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EFFECTS OF SPRING AND INERTIA DEVICES ON THE LONGITUDINAL STABILITY OF AIRCRAFT

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JULY 1953

WRIGHT AIR DEVELOPMENT CENTER

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WADC TECHNICAL REPORT 53-350

EFFECTS OF SPRING AND INERTIA DEVICES ON THE LONGITUDINAL STABILITY OF AIRCRAFT

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July 1953

Aircraft Laboratory
Contract AF 33(616)-70
RDO No. 458-414

Wright Air Development Center
Air Research and Development Command
United States Air Force
Wright-Patterson Air Force Base, Ohio

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FOREWORD

The research reported herein was conducted by Princeton University under Air Force Contract 33 (616) - 70 during the period 1 April to 1 August 1953. Work was monitored by the Aircraft Laboratory, Directorate of Laboratories, Wright Air Development Center, with Mr. C. B. Westbrook serving as project engineer. The pertinent Research and Development Order is 458-414, "General Investigations of Aircraft and Missile Stability and Control."

Special acknowledgement is extended to Lt Charles B. Smith, USN, for his outstanding performance as flight-test pilot for this project, Mr Robert Cooper for his excellent efforts in the modification of the test aircraft, Mr. Harry Williams for his valuable assistance in the design of the instrumentation system, and Mr. Anthony Carnevale for his contribution as data recorder.

Abstract

An investigation was conducted to determine the effects of spring and inertia devices on the longitudinal static and dynamic stability of aircraft. Theoretical studies and flight tests of a modified Navion airplane were completed. It was concluded that introduction of bobweights in the elevator control system consistently improved the phugoid damping and downsprings had a favorable effect when limited to a light or moderate application.

PUBLICATION REVIEW

This report has been reviewed and is approved.

E H Schwartz

For D. D. McKEE
Colonel, USAF
Chief, Aircraft Laboratory
Directorate of Laboratories

FIGURES

LIST OF FIGURES

I. SUMMARY

Theoretical studies and flight test measurements of the effects of bobweights and downsprings on static and dynamic stability have been completed. An analytical analysis shows the springy tab is equivalent to the downspring. The test article was a modified Navion type airplane considered in cruising flight. Theoretical results based on estimated characteristics were initially obtained for complete ranges of control device forces and center of gravity displacements in order to establish qualitatively the significant trends to be verified by actual measurement. A flight region selected on the basis of the analytical computations was then explored in a series of dynamic tests. Static tests were conducted in conjunction with this phase to determine the reference static stability for each dynamic test condition. Random checks of the dynamic runs were also made with an analog computer utilizing aerodynamic data extracted from the static flight results.

The analytical phase indicated that the effects of force devices on the longitudinal dynamics could be expressed as a function of their control of just two principal static parameters -- the stick-free maneuvering margin and the difference between the stick-free maneuver and neutral points. The first is regulated by bobweight and center of gravity location and the second is regulated by downspring or springy tab solely. The increment $N_m' - N_o'$ is referred to as the static damping parameter since this term is dependent upon the basic damping in pitch for airplanes without auxiliary control devices and in the subject investigation it was found to have a strong influence on the damping characteristics. It was established that the phugoid mode was of predominant importance in the static stability region under study. All pertinent phugoid trends were substantiated by the flight data and a reasonable agreement was shown with regard to the magnitude of the damping and period. In addition qualitative comparisons of the analog solutions and the flight

results were satisfactory.

In general, it was found that increasing the stick-free static stability at rearward displaced centers of gravity by introducing bobweight forces had consistently favorable effects on the phugoid. The mode changed from a pure divergence to a damped oscillation at low values of downspring force (positive damping parameter) and from a pure divergence to an undamped oscillation at high values of downspring force. A moderate application of downspring to restore level-flight static stability improved the phugoid damping from a pure divergence to a damped oscillation when the spring forces were limited to maintain a positive damping parameter. Further increases in downspring force sufficient to cause the damping parameter to become negative resulted in rapidly divergent oscillations. This effect was particularly pronounced at low positive maneuver margins. In all cases a damped phugoid was obtained only for a positive damping parameter in conjunction with a moderate maneuver margin.

Practical limits for application of bobweights and downsprings were found to depend upon the basic elevator and tab power and thereby imposed restrictions on the center of gravity displacement which could be corrected by these control modifications. Other considerations made evident in the test program were the possibility of adverse effects due to the increases in elevator inertia and ground-handling stick forces that occur when auxiliary control devices of considerable strength are used.

Most of the flight testing was conducted with the center of gravity located aft of the stick-fixed neutral point so that the airplane had an unstable elevator deflection gradient. It is of interest to note that throughout the program the pilot had no impression in the handling qualities of this static instability. The concept that only the stick-free stability has importance in level-flight operation would require further comprehensive testing for confirmation.

II. INTRODUCTION

Modern trends in the design of high performance aircraft have made it increasingly difficult to achieve center of gravity positions located sufficiently forward to meet requirements for satisfactory static stability force gradients. Therefore, artificial means of improving static stability, although used fairly extensively in the past, have become more prevalent. The stability change is accomplished by introducing constant forces in the longitudinal control system with devices such as bobwieghts, downsprings and springy tabs. The usable center of gravity range is generally limited by the maximum allowable maneuvering force gradient and the minimum allowable level-flight force gradient. Through the application of downsprings or springy tabs, the usable center of gravity range can be increased in magnitude and with bobweights the absolute values of each gradient can be increased or restored. In the past, effects of these devices on the longitudinal dynamic stability were not considered particularly important since generally the basic airplane characteristics were such that the short period mode was heavily damped and the phugoid period was long enough to make its damping relatively inconsequential. Current design trends however, have also led to numerous instances of unsatisfactory phugoid damping so that now a new need exists for information on how the longitudinal dynamics are modified by auxiliary control devices. Since relatively little was known about these effects this program was undertaken in an effort to provide actual test data.

It was thought feasible to use a small light airplane to establish generalized results in a qualitative form. Although quantitative effects of bobweights and downsprings depend upon the particular characteristics of the airplane under consideration, trends could be determined which would be of value in application to subsonic non-boost control aircraft in general. Results of such an investigation would be useful in the design of force additive devices for the satisfaction of static requirements as well as to afford some means of correcting inherent dynamic difficulties encountered without force installations. In addition, as phugoid requirements for satisfactory flying qualities are not as yet established, information on the practical limits for marginal dynamic characteristics is of value.

III. DESCRIPTION OF AIRPLANE

The flight research article, a Navion airplane, is a single-engine, low-wing monoplane provided with a retractable tricycle landing gear and an adjustable pitch propeller. A general view of the test airplane is shown in Figure 1 and the physical characteristics are given in Appendix I. The basic elevator configuration consists of a 35% full-span surface without aerodynamic balance. Wheel-type dual controls are included and the elevators have adjustable trim tabs on each panel.

In order to minimize the destabilizing center of gravity displacements required for test purposes the horizontal tail area was reduced. This was accomplished by decreasing the tail span 34 inches to obtain an 18% reduction in total area. The modified horizontal tail is shown in comparison with the original configuration in Figure 2. After modification, the elevator was statically unbalanced with a trailing edge down hinge moment equivalent to a push stick force of approximately 8 lbs. Adjustments were made to the stabilizer incidence setting to compensate for trim changes due to the horizontal tail redesign. The elevator trim tab area was increased by a trailing edge extension to enable trim of heavy force devices and the full down elevator deflection was increased by a few degrees to improve the control effectiveness for dynamic testing. The friction level in the longitudinal control system was reduced to approximately 1 lb. and the rear seats were removed to allow installation of the instrumentation.

IV. WEIGHT AND BALANCE

After installation of all instrumentation and modification of the tail configuration the Navion gross weight was 2680 lbs. with full fuel, oil and a crew of two. The center of gravity location for this loading was at 27% MAC.

It was estimated that center of gravity locations approaching 40% MAC would be desirable for the test program. To achieve these aft positions it was necessary to install fixed ballast in the fuselage under the dorsal fin as well as transfer movable ballast within the cabin following take-off. Fixed ballast was increased in increments of 20 lbs. to a maximum of 80 lbs. and movable ballast weighing 125 lbs. was displaced in varying increments through a distance of 8 ft. during flight to attain center of gravity locations between 27% and 39%. Thus, the maximum take-off weights were approximately 2900 lbs. in the course of the program. Center of gravity locations aft of 39% were not considered reasonable for the static tests based on considerations of overload gross weights, ground handling, structural limitations and ease of ballast transfer. Dynamic tests were restricted to center of gravity locations forward of 36% MAC due to a deficiency of down elevator deflection required for recovering from severe nose-up attitudes. Records were kept on balance changes in order to accurately compute take-off weights and center of gravity locations for each flight. Effects on the balance of fuel consumption during flight were found to be negligible.

V. BOBWEIGHT AND DOWNSPRING SYSTEMS

A rigid arm was fixed normal to the control column so that through a reverse leverage system a positive (tending to depress the elevator) bobweight moment could be applied and by attaching a preloaded spring from the canopy to this member a positive downspring moment could be introduced as well. The test installation is shown in Figure 3. Before the test phase was begun the elevator was statically balanced by attaching a weight directly to the control column arm to correct the trailing edge heaviness condition. Both the bobweight and downspring systems were designed so that the moment exerted on the elevator was essentially constant in the test range of deflections.

Positive bobweights and downspring moments, and combinations thereof, were applied progressively in equal increments, each equivalent to a stick force of 8 lbs. This was done using a series of similar weights and a series of tension springs of equal preload. The magnitude of incremental force was selected as that affording a static stability variation consistent with convenient changes in airplane balance.

VI. INSTRUMENTATION

Special test instrumentation was installed in the Navion to accurately measure airspeed, control force, normal acceleration and elevator deflection. For the static tests readings were visually recorded and for dynamic testing a photorecorder was used including a 16 mm gun camera to film an airspeed indicator and a stop watch.

Airspeed data were obtained with a pitot-static head located on a wing boom. Calibrations indicated the corrections for instrument and position error were negligible. The control force applied to the wheel was measured with a strain gage system to give current readings which were proportional to wheel force and accurate to 0.5 lb. The accelerometer calibrated to read directly in "g" units consisted of a freely suspended spring and mass enclosed in a glass tube and was accurate to .01 "g". Elevator positions were measured to within 0.1 degree with a dual autosyn system made up of a master unit attached through a gear train to the elevator torque tube which drove a repeater indicator in the cabin. Stick force and elevator deflection calibrations were kept up to date throughout the test phase to minimize error due to sensitivity changes and drift.

VII. THEORETICAL ANALYSIS

A. Static Stability

The longitudinal stick-free stability of aircraft exhibiting excessively low control force gradients may be improved by introducing artificial forces which are independent of airspeed. This is accomplished by placing devices such as bobweights and/or downsprings in the longitudinal control system to produce constant mechanical hinge moments tending to increase the positive floating angle of the elevator. Under unaccelerated flight conditions the stability changes resulting from each device are equivalent. When the airplane develops normal accelerations in maneuvering flight the bobweight moment, being subject to inertial effects, increases as a linear function of the acceleration whereas the downspring moment remains unchanged from the level-flight condition. The static stability effects of bobweights and downsprings are readily calculated for both level-flight and maneuvering flight operation.

The change in elevator floating angle caused by the control devices is given by

$$(\Delta\delta)_{FREE} = -\frac{(C_{h_0})_{BW,DS}}{C_{h_\delta}} \tag{1}$$

where the hinge moment coefficient developed mechanically is

$$(C_{h_0})_{BW,DS} = \frac{(HM)_{BW,DS}}{q S_e C_e} \tag{2}$$

The airplane pitching moment coefficient corresponding to the change in floating angle is

$$(\Delta C_m)_{FREE} = C_{m_\delta} (\Delta\delta)_{FREE} \tag{3}$$

Combining these relations yields

$$(\Delta C_m)_{FREE} = -\frac{(HM)_{BW,DS}\, C_{m_\delta}}{q S_e C_e C_{h_\delta}} \tag{4}$$

Stability contributions, expressed as stick-free neutral point displacements, are now evaluated by finding the rate of change of pitching moment with lift coefficient. The lift coefficient is defined in general form by

$$C_L = \frac{W/S\, n}{q} \tag{5}$$

In unaccelerated flight the load factor is maintained constant at $n = 1$ and the airspeed considered as a variable. Consequently, the pitching moment in terms of lift coefficient is

$$(\Delta C_m)_{n=1} = -\frac{(HM)_{BW,DS}\, C_{m_\delta}\, C_L}{W/S\; S_e\, C_e\, C_{h_\delta}} \tag{6}$$

and taking the derivative with respect to C_L gives

$$\left(\frac{dC_m}{dC_L}\right)_{n=1} = -\frac{(HM)_{BW,DS}\, C_{m_\delta}}{W/S\; S_e\, C_e\, C_{h_\delta}} \tag{7}$$

This expression then represents the shift in stick-free neutral point produced by either a bobweight or downspring hinge moment.

In maneuvering flight the airspeed is held fixed at some trim value and the angle of attack or normal load factor is varied. Referring to the general moment equation it can be seen that for the downspring there is no change in pitching moment for this condition and therefore the downspring has no effect on maneuvering stability. In the case of the bobweight, the change in moment coefficient is

$$(\Delta C_m)_{V=V_{TRIM}} = -\frac{(HM)_{BW}\, C_{m_\delta}}{q\; S_e\, C_e\, C_{h_\delta}} \tag{8}$$

and the derivative is

$$\left(\frac{dC_m}{dC_L}\right)_{V=V_{TRIM}} = -\frac{C_{m_\delta}}{q\; S_e\, C_e\, C_{h_\delta}} \times \frac{d(HM)_{BW}}{dC_L} \tag{9}$$

The rate of change of bobweight hinge moment with lift coefficient can be expressed as

$$\frac{d(HM)_{BW}}{dC_L} = \frac{d(HM)_{BW}}{dn} \times \frac{dn}{dC_L} \qquad (10)$$

where

$$\frac{d(HM)_{BW}}{dn} = (HM)_{BW(n=1)} \qquad (11)$$

and

$$\frac{dn}{dC_L} = \frac{q}{W/S} \qquad (12)$$

Substituting to obtain the final form,

$$\left(\frac{dC_m}{dC_L}\right)_{V=V_{TRIM}} = -\frac{(HM)_{BW}\, C_{m_\delta}}{W/S\; S_e\, C_e\, C_{h_\delta}} \qquad (13)$$

This term then represents the shift in stick-free maneuver point due to the bobweight and is seen to be identical to the shift in neutral point obtained with either a bobweight or downspring of equal strength. Therefore it may be stated for future reference, the bobweight does not alter the basic difference between the neutral and maneuver point locations whereas the downspring does.

Another auxiliary control device which has the very same stability effect as the downspring is the springy tab. The only distinction between the two is the fact that the downspring produces a stick force noticeable in ground handling and the springy tab has no effect at zero airspeed. This mechanism was not evaluated in the reported study since it was demonstrated that the dynamics depend on the static stability changes produced by the devices. On this basis if the springy tab is properly designed it should have the same influence on the dynamic characteristics as the downspring when producing the same change in static stability. The static equivalence between the springy tab and the downspring may be shown analytically.

The springy tab is a free elevator tab in combination with a spring which exerts a constant moment about the tab hinge line. At zero airspeed the tab is at its full up deflection and this position is maintained until the the airspeed increases to a particular value where the spring torque is balanced. For speeds in excess of this value the tab will deflect downward to some equilibrium floating angle. The hinge moment about the tab hinge line is given by

$$(HM)_{TAB} = K + C_{h_{t_{\delta_t}}} \delta_t \, q \, S_t \, C_t \tag{14}$$

where K is the spring torque in lb.-ft. and $C_{h_{t_{\delta_t}}}$ is the rate of change of tab hinge moment with tab deflection. Setting this expression equal to zero and solving for δ_t yields the tab floating angle,

$$\delta_t = - \frac{K}{C_{h_{t_{\delta_t}}} \, q \, S_t \, C_t} \tag{15}$$

The elevator hinge moment due to the tab deflection is

$$HM = C_{h_{e_{\delta_t}}} \delta_t \, q \, S_e \, C_e \tag{16}$$

where $C_{h_{e_{\delta_t}}}$ is the rate of change of elevator hinge moment with tab deflection. Substituting for the tab deflection yields

$$HM = - \frac{K \, C_{h_{e_{\delta_t}}} S_e \, C_e}{C_{h_{t_{\delta_t}}} S_t \, C_t} \tag{17}$$

This expression which is independent of airspeed shows that the elevator hinge moment produced by the springy tab is constant just as is that of the downspring. And when the springy tab installation is designed so as not to introduce weight moments about the elevator hinge line it has no effect on the maneuvering stability as in the case of the downspring.

B. Dynamic Stability

1. Exact Solution

The general longitudinal equations of motion of an airplane with the elevator free to rotate are derived in reference 1. In non-dimensional form the four linear differential equations are

Drag

$$(C_D + d)u - \frac{1}{2}(C_L - C_{D_\alpha})\alpha + \frac{C_L}{2}\theta = 0 \qquad (18)$$

Lift

$$C_L u + (\frac{1}{2}C_{L_\alpha} + d)\alpha - d\theta = 0 \qquad (19)$$

Pitching Moment

$$C_{m_u} u + (C_{m_\alpha} + C_{m_{d\alpha}} d)\alpha + (C_{m_{d\theta}} d - h d^2)\theta \qquad (20)$$
$$+ (C_{m_\delta} + C_{m_{d\delta}} d)\delta = 0$$

Elevator Hinge Moment

$$C_{h_u} u + [C_{h_\alpha} - (h_1 - C_{h_{d\alpha}})d]\alpha + [(h_1 + C_{h_{d\theta}})d \qquad (21)$$
$$- (h_2 + l_1)d^2]\theta + (C_{h_\delta} + C_{h_{d\delta}} d - h_2 d^2)\delta = 0$$

All the relations are referred to a time base of t/τ and the operator is defined as

$$d = \frac{d}{d(t/\tau)}$$

The qualifying assumptions involved in the development of these expressions are that the motion generated in response to some initial disturbance or control input is small in amplitude, motions depend on displacement and velocity dependent derivatives only and not acceleration terms and also that these partial derivatives or aerodynamic characteristics are constants for a given airplane thus making the differential equations linear. These are major considerations in the analytical treatment of the flight dynamics and the extent to which these conditions apply determines the degree of agreement with the actual airplane motion.

Appropriate modifications to the general equations are first made for the subject study. The pitching moment due to the rate of elevator deflection, $C_{m_{d\delta}}$, was estimated for the Navion and found to be negligible. The change in pitching moment with flight velocity, C_{m_u}, may be disregarded at this point since compressibility effects are not involved and power contributions to the pitching moment are usually small. At a later stage in the analysis it is shown how possible power effects can be taken into account. Velocity dependent hinge moment terms encompassed by C_{h_u} would normally be neglected for the same reasons. However, in this study the control devices produce an effective C_{h_u}. The equations of motion as given are for no inputs; that is they represent motion in return to trim following an instantaneous disturbance. Hinge moments from force devices constitute a steady elevator input as a right-hand term in the elevator equation. The hinge moment derivative is then a function of the velocity and can be derived consistent with the general equations as follows:

$$(HM)_{GW\,DS} = C_{h_0} \tfrac{1}{2} \rho V^2 S_e C_e \tag{22}$$

Finding the partial derivative with respect to velocity

$$\frac{\partial HM}{\partial V} = C_{h_0} \rho V S_e C_e \tag{23}$$

Dividing by $\frac{1}{2}\rho V^2 S_e C_e$ and multiplying by ΔV yields $2C_{h_0} u$ as the right-hand term. Transposing in the elevator equation, it is apparent that $C_{h_u} = -2C_{h_0}$. After making the preceding changes the complete equations become

Drag

$$(C_D + d)u - \tfrac{1}{2}(C_L - C_{D_\alpha})\alpha + \frac{C_L}{2}\theta = 0 \tag{24}$$

Lift

$$C_L u + \left(\tfrac{1}{2}C_{L_\alpha} + d\right)\alpha - d\theta = 0 \tag{25}$$

Pitching Moment

$$(C_{m_\alpha} + C_{m_{d\alpha}} d)\alpha + (C_{m_{d\theta}} d - I_1 d^2)\theta + C_{m_\delta}\delta = 0 \tag{26}$$

Elevator Hinge Moment

$$-2C_{h_0}u + [C_{h_\alpha} - (h_1 - C_{h_{d\alpha}})d]\alpha + [(h_1 + C_{h_{d\theta}})d - (h_2 + l_1)d^2]\theta + (C_{h_\delta} + C_{h_{d\delta}} d - h_2 d^2)\delta = 0 \tag{27}$$

A simultaneous solution of the revised equations follows the standard procedure given in reference 1 and yields a sixth order characteristic equation from which the period and damping of each of the three modes of motion can be determined. Usually these equations are solved in two separate phases so that fourth order characteristics can be obtained instead of the cumbersome sixth order form. In the first stage it is assumed that the elevator is fixed, thereby eliminating the elevator equation and elevator terms from the remaining equations. This approach affords an examination of the short period and the phugoid modes. The second stage is based on the velocity considered constant, which leads to the exclusion of all velocity terms and a characteristic equation representing the short period and elevator modes. Neither of these simplifications was utilized because particular interest was centered on the phugoid as affected by the free elevator; and the usually accepted notion of the phugoid being relatively independent of the elevator motion was not believed to be valid for systems involving fairly strong hinge moment devices. On this basis a preliminary analysis was undertaken, applying the more complete solution, to study the interdependence of the various derivatives and enable the further reduction of the sixth order form. A number of solutions were made for the modified Navion using estimated aerodynamic characteristics and considering the airplane trimmed in level flight at an indicated airspeed of 110 mph and an altitude of 8,000 feet. Solutions for various static stability margins were obtained both analytically and by an analog computer. In the computer phase of the work the analog circuitry duplicated the revised equations as given and combined them into a single dynamic system. The coefficients of the characteristic equation required for the analytical computations were determined and they are defined as follows:

$$\lambda^6 + A\lambda^5 + B\lambda^4 + C\lambda^3 + D\lambda^2 + E\lambda + F = 0 \tag{28}$$

$$A = C_D + C_{L_\alpha}/2 - C_{h_{d\delta}}/h_2 - \frac{1}{h}\left(C_{m_{d\theta}} + C_{m_{d\alpha}}\right) \tag{29}$$

$$\begin{aligned} B = {} & C_{m_\delta}(h_2 + l_1)/hh_2 + C_{L_\alpha}/2\left(C_D - C_{h_{d\delta}}/h_2 - C_{m_{d\theta}}/h\right) \\ & + \frac{1}{hh_2}\left(C_{h_{d\delta}} - h_2 C_D\right)\left(C_{m_{d\theta}} + C_{m_{d\alpha}}\right) - \frac{1}{h_2}\left(C_{h_\delta} + C_D C_{h_{d\delta}}\right) \\ & - C_{m_\alpha}/h + C_L/2\left(C_L - C_{D_\alpha}\right) \end{aligned} \tag{30}$$

$$\begin{aligned} C = {} & \frac{1}{hh_2}\Big\{ C_{m_\delta}\big[(h_2 + l_1)(C_D + C_{L_\alpha}/2) + (h_1 - C_{h_{d\alpha}}) \\ & - (h_1 + C_{h_{d\theta}})\big] + (C_{m_{d\theta}} + C_{m_{d\alpha}})(C_{h_\delta} + C_D C_{h_{d\delta}}) \\ & - C_{m_{d\alpha}} h_2 C_L^2/2 + C_{m_\alpha}(C_{h_{d\delta}} - h_2 C_D)\Big\} \\ & + C_{L_\alpha}/2\big[C_{h_{d\delta}}(C_{m_{d\theta}} - h C_D)/hh_2 - C_{h_\delta}/h_2 \\ & - C_D C_{m_{d\theta}}/h\big] - C_L/2(C_L - C_{D_\alpha})(C_{h_{d\delta}}/h_2 + C_{m_{d\theta}}/h) \\ & - C_D C_{h_\delta}/h_2 \end{aligned} \tag{31}$$

$$\begin{aligned} D = {} & \frac{1}{hh_2}\Big\{ C_{L_\alpha}/2\Big\{C_{m_\delta}\big[C_D(h_2 + l_1) - (h_1 + C_{h_{d\theta}})\big] \\ & + C_{m_{d\theta}}(C_{h_\delta} + C_D C_{h_{d\delta}}) - C_D C_{h_\delta} h\Big\} + C_L/2(C_L - C_{D_\alpha}) \\ & \big[C_{m_\delta}(h_2 + l_1) - C_{h_\delta} h + C_{h_{d\delta}} C_{m_{d\theta}}\big] + C_{m_{d\alpha}}(C_D C_{h_\delta} \\ & + C_{h_{d\delta}} C_L^2/2) + C_{m_\alpha}(C_{h_\delta} + C_D C_{h_{d\delta}} - h_2 C_L^2/2) \\ & + C_{m_{d\theta}} C_D C_{h_\delta} + C_{m_\delta}\Big\{C_D\big[(h_1 - C_{h_{d\alpha}}) - (h_1 + C_{h_{d\theta}})\big] \\ & - C_{h_\alpha}\Big\}\Big\} \end{aligned} \tag{32}$$

$$\begin{aligned} E = {} & \frac{1}{hh_2}\Big\{\frac{1}{2}(C_L - C_{D_\alpha})\Big\{C_{m_\delta}\big[2C_{h_0} - C_L(h_1 + C_{h_{d\theta}})\big] \\ & + C_{m_{d\theta}} C_{h_\delta} C_L\Big\} - C_{L_\alpha} C_D/2\big[C_{m_\delta}(h_1 + C_{h_{d\theta}}) \\ & - C_{h_\delta} C_{m_{d\theta}}\big] + C_{m_\alpha}(C_D C_{h_\delta} + C_{h_{d\delta}} C_L^2/2) + C_{m_\delta}\Big\{C_L/2 \\ & \big[C_L(h_1 - C_{h_{d\alpha}}) - 2C_{h_0}\big] - C_D C_{h_\alpha}\Big\} + C_{h_\delta} C_{m_{d\alpha}} C_L^2/2\Big\} \end{aligned} \tag{33}$$

$$F = C_L / 2hh_2 \left[-C_{m_\delta} \left(C_{h_\alpha} C_L + C_{L_\alpha} C_{h_c} \right) + C_{h_\delta} C_{m_\alpha} C_L \right] \quad (34)$$

2. Simplified Solution

An examination of the exact solution results indicated that the elevator mode was sufficiently damped to permit the omission of the damping and inertia terms, $C_{h_{d\delta}}$ and h_2 respectively, from the hinge moment equation with a negligible loss in accuracy. This condition applied even when bobweight systems producing large hinge moments and accompanying increases in elevator inertia were considered. With the elevator equation simplified the characteristic reduces to a fourth order which can be solved fairly conveniently by a number of standard mathematical methods. It should be pointed out that the simplification eliminates the elevator mode but does include the effects of the free elevator on the remaining short period and phugoid modes. The coefficients of the quartic in terms of the familiar aerodynamic quantities were found to be

$$\lambda^4 + A\lambda^3 + B\lambda^2 + C\lambda + D = 0 \quad (35)$$

$$A = C_D + C_{L_\alpha}/2 - \frac{1}{h - \frac{C_{m_\delta}}{C_{h_\delta}}(h_2 + l_1)} \left[C_{m_{d\theta}} + C_{m_{d\alpha}} - \frac{C_{m_\delta}}{C_{h_\delta}} \left(C_{h_{d\theta}} + C_{h_{d\alpha}} \right) \right] \quad (36)$$

$$B = \frac{C_L}{2}\left(C_L - C_{D_\alpha}\right) + C_D C_{L_\alpha}/2 - \frac{1}{h - \frac{C_{m_\delta}}{C_{h_\delta}}\left(h_2 + \ell_1\right)} \left\{C_{m_\alpha} - \frac{C_{m_\delta}}{C_{h_\delta}} C_{h_\alpha} + \left(C_D + C_{L_\alpha}/2\right)\left[C_{m_{d\theta}} - \frac{C_{m_\delta}}{C_{h_\delta}}\left(h_1 + C_{h_{d\theta}}\right)\right] + C_D\left[C_{m_{d\alpha}} + \frac{C_{m_\delta}}{C_{h_\delta}}\left(h_1 - C_{h_{d\alpha}}\right)\right]\right\} \tag{37}$$

$$C = - \frac{1}{h - \frac{C_{m_\delta}}{C_{h_\delta}}\left(h_2 + \ell_1\right)} \left\{C_D \left\{C_{m_\alpha} - \frac{C_{m_\delta}}{C_{h_\delta}} C_{h_\alpha} + C_{L_\alpha}/2\left[C_{m_{d\theta}} - \frac{C_{m_\delta}}{C_{h_\delta}}\left(h_1 + C_{h_{d\theta}}\right)\right]\right\} - C_L C_{D_\alpha} \left\{\frac{1}{2}\left[C_{m_{d\theta}} - \frac{C_{m_\delta}}{C_{h_\delta}}\left(h_1 + C_{h_{d\theta}}\right)\right] + \frac{C_{h_c} C_{m_\delta}}{C_L C_{h_\delta}}\right\} + \frac{C_L^2}{2}\left[C_{m_{d\theta}} + C_{m_{d\alpha}} - \frac{C_{m_\delta}}{C_{h_\delta}}\left(C_{h_{d\theta}} + C_{h_{d\alpha}}\right)\right]\right\} \tag{38}$$

$$D = - \frac{1}{h - \frac{C_{m_\delta}}{C_{h_\delta}}\left(h_2 + \ell_1\right)} \frac{C_L^2}{2}\left(C_{m_\alpha} - \frac{C_{m_\delta}}{C_{h_\delta}} C_{h_\alpha} - \frac{C_{L_\alpha} C_{h_c} C_{m_\delta}}{C_L C_{h_\delta}}\right) \tag{39}$$

Effects of force systems are accounted for by: C_{h_c} , the hinge moment coefficient due to devices and in the general sense that resulting as well from either aerodynamic causes or elevator mass unbalance (effective bobweight), h_1 the elevator mass unbalance as acted upon by the airplane's linear acceleration, and ℓ_1 , the elevator mass unbalance as acted upon by the airplane's angular acceleration. Since the systems under study are essentially stick-free static stability controls, a rearrangement of the coefficient equations in terms of familiar static parameters and simplified dynamic terms is desirable. This would facilitate the dynamic analysis by clearly indicating the contributory effects of the static stability margins and by reducing the lengthy mathematical computations.

Applying the relationships, as given in reference 1,

$$C_{m_{d\epsilon}} = \frac{C_{m_\delta}}{\tau_e} \frac{l_t}{\mu c} \tag{40}$$

$$C_{m_{d\alpha}} = C_{m_{d\epsilon}} \frac{d\epsilon}{d\alpha} \tag{41}$$

$$C_{h_{d\epsilon}} = C_{h_{\alpha_t}} \frac{l_t}{\mu c} \tag{42}$$

$$C_{h_{d\alpha}} = C_{h_{d\epsilon}} \frac{d\epsilon}{d\alpha} \tag{43}$$

results in the following equality

$$\begin{aligned} &C_{m_{d\epsilon}} + C_{m_{d\alpha}} - \frac{C_{m_\delta}}{C_{h_\delta}}\left(C_{h_{d\epsilon}} + C_{h_{d\alpha}}\right) \\ &= \left(C_{m_{d\epsilon}} + C_{m_{d\alpha}}\right)\left(1 - \tau_e \frac{C_{h_{\alpha_t}}}{C_{h_\delta}}\right) \end{aligned} \tag{44}$$

This value is seen to be a measure of the aerodynamic damping of the airplane as modified by the free elevator and is referred to as $\left(C_{m_{d\epsilon}} + C_{m_{d\alpha}}\right)'$. In a like manner, the quantity $h_1 - \frac{C_{m_\delta}}{C_{h_\delta}}\left(h_2 + l_1\right)$ is identified as the effective airplane inertia when modified by a free elevator including mass unbalance and is denoted by h'.

The stick-free level-flight and maneuver margins can be expressed as a function of repeated groupings of related terms in the coefficient equations. From reference 1, freeing the elevator shifts the stick-fixed neutral point forward as given by

$$N_0' - X_{CG} = \left(N_0 - X_{CG}\right) - \left(N_0 - N_0'\right) \tag{45}$$

where

$$N_0 - X_{cg} = -\frac{C_{m_\alpha}}{C_{L_\alpha}} \tag{46}$$

$$N_0 - N_0' = -\frac{C_{m_\delta} C_{h_\alpha}}{C_{L_\alpha} C_{h_\delta}} \tag{47}$$

Considering the dynamic modes at essentially unaccelerated flight, $C_L = \frac{W/S}{q}$, and the rearward shift in neutral point caused by a bobweight or downspring can be developed in coefficient form as

$$\Delta(N_0')_{BW,DS} = -(\Delta C_{m_{C_L}})_{BW,DS} = \frac{(HM)_{BW,DS}\, C_{m_\delta}}{W/S\; S_e\, c_e\, C_{h_\delta}} \tag{48}$$

$$(HM)_{BW,DS} = C_{h_c}\, q\, S_e\, c_e \tag{49}$$

$$\Delta(N_0')_{BW,DS} = \frac{C_{h_c}\, C_{m_\delta}}{C_L\, C_{h_\delta}} \tag{50}$$

Thus, the general expression for the level-flight stability margin may be written as

$$N_0' - X_{cg} = -\frac{C_{m_\alpha}}{C_{L_\alpha}} + \frac{C_{m_\delta} C_{h_\alpha}}{C_{L_\alpha} C_{h_\delta}} + \frac{C_{h_c} C_{m_\delta}}{C_L\, C_{h_\delta}} \tag{51}$$

$$N_0' - X_{cg} = -\frac{1}{C_{L_\alpha}}\left(C_{m_\alpha} - \frac{C_{m_\delta} C_{h_\alpha}}{C_{h_\delta}} - \frac{C_{L_\alpha} C_{h_c} C_{m_\delta}}{C_L\, C_{h_\delta}}\right) \tag{52}$$

The rearward displacement of the stick-free maneuver point from the neutral point is given in reference 1 as

$$N_m' - N_0' = \frac{l_t\, \rho\, g}{2\; W/S}\,\frac{C_{m_\delta}}{C_{h_\delta}}\left(C_{h_{\alpha_t}} - \frac{11\; C_{h_\delta}}{\tau_e}\right) \tag{53}$$

If the damping contributed by components other than the horizontal tail is neglected and the relations for $C_{m_{d\theta}}$, $C_{m_{d\alpha}}$ and $\mu = \frac{W/S}{g \rho c}$ are applied, this margin becomes

$$N_m' - N_0' = -\frac{1}{2}\left(C_{m_{d\theta}} - \frac{C_{m_\delta}}{C_{h_\delta}}\, C_{h_{d\theta}}\right) \tag{54}$$

The shift in the maneuver point due to bobweight or mass unbalance effects can be expressed as

$$\Delta(N_m')_{BW} = \frac{C_{h_o} C_{m_\delta}}{C_L C_{h_\delta}} = \frac{1}{2} \frac{C_{m_\delta}}{C_{h_\delta}} h_1 \tag{55}$$

by using the following equalities:

$$(HM)_{BW} = g\, m_e x_e \tag{56}$$

$$h_1 = \frac{2 \mu_e x_e}{C \mu} \tag{57}$$

$$\mu_e = \frac{m_e}{\rho S_e C_e} \tag{58}$$

And including the neutral point displacement due to bobweights or downsprings as previously given, the following generalized representation of $N_m' - N_o'$ is obtained:

$$N_m' - N_o' = -\left\{ \frac{1}{2} \left[C_{m_{d\theta}} - \frac{C_{m_\delta}}{C_{h_\delta}} \left(h_1 + C_{h_{d\theta}} \right) \right] + \frac{C_{h_c} C_{m_\delta}}{C_L C_{h_\delta}} \right\} \tag{59}$$

For mass balanced elevator systems employing a downspring only, $h_1 = 0$ and when a bobweight is introduced its contribution to the C_{h_c} term is nullified by the h_1 term as shown. Consequently this form is presented to retain the general meaning of C_{h_o} as the sum of applied hinge moment coefficients from all causes.

The stick-free maneuver margin can now be determined from

$$N_m' - X_{CG} = (N_o' - X_{CG}) + (N_m' - N_o') \tag{60}$$

and using the derived relationships,

$$N_m' - X_{CG} = -\frac{1}{C_{L_\alpha}} \left\{ C_{m_\alpha} - \frac{C_{m_\delta}}{C_{h_\delta}} C_{h_\alpha} + \frac{1}{2} C_{L_\alpha} \left[C_{m_{d\theta}} - \frac{C_{m_\delta}}{C_{h_\delta}} \left(h_1 + C_{h_{d\theta}} \right) \right] \right\} \tag{61}$$

By substituting all the foregoing static stability and damping equivalences in the coefficient equations for the stability quartic, the four expressions reduce to

$$A = C_D + \frac{C_{L_\alpha}}{2} - \frac{1}{h'}\left(C_{m_{d\alpha}} + C_{m_{d\theta}}\right)' \tag{62}$$

$$B = \frac{C_L}{2}\left(C_L - C_{D_\alpha}\right) + \frac{C_D C_{L_\alpha}}{2} + \frac{1}{h'}\Big[C_{L_\alpha}\left(N_m' - X_{cg}\right) - C_D\left(C_{m_{d\theta}} + C_{m_{d\alpha}}\right)'\Big] \tag{63}$$

$$C = \frac{1}{h'}\Big[C_D C_{L_\alpha}\left(N_m' - X_{cg}\right) - C_L C_{D_\alpha}\left(N_m' - N_0'\right) - \frac{C_L^2}{2}\left(C_{m_{d\theta}} + C_{m_{d\alpha}}\right)'\Big] \tag{64}$$

$$D = \frac{1}{h'}\left(\frac{C_L^2 C_{L_\alpha}}{2}\right)\left(N_0' - X_{cg}\right) \tag{65}$$

The equations can be somewhat further simplified by omitting terms which are small for the conditions of a particular study. The selection of cruising or high speed operation for an airplane with an unswept moderate aspect ratio wing will permit C_D to be neglected relative to $\frac{1}{2} C_{L_\alpha}$ as well as the product of C_D and other comparitively small terms (such as the last quantity in the equation for B). In addition, as a first approximation ℓ_1 may be considered as zero for balanced elevators with bobweight systems located near the center of gravity and h_2 may be neglected relative to h. These changes to the coefficient equations were employed in the subject investigation. Power effects may be taken into account by including the C_{m_u} term in the pitching moment equation of motion. This results in the addition of the increment $\Delta(N_0') = \frac{C_{m_u}}{2 C_L}$ to the expression for the level-flight stability margin. As the maneuvering stability applies to constant velocity operation it is unaffected by power variations.

3. Theoretical Results

An extensive computational program was completed using the simplified solution to obtain the short period and phugoid mode characteristics as a function of the independent variables, $N_m' - X_{CG}$ and $N_m' - N_c'$. With these parameters as the prime criteria for the theoretical analysis the effects of center of gravity displacement, bobweights and downsprings can be isolated for study in a manner consistent with the objectives of the test program. As has been demonstrated for a given airplane, the maneuver margin can be controlled by a combination of center of gravity location and bobweight strength and the difference between the maneuver and neutral points can be controlled solely by a downspring. For convenience, the increment $N_m' - N_c'$ is called the static damping parameter since this quantity is dependent upon the aerodynamic damping in pitch for airplanes without control devices and it also is an important factor affecting the damping of the dynamic characteristics.

A single operating condition was considered for the modified Navion airplane. This was trimmed level flight at an indicated airspeed of 110 mph and an altitude of 8,000 feet with the gross weight assumed constant at 2,500 lbs. Values of $N_m' - X_{CG}$ and $N_m' - N_o'$ were selected (thus fixing $N_o' - X_{CG}$) covering a wide range of stability margins and the coefficients of the characteristic equation were then determined using estimated values of the other aerodynamic terms in the simplified equations. The approximations for the aerodynamic and physical parameters applied in this analysis are given in Appendix II. Solutions for the roots of approximately eighty stability quartics involved were obtained by a semi-empirical method which allowed a good degree of accuracy without incurring any unreasonably lengthy computational work. Conversions were made of negative real roots and negative real parts of complex roots to the reciprocal of the time to damp to half amplitude, positive real roots and positive real parts to the reciprocal of the time to diverge to double amplitude, and the imaginary parts of complex roots to oscillatory period. Inverse quantities are used, similar to root loci, so that the damping characteristics can be plotted as continuous functions through the neutral stability boundary, thereby simplifying the definition of all trends. The complete theoretical results are presented in Figure 4 in the form of damping and period as a function of $N_m' - X_{CG}$ for various fixed values of $N_m' - N_0'$. The stick-free maneuver margin range extends from large positive or highly stable values to large negative or highly unstable ones. The magnitude of the static damping parameter varies from .04, comparable to the modified Navion configuration without downspring, to -.20, that obtained with a very strong downspring. Oscillatory motion, determined by real parts of complex roots, is identified by dotted lines and aperiodic motion, determined from real roots, is identified by solid lines.

An examination of the damping characteristics in the upper part of the plot reveals a number of significant facts. First, the short period mode has no importance in the subject study. In almost the entire stable maneuver margin range this mode is independent of both parameters and maintains a constant damping characteristic of the Navion. At maneuver margins less than $N_m' - X_{CG} = .048$, it is shown as a convergent aperiodic but actually has no bearing on the resultant airplane motion because the phugoid mode is then moderately or highly divergent and thus it has the predominating effect. Second, the suspicions that prompted the program are substantiated since both bobweights and downsprings have a considerable influence on airplane dynamics. This is made evident by the phugoid curve family which rapidly deteriorates from the normal damped oscillation at moderately stable maneuver margins to highly unstable oscillations and eventually pure divergences at reduced levels of stability. And finally, the quantity $N_m' - N_0'$ is seen to represent a generalized parameter of importance in the study of dynamic stability effects. Of only academic interest are the indications that the short period and phugoid modes are joined by two aperiodic branches which subsequently develop into a single oscillatory mode. These motions are completely dominated, as is the convergent short period branch, by the concurrent highly divergent phugoid.

The period trends shown in the lower part of the plot correspond as would be expected to the associated damping curves -- in the sense that as the oscillation becomes increasingly divergent the period increases, asymptoticaly approaching infinity at the point where the aperiodic motion originates. With reference to the normal short period mode, decreasing the maneuver margin increases the period and variations in the static damping parameter alter the phugoid slightly.

It is evident from the results presented in Figure 4 that the correction of rearward center of gravity locations by progressively applying bobweights has a continuously favorable effect on the phugoid damping. Whereas when downsprings are introduced the damping is improved until some optimum spring strength is reached and then additional increments of downspring cause the damping to become progressively reduced. The overall trends indicate that the bobweight is a considerably more powerful device in the control of the dynamic characteristics than is the downspring. This is clearly borne out by the fact that as the maneuver margin is made less stable the phugoid rapidly increases in divergence. Changes in the static damping parameter displace these variations but do not alter the general nature of the unstable trend. In order to more conveniently isolate these effects with reference to stability boundaries and correlate them with the flight data, crossplots of the phugoid curves were made with lines of constant damping times and period constructed on a $N_m' - N_o'$ versus $N_m' - X_{cg}$ grid. The data were obtained by interpolation and are limited to the portion of the complete plot which could practically be explored in the flight program. The damping crossplot is given in Figure 5 and the period crossplot is given in Figure 6.

Three separate, fairly well defined stability regions were established as seen in Figure 5. These consist of aperiodic divergence, damped oscillations and undamped or divergent oscillations. The important fact indicated by this plot is that if a damped or essentially neutrally stable phugoid is to be maintained the maneuver margin must be stable and the static damping parameter must be positive (unless the maneuvering stability is moderately high in which case sufficient spring may be introduced to cause the damping parameter to become negative). It can be seen that increasing the bobweight strength or positive $N_m' - X_{CG}$ values displaces the phugoid from the pure divergent region to either the damped oscillatory one, at positive $N_m' - N_o'$ values representing little or no downspring, or to the undamped oscillatory one, at negative $N_m' - N_c'$ values representing moderate or heavy downsprings. Increasing the downspring magnitude when the maneuver margin is moderately or highly stable displaces the phugoid from a damped oscillatory region to an undamped one. At low positive maneuver margins adding downspring displaces the phugoid from the divergent aperiodic region to first the damped oscillatory one and finally to the undamped oscillatory region. At negative maneuver margins the displacement is from the divergent aperiodic region to the undamped oscillatory region.

The period crossplot of Figure 6 shows that the trends of the constant period lines are consistent with that of the aperiodic boundary as would be expected. In the oscillatory region considered the period varies from 12 to 60 seconds.

4. Analog Computer Studies

A number of theoretical solutions of the modified Navion dynamics were also obtained by analog computer in order to observe the combined-mode transient response of the airplane, including any peculiarities of the overall motion not evident in the analytical treatment, as well as to make spot checks of the responses measured in the test phase. For this purpose aerodynamic derivatives extracted from static flight data were used in conjunction with the complete equations of motion. The experimentally derived quantities and the reduction methods applied are presented in Appendix III. Phugoid responses to constant elevator inputs were obtained for various force device arrangements and the same operating conditions were considered as in the analytical analysis. The results for six different dynamic runs, transformed into comparable inputs, are presented in the summary of flight data for comparative study.

VIII. FLIGHT TEST PROCEDURE

A. Static Testing

A series of static stability tests were conducted at various center of gravity positions with different magnitudes of bobweight and downspring in order to determine the stick-free static stability characteristics required for the dynamic analysis. Force gradients were measured for both level-flight and maneuvering flight to establish the effectiveness of the force devices in the control of the stick-free neutral and maneuver point locations For the sake of completeness elevator deflection variations were also measured to obtain the stick-fixed neutral and maneuver point locations. All straight and level runs were made at indicated airspeeds ranging from 80 to 140 mph with the control force trimmed at 110 mph. The maneuvering tests were made in steady accelerated turns flown at normal load factors ranging from 1.2 "g"s to 2.0 "g"s with the indicated airspeed held constant at 110 mph and the control force trimmed for straight and level flight. An average pressure altitude of 5,000 feet was maintained during all testing.

Speed points and accelerated turns were carefully stabilized, gradients were given rough checks in flight and repeat runs were made where necessary. Airspeed, normal load factor and indicator readings of elevator deflection and stick force were visually recorded during the tests.

Preliminary flight checks were made to establish the constancy of the bobweight and downspring hinge moments over the test range of stick deflection. It was also confirmed that the downspring did not affect the maneuvering stability. Sufficient data were then taken to accurately locate: the neutral and maneuver points for the balanced elevator without devices, the neutral points for bobweight and downspring combinations inducing up to 32 lbs. of equivalent stick force, and the maneuver points for bobweight strengths up to 24 lbs., all in control force increments of 8 lbs. each.

B. Dynamic Testing

The general method of dynamic testing employed is that duplicating as closely as possible the theoretical case of transient response to a step input in elevator control. This was done by trimming the airplane at an indicated airspeed of 110 mph in straight and level flight at 5,000 ft., then applying a sufficient steady control force to attain equilibrium at a speed between 2 and 5 mph above or below the trim velocity and then instantaneously releasing the stick. Airspeed time histories were obtained by photorecorder for the full duration of each phugoid response. Considerable care was taken to achieve the trim speed accurately and avoid perceptible pitch inputs and gust disturbances so that true responses could be recorded.

In order to substantiate the significant theoretical trends, it was planned to explore as great a portion of the static stability grid of Figure 5 as possible. The actual test conditions were somewhat limited by several practical considerations. Center of gravity locations were restricted to positions forward of 36.3% MAC due to a critical shortage of down elevator deflection required for recovery from rapid divergences. The positions for the dynamic tests were at approximately 32%, 34% and 36% MAC. In addition, the maximum effectiveness of the modified elevator tab limited the total control force imposed by the devices to 32 lbs. Individual bobweight and downspring forces were varied from 0 to 24 lbs. and numerous phugoid responses were recorded for various combinations of each at the three center of gravity locations.

IX. FLIGHT DATA ANALYSIS

A. Static Stability

Indicator readings of stick force and elevator deflection were transformed to actual values using current calibrations. Lift coefficients and free stream dynamic pressures were computed for the level-flight runs from the take-off weight and the indicated airspeed readings. Plots were then constructed of: F_s/q versus C_L for each combination of center of gravity and total control force due to the bobweight and downspring, F_s versus n for each combination of center of gravity and bobweight force, and δ versus C_L and δ versus n for each center of gravity. The slopes of all the faired variations or gradients representing a measure of the static stability, were then measured. Typical flight data are shown in Figure 7. It can be seen that linearity is obtained in the stick-fixed curves and the maneuver force gradient. With regard to the level-flight force characteristics some variation in C_{h_δ} with elevator deflection is evident. The deviation from the straight slope for other tests varied more or less from the example presented depending upon the particular range of elevator deflections involved. By taking the slope of the F_s/q curve at trim this tendency does not impair the accuracy of the stability measurement.

All force and elevator gradients were compiled in summary plots as a function of center of gravity location. These results are given in Figure 8. The stick-fixed and stick-free neutral and maneuver points are indicated at the zero gradient intercepts. To aid in the analysis the curves were faired in accordance with the required relationship between the slopes of $dF_s/q/dC_L$ and dF_s/dn versus center of gravity. A similar procedure was followed as a guide in fairing the test points for the case of $d\delta/dC_L$ and $d\delta/dn$ as a function of center of gravity. It can be shown that

$$\frac{d^2 F_s/q}{dC_L\, dX_{cg}} \times \frac{W}{S} = \frac{d^2 F_s}{dn\, dX_{cg}} \tag{66}$$

$$\frac{d^2 \delta}{dC_L\, dX_{cg}} \times C_L = \frac{d^2 \delta}{dn\, dX_{cg}} \tag{67}$$

where $\frac{W}{S}$, the average wing loading is 15.2 lbs./ft.^{2} and C_L, the trimmed lift coefficient, is 0.49. The overall correlation of the static results is satisfactory; showing relatively little data scatter, linear stability gradients and agreement between the neutral and maneuver point displacements caused by equal increments of device force. The static stability characteristics are therefore considered as having been accurately determined.

B. Dynamic Stability

Approximately eighty phugoid runs were recorded for twenty-nine different combinations of maneuver margin and static damping parameter. Faired envelopes of each oscillatory response were constructed in order to qualitatively evaluate the damping and period characteristics. It was evident that gust inputs during the course of the motion had caused discrepancies among the various records for the same test conditions. A judicious selection of the most reasonable indications was made on the basis of a cyclic consistency approaching a constant period and a continuous change in amplitude. Aperiodic motions being of short duration were not as subject to gust effects and showed satisfactory repeatability. The velocity time histories selected for each condition of static stability are presented in Figures 9 through 15. For comparison purposes most of the original data were proportionately transformed into velocity variations for a positive input of 5 mph. The actual mean velocity differed slightly from the 110 mph value due to the residual friction level and the difficulty in trimming when the force gradients are low.

Dynamic results have been arranged in static stability groupings to facilitate the correlation of the flight data with the theoretical trends previously described. The dynamic equivalence of bobweight and center of gravity effects on the maneuver margin is first demonstrated by a series of phugoid modes shown in Figure 9. In this study different combinations of bobweight and center of gravity giving approximately the same maneuver margin are compared for various static damping parameters. It can be seen that for a given value of downspring, equivalent modes are obtained when the maneuver margins are comparable regardless of the bobweight magnitude and center of gravity location. The discrepancies shown are as expected considering the actual differences in maneuver margins in each pair. In Figures 10 through 12 the static damping parameter is held constant at various values and the maneuver margins are progressively reduced. It is evident that reductions in the maneuvering stability have continuously deleterious effects on the phugoid mode. This trend is unaffected by the static damping parameter as shown for the zero downspring force in Figure 10, the moderate spring in Figure 11 and the heavy spring in Figure 12. In Figures 13 through 15 the maneuver margins are maintained approximately constant at several levels and the static damping parameter is progressively reduced by steadily increasing the downspring force. In Figure 13 it is indicated that when the maneuver margin is high the introduction of downspring forces has an adverse effect, changing the phugoid from a damped oscillation to an undamped oscillation. Figures 14 and 15 demonstrate the improvement in the dynamics attainable at low positive maneuver margins with a moderate application of downspring. It is apparent from the grouping in Figure 14 that further increases in downspring force have the effect of partially nullifying the original gains. On the basis of the foregoing results the theoretical effects of force devices on the dynamics are considered as having been qualitatively confirmed in the flight test phase.

For general interest the damping time and period of each phugoid response was measured and a quantitative comparison with the theoretical results was made. These results are included in Figures 5 and 6. This presentation offers an alternate indication of the qualitative agreement with theory and also shows an unexpected agreement in a quantitative sense. It was originally thought that the necessary theoretical assumption of motion limited to small perturbations and purely linear aerodynamic effects would make a quantitative check unlikely. The analog computer comparisons that are included in Figures 10, 11, 12 and 14 are also satisfactory considering these same factors.

X. CONCLUSIONS

When auxiliary control force devices such as bobweights and downsprings are used to obtain satisfactory static stability, consideration should be given to the resultant effects on the phugoid mode. Conversely, it may be possible to correct inherently undesirable phugoid characteristics by a judicious application of force devices.

A bobweight will have no adverse influence on the phugoid and is likely to produce appreciable improvement in the damping. A limited use of downspring may have favorable effects on the phugoid but an excessive application will induce strong instabilities. On the dynamic basis the bobweight is preferable to the downspring as a static stability control device.

After considerable flight operation with the center of gravity located aft of the stick-fixed neutral point the pilot concluded that no adverse effects due to the static instability were noticeable.

XI. RECOMMENDATIONS

To enlarge the scope of the general study of the phugoid it is recommended that stabilization means such as pitch damping mechanisms be investigated. It would also be advisable to extend the present study to include flight conditions other than cruising operation. In order to increase the practicality of the results derived from phugoid studies it is recommended that minimum requirements for satisfactory flying qualities be established for this mode.

XII. REFERENCES

1. Perkins, C. D. and Hage, R., "Airplane Performance Stability and Control" John Wiley and Sons, Inc. 1949.

XIII. NOMENCLATURE

c, MAC mean aerodynamic chord of wing, ft.

c_e root-mean-square chord of elevator, ft.

c_t root-mean-square chord of elevator trim tab, ft.

C_D airplane drag coefficient

C_h elevator hinge moment coefficient

C_L airplane lift coefficient

C_m pitching moment coefficient about airplane center of gravity

C_{h_0} applied hinge moment coefficient

F_s stick control force, positive for push, lbs.

F_{BW} stick force equivalent to bobweight hinge moment, lbs.

F_{DS} stick force equivalent to downspring hinge moment, lbs.

g acceleration due to gravity (32.2 ft./sec^2.)

h airplane inertia term $\left(\frac{2k_y^2}{\mu c^2}\right)$

h_1 elevator mass unbalance effect $\left(\frac{2m_e x_e}{\mu c}\right)$

h_2 elevator inertia term $\left(\frac{2m_e k_e^2}{\mu^2 c^2}\right)$

HM elevator hinge moment, positive when deflecting the trailing edge downward, lb.-ft.

k_e elevator radius of gyration about the hinge line, ft.

k_y airplane radius of gyration about the pitch axis, ft.

l_1 elevator mass unbalance term as affected by airplane pitching acceleration $\left(\frac{2m_e x_e l_t}{\mu^2 c^2}\right)$

l_t horizontal tail moment arm, ft.

m mass of airplane, slugs

m_e mass of elevator, slugs

N_0 stick-fixed neutral point location in terms of c

N_0' stick-free neutral point location in terms of c

N_m	stick-fixed maneuver point location in terms of C
N_m'	stick-free maneuver point location in terms of C
n	normal load factor, acceleration in terms of g
P	period of oscillation, sec.
q	free stream dynamic pressure, lbs./ft.^{2}
S	wing area, ft.^{2}
S_e	elevator area, ft.^{2}
S_t	elevator tab area, ft.^{2}
$T_{1/2}$	time required for oscillation to damp to half amplitude, sec.
T_2	time required for oscillation to double amplitude, sec.
t	time, sec.
u	flight velocity ratio $\left(\frac{\Delta V}{V}\right)$
V	true flight velocity, ft./sec.
ΔV	change in true velocity from trimmed value, ft./sec.
V_c	calibrated airspeed, ft./sec.
ΔV_c	change in calibrated airspeed from trimmed value, ft./sec.
W	airplane gross weight, lbs.
W/S	wing loading, lbs./ft.^{2}
X_{cg}	center of gravity location in terms of C
x_e	distance from hinge line to center of gravity of equivalent elevator mass, positive when aft of hinge line, ft.
α	airframe angle of attack, rad.
α_t	horizontal tail angle of attack, rad.
δ	elevator deflection, positive for trailing edge down, rad.
δ_t	trim tab deflection, positive for trailing edge down, rad.
ϵ	angle of downwash over horizontal tail, rad.
θ	angle of pitch of airplane, rad.
λ	root of stability equation

μ relative density term of airplane $\left(\frac{m}{\rho S c}\right)$

μ_e relative density term of elevator $\left(\frac{m_e}{\rho S_e c_e}\right)$

ρ free stream air density, slugs/ft.^{3}

τ time factor $\left(\frac{m}{\rho S V}\right)$

τ_e elevator effectiveness $\left(\frac{d\alpha_t}{d\delta}\right)$

Whenever u, α, δ, θ, $d\alpha$, $d\delta$ and $d\theta$ are used as subscripts, a derivative is indicated. For example, $C_{h_\alpha} = \frac{\partial C_h}{\partial \alpha}$ and $C_{h_{d\alpha}} = \frac{\partial C_h}{\partial (d\alpha)}$

$N_m' - N_o'$ static damping parameter.

APPENDIX I

PHYSICAL CHARACTERISTICS OF MODIFIED NAVION AIRPLANE

Wing	
Area	184.2 $ft.^2$
Mean Aerodynamic Chord	5.70 ft.
Aspect Ratio	6.04
Span	33.36 ft.
Root Chord	7.22 ft.
Tip Chord	3.83 ft.
Airfoil Section	
Root	NACA 4415 R
Tip	NACA 6410 R
Horizontal Tail	
Area	35.2 $ft.^2$
Mean Chord	3.48 ft.
Aspect Ratio	2.96
Span	10.22 ft.
Root Chord	4.00 ft.
Tip Chord	2.89 ft.
Tail Moment Arm	14.93 ft.
Elevator	
Area	12.3 $ft.^2$
Root-Mean-Square Chord	1.23 ft.
Span	10.22 ft.
Root Chord	1.40 ft.
Tip Chord	1.03 ft.
Aerodynamic Balance	none

APPENDIX II

ESTIMATED PARAMETERS APPLIED IN THE ANALYTICAL ANALYSIS

Operating Conditions

V_c = 110 mph

Alt. = 8,000 ft.

W = 2,500 lbs.

C_L = .556

C_D = .0443

C_{L_α} = 4.58 rad.$^{-1}$

C_{D_α} = .326 rad.$^{-1}$

C_{m_δ} = -.674 rad.$^{-1}$

$C_{m_{\dot{\alpha}}}$ = -.0818 rad.$^{-1}$

$C_{m_{d\alpha}}$ = -.0312 rad.$^{-1}$

$C_{h_{\alpha_t}}$ = -.206 rad.$^{-1}$

C_{h_δ} = -.607 rad.$^{-1}$

τ_e = .55

h = .025

h_2 = 0

ℓ_1 = 0

APPENDIX III

EXPERIMENTAL DERIVATION OF PARAMETERS APPLIED IN ANALOG COMPUTER STUDIES

Operating Conditions

$$V_c = 110 \text{ mph}$$

$$\text{Alt.} = 5{,}000 \text{ ft.}$$

$$W = 2{,}800 \text{ lbs.}$$

C_{L_α}, C_L

Based on the wing planform and section characteristics the airplane lift curve slope is estimated as

$$C_{L_\alpha} = 4.58 \text{ rad.}^{-1}$$

The trimmed lift coefficient is

$$C_L = \frac{W/S}{q} \qquad \text{(III-1)}$$

$$W/S = 15.2 \text{ lbs./ft.}^2$$

$$q = \frac{(110)^2}{391}$$

$$= 31.0 \text{ lbs./ft.}^2$$

$$C_L = \frac{15.2}{31.0}$$

$$= .49$$

C_{D_α}, C_D

The drag coefficient derivative is given by

$$C_{D_\alpha} = C_{D_{C_L}} \times C_{L_\alpha} \qquad \text{(III-2)}$$

The airplane drag equation as determined from gliding flight tests at windmilling power and from the estimated propeller characteristics is

$$C_D = .0290 + .0605\, C_L^2 \qquad \text{(III-3)}$$

$$C_{D_{C_L}} = .121\, C_L \qquad \text{(III-4)}$$

$$C_{D_\alpha} = .121 \times .49 \times 4.58$$
$$= .272 \text{ rad.}^{-1}$$

The drag coefficient considered in the equations of motion represents the variation of force along the flight axis with velocity and consequently the contribution of thrust changes with airspeed should be included.

$$\text{Effective } C_D = C_D - \frac{dT}{dV} \times \frac{1}{\rho S V} \qquad \text{(III-5)}$$
$$C_D = .0290 + .0605\,(.49)^2$$
$$= .0435$$

Power-on glide and climb tests were conducted to establish the thrust derivative term for the given operating conditions.

$$\frac{dT}{dV} \times \frac{1}{\rho S V} = -.0120$$
$$\text{Effective } C_D = .0435 + .0120$$
$$= .0555$$

$\underline{C_{m_\alpha}}$

The pitching moment derivative with respect to angle of attack is computed from the stick-fixed neutral point for a specified center of gravity location.

$$C_{m_\alpha} = -C_{L_\alpha}\,(N_o - X_{CG}) \qquad \text{(III-6)}$$
$$= -4.58\,(.308 - X_{CG})$$
$$= 4.58\,X_{CG} - 1.410 \text{ (rad.}^{-1})$$

$\underline{C_{m_\delta}}$

The elevator effectiveness is determined from the stick-fixed static stability tests.

Level Flight Data

$$C_{m_\delta} = -57.3 \Big/ \frac{d^2\delta}{dC_L\, dX_{CG}} \qquad \text{(III-7)}$$

$$\frac{d^2\delta}{dC_L\, dX_{CG}} = 58.6 \text{ deg.}$$

$$C_{m_\delta} = -\frac{57.3}{58.6}$$

$$= -\ .978 \text{ rad.}^{-1}$$

Maneuvering Flight Data

$$C_{m_\delta} = -\frac{57.3}{\dfrac{d^2\delta}{dn\, dX_{CG}}} \qquad \text{(III-8)}$$

$$\frac{d^2\delta}{dn\, dX_{CG}} = 29.3 \text{ deg.}$$

$$C_{m_\delta} = -\frac{57.3 \times .49}{29.3}$$

$$= -.958 \text{ rad.}^{-1}$$

Average $\quad C_{m_\delta} = -\ .968 \text{ rad.}^{-1}$

$\underline{C_{h_\delta}}$

The elevator hinge moment variation with elevator deflection is determined from the stick-free static stability tests.

Level Flight Data

$$C_{h_\delta} = \frac{C_{m_\delta}}{G\, S_e\, C_e\, \eta_t} \times \frac{d^2 F_s/q}{dC_L\, dX_{CG}} \qquad \text{(III-9)}$$

G, the control system gearing ratio = 1.05 ft.$^{-1}$

η_t, the tail efficiency = 0.9

$$S_e = 12.3 \text{ ft.}^2$$

$$C_e = 1.23 \text{ ft.}$$

$$\frac{d^2 F_s/q}{dC_L\, dX_{CG}} = 20.2 \text{ ft.}^2$$

$$C_{h_\delta} = -\frac{.968 \times 20.2}{1.05 \times 12.3 \times 1.23 \times 0.9}$$

$$C_{h_\delta} = -1.370 \text{ rad.}^{-1}$$

Maneuvering Flight Data

$$C_{h_\delta} = \frac{C_{m_\delta}}{W/S \; G \; S_e C_e \eta_t} \times \frac{d^2 F_s}{dn \, dX_{cg}} \qquad \text{(III-10)}$$

$$\frac{d^2 F_s}{dn \, dX_{cg}} = 307.5 \text{ lbs.}$$

$$C_{h_\delta} = \frac{-.968 \times 307.5}{15.2 \times 1.05 \times 12.3 \times 1.23 \times 0.9}$$

$$= -1.370 \text{ rad.}^{-1}$$

$\underline{C_{h_\alpha}}$

The elevator hinge moment variation with angle of attack was obtained from the measured difference between the stick-fixed and the stick-free neutral points.

$$C_{h_\alpha} = -\frac{C_{L_\alpha} C_{h_\delta}}{C_{m_\delta}} (N_o - N_o') \qquad \text{(III-11)}$$

$$N_o - N_o' = .308 - .301$$

$$= .007$$

$$C_{h_\alpha} = -\frac{4.58 \times 1.370 \times .007}{.968}$$

$$= -.0454 \text{ rad.}^{-1}$$

$\underline{C_{m_{d\theta}}}$

The damping in pitch was determined by measuring the difference in elevator deflection required for trim between level flight operation and steady maneuvering flight at n =1.5. Incremental elevator deflections at various lift coefficients were then converted to pitching moment increments at given pitch velocities by applying

$$\Delta C_m = C_{m_\delta} \times \Delta \delta \qquad \text{(III-12)}$$

$$\dot{q} = g\left(n - \frac{1}{n}\right)\sqrt{\frac{\rho C_L}{2 W/S\, n}} \qquad \text{(III-13)}$$

where $\dot{q}$ = the pitching velocity, rad./sec.

ρ = .00205 slugs/ft.^{3}

A plot of ΔC_m as a function of $\dot{q}$ was constructed yielding a linear variation with a slope of $\frac{dC_m}{d\dot{q}}$ = -.119 sec./rad. And in non-dimensional form the damping derivative is

$$C_{m_{d\theta}} = \frac{dC_m}{d\dot{q}} \times \frac{1}{\tau} \qquad \text{(III-14)}$$

τ = 1.32 sec.

$$C_{m_{d\theta}} = -\frac{.119}{1.32}$$

$$= -.0902 \text{ rad.}^{-1}$$

$\underline{C_{m_{d\alpha}}}$

The damping due to downwash lag is given by

$$C_{m_{d\alpha}} = C_{m_{d\theta}} \times \frac{d\epsilon}{d\alpha} \qquad \text{(III-15)}$$

The downwash derivative is estimated to be

$$\frac{d\epsilon}{d\alpha} = .382$$

$$C_{m_{d\alpha}} = -.0902 \times .382$$

$$= -.0345 \text{ rad.}^{-1}$$

$\underline{C_{h_{d\theta}}}$

The change in hinge moment with pitching velocity is computed from

$$C_{h_{d\theta}} = C_{h_{\alpha_t}} \frac{l_t}{\mu c} \qquad \text{(III-16)}$$

$$C_{h_{\alpha_t}} = \frac{C_{h_\alpha}}{1 - \frac{d\epsilon}{d\alpha}} \qquad \text{(III-17)}$$

l_t = 14.93 ft.

$$\mu = 40.9$$

$$C = 5.70 \text{ ft.}$$

$$C_{h_{d\theta}} = -\frac{.0454 \times 14.93}{(1-.382)\ 40.9 \times 5.70}$$

$$= -.00470 \text{ rad.}^{-1}$$

$C_{h_{d\alpha}}$

The hinge moment due to downwash lag is obtained from

$$C_{h_{d\alpha}} = C_{h_{d\theta}} \frac{d\epsilon}{d\alpha} \tag{III-18}$$

$$= -.00470 \times .382$$

$$= -.00180 \text{ rad.}^{-1}$$

$C_{h_{d\delta}}$

The elevator damping derivative is calculated theoretically by the method presented in reference 1.

$$C_{h_{d\delta}} = -\frac{C_t}{2\mu C}\left[C + D(C_{L_\alpha})_t\right] \tag{III-19}$$

C_t, the mean chord of the horizontal tail = 3.48 ft.

$(C_{L_\alpha})_t$, the estimated horizontal tail lift curve slope = 2.58 rad.$^{-1}$

C and D, constants dependent upon the chordwise hinge line location, are 1.35 and .08, respectively.

$$C_{h_{d\delta}} = -\frac{3.48}{2 \times 40.9 \times 5.70}\left(1.35 + .08 \times 2.58\right)$$

$$= -\ .0116 \text{ rad.}^{-1}$$

C_{h_o}

The applied hinge moment coefficient due to force devices is given by

$$C_{h_o} = \frac{F_{BW,DS}}{G\, q\, S_e C_e} \tag{III-20}$$

$$= \frac{F_{BW,DS}}{1.05 \times 31.0 \times 12.3 \times 1.23}$$

$$= .00203\ F_{BW,DS}$$

$\underline{h}$

The airplane inertia parameter was obtained from the measured moment of inertia about the jack points. After applying corrections for the actual test loading conditions and the transfer to the center of gravity, the moment of inertia was determined as I_{CG} = 3,800 slug-ft.^{2}

$$h = \frac{2\, g\, I_{CG}}{W \mu C^2} \tag{III-21}$$

$$= \frac{2 \times 32.2 \times 3{,}800}{2{,}800 \times 40.9\ (5.70)^2}$$

$$= .0658$$

$\underline{h_2}$

The effective elevator inertia parameter was determined from estimations of the basic elevator moment of inertia and the contribution of the bob-weight installation including the constant increment required to initially balance the control system.

Basic I = 0.2 slug-ft.^{2}

Neg. Bobweight I = 0.8 slug-ft.^{2}

Pos. Bobweight I = 0.2 F_{BW} (slug-ft.^{2})

$$I_e = 0.2 + 0.8 + 0.2\ F_{BW} \tag{III-22}$$

$$= 1.0 + 0.2\ F_{BW}\ (\text{slug-ft.}^2)$$

$$h_2 = \frac{2 I_e}{\rho\, S_e C_e (\mu C)^2} \tag{III-23}$$

$$= \frac{2\ (1.0 + 0.2\ F_{BW})}{.00205 \times 12.3 \times 1.23\ (40.9 \times 5.70)^2}$$

$$= .0012 + .00024\ F_{BW}$$

$\underline{h_1}$

The elevator mass unbalance term is computed from the bobweight effect.

$$h_1 = \frac{2 F_{BW}}{G\, g\, \rho\, S_e C_e \mu C} \tag{III-24}$$

$$h_1 = \frac{2 F_{BW}}{1.05 \times 32.2 \times .00205 \times 12.3 \times 1.23 \times 40.9 \times 5.70}$$

$$= .0082 \, F_{BW}$$

$\underline{\ell_1}$

The parameter representing the mass unbalance as affected by the airplane pitching acceleration is computed from

$$\ell_1 = \frac{\ell_{BW}}{\mu C} \times h_1 \qquad \text{(III-25)}$$

where ℓ_{BW}, the displacement of the bobweight center of gravity aft of the airplane center of gravity, = 2.0 ft.

$$\ell_1 = \frac{2 \times .0082 \times F_{BW}}{40.9 \times 5.70}$$

$$= .00071 F_{BW}$$

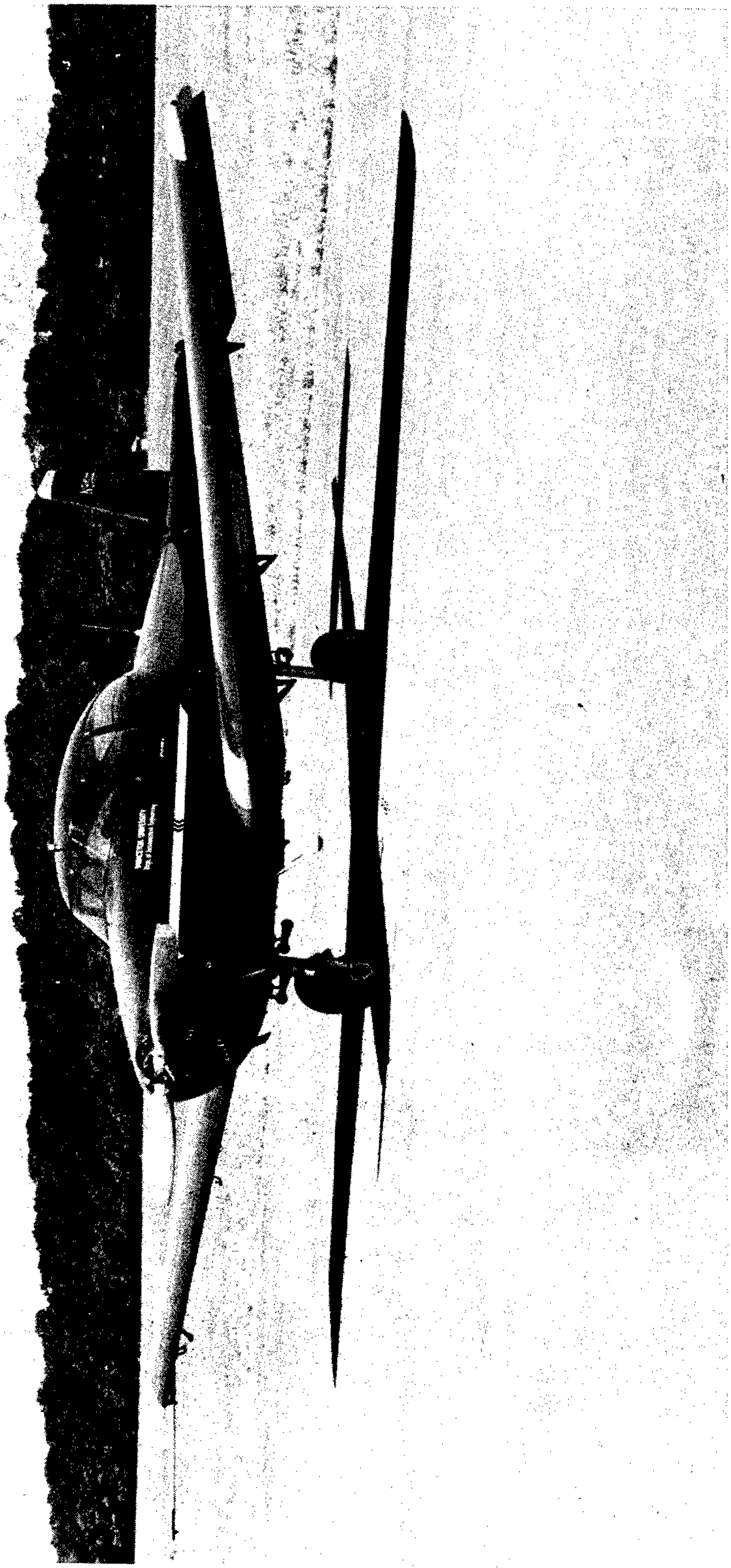


Figure 1. Modified Navion Airplane, General View of Test Airplane

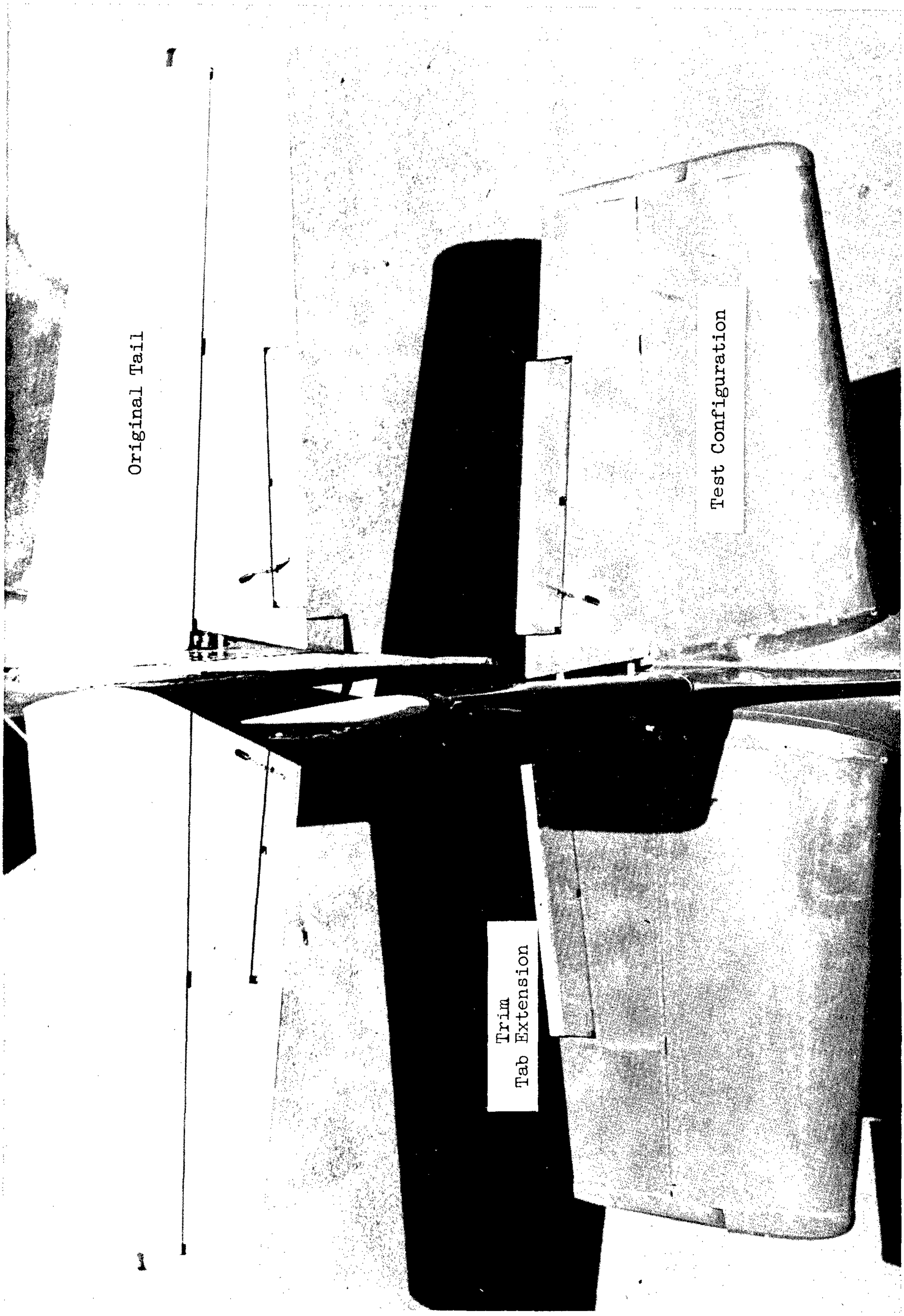


Figure 2. Modified Navion Airplane, Modified Horizontal Tail

MODIFIED NAVION AIRPLANE

DIAGRAM OF BOBWEIGHT-DOWNSPRING TEST INSTALLATION

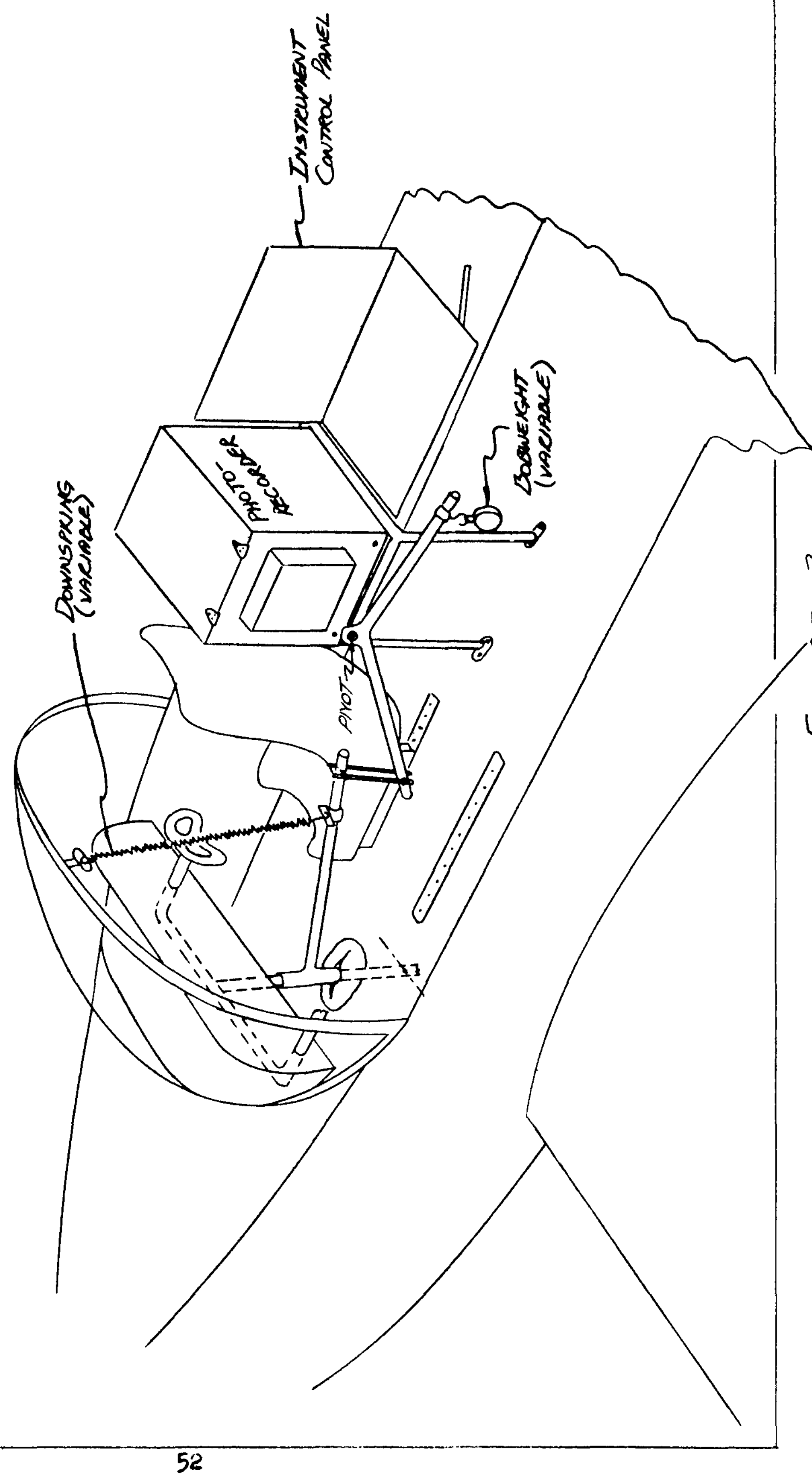


FIGURE 3

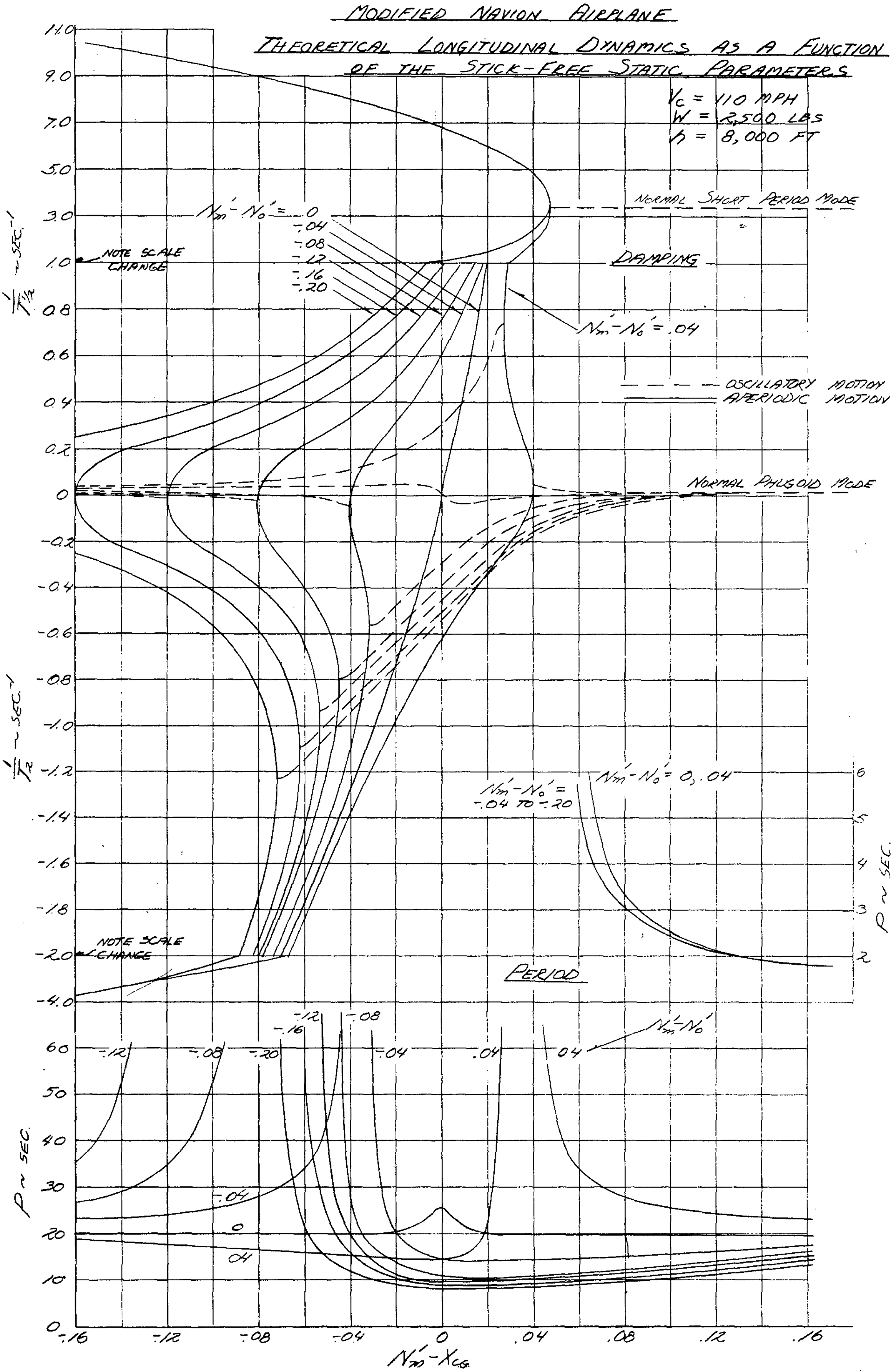
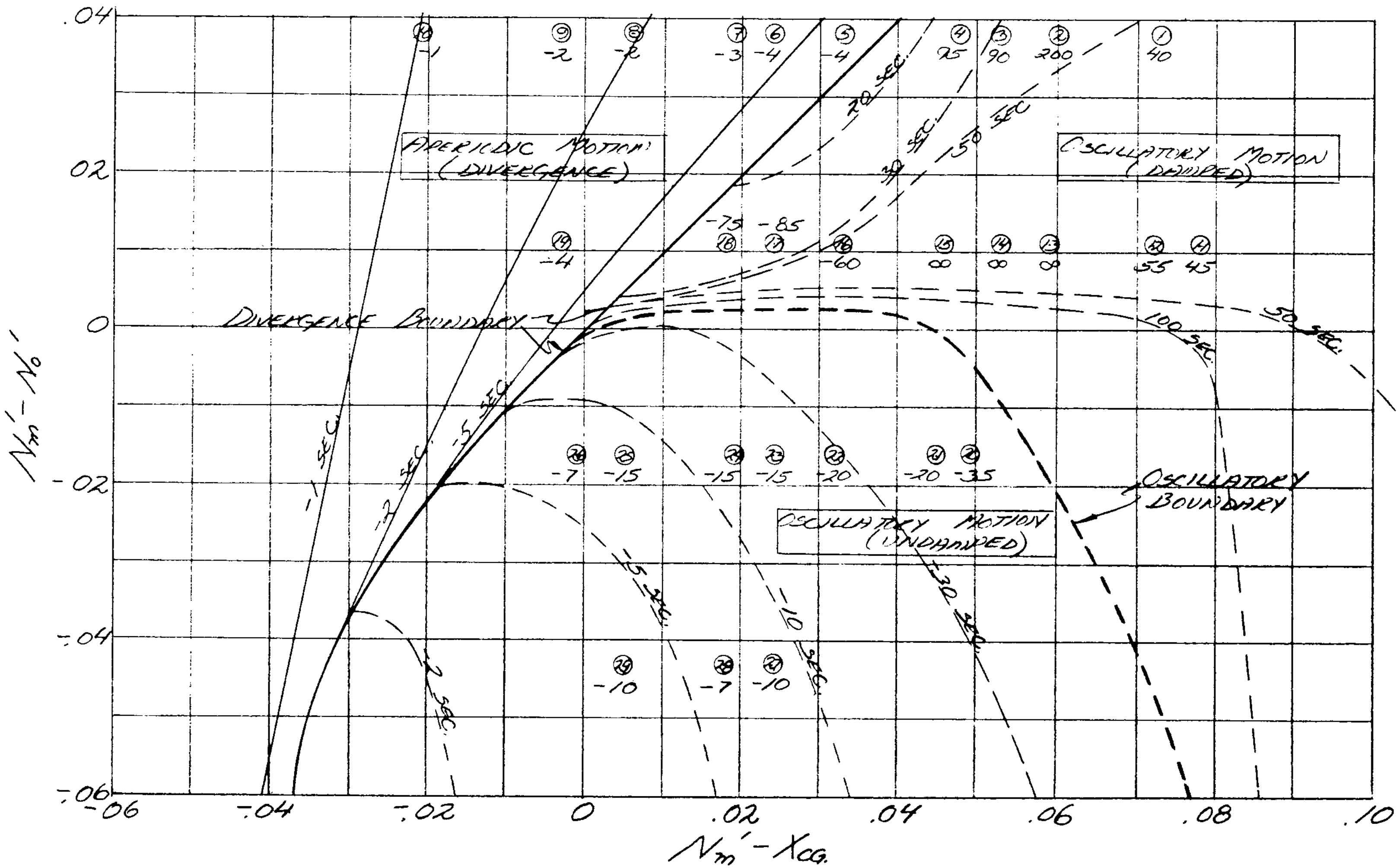

MODIFIED NAVION AIRPLANE
THEORETICAL LONGITUDINAL DYNAMICS AS A FUNCTION OF THE STICK-FREE STATIC PARAMETERS
V_c = 110 MPH
W = 2,500 LBS
h = 8,000 FT
NORMAL SHORT PERIOD MODE
DAMPING
NOTE SCALE CHANGE
OSCILLATORY MOTION
APERIODIC MOTION
NORMAL PHUGOID MODE
PERIOD
$N_m' - N_o' = 0, .04$
$N_m' - N_o' = -.04$ TO $-.20$
$N_m' - N_o' = .04$
$N_m' - N_o'$
P ~ SEC.
$N_m' - X_{CG}$

FIGURE 4

MODIFIED NAVION AIRPLANE

THEORETICAL CURVES OF CONSTANT PHUGOID DAMPING AS A FUNCTION OF THE STATIC STABILITY PARAMETERS, INCLUDING FLIGHT TEST RESULTS

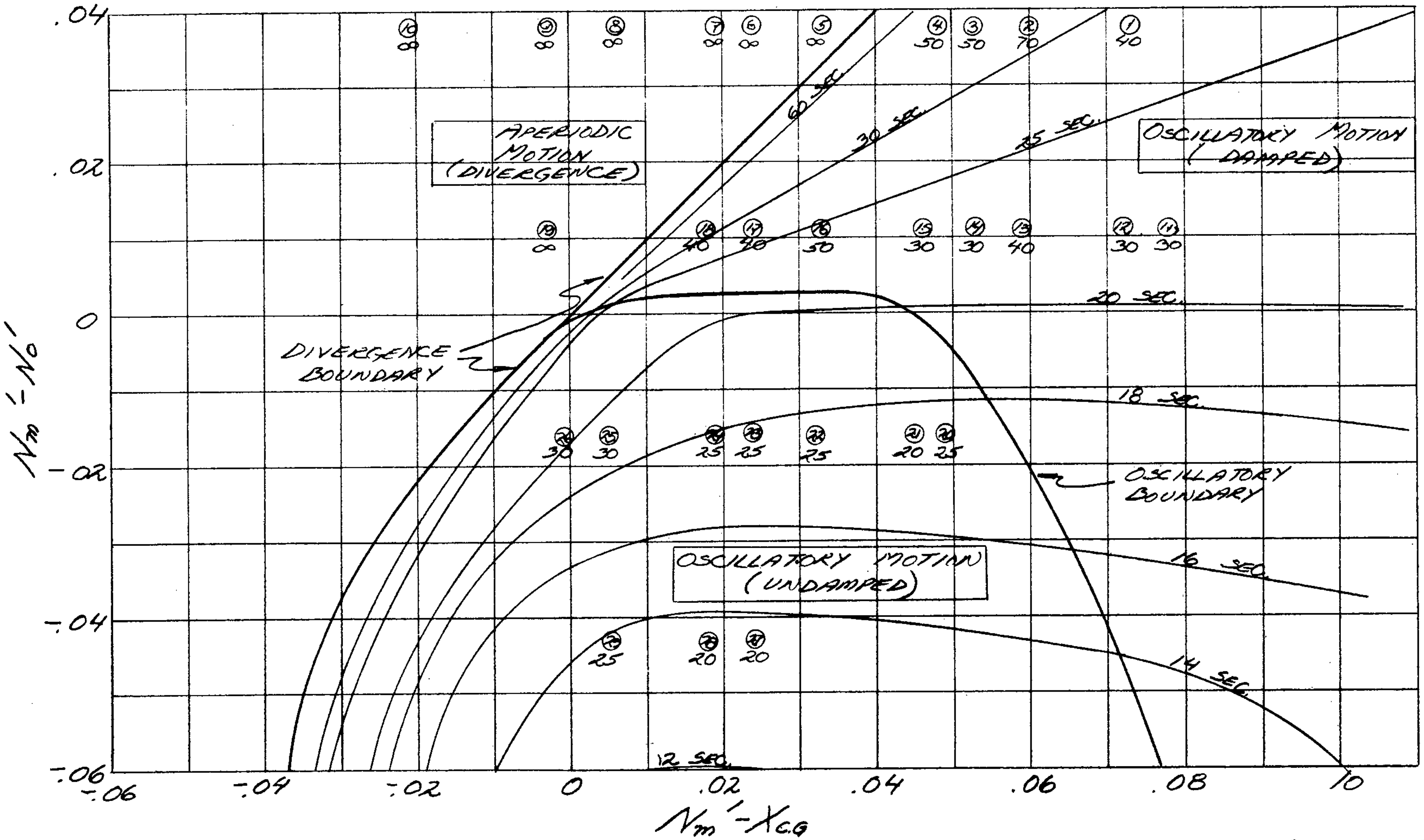


NOTE: DAMPING NOTATION IS TIME TO HALF AMPLITUDE, NEGATIVE VALUES DENOTE TIME TO DOUBLE AMPLITUDE. FLIGHT TEST PTS INDICATED BY CIRCLED RUN NUMBERS, ADJACENT VALUES ARE DAMPING TIMES

FIGURE 5

MODIFIED NAVION AIRPLANE

THEORETICAL CURVES OF CONSTANT PHUGOID PERIOD AS A FUNCTION OF THE STATIC STABILITY PARAMETERS, INCLUDING FLIGHT TEST RESULTS



NOTE: FLIGHT TEST PTS INDICATED BY CIRCLED RUN NUMBERS, ADJACENT VALUES ARE FLIGHT TEST PERIODS

FIGURE 6

MODIFIED NAVION AIRPLANE

TYPICAL FLIGHT DATA FOR STATIC LONGITUDINAL STABILITY AND CONTROL TESTS

$X_{CG} = .321$, TRIM $V_C = 110$ MPH, $h = 5{,}000$ FT., $W = 2800$ LBS.

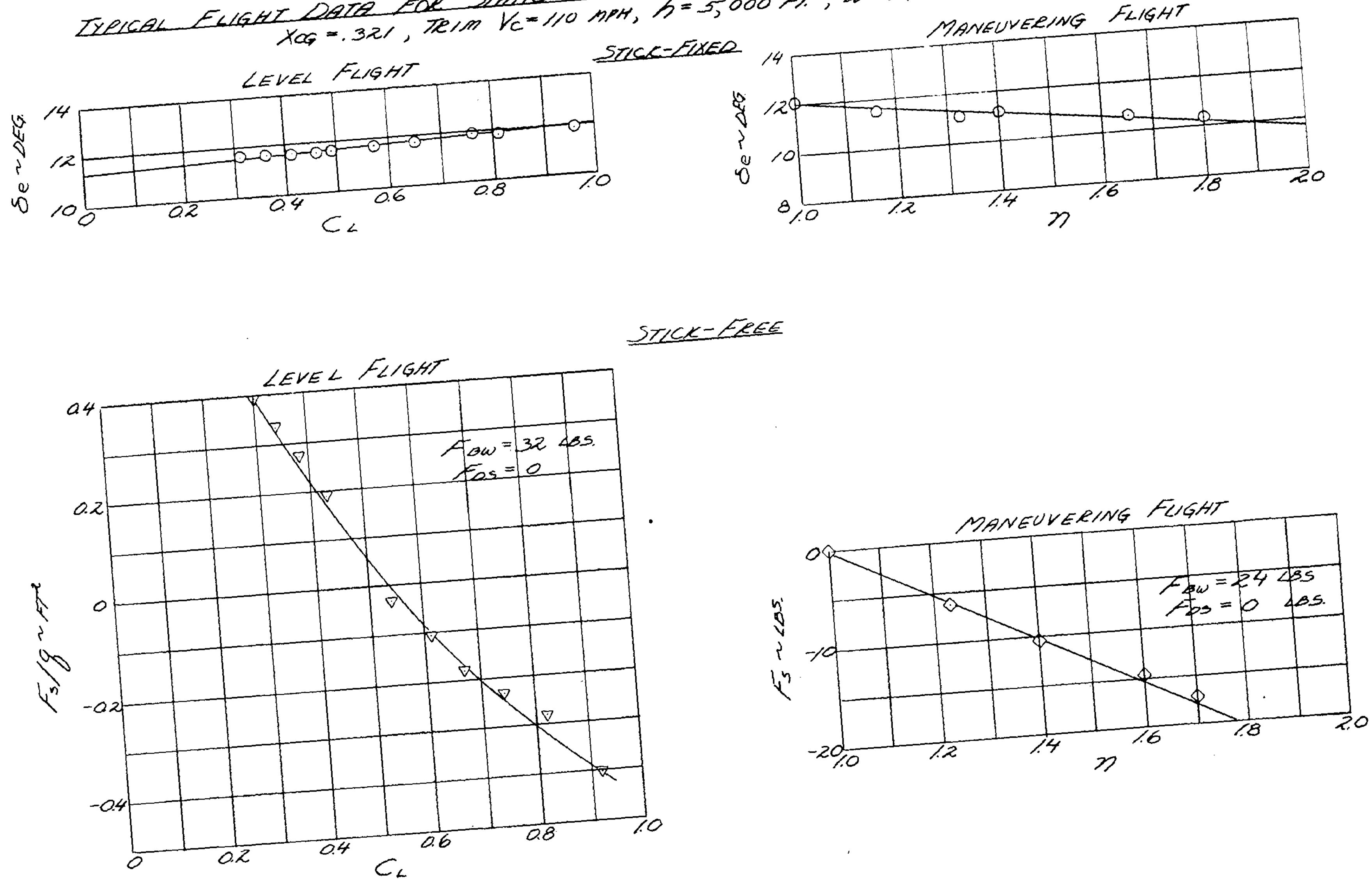


FIGURE 7

MODIFIED NAVION AIRPLANE

STATIC LONGITUDINAL STABILITY AND CONTROL CHARACTERISTICS BASED ON FLIGHT TEST DATA

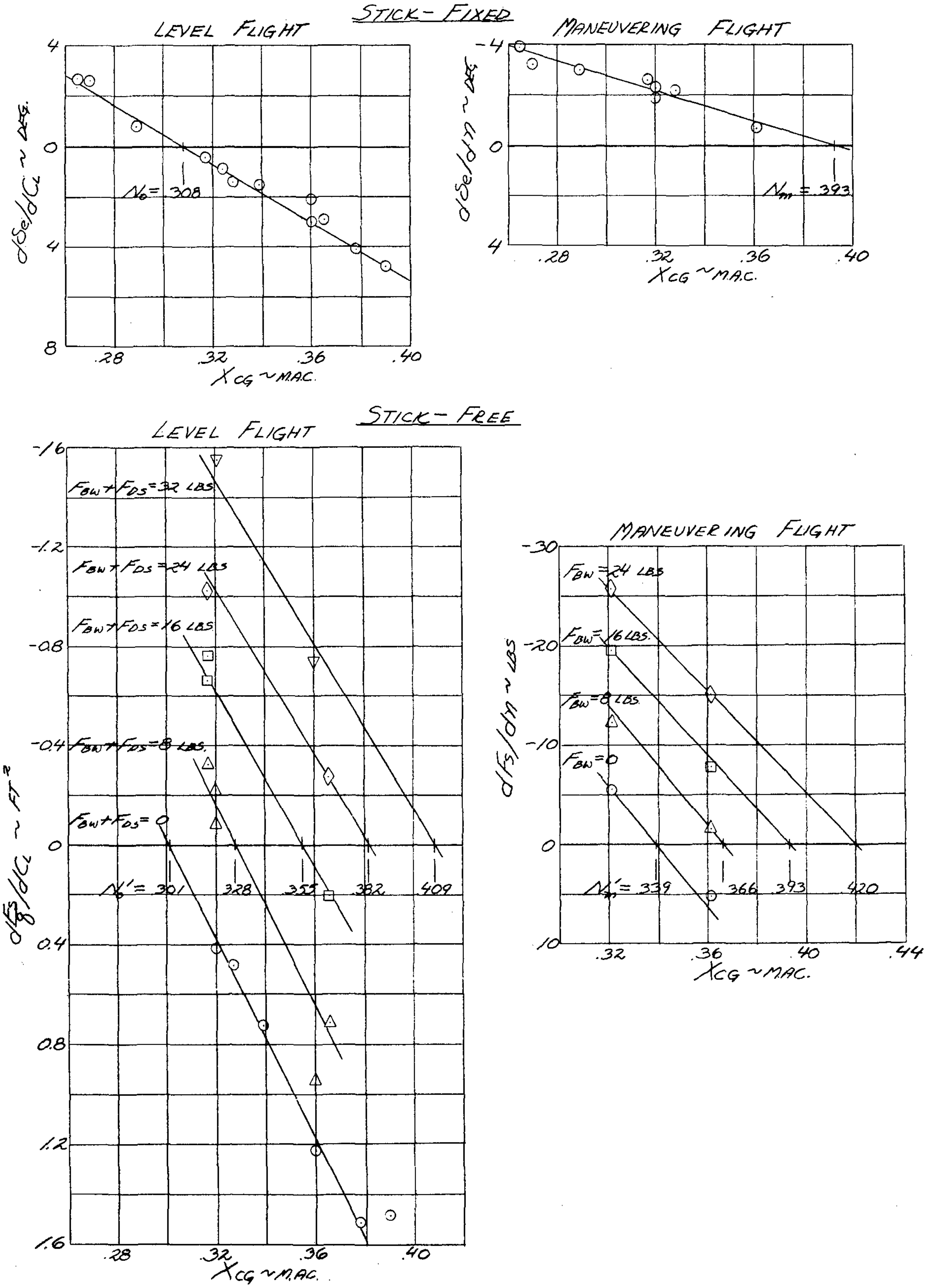


FIGURE 8

MODIFIED NAVION AIRPLANE
PHUGOID MODES AT COMPARABLE MANEUVER MARGINS FOR VARIOUS STATIC DAMPING PARAMETERS, BASED ON FLT. TEST DATA

TRIM V_c = 110 MPH, h = 5,000 FT, W = 2,800 LBS

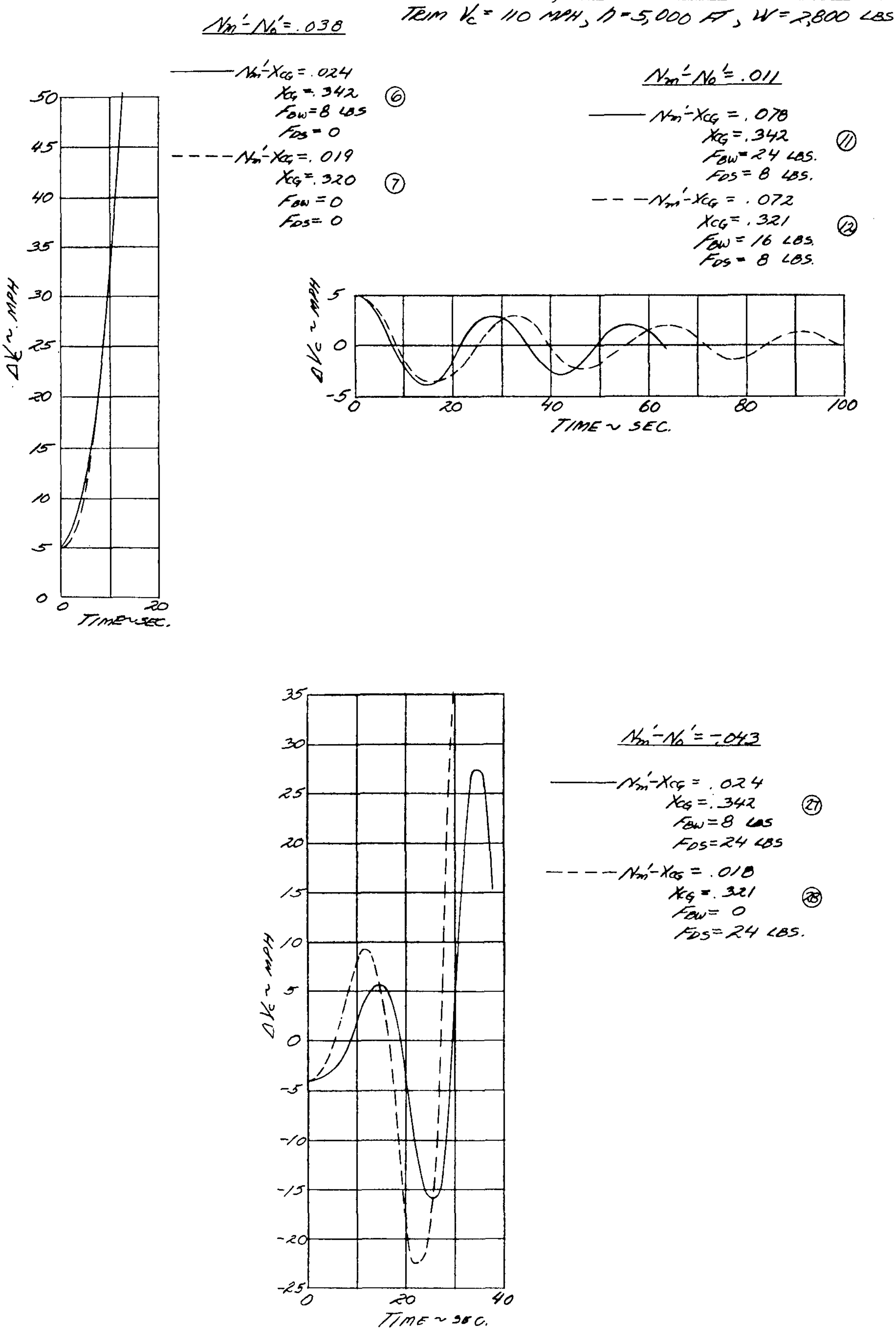


FIGURE 9

MODIFIED NAVION AIRPLANE

PHUGOID MODES FOR VARIOUS STATIC MANEUVER MARGINS

BASED ON FLIGHT TEST DATA

TRIM V_c = 110 MPH, h = 5,000 FT, W = 2,800 LBS

$N_m' - N_o'$ = .038

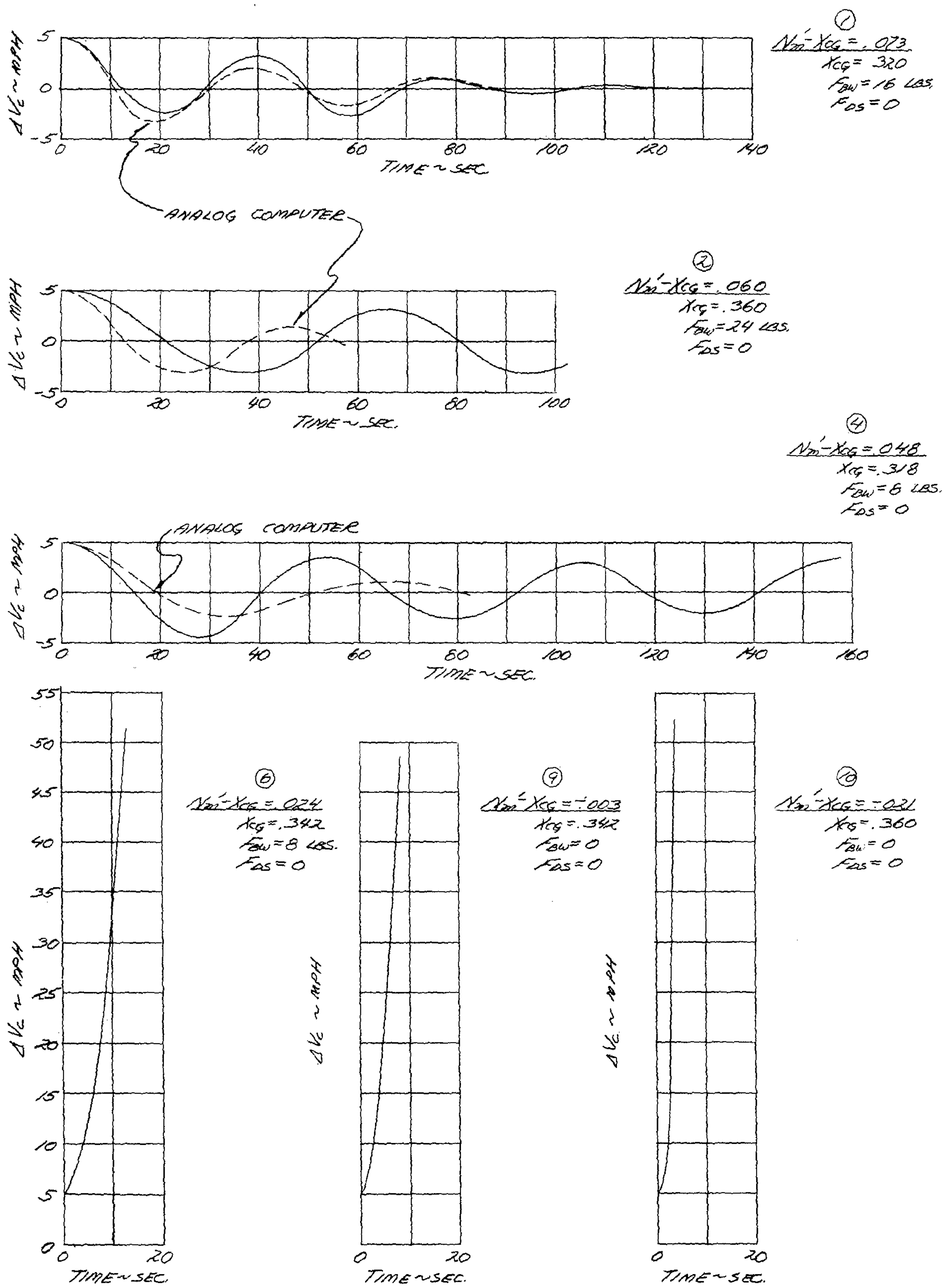


FIGURE 10

MODIFIED NAVION AIRPLANE

PHUGOID MODES FOR VARIOUS STATIC MANEUVER MARGINS

BASED ON FLIGHT TEST DATA

$N_m' - N_0' = .011$

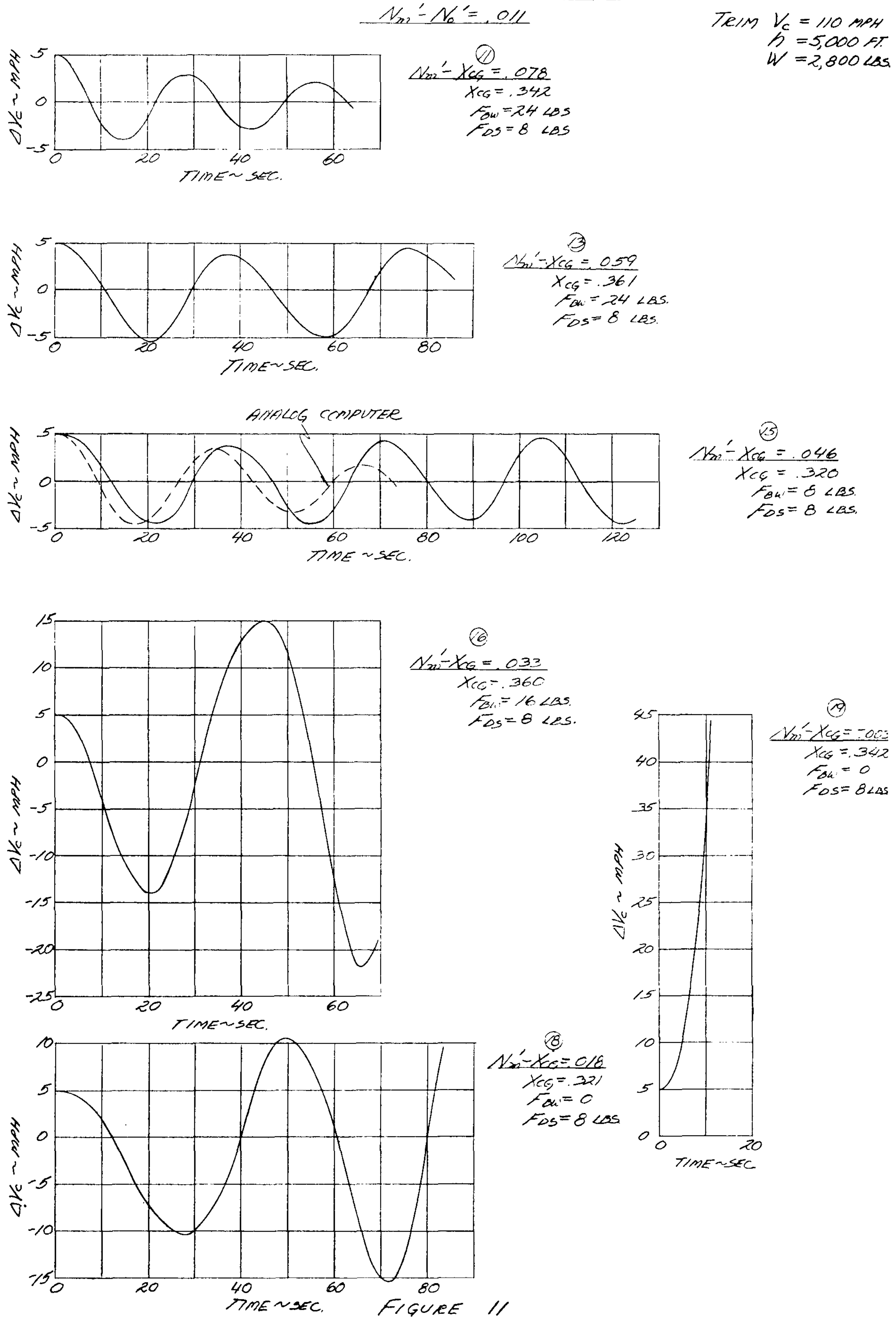


FIGURE 11

MODIFIED NAVION AIRPLANE

PHUGOID MODES FOR VARIOUS STATIC MANEUVER MARGINS

BASED ON FLIGHT TEST DATA

TRIM $V_c = 110$ MPH, $h = 5{,}000$ FT., $W = 2800$ LBS.

$N_m' - N_o' = -.016$

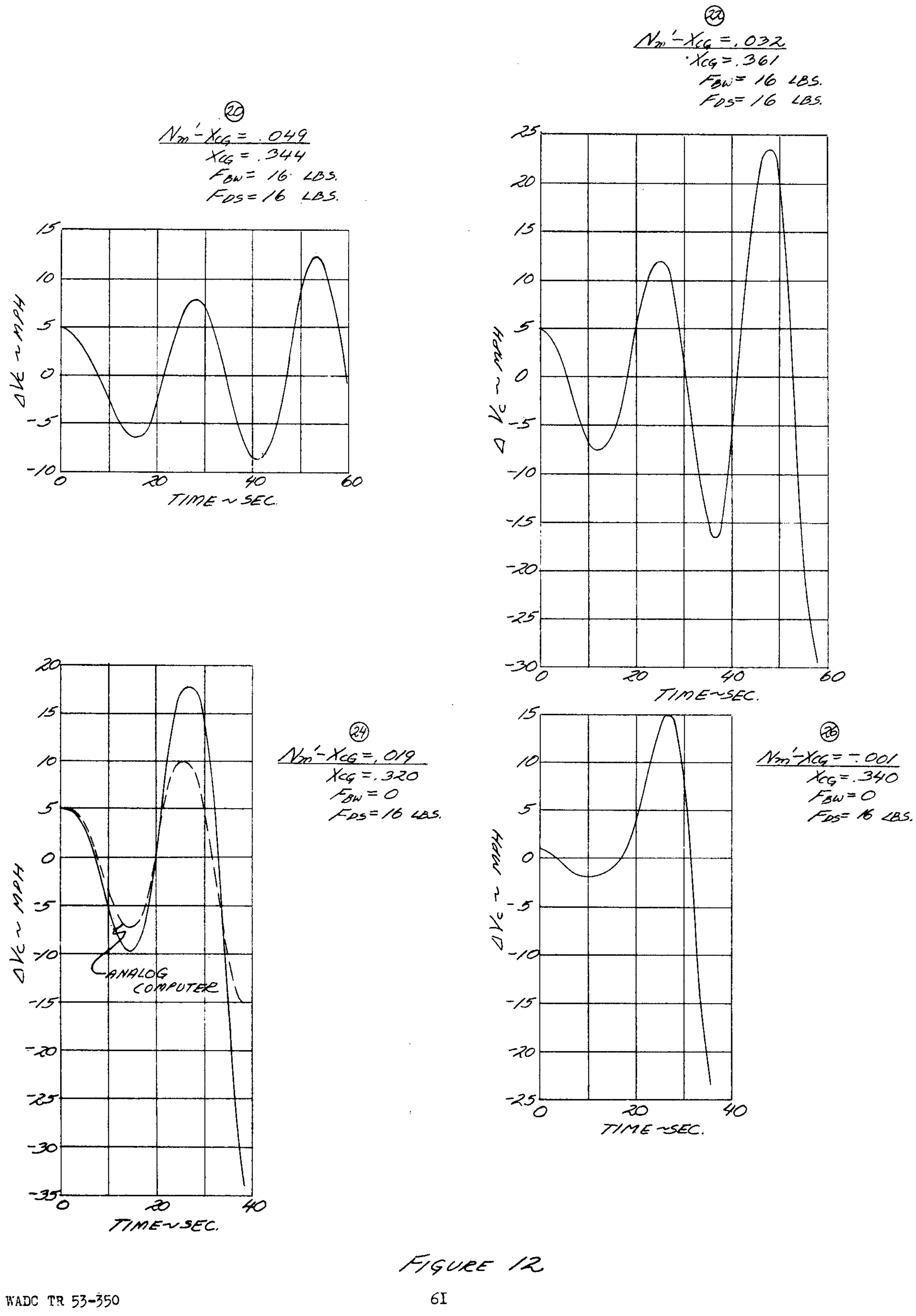
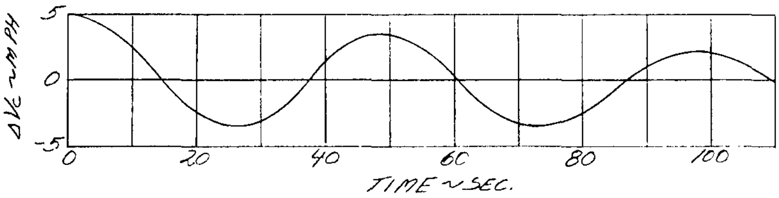


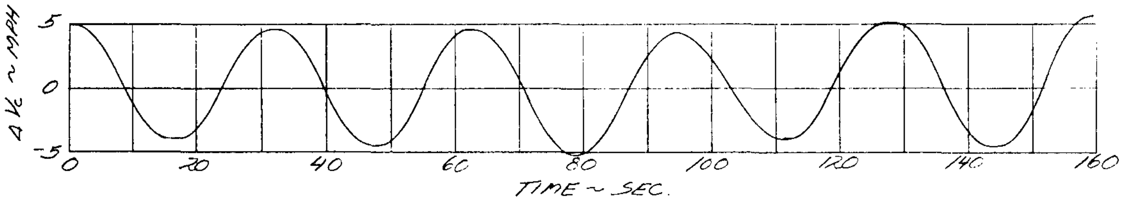
FIGURE 12

MODIFIED NAVION AIRPLANE
PHUGOID MODES FOR VARIOUS STATIC DAMPING PARAMETERS
BASED ON FLIGHT TEST DATA
TRIM $V_C = 110$ MPH, $h = 5{,}000$ FT., $W = 2{,}800$ LBS.
$N_m' - X_{CG} \approx .050$

③
$N_m' - N_o' = .038$
$X_{CG} = .340$
$F_{BW} = 16$ LBS.
$F_{DS} = 0$



⑭
$N_m' - N_o' = .011$
$X_{CG} = .340$
$F_{BW} = 16$ LBS.
$F_{DS} = 8$ LBS.



㉑
$N_m' - N_o' = -.016$
$X_{CG} = .321$
$F_{BW} = 8$ LBS
$F_{DS} = 16$ LBS.

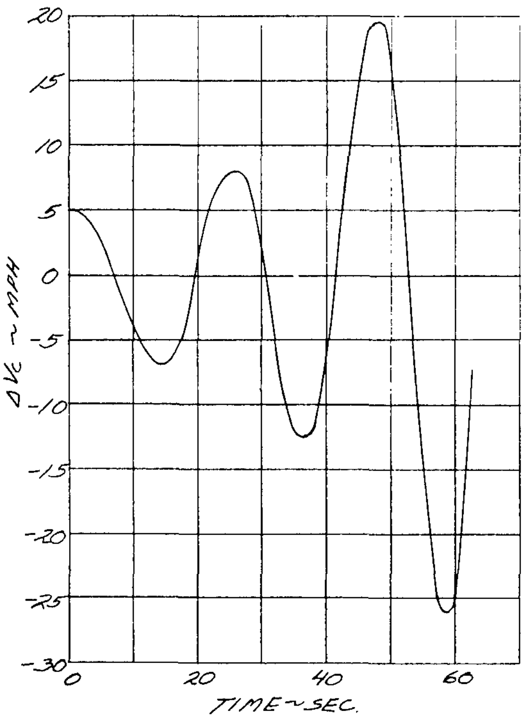


FIGURE 13

MODIFIED NAVION AIRPLANE
PHUGOID MODES FOR VARIOUS STATIC DAMPING PARAMETERS
BASED ON FLIGHT TEST DATA

TRIM $V_c = 110$ MPH, $h = 5{,}000$ FT., $W = 2800$ LBS.

$N_m' - X_{CG} \approx .026$

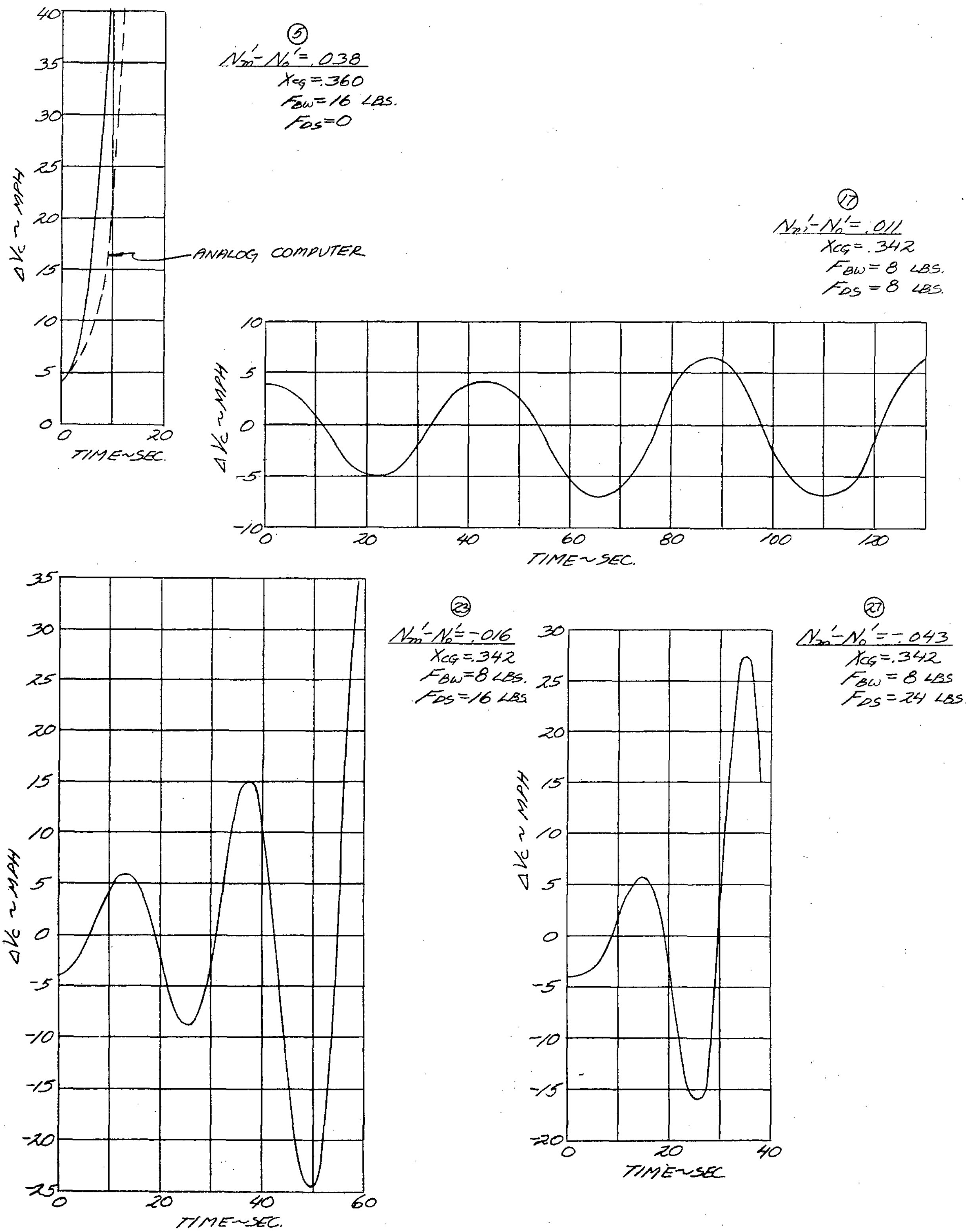


FIGURE 14

MODIFIED NAVION AIRPLANE

PHUGOID MODES FOR VARIOUS STATIC DAMPING PARAMETERS

BASED ON FLIGHT TEST DATA

$N_m' - X_{CG} \approx .003$

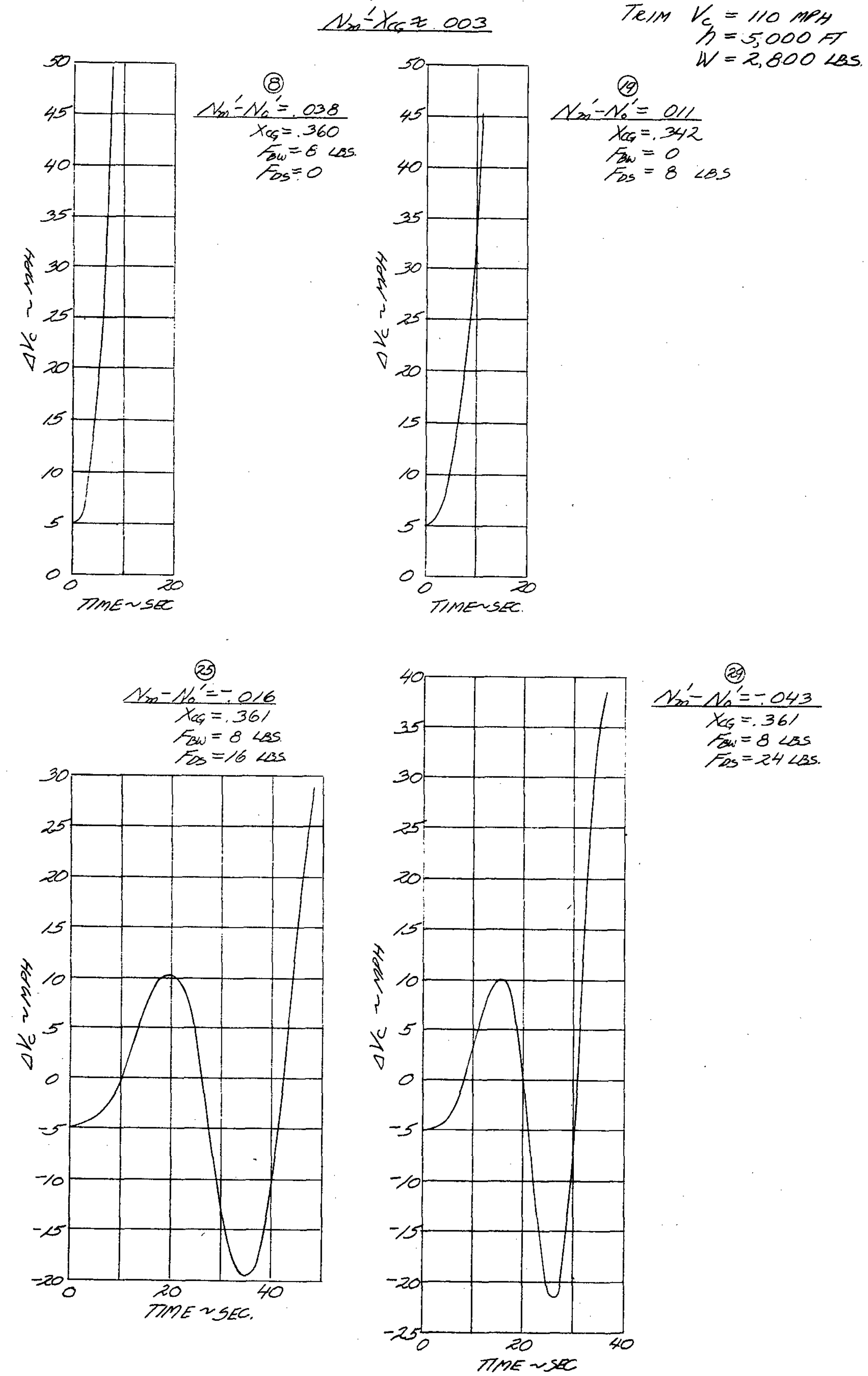


FIGURE 15