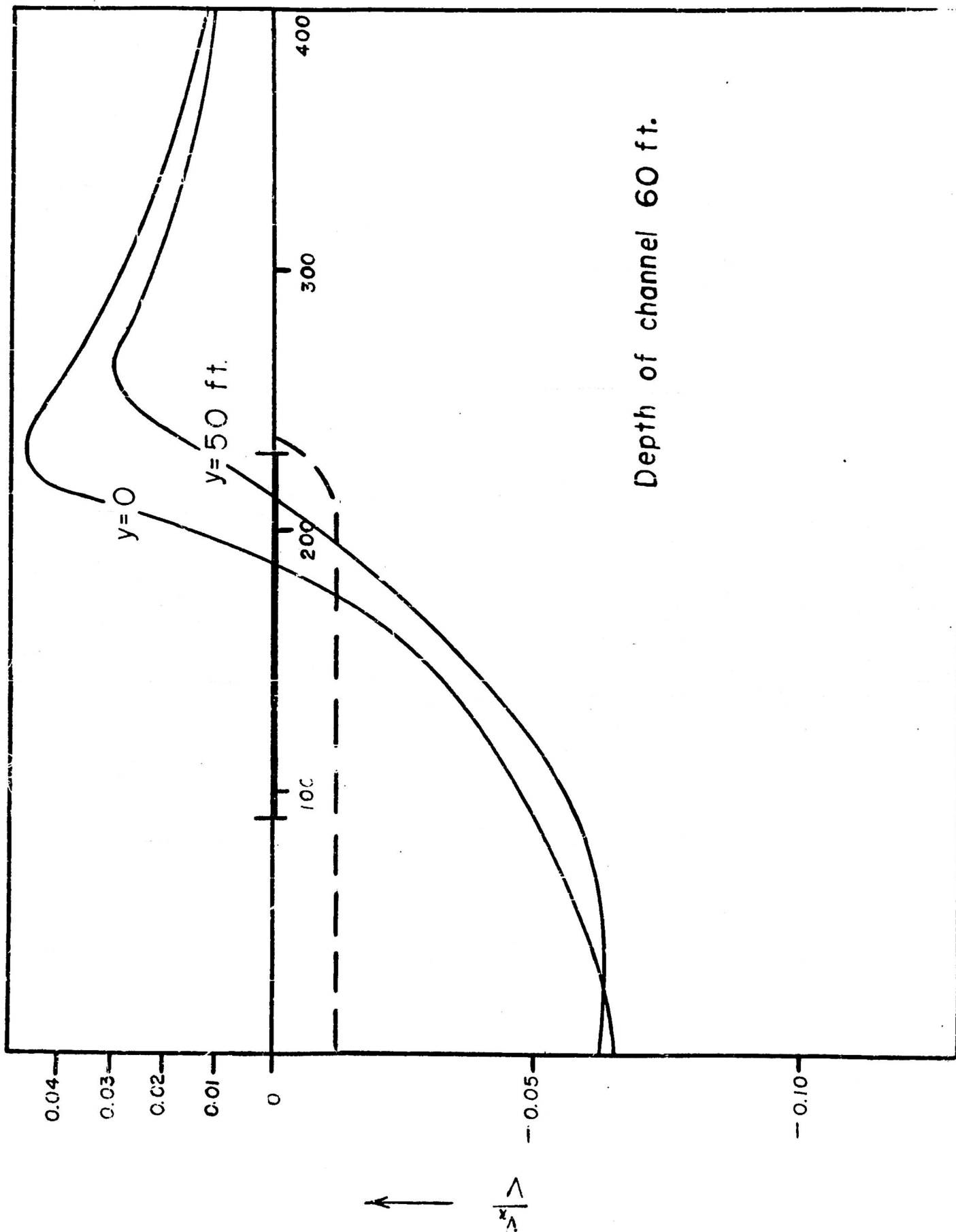


Fig. 25

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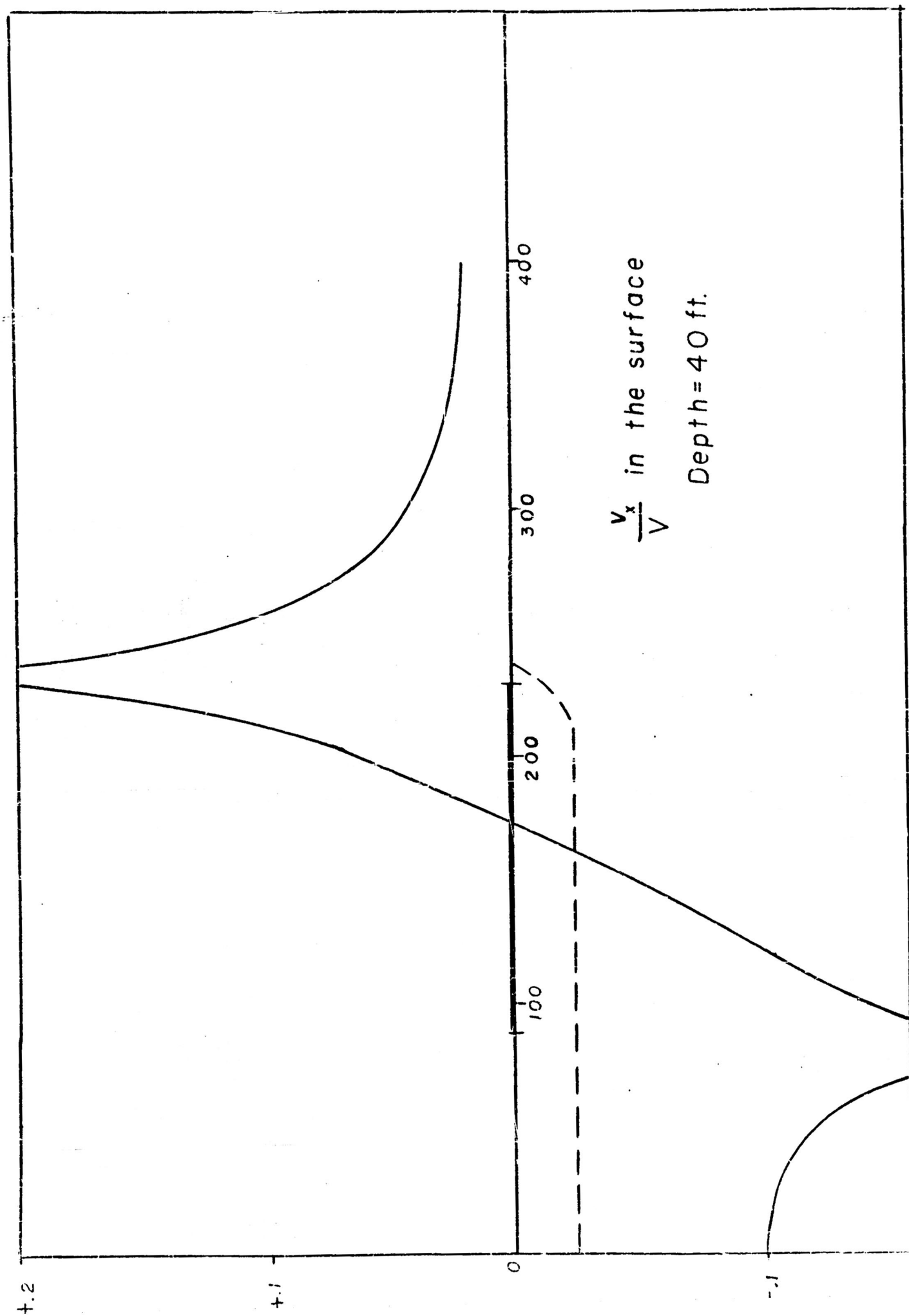
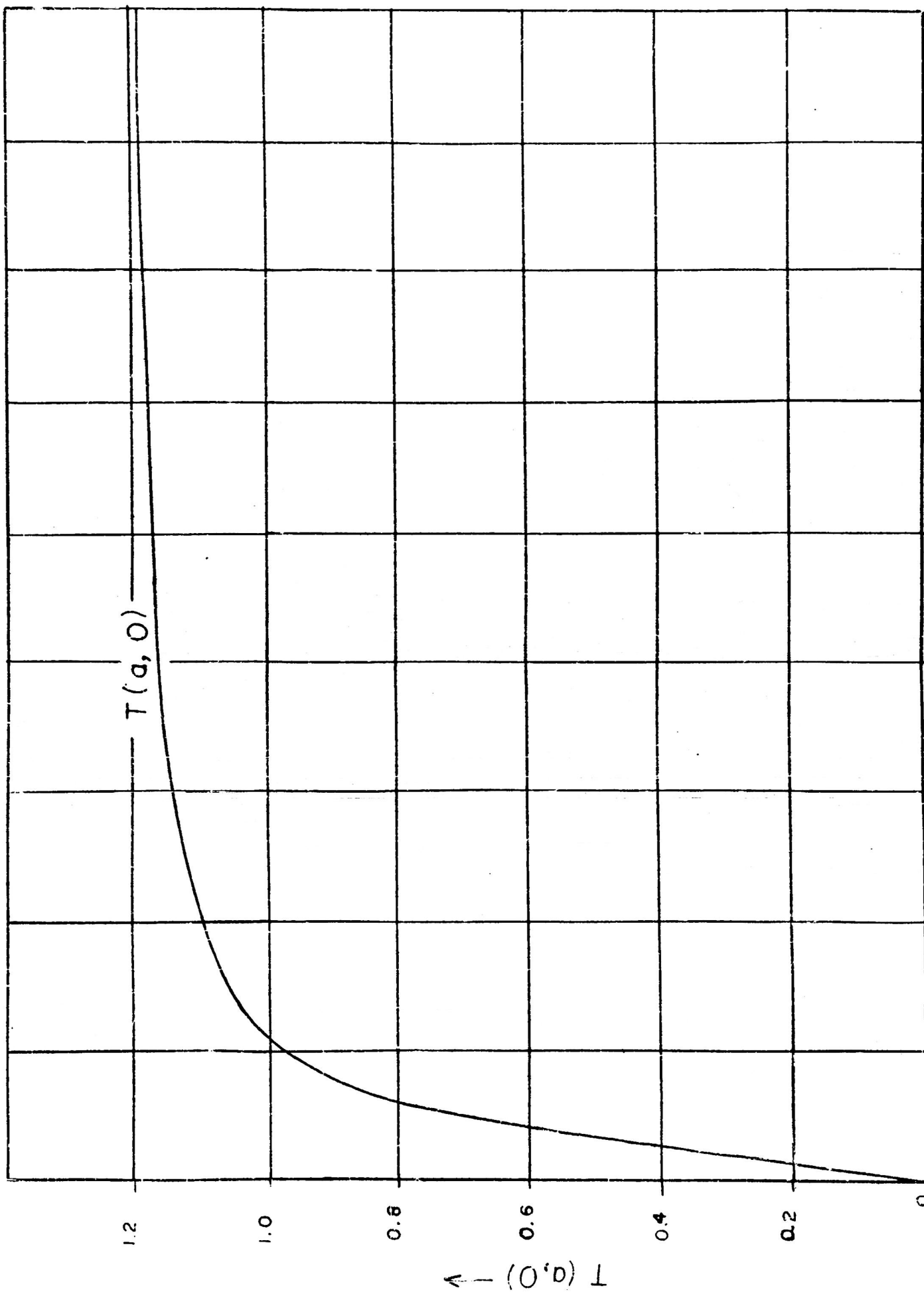


Fig. 26

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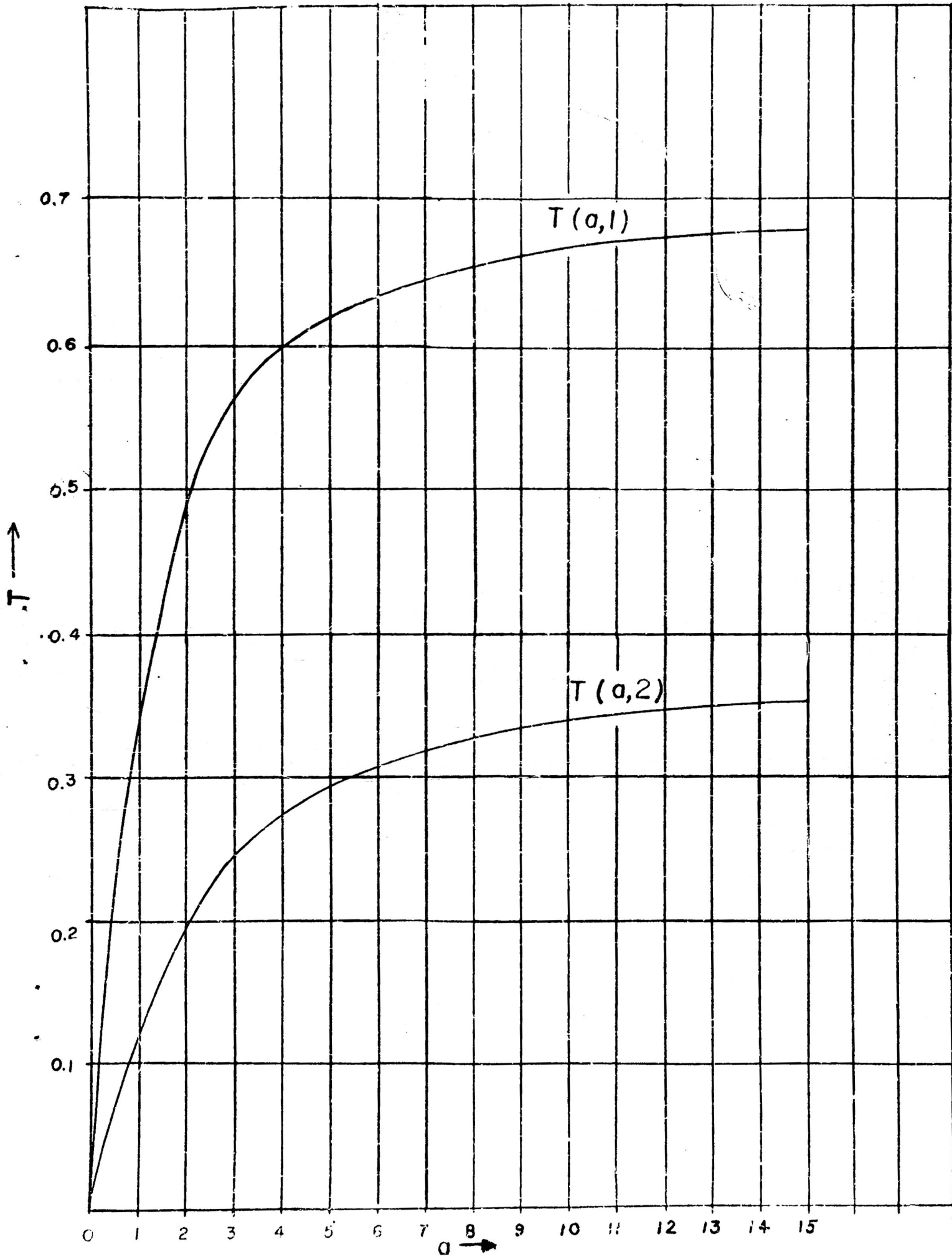
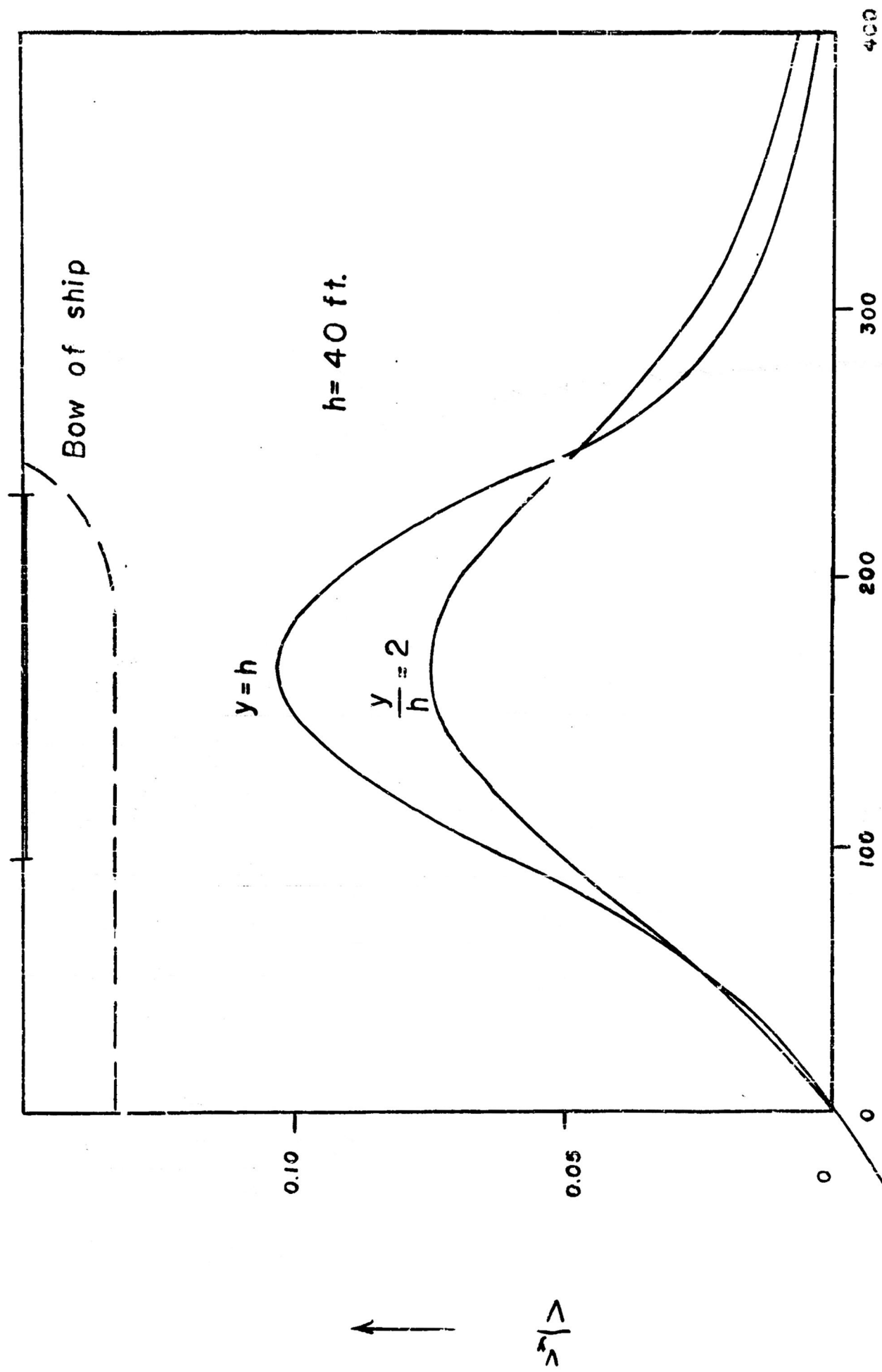


Fig. 28

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Negative pattern
for stern end of ship

Feet from ship center →

Fig. 29

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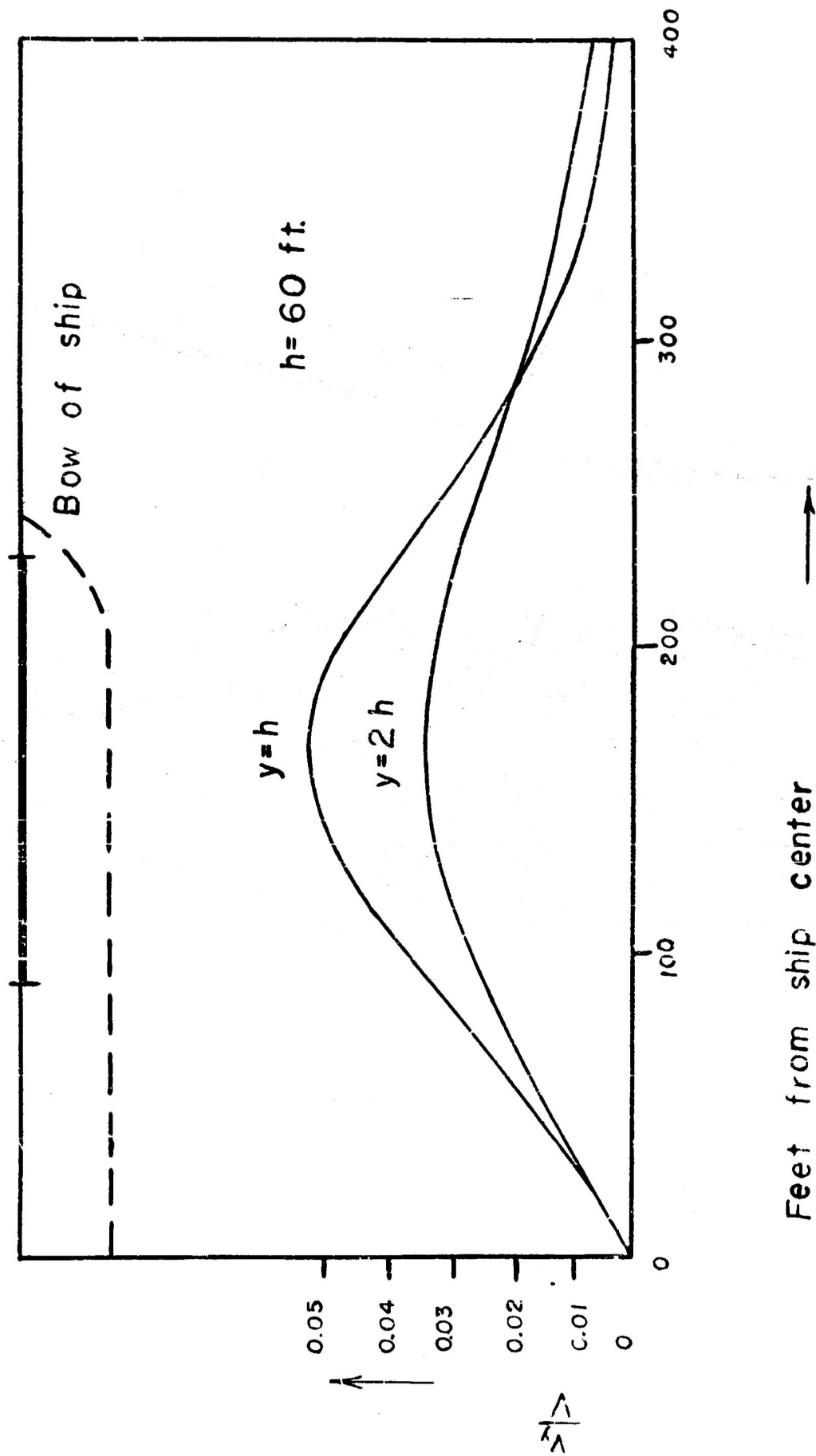
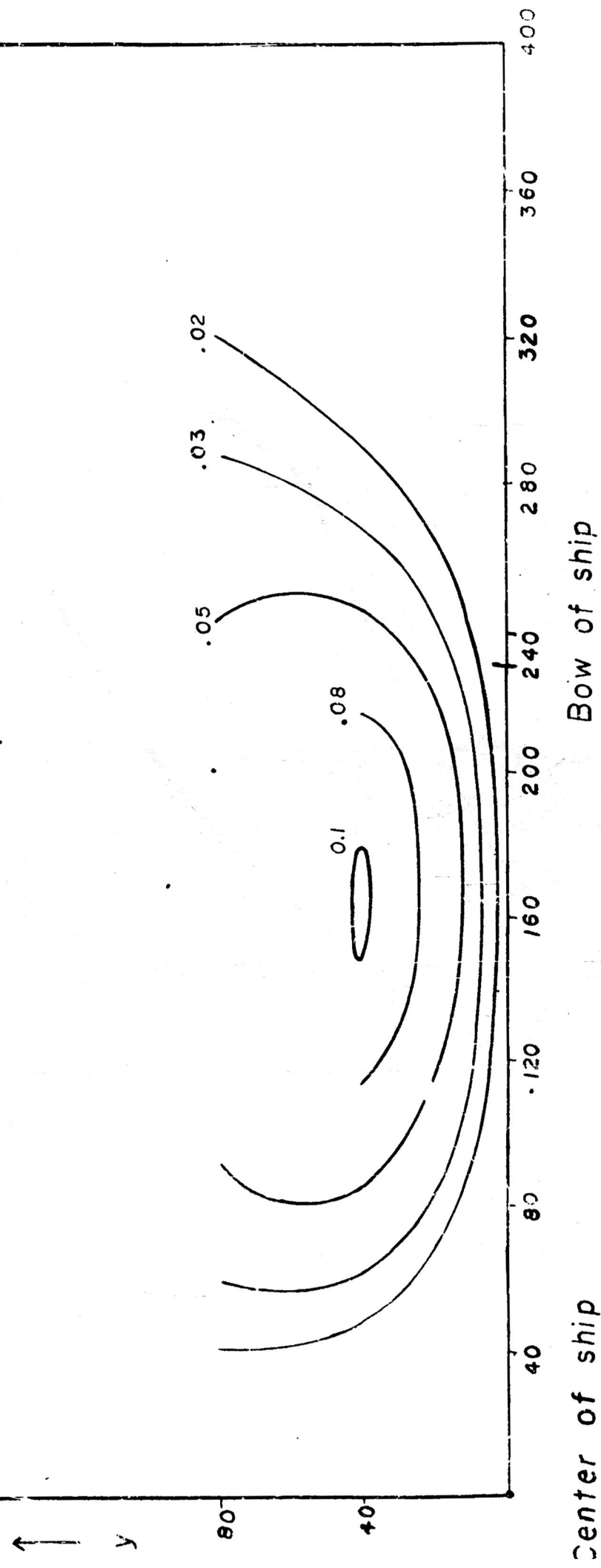


Fig. 30

Lines of constant $\frac{v_y}{V}$ on bottom at depth 40 ft.



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