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THE MUNICIPAL UNIVERSITY OF WICHITA

REVIEW OF A SYSTEMATIC, THEORETICAL
INVESTIGATION OF JET PUMPS

by H.B. Helmbold

Engineering Study No. 122

for the Office of Naval Research
Contract N-onr 201(01)

November 1953
University of Wichita
School of Engineering
Wichita, Kansas



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SUMMARY

A survey of four studies on basic jet-pump theory is given in order to clarify the admissibility and the indispensability of the underlying simplificatory assumptions.

The first study showed that the efficiency of a mixing process can be defined only with reference to an ideal final state of perfect mixture. This ideal efficiency can be theoretically predicted for two special types of mixing tubes, viz. constant-diameter and constant-pressure. The latter type was found superior, especially at low induced-flow velocities.

Instead of solving the differential equations of the mixing problem, a set of integral relations can be solved if a sharp delimitation between mixing zone and induced flow can be maintained. The criterion for the occurrence of reverse flow in a constant-pressure mixing tube is predicted from the second study by use of integral expressions.

The third study involves a major simplification by assuming universal excess velocity profiles in the mixing zone. Though lacking in accuracy the results permit determination of the characteristic differences between the behavior of the constant-diameter and constant-pressure mixing process.

Two exact solutions are known for constant-pressure mixing in the limiting cases of a jet issuing into still air and of a vanishing jet excess velocity in moving air. These solutions are used in the fourth study as a basis for an approximate solution in the more general case of a finite jet to secondary air velocity ratio utilizing the energy equation and the concept of a mixing length. This investigation was restricted to the section where the mixing zone is still surrounded by a uniform potential flow and it is shown that the major portion of energy transformation is performed in this section of the mixing tube.

I. Comparison of Perfect Mixing Processes.

An ideal efficiency can be defined on the assumption that the final state of perfect mixing (a uniform velocity distribution) can be attained. Theoretically this involves an infinite length of the mixing tube, but fortunately the equalizing effect of the turbulent shearing stresses on the velocity distribution is usually strong enough to produce an almost uniform velocity distribution within a distance of, say, 10 mixing-tube diameters from the jet orifice. When this cannot be performed with a reasonable length of the mixing tube, a certain degree of non-uniformity will remain and it is then not permissible to calculate the efficiency by using the total energy at the end of the tube. In this case a correct evaluation of the efficiency can be made only by a theoretical continuation of the mixing process downstream to infinity on the assumption that no shearing stresses act on the wall of the mixing-tube continuation. This remark is especially important when the optimum length of a mixing tube is desired. Practicing engineers have defined an allegedly optimum length of a cylindrical (constant-diameter) mixing tube by the position of the axial static-pressure distribution maximum. (Such a maximum exists only because of the frictional losses at the tube wall. The energy distribution in this cross section is far from being uniform.) This definition is merely arbitrary and conventional, because the purpose of a mixing tube is not to raise the static pressure (this can be performed without any mixing process by a diffuser), but to impart a uniformly distributed increase of total pressure to the secondary air. Actually, there is no optimum length of a mixing tube, because the total energy of jet plus secondary air decreases monotonically from the initial cross section downstream to infinity. The only course is to agree by convention on a small arbitrary degree of non-uniformity in the final energy distribution and then investigate the influence of working conditions and design characteristics of the tube needed to attain this standardized degree of uniformity.

The theoretical calculation of ideal efficiency can easily be done for two special types of mixing tubes, i.e., the constant-diameter mixing tube and the constant-pressure mixing tube. For simplicity, this calculation has been performed for incompressible flow only, the results being immediately needed for further steps of the investigation. There should be, however, no difficulty in extending them to the case of compressible flow. These results are presented in two diagrams showing the ideal efficiencies as functions of the initial velocity ratio (secondary flow to jet) with the

mass ratio as a parameter (Figs. 1 and 2). Curves of constant net-power (total-pressure) coefficient were superimposed to permit a more consistent comparison of the two types of mixing tubes. The constant-pressure jet pump was assumed to be a combination of a (sufficiently long) constant-pressure mixing tube and an ideal diffuser with a final cross-section equal to the initial cross-section to make it directly comparable to the corresponding cylindrical mixing tube. With both types of jet pumps the efficiency increases for increasing velocity ratio at a given mass ratio. This result means that the jet velocity must not exceed the secondary-flow velocity too much for best efficiency.

In most applications (e.g., a circulation control system in an airplane) the efficiency of the mixing process is not as adequate for the technical evaluation of the jet-pump performance as the mass ratio. Therefore, an additional diagram (Fig. 3) was prepared to permit the determination of the mass ratio for given values of the initial velocity ratio and total-pressure transmission ratio k for constant-pressure mixing tubes. This total-pressure transmission ratio is defined as the ratio of total-pressure increase in the secondary air to the excess total pressure in the jet.

The ideal efficiency of the constant-pressure jet pump is always greater than for the cylindrical tube. (It must be kept in mind that an individual constant-pressure jet pump justifies its name only when it is operating at its design velocity ratio. Therefore, the preceding statement is only true for a family of constant-pressure jet pumps at their design conditions.) The economical superiority of the constant-pressure jet pump is restricted in velocity ratio, because at high velocity ratios the small increase of ideal efficiency is annihilated by the energy loss in the diffuser. In the lower range of velocity ratios, the success or failure of jet-pump installations may depend on the decision to drop the cylindrical type and convert to the constant-pressure design. This makes a thorough study of constant-pressure mixing-tube design a necessity.

The constant-pressure jet pump is not an optimum design. A jet pump with axially decreasing static-pressure in the mixing tube would certainly be more economical, but in considering this possibility, there are two difficulties. As with the constant-pressure mixing tube a diffuser is needed and at least the length, if not the energy loss, of this diffuser in most cases would be prohibitive even with very low velocity ratios. The other difficulty is that at the present time the ideal efficiency and the shape of the mixing tube cannot be calculated since no known method exists for computing the correlation between pressure distribution and geometrical shape of the mixing tube, except for the two special cases dealt with here. (In these cases the axial component of the pressure force on the tube wall vanishes).

The economical superiority of a constant-pressure or decreasing-pressure mixing tube can easily be understood. The energy loss in the mixing process is the work done by the turbulent shearing stresses in the mixing zone. This work can be decreased by reducing the difference between the velocities in the core of the mixing zone and in the secondary flow. A downstream pressure increase, as in the cylindrical mixing tube, decreases the velocity of the weaker secondary flow more than the velocity of the stronger jet with a resultant adverse effect on this velocity difference. Inversely, a downstream pressure decrease supports the equalizing effect of the mixing process and so reduces the work to be done by the turbulent shearing stresses.

II. Integral Relations of Mixing Processes.

The flow in a slender mixing tube at sufficiently high Reynolds numbers has properties that permit the application of Prandtl's boundary-layer equations. A mixing zone containing strong vorticity can be distinguished from the surrounding potential flow in the first portion of the mixing tube (in this part of the tube the term 'free mixing zone' is used). The secondary flow is practically uniform, the static pressure is constant over any cross-section, and the radial velocities are small compared to the axial velocities. The boundary-layer equations must be supplemented by a mathematical setup for the relationship between the turbulent shearing stress and the radial velocity gradient. In order to study the general behavior of the mixing process and the mixing-tube design resulting therefrom, it may be sufficient to satisfy certain integral relations for an entire cross-section instead of the differential equation at any point in the cross-section (this is known as the integral method of boundary-layer theory). The primary integral conditions to be fulfilled are continuity (the mass flux or, with an incompressible flow, the volume flux must be constant along the tube), momentum (the change of momentum flux must be in equilibrium with the resultant pressure forces on the tube wall if shearing stresses at the wall are to be neglected) and energy (the loss of total energy in the steady average flow equals the work done by the shearing stresses produced by the superimposed turbulent motion). This yields a system of three equations for the determination of three unknown functions which will be dealt with in Section IV. At first only considerations of continuity and momentum flux need be used to state the relationship between the basic and maximum excess velocities and the mixing zone radius. The main results are these:

a) Within a constant-diameter mixing tube there is an axial pressure rise and, by Bernoulli's Law, a corresponding velocity decrease in the secondary flow. If the initial dynamic pressure in the secondary flow is not strong enough to overcome this axial static-pressure increase until the

boundary of the mixing zone reaches the tube wall, a reverse flow in the secondary flow will originate near the wall to form a ring vortex with additional energy loss. This critical working condition is characterized by a specific value of the net power coefficient that is dependent on the shape of the non-dimensional excess velocity profile at the end of the free mixing zone. Since that shape is restricted within relatively narrow limits the critical net-power coefficient (C_p^*) is nearly constant, its order of magnitude being about 5 (Figure 1).

b) With a constant-pressure mixing tube the condition of continuity determines the mixing-tube radius as a function of the mixing-zone radius and the ratio of maximum excess velocity to secondary-air velocity to finally determine where the mixing zone for the first time fills the whole mixing tube (end of the free mixing zone).

III. Simplified Theory of Mixing-Zone Spreading.

An important simplification of the theoretical treatment is attained by assuming the jet to be issuing from a point orifice instead from a finite orifice area. This latter case was investigated by W. Szablewski through the initial length of the jet where a potential core still exists; his results are applicable for a rough computation of the finite-jet orifice position in a constant-pressure mixing tube. Finally, this position can be corrected empirically by slight longitudinal adjustments to obtain a constant axial static-pressure distribution through the mixing-zone length. If an exact account was made for the finiteness of the real jet orifice, another parameter would be introduced into the problem, (nozzle radius to excess-momentum radius ratio,) and would create an unnecessary complication.

An attempt to further simplification was made by dispensing with the energy equation and replacing it by the assumption that the excess-velocity profiles in the free mixing zone are affine. It was appreciated that this assumption would not be strictly valid, but it was hoped that a satisfactory description of jet diffusion in a mixing tube would be attained in this way since it was known that the shape variation of the non-dimensional, excess-velocity profiles is restricted to rather narrow limits. Therefore, an intermediate shape was chosen as a compromise for the general case. The form parameters of the excess-velocity profile thus appeared as universal constants. As shown later by a more exact treatment of the problem (Section IV) the qualitative predictions of this simplification were dependable and afforded a reasonable comparison of the two cases (Constant diameter and constant pressure), but under certain conditions the quantitative error became greater than permissible. Results from this investigation were as follows:

a) Principally the jet diffusion in the cylindrical mixing tube is characterized by the net-power coefficient. As mentioned previously, a critical net-power coefficient exists as a condition for the beginning of reverse flow. For the chosen excess-velocity profile the critical net-power coefficient (C_p^*) was 4.56.

b) Jet diffusion in the constant-pressure mixing tube can be represented by single curves for the mixing-zone radius, the ratio of maximum excess to basic velocity and other relevant functions if the coordinates (axial distance and mixing zone radius) are referred to the characteristic length of the problem, namely, the excess-momentum radius of the mixing zone (constant in this special case). The initial mixing-tube radius is not a characteristic length, but only specified length without pertinence to the behavior of the flow. Its choice merely determines the stream surface used as the tube wall.

c) In this simplified study the momentum-exchange setup (Prandtl 1942) for the relationship between turbulent shearing stresses and radial velocity gradient was used because of its simplicity. Exact solutions of the boundary-layer equations for jet diffusion problems yield an indefinite width (diameter) of the mixing zone when this setup is applied, but an excess-velocity profile of finite width was chosen for the evaluation of the integral relations. This inconsistency was justified by keeping the relations derived from the two different assumptions separated. Thus, integrations across the mixing zone have to be done on the basis of the chosen excess-velocity profile and integrations along the mixing-tube have to be done on the basis of the adapted boundary-layer equation containing the momentum-exchange setup. This second integration involves a non-dimensional spreading constant which has to be normalized in accordance with experimental results from tests on a free jet. The latest results of Reichardt were used, but the test results are dependent on the initial Reynolds number of the jet (based on the nozzle diameter). Unfortunately, this Reynolds number never has been varied systematically in the experiments to show its influence on the results.

IV. Approximate Theory of Jet Diffusion in a Constant-Pressure Mixing Tube.

In order to get dependable information on constant-pressure mixing an advanced investigation was carried through on the basis described in Section II by starting from the exact solutions of the boundary-layer equations for jet problems. Such solutions exist for incompressible flow under constant pressure only and they are restricted to special cases when either the secondary air is at rest at infinity (free jet) or the secondary flow is but slightly slower than the jet (weak jet). These are the limiting cases of the more general problem. The solutions are available both for the

'mixing-length' setup (Prandtl 1925) and for the previously mentioned momentum-exchange setup. But if the concept of a finite mixing zone within a surrounding potential flow is kept, only the solutions based on the 'mixing-length' setup can be utilized.

With a free jet the secondary air at infinity will be at rest only when the flow is restricted to a half-space by the plane wall from which the jet issues (without this restriction the infinite sum of sinks representing the effect of the mixing zone on the secondary air would induce a finite axial velocity of the secondary air everywhere). Fortunately, this is only a singular property of the free jet. The error involved in using the free-jet, half-space solution for a point orifice as a limiting case is expected to be of negligible importance for the general problem with a finite secondary-flow velocity and at distances from the point orifice corresponding to the location of the finite jet orifice.

The energy condition was restituted so that the hypothesis of affine excess-velocity profiles could be dropped. For purposes of analytical calculation the excess-velocity profiles were simply derived from the exactly known profiles of the limiting cases by linear interpolation which was permissible since the profiles of the limiting cases are not very different from one another. Now any individual shape of intermediate excess-velocity profile is characterized by a form parameter depending on the ratio of maximum excess to secondary air velocity. The next steps were to solve the system of conditions described in Section II for the function connecting this form parameter with the said velocity ratio in the vicinity of both limiting cases, i.e. with small velocity-ratio values and with small values of its reciprocal, to the first and second orders of approximation. Finally, the two sets of approximations were interpolated by a simple formula for the form parameter and the integration of the axis condition was numerically performed on the basis of the complete set of exact integral relations. In comparison with the results of the simplified theory the improvement wrought by the more rigorous procedure was considerable, showing more important influence of the axial excess-velocity profile variation than originally expected.

In these investigations the 'mixing length' was always assumed constant across the mixing zone until the boundary of the mixing zone reaches the wall. It is not known exactly what the effect would be on the 'mixing length' as the boundary approaches the wall, but certainly the wall's presence is a restriction of the mixing zone radial fluctuations probably resulting in a slight decrease of the 'mixing length' and a corresponding slight deformation of the excess-velocity profile near the boundary of the mixing zone.

This problem becomes serious in the second part of a mixing tube where the mixing zone completely fills the tube. A preliminary study indicated that the 'mixing length' near the wall would behave like the square root of the distance to the wall. But at the present time the flow problem of the guided or restricted mixing process is far from being solved. Therefore it is a favorable fact that in most cases when the design of a constant-pressure jet pump is indicated by considerations of efficiency, the contribution of the remainder of the mixing tube to the transformation of steady mean flow energy to energy of turbulent motion is so small that it doesn't matter much if this transformation is done under favorable or adverse static-pressure conditions.

This may be illustrated by the results of energy calculations on two constant-pressure jet pump designs. Both of them have an initial area ratio (jet to secondary air) of one percent, but the first model is designed for a net-power coefficient of 2.0 and a corresponding initial velocity ratio (secondary air-jet) of 9.9 percent while the second one for a net-power coefficient of 5.0 and an initial velocity ratio of 6.9 percent. The ideal efficiencies are 25.2 percent and 24.0 percent, respectively (instead of 17.4 percent and 11.4 percent for the cylindrical mixing tubes for corresponding C_p values). This means that 74.5 percent and 76.0 percent of the initial total excess energy of the jet is transformed into turbulent energy. Now taking these ideal efficiencies as reference quantities, 90.9 percent and 80.8 percent, respectively, of the jet energy will be theoretically transformed under constant pressure in the first section of the mixing tube (free mixing zone). This means that only 9.1 percent and 19.2 percent of the jet energy remain to be transformed under conditions not as favorable as the condition of constant pressure.

However, the second section of the mixing tube still remains a worthy object of further investigation because of its relatively great length when the hitherto neglected frictional losses at the tube wall are considered.

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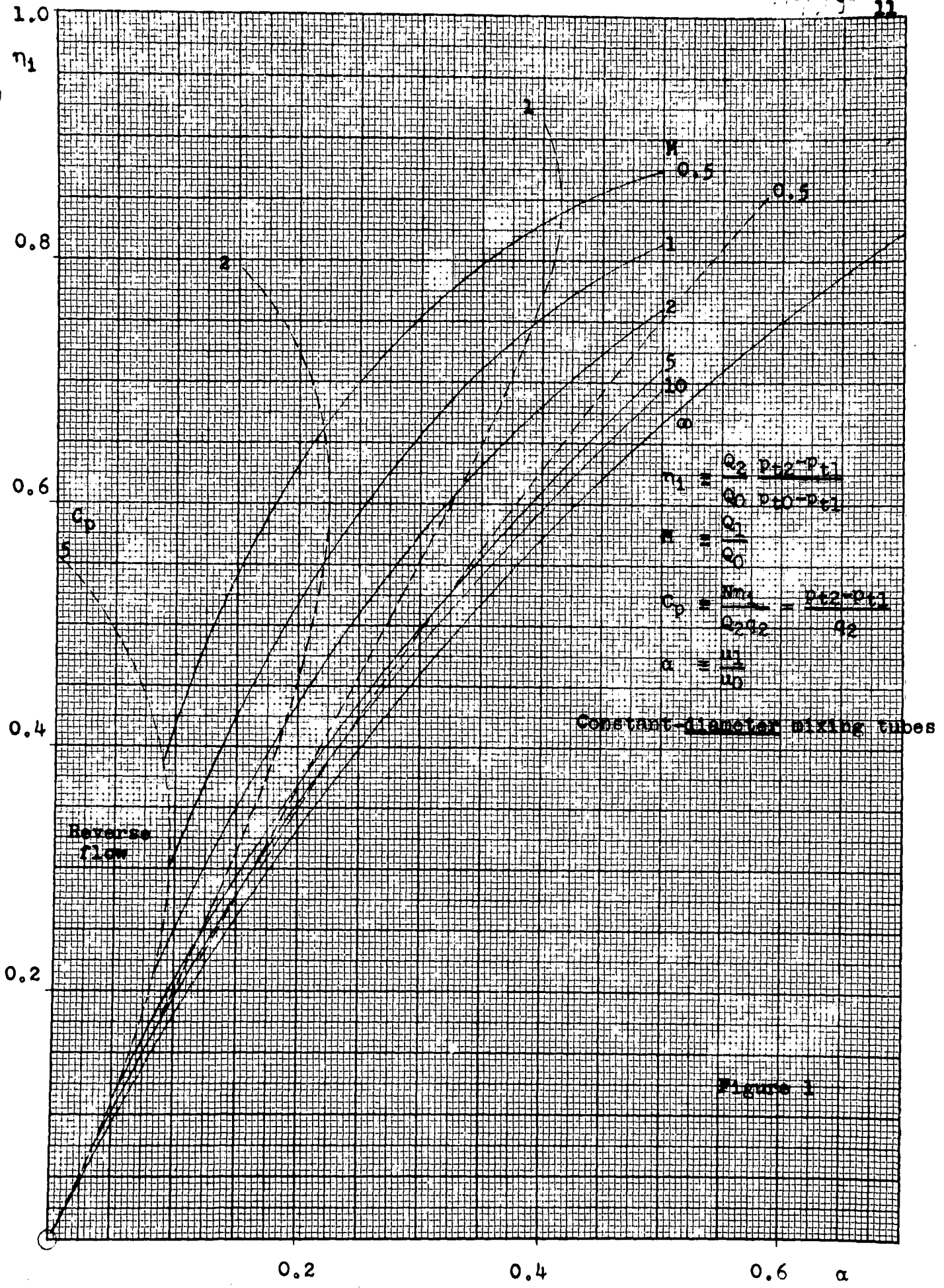
NOMENCLATURE

$c_p = \frac{P_{t2} - P_{t1}}{q_2}$	net power coefficient
$\eta_1 = k(1+M)$	ideal efficiency
$k = \frac{P_{t2} - P_{t1}}{P_{t0} - P_{t1}}$	total-pressure transmission ratio
$M = \frac{Q_1}{Q_0} = \frac{S_1 u_1}{S_0 u_0}$	secondary to primary fluid mass ratio
$\alpha = \frac{u_1}{u_0}$	secondary to primary fluid velocity ratio
P_t	total pressure
q	dynamic pressure
Q	quantity flow
S	cross-sectional area
u	axial velocity

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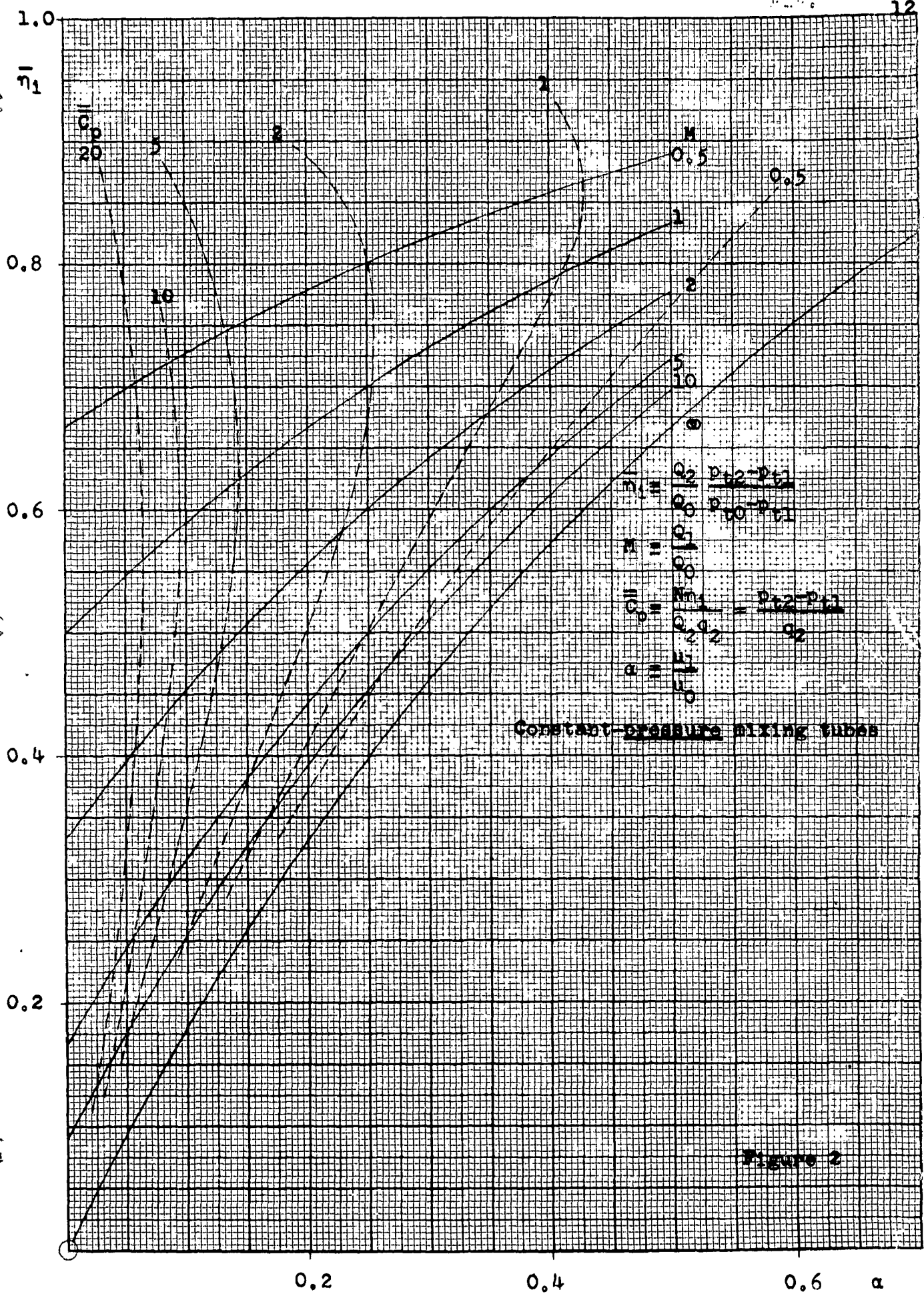
0	jet exit conditions
1	secondary fluid conditions at the initial plane
2	mixed fluid conditions at the end of the diffuser at infinity downstream (bar is added to the variable to denote same conditions at infinity downstream, but at the end of the mixing tube)

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Constant-pressure mixing tubes

Figure 2