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Regularities in the Change of Binding Energy of Nucleons in Nuclei

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By a comparison, based on experimental data, of the binding energies of the "last" nucleons in a nucleus, we previously[1] established the existence of the following regularities: (a) the binding energy of the last neutron increases with the number of protons in a nucleus and decreases with the number of neutrons in a nucleus; (b) the binding energy of the last proton increases with the number of neutrons in a nucleus and decreases with the number of protons in a nucleus. In a paper published later, Way and Wood[2] described these regularities, but only for heavy nuclei. Improvement of the experimental values showed that there are as yet no exceptions to these regularities and that they are, therefore, probably universal. In our previous work, we have shown that the change of binding energy is greatest for light nuclei and decreases gradually as we go to heavier nuclei.

The changes in binding energy of nucleons were also studied quantitatively. The binding energies of the last nucleons were taken from binding energy tables compiled by the present author on the basis of mass-spectrographic measurements, energies of nuclear reactions, energies of radioactive transformations, and other experimental data. These tables were based on experimental material published before July 1, 1952. The part of the table which deals with heavy nuclei has been published by the present author.[3]

These experimental binding energies were used to compute the average increase in the binding energy i_n of a neutron caused by the addition of a proton to the nucleus, and the average increase in the binding energy i_p of a proton caused by the addition of a neutron to the nucleus. The values i_n and i_p were found from the formulas

$$i_n = \frac{e_{2n}(Z+K, N) - e_{2n}(Z, N)}{2K}, \qquad i_p = \frac{e_{2p}(Z, N+K) - e_{2p}(Z, N)}{2K}, \tag{1}$$

where $e_{2n}(Z, N)$ and $e_{2p}(Z, N)$ are the binding energies of the last pairs of neutrons and protons in the nucleus with Z protons and N neutrons (see, for instance, Eqs. (12) and (13) in reference 3).

Fig. 1 shows the hyperbola

$$i_n = \frac{24}{N}, \tag{2}$$

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and Fig. 2 the hyperbola

$$l_p = \frac{24}{Z}; \tag{3}$$

the points are the experimental values.

We see from Figs. 1 and 2 that the curves correspond very satisfactorily to the experimental points.

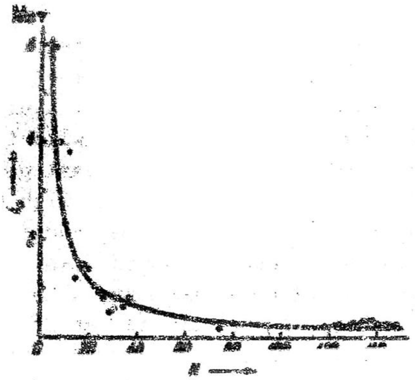

Fig. 1.

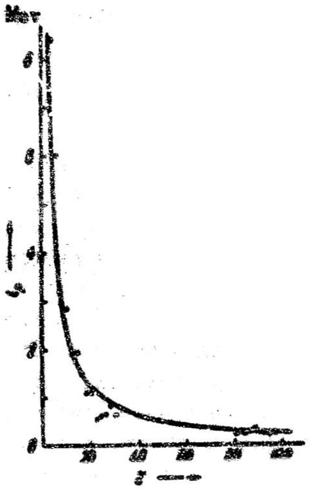

Fig. 2.

The same experimental data were used to calculate the average decrease in the binding energy of the neutron d_n caused by the addition of one neutron to the nucleus, and the average decrease of the binding energy of a proton d'_p caused by the addition of one proton to the nucleus. The values d_n and d'_p were computed from the formulas

$$d_n = \frac{e_{2n}(Z, N) - e_{2n}(Z, N+K)}{2K}, \qquad d'_p = \frac{e_{2p}(Z, N) - e_{2p}(Z+K, N)}{2K}. \tag{4}$$

Since the decrease in binding energy of the protons with increasing number of protons in the nucleus depends not only on the nuclear forces but also on the electrostatic forces, we computed the change of the electrostatic energy E_s from the formula

$$E_s = \frac{e^2}{r_n}, \tag{5}$$

where e is the value of the elementary electric charge, r_n is the radius of the nucleus calculated from the formula

$$r_n = r_0\sqrt[3]{A}, \tag{5'}$$

where A is the mass number and r_0 is a constant taken to be 1.38×10^{-13} cm, in accordance with the experimental data in reference 4.

Fig. 3 shows the curve

$$d_n = \frac{20}{Z}, \tag{6}$$

and Fig. 4 the curve

$$d'_p - E_e = \frac{20}{N}; \qquad (7)$$

the points are the experimental values.

The agreement of the experimental points with the curves is poorer than in Figs. 1 and 2, but is still satisfactory. Fig. 4 shows that the electrostatic correction improves the distribution of the points. If the nuclear radius r_n increased more slowly than according to (5'), i.e., if the density of heavy nuclei were greater than that of light nuclei, the experimental points in Fig. 4 would be closer to the curve.

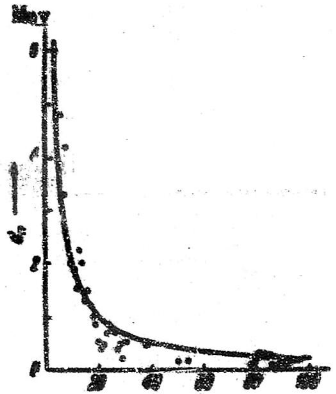

Fig. 3.

Eqs. (2) and (3), on the one hand, and (6) and (7), on the other, show to a certain degree the charge independence of the binding energy of the nucleons in the nucleus. In order to achieve this charge independence, it is sufficient that the nuclear interaction of protons with protons (p – p) be identical with the interaction of neutrons with neutrons (n – n). This does not contradict the conclusions drawn from experiments on the scattering of protons and neutrons in references 5 and 6. Indeed, scattering experiments show that the neutron-proton interaction at high energies (> 30 Mev) is somewhat different from the proton-proton interaction. But these data give no statement about a difference between the p – p and n – n interactions.

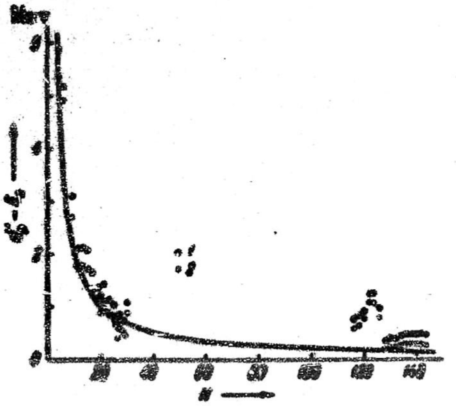

Fig. 4. 1 – experimental values of d_p, 2 – experimental values of d'_p after subtracting the correction for the electrostatic energy.

The binding energy of a nucleon is the effective energy of interaction of the whole nucleus with the nucleon and therefore depends on many factors. This energy depends not only on the nuclear force law, but also on the state of all the nucleons in the nucleus, i.e., on orbital angular momenta, symmetry, spins, and so forth. Still, the existence of such a simple dependence gives us grounds to suppose that it is caused by a basic factor, possibly by the law of nuclear forces. The deviations of the experimental points from the curves have the same order of magnitude as the errors, but some of them, nevertheless, can be explained in a different way. In Fig. 4, in particular, we can see clearly a sharp peak of experimental points at N = 126, which can be explained only by the effect of the closing of the proton-neutron shell in the vicinity of the nucleus $_{82}Pb^{208}$. On the other graphs, irregularities also seem to appear in the same region. Thus, the shells affect very little the dependences described by Eqs. (2), (3), (6), and (7).

The analysis of experimental regularities presented here and its further refinement should be continued as new, more precise experimental data become available. Every theory of nuclear forces and theory of nuclear structure, along with the explanation of other facts, will also have to account for these quite simple regularities.

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