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DEPARTMENT OF CIVIL ENGINEERING

APPLICATION OF GREEN'S METHOD IN DERIVING APPROXIMATE THEORIES OF ELASTICITY

by

G. HERRMANN

Office of Naval Research Project NR-064-388
Contract Nonr-266(09)
Technical Report No. 13
CU-14-54-ONR-266(09)-CE
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ABSTRACT

G. Green's procedure of deriving the three-dimensional linear equations of the theory of elasticity, is used as a guide in deducing an approximate theory of longitudinal motions of an elastic rod which contains the effects of radial inertia and radial shear deformation. At various stages of the development, attention is drawn to the analogies between the three-dimensional and the approximate theory. It is noted in particular, that the principle of conservation of energy imposes restrictions on possible stress-strain relations. These restrictions, which were overlooked in a previous paper by Mindlin and Herrmann, are taken into account here, thus altering the displacement equations of motion, but leaving the wave velocities in an infinite rod unchanged.

APPLICATION OF GREEN'S METHOD
IN DERIVING APPROXIMATE THEORIES OF ELASTICITY

Introduction

Various procedures have been adopted in establishing approximate linear theories of equilibrium and motion of elastic bodies, one or two of whose dimensions are small in comparison with the others.

One possibility consists in applying Newton's second law with respect to an element of the body, restricting possible deformations or stress distributions. It was followed, for example, by Bernoulli and Euler[1] in deriving what is termed today the elementary beam theory.

Another possibility consists in operating in appropriate fashions upon the general three-dimensional equations, as was demonstrated by Boussinesq [2] in deriving classical plate equations.

More recently, plate theories containing the influence of rotatory inertia and shear deformation were derived by various means by E. Reissner [3], [4], A. E. Green [5], R. D. Mindlin [6] and a similar shell theory by F. B. Hildebrand, E. Reissner and G. B. Thomas [7].

In deriving approximate equations of coupled longitudinal and radial motions in an elastic rod, Mindlin and Herrmann [8] have employed the three-dimensional energy expressions and stress-strain relations.

The purpose of the present paper is to demonstrate, with the example of a rod theory, how G. Green's method of deriving the general three-dimensional theory, may be used in deriving approximate theories of motion.

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1 See Historical Introduction in reference [1]. Numbers in brackets refer to the Bibliography at the end of the paper.

Shortly after the three-dimensional theory had been established, G. Green [9] proposed a new method of deriving these equations from the principle of conservation of energy. Later, Lord Kelvin [10] elaborated on the justification of Green's procedure within the framework of the laws of thermodynamics; but ever since its inception, Green's approach has remained a controversial issue. De Saint-Venant [11] termed it only a chain of arbitrary assumptions. In recent times, despite its elegance, it has been criticized severely by Truesdell [12], chiefly because of its inability to encompass as many phenomena as an approach starting from the Newtonian laws. Even though Green's formalism does not provide full physical insight, it has merits in its expediency and economy in arriving at a formally complete and consistent theory, within the framework of starting assumptions. The term "consistency" of an approximate theory is used here with particular reference to the principle of conservation of energy. In the absence of heat, the "stresses" of any approximate theory must be derivable from a potential, which imposes restrictions on possible stress-strain relations. This type of restriction leads to the requirement that the differential operator matrix of the displacement equations of equilibrium or of motion must be symmetric, as was pointed out by Vlasov [13]. For example, Vlasov has shown that Love's shell theory[2] does not satisfy the requirement.

This restriction was also overlooked in the recent paper by Mindlin and Herrmann [8], where certain constants were introduced into the stress-displacement relations in a manner which made the differential operator matrix of the displacement equations of motion asymmetric.

In the present paper it is shown how these constants may be introduced into the stress-displacement relations so as to keep the matrix symmetric.

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2 See chapter 24 of reference [1].

Even though the displacement equations of motion differ from those of [8], the velocity equation is the same, that is, the wave velocities have the same values as all those presented in [8].

In the first section of the present paper, G. Green's method is used in deriving the equations of three-dimensional linear elasticity in a manner employed by Professor R. D. Mindlin in the seminar in theory of elasticity at Columbia University.

In the second section, an approximate theory of compressional motions in an elastic solid rod is derived, using Green's method as a guide. The starting assumptions are the same as those in [8] and at various stages of the development attention is drawn to analogies with the general three-dimensional theory.

Classical Linear Theory of Elasticity

We consider a volume V of a body of mass density ρ, bounded by a surface S. We shall assume that the total energy in V is composed of the sum of the kinetic energy T and the internal energy W, which are, in the linear theory

$$T = \int_V T^* dv \tag{1}$$

$$W = \int_V W^* dv \tag{2}$$

where dv is the element of volume, T^* the kinetic energy per unit of volume and W^* the internal energy per unit of volume.

Let the body be acted upon by body forces per unit of mass $\underset{\sim}{F}$ and surface tractions per unit of area $\underset{\sim}{T}_n$, n being the outward drawn normal to the tangent plane and let $\underset{\sim}{u}$ be the displacement produced.

Neglecting thermal effects, the law of conservation of energy is then[3]

$$\dot{T} + \dot{W} = \int_V \rho \underset{\sim}{F} \cdot \dot{\underset{\sim}{u}}\, dv + \int_S \underset{\sim}{T}_n \cdot \dot{\underset{\sim}{u}}\, ds \tag{3}$$

That is, the rate of increase of the total energy in the body is equal to the rate at which external forces do work on the body.

Let $\underset{\sim}{\varepsilon}$ designate the strain dyadic, which is the symmetric part of the deformation dyadic $\underset{\sim}{\nabla} \underset{\sim}{u}$, that is,

$$\underset{\sim}{\varepsilon} = \frac{1}{2}\left(\underset{\sim}{\nabla}\underset{\sim}{u} + \underset{\sim}{u}\underset{\sim}{\nabla}\right) \tag{4}$$

Following Green, W^* is assumed to be a single-valued function of $\underset{\sim}{\varepsilon}$

$$W^* = W^*(\underset{\sim}{\varepsilon}) \tag{5}$$

The time rate of change of W^* will thus be

$$\dot{W}^* = \frac{\partial W^*}{\partial \underset{\sim}{\varepsilon}} : \dot{\underset{\sim}{\varepsilon}} \tag{6}$$

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3 The notation employed in this section is that of reference [14].

The kinetic energy density T^* is given in the linear theory by

$$T^* = \frac{1}{2}\rho\,\underset{\sim}{\dot{u}}\cdot\underset{\sim}{\dot{u}} \tag{7}$$

and its time rate of change by

$$\dot{T}^* = \rho\,\underset{\sim}{\dot{u}}\cdot\underset{\sim}{\ddot{u}} \tag{8}$$

Substituting (6) and (8) into (3) we have

$$\int_V \left[\rho\,\underset{\sim}{\dot{u}}\cdot\underset{\sim}{\ddot{u}} + \frac{\partial W}{\partial \underset{\sim}{\varepsilon}} : \underset{\sim}{\dot{\varepsilon}}\right] dv = \int_V \rho\,\underset{\sim}{F}\cdot\underset{\sim}{\dot{u}}\,dv + \int_S \underset{\sim}{T}_n\cdot\underset{\sim}{\dot{u}}\,ds \tag{9}$$

Since $\dfrac{\partial W}{\partial \underset{\sim}{\varepsilon}}$ is symmetric

$$\frac{\partial W}{\partial \underset{\sim}{\varepsilon}} : \underset{\sim}{\dot{\varepsilon}} = \frac{\partial W}{\partial \underset{\sim}{\varepsilon}} : \underset{\sim}{\nabla}\underset{\sim}{\dot{u}} \tag{10}$$

because $\underset{\sim}{\varepsilon}$ is the symmetric part of $\underset{\sim}{\nabla}\underset{\sim}{u}$.

With the aid of the expansion

$$\underset{\sim}{\nabla}\cdot(\underset{\sim}{\Phi}\cdot\underset{\sim}{a}) = (\underset{\sim}{\nabla}\cdot\underset{\sim}{\Phi})\cdot\underset{\sim}{a} + \underset{\sim}{\Phi} : \underset{\sim}{\nabla}\underset{\sim}{a} \tag{11}$$

where $\underset{\sim}{\Phi}$ is a dyadic and $\underset{\sim}{a}$ a vector, and the transformation

$$\int_V \underset{\sim}{\nabla}\cdot\underset{\sim}{\Phi}\,dv = \int_S \underset{\sim}{n}\cdot\underset{\sim}{\Phi}\,ds \tag{12}$$

$\underset{\sim}{n}$ being the outward drawn normal unit vector, (9) becomes

$$\int_V \left[\rho\,\underset{\sim}{\dot{u}}\cdot\underset{\sim}{\ddot{u}} - \left(\underset{\sim}{\nabla}\cdot\frac{\partial W^*}{\partial \underset{\sim}{\varepsilon}}\right)\cdot\underset{\sim}{\dot{u}}\right] dv = \int_V \rho\,\underset{\sim}{F}\cdot\underset{\sim}{\dot{u}}\,dv$$

$$\int_S \left[\underset{\sim}{T}_n\cdot\underset{\sim}{\dot{u}} - \underset{\sim}{n}\cdot\frac{\partial W^*}{\partial \underset{\sim}{\varepsilon}}\cdot\underset{\sim}{\dot{u}}\right] ds \tag{13}$$

which must hold for any volume V and for all velocities $\underset{\sim}{\dot{u}}$, hence,

$$\underset{\sim}{\nabla}\cdot\frac{\partial W^*}{\partial \underset{\sim}{\varepsilon}} = \rho(\underset{\sim}{F} + \underset{\sim}{\ddot{u}}) \qquad \text{in } V \tag{14}$$

and

$$\underset{\sim}{n}\cdot\frac{\partial W^*}{\partial \underset{\sim}{\varepsilon}} = \underset{\sim}{T}_n \qquad \text{on } S \tag{15}$$

Relation (15) defines the dyadic $\frac{\partial W^*}{\partial \underset{\sim}{\varepsilon}}$, which is called the stress and is designated by the symbol $\underset{\sim}{\sigma}$. It is that dyadic whose scalar product with $\underset{\sim}{n}$, gives the traction $\underset{\sim}{T}_n$ across the plane whose outward drawn normal is $\underset{\sim}{n}$. Equation (14) is, then, the stress equation of motion.

In order to insure uniqueness of solutions of the stress equations of motion, it is necessary to establish an explicit stress-strain relation, whose existence is already implicit in (3) and (5). To this end, we assume W^* to be analytical and expand W^* into a power series of $\underset{\sim}{\varepsilon}$. Because of the assumed existence of a natural, unstrained state, the linear term in the series vanishes. Further, the third and higher powers of the expansion are neglected on account of the smallness of $\underset{\sim}{\varepsilon}$. We have thus a quadratic expression for the strain energy

$$W^* = \frac{1}{2}\, \underset{\sim}{\varepsilon} : \overset{4}{\underset{\sim}{C}} : \underset{\sim}{\varepsilon} \tag{16}$$

where $\overset{4}{\underset{\sim}{C}}$ is the elasticity tetradic. Equation (16) results in a linear stress-strain relation

$$\frac{\partial W^*}{\partial \underset{\sim}{\varepsilon}} \equiv \underset{\sim}{\sigma} = \overset{4}{C} : \underset{\sim}{\varepsilon} \tag{17}$$

Because of the symmetry of $\underset{\sim}{\sigma}$ and $\underset{\sim}{\varepsilon}$ the number of independent constants $c_{ijk\ell}$ which constitute the tetradic $\overset{4}{\underset{\sim}{C}}$ is 36. Since W^* is a single-valued function, this number is reduced further to 21.

Having thus the definition of stress (15), the stress equations of motion (14) and the stress-strain relations (17), appropriate boundary and initial conditions of the theory should be established, i.e., conditions to be specified on the boundary of the region and at the initial instant, which assure a unique solution.

By an argument due to Neumann[4], it may be shown that, in the absence

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4 See p. 176 in reference [1].

of singularities and displacement discontinuities, uniqueness may be assured by specifying (1) one member of each of the three products in $\underset{\sim}{T}_n \cdot \underset{\sim}{u}$ at each point of the surface and (2) the initial displacement and velocity throughout the volume.

A One-Dimensional Theory of Compressional Motions in an Elastic Rod

To illustrate the application of Green's method to an approximate linear theory, a one-dimensional theory governing the compressional motions in an elastic rod will be derived.

Wishing to include in this theory more than is contained in the classical elementary theory, we shall start out from the same assumptions as in paper [8], taking into account the effects of radial inertia and radial shear deformation.

We consider a rod of circular cross-section, $r = a$, and approximate axially symmetric components of displacement u_r, u_θ, u_z, referred to cylindrical coordinates r, θ, z, by the forms

$$\begin{aligned} u_r &\rightarrow r a^{-1} u(z,t) \\ u_\theta &= 0 \\ u_z &\rightarrow w(z,t) \end{aligned} \tag{18}$$

The r-dependence in these approximations has been specified completely, so that integration with respect to r may be performed, thus eliminating r from the resulting equations and reducing the number of space coordinates on which the displacements depend.

From the mathematical point of view we are led to the above approximations by expanding the components of displacement into power series of r, retaining only the first term of each expansion. From the physical point of view the above approximations are justified for the study of compressional motions, because they accommodate the limiting form of the wave motion for long waves in the general three-dimensional theory.

The kinetic energy T of a volume V is, in cylindrical coordinates, given by

$$T = \frac{1}{2} \int_V \rho \left(\dot{u}_r^2 + \dot{u}_\theta^2 + \dot{u}_z^2 \right) dv \tag{19}$$

and its time rate of change

$$\dot{T} = \int_V \int \rho \left(\dot{u}_r \ddot{u}_r + \dot{u}_\theta \ddot{u}_\theta + \dot{u}_z \ddot{u}_z \right) dv \tag{20}$$

An approximation $\bar{T}$ to the kinetic energy T is obtained by substituting the approximations to the velocities from (18). We obtain

$$\dot{\bar{T}} = \int_0^{2\pi}\int_0^a\int_b^c \rho \left(r^2 a^{-2} \dot{u}\ddot{u} + \dot{w}\ddot{w} \right) r\,dr\,d\theta\,dz \tag{21}$$

and carrying out the integration

$$\dot{\bar{T}} = 2\pi \int_b^c \rho \left(a^2 \dot{u}\ddot{u}/4 + a^2 \dot{w}\ddot{w}/2 \right) dz \tag{22}$$

where $c - b$ is the length of the rod.

The corresponding approximation $\bar{\underset{\sim}{\varepsilon}}$ to the strain $\underset{\sim}{\varepsilon}$, calculated on the basis of the approximations to the displacements (18), possesses, in cylindrical coordinates, three different components, namely $\frac{u}{a}$, $ru'/2a$, w', the prime indicating partial differentiation with respect to z.

The approximation $\bar{W}^*$ to the internal energy W^*, will thus be, following Green,

$$\bar{W}^* = \bar{W}^* \left(u/a,\ ru'/2a,\ w' \right) \tag{23}$$

and the time rate of change of the total internal energy $\bar{W}$

$$\dot{\bar{W}} = \int_0^{2\pi}\int_0^a\int_b^c \left(\frac{\partial \bar{W}^*}{\partial \frac{u}{a}} \frac{\dot{u}}{a} + \frac{\partial \bar{W}^*}{\partial (\frac{r}{2a} u')} \frac{r}{2a} \dot{u}' + \frac{\partial \bar{W}^*}{\partial w'} \dot{w}' \right) r\,dr\,d\theta\,dz \tag{24}$$

or

$$\dot{\bar{W}} = 2\pi \int_b^c \left(\bar{P}\dot{u}/a + Q\dot{u}' + P_z \dot{w}' \right) dz \tag{25}$$

where

$$\left.\begin{aligned} \bar{P} &= \int_0^a \frac{\partial \overline{W}^*}{\partial\left(\frac{u}{a}\right)}\, r\, dr \\ Q &= \int_0^a \frac{\partial \overline{W}^*}{\partial\left(\frac{1}{2}\frac{r}{a}u'\right)}\, \frac{r^2}{2a}\, dr \\ P_z &= \int_0^a \frac{\partial \overline{W}^*}{\partial w'}\, r\, dr \end{aligned}\right\} \tag{26}$$

In analogy to the three-dimensional theory, the integrals of the set (26) shall be termed "rod-stresses" of the present theory.

Having the expressions (22), (25) for the time rate of change of the kinetic energy $\overline{T}$ and the internal energy $\overline{W}$, respectively, we proceed to formulate the principle of conservation of energy for a rod of finite length $c-b$, expressing that the time rate of change of internal energy must equal the rate at which tractions do work in the displacement of the surface points, assuming the absence of body forces.

Due to the assumption of axially symmetric deformation, the tangential traction T_s, at the end sections of the rod, will have radial components only, independent of θ. The tangential traction Z on the cylindrical surface of the rod will have components in the axial direction only and also independent of θ. The normal component of traction T_z on the end section of the rod and the normal component of traction, R, on the cylindrical surface of the rod will be also independent of θ.

The rate at which these tractions do work in the assumed displacements (18) is given by the expression

$$2\pi a \int_b^c (R\dot{u} + Z\dot{w})\, dz + 2\pi \int_0^a \Big[(T_s r a^{-1} \dot{u} + T_z \dot{w}) \Big]_b^c r\, dr \tag{27}$$

and the principle of conservation of energy takes the form

$$2\pi\int_b^c \rho(a^2\dot{u}\ddot{u}/4 + a^2\dot{w}\ddot{w}/2)\,dz + 2\pi\int_b^c (\bar{P}\dot{u}/a + Q\dot{u}' + P_z\dot{w}')\,dz$$
$$= 2\pi a\int_b^c (R\dot{u} + Z\dot{w})\,dz + 2\pi\int_0^a \left[(T_s r a^{-1}\dot{u} + T_z\dot{w})\right]_b^c r\,dr \tag{28}$$

Upon integration by parts of the last two terms in the second integral and upon rearrangement, we obtain

$$\int_b^c \left\{\dot{u}\left[\rho a^2\ddot{u}/4 + \bar{P}/a - Q' - aR\right] + \dot{w}\left[\rho a^2\ddot{w}/2 - P_z' - aZ\right]\right\} dz$$
$$= \dot{u}\left[\int_0^a T_s r^2 a^{-1}\,dr - Q\right]_b^c + \dot{w}\left[\int_0^a T_z r\,dr - P_z\right]_b^c \tag{29}$$

This relation corresponds to Equation (13) of the three-dimensional theory. By an analogous argument, we obtain

$$\left.\begin{aligned} Q' - a^{-1}\bar{P} + aR &= \rho a^2\ddot{u}/4 \\ P_z' + aZ &= \rho a^2\ddot{w}/2 \end{aligned}\right\} \tag{30}$$

$$\left.\begin{aligned} Q &= \int_0^a T_s r^2 a^{-1}\,dr \\ P_z &= \int_0^a T_z r\,dr \end{aligned}\right\} \tag{31}$$

The first two equations (30) are the stress equations of motion, which are the same as those of paper [8], provided $\bar{P}$ is set equal to $P_r + P_\theta$, defined in [8]. The last two equations define the two bar-stresses Q and P_z of the theory and correspond to Equation (15) of the three-dimensional theory.

It should be noted that the method used here does not allow for an identification of the quantity $\bar{P}$ in terms of components of traction.

Comparing (31) with (26) we obtain

$$\left.\begin{aligned} \frac{\partial \bar{W}^*}{\partial(\frac{1}{2}\frac{r}{a}u')} &= T_s \\ \frac{\partial \bar{W}^*}{\partial w'} &= T_z \end{aligned}\right\} \tag{32}$$

but there is no possibility of relating $\dfrac{\partial \bar{W}^*}{\partial(\frac{u}{a})}$ to a component of traction.

Noteworthy is also that the surface tractions R and Z enter the differential equation of motion in a manner analogous to the appearance of body force in the three-dimensional equations.

The boundary and initial conditions, found by an argument of uniqueness, analogous to the one of three-dimensional theory, are, for the present theory, that it is sufficient to specify

(1) at each point along the length of the bar, one factor of each of the products $\dot{u}R$ and $\dot{w}P_z$

(2) at each end of the bar, one member of each of the products $\dot{u}Q$ and $\dot{w}P_z$

(3) throughout the length of the bar, the initial displacements and velocities.

We proceed to establish the stress-displacement relations, by expanding $\overline{W}^*$, just as W^* was expanded in the three-dimensional theory. Retaining only the quadratic terms of the expansion, we have

$$\overline{W}^* = \alpha_1\left(\frac{u}{a}\right)^2 + \alpha_2\left(\frac{r}{2a}u'\right)^2 + \alpha_3\left(w'\right)^2 + \alpha_4\frac{u}{a}\frac{r}{2a}u' + \alpha_5\frac{u}{a}w' + \alpha_6\frac{r}{2a}u'w' \tag{33}$$

which yields the stress-displacement relations, in accordance with (26), carrying out the integration over the radius r

$$\begin{aligned} \overline{P} &= \alpha_1 a u + \alpha_4\frac{a^2}{6}u' + \alpha_5\frac{a^2}{2}w' \\ Q &= \alpha_2\frac{a^2}{4}u' + \alpha_4\frac{a}{3}u + \alpha_6\frac{a^2}{3}w' \\ P_z &= \alpha_3 a^2 w' + \alpha_5\frac{a}{2}u + \alpha_6\frac{a^2}{6}u' \end{aligned} \tag{34}$$

This completes the formal development of the theory by the method of Green. The 6 constants α_i play the role of the 21 elastic constants c_{ijkl} of the three-dimensional theory. It would not be difficult but it is cumbersome to express α_i in terms of c_{ijkl}. It would be necessary to write down, in cylindrical coordinates, the expression for W^* in terms of the components of strain, substitute the approximations (18), obtaining thus $\overline{W}^*$ and compare with $\overline{W}^*$ given by (33).

If we consider an isotropic rod, characterized elastically by means of Lamé's constants λ and μ, the following values of α_i are readily obtained, in carrying out the procedure outlined above:

$$\left.\begin{aligned} \alpha_1 &= \underline{2}\,(\lambda+\mu) \\ \alpha_2 &= 2\mu \\ \alpha_3 &= \tfrac{1}{2}(\lambda+2\mu) \\ \alpha_4 &= 0 \\ \alpha_5 &= 2\lambda \\ \alpha_6 &= 0 \end{aligned}\right\} \qquad (35)$$

In paper [8] it has been demonstrated that the present theory predicts wave velocities of first mode which are always higher than those obtained from the general three-dimensional theory.

One possibility of improving the present theory without rendering it more complicated consists in altering the values of α_i, without changing the form of the displacements. Formally, the constants α_i will be determined not in terms of the elastic constants, λ and μ, of the isotropic rod, i.e., matching the strain energy, as was done above, but by matching a number of solutions of basic dynamic problems of the rod, obtained within the present theory and within the three-dimensional theory. This procedure means physically the attempt to diminish the error inherent in the approximate forms of the displacements by making the rod appropriately anisotropic.

Determining α_i in this fashion, we note first that the strain energy expression (33) must be invariant to a reversal of the sense of the z-axis; thus

$$\left.\begin{aligned} \alpha_4 &= 0 \\ \alpha_6 &= 0 \end{aligned}\right\} \qquad (36)$$

just as in the isotropic bar.

Considering the problem of pure radial vibration of an infinite isotropic rod, it may be readily verified that the frequency, from the three-dimensional theory, is given by

$$p^2 = \frac{\eta^2}{a^2}\frac{(\lambda+2\mu)}{\rho} \tag{37}$$

where η is the lowest root of

$$\frac{\lambda+2\mu}{2\mu}\,\eta\, J_0(\eta) = J_1(\eta) \tag{38}$$

J_0, J_1 being Bessel functions of order zero and one respectively.

In the present theory, the frequency of pure radial vibration is obtained, from

$$-\alpha_1 u = \frac{\rho a^2}{4}\ddot{u} \tag{39}$$

as

$$\bar{p}^2 = \frac{4\alpha_1}{\rho a^2} \tag{40}$$

The two frequencies are equal if

$$\alpha_1 = \frac{\eta^2(\lambda+2\mu)}{4} \tag{41}$$

Wishing to determine the velocity of compressional waves, travelling in the infinite rod, we seek solutions in the form

$$\begin{aligned} u &= Ae^{i\gamma(z-ct)} \\ w &= Be^{i\gamma(z-ct)} \end{aligned} \tag{42}$$

with $R = Z = 0$

where $\gamma = 2\pi/L$, and c the wave-velocity.

The equations of motion reduce then to two homogeneous, linear, algebraic equations in A and B , whose determinant, set equal to zero, furnishes the frequency or velocity equation

$$\left[\alpha_2/\rho + 4\alpha_1/a^2\gamma^2\rho - c^2\right]\left[2\alpha_3/\rho - c^2\right] - 2\alpha_5^2/a^2\gamma^2\rho = 0 \tag{43}$$

For long waves ($a\gamma \to 0$), Equation (43) yields

$$c^2 = \frac{8\alpha_1\alpha_3 - 2\alpha_5^2}{4\alpha_1\rho}, \qquad c^2 = \infty \tag{44}$$

while the corresponding exact solutions are

$$c^2 = E/\rho, \qquad c^2 = \infty \tag{45}$$

Matching the lower velocity we obtain

$$E = 2\alpha_3 - \alpha_5^2/2\alpha_1 \tag{46}$$

We note that Equation (46) is satisfied by the values of α_i in the isotropic rod given by (35). But since α_1 has now been altered in accordance with (41), α_5 has to be changed also, to retain the validity of (46). Expressing the altered value of $\alpha_1 = \chi_1^2(\lambda+\mu)$ as a multiple of its original value $(\lambda+\mu)$, the original value of α_5 has to be changed to $\alpha_5 = 2\lambda\chi_1$, and the original value of α_3 has to be retained. χ_1^2 is determined, in virtue of (41), by

$$\chi_1^2 = \frac{\eta^2(\lambda+2\mu)}{8(\lambda+\mu)} \tag{47}$$

For short waves ($a\gamma \to \infty$), Equation (43) yields

$$c^2 = \alpha_2/\rho, \qquad c^2 = 2\alpha_3/\rho \tag{48}$$

while the corresponding exact solutions are

$$c^2 = \varkappa^2\mu/\rho, \qquad c^2 = \mu/\rho \tag{49}$$

the first being the Rayleigh wave velocity where $\varkappa^2$ is the root of [5]

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[5] See p. 308 in reference [1].

$$4\left[(1-\alpha\chi^2)(1-\chi^2)\right]^{\frac{1}{2}} = (2-\chi^2)^{\frac{1}{2}}, \qquad 0<\chi<1 \tag{50}$$

and

$$\alpha = \frac{(1-2\nu)}{2(1-\nu)} \tag{51}$$

Since α_3 is already determined, we can match only the lower velocity, obtaining for α_2

$$\alpha_2 = \chi^2\mu \tag{52}$$

The stress-strain relations of the present theory become thus, in terms of λ, μ

$$\left.\begin{aligned} \bar{P} &= 2\chi_1^2 a(\lambda+\mu)u + \chi_1 a^2\lambda w' \\ 2P_z &= 2\chi_1 a\lambda u + (\lambda+2\mu)a^2 w' \\ 4Q &= \chi^2\mu a^2 u' \end{aligned}\right\} \tag{53}$$

It may be noted that χ_1 has been determined in a different manner in paper [8].

Comparison of the stress-displacement relations (53) with those of paper [8] indicates that the first and the second relations are different, while the third is identical.

From the stress equations of motion (30), with the stress-displacement relations (53), we obtain the displacement equations of motion

$$\begin{aligned} \chi^2 a^2\mu u''/4 - 2\chi_1^2(\lambda+\mu)u - \lambda\chi_1 a w' + aR &= \rho a^2\ddot{u}/4 \\ \lambda\chi_1 a u' + (\lambda+2\mu)a^2 w''/2 + aZ &= \rho a^2\ddot{w}/2 \end{aligned} \tag{54}$$

It is observed, that the differential operator matrix of u and w is symmetric. The operator on w in the first equation is the same as the operator on u in the second equation, except for an immaterial opposite sign. This is not the case in the displacement equations of motion in paper [8]. However, the equation of wave velocities in an infinite rod, deduced from (54), is the same as the one of paper [8]. Thus, all the calculations of wave velocities plotted in [8], retain their validity within the present, modified theory.

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