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Technical Memorandum

**LAWS OF DYNAMIC SIMILITUDE  
FOR  
AERIAL PICKUP MODELING**

by  
P. S. Chase

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**FEASIBILITY OF AERIAL PICKUP SYSTEMS  
Office of Naval Research Contract Nonr-1279(00)  
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Date: 9/27/54 J. W. Johnston  
By direction of  
Chief of Naval Research (Code 461)

**Technical Memorandum**

**Laws of Dynamic Similitude  
for Aerial Pickup Modeling**

by

**P. S. Chase**

**October 27, 1953**

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Abstract

Similarity laws for the dynamic modeling of object trajectories and accelerations in various aerial pickup systems are presented.

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### Introduction

In evaluating the feasibility of certain aerial pickup systems, the need has arisen to extrapolate observed results from experiments at less than full scale to those which may be expected in the full-scale prototype. In the future it may be necessary to conduct model experiments on systems as yet untested.

The interpretation of model experimental data may be accomplished with the similarity laws involved. These laws are tabulated here for the dynamic modeling of aerial pickup phenomena.

### Analysis

The problem of modeling static situations has been described in Ref. (a) and elsewhere. Modeling of certain specific dynamic phenomena has been described in Refs. (b) and (c). In the present discussion the techniques developed in these references are extended to the specific case of dynamic modeling for aerial pickup systems.

It is of prime importance that geometrical similarities be preserved between full-scale and the model; i. e., cable length, cable diameter, object dimensions, etc., must all be scaled by the same ratio.

Assume the relationship between the linear dimension (L) of full-scale and model is given by the ratio R. Then denoting full-scale conditions by the subscript ( )<sub>fs</sub> and model conditions by ( )<sub>m</sub>.

$$\frac{L_{fs}}{L_m} = R$$

Also assume that the ratio of velocity (V) between full-scale and model conditions is established by some criterion as yet unspecified. Therefore

$$\frac{V_{fs}}{V_m} = K$$

It is necessary to maintain the ratio of inertia forces to aerodynamic forces constant in the prototype and the model in order to preserve the similarity of vector force diagrams for any component. Therefore, since inertia forces are equal to mass (m) times acceleration (a) and aerodynamic forces are proportional to a force coefficient (C), the density (d) of the fluid, the velocity squared, and a length squared,

$$\frac{m_{fs} a_{fs}}{C_{fs} d_{fs} V_{fs}^2 L_{fs}^2} = \frac{m_m a_m}{C_m d_m V_m^2 L_m^2}$$

and the relationship between accelerations becomes:

$$\begin{aligned} \frac{a_{fs}}{a_m} &= \frac{m_m}{m_{fs}} \frac{C_{fs}}{C_m} \left( \frac{d_{fs}}{d_m} \right) \text{fluid} \frac{V_{fs}^2}{V_m^2} \frac{L_{fs}^2}{L_m^2} \\ &= \frac{m_m}{m_{fs}} \frac{C_{fs}}{C_m} \left( \frac{d_{fs}}{d_m} \right) \text{fluid} K^2 R^2 \\ \text{but } \frac{m_m}{m_{fs}} &= \left( \frac{d_m}{d_{fs}} \right) \text{solid} \frac{L_m^3}{L_{fs}^3} \\ &= \left( \frac{d_m}{d_{fs}} \right) \text{solid} \frac{1}{R^3} \end{aligned}$$

and 
$$\frac{a_{fs}}{a_m} = \left(\frac{d_m}{d_{fs}}\right)_{\text{solid}} \left(\frac{d_{fs}}{d_m}\right)_{\text{fluid}} \frac{C_{fs}}{C_m} \frac{K^2}{R}$$

Since time (t) equals velocity divided by acceleration the time relationship is,

$$\begin{aligned} \frac{t_{fs}}{t_m} &= \frac{V_{fs}}{a_{fs}} \frac{a_m}{V_m} \\ &= \left(\frac{d_{fs}}{d_m}\right)_{\text{solid}} \left(\frac{d_m}{d_{fs}}\right)_{\text{fluid}} \frac{C_m}{C_{fs}} \frac{R}{K} \end{aligned}$$

If it is of interest to investigate angular velocities, the criterion is that helix angles in the full-scale phenomena be duplicated in the model phenomena. Helix angle is equal to angular velocity (W), times a characteristic length, divided by velocity. Therefore,

$$\frac{W_{fs} L_{fs}}{V_{fs}} = \frac{W_m L_m}{V_m}$$

and 
$$\frac{W_{fs}}{W_m} = \frac{K}{R}$$

Since angular acceleration (A) is equal to linear acceleration divided by a characteristic length, the ratio of angular accelerations becomes:

$$\frac{A_{fs}}{A_m} = \frac{a_{fs}}{L_{fs}} \frac{L_m}{a_m}$$

and 
$$\frac{A_{fs}}{A_m} = \left(\frac{d_m}{d_{fs}}\right)_{\text{solid}} \left(\frac{d_{fs}}{d_m}\right)_{\text{fluid}} \frac{C_{fs}}{C_m} \frac{K^2}{R^2}$$

Although it does not appear to be of great importance in the present case, it has been shown in Ref. (b) that the relationship of moment of inertia between the full-scale prototype components and those of the model must be

$$\frac{I_{fs}}{I_m} = R^5 \left(\frac{d_{fs}}{d_m}\right)_{\text{Solid}}$$

### Discussion

It will be noted that in modeling aerial pickup systems it is most convenient if the acceleration of gravity is identical in the full-scale case and the model case, although for exact modeling it should be scaled up in accordance with the previously given law of similarity for linear accelerations. Therefore vertical heights of observed trajectories in the model are too large and must be corrected accordingly by subtracting the displacement error due to gravity. If the times of action in the model are small, the modeling error due to gravity will be negligible.

The ratio of velocity between full-scale and model may now be considered, since it may be determined as a matter of convenience, but only within a certain range which will be specified by the Reynolds number of full-scale and model tests. It is necessary to duplicate Reynolds number (the ratio of inertia to viscous forces in the fluid) in order to obtain exact geometrical similarity of flow conditions.

The various types of systems to be investigated will determine the relative importance of the factors described above. In some instances, it may be more important to duplicate inertia effects and in others aerodynamic effects. However, the dynamic similarity laws are not incompatible, and for a large number of systems of interest, very good modeling should be possible.

If it is assumed that model experiments will be made in air,

$$\left( \frac{d_{fs}}{d_m} \right)_{\text{fluid}} = 1,$$

and if identical materials are used in full-scale and model components,

$$\left( \frac{d_{fs}}{d_m} \right)_{\text{solid}} = 1$$

If Reynolds numbers are duplicated

$$C_{fs} = C_m$$

and the laws of similarity may be summarized as follows:

$$\frac{L_{fs}}{L_m} = R$$

$$\frac{V_{fs}}{V_m} = K$$

$$\frac{a_{fs}}{a_m} = \frac{K^2}{R}$$

$$\frac{t_{fs}}{t_m} = \frac{R}{K}$$

$$\frac{W_{fs}}{W_m} = \frac{K}{R}$$

$$\frac{A_{fs}}{A_m} = \frac{K^2}{R^2}$$

$$\frac{I_{fs}}{I_m} = R^5$$

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- (a) Taylor, D. W., "Some Aspects of the Comparison of Model and Full-Scale Tests," NACA Report 219, 1926.
- (b) Scherberg, M., and Rhode, R. V., "Mass Distribution and Performance of Free Flight Models", NACA Technical Note 268, October 1927.
- (c) Neihouse, A. I., and Pepoon, P. W., "Dynamic Similitude Between a Model and Full-Scale Body for Model Investigation at Full-Scale Mach Number", NACA Technical Note 2062, March 1950.

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