

Errata for Technical Report #13

Sampling Plans for Inspection by Variables

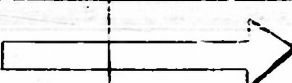
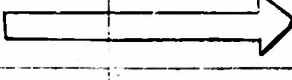
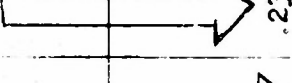
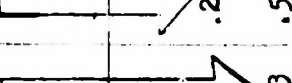
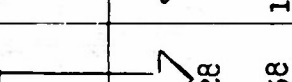
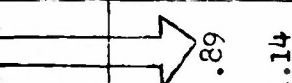
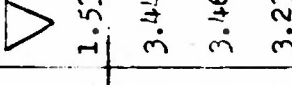
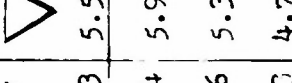
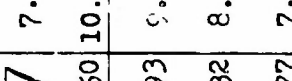
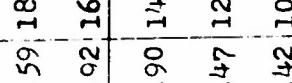
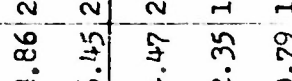
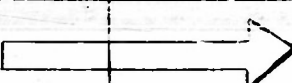
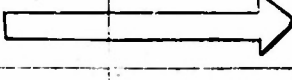
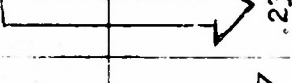
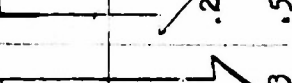
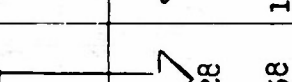
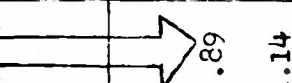
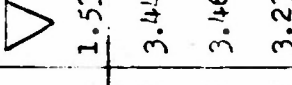
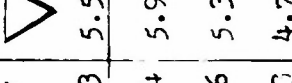
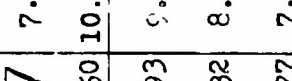
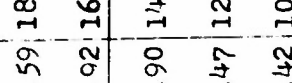
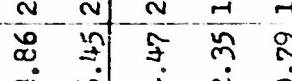
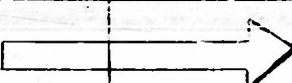
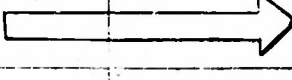
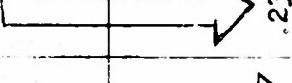
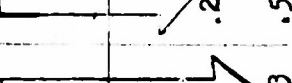
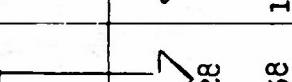
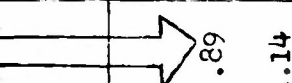
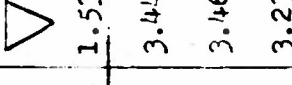
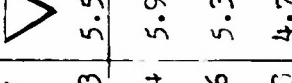
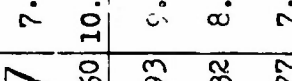
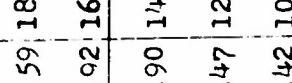
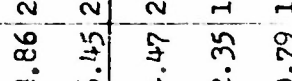
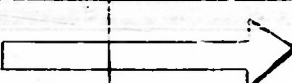
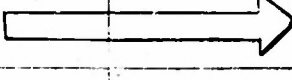
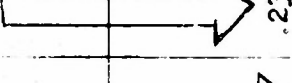
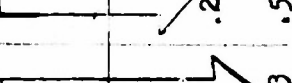
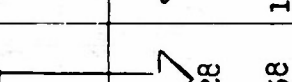
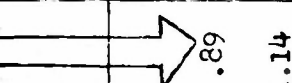
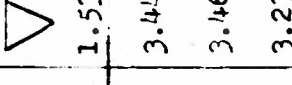
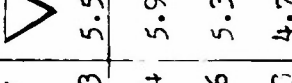
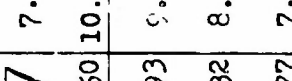
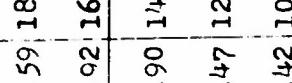
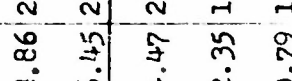
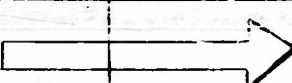
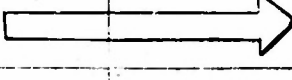
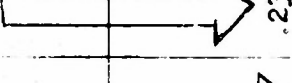
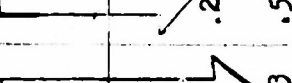
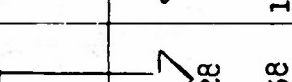
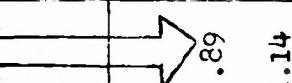
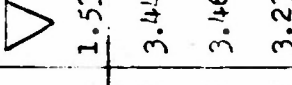
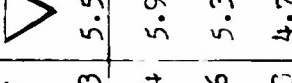
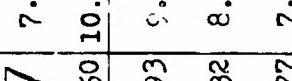
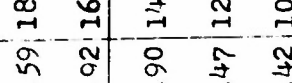
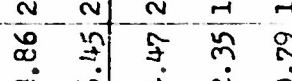
by

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**Replace Table V - Master Table for Normal and Tightened Inspection
for Sampling Plans Based on the Average Range - with the enclosed table -
Corrected Table V.**

July 15, 1955

Corrected Table V Master Table for Sampling Plans Based on the Average Range

Sample size code letter	Sam-ple size	h fac-tor	Acceptable Quality Levels (normal inspection)													
			.04 p*	.065 p*	.10 p*	.15 p*	.25 p*	.40 p*	.65 p*	1.00 p*	1.50 p*	2.50 p*	4.00 p*	6.50 p*	10.00 p*	15.00 p*
B	3	1.910														
C	4	2.234														
D	5	2.474														
E	7	2.830														
F	10	2.405														
G	15	2.379	.061	.136	.253	.430	.786	1.30	2.10	3.11	4.44	6.76	9.76	14.09	19.30	
H	25	2.358	.125	.214	.336	.506	.827	1.27	1.95	2.92	3.96	5.98	8.65	12.59	17.48	
I	30	2.353	.147	.240	.366	.537	.856	1.29	1.96	2.81	3.92	5.88	8.50	12.36	17.19	
J	35	2.349	.165	.261	.391	.564	.883	1.33	1.98	2.82	3.90	5.85	8.42	12.24	17.03	
K	40	2.346	.160	.252	.375	.539	.842	1.25	1.88	2.69	3.73	5.61	8.11	11.84	16.55	
L	50	2.342	.169	.261	.381	.542	.838	1.25	1.60	2.63	3.64	5.47	7.91	11.57	16.20	
M	60	2.339	.158	.244	.356	.504	.781	1.16	1.74	2.47	3.44	5.17	7.54	11.10	15.64	
N	85	2.335	.156	.242	.350	.493	.755	1.12	1.67	2.37	3.30	4.97	7.27	10.73	15.17	
O	115	2.330	.153	.230	.333	.468	.718	1.06	1.58	2.25	3.14	4.76	6.99	10.37	14.74	
P	175	2.333	.130	.210	.303	.427	.655	.972	1.46	2.08	2.93	4.47	6.60	9.89	14.15	
Q	230	2.333	.142	.215	.308	.432	.661	.976	1.47	2.08	2.92	4.46	6.57	9.84	14.10	

All AQL and table values are in percent defective.

SAMPLING PLANS FOR INSPECTION BY VARIABLES

BY

GERALD J. LIEBERMAN AND GEORGE J. RESNIKOFF

TECHNICAL REPORT NO. 13

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FOR

OFFICE OF NAVAL RESEARCH

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Sampling Plans for Inspection by Variables

By

Gerald J. Lieberman and George J. Resnikoff

I. Introduction.

In lot-by-lot acceptance sampling by attributes each item of a sample drawn from a lot of manufactured items is classified simply as defective or non-defective. A random sample is drawn from the lot and the lot is either accepted or rejected depending solely upon the number of defectives in the sample. When the classification into defective and non-defective is made on the basis of the measurement of a variable quality characteristic, and the decision to accept or reject the lot is a function of these measurements (as opposed to the number of defectives) such inspection procedures are known as sampling inspection by variables. Since inspection by variables makes greater use of the information concerning the lot than does inspection by attributes, whenever the testing of the individual items involves measurement, variables plans require smaller sample sizes to furnish the same degree of protection than do attributes plans. Variables sampling plans pertain to a single quality characteristic, and it is usually assumed that measurements of this quality characteristic are independent, identically distributed normal random variables. Such an assumption will be made throughout the paper.

Sampling inspection by variables is divided into three categories, known standard deviation plans, unknown standard deviation plans, and

average range plans. Known standard deviation plans are based upon the sample mean and the known standard deviation. Unknown standard deviation plans are based upon the sample mean and the sample standard deviation. Average range plans are based upon the sample mean and the average range of k subgroups of r items (the sample size is then kr).

The purpose of this paper is to present a matching collection of variables sampling plans based on known standard deviation, unknown standard deviation, and average range. These plans are classified according to sample size and Acceptable Quality Level (AQL). The probability of acceptance at the AQL was done to be consistent with MIL STD 105A.

The plans presented below have the following properties which differ from those plans already published:

1. Each OC curve shown represents a variables sampling plan based on known standard deviation, unknown standard deviation, and average range. In other words, if the user chooses an OC curve, he has at his disposal the choice of the three types of plans, guaranteeing, for all practical purposes, the same protection.
2. The usual acceptance-rejection procedure in lot-by-lot sampling inspection by attributes is to draw a random sample and accept the lot if the number of defectives is sufficiently small. This is equivalent to the procedure of estimating the percentage defective in the lot by its best estimate, namely, the percentage defective in the sample, and accepting the lot if this estimate is small. Similarly, sampling inspection by variables can be viewed in the same manner. In this paper, the acceptance-rejection

procedure is based upon estimating the percentage defective in the lot^{1/} with acceptance occurring if and only if the estimate is not too large.

3. All existing sampling procedures for two-sided specification limits have the disadvantage that the probability of accepting a submitted lot with given percentage defective p does not depend on p alone, but is also a function of the actual lot mean μ , or equivalently, on the actual division of the percentage defective into components lying above and below the specifications U and L ^{2/}. For this reason, it is not possible to compute a single OC curve giving the operating characteristics of a two-sided plan. Such plans do not yield a constant probability of acceptance for a given percentage defective, but rather a spectrum of probabilities. Computing OC curves for all possible divisions of p results in a band of curves. The two-sided procedure presented is consistent with (2). The percentage below the lower specification is estimated and the percentage above the upper specification is estimated. The lot is accepted if and only if the sum of these estimates is sufficiently small.

This procedure has the property that the band of OC curves due to all possible divisions of the percentage defective is so

^{1/} The estimates used for the known and unknown standard deviation cases are the uniformly minimum variance unbiased estimates of the percentage defective in the lot.

^{2/} The two-sided regions for unknown standard deviation plans in Sampling Inspection by Variables, by Bowker and Goode, McGraw-Hill, 1952, are equivalent to those presented here. Both pieces of work stem from research conducted at the Applied Mathematics and Statistics Laboratory, Stanford University, during the summer of 1951.

narrow as to be, for practical purposes, a single curve.

Furthermore, it then follows that the OC curve of the related-one-sided test very closely approximates the OC band of the two-sided test.

II. General Inspection Criteria.

Associated with each inspection characteristic are the design specifications. If only an upper specification limit is given, the item is considered defective if its measurement exceeds U. If only a lower specification is required, the item is considered defective if its measurement is smaller than L. If both upper and lower specification limits are specified, the item is considered defective if its measurement either exceeds U or is smaller than L.

If the percentage defective of a submitted lot is known, no sampling inspection is necessary to determine whether or not the lot is to be accepted. If the percentage is sufficiently small, the lot is accepted, otherwise rejected. Since such knowledge about the percent defective is rare, a logical procedure is to estimate this percentage defective from a sample, and accept or reject the lot on the basis of this estimate. A sampling plan is then described as consisting of the sample size n ; a method of estimating the percentage defective, and a maximum allowable estimated percentage defective allowing acceptance of the lot, p^* . If an upper specification limit is given, the percentage above this limit, $\hat{p}_U$, is estimated from the sample of size n . If $\hat{p}_U \leq p^*$, the lot is accepted. If a double specification limit is given, both $\hat{p}_U$ and $\hat{p}_L$ are computed. If $\hat{p} = \hat{p}_U + \hat{p}_L \leq p^*$, the lot is accepted.

In the ensuing sections sampling plans based on known standard

deviation, unknown standard deviation, and average range will be presented. These plans will be indexed according to code letter^{1/} and AQL. The OC curves for the corresponding plans (same code letter and AQL) based on known standard deviation, unknown standard deviation, and average range are essentially the same and are presented in Figures 1-16.

III. Known Standard Deviation Plans.

In this section, it will be assumed that the standard deviation σ of the characteristic measurements is known. A complete set of sampling plans indexed according to AQL and code letter is given in Table 1. Operating characteristic curves for these plans are provided in the appropriate figures.

Before presenting the general inspection procedure for known standard deviation plans, some discussion about two-sided specifications is in order. For this case information is available about the percentage defective in the lot before a sample is drawn. There is a sufficiently small value of $\frac{U-L}{\sigma}$ which is a function only of the AQL that assures incoming quality worse than the AQL. Consequently, before sampling, $\frac{U-L}{\sigma}$ should be calculated and the lot rejected (with no sample taken) if this value is smaller than the minimum value of $\frac{U-L}{\sigma}$ on the grounds that the incoming quality is poorer than the AQL. Minimum values of $\frac{U-L}{\sigma}$ can be found in Table 1. Intuitively, this implies that the combination of narrow specification limits and large standard deviation insure against the submittal of good quality.

^{1/} The code letters run from B to Q, omitting A. Again this was done so that there would be somewhat of a semblance between these tables and the corresponding ones in MIL STD 105A.

The inspection procedure is as follows:

1. Draw a random sample of n items and compute $\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$.
2. a. For an upper specification limit compute $C'_U = (\frac{U-\bar{x}}{\sigma})v$
 where $v = \sqrt{\frac{n}{n-1}}$.
 b. For a lower specification limit compute $C'_L = (\frac{\bar{x}-L}{\sigma})v$
 where $v = \sqrt{\frac{n}{n-1}}$.
 c. For a double specification limit, compute both
 $C'_U = (\frac{U-\bar{x}}{\sigma})v$ and $C'_L = (\frac{\bar{x}-L}{\sigma})v$ if $\frac{U-L}{\sigma}$ is greater than the min.
 Otherwise, the lot is rejected before a sample is drawn.
3. Enter Table II with C'_U and/or C'_L and read out p_U and/or p_L
 whichever is applicable.^{1/}
4. For an upper specification limit accept the lot if $\hat{p}_U \leq p^*$.
 For a lower specification limit accept the lot if $\hat{p}_L \leq p^*$.
 For a double specification limit accept the lot if $\hat{p}_U + \hat{p}_L \leq p^*$.

Example.

The specified minimum yield point for certain steel castings is 55000 psi. The standard deviation is known to be $\sigma = 3000$ psi. A 1% AQL plan is to be used with a sample size of 6. The yield points of the sample specimens are

62000; 61000; 68500; 59500; 65500; 63900.

The following are computed from the data:

1. $\bar{x} = 63400$.
- 2b. $C'_L = (\frac{63400-55000}{3000})(1.095) = 3.07$.
3. From Table II $\hat{p}_L = .107\%$.

^{1/} It is shown in Section VI-3 that these estimates are the uniformly minimum variance unbiased estimates of the true percentage defective.

4. From Table I p^* is 2.57%. Hence the lot is accepted.

IV. Unknown Standard Deviation Plans.

In this section it will be assumed that the standard deviation of the characteristic measurement is unknown. A complete set of sampling plans indexed according to AQL and code letter is given in Table III. Operating Characteristics Curves for these plans are provided in the appropriate figures.

The inspection procedure is as follows:

1. Draw a random sample of n items and compute $\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$
and $s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$.
2. a. For an upper specification limit, compute $C_U = \frac{U - \bar{x}}{s}$.
b. For a lower specification limit, compute $C_L = \frac{\bar{x} - L}{s}$.
c. For a double specification limit, compute both $C_U = \frac{U - \bar{x}}{s}$ and $C_L = \frac{\bar{x} - L}{s}$.
3. Enter Table IV with C_U and/or C_L and read out $\hat{p}_U$ and/or $\hat{p}_L$ whichever is applicable^{1/}.
4. For an upper specification limit accept the lot if $\hat{p}_U \leq p^*$.
For a lower specification limit accept the lot if $\hat{p}_L \leq p^*$.
For a double specification limit accept the lot if $\hat{p}_U + \hat{p}_L \leq p^*$.

Example.

The specifications for electrical resistance of a certain electrical component is $U = 660$ ohms and $L = 620$ ohms. A .4% AQL plan is to be used with a sample of size 10. The data are as follows:

^{1/} It is shown in Section VI-4 that these estimates are the uniformly minimum variance unbiased estimates of the true percentage defective.

639, 640, 650, 647, 662, 637, 652, 643, 657, 649.

The following are computed from the data:

1. $\bar{x} = 647.6$ $s = \sqrt{65.38} = 8.04.$

2c. $C_L = \frac{647.6-620}{8.04} = 3.44$

$$C_U = \frac{660-647.6}{8.04} = 1.54.$$

3. From Table IV

$$\hat{p}_L = 0$$

$$\hat{p}_U = 5.31\%$$

$$\hat{p} = \hat{p}_L + \hat{p}_U = 5.31\%.$$

4. From Table III

$$p^* = 1.30\%.$$

Since $\hat{p} > p^*$, the lot is rejected, furthermore, for possible review board evidence, the best estimate of the percentage defective in the lot is 5.31%. Furthermore, it is evident that all the trouble lies in the upper specification limit.

V. Average Range Plans.

In this section it will be assumed that the standard deviation of the characteristic measurement is unknown. A complete set of sampling plans indexed according to AQL and code letter is given in Table V. Operating characteristic curves for these plans are provided in the appropriate figures.

The inspection procedure is as follows:

1. Draw a random sample of n items and group the measurements into k subgroups of 5 (where $n = 5k$) with the exception of sample sizes of 3, 4, and 7. For these sample sizes use a single subgroup of 3, 4, and 7 respectively. For each

subgroup compute the range (R_i). Determine the average range $\bar{R} = (\sum_{i=1}^k R_i)/k$. Calculate $\bar{x} = (\sum_{i=1}^n x_i)/n$.

2. a. For an upper specification limit, compute $C_U'' = (\frac{U-\bar{x}}{\bar{R}})h \cdot \frac{1}{\sqrt{2}}$
- b. For a lower specification limit, compute $C_L'' = (\frac{\bar{x}-L}{\bar{R}})h \cdot \frac{1}{\sqrt{2}}$
- c. For a double specification limit, compute both

$$C_U'' = (\frac{U-\bar{x}}{\bar{R}})h \text{ and } C_L'' = (\frac{\bar{x}-L}{\bar{R}})h \cdot \frac{1}{\sqrt{2}}$$

3. Enter Table VI with C_U'' and/or C_L'' and read out $\hat{p}_U$ and/or $\hat{p}_L$ whichever is applicable.
4. For an upper specification limit accept the lot if $\hat{p}_U \leq p^*$.
For a lower specification limit accept the lot if $\hat{p}_L \leq p^*$.
For a double specification limit accept the lot if $\hat{p}_U + \hat{p}_L \leq p^*$.

Example.

The maximum temperature of operation for a certain device is specified as 180°. A 4% AQL plan is to be used with a sample of size 15. The sample items have operative temperatures of

178, 175, 174, 158, 172	$R_1 = 20$
177, 166, 172, 167, 163	$R_2 = 14$
174, 173, 162, 182, 170	$R_3 = 20$

The following are computed from the data:

1. $\bar{x} = 170.87$.
- 2b. $C_L'' = (\frac{180-170.87}{18}) 2.394 = 1.21$.
3. $\hat{p}_L = 11.08\%$
4. From Table VI
 $p^* = 9.61\%$.

Since $\hat{p}_L > p^*$ the lot is rejected.

$\frac{1}{\sqrt{2}}$ h is a factor found in Table V which is a function of the sample code letter.

Table ~~XXX~~ Master Table for Normal and Tightened Inspection for Plans Based on Known Standard Deviation

Code letter		Acceptable Quality Levels (normal inspection)																				
		.065			.10			.15			.25			.40			.65					
		n	p	v	n	p	v	n	p	v	n	p	v	n	p	v	n	p	v			
B																						
C																						
D																						
E																						
F																						
G	3	.079	1.225	3	.114	1.225	4	.230	1.155	4	.399	1.155	4	.681	1.155	5	1.05	1.118	5	1.76	1.118	
H	4	.111	1.155	4	.161	1.155	5	.296	1.118	5	.445	1.118	6	.721	1.095	6	1.14	1.095	7	1.75	1.080	
I	5	.130	1.118	6	.230	1.095	6	.321	1.095	6	.478	1.095	7	.736	1.080	8	1.14	1.069	8	1.80	1.069	
J	6	.145	1.095	6	.234	1.095	7	.343	1.080	7	.507	1.080	8	.791	1.065	9	1.18	1.061	10	1.79	1.054	
K	7	.161	1.080	7	.226	1.080	8	.330	1.069	9	.469	1.061	9	.760	1.061	10	1.14	1.054	11	1.73	1.049	
L	8	.153	1.069	8	.243	1.069	9	.351	1.061	10	.494	1.054	11	.768	1.049	12	1.15	1.045	13	1.74	1.041	
M	10	.161	1.054	11	.217	1.049	11	.326	1.049	12	.461	1.045	13	.721	1.041	14	1.08	1.038	16	1.62	1.033	
N	14	.198	1.038	15	.211	1.035	16	.308	1.033	17	.438	1.031	19	.673	1.027	21	1.00	1.025	23	1.51	1.023	
O	19	.134	1.027	20	.207	1.026	22	.296	1.024	23	.423	1.023	25	.655	1.021	27	.980	1.019	30	1.47	1.017	
P	27	.129	1.019	30	.193	1.017	31	.283	1.017	34	.397	1.015	37	.615	1.014	40	.921	1.013	44	1.39	1.012	
Q	37	.130	1.014	40	.196	1.013	42	.285	1.012	45	.402	1.011	49	.620	1.010	54	.920	1.009	59	1.39	1.009	
		.065			.10			.15			.25			.40			.65			1.00		
Acceptable Quality Levels (tightened inspection)																						

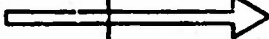
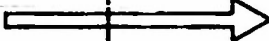
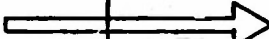
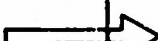
Table IV-d Master Table for Normal and Tightened Inspection for Plans Based on Known Standard Deviation (continued)

Min. Value		Acceptable Quality Levels (normal inspection)																							
H-L		H-L																							
C		C																							
5.15		4.48																							
1.00		1.50																							
2.50		4.00																							
6.50		10.00																							
15.00																									
B	2	2.73	1.414	2	3.90	1.414	2	6.11	1.414	2	9.27	1.414	2	17.74	1.225	3	24.22	1.225	4	33.67	1.225				
D	2	2.23	1.414	2	3.60	1.414	3	7.96	1.225	3	10.79	1.225	3	15.60	1.225	4	22.97	1.155	4	31.01	1.155				
E	3	2.76	1.225	3	3.85	1.225	4	6.99	1.155	4	9.97	1.155	5	15.21	1.118	5	20.80	1.118	6	28.64	1.095				
F	4	2.58	1.155	4	3.87	1.155	5	6.05	1.118	5	8.92	1.118	6	13.89	1.095	7	19.46	1.080	8	26.64	1.069				
G	6	2.57	1.095	6	3.77	1.095	7	5.83	1.080	8	8.62	1.069	9	12.88	1.061	11	17.88	1.049	12	24.88	1.045				
H	7	2.62	1.080	8	3.68	1.069	9	5.68	1.061	10	8.43	1.054	12	12.35	1.045	14	17.36	1.038	16	23.96	1.033				
I	9	2.59	1.061	10	3.63	1.054	11	5.60	1.049	13	8.13	1.041	15	12.04	1.033	17	17.05	1.031	20	23.43	1.026				
J	11	2.97	1.049	12	3.61	1.045	13	5.58	1.041	15	8.15	1.033	18	11.88	1.029	21	15.71	1.025	24	23.13	1.022				
K	12	2.49	1.045	14	3.43	1.038	15	5.34	1.035	18	7.72	1.029	20	11.97	1.026	24	16.33	1.022	27	22.63	1.019				
L	14	2.51	1.038	15	3.54	1.035	18	5.29	1.029	20	7.80	1.026	23	11.96	1.023	27	16.27	1.019	31	22.57	1.017				
M	17	2.35	1.031	19	3.28	1.027	22	4.98	1.024	25	7.34	1.021	29	10.93	1.018	33	15.61	1.016	38	21.77	1.013				
N	25	2.19	1.021	28	3.05	1.018	32	4.68	1.016	36	6.95	1.014	42	10.40	1.012	49	14.87	1.010	56	20.90	1.009				
O	33	2.12	1.016	36	2.99	1.014	42	4.55	1.012	46	6.75	1.011	55	10.17	1.009	64	14.58	1.008	75	20.45	1.007				
P	49	2.00	1.010	54	2.82	1.009	61	4.35	1.008	70	6.48	1.007	82	9.76	1.006	95	14.09	1.005	111	19.90	1.005				
Q	65	2.00	1.008	71	2.82	1.007	81	4.34	1.006	93	6.46	1.005	109	9.73	1.005	127	14.02	1.004	147	19.34	1.003				
1.50		2.50		4.00		6.50		10.00		15.00															
Acceptable Quality Levels (tightened inspection)																									

Table for Estimating the Lot Percentage Defective for Plans Based on Known Standard Deviation

[illegible]

Table 27-6 Master Table for Normal and Tightened Inspection for Sampling Plans Based on Unknown Standard Deviation

Sample size code letter	Sam- ple size	Acceptable Quality Levels (normal inspection)														
		.04	.065	.10	.15	.25	.40	.65	1.00	1.50	2.50	4.00	6.50	10.00	15.00	
		p*	p*	p*	p*	p*	p*	p*	p*	p*	p*	p*	p*	p*	p*	
B	3									7.59	18.86	26.94	33.69	40.47		
C	4							1.53	5.50	10.92	16.45	22.85	29.45	36.90		
D	5							1.33	3.32	5.83	9.80	14.39	20.19	26.56	33.99	
E	7					0.422	1.06	2.14	3.55	5.35	8.40	12.20	17.35	23.29	30.50	
F	10				0.349	0.716	1.30	2.17	3.26	4.77	7.29	10.54	15.17	20.74	27.57	
G	15	0.099	0.186	0.312	0.503	0.818	1.31	2.11	3.05	4.31	6.56	9.46	13.71	18.94	25.61	
H	20	0.135	0.228	0.365	0.544	0.846	1.29	2.05	2.95	4.09	6.17	8.92	12.99	18.03	24.53	
I	25	0.155	0.250	0.380	0.551	0.877	1.29	2.00	2.86	3.97	5.97	8.63	12.57	17.51	23.97	
J	30	0.179	0.280	0.413	0.581	0.879	1.29	1.98	2.83	3.91	5.86	8.47	12.36	17.24	23.58	
K	35	0.170	0.264	0.388	0.535	0.847	1.23	1.87	2.68	3.70	5.57	8.10	11.87	16.65	22.91	
L	40	0.179	0.275	0.401	0.566	0.873	1.26	1.88	2.71	3.72	5.58	8.09	11.85	16.61	22.86	
M	50	0.163	0.250	0.363	0.503	0.789	1.17	1.71	2.49	3.45	5.20	7.61	11.23	15.87	22.00	
N	75	0.147	0.228	0.330	0.467	0.720	1.07	1.60	2.29	3.20	4.87	7.15	10.63	15.13	21.11	
O	100	0.145	0.220	0.317	0.447	0.689	1.02	1.53	2.20	3.07	4.69	6.91	10.32	14.75	20.66	
P	150	0.134	0.203	0.293	0.413	0.638	0.949	1.43	2.05	2.89	4.43	6.57	9.38	14.20	20.02	
Q	200	0.135	0.204	0.294	0.414	0.637	0.945	1.42	2.04	2.87	4.40	6.53	9.81	14.12	19.92	
		.065	.10	.15	.25	.40	.65	1.00	1.50	2.50	4.00	6.50	10.00	15.00		
		Acceptable Quality Levels (tightened inspection)														

All AQL and table values are in percent defective.

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TABLE FOR INTERESTING AND FOR PROPORTION FOR PLANS BASED ON MEANS OF PROPORTION

Sample Mean Data Table	Sample Mean Data Table															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00
1	47.24	46.47	45.70	44.93	44.16	43.39	42.62	41.85	41.08	40.31	39.54	38.77	38.00	37.23	36.46	35.69
2	44.48	43.71	42.94	42.17	41.40	40.63	39.86	39.09	38.32	37.55	36.78	36.01	35.24	34.47	33.70	32.93
3	41.72	40.95	40.18	39.41	38.64	37.87	37.10	36.33	35.56	34.79	34.02	33.25	32.48	31.71	30.94	30.17
4	38.96	38.19	37.42	36.65	35.88	35.11	34.34	33.57	32.80	32.03	31.26	30.49	29.72	28.95	28.18	27.41
5	36.20	35.43	34.66	33.89	33.12	32.35	31.58	30.81	30.04	29.27	28.50	27.73	26.96	26.19	25.42	24.65
6	33.44	32.67	31.90	31.13	30.36	29.59	28.82	28.05	27.28	26.51	25.74	24.97	24.20	23.43	22.66	21.89
7	30.68	29.91	29.14	28.37	27.60	26.83	26.06	25.29	24.52	23.75	22.98	22.21	21.44	20.67	19.90	19.13
8	27.92	27.15	26.38	25.61	24.84	24.07	23.30	22.53	21.76	20.99	20.22	19.45	18.68	17.91	17.14	16.37
9	25.16	24.39	23.62	22.85	22.08	21.31	20.54	19.77	19.00	18.23	17.46	16.69	15.92	15.15	14.38	13.61
10	22.40	21.63	20.86	20.09	19.32	18.55	17.78	17.01	16.24	15.47	14.70	13.93	13.16	12.39	11.62	10.85
11	19.64	18.87	18.10	17.33	16.56	15.79	15.02	14.25	13.48	12.71	11.94	11.17	10.40	9.63	8.86	8.09
12	16.88	16.11	15.34	14.57	13.80	13.03	12.26	11.49	10.72	9.95	9.18	8.41	7.64	6.87	6.10	5.33
13	14.12	13.35	12.58	11.81	11.04	10.27	9.50	8.73	7.96	7.19	6.42	5.65	4.88	4.11	3.34	2.57
14	11.36	10.59	9.82	9.05	8.28	7.51	6.74	5.97	5.20	4.43	3.66	2.89	2.12	1.35	0.58	-0.19
15	8.60	7.83	7.06	6.29	5.52	4.75	3.98	3.21	2.44	1.67	0.90	0.13	-0.64	-1.41	-2.18	-2.95
16	5.84	5.07	4.30	3.53	2.76	1.99	1.22	0.45	-0.32	-1.09	-1.86	-2.63	-3.40	-4.17	-4.94	-5.71
17	3.08	2.31	1.54	0.77	-0.00	-0.77	-1.54	-2.31	-3.08	-3.85	-4.62	-5.39	-6.16	-6.93	-7.70	-8.47
18	0.32	-0.45	-1.22	-1.99	-2.76	-3.53	-4.30	-5.07	-5.84	-6.61	-7.38	-8.15	-8.92	-9.69	-10.46	-11.23
19	-2.44	-3.21	-3.98	-4.75	-5.52	-6.29	-7.06	-7.83	-8.60	-9.37	-10.14	-10.91	-11.68	-12.45	-13.22	-13.99
20	-5.16	-5.93	-6.70	-7.47	-8.24	-9.01	-9.78	-10.55	-11.32	-12.09	-12.86	-13.63	-14.40	-15.17	-15.94	-16.71
21	-7.88	-8.65	-9.42	-10.19	-10.96	-11.73	-12.50	-13.27	-14.04	-14.81	-15.58	-16.35	-17.12	-17.89	-18.66	-19.43
22	-10.60	-11.37	-12.14	-12.91	-13.68	-14.45	-15.22	-15.99	-16.76	-17.53	-18.30	-19.07	-19.84	-20.61	-21.38	-22.15
23	-13.32	-14.09	-14.86	-15.63	-16.40	-17.17	-17.94	-18.71	-19.48	-20.25	-21.02	-21.79	-22.56	-23.33	-24.10	-24.87
24	-16.04	-16.81	-17.58	-18.35	-19.12	-19.89	-20.66	-21.43	-22.20	-22.97	-23.74	-24.51	-25.28	-26.05	-26.82	-27.59
25	-18.76	-19.53	-20.30	-21.07	-21.84	-22.61	-23.38	-24.15	-24.92	-25.69	-26.46	-27.23	-28.00	-28.77	-29.54	-30.31
26	-21.48	-22.25	-23.02	-23.79	-24.56	-25.33	-26.10	-26.87	-27.64	-28.41	-29.18	-29.95	-30.72	-31.49	-32.26	-33.03
27	-24.20	-24.97	-25.74	-26.51	-27.28	-28.05	-28.82	-29.59	-30.36	-31.13	-31.90	-32.67	-33.44	-34.21	-34.98	-35.75
28	-26.92	-27.69	-28.46	-29.23	-30.00	-30.77	-31.54	-32.31	-33.08	-33.85	-34.62	-35.39	-36.16	-36.93	-37.70	-38.47
29	-29.64	-30.41	-31.18	-31.95	-32.72	-33.49	-34.26	-35.03	-35.80	-36.57	-37.34	-38.11	-38.88	-39.65	-40.42	-41.19
30	-32.36	-33.13	-33.90	-34.67	-35.44	-36.21	-36.98	-37.75	-38.52	-39.29	-40.06	-40.83	-41.60	-42.37	-43.14	-43.91
31	-35.08	-35.85	-36.62	-37.39	-38.16	-38.93	-39.70	-40.47	-41.24	-42.01	-42.78	-43.55	-44.32	-45.09	-45.86	-46.63
32	-37.80	-38.57	-39.34	-40.11	-40.88	-41.65	-42.42	-43.19	-43.96	-44.73	-45.50	-46.27	-47.04	-47.81	-48.58	-49.35
33	-40.52	-41.29	-42.06	-42.83	-43.60	-44.37	-45.14	-45.91	-46.68	-47.45	-48.22	-48.99	-49.76	-50.53	-51.30	-52.07
34	-43.24	-44.01	-44.78	-45.55	-46.32	-47.09	-47.86	-48.63	-49.40	-50.17	-50.94	-51.71	-52.48	-53.25	-54.02	-54.79
35	-45.96	-46.73	-47.50	-48.27	-49.04	-49.81	-50.58	-51.35	-52.12	-52.89	-53.66	-54.43	-55.20	-55.97	-56.74	-57.51
36	-48.68	-49.45	-50.22	-50.99	-51.76	-52.53	-53.30	-54.07	-54.84	-55.61	-56.38	-57.15	-57.92	-58.69	-59.46	-60.23
37	-51.40	-52.17	-52.94	-53.71	-54.48	-55.25	-56.02	-56.79	-57.56	-58.33	-59.10	-59.87	-60.64	-61.41	-62.18	-62.95
38	-54.12	-54.89	-55.66	-56.43	-57.20	-57.97	-58.74	-59.51	-60.28	-61.05	-61.82	-62.59	-63.36	-64.13	-64.90	-65.67
39	-56.84	-57.61	-58.38	-59.15	-59.92	-60.69	-61.46	-62.23	-63.00	-63.77	-64.54	-65.31	-66.08	-66.85	-67.62	-68.39
40	-59.56	-60.33	-61.10	-61.87	-62.64	-63.41	-64.18	-64.95	-65.72	-66.49	-67.26	-68.03	-68.80	-69.57	-70.34	-71.11
41	-62.28	-63.05	-63.82	-64.59	-65.36	-66.13	-66.90	-67.67	-68.44	-69.21	-69.98	-70.75	-71.52	-72.29	-73.06	-73.83
42	-65.00	-65.77	-66.54	-67.31	-68.08	-68.85	-69.62	-70.39	-71.16	-71.93	-72.70	-73.47	-74.24	-75.01	-75.78	-76.55
43	-67.72	-68.49	-69.26	-70.03	-70.80	-71.57	-72.34	-73.11	-73.88	-74.65	-75.42	-76.19	-76.96	-77.73	-78.50	-79.27
44	-70.44	-71.21	-71.98	-72.75	-73.52	-74.29	-75.06	-75.83	-76.60	-77.37	-78.14	-78.91	-79.68	-80.45	-81.22	-81.99
45	-73.16	-73.93	-74.70	-75.47	-76.24	-77.01	-77.78	-78.55	-79.32	-80.09	-80.86	-81.63	-82.40	-83.17	-83.94	-84.71
46	-75.88	-76.65	-77.42	-78.19	-78.96	-79.73	-80.50	-81.27	-82.04	-82.81	-83.58	-84.35	-85.12	-85.89	-86.66	-87.43
47	-78.60	-79.37	-80.14	-80.91	-81.68	-82.45	-83.22	-83.99	-84.76	-85.53	-86.30	-87.07	-87.84	-88.61	-89.38	-90.15
48	-81.32	-82.09	-82.86	-83.63	-84.40	-85.17	-85.94	-86.71	-87.48	-88.25	-89.02	-89.79	-90.56	-91.33	-92.10	-92.87
49	-84.04	-84.81	-85.58	-86.35	-87.12	-87.89	-88.66	-89.43	-90.20	-90.97	-91.74	-92.51	-93.28	-94.05	-94.82	-95.59
50	-86.76	-87.53	-88.30	-89.07	-89.84	-90.61	-91.38	-92.15	-92.92	-93.69	-94.46	-95.23	-96.00	-96.77	-97.54	-98.31
51	-89.48	-90.25	-91.02	-91.79	-92.56	-93.33	-94.10	-94.87	-95.64	-96.41	-97.18	-97.95	-98.72	-99.49	-100.26	-101.03
52	-92.20	-92.97	-93.74	-94.51	-95.28	-96.05	-96.82	-97.59	-98.36	-99.13	-99.90	-100.67	-101.44	-102.21	-102.98	-103.75
53	-94.92	-95.69	-96.46	-97.23	-98.00	-98.77	-99.54	-100.31	-101.08	-101.85	-102.62	-103.39	-104.16	-104.93	-105.70	-106.47
54	-97.64	-98.41	-99.18	-99.95	-100.72	-101.49	-102.26	-103.03	-103.80	-104.57	-105.34	-106.11	-106.88	-107.65	-108.42	-109.19
55	-100.36	-101.13	-101.90	-102.67	-103.44	-104.21	-104.98	-105.75	-106.52	-107.29	-108.06	-108.83	-109.60	-110.37	-111.14	-111.91
56	-103.08	-103.85	-104.62	-105.39	-106.16	-106.93	-107.70	-108.47	-109.24	-110.01	-110.78	-111.55	-112.32	-113.09	-113.86	-114.63
57	-105.80	-106.57	-107.34	-108.11	-108.88	-109.65	-110.42	-111.19	-111.96	-112.73	-113.50	-114.27	-115.04	-115.81	-116.58	-117.35
58	-108.52	-109.29	-110.06	-110.83	-111.60	-112.37	-113.14	-113.91	-114.68	-115.45	-116.22	-116.99	-117.76	-118.53	-119.30	-120.07
59	-111.24	-112.01	-112.78	-113.55	-114.32	-115.09	-115.86	-116.63	-117.40	-118.17	-118.94	-119.71	-120.48	-121.25	-122.02	-122.79
60	-113.96	-114.73	-115.50	-116.27	-117.04	-117.81	-118.58	-119.35	-120.12	-120.89	-121.66	-122.43	-123.20	-123.97	-124.74	-125.51

Values indicated are read in percent.

IV

TABLE 10000. TABLE FOR ESTIMATING THE LOG PROBABILITIES FOR PLAIN BASED ON MEASURED PROBABILITY DISTRIBUTION (continued)

z		Sample Size Code Letter																				Sample Size Code Letter																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																													
		A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	aa	ab	ac	ad	ae	af	ag	ah	ai	aj	ak	al	am	an	ao	ap	aq	ar	as	at	au	av	aw	ax	ay	az																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																														
1.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.0

NOTE: PERCENTAGE OF THE TOTAL STANDARD DEVIATION (continued)

[illegible]

TABLE VIII
TABLE FOR ESTIMATING THE LOT PERCENTAGE FOR PLANS BASED ON UNKNOWN STANDARD DEVIATION (continued)[illegible]

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TABLE VII-9. TABLE FOR ESTIMATING THE LOT PERCENTAGE FOR PLANS BASED ON MEASURED SPACINGS (continued)

[illegible]

Table Y Master Table for Normal and Tightened Inspection for Sampling Plans Based on the Average Range

Sample size code letter	Sample size	h factor	Acceptable Quality Levels (normal inspection)												
			.04	.065	.10	.15	.25	.40	.65	1.00	1.50	2.50	4.00	6.50	10.00
			p*	p*	p*	p*	p*	p*	p*	p*	p*	p*	p*	p*	p*
B	3	1.910										7.59	18.86	26.94	33.69
C	4	2.243					0.420	0.580	1.54	2.89	5.50	10.92	16.45	22.86	29.45
D	5	2.607					0.175	0.990	1.85	2.98	4.47	8.10	12.73	20.57	25.30
E	7	2.907					0.053	0.123	0.401	1.24	3.02	6.62	9.61	13.92	19.16
F	10	2.439					0.233	0.208	0.496	1.25	2.79	5.94	8.60	12.53	17.42
G	15	2.394	0.053	0.123	0.233	0.401	0.740	1.24	2.02	3.02	4.32	6.62	9.61	13.92	19.16
H	25	2.363	0.121	0.208	0.328	0.496	0.812	1.25	1.93	2.79	3.93	5.94	8.60	12.53	17.42
I	30	2.356	0.144	0.236	0.360	0.530	0.846	1.28	1.95	2.79	3.89	5.85	8.47	12.32	17.15
J	35	2.351	0.162	0.258	0.387	0.558	0.876	1.32	1.97	2.81	3.89	5.82	8.40	12.21	17.00
K	40	2.348	0.159	0.250	0.372	0.535	0.836	1.25	1.87	2.68	3.72	5.59	8.09	11.82	16.53
L	50	2.343	0.168	0.260	0.380	0.539	0.834	1.24	1.60	2.62	3.63	5.46	7.89	11.55	16.19
M	60	2.340	0.157	0.243	0.355	0.503	0.778	1.16	1.74	2.47	3.43	5.17	7.53	11.09	15.63
N	85	2.336	0.156	0.241	0.349	0.493	0.754	1.12	1.67	2.37	3.29	4.97	7.27	10.72	15.17
O	115	2.333	0.153	0.230	0.333	0.467	0.717	1.06	1.58	2.25	3.14	4.76	6.99	10.37	14.74
P	175	2.332	0.139	0.210	0.303	0.427	0.655	0.972	1.46	2.08	2.93	4.47	6.60	9.89	14.15
Q	230	2.330	0.142	0.215	0.308	0.432	0.661	0.976	1.47	2.08	2.92	4.46	6.57	9.84	14.10
			0.065	.10	.15	.25	.40	.65	1.00	1.50	2.50	4.00	6.50	10.00	15.00
			Acceptable Quality Levels (tightened inspection)												

All AQL and table values are in percent defective.

TABLE FOR ESTIMATING THE LOS PERCENTAGE FOR PAIRS BASED ON AVERAGE RUMPS

C	Sample Size: Code Letter										Sample Size: Code Letter									
	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
0	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00
1	47.24	46.47	45.44	44.29	42.97	41.48	39.84	38.07	36.19	34.19	32.07	29.84	27.51	25.09	22.59	20.00	17.34	14.54	11.61	8.56
2	44.46	43.33	42.20	40.84	39.34	37.71	35.94	34.07	32.19	30.19	28.07	25.84	23.51	21.09	18.59	16.00	13.34	10.54	7.61	4.56
3	41.68	40.33	39.00	37.54	35.94	34.31	32.54	30.67	28.79	26.79	24.67	22.44	20.11	17.69	15.19	12.60	10.00	7.24	4.31	1.26
4	38.90	37.44	36.00	34.44	32.84	31.21	29.44	27.57	25.69	23.69	21.57	19.34	17.01	14.59	12.19	9.60	7.00	4.24	1.31	0.00
5	36.12	34.67	33.23	31.57	29.97	28.34	26.57	24.70	22.82	20.82	18.70	16.47	14.14	11.79	9.39	6.90	4.30	1.54	0.00	0.00
6	33.34	31.89	30.45	28.79	27.19	25.56	23.79	21.92	20.04	18.04	15.92	13.69	11.36	9.01	6.61	4.10	1.54	0.00	0.00	0.00
7	30.56	29.11	27.67	26.01	24.41	22.78	21.01	19.14	17.26	15.26	13.14	10.91	8.58	6.23	3.83	1.32	0.00	0.00	0.00	0.00
8	27.78	26.33	24.89	23.23	21.63	20.00	18.33	16.56	14.68	12.68	10.56	8.33	6.00	3.60	1.19	0.00	0.00	0.00	0.00	0.00
9	25.00	23.55	22.11	20.45	18.85	17.22	15.55	13.78	11.90	10.00	8.00	5.98	3.85	1.61	0.00	0.00	0.00	0.00	0.00	0.00
10	22.22	20.77	19.33	17.67	16.07	14.44	12.77	11.00	9.22	7.44	5.66	3.83	2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
11	19.44	17.99	16.55	14.89	13.29	11.66	10.00	8.33	6.55	4.77	3.00	1.26	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
12	16.66	15.21	13.77	12.11	10.51	8.88	7.22	5.55	3.77	2.00	0.26	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
13	13.88	12.43	11.00	9.33	7.73	6.10	4.44	2.77	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
14	11.10	9.65	8.21	6.55	4.95	3.32	1.66	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
15	8.32	6.87	5.43	3.77	2.17	0.54	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
16	5.54	4.09	2.65	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
17	2.76	1.31	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
23	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
24	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
26	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
27	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
28	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
29	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
31	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
32	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
34	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
35	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
36	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
37	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
38	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
39	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
41	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
42	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
43	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
44	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
45	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
46	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
47	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
48	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
49	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Values stipulated are read in percent.

TABLE FOR DETERMINING THE LOT PERCENTAGE FOR PLANS BASED ON AVERAGE RANGE (continued)

By 4 or 4		Sample Size Code Letter															
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	
1.10	9.84	13.33	13.46	13.59	13.72	13.85	13.98	14.11	14.24	14.37	14.50	14.63	14.76	14.89	15.02	15.15	
1.11	9.89	13.00	13.13	13.26	13.39	13.52	13.65	13.78	13.91	14.04	14.17	14.30	14.43	14.56	14.69	14.82	
1.12	7.08	12.67	12.80	12.93	13.06	13.19	13.32	13.45	13.58	13.71	13.84	13.97	14.10	14.23	14.36	14.49	
1.13	6.40	12.33	12.46	12.59	12.72	12.85	12.98	13.11	13.24	13.37	13.50	13.63	13.76	13.89	14.02	14.15	
1.14	5.08	12.00	12.13	12.26	12.39	12.52	12.65	12.78	12.91	13.04	13.17	13.30	13.43	13.56	13.69	13.82	
1.15	4.89	11.67	11.80	11.93	12.06	12.19	12.32	12.45	12.58	12.71	12.84	12.97	13.10	13.23	13.36	13.49	
1.16	4.00	11.33	11.46	11.59	11.72	11.85	11.98	12.11	12.24	12.37	12.50	12.63	12.76	12.89	13.02	13.15	
1.17	4.00	11.00	11.13	11.26	11.39	11.52	11.65	11.78	11.91	12.04	12.17	12.30	12.43	12.56	12.69	12.82	
1.18	4.00	10.67	10.80	10.93	11.06	11.19	11.32	11.45	11.58	11.71	11.84	11.97	12.10	12.23	12.36	12.49	
1.19	4.00	10.33	10.46	10.59	10.72	10.85	10.98	11.11	11.24	11.37	11.50	11.63	11.76	11.89	12.02	12.15	
1.20	4.00	10.00	10.13	10.26	10.39	10.52	10.65	10.78	10.91	11.04	11.17	11.30	11.43	11.56	11.69	11.82	
1.21	4.00	9.67	9.80	9.93	10.06	10.19	10.32	10.45	10.58	10.71	10.84	10.97	11.10	11.23	11.36	11.49	
1.22	4.00	9.33	9.46	9.59	9.72	9.85	9.98	10.11	10.24	10.37	10.50	10.63	10.76	10.89	11.02	11.15	
1.23	4.00	9.00	9.13	9.26	9.39	9.52	9.65	9.78	9.91	10.04	10.17	10.30	10.43	10.56	10.69	10.82	
1.24	4.00	8.67	8.80	8.93	9.06	9.19	9.32	9.45	9.58	9.71	9.84	9.97	10.10	10.23	10.36	10.49	
1.25	4.00	8.33	8.46	8.59	8.72	8.85	8.98	9.11	9.24	9.37	9.50	9.63	9.76	9.89	10.02	10.15	
1.26	4.00	8.00	8.13	8.26	8.39	8.52	8.65	8.78	8.91	9.04	9.17	9.30	9.43	9.56	9.69	9.82	
1.27	4.00	7.67	7.80	7.93	8.06	8.19	8.32	8.45	8.58	8.71	8.84	8.97	9.10	9.23	9.36	9.49	
1.28	4.00	7.33	7.46	7.59	7.72	7.85	7.98	8.11	8.24	8.37	8.50	8.63	8.76	8.89	9.02	9.15	
1.29	4.00	7.00	7.13	7.26	7.39	7.52	7.65	7.78	7.91	8.04	8.17	8.30	8.43	8.56	8.69	8.82	
1.30	4.00	6.67	6.80	6.93	7.06	7.19	7.32	7.45	7.58	7.71	7.84	7.97	8.10	8.23	8.36	8.49	
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TABLE 10.—TABLE FOR ESTIMATING THE LOT PERCENTAGE FOR PLANS BASED ON AVERAGE RANGE (continued.)

[illegible]

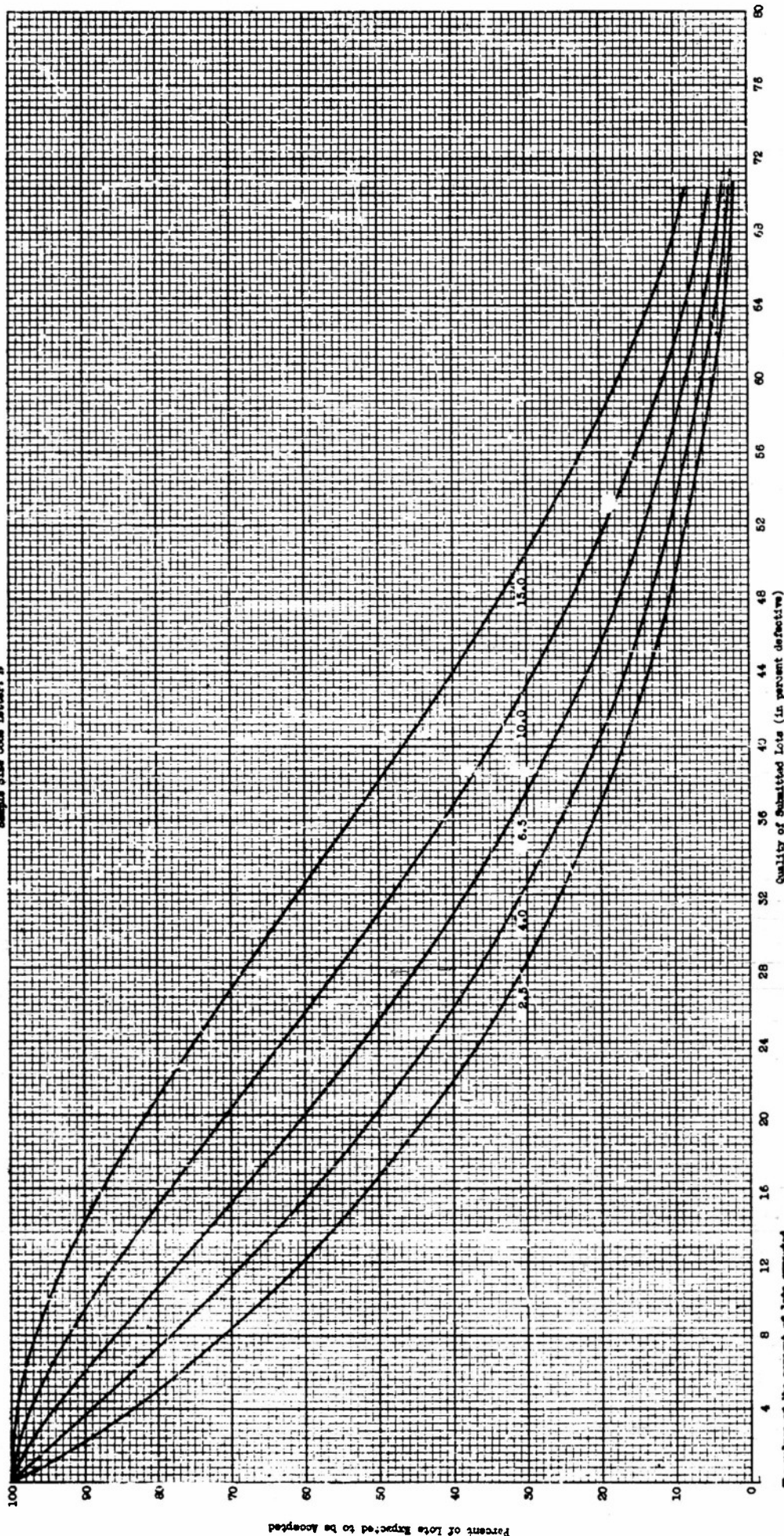
Fig. 1

Sampling Plans for Sample Size Code Letter: B

OPERATING CHARACTERISTIC CURVES FOR SAMPLING PLANS BASED ON UNKNOWN STANDARD DEVIATION

(Curves for sampling plans based on average range and known standard deviations are essentially equivalent)

Sample Size Code Letter: B

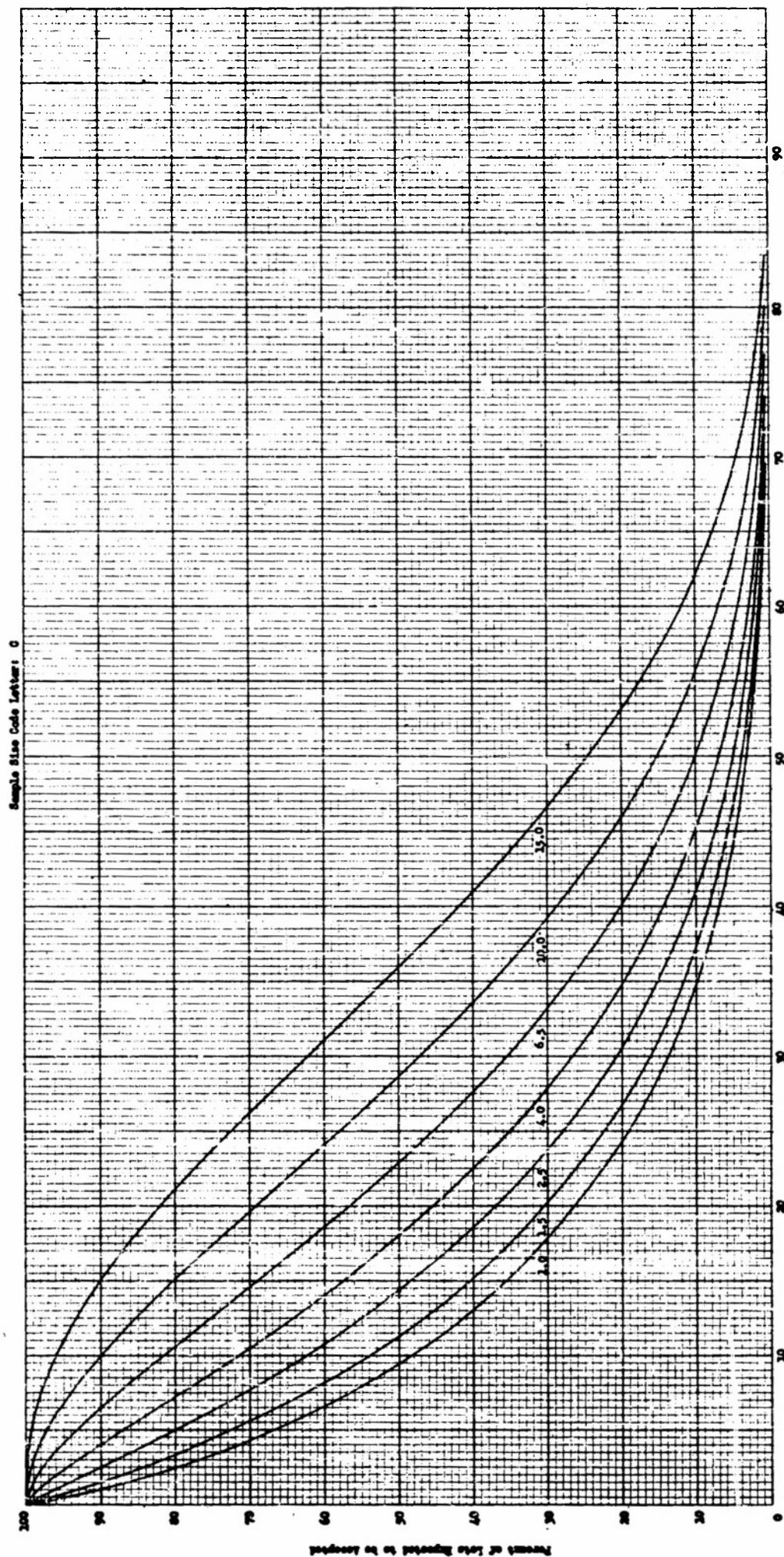


The values of the percent of lots expected to be accepted are valid only when the submitted items follow a normal distribution.

Note: Figures on curves are Acceptance Quality Levels for normal inspection

Fig 2

Sampling Plans for Sample Size Letter: G - Continued
 OPERATING CHARACTERISTIC CURVES FOR SAMPLING PLANS BASED ON UNKNOWN STANDARD DEVIATION
 (Curves for sampling plans based on average range and known standard deviations are essentially equivalent)



Quality of Submitted Lots (in percent defective)
 Notes: Figures on curves are Acceptance Quality Levels for normal inspection

The values of the percent of lots expected to be accepted are valid only when the submitted lots follow a normal distribution.

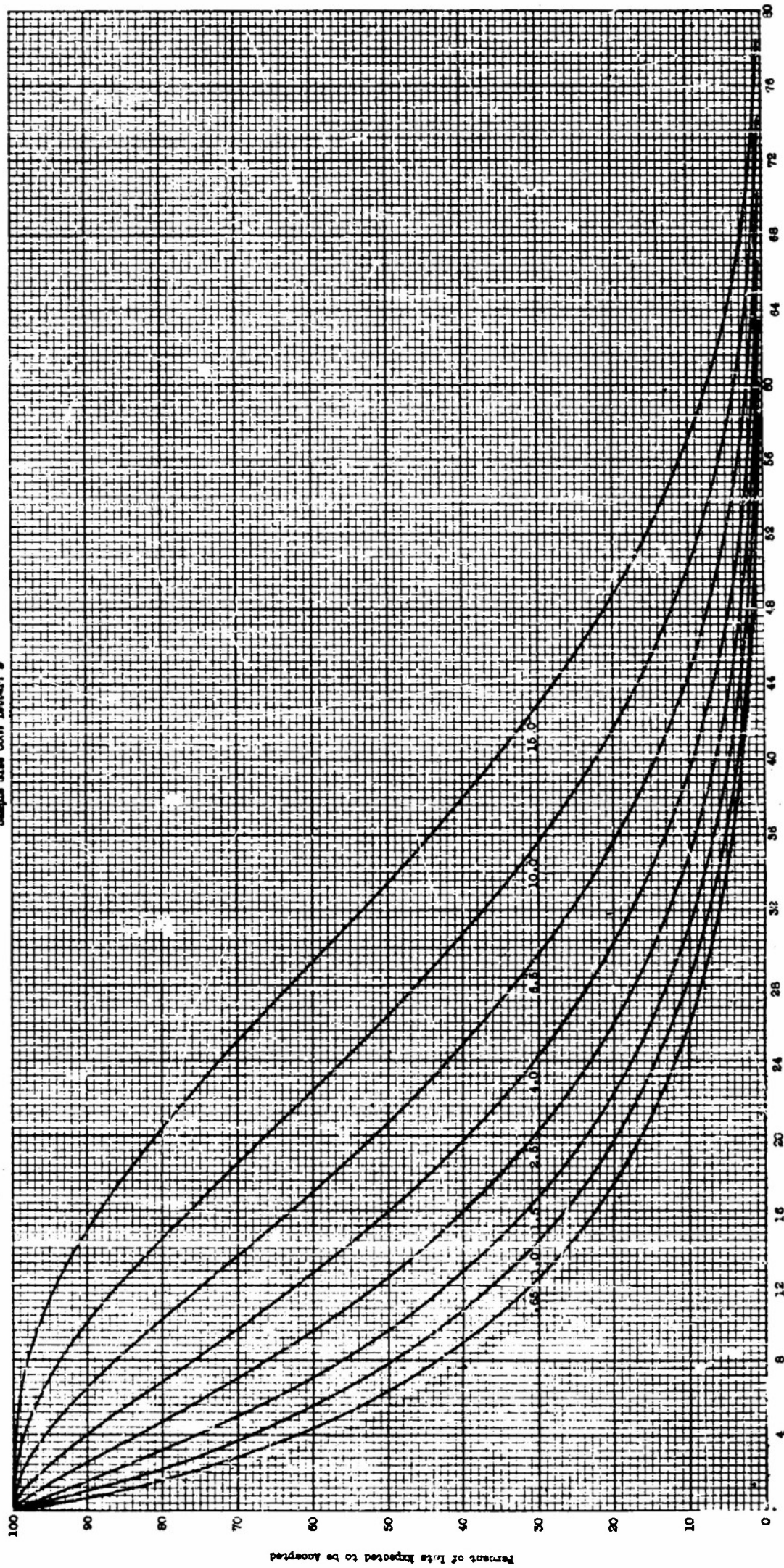
Fig. 3

Sampling Plans for Sample Size Code Letter: B—Continued

OPERATING CHARACTERISTIC CURVES FOR SAMPLING PLANS BASED ON UNKNOWN STANDARD DEVIATION

(Curves for sampling plans based on average runs and known standard deviations are essentially equivalent)

Sample Size Code Letter: B

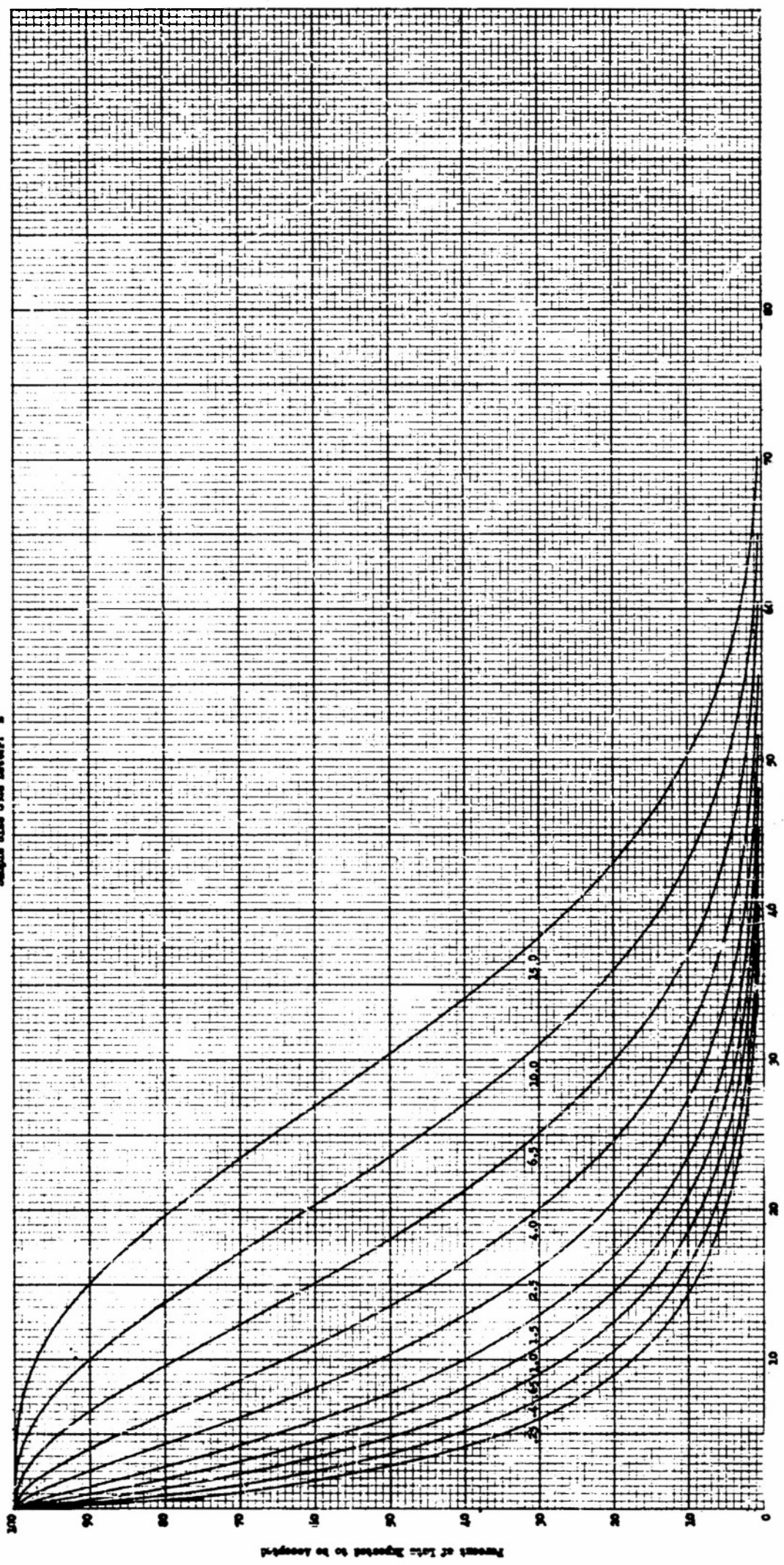


Note: Figures on curves are acceptance quality levels for normal inspection

The values of the percent of lots expected to be accepted are valid only when the submitted lots follow a normal distribution.

Fig. 4

Sampling Plans for Sample Size Code Letter: B - Continued
 OPERATING CHARACTERISTIC CURVES FOR SAMPLE PLANS BASED ON UNKNOWN STANDARD DEVIATION
 (Curves for sampling plans based on average range and known standard deviations are essentially equivalent)
 Sample Size Code Letter: B



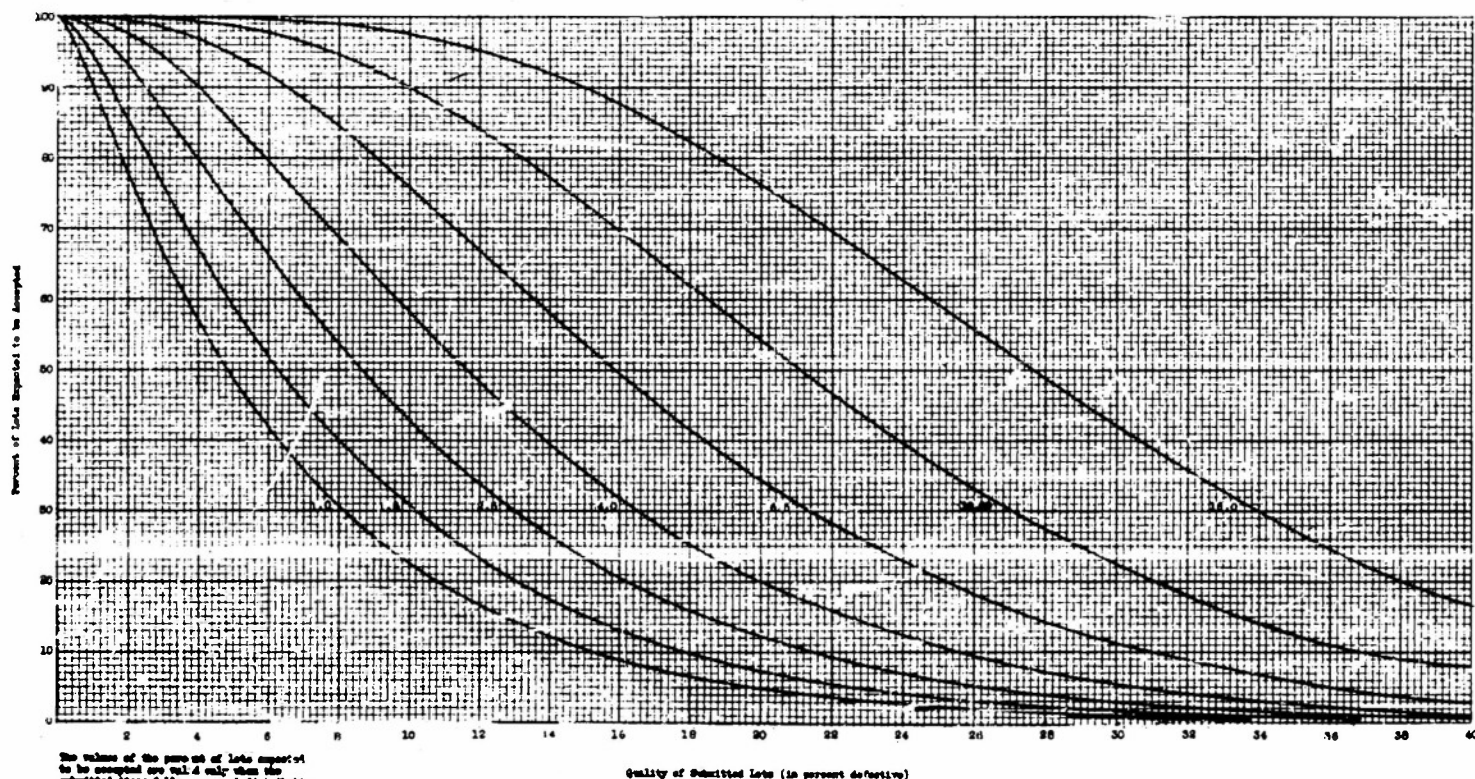
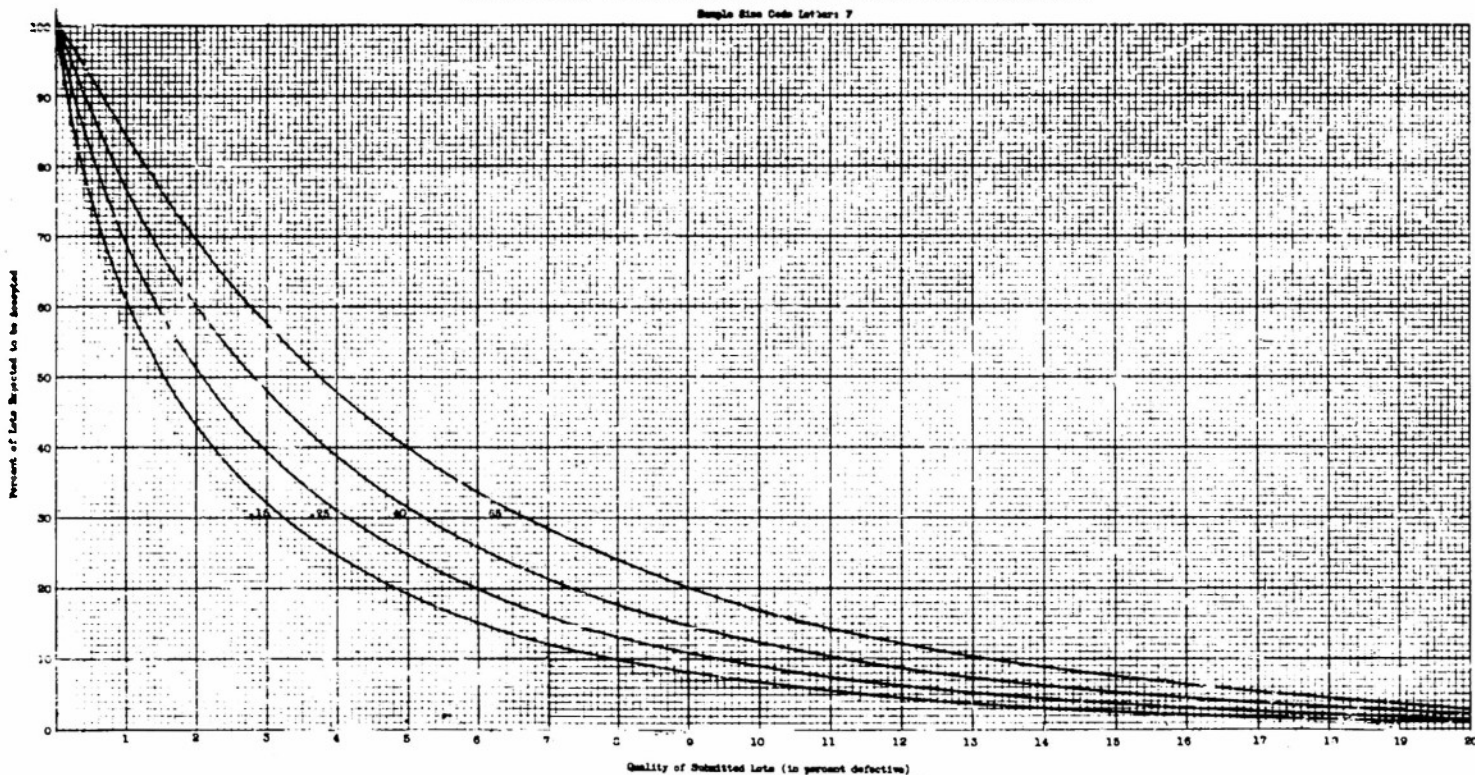
The values of the percent of lots accepted to be accepted are valid only when the submitted items follow a normal distribution.

Notes: Figures on curves are Acceptance Quality Levels for normal inspection.

Fig. 5

Sampling Plans for Sample Size Code Letter: F—Continued
 (Sampling Characteristics Curves for Sampling Plans Based on Known Standard Deviation
 (Curves for sampling plans based on average range and known standard deviations are essentially equivalent))

Sample Size Code Letter: F

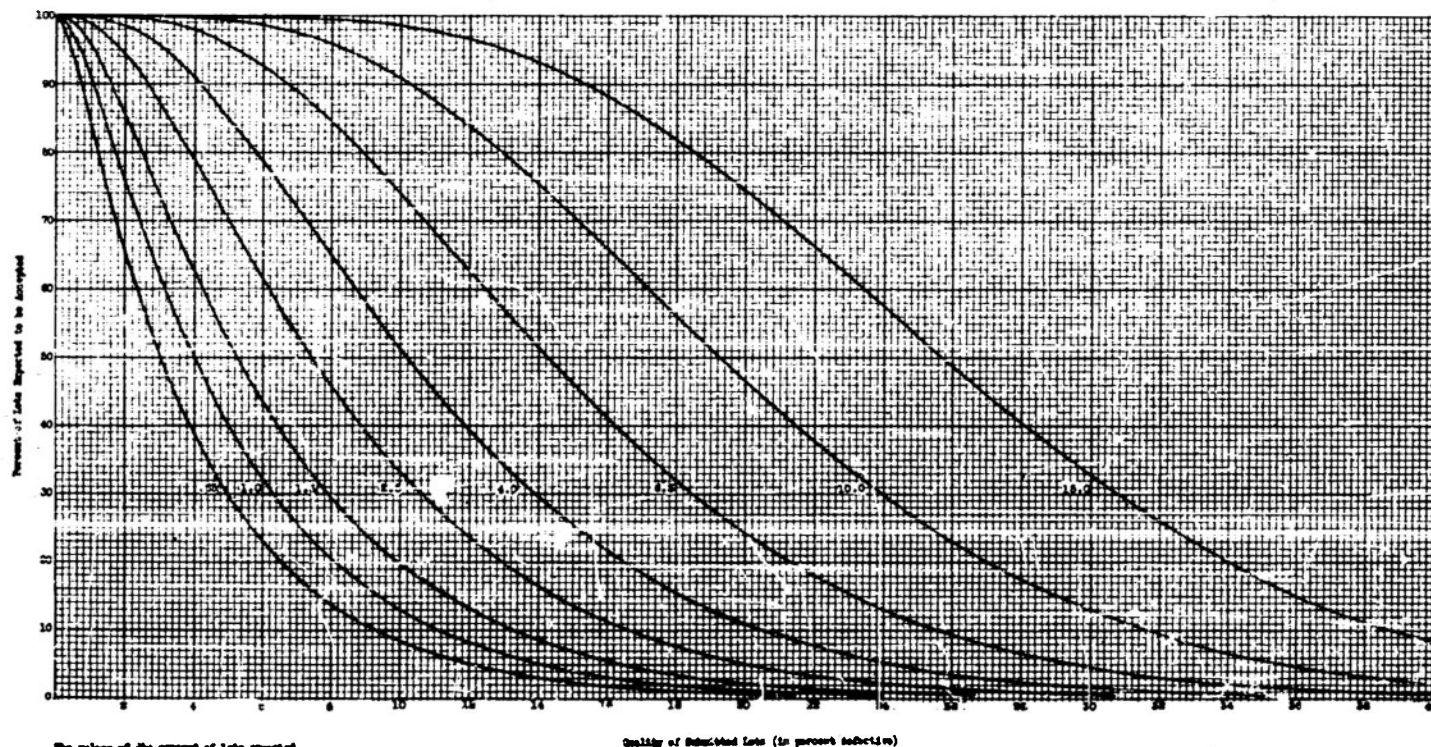
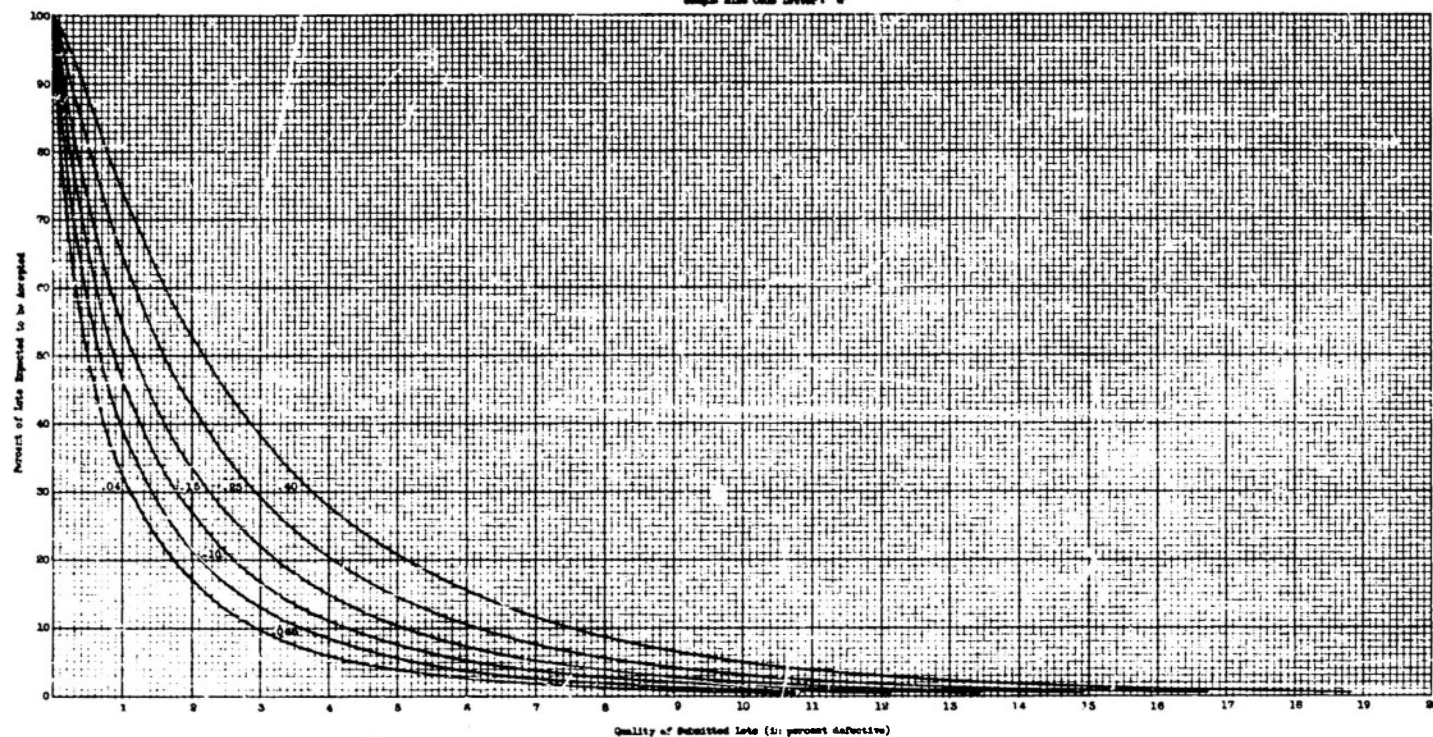


The values of the percent of lots expected to be accepted are valid only when the submitted lots follow a normal distribution.

Note: Figures on curves are Acceptance Quality Levels for normal inspection

Fig. 6

Sampling Plans for Sample Size Code Letter: G—Continued
OPERATING CHARACTERISTIC CURVES FOR SAMPLING PLANS BASED ON NORMAL DISTRIBUTION
(Curves for sampling plans based on average range and known standard deviation or essentially equivalent)
Sample Size Code Letter: G

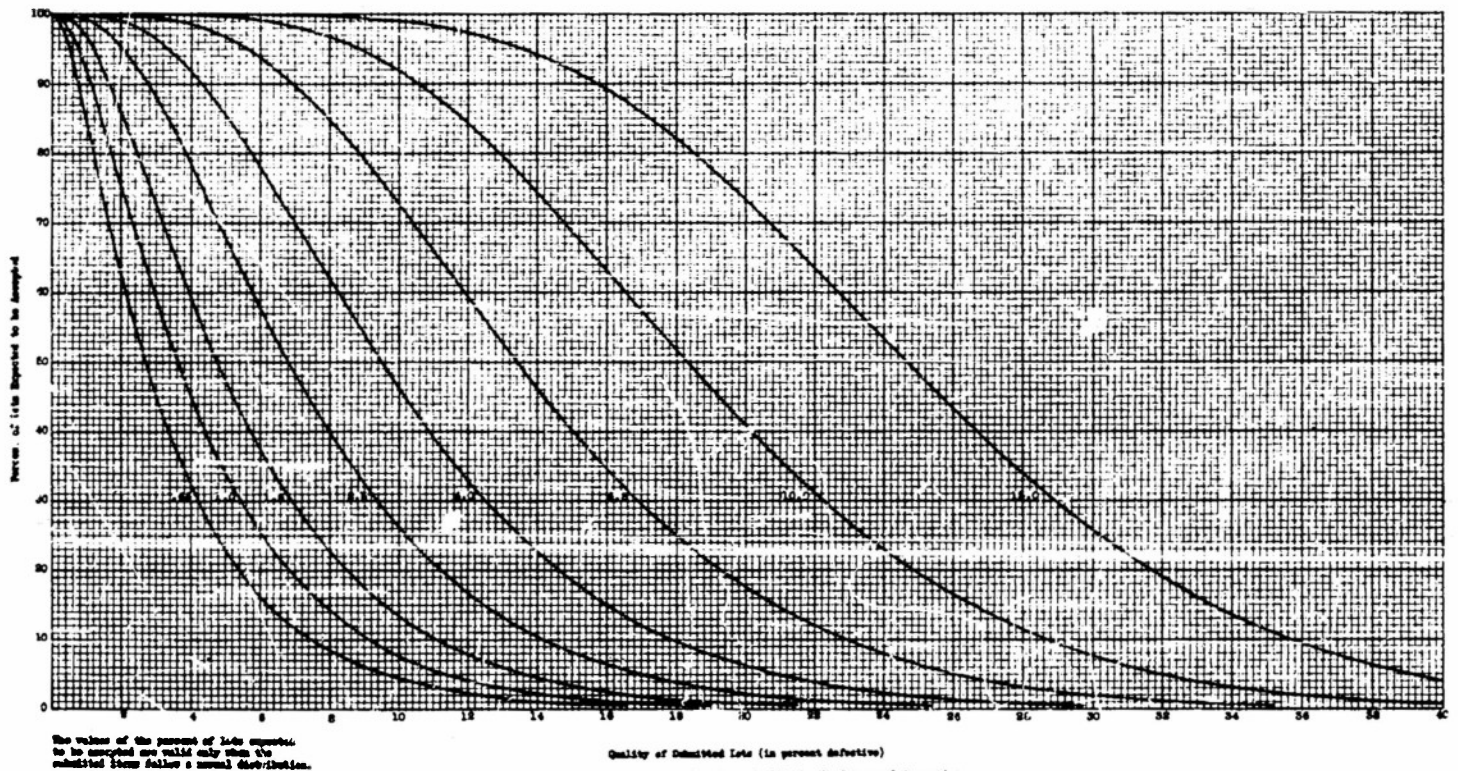
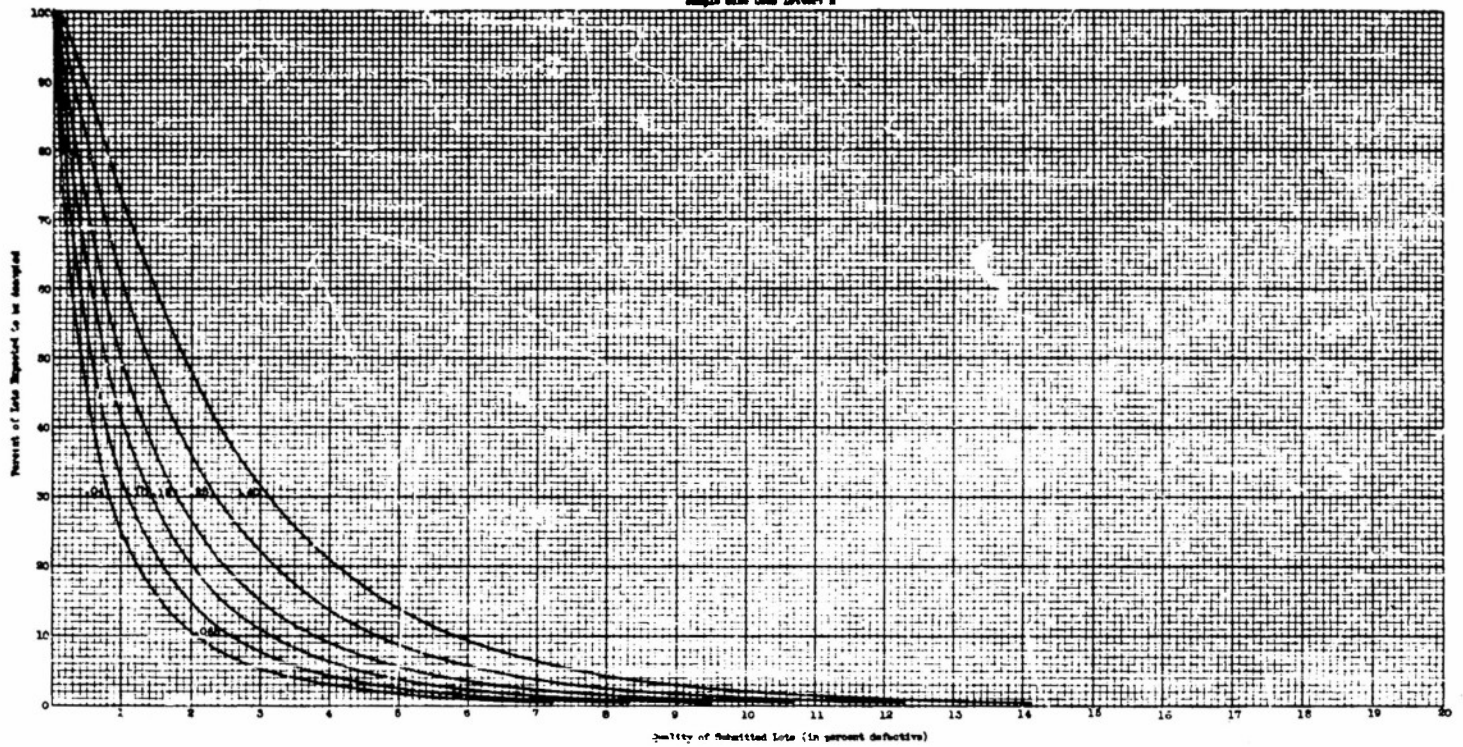


The values of the percent of lots expected to be accepted are valid only when the submitted lots follow a normal distribution.

Note: Figures on curves are acceptance quality levels for normal inspection.

Fig. 7

Operating Characteristic Curves for Sampling Plans Based on Known Standard Deviation
(Curves for sampling plans based on average range and known standard deviations are essentially equivalent)
Sample Size Code Letter: H



The values of the percent of lots expected to be accepted are valid only when the submitted lots follow a normal distribution.

Note: Figures on curves are "Acceptance Quality Levels" for normal inspection

Table VI-2. Sampling Plan for Sample Size Code Letter: I—Continued
 OPERATING CHARACTERISTIC CURVES FOR SAMPLING PLANS BASED ON KNOWN STANDARD DEVIATION
 (Curves for sampling plans based on average range and known standard deviations are essentially equivalent)
 Sample Size Code Letter: I

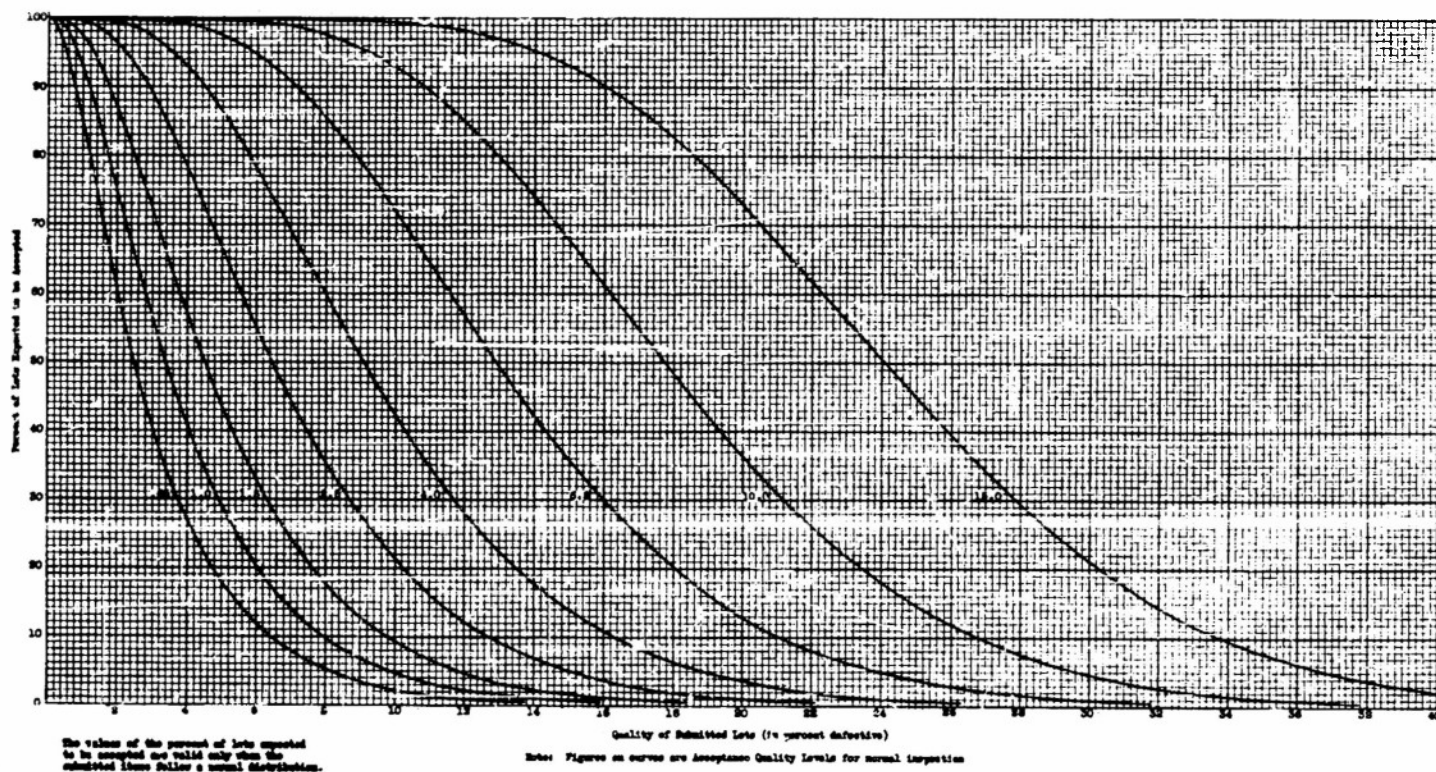
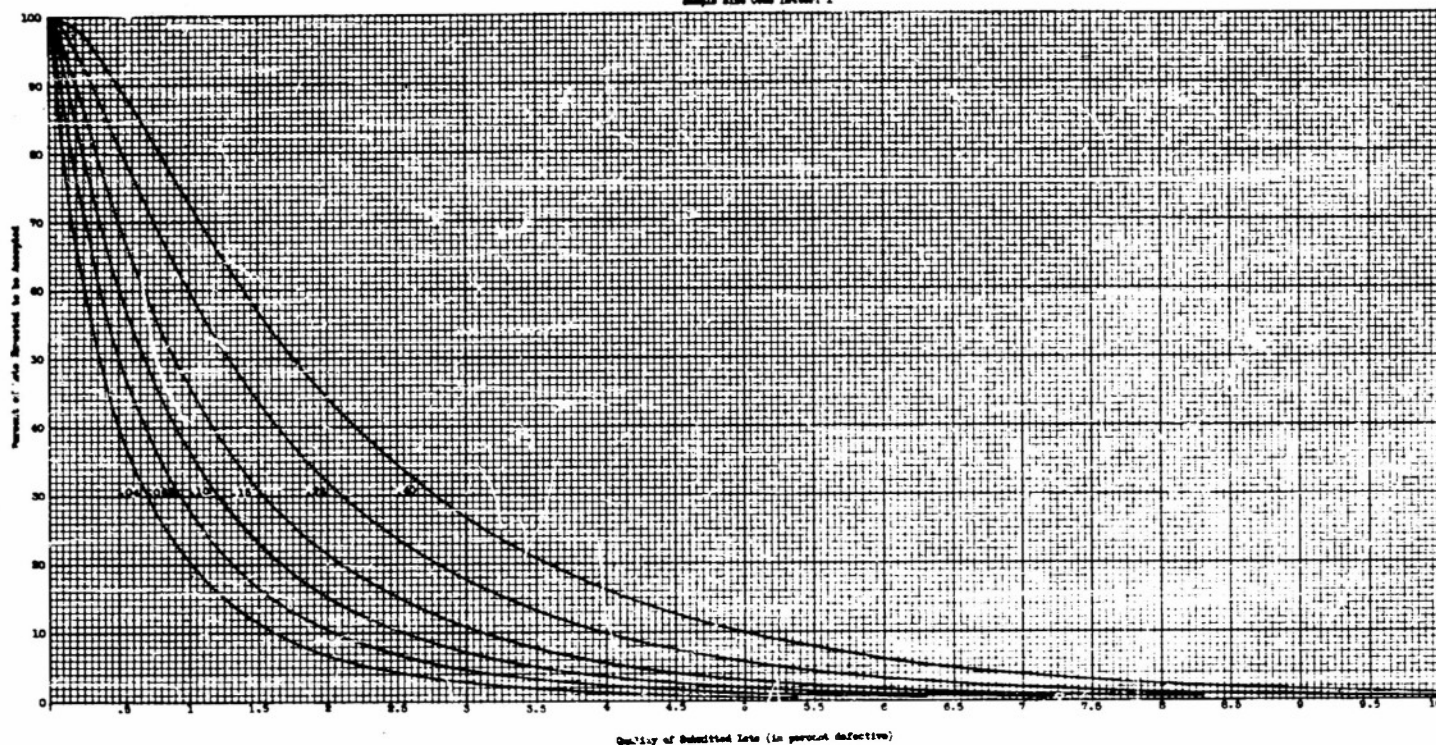
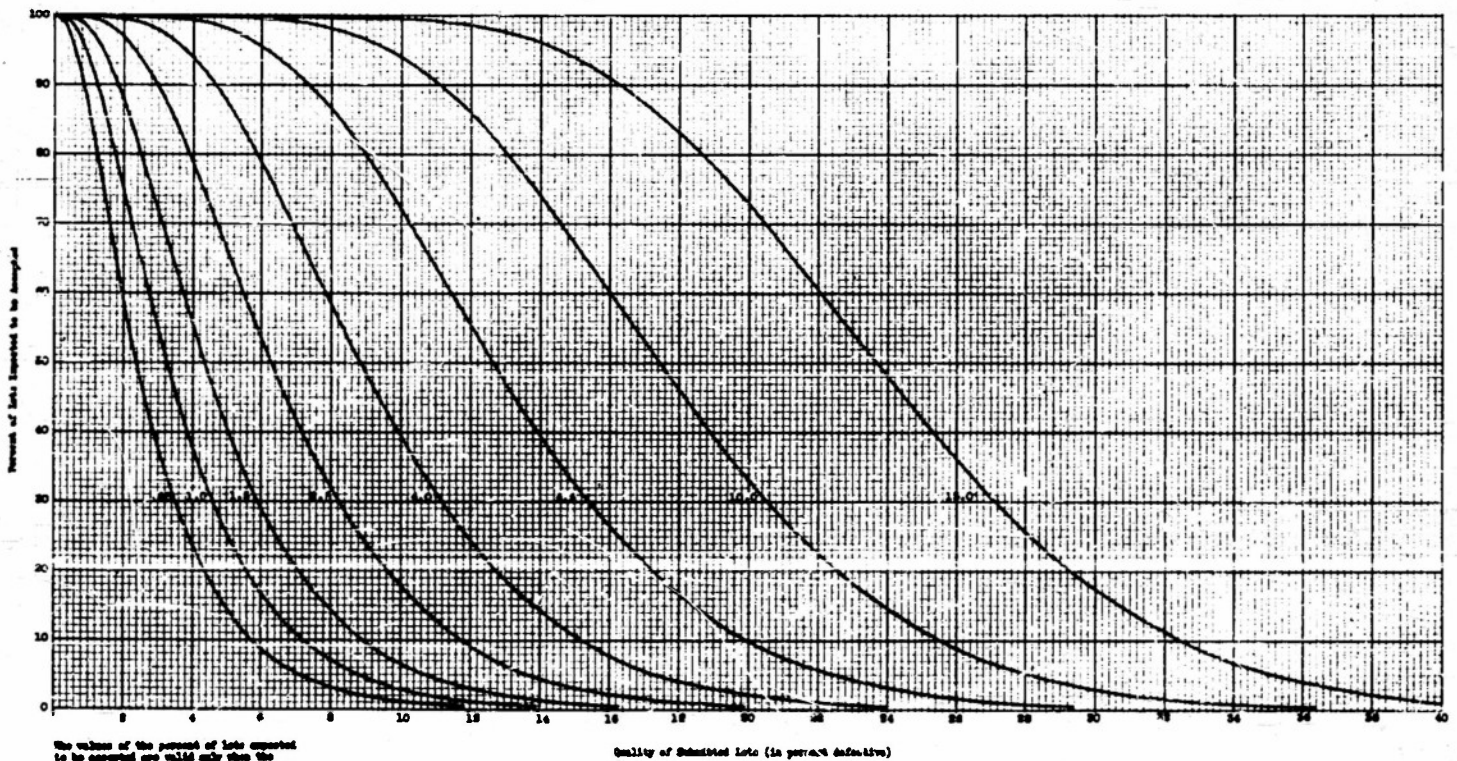
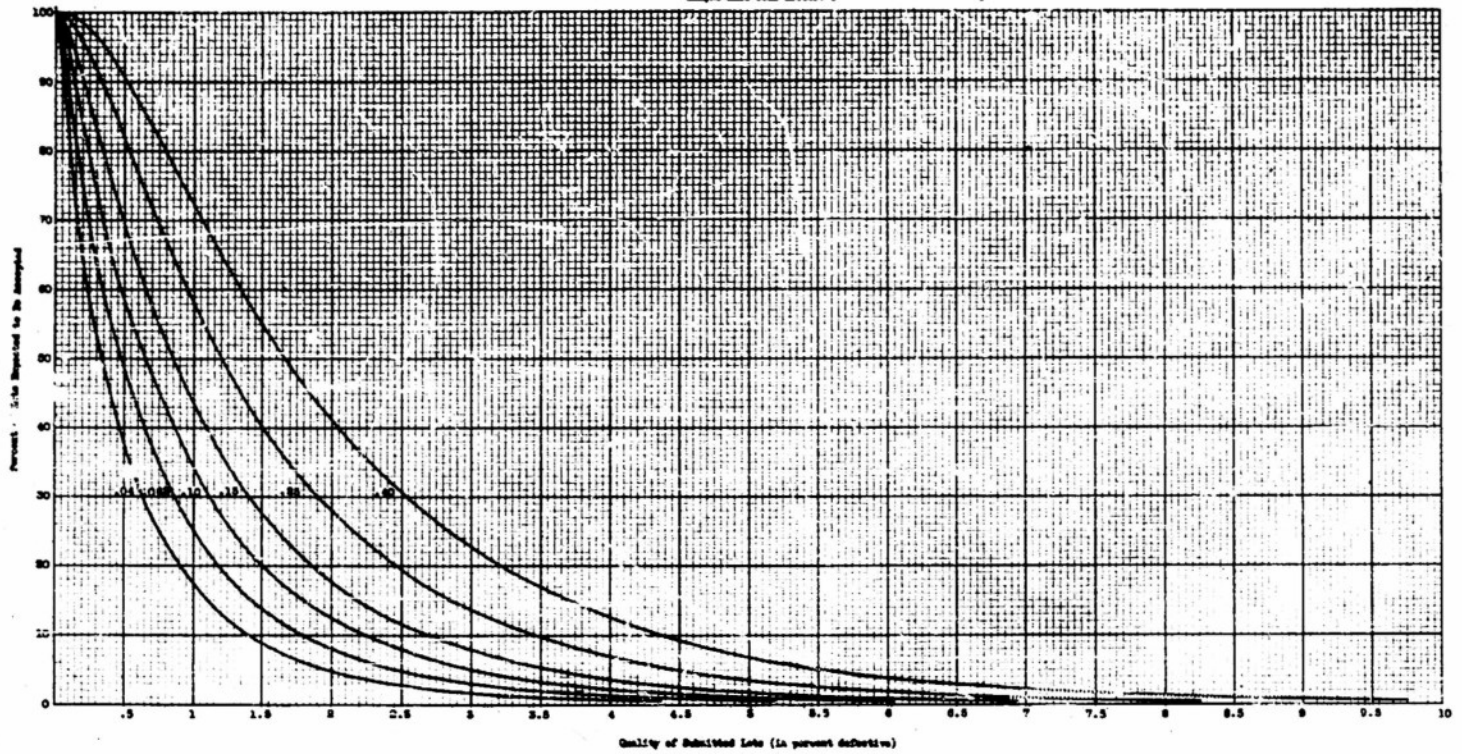


Fig. 9

Sampling Plans for Sample Size Code Letters: J—Continued
OPERATING CHARACTERISTIC CURVES FOR SAMPLING PLANS BASED ON UNKNOWN STANDARD DEVIATION
(Curves for sampling plans based on average range and known standard deviations are essentially equivalent)
Sample Size Code Letters: J



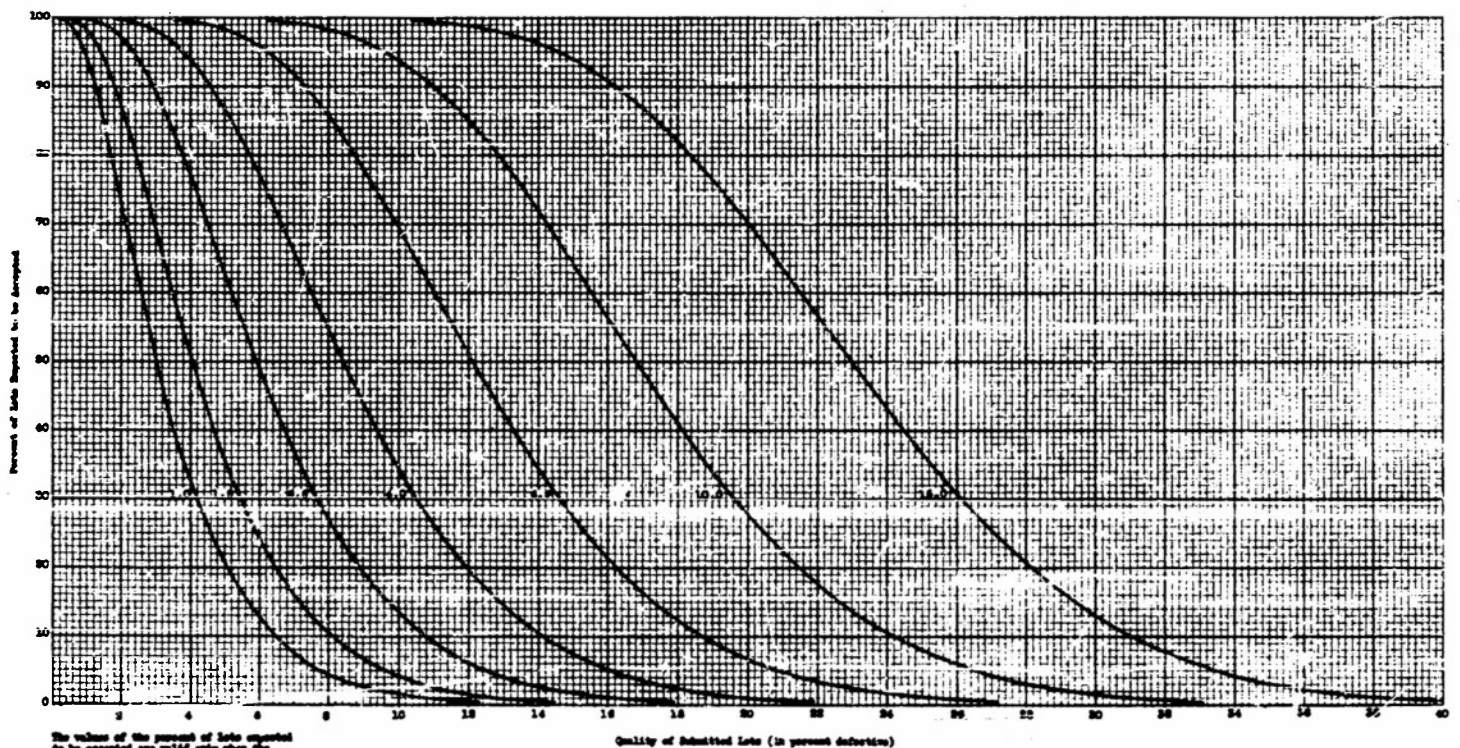
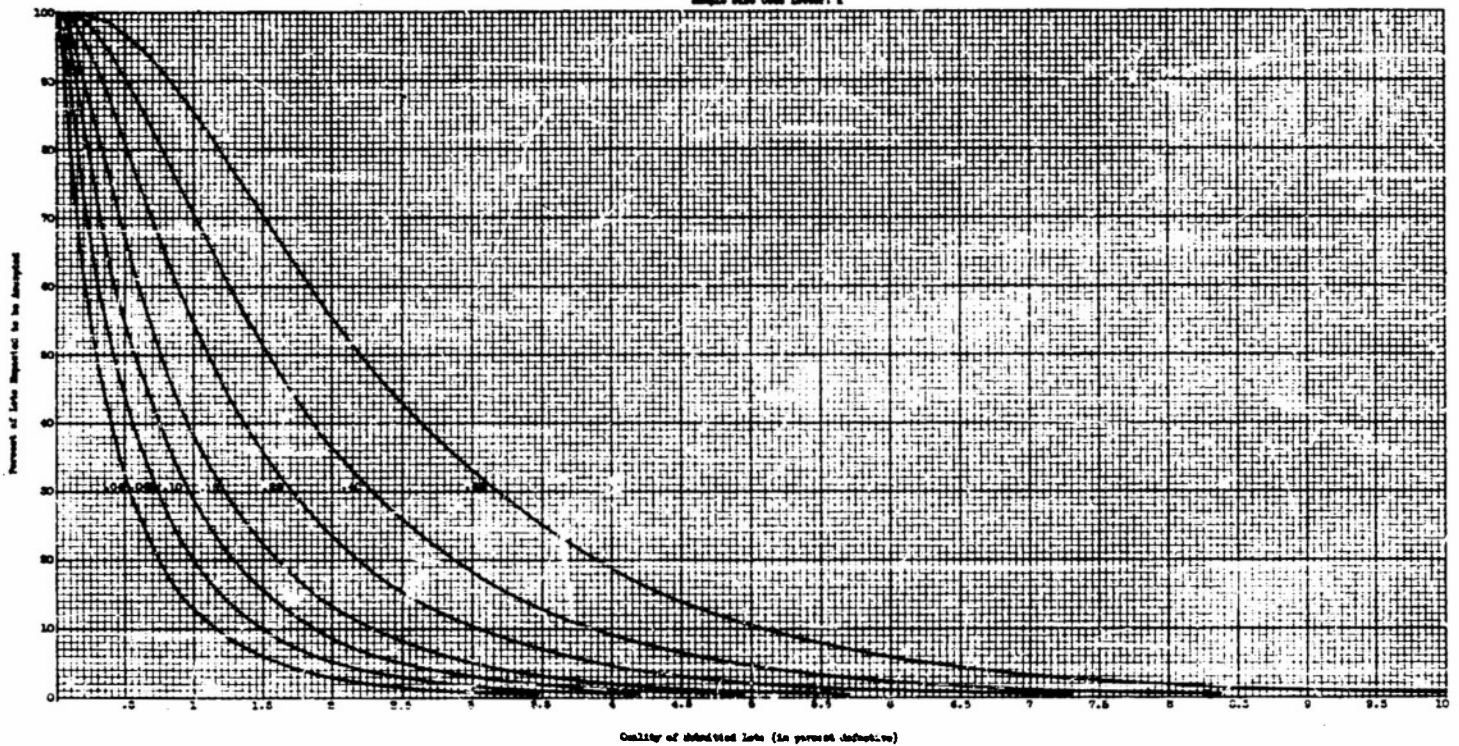
The values of the percent of lots accepted to be accepted are valid only when the submitted lots follow a normal distribution.

Note: Figures on curves are acceptance quality levels for normal inspection

Fig. 10

~~Sampling Plans for Single Size Code Letter: E—Continued~~
 CURVES FOR DETERMINING CURVES FOR SAMPLING PLANS BASED ON NORMAL DISTRIBUTION
 (Curves for sampling plans based on average range and known standard deviations are essentially equivalent)

Single Size Code Letter: E

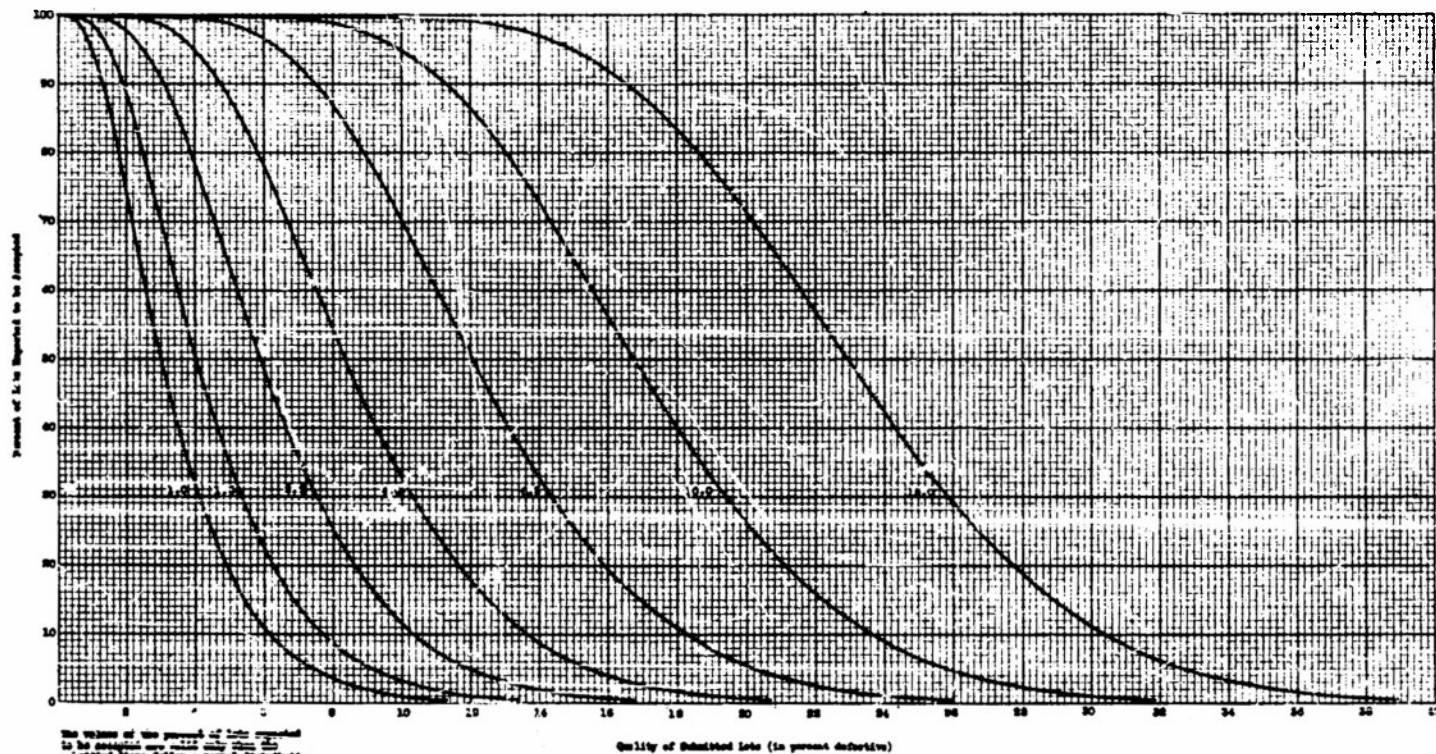
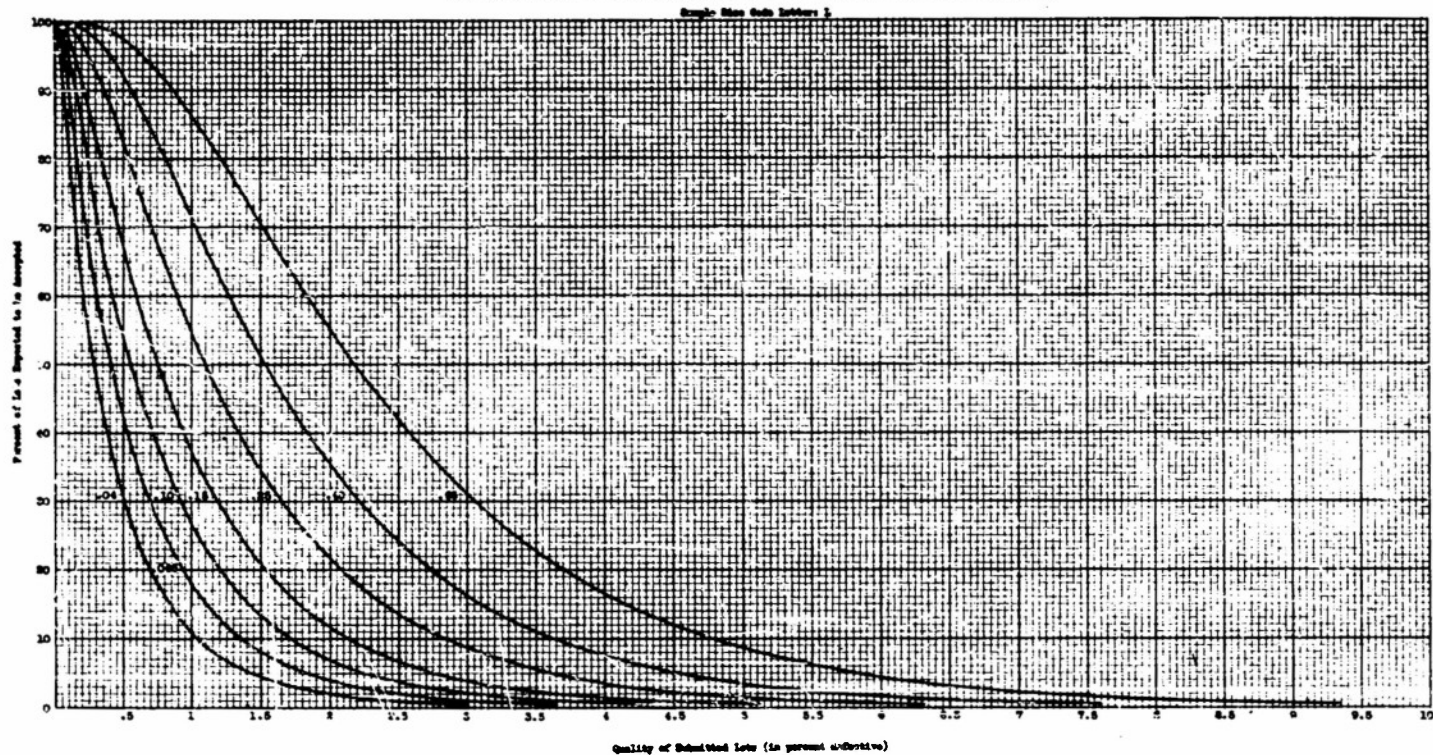


The values of the percent of lots accepted to be accepted are valid only when the submitted lots follow a normal distribution.

Note: Figures on curves are acceptance quality levels for usual inspection

Fig. 11

RECOMMENDED Sampling Plans for Single Size Lots: *Continued*
 OPERATING CHARACTERISTICS: THESE TWO SAMPLING PLANS BASED ON VARIOUS ASSUMED DEFECTIVE
 (Curves for sampling plans based on average range and known standard deviations are essentially equivalent)



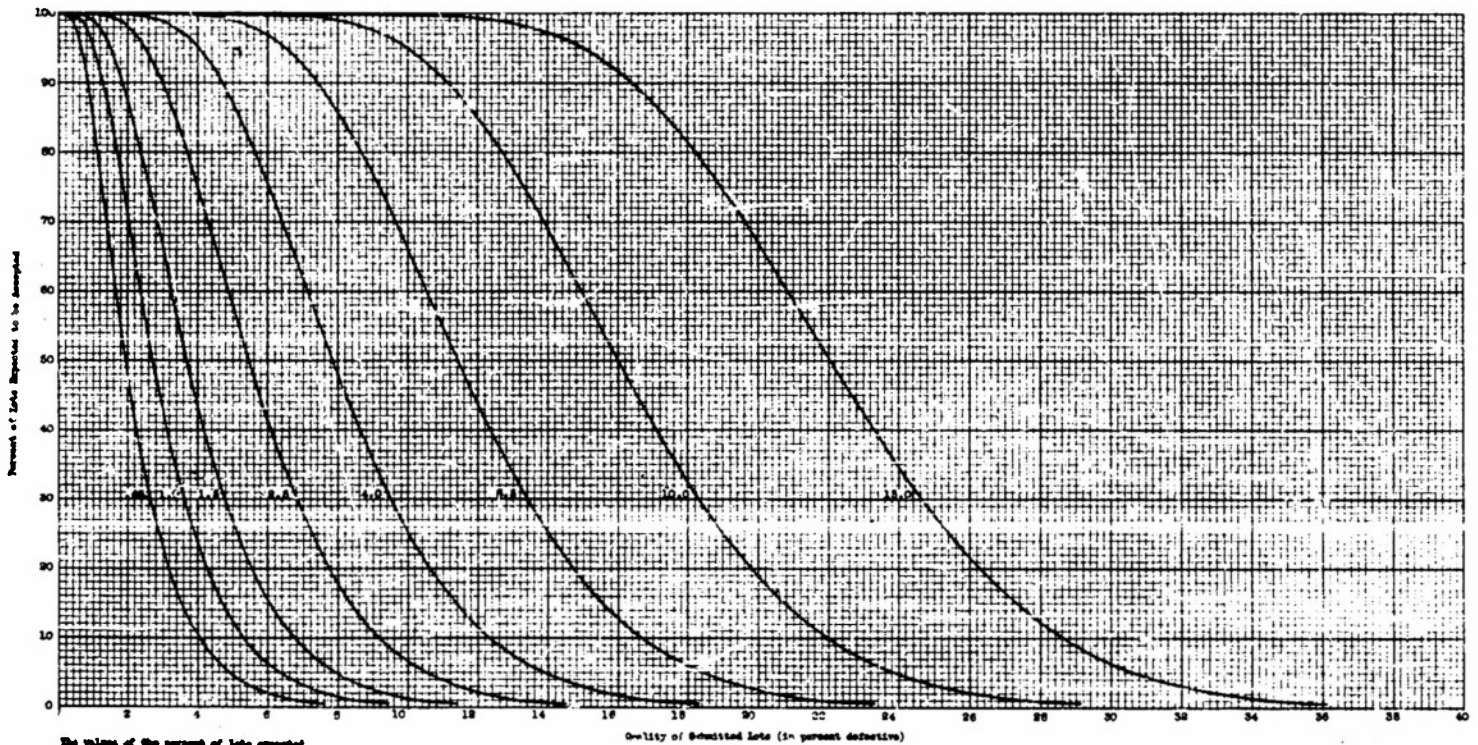
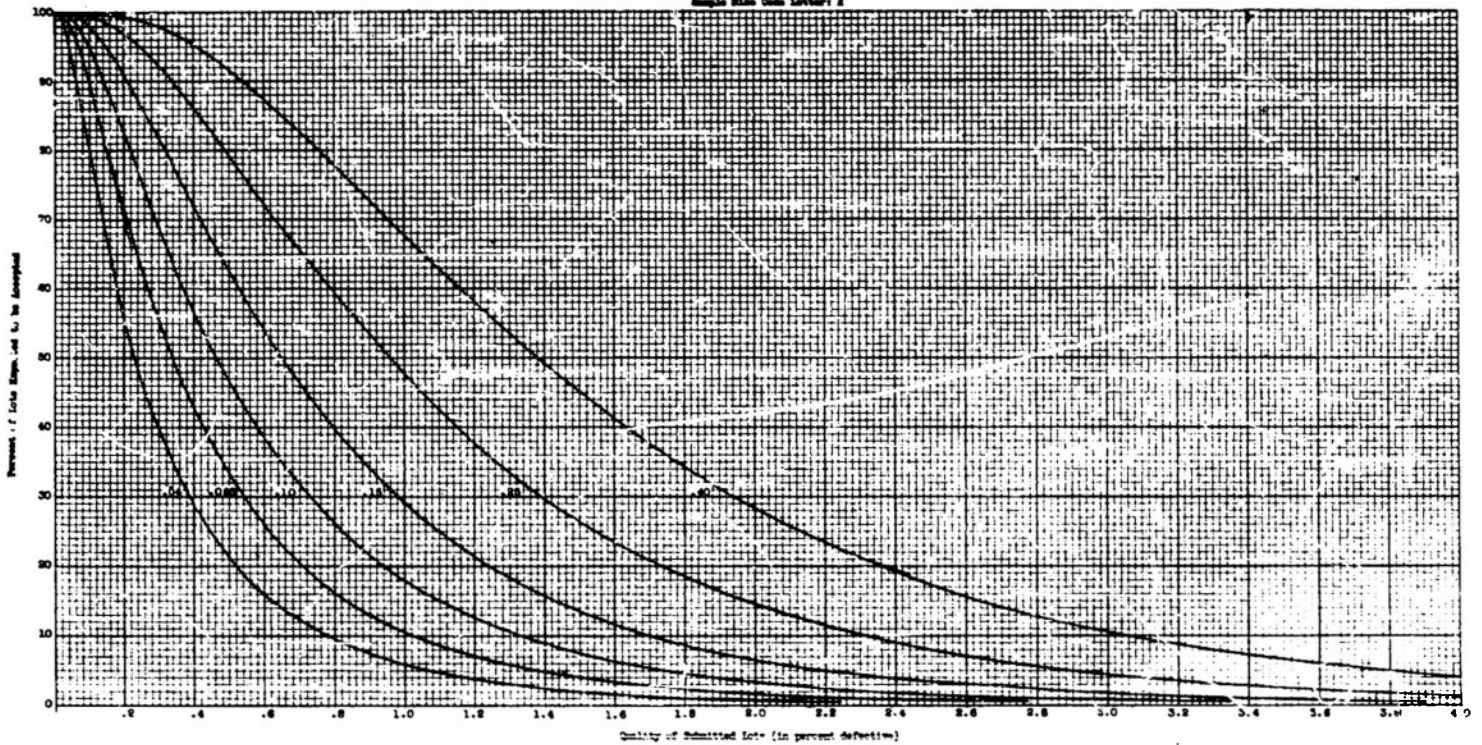
The values of the percent of lots accepted to be inspected are based on the assumption that the submitted lots follow a normal distribution.

Note: Figures on curves are Acceptance Quality Levels for normal inspection

Fig. 12

Table 12-10- Sampling Plans for Single Size Code Letters: H-Continued
OPERATING CHARACTERISTIC CURVES FOR SAMPLING PLANS BASED ON UNKNOWN STANDARD DEVIATIONS
(Curves for sampling plans based on average range and known standard deviations are essentially equivalent)

Single Size Code Letter: H



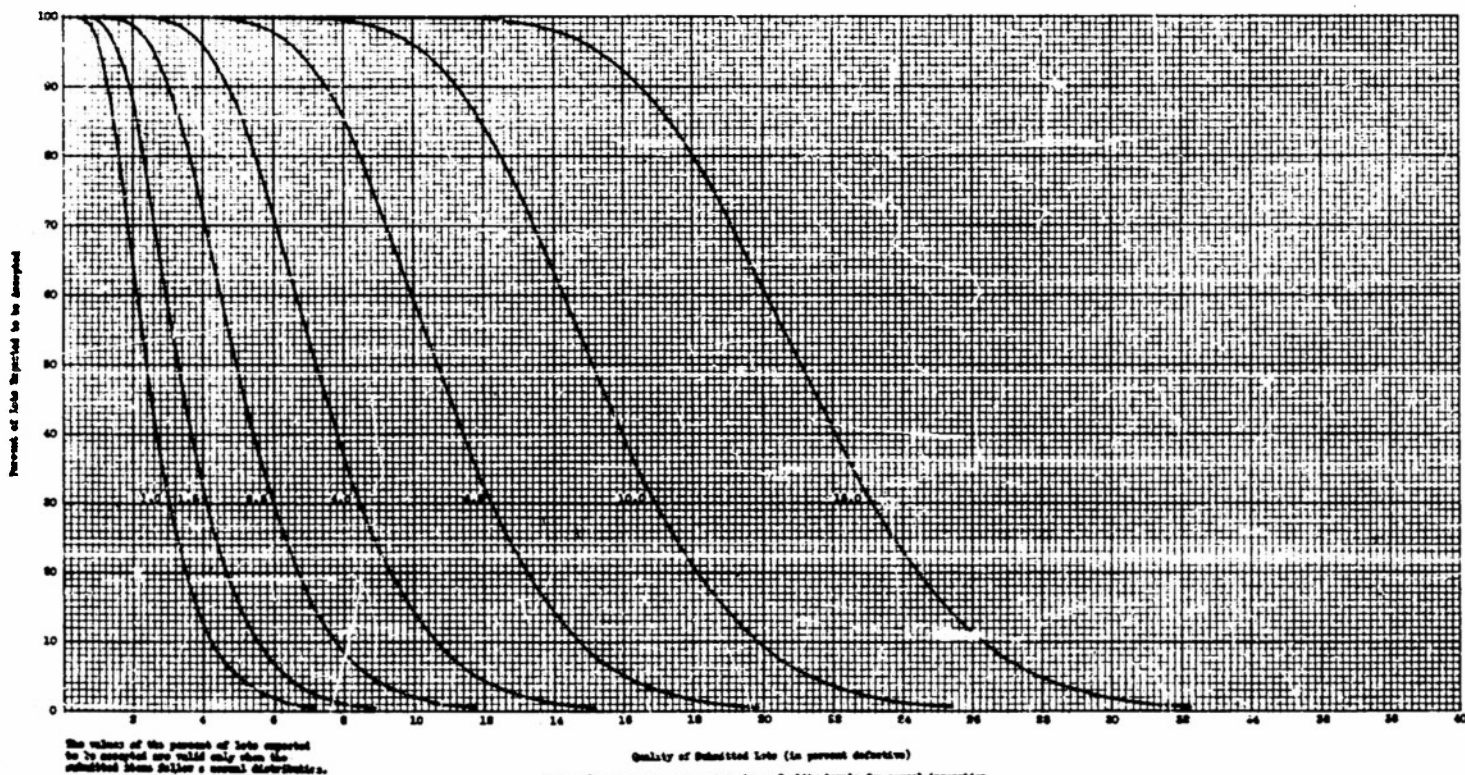
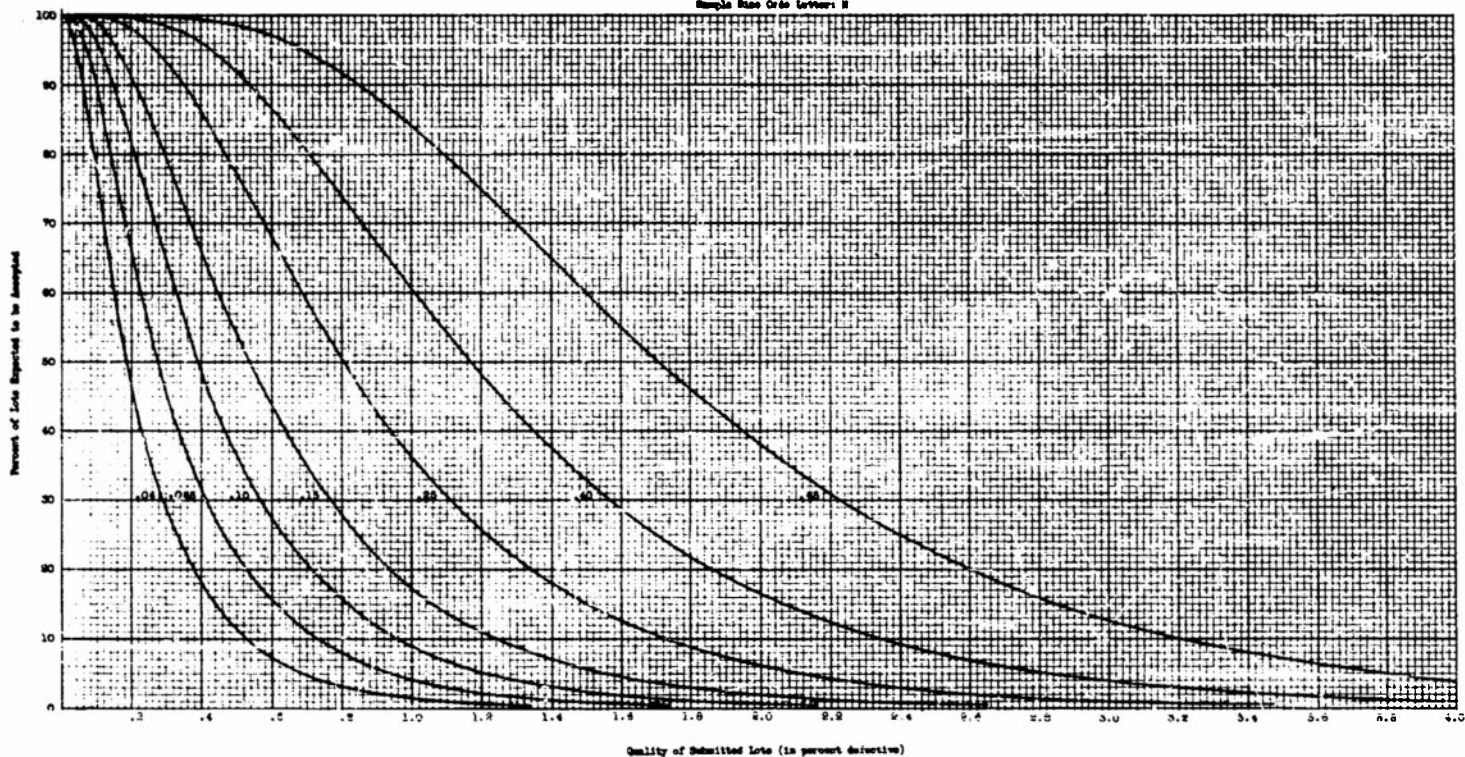
The values of the percent of lots expected to be accepted are valid only when the submitted lots follow a normal distribution.

Note: Figures on curves are Acceptance Quality Levels for normal inspection

Fig. 13

Sampling Plans for Simple Size Code Letters: B-Continued
OPERATING CHARACTERISTIC CURVES FOR SAMPLING PLANS BASED ON UNKNOWN STANDARD DEVIATION
(Curves for sampling plans based on average range and known standard deviations are essentially equivalent)

Simple Size Code Letter: B

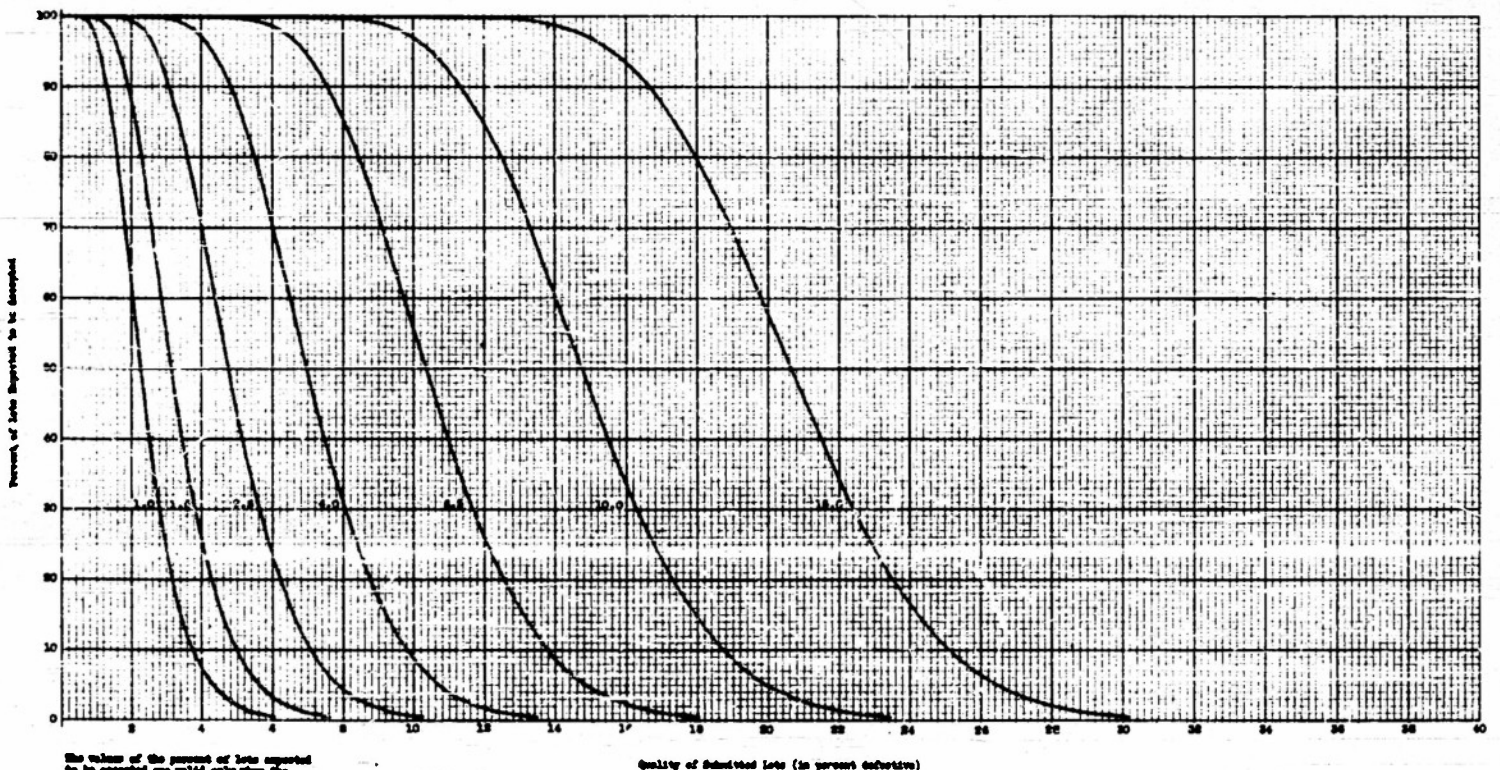
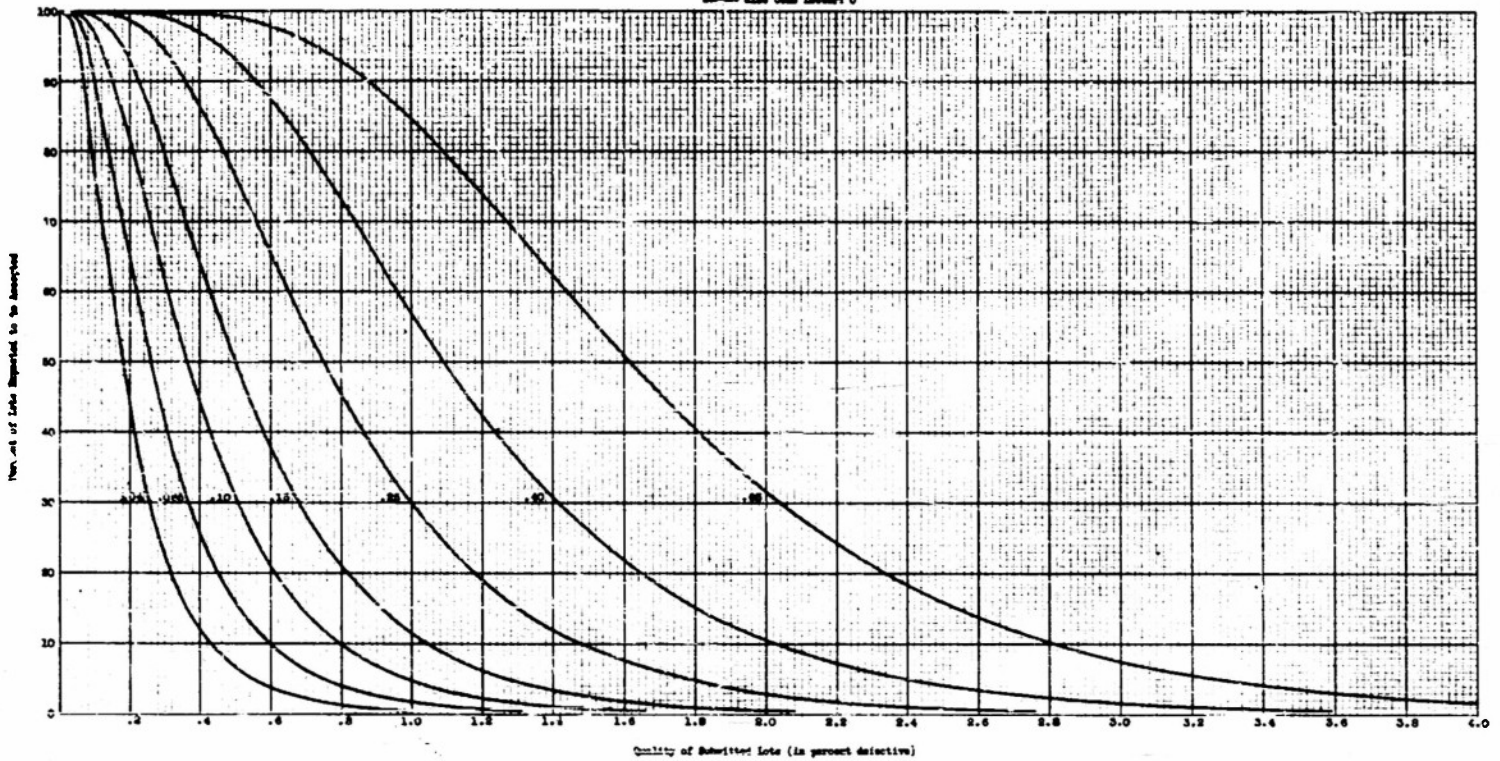


The values of the percent of lots expected to be accepted are valid only when the submitted lots follow a normal distribution.

Note: Figures on curves are acceptance quality levels for normal inspection

Fig. 14

Sampling Plan for Sample Size Code Letter: O—Continued
 OPERATING CHARACTERISTIC CURVES FOR SAMPLED PLANS BASED ON NORMAL STANDARD DEVIATION
 (Curves for sampling plans based on average range and known standard deviations are essentially equivalent)
 Sample Size Code Letter: O

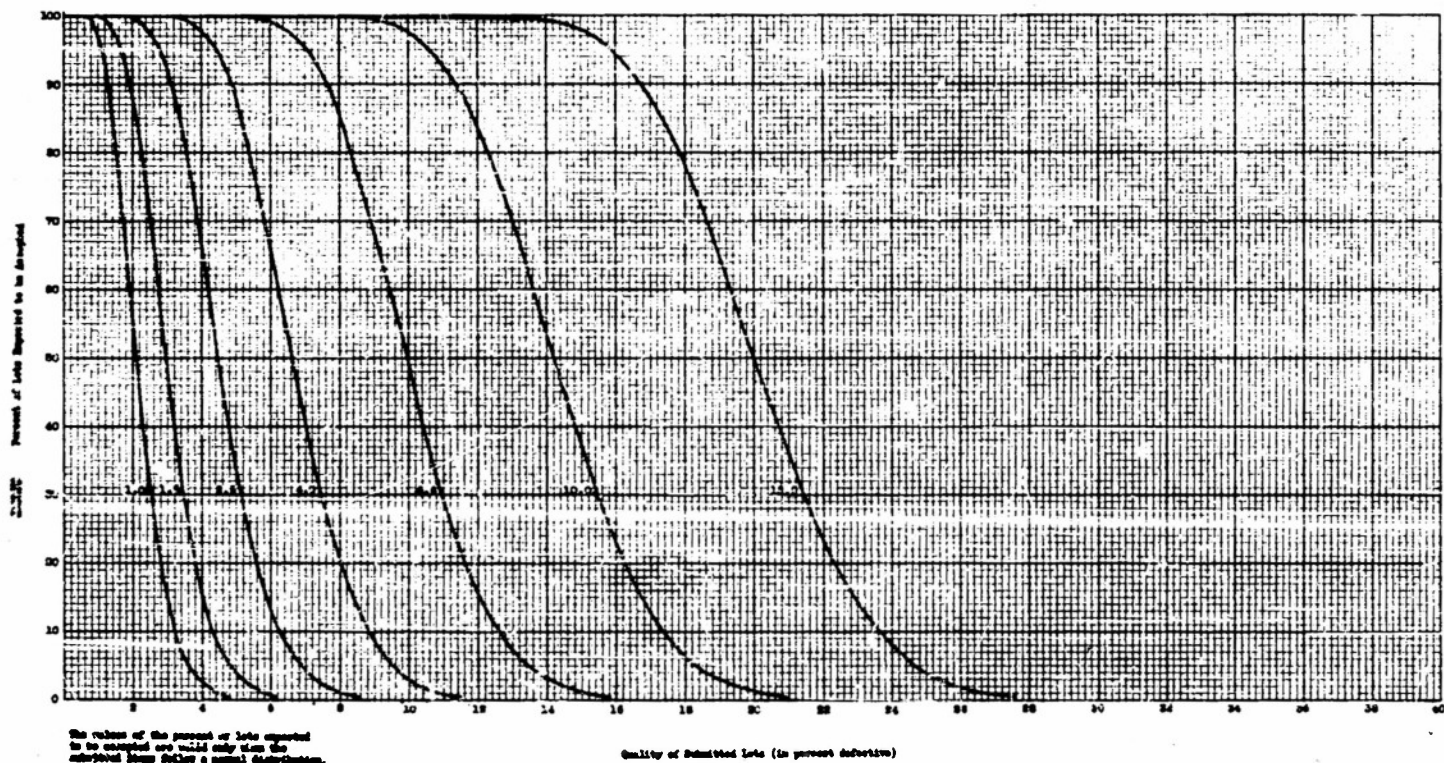
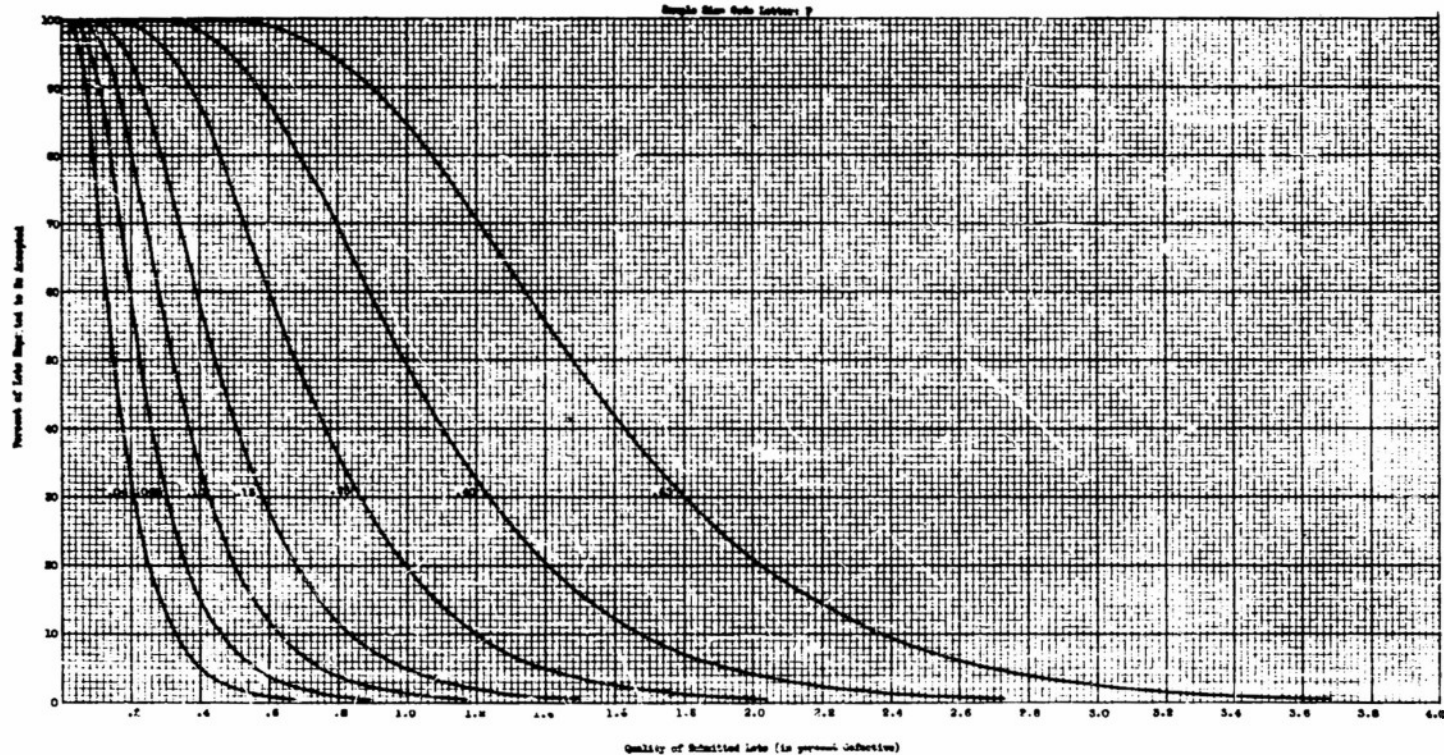


The values of the percent of lots accepted to be accepted are valid only when the submitted lots follow a normal distribution.

Note: Figures on curves are acceptance quality levels for normal inspection.

Fig 15

Sampling Plan P, Single Size Lots Letter P-Continued
 OPERATING CHARACTERISTICS CURVES FOR SAMPLING PLAN P, 1% AQL OF UNKNOWN STANDARD DEVIATION
 (Curves for sampling plans based on average range and known standard deviations are essentially equivalent)



The values of the percent of lots expected to be accepted are valid only when the submitted lots follow a normal distribution.

Note: Figures on curves are acceptance quality levels for normal inspection.

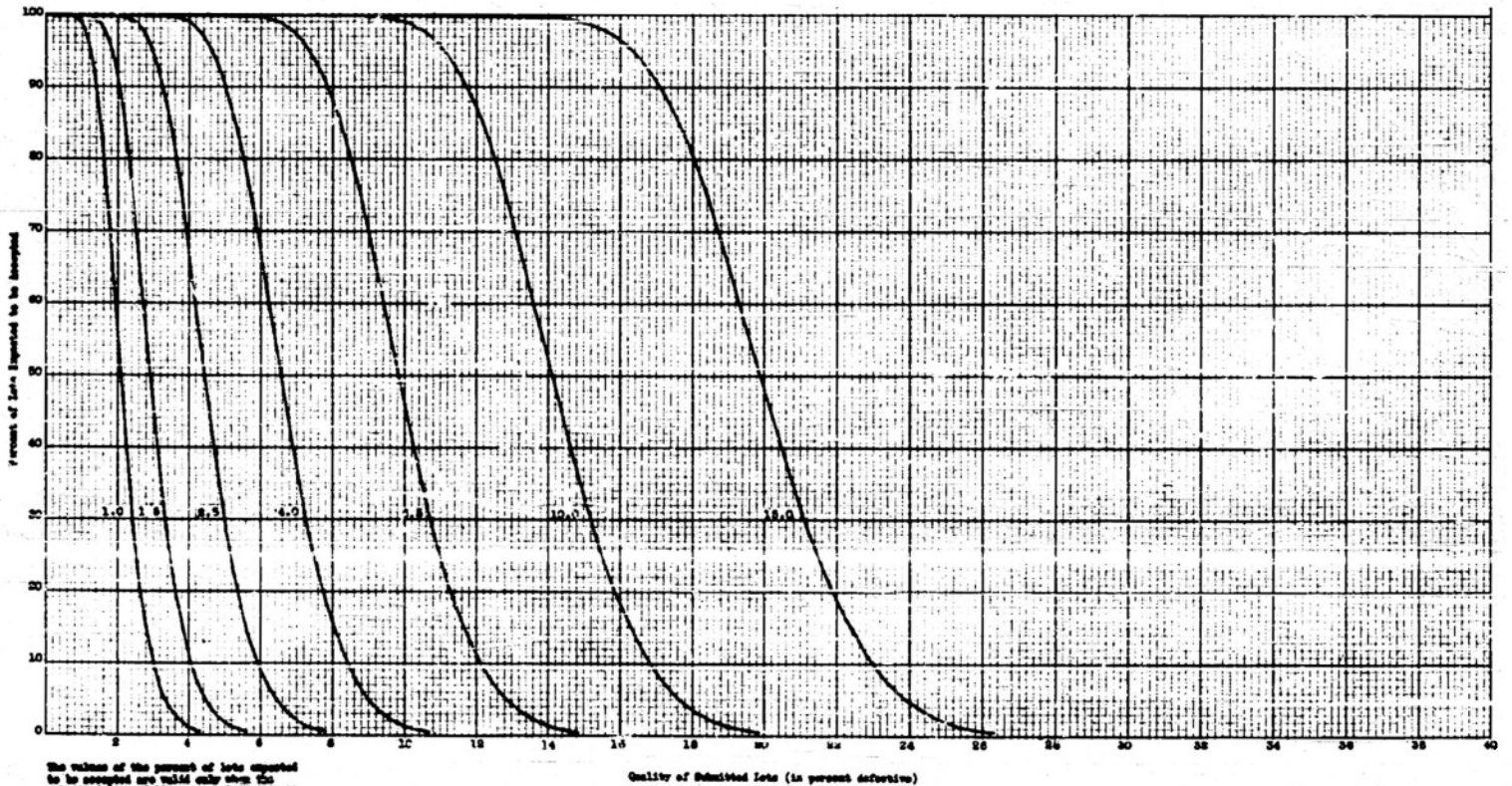
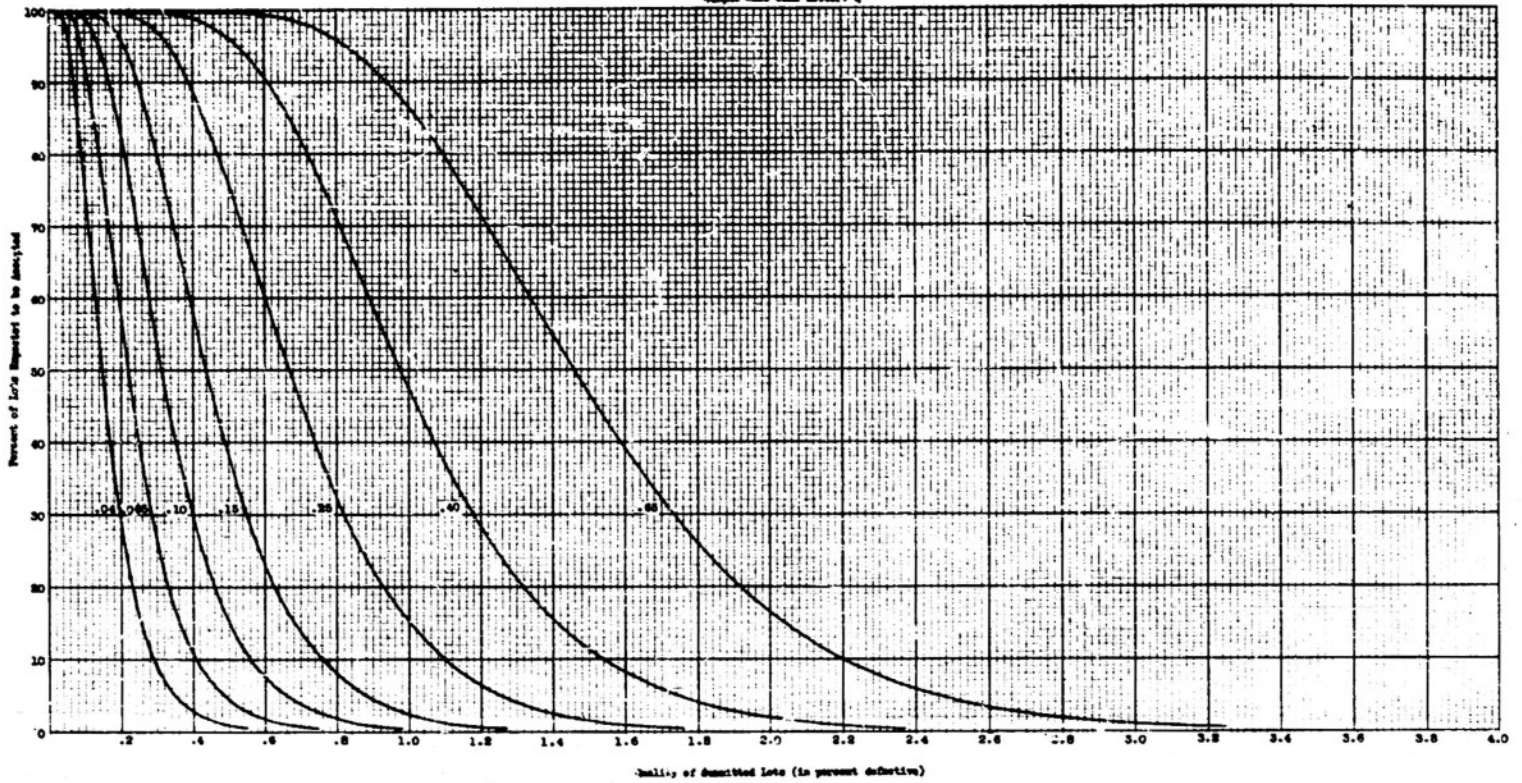
Fig. 16

Sampling Plans for Single Size Code Letters: G-Continued

OPERATION CHARACTERISTIC CURVES FOR SAMPLING PLANS BASED ON VARIATION FROM SPECIFICATION

(Curves for sampling plans based on average range and known standard deviations are essentially equivalent)

Single Size Code Letters: G



The values of the percent of lots expected to be accepted are valid only when the submitted lots follow a normal distribution.

Note: Figures 11-16 curves are Acceptance Quality Levels for normal inspection

VI-1. General Theory.

D. Blackwell [1] has shown that if x has density $p_{\theta}(x)$, g is any unbiased estimate of θ , and T is a sufficient statistic for θ , then $E(g|T)$ is also unbiased, and furthermore, has a variance no greater than that of g . Lehmann and Scheffé [8] have extended this result and have shown that if T is complete^{2/}, then every estimable function^{3/} $h(\theta)$ possesses an unbiased estimate with uniformly smallest variance and this estimate is the unique unbiased estimate of $g(\theta)$ which is a function of T .

It has already been shown [8] that the sufficient statistics for the normal distribution are complete. Furthermore, it is evident that there exists an unbiased estimate of the fraction of a normal population lying outside a fixed interval, e.g. the fraction of independent observations in a sample from this normal population which lie outside this interval. Consequently, the fraction of a normal population lying outside a fixed interval (p) is an estimable function and has an unbiased estimate with uniformly smallest variance, and this estimate is the unique unbiased estimate of p which is a function of the sufficient statistics.

^{1/} The idea of basing the two-sided acceptance procedure on the uniformly minimum variance unbiased estimate of the fraction defective stems from research carried out at the Applied Math. and Stat. Lab. by a group including Professors A. H. Bowker, H. Rubin, and L. E. Moses.

^{2/} A statistic T (more properly a family of distributions of T) is said to be complete if $E_{\theta}f(T) = 0$ on all $\theta \in r$ implies $f(T) = 0$ except possibly on a set N for which $p_{\theta}(N) = 0$ for all $\theta \in r$.

^{3/} Given a random variable x with density $p_{\theta}(x)$, a function $g(\theta)$ is said to be estimable if there exists a function $h(x)$ such that $E_{\theta}h(x) = g(\theta)$.

We are concerned with estimating the parameter

$$p = 1 - \int_L^U \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2} \frac{(z-\mu)^2}{\sigma^2}} dz .$$

Two cases will be considered; namely, (1) μ unknown, σ known (2) both μ and σ unknown.

Let $x_1, x_2, \dots, x_n$ be independent random variables from the above normal distribution. Define $\tilde{p}'$ as the usual attribute estimate of the fraction defective i.e., the ratio of the number of defective items to the sample size. It is evident that $\tilde{p}'$ is unbiased. Hence it follows from Blackwell [1] and Lehmann and Scheffé's [8] results that $\hat{p} = E(\tilde{p}'|T)$ is the unique uniformly minimum variance (UMV) unbiased estimate of p , where T are the sufficient statistics for the normal distribution. Since $\tilde{p}'$ is the sum of independent identically distributed random variables taking on the values 0 and 1, it is evident that $\hat{p} = E(\tilde{p}'|T)$ is equivalent to $E(\tilde{p}|T)$ where $\tilde{p}$ is defined as follows: Let y be any one of the observations ($x_1, x_2, \dots, x_n$) say x_1 .

$$(1) \quad \begin{aligned} \tilde{p}(y_1, x_2, x_3, \dots, x_n) &= 0 && \text{if } L \leq y \leq U \\ \tilde{p}(y_1, x_2, x_3, \dots, x_n) &= 1 && \text{otherwise.} \end{aligned}$$

VI-2. The Estimate When the Population Variance is Known.

If the observations are drawn from a normal population with unknown mean μ , and known variance σ^2 , then $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$ is a sufficient statistic. In this case $\hat{p} = E(\tilde{p}|\bar{x})$ is the UMV unbiased estimate of p .

If $\tilde{p}$ is the estimate (1) then

$$\hat{p}(\bar{x}) = E(\tilde{p}|\bar{x}) = \Pr\{\tilde{p} = 1|\bar{x}\} = 1 - \Pr\{L \leq y \leq U|\bar{x}\} = 1 - \int_L^U \frac{g(y|\bar{x})}{h(\bar{x})} dy$$

where $g(y, \bar{x})$ is the joint probability density of y and $\bar{x}$, and $h(\bar{x})$ is the probability density of $\bar{x}$.

Consider the joint probability density of y and $\bar{x}' = \sum_{i=2}^n \frac{x_i}{n-1}$

$$\frac{\sqrt{n-1}}{2\pi\sigma^2} e^{-\frac{1}{2\sigma^2} \{(y-\mu)^2 + (n-1)(\bar{x}'-\mu)^2\}}$$

The transformation

$$\bar{x} = \frac{(n-1)\bar{x}'}{n} + \frac{y}{n}$$

leads to

$$g(y, \bar{x}) = \frac{n}{\sqrt{n-1} 2\pi\sigma^2} e^{-\frac{n}{2\sigma^2} \left\{ (y-\mu)^2 + \frac{1}{n-1} (n\bar{x}-y-\mu)^2 \right\}}$$

and division by

$$h(\bar{x}) = \frac{\sqrt{n}}{\sqrt{2\pi}\sigma} e^{-\frac{n}{2\sigma^2} (\bar{x}-\mu)^2}$$

results in

$$\frac{g(y, \bar{x})}{h(\bar{x})} = \sqrt{\frac{n}{n-1}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{n}{2(n-1)\sigma^2} (y-\bar{x})^2}.$$

Therefore

$$\begin{aligned} \hat{p}(\bar{x}) &= 1 - \int_L^U \sqrt{\frac{n}{n-1}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{n}{2(n-1)\sigma^2} (y-\bar{x})^2} dy \\ &= \int_{-\infty}^{\sqrt{\frac{n}{n-1}} \frac{L-\bar{x}}{\sigma}} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt + \int_{\sqrt{\frac{n}{n-1}} \frac{U-\bar{x}}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt. \end{aligned}$$

VI-3. The Known- σ Acceptance Criterion.

The acceptance procedure is formulated as follows: Accept a lot of items if in the sample $\hat{p} \leq p^*$ where p^* is so chosen that if the population percentage defective is p , i.e., if the portion of the population lying outside (U, L) is p , then the probability of acceptance will be L_p . It is shown that in the one-sided case this is equivalent to the well-known and widely used procedure: Accept if $\bar{x} \leq U - k\sigma$.

Let K_ϵ be defined by

$$\int_{K_\epsilon}^{\infty} \frac{e^{-\frac{t^2}{2}}}{\sqrt{2\pi}} dt = \epsilon.$$

In the one-sided case, when $L = -\infty$, then

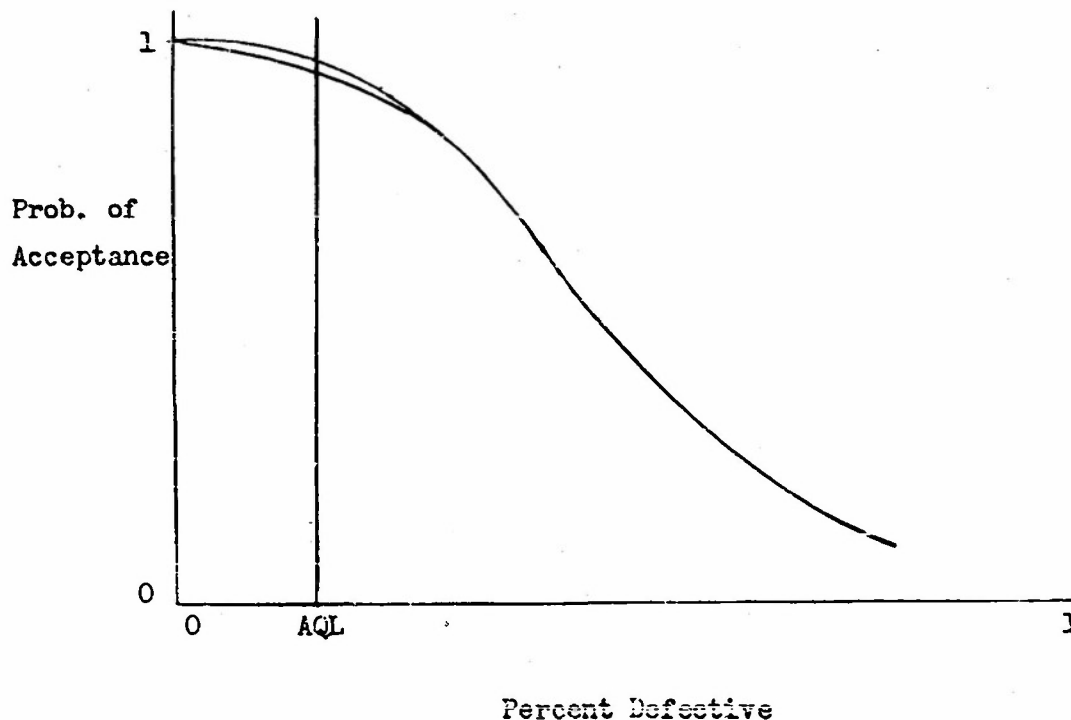
$$\hat{p} = \int_{\frac{\sqrt{\frac{n}{n-1}} \frac{U-\bar{x}}{\sigma}}}{\infty} \leq p^*$$

we must have

$$\sqrt{\frac{n}{n-1}} \frac{U-\bar{x}}{\sigma} \geq K_{p^*} \quad \text{or} \quad \bar{x} \leq U - \sqrt{\frac{n-1}{n}} K_{p^*} \sigma .$$

If we take $k = \sqrt{\frac{n-1}{n}} K_{p^*}$ the OC curves of the two acceptance procedures will be identical.

For the case where there is a double specification limit and the standard deviation is known, it is evident that if $\frac{U-L}{\sigma} < 2 K_{\frac{AQL}{2}}$, the incoming value of p , the percentage defective, is greater than the AQL. Consequently, the lot should be rejected without a sample being drawn. The OC curve of this two sided test takes the form of a band, the lower bound of which is the one sided OC curve. For incoming quality which is no worse than the AQL the upper bound corresponds to equal division of the defective below the lower specification limit and above the upper specification limit. The band here is relatively wide, but this is desirable since this gives "good" quality a better chance of acceptance. For quality worse than the AQL, the above restriction on $\frac{U-L}{\sigma}$ eliminates certain divisions, including equal division, so that the band quickly tends to the one sided curve. A diagram of the OC band is as follows:



VI-4. The Estimate of p When the Population Variance is Unknown.

If the observations are drawn from a normal population with unknown mean μ and unknown variance σ^2 , then the pair of sample values $\bar{x}$, and $\sum (x_i - \bar{x})^2$ are sufficient statistics. In this case $\hat{p}(\bar{x}, S^2) = E(\tilde{p} | \bar{x}, S^2)$ where

$$\bar{x} = \sum_{i=1}^n \frac{x_i}{n} \quad S^2 = \sum_{i=1}^n (x_i - \bar{x})^2 .$$

If $\tilde{p}$ is the estimate (1) then

$$\hat{p}(\bar{x}, S^2) = \Pr \{ \tilde{p} = 1 | \bar{x}, S^2 \} = 1 - \int_L^U \frac{f(y, \bar{x}, S^2)}{h(\bar{x}, S^2)} dy$$

where $f(y, \bar{x}, S^2)$ is the joint probability density of y , one of the observations in the sample, the sample mean $\bar{x}$, and the sample sum of squares S^2 , and $h(\bar{x}, S^2)$ is the joint probability density of $\bar{x}$ and S^2 .

It is well known that $h(\bar{x}, S^2)$ is given by

$$h(\bar{x}, S^2) = \frac{\sqrt{n} (S^2)^{\frac{n-3}{2}} e^{-\frac{n}{2\sigma^2}(\bar{x}-\mu)^2 - \frac{S^2}{2\sigma^2}}}{\sqrt{2\pi} \sigma \Gamma(\frac{n-1}{2}) (2\sigma^2)^{\frac{n-1}{2}}}.$$

To find $f(y, \bar{x}, S^2)$ consider the joint density of the sample. This may be expressed as the joint density of the mutually independent sample statistics y , $\bar{x}'$, and S'^2 where

$$\bar{x}' = \sum_{i=2}^n \frac{x_i}{n-1} \quad S'^2 = \sum_{i=2}^n (x_i - \bar{x}')^2$$

$$(2) \quad \left(\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(y-\mu)^2} \right) \cdot \left(\frac{n-1}{\sqrt{2\pi}\sigma} e^{-\frac{n-1}{2\sigma^2}(\bar{x}'-\mu)^2} \right) \cdot \left(\frac{(S'^2)^{\frac{n-4}{2}}}{\Gamma(\frac{n-2}{2})(2\sigma^2)^{\frac{n-2}{2}}} e^{-\frac{1}{2\sigma^2}S'^2} \right).$$

Now let

$$\bar{x}' = \frac{n\bar{x}}{n-1} - \frac{y}{n-1}$$

$$S'^2 = S^2 - \frac{n}{n-1} (\bar{x}-y)^2$$

$$y = y \quad .$$

Under this transformation the expression (2) becomes

$$(3) \quad K \left\{ S^2 - \frac{n}{n-1} (\bar{x}-y)^2 \right\}^{\frac{n-4}{2}} e^{-\frac{n-1}{2\sigma^2} \left\{ \frac{(y-\mu)^2}{n-1} + \left(\frac{n\bar{x}-y}{n-1} - \mu \right)^2 + S^2 - \frac{n}{n-1} (\bar{x}-y)^2 \right\}}$$

where

$$K = \frac{n}{\sqrt{n-1} 2\pi\sigma (2\sigma^2)^{\frac{n-2}{2}} \Gamma(\frac{n-2}{2})}$$

and the variables are subject to the restrictions

$$-\infty < y < \infty$$

$$0 \leq S^2 \leq \infty$$

$$\left| \frac{\bar{x}-y}{S} \right| \leq \sqrt{\frac{n-1}{n}} \quad .$$

Dividing the expression (3) by $h(\bar{x}, S^2)$ results in the conditional density of y , given $\bar{x}$ and S^2 .

$$(4) \quad \sqrt{\frac{n}{n-1}} \frac{\Gamma(\frac{n-1}{2})}{\sqrt{\pi} \Gamma(\frac{n-2}{2})} \frac{1}{S} \left\{ 1 - (\frac{\bar{x}-y}{S})^2 \frac{n}{n-1} \right\}^{\frac{n-4}{2}}.$$

If now the transformation $z = \frac{1}{2} + \frac{1}{2} \frac{\bar{x}-y}{S} \sqrt{\frac{n}{n-1}}$ is made the density of the random variable z is obtained:

$$(5) \quad g(z) = \frac{\Gamma(\frac{n-2}{2})}{\Gamma(\frac{n-2}{2}) \Gamma(\frac{n-2}{2})} z^{\frac{(n-2)}{2}-1} (1-z)^{\frac{(n-2)}{2}-1} \quad 0 \leq z \leq 1.$$

This will be recognized as a symmetrical Beta distribution with parameters $\frac{n}{2} - 1$. Hereafter, $g(z)dz$ will be denoted by $d\beta(\frac{n}{2} - 1)$.

Since $\hat{p}(\bar{x}, S^2) = \Pr \{y \geq U | \bar{x}, S^2\} + \Pr \{y < L | \bar{x}, S^2\}$ then in terms of z

$$\begin{aligned} \hat{p}(\bar{x}, S^2) &= \Pr \left\{ z \leq \frac{1}{2} - \frac{1}{2} \frac{U-\bar{x}}{S} \sqrt{\frac{n}{n-1}} \right\} + \Pr \left\{ z \geq \frac{1}{2} - \frac{1}{2} \frac{\bar{x}-L}{S} \sqrt{\frac{n}{n-1}} \right\} \\ &= \int_0^{\max[0, \frac{1}{2} - \frac{1}{2} \frac{U-\bar{x}}{S} \sqrt{\frac{n}{n-1}}]} d\beta(\frac{n}{2}-1) + \int_{\max[0, \frac{1}{2} - \frac{1}{2} \frac{\bar{x}-L}{S} \sqrt{\frac{n}{n-1}}]}^1 d\beta(\frac{n}{2}-1). \end{aligned}$$

VI-5. The Unknown- Acceptance Criterion.

Just as in the case where the population variance is known the acceptance procedure is: Accept if $\hat{p}(\bar{x}, S^2) \leq p^*$ where p^* is so chosen that if the population fraction defective is p , the probability of acceptance will be L_p . It is shown that in the one-sided case, i.e., where there is only one specification limit, say U , this procedure is equivalent to the well-known and widely used test $\bar{x} + ks \leq U$, where

$$s = S/\sqrt{n-1}.$$

Let β_{p^*} be defined by $\int_0^{\beta_{p^*}} d\beta \left(\frac{n}{2} - 1\right)$, then $\hat{p} \leq p^*$ if and only if $\frac{1}{2} - \frac{1}{2} \sqrt{\frac{n}{n-1}} \frac{U-\bar{x}}{S} \leq \beta_{p^*}$ and since $s = S/\sqrt{n-1}$ this is equivalent to

$$\bar{x} + \frac{n-1}{\sqrt{n}} (1-2\beta_{p^*})s \leq U.$$

If k is taken equal to $\frac{n-1}{\sqrt{n}} (1-2\beta_{p^*})$ the procedures $\hat{p} \leq p^*$ and $\bar{x} + ks \leq U$ will have identical OC curves. In this paper the acceptance criteria $\hat{p}$ based on the statistic $\frac{U-\bar{x}}{s}$ are tabled rather than $\frac{U-\bar{x}}{S}$, to conform with the common use of the square root of the unbiased estimator of σ^2 .

For the case where there is a double specification limit one of the authors made numerical investigations [11] of the OC curves for various divisions of the percentage defective. It was evident that the band was so narrow that for all practical purposes it can be assumed to be a single OC curve, i.e., the OC curve of the one sided plan.

VI-6. The Acceptance Procedure and Estimate Based on the Range.

Since the pair of sample statistics $(\bar{x}, \bar{R})$, where $\bar{R}$ is the sample range or average range^{1/} is not sufficient for the normal distribution when the mean and variance are both unknown, no uniformly minimum variance estimate of p which is a function $\bar{x}$ and $\bar{R}$ can be derived. In this paper the following estimate of $p(\bar{x}, \bar{R})$ is used:

^{1/} In this paper, whenever possible the subgroup size is taken as 5, so that $\bar{R}$ is the average range of m subgroups of S (the sample consisting of Sm observations). For sample sizes of 3, 4, and 7, $\bar{R}$ is taken as the range of the sample.

$$p(\bar{x}, \bar{R}) = \int_0^{\max[0, \frac{1}{2} - \frac{1}{2} \frac{\alpha}{\gamma} (\frac{U-\bar{x}}{\bar{R}})]} d\beta [\frac{\gamma+1}{2} - 1] + \int_0^{\max[0, \frac{1}{2} - \frac{1}{2} \frac{\alpha}{\gamma} (\frac{\bar{x}-L}{\bar{R}})]} d\beta [\frac{\gamma+1}{2} - 1]$$

where α and γ are constants for fixed n , which will be explained in a subsequent paragraph of this section.

No difficulty is encountered in connection with OC curves (which were obtained by numerical integration) for the acceptance procedure $\hat{p}(\bar{x}, \bar{R}) \leq p^*$ since $\Pr\{\hat{p} \leq p^*\} = \Pr\{\bar{x} + k\bar{R} \leq U\}$ whenever $k = \frac{\gamma}{\alpha} (1 - 2\beta_{p^*})$. The procedure to accept if $\bar{x} + k\bar{R} \leq U$ is of course the one commonly used in place of the sample standard deviation in variables sampling.

A heuristic justification for the statistic $\hat{p}(\bar{x}, \bar{R})$ as an estimate of p is the following. It has been verified by numerical investigation that the statistic $\alpha \frac{U-\bar{x}}{\bar{R}}$ is approximately distributed as non-central t with degrees of freedom γ and eccentricity $\sqrt{\gamma+1} \frac{U-\mu}{\sigma}$, whenever μ and σ are such that $\frac{U-\mu}{\sigma}$ is sufficiently large, i.e., when the population fraction defective is small. The numbers α and γ are constants for each fixed sample size n and are determined as follows. Under the assumption that $\alpha \frac{U-\bar{x}}{\bar{R}}$ is distributed as non-central $t_{\gamma, \sqrt{\gamma+1} K_p}$ for small p then

$$\Pr\left\{\bar{x} + \frac{k}{\alpha} \bar{R} \leq U\right\} = \Pr\left\{\bar{x}_{\gamma+1} + \frac{k}{\sqrt{\gamma+1}} s_{\gamma} \leq U\right\}$$

where $\bar{x}_{\gamma+1}$ and s_{γ} are the sample mean and sample standard deviation based on γ observations. Equating the first two moments of the statistics $\bar{x} + \frac{k}{\alpha} \bar{R}$ and $\bar{x}_{\gamma+1} + \frac{k}{\sqrt{\gamma+1}} s_{\gamma}$ and solving for α and γ gives α as a function of γ and γ as a function of n and k . In

fact $\alpha = \frac{d_2 \sqrt{\nu}}{c_2}$ where d_2 is defined by $EH = d_2 \sigma$ and c_2 is defined by $E\sqrt{\frac{S}{n}} = c_2 \sigma$. In order to make the constants independent of k the values for the limiting case, i.e., when p goes to zero, were chosen^{1/}

This approximation was verified by the authors by numerical integration for the statistic $\frac{U-\bar{x}}{\bar{R}}$ and reference to tables of the non-central t . It was found to be very good for both large and small n for most of the AQL values used in this report. It is especially good for small n regardless of p . For the larger values of p where n is large the approximation did not hold with great accuracy. However, it could have been improved by making ν , and hence α , functions of p as well as of n . This was not done because this would have entailed increasing the number of tables of estimates, thereby complicating use of the tables by sheer bulk. Furthermore, the effect of this on the estimate $\hat{p}(\bar{x}, \bar{R})$ would have been slight since on the average the sample values of $\frac{U-\bar{x}}{\bar{R}}$ fall in the region of the tables where the estimate $\hat{p}$ changes very little as n changes.

Essentially the above argument indicates that $\hat{p}(\bar{x}, \bar{R})$ is approximately distributed as $\hat{p}(x_{\nu+1}, s_{\nu})$. Hence $\hat{p}(\bar{x}, \bar{R})$ has sampling properties similar to that of $\hat{p}(\bar{x}_{\nu}, s_{\nu})$ where the effective number of observations is $\nu+1$.

^{1/} This value of ν obtained in this way is exactly that obtained by Patnaik who suggested that $\sqrt{n} \frac{U-\bar{x}}{\bar{R}}$ may be approximated by non-central t with degrees of freedom ν but with eccentricity $\sqrt{n} \frac{U-\bar{x}}{\bar{R}}$. This approximation is excellent and will hold for all p but would not fit into the framework of the proposed estimate $\hat{p}(\bar{x}, \bar{R})$. See P. B. Patnaik, The use of mean range as an estimator of variance in statistical test, Biometrika, vol. 37, June 1950.

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