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REPORT No. 892

**Mechanism For Balancing
A Rigid Body
Off Its Center of Gravity**

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DEPARTMENT OF THE ARMY PROJECT No. 503-05-005
ORDNANCE RESEARCH AND DEVELOPMENT PROJECT No. TB3-0230

BALLISTIC RESEARCH LABORATORIES



ABERDEEN PROVING GROUND, MARYLAND

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JANUARY 1954

S. Zarodny/ow
Aberdeen Proving Ground, Md.
January 1954

MECHANISM FOR BALANCING A RIGID BODY OFF ITS CENTER OF GRAVITY

ABSTRACT

A mechanism is outlined which will make it possible to pivot a gun about its muzzle, without providing any mass in front of the pivot. The mechanism furnishes, in effect, a true balance, rather than a mere equilibrium; i.e., a transverse displacement of the pivot will only move the gun parallel to itself, without turning it. An important feature of this system is that the forces causing a rotation of the center of gravity of this system about the pivot may be applied in such a way that no rotation of gun itself is produced; so that, while some servomechanism is still necessary, the perfection of this servomechanism is not essential.

The mechanism consists of two additional masses, termed centerpiece and counterweight, pivoting about the same pivot as the gun and so linked that the centerpiece divides the angle between the gun and the counterweight in a fixed proportion. The mass of these two pieces must be of the order of four times the mass of the gun (unless the pivot can be moved rearward); but this may be not at all impractical, considering the other weights which are necessary in such an installation in any case.

INTRODUCTION

Dr. Kent has pointed out that the drag produced by flexible guns protruding from the fuselage of a bomber is a serious impediment to the performance of the bomber; and that therefore there exists a need for a gun that will pivot about its muzzle, requiring neither a large blister, nor a large opening in the fuselage.

A complete design of such an installation is a very large job that is outside the scope of this paper. Yet it has been possible to make a contribution from the basic theoretical point of view: viz., it has been possible to outline a mechanism which will solve the most important difficulties presented by this problem. This mechanism may form a helpful basis not only for the further work on this problem, but also for commencing a new approach to several other related problems (such as ground-to-air machine gun fire, and the gyroscopic stabilization of guns in tanks).

ANALYSIS OF THE PROBLEM

From the viewpoint of statics, the most obvious requirement is to counteract the preponderance of the gun. In itself, this is hardly a problem. It would be a mistake, however, to disregard the dynamical considerations. A flexible gun (whether manually or automatically controlled) is useless unless it can be operated rapidly and with great accuracy. The circumstances which may impede the operation and ruin the accuracy of the gun are many and involved, and this field of study is just beginning to be explored; yet one thing is certain: their effects are aggravated when the gun is not balanced on its trunnions. It is therefore essential that in a design all such known difficulties be recognized from the start, and that they be solved in a fundamental, possibly radical, way. A development that will achieve satisfactory results will be difficult even with such a foresighted approach; with a mere extension of the presently conventional methods the failure of such a development seems probable.

While the correction of the preponderance of an unbalanced gun must be an essential feature of such a design, it is not the complete answer, since the airplane can generally have transverse - linear and angular - accelerations. The methods of such correction may be divided, crudely, into two classes:

- (1) Methods whereby an attempt is made to control the displacement, or the attitude, of the gun; and such forces are applied, and for such length of time, as will achieve this. An example is any irreversible (e.g., worm gear) elevating mechanism of an unbalanced gun. With such methods transverse linear accelerations of the airplane theoretically do not matter, for they cause the gun to move only parallel to itself; but any transverse angular acceleration of the airplane - except in so far as it may be corrected by a servomechanism - will spoil the aim. Operation is possible only if a fairly large power source, with a high-speed drive, and with a sensitive and nearly perfect servomechanism, is

used. The practicality of the result is clearly dependent upon the characteristics of the servomechanism, and absolute perfection is unattainable. Once such a method of correcting the preponderance is accepted in principle, the problem is clearly defined as one for the specialists in servomechanisms; on our part, however, this method appears to be failing to tackle the problem in a fundamental way, and we shall not consider it further in this paper.

(2) Methods whereby an attempt is made to provide a steady force balancing the preponderance of the gun, but the question of whether the force provided is exact, and whether the attitude of the gun is correct, are left to a separate control mechanism. This is the meaning of "equilibration" in ordnance. The desired (approximate) steady force may be furnished, for instance, by long soft springs; or by cylinders operating under a controlled pressure, or connected to large elastic reservoirs. Then a manual operation is easy. If the force is indeed "steady", the transverse angular accelerations of airplane theoretically leave the attitude of the gun unchanged; but the immunity of the attitude of the gun from the transverse linear accelerations of the pivot now requires also a true balance. It is to this point that the remainder of this paper is devoted.

Conventionally, these methods are combined. Thus, in the gyroscopic stabilization of a tank gun the elements of both these classes are readily discernible (power drive and servomechanism; equilibration and balance).

With a true balance (as distinguished from mere equilibration) the gun is theoretically immune to both linear and angular transverse accelerations. It is necessary to recognize that the balance is even more important than that: what it does, is to reduce the load on all other mechanisms associated with the laying of the gun. We approach this design with the attitude that the lack of balance is a fundamental handicap, and that this balance must be regained.

Whatever the reasons for the disturbances of the gun's attitude from the desired one, some such disturbances will occur. For the most part, they are corrected manually, i.e., by the process of "operating the gun". Yet some of these disturbances can be corrected automatically, by a servomechanism (if the desired attitude is "steady"); this automatic aspect of operation is called stabilization. While a discussion of the principles underlying the functioning of servomechanisms is outside of the scope of this paper, it should be basically recognized that a stabilizing mechanism is amply justified if given only a crude job to do, but must be extremely elaborate and powerful if it must do such a delicate job as to assist in aiming the gun. To give an important example, to the knowledge of this writer, the gyroscopic stabilization of the tank guns still has not achieved such degree of perfection as to make it unnecessary to stop the tank for accurate fire.

The operation of the gun (i.e., aiming) is far from being the simple thing we are wont to imagine it to be. Without going into a discussion

of the number of known problems of operation of a gun,^{*} we propose that our design must take cognizance of this one requirement: All the force applied by the man (or aiming mechanism) should accelerate the gun, and not be absorbed by the trunnions or pivot.

Our suggestion, then, is this: let's leave the delicate job of aiming to the man (or to some separate control mechanism); but let's freely introduce auxiliary servomechanisms (say, for equilibrating and/or stabilizing) if their job is a crude one; and let's require that these crude servomechanisms do not interfere with the controls. Now, it may still be necessary to have an elaborate (sensitive and powerful) control mechanism; but let's require that this mechanism is completely relieved from such extraneous and difficult jobs as might arise out of the necessity to compensate for the lack of balance.

OUTLINE OF SOLUTION IN ONE DEGREE OF FREEDOM

The general idea is to make the gun a part of such a system (all located behind the pivot) that with a transverse acceleration of the pivot the gun will move only parallel to itself, as though its center of gravity were at the pivot.

Let us provide two masses, which we shall call counterweight (1) and centerpiece (2), in addition to the gun (3), all pivoted on the same pivot as the gun, and so linked that the centerpiece always divides the angle between the gun and counterweight in a fixed proportion. Let the centerpiece have its center of percussion, with respect to the pivot, as far from the pivot as permissible; and let the center of percussion of the counterweight be quite close to the pivot. This "triple scissor" arrangement is illustrated schematically in Fig. 1, and those versed in the art will have no difficulty in replacing that sketch by a more practical equivalent design.

For the time being let us disregard gravity. Consider then what happens if the pivot is suddenly forced down (in the plane of Fig. 1). The gun tends to turn (viz., would turn if mounted alone) about its center of percussion, which is some $2/3$ of its length from the pivot. The centerpiece will tend to turn through some smaller angle than the gun; and the counterweight will tend to turn through a larger angle. The linkage then will exert a torque which tends to resist the separation of the centerpiece and counterweight and tends to turn the gun in the opposite direction.

* There has been recently discovered (cf. BRL TN 353) an apparently quite complicated interplay (termed "gyroscopics of tracking") of the psychological (or servomechanism) aspects of handling a gun on a conventional mount with dynamics of the gun. It is natural that we should demand that our design be free from such processes. The existing suggestions for avoiding such processes seem to call for a servomechanism, but only for a crude, "cleaning-up" one. This matter is mentioned here only because it furnishes an added justification for the balance-regaining mechanism here suggested.

In Appendix I the equations of motion of this system are derived, with the natural assumption that the angles involved are small. In Appendix II it is shown that these masses can be so proportioned and located that the gun will only move parallel to itself, as though its center of gravity were at pivot.

Obviously, the "cost" of this system is (in part) the introduction of these masses. The minimum necessary additional mass is given by

$$(m_2 + m_1)/m_3 = 7.71(1 + 1/S)(r_3/r_2),$$

when m 's are the masses, r 's are the distances from the pivot to the centers of gravity of the corresponding masses, and S is the linkage ratio. For instance, if the pivot is $1/8$ of the gun length from the muzzle ($r_3 = 3/8$), if the center of gravity of the centerpiece is at the breech of the gun ($r_2 = 7/8$), and if $S = 5$, the added mass is only 3.97 times the weight of the gun. This is not as prohibitive as it might appear at the first glance: we must reflect that this is only a part of the total weight of gun installation that must also include the weight of the framework, pivot, operator, ammunition, as well as the sighting, equilibrating and control mechanisms, which are necessary in either case. In fact, with this system certain auxiliary mechanisms may be remarkably crude and primitive - which may go a long way toward offsetting this addition of mass. The overriding consideration, of course, is simply that this system makes a certain performance possible that would not have been possible otherwise. This system may find other applications (e.g., tank guns), where the unbalance will be introduced purposely, and the added weight will be negligible.

The system possesses a number of other interesting features. They are discussed, by way of a numerical example, in Appendix III. In that example $S = 3$, the gun is mounted at the muzzle ($r_3/r_2 = 1/2$), and the added weight is $5.142 m_3$.

The most important feature is that the gun in this system is already equilibrated: indeed, this can be readily seen from the fact that the effect of gravity could be produced by accelerating the pivot upward at 32.2 ft/sec^2 . More specifically, if the scissors are held closed in the horizontal position and are suddenly released, it is the counterweight and the centerpiece which will "fall": the gun will remain stationary, being supported, so to say, by an internal recoil of the counterweight and centerpiece. The system in effect diverts the disturbances (imperfections of the equilibrating force, as well as the transverse linear acceleration of the pivot) into a storage (the opening of the scissors), where these disturbances can be counteracted subsequently, at relative leisure.

The scissors are opened not only by the accelerations of the pivot, or by the errors of the equilibrating mechanism, but also in the process of operating (i.e., turning) the gun. It is therefore necessary that some forces act so as to close the scissors; in fact, these forces may also accomplish the equilibration.

Now, the second important advantage of this system is that such external forces (i.e., forces between the system and the airplane frame-

work) may be so applied that they will not affect the motion of the gun. Again, the system acts as a storage for the imperfection of these forces: if these forces are not exactly those desired, the scissors merely open through some larger angle (or overshoot when they are closed), but the gun remains steady.

These forces will be provided by two mechanisms: one is the equilibrating mechanism (essentially a long soft spring, whose pull is divided in a certain definite proportion between the three masses); the other is a servomechanism, which senses the angle of opening of the scissors, and applies, accordingly, some (rapidly varying) force - divided in a definite proportion between the three masses - between the system and the airplane framework. The first mechanism may be simple indeed: its imperfections are taken care of by the second mechanism, and it must be reasonably good merely in order to reduce the load upon the second mechanism. The servomechanism will be slightly more complicated. As far as the generation of the force is concerned, it must be of the kind which attempts to enforce a certain displacement (in this case, the zero angle of opening of the scissors), and which can never be perfect; but as far as the transmission of this (variable) force is concerned, it must be of the kind which transmits a given force essentially independently of the displacement (this is not difficult, an example being an ordinary squirrel-cage motor). Obviously, the function of this mechanism is rather analogous to that of a stabilizing mechanism; however, the important difference is that this scissors-closing mechanism need only be good enough to prevent an excessive hunting of the counterweight and the centerpiece about the stationary gun, as well as to prevent the scissors from opening through angles so large that the mathematics of Appendix I would cease to apply (while a stabilizing mechanism, as conventionally conceived, must be very accurate).

The immunity of this system to the imperfection of its internal (equilibrating and scissors-closing) mechanisms does not extend to the external, control, mechanism. This control mechanism, naturally, must be as good as the contemporary state of art can make it (or, as the accuracy of the gun and of the sighting and predicting mechanisms demand). As far as this mechanism is concerned our system - as here described, so far, in one degree of freedom* - has just this advantage: the control mechanism is relieved from the extraneous job of compensating for the unbalance of the gun.

* It should be noted that at the time of this writing (1953) this one-degree-of-freedom formulation of the problem of operation of a gun (or telescope, radar, etc.) seems to be a standard practice. The consideration of the interplay between the two degrees of freedom (e.g., azimuth and elevation) amounts to the consideration of the gyroscopics of tracking.

Other interesting features in the example of Appendix III are as follows. While the mass, the preponderance and the moment of inertia of the system are increased in ratios of 6.142, 4.269 and 5.133, the gun in operation feels, surprisingly, only 1.58 times as heavy as it would have felt if mounted alone. Since the transverse acceleration of the pivot involves the internal recoil of pieces against each other, and since the force applied at the pivot gives rise to some rotational (as well as translational) kinetic energy, the whole system appears to the airplane structure (at the pivot, as far as vibration of the pivot is concerned) to be 9.06 times as heavy as the gun mounted alone, with its muzzle at the pivot, would have seemed; incidentally, this is 2.35 times as heavy as the gun would have appeared if mounted with its c.g. at the pivot. Therefore, with a soft-spring suspension of the pivot in the airplane framework, the pivot might in effect be quite steady and insensitive to the vibrations of the framework. The angle of the opening of the scissors, which must be sensed by the scissors-closing servo-mechanism, is 3.19 times the angle through which the gun is operated (or, 3.88 times the angle through which the gun would have turned, by the pivot acceleration or by the imperfection of equilibration, if mounted alone); i.e., the controls of this scissors-closing mechanism need be much less sensitive than the controls of the conventional gyro-stabilizing mechanism would have been. Two schemes of dividing the scissors-closing torque between the three pieces are given on Fig. 2; any combination of these two schemes is possible. The equilibrating torque must be divided between pieces 1, 2, and 3 in the ratio .1552/.6095/.2343; but it will accomplish the equilibration (i.e., zero acceleration of all three pieces) only if the total force is exact. The system may accelerate as a rigid unit only if the torque is divided in the certain different ratios (e.g., .0543/.6381/.3076). Thus an error in the total equilibrating force will open the scissors, and therefore will presently be counteracted by the scissors-closing mechanism.

OUTLINE OF THE POSSIBILITIES

The outlined mechanism, it is believed, solves the principal problem of the design in question, viz., the regaining of the balance of a muzzle-mounted gun. It appears that the additional problems remaining are difficult, but not insuperable.

The first one of these is the extension of this mechanism to provide two rotational degrees of freedom to the gun. Obviously, the same counterweight and the same centerpiece will be used both for azimuth and elevation. A very crude sketch of a possible arrangement is given in Fig. 3. The centerpiece will have the form of a large cone, with its apex at the pivot (or slightly in front of the pivot), and with its mass concentrated mainly in its base, as a ring about, and slightly behind, the breech; its diameter must be as large as can be permitted. The counterweight will have the form of a ring surrounding this cone and mounted on the larger end of another (light) coaxial cone on the additional gimbals (for the reasons explained in Appendix II). At the first glance, the business of providing two (and perhaps, three!) rotational degrees of freedom to the gun and two coaxial (and "copical") cones might call for a formidable system of gimbals. In

practice, the situation is much simplified by the fact that the angles within the system are small, and a number of relatively simple arrangements (with flexible members instead of with shafts and bearings) are possible. If the angular rates of the gun are slow and "smooth", and in particular, if the axial component of the angular velocity of the gun is small (as is probably the case in the fire against a pursuing fighter, with automatic controls and with aided tracking), it is probable that the problems of gyroscopics of tracking will not arise. The gyroscopics of tracking, in any case, is outside the scope of this paper; but it would seem wise to give some consideration to that process if the design of such an installation is undertaken.

Needless to say, the design will pose a number of additional problems, and will call for a cooperation of a large number of specialists.

It is desired to emphasize that the kinematic system here outlined may find applicability in a number of problems other than a flexible gun in the fuselage of a bomber.


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APPENDIX I

EQUATIONS OF MOTION

Given three rod-like, originally coincident, rigid bodies; all mounted at the same pivot. Let subscript 1 refer to counterweight, 2 to the centerpiece, 3 to the gun. Let m_j = mass, r_j = distance from the pivot to the center of gravity, I_j = moment of inertia about center of gravity; also, $M_j = m_j r_j$ = preponderance, and $I_j^! = I_j + m_j r_j^2$ = moment of inertia about the pivot. Let a transverse force F act down at the pivot, and let pure couples T_j act on each body. Let the displacement of the pivot be x , and the displacement of each center of gravity, X_j ; let each piece turn through an angle ϕ_j . Introduce internal restraints such that $\phi_1 - \phi_2 = S(\phi_2 - \phi_3)$, S a constant.

Take as the independent variables the generalized coordinates (q's) x , $\delta = \phi_2$ and $\gamma = \phi_2 - \phi_3$, so that

$$\phi_1 = \delta + S\gamma$$

$$\phi_2 = \delta$$

$$\phi_3 = \delta - \gamma$$

If the ϕ 's are sufficiently small,

$$X_1 = x - r_1(\delta + S\gamma)$$

$$X_2 = x - r_2\delta$$

$$X_3 = x - r_3(\delta - \gamma)$$

From

$Fdx + T_1 d(\delta + S\gamma) + T_2 d\delta + T_3 d(\delta - \gamma) = Q_x dx + Q_\delta d\delta + Q_\gamma d\gamma$
we conclude that the generalized forces (Q's) are

$$Q_x = F$$

$$Q_\delta = T_1 + T_2 + T_3$$

$$Q_\gamma = ST_1 - T_3$$

The kinetic energy of the system, $T = \sum (m\dot{x}^2/2 + I\dot{\phi}^2/2)$, is

$$m_1(\dot{x} - r_1\dot{\delta} - r_1S\dot{\gamma})^2/2 + m_2(\dot{x} - r_2\dot{\delta})^2/2 + m_3(\dot{x} - r_3\dot{\delta} + r_3\dot{\gamma})^2/2 + I_1(\dot{\delta} + S\dot{\gamma})^2/2 + I_2\dot{\delta}^2/2 + I_3(\dot{\delta} - \dot{\gamma})^2/2$$

The equations of motion, $(d/dt)(\partial T/\partial \dot{q}_j) - \partial T/\partial q_j = Q_j$, become (upon the simplification of the notation)

$$\begin{aligned}(\sum M)x'' + (M_3 - SM_1)\gamma'' + (-\sum M)\delta'' &= F \\(M_3 - SM_1)x'' + (I_1 S^2 + I_3)\gamma'' + (I_1 S - I_3)\delta'' &= Q_\gamma \\(-\sum M)x'' + (I_1 S - I_3)\gamma'' + (\sum I')\delta'' &= Q_\delta\end{aligned}$$

It is now our object to proportion m_j , r_j , I_j that $\delta_3'' = 0$ when F alone is applied (with $Q_\gamma = Q_\delta = 0$). This means $\gamma'' = \delta''$, i.e., of particular interest are the minors of the first row, second and third column. They are

$$\begin{aligned}\Delta_{12} &= I_1'(M_2 S + M_3(S + 1)) + I_2'(M_3 - SM_1) + I_3'(-M_1(S + 1) - M_2) \\ \text{and } \Delta_{13} &= I_1'(M_2 S(S + 1) + M_3(S + 1)^2) + I_3'(M_1(S + 1) + M_2)\end{aligned}$$

The requirement that $\gamma'' = \delta''$, or that $-\Delta_{12} = \Delta_{13}$, yields

$$I_1'(M_2 S(S + 1) + M_3(S + 1)^2) + I_2'(M_3 - SM_1) = 0$$

It is natural that I_3' here drops out, since with $\delta_3'' = 0$ it no longer matters.

APPENDIX II

MINIMIZATION OF MASS

Taking m_3 and r_3 as units, and putting $I_j = k_j m_j r_j^2$, the requirement of $\theta_3 = 0$ can be put in a form in which the variables (viz., the parameters of the design) are separated:

$$\frac{m_2 r_2^S + S + 1}{k_2 m_2 r_2^2} (S + 1) = \frac{m_1 r_1^S - 1}{k_1 m_1 r_1^2} = E, \text{ say}^*$$

For a given E (viz., for any specified combination of k_2 , m_2 , r_2 and S) the mass of the counterweight

$$m_1 = 1/(r_1^S - E k_1 r_1^2)$$

has a definite minimum, of $4Ek_1/S^2$, at $r_1 = S/2Ek_1$.

On the other hand (for any combination of k_1 , m_1 , r_1 and S) the mass of the centerpiece

$$m_2 = (S + 1)^2 / (Ek_2 r_2^2 - r_2 S(S + 1))$$

is positive only for $r_2 > S(S + 1)/Ek_2$, and to minimize it, r_2 must be as large as the space limitations permit. It is obvious, incidentally, that because of the large r_2 it is the centerpiece that must approach the action of an "immovable body" (or at least, not-turnable body), and it is to it that gyroscopic stabilization, if and when considered, would best be applied. Unfortunately, the possibilities in that connection do not appear good: it will be presently shown that most of the added mass goes into the counterweight.

Letting $r_1 = S/2Ek_1$, and r_2 be the maximum permitted, the added weight becomes a function of the separation constant E :

$$m_1 + m_2 = 4Ek_1/S^2 + (S + 1)^2 / (Ek_2 r_2^2 - r_2 S(S + 1))$$

This function has a minimum, at $E = S(S+1)(1 + 0.5\sqrt{k_2/k_1})/r_2 k_2$, which is

$$\min(m_1 + m_2) = 4(1 + 1/S)(k_1/k_2 + \sqrt{k_1/k_2})/r_2$$

Hence k_1 must be a minimum (viz., 1); i.e., the counterweight must act as a point mass, or a mass freely pivoted at its center of gravity.

* Note that E must be positive.

It follows, incidentally, that the counterweight cannot be replaced by an assembly of light, hard-spinning gyroscopes.

On the other hand, k_2 , r_2 and S must be as large as possible. The requirement r_2 large was obvious before. The requirement of large k_2 (which, incidentally, is in agreement with the concept of the centerpiece as an approach to a not-turnable body), unfortunately, is contradictory to the more important requirement of large r_2 : the mass of the centerpiece is already concentrated as far behind the pivot as permitted; to increase k_2 it is possible only to spread this mass away from the centerline of the centerpiece. The volume limitations on the centerpiece will probably make it difficult to get k_2 much larger than, say, 1.05. In that case the minimum additional mass is

$$7.71(1 + 1/S)/r_2$$

A large linkage ratio S , apparently, improves the leverage with which the counterweight acts upon the gun. Unfortunately, these returns are diminishing, and it apparently would not pay to make S much more than, say, 5.

It can be also shown that the ratio $m_1/m_2 = 2\sqrt{k_1/k_2}(1 + .5\sqrt{k_1/k_2})$ i.e., a little less than 3.

APPENDIX III

PERFORMANCE OF THE SYSTEM

For the sake of simplicity, let us consider a typical numerical example. Let $S = 3$, $k_1 = 1$, $k_2 = 1.05$, $k_3 = 1.35$ (gun much like a uniform thin rod) and $r_2 = 2$ (e.g., the gun is pivoted at the muzzle and the center of gravity of the centerpiece is at the breech of the gun). Applying the formulas of Appendix II, we have the characteristics of the system as follows:

j	m_j	$\% \sum m$	r_j	M_j	$\% \sum M$	k_j	I_j^*	$\% \sum I^*$
1	3.841	62.54	.1736	.667	15.62	1	.116	1.67
2	1.301	21.18	2	2.602	60.95	1.05	5.464	78.85
3	1	16.28	1	1	23.43	1.35	1.35	19.48
	6.142	100.00		4.269	100.00		6.930	100.00

The matrix of Appendix I* is

$$\begin{array}{ccc} 6.142 & - 1.000 & - 4.269 \\ - 1.000 & 2.392 & - 1.003 \\ - 4.269 & - 1.003 & 6.930 \end{array}$$

The solution is

$$\begin{aligned} x'' &= .4260 F + .3068 Q_\gamma + .3068 Q_\delta \\ \gamma'' &= .3068 F + .6660 Q_\gamma + .2854 Q_\delta \\ \delta'' &= .3068 F + .2854 Q_\gamma + .3746 Q_\delta \end{aligned}$$

Transverse acceleration of the pivot. Let $F \neq 0$, $T_1 = T_2 = T_3 = 0$. Then $Q_\gamma = Q_\delta = 0$, and, obviously, $\gamma'' = \delta''$, or $\theta_3'' = 0$; i.e., the gun moves only parallel to itself, as expected.

To visualize the functioning of the system, it is of interest to compute, for each one of the component pieces, the following quantities: the distance from the pivot to the effective center of percussion, $p = x''/\theta''$; the displacement of the center of gravity, $X'' = x'' - r\theta''$; and

* To avoid the annoying truncation errors, the following calculations are based on the exact (not rounded) figures.

the product mX'' . Furthermore, it is of interest to compare these quantities with those which would have been the case if each one of these pieces were mounted separately (i.e., if the linkage were disconnected), with force F_j acting on it at the pivot; in that case $p = I'/M_j$

$\phi'' = F_j p / (I + m(p - r)^2)$; $x'' = p\phi''$, etc. This comparison is as follows:

Piece	x''	ϕ''	p	X''	mX''
1, in system	.4260 F	1.2272 F	.347	.2130 F	.8181 F
2 " "	.4260 F	.3068 F	1.389	-.1876 F	-.2441 F
3 " "	.4260 F	0	infinite	.4260 F	.4260 F
					1.000 F
1, alone	infinite	infinite	.1736	0	0
2 "	16.15 F_2	7.692 F_2	2.100	.769 F_2	1.000 F_2
3 "	3.86 F_3	2.86 F_3	1.35	1.000 F_3	1.000 F_3

As far as airplane frame (at the pivot) is concerned, the effective mass of the system is $1/.4260 = 2.35$ times the mass of the gun; while if the gun were mounted alone with the muzzle in the pivot, its effective mass would have been $1/3.86 = .259$ its real mass. Thus the system feels, at the pivot, $3.86/.4260 = 9.06$ times as heavy (or rather, stiff) as the muzzle-mounted gun alone.

As far as the sensitivity of the scissors-closing servomechanism (which should be compared with the stabilizing mechanism of a conventional installation) is concerned, for the same force applied at the pivot the angle of opening of the scissors is only $1.227/2.86 = .43$ of the angle through which the gun would have turned if mounted alone; this, of course, merely reflects the stiffness of the system at the pivot. More realistic is the comparison of the angles for a given forced displacement of the pivot: this ratio then is $1.35/.347 = 3.89$.

An exact evaluation of the forces of restraint (necessary when the detailed design of the linkage is undertaken), with Lagrangian multipliers, will call for a system of nine equations; but a qualitative understanding of the nature of these forces can be got by the mere inspection of the above table.

With a downward F , the linkage exerts (on Fig. 1) a rightward torque on the gun. Therefore, the gun reaction must tend to lift the assembly of the centerpiece and counterweight. Furthermore, the bottom half of the scissors (the angle $\phi_2 - \phi_3$) is forced to open because the upper half (the angle $\phi_1 - \phi_2$) is being opened; hence, the resistance of the lower half will tend to close the upper half, i.e., the centerpiece and the counterweight are being squeezed by the linkage together. As a result,

the center of gravity of the centerpiece moves up rather than down (note the negative X_2''). The center of gravity of the counterweight, however, moves down rather hard: its acceleration is only half as great as that of the gun (compare X_1'' with X_3''), but because of its great mass this downward acceleration absorbs well-nigh all of the applied force (.8181 F); the increase of this force by the factor 9.06 comes from this downward force on the counterweight, from the increase of the downward force on the gun (to .4260 F) and from the "internal recoil" of the centerpiece (-.2441 F).

The upward motion of the centerpiece, coupled with its rotation, moves its effective center of percussion toward the pivot (from 2.100 to 1.309); while the effective center of percussion of the counterweight moves, because of the downward motion of the counterweight, away from the pivot (from .1736 to .347). As these two centers of percussion move toward each other, the angle $\phi_1 - \phi_2$ decreases; and particularly, with large S, the angle $\phi_2 - \phi_3 = (\phi_1 - \phi_2)/S$ decreases even more. Therein, apparently, lie the limitations of the linkage, as far as the increase of S is concerned.

Stationary Pivot. In the following we shall consider the pivot stationary; i.e., we shall assume a presence of such a reaction F at the pivot that $x'' = 0$. Then the solution of the equations of motion is simplified to

$$\gamma'' = .4445 Q_\gamma + .0644 Q_\delta$$

$$\delta'' = .0644 Q_\gamma + .1538 Q_\delta$$

It will now no longer matter whether the torques T_1 , T_2 and T_3 are applied as pure couples, or as a certain force at a certain arm: for that force can be transferred to the pivot, where it is absorbed in the reaction of the pivot, and only its moment matters.

Balance. Let the applied torques be those of the preponderance of each piece; i.e., let $T_1 = M_1$, $T_2 = M_2$, $T_3 = M_3$ (here we are putting the acceleration of gravity $g = 1$, and are reversing the positive direction of these torques; this, however, will cause no difficulty). Then $Q_\gamma = 3 \times .667 - 1 = 1$, and $Q_\delta = 4.269$. Therefore

$$\gamma'' = .4445 \times 1 + .0644 \times 4.269 = .720$$

$$\delta'' = .0644 \times 1 + .1538 \times 4.269 = .720$$

Thus $\phi_3'' = \delta'' - \gamma''$ is indeed zero, i.e., the gun remains stationary, as expected. It is interesting here to make certain checks and comparisons: viz., to compute, for a number of cases, the following quantities: ϕ'' ; $X'' = r\phi''$; mX'' ; $f = m - mX''$; and also, $I = I' mr^2$ and $fr/I = \phi''$, as a check.

Piece	m	r	M	I'	I	ϕ''	X''	mX''	f
1, alone	3.841	.1736	.667	.116	0	5.76	1	3.841	0
2, "	1.301	2	2.602	5.464	.260	.476	.952	1.239	.062
3, "	1	1	1	1.35	.35	.741	.741	<u>.741</u>	<u>.259</u>
c.g. of 3 pieces		.695				1.364	.948	5.821 +	.321
									= 6.142
1, in system						2.88	.50	1.920	(1.921)
2, " "						.72	1.44	1.873	(-.572)
3, " "						0	0	<u>0</u>	<u>(1)</u>
c.g. of system						.889	.618	3.793 +	2.349
									= 6.142
assembly, locked	6.142	.695	4.269	6.930	3.963	.616	.428	2.629 +	3.513
									= 6.142

Thus, the angle which must be sensed by the scissors-closing mechanism (which can also do the equilibrating) is $(2.88 - 0) / .741 = 3.88$ times as great as it would be in the case of the conventional gyroscopic stabilization. The angular acceleration of the counterweight however, is only half of what it would have had if free (2.88 vs. 5.76). The acceleration of the center of gravity of the system (.618) is less than it would have been without the linkage (.948), and slightly less than of gun alone (.741); but more than it would have been with the system locked, as one rigid body (.428). This is due to the variation of the pivot reaction f, which would have been only .321 without linkage, is 2.349 with the linkage, and would be 3.513 with the linkage locked. It is curious that the centerpiece at the pivot is forced up, rather than down ($f = -.572$).

Since the effect of gravity is pretty much the same thing as the transverse acceleration of the pivot, this table is rather a mere refinement of the preceding one. Thus, we have previously met 2.349 as 2.35, and the ratio of the pivot reactions $2.349 / .259$ is 9.06 as before. In fact, from the expression for x'' (putting it equal to 0), we have the (internal) pivot reaction as $F = (-.3068 Q_v - .3068 Q_g) / .4260 = -.3068 \times 5.269 / .4260 = -3.793$, i.e., a downward force of 3.793 must be applied at the pivot (to keep it from rising) if the preponderances were pure couples; but the preponderances are due to the forces of gravity which, transferred to the pivot, represent a downward force of 6.142; hence the upward reaction of the pivot is $6.142 - 3.793 = 2.349$, as shown in the table.

Equilibration. The fact that the scissors open when T's are proportional to M's should not be confused with the fact that the applied torques must be proportional - in fact, equal and opposite - to M's if the scissors

are to remain closed. We have just considered the gravitational torques as "applied" torques; if we now apply, externally, equal and opposite torques, all T's are zero and the system remains stationary. As the first table of this Appendix shows, the total torque applied to pieces 1,2,3, must be divided in the ratio .1562/.6095/.2343; such a division can be easily accomplished exactly (say by various mobile arrangements). It is more difficult to assure that the total equilibrating force is exact; if it is not (but is divided in the proper proportion), the resultant torques on each piece are in the same proportion; therefore (as we have just seen) the scissors will open, but the gun will not be affected.

Closing of the Scissors. The scheme just described, obviously, is sufficient for closing the scissors: e.g., if the counterweight is falling with respect to the gun, we need merely to increase the total equilibrating force. This scheme, however, is not the only one that can be used by the correcting servomechanism (even though it is the one to which the equilibrating mechanism must try to adhere).

Generally, the requirement that the correction does not disturb the gun amounts to

$$\begin{aligned}\phi_3'' &= \delta'' - \gamma'' (.0644 - .4445)Q_Y + (.1538 - .0644)Q_G \\ &= -.3801 Q_Y + .0894 Q_G = 0,\end{aligned}$$

i.e., to specifying $Q_Y/Q_G = .0894/.3801$. More specifically, this means

$$\begin{aligned}- .3801(3T_1 - T_3) + .0894(T_1 + T_2 + T_3) &= \\ - 1.0509 T_1 + .0894 T_2 + .4695 T_3 &= 0,\end{aligned}$$

of which the ratio $T_1/T_2/T_3 = .1562/.6095/.2343$ is just one out of infinity of solutions. In particular, two simpler alternatives are interesting.

The most obvious one is to apply the correcting torques to counterweight and to centerpiece, but not the gun. Let, then, $T_1 \neq 0$, $T_2 \neq 0$, $T_3 = 0$. Then $T_1/T_2 = .0894/1.0509 = .0851$; i.e., to total correcting torque $T' = T_1 + T_2$ must be so divided that $T_1 = .0851T'/1.0851 = .0784 T'$, and $T_2 = .9216 T'$. It seems curious that most of the correcting torque must be applied to our would-be immovable body: off-hand, it would seem, most of it should be applied to the more conspicuously-moving counterweight!

Since the angle sensed by the correcting mechanism most probably will be the largest angle in the system, viz., $\phi_1 - \phi_3$, the correcting torque can be applied (even though less obviously) to the counterweight

and the gun, but not the centerpiece. Let then, $T_1 \neq 0$, $T_2 = 0$, $T_3 \neq 0$. Then (it is easy to show) the applied torque must be divided in the ratio .3088/0/.6912.

It is of interest to consider also, in this case, the angular acceleration

$$\delta'' = .3470 T_1 + .1538 T_2 - .0394 T_3.$$

If we visualize the division of the total external force accomplished by a sort of mobile (an arrangement of balance beams), this total force is obviously Q_δ . It may well be that that arrangement is preferable which gives a larger δ'' for a given Q_δ . A sample of such a comparison is given below:

$T_1/T_2/T_3$	δ''/Q_δ
.1562/.6095/.2343	.1387
.0784/.9216/ 0	.1689
.3088/ 0 / .6912	.0799,

i.e., of the three schemes described the second one is both the most obvious and the most efficient. It seems an amusing paradox that this ratio (δ''/Q_δ) may be anything, e.g., a finite δ'' may be produced by a negligible total applied force. Indeed, Q_γ is, in a sense, an "internal" torque: the airplane frame feels only Q_δ . The fact of the matter, of course, is that the angle through which this total applied torque (Q_δ) acts is not δ ; it is a proper combination of δ and γ . The various schemes of dividing Q_δ amount simply to a change of leverage of the total equilibrating force - which, indeed, may be a useful degree of freedom in the design.

Operation. To move the gun, the obvious thing to do is to apply all external torque to the gun, and leave the other two pieces "free". Let, then, $T_1 = T_2 = 0$ and $T_3 \neq 0$. Then $Q_\gamma = -T_3$, $Q_\delta = T_3$; $\gamma'' = -.3801 T_3$, $\delta'' = .0894 T_3$; and

$$\phi_1'' = -1.0509 T_3$$

$$\phi_2'' = +0.0894 T_3$$

$$\phi_3'' = +0.4695 T_3$$

Thus, as the gun is moved through an angle ϕ , the counterweight is pushed by the linkage hard in the opposite direction; and the centerpiece moves

slightly in the same direction as the gun (mainly because of the "recoil" of the counterweight). If mounted alone, the gun would have moved at $\phi'' = T_3/1.35 = .7430 T_3$. Thus the gun in this system feels $.7430/.4695 = 1.58$ times as heavy as it would if mounted alone. This increase in inertia might be regrettable if great speed of operation is desired but it might be advantageous if the main problem of the operation of the gun is that of unwanted vibrations (in which case the heavier gun is steadier). This increase seems to be pleasantly small, considering that the mass of the system is increased in the ratio 6.142, the preponderance in the ratio 4.269, and the sum of the moments of inertia, in the ratio $6.93/1.35 = 5.13$.

Obviously, a vector can be constructed which would divide the angle $\phi_1 - \phi_2$ in a certain fixed proportion, and which would not move when the gun is moved. This proportion, in fact, is $.0894/1.0509$, i.e., $.0784/.9216$, that we have met before; i.e., it is the vector to which the correcting, or scissors-closing, torque must be applied if it is not to affect the gun. Similarly, this vector divides the angle $\phi_1 - \phi_3$ in the ratio $.4695/1.0509$, i.e., $.3088/.6912$. The scissors open with the acceleration $\phi_1'' - \phi_3''$, or $1.5304 T_3$, i.e., through an angle $1.5304/.4795 = 3.19$ times as great as the angle through which the gun is operated.

While the total operating torque may be applied to the system in many ways other than in the ratio 0/0/1, it is interesting to show that this most obvious ratio is also the most efficient one under certain natural assumptions: if the motion is started from standstill with the scissors closed, and if the applied torques are, for an element of time, constant, we may require that the kinetic energy imparted to the two auxiliary pieces be minimum, for a given kinetic energy imparted to the gun. The increments of the angles ϕ_j are then proportional to the angular accelerations ϕ_j'' , and the kinetic energy imparted to each piece is proportional to the product $T_j \phi_j''$. It can then be shown that

$$T_3 \phi_3'' - (T_1 \phi_1'' + T_2 \phi_2'') = .4695 T_3^2 - 4.5407 T_1^2 - .1538 T_2^2 - .6940 T_1 T_2,$$

which is maximum at $T_1 = T_2 = 0$.

Conversely, should it be desired to open or close the scissors without regard to the motion of the gun, all external torque should be applied to counterweight.

Also, if all external torque is applied to the centerpiece, the scissors will open: since it is easier to turn the counterweight, the gun will lag, and the counterweight will move ahead.

Locked System. While a motion of the system as a rigid body (e.g., with the linkage locked in the closed-scissors position) might not be absolutely necessary, it is interesting to inspect this motion as a check. Let $Q_5 = 1$. The requirement is that $\gamma'' = 0$, i.e., $Q_\gamma = -.0644/.4445 = -.1449$.

Since $Q_\gamma = 3T_1 - T_3$, this means $T_3 = .1449 + 3T_1$. Again, an infinity of solutions is possible. Of these perhaps the most obvious one is to make each T_j proportional to its I_j ; this results in the ratio .0167/.7885/.1948, and is the only scheme with which the linkage transmits no torque. Another obvious scheme is to consider the applied torque as consisting of a T_3 and such a correcting torque T' (acting according to the scheme .0784/.9216/0, say) as is necessary to counteract the opening of the scissors produced by T_3 ; this results in the ratio .0543/.6381/.3076, and with all such other schemes the linkage exerts certain torques (in effect, the "internal" torque Q_γ) which, however, do no work. These schemes satisfy not only the requirement that $\gamma'' = 0$, but also the requirement that $\delta'' = Q_\delta/6.93$. Some systematization of these (as well as scissors-closing) schemes is given in Fig. 2.

Remarks. Throughout this discussion no mention has been made of the angle of elevation of the gun. Indeed, the direction of the basic line $\phi = 0$ could be quite arbitrary, except for this one refinement: if the angle of elevation θ is large in magnitude, and if the scissors are simultaneously open, the gravitational preponderance of each piece is $M_j \cos(\theta + \phi_j)$; i.e., the ratio of the preponderances is not simply $M_1/M_2/M_3$, but involves also δ and γ . Our neglect of this circumstance amounted to the assumption that either θ is not large, or δ and γ are indeed so small that they are negligible in comparison with θ . If these assumptions are to be rescinded, a perturbation of this theory might be desired; it is possible that under certain circumstances the imperfections of the equilibration might be felt at the gun. These aspects of the problem, however, are minor and overcomable.

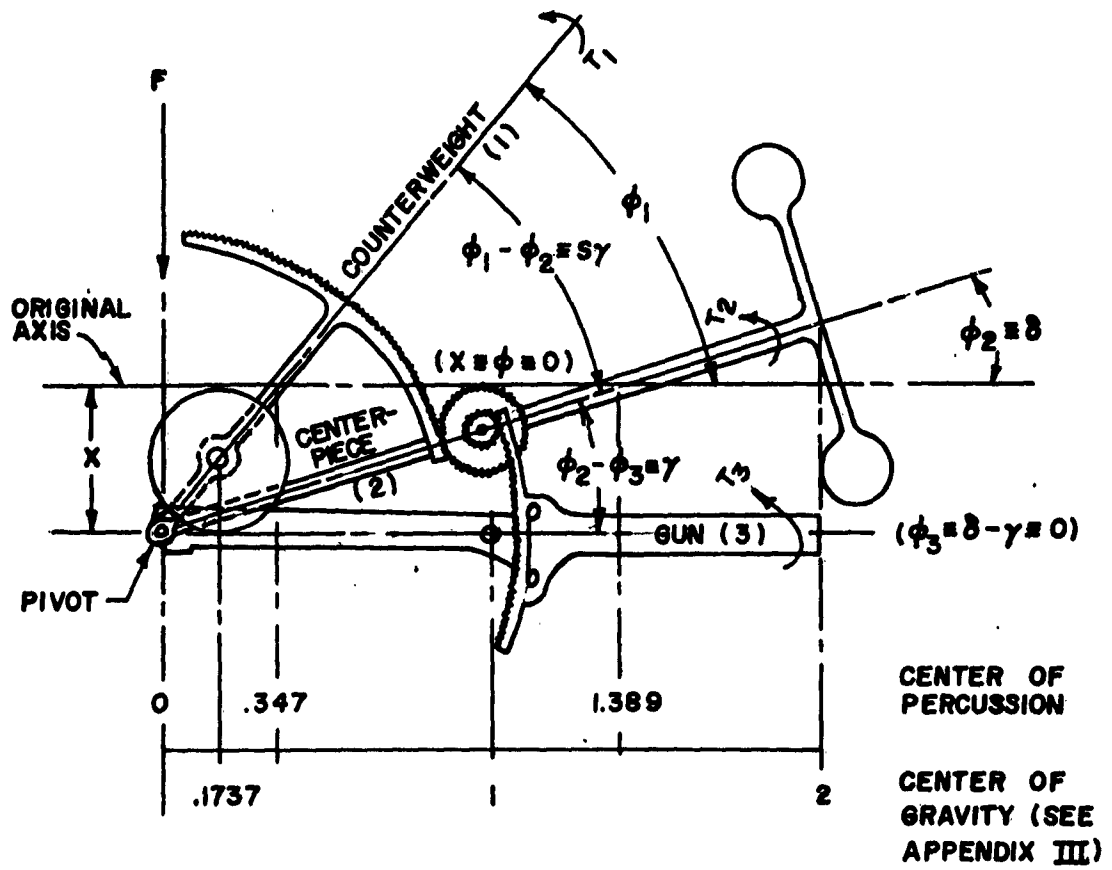
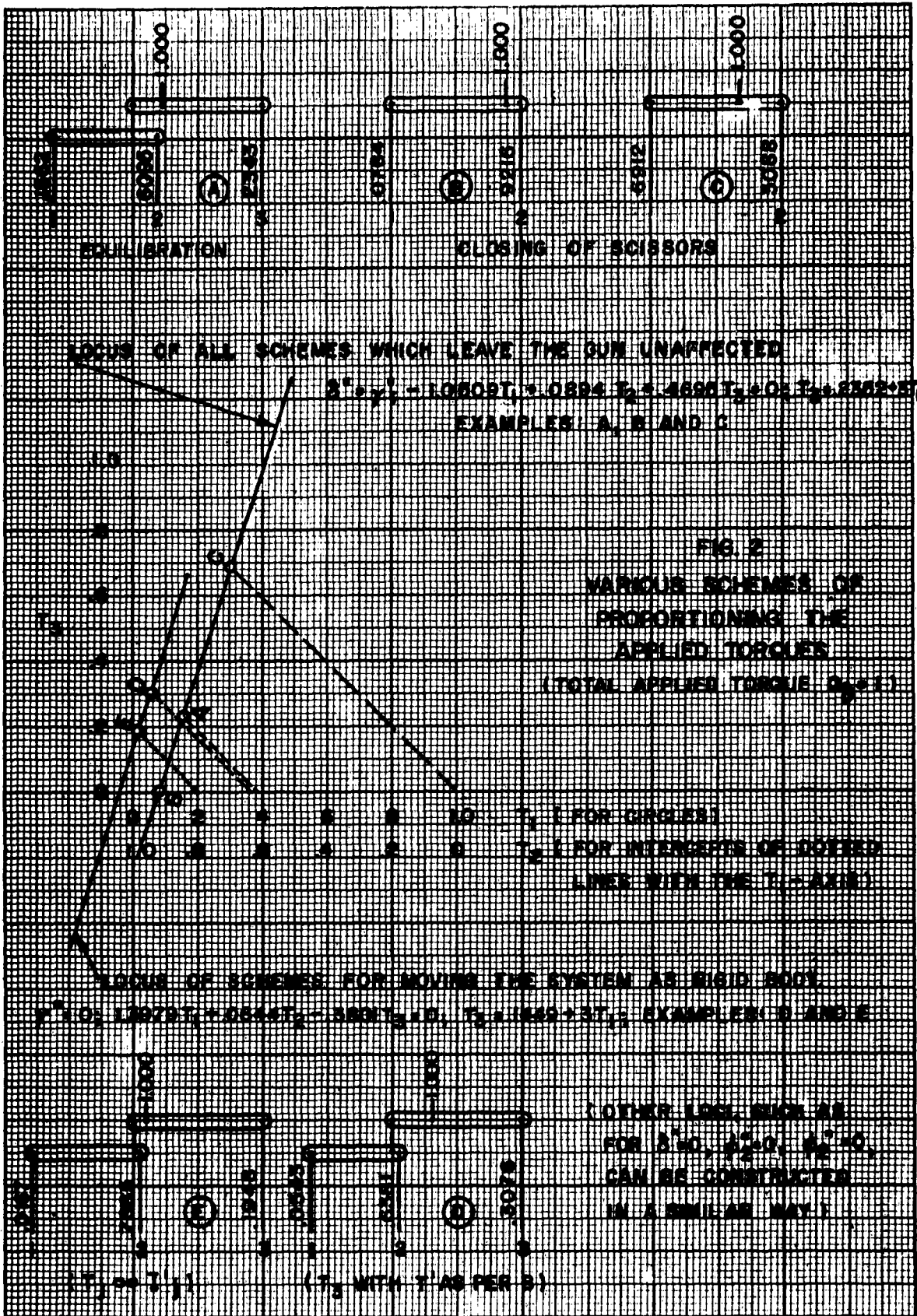
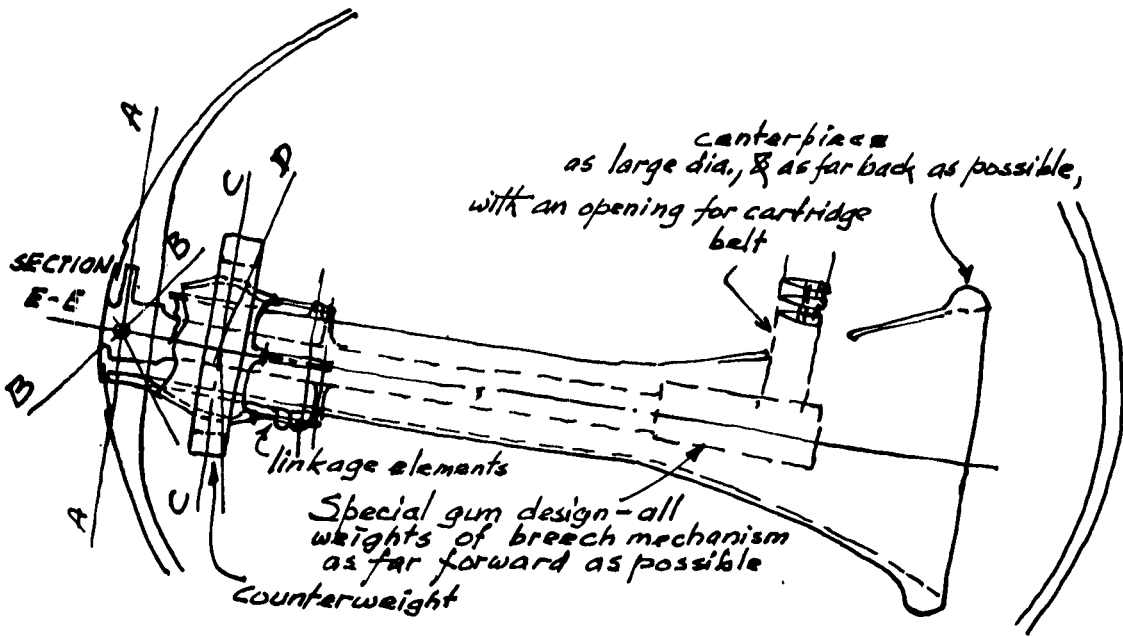


FIG. I

CRUDE SKETCH OF TRIPLE SCISSORS
MECHANISM (ANGLES GROSSLY EXAGGERATED)

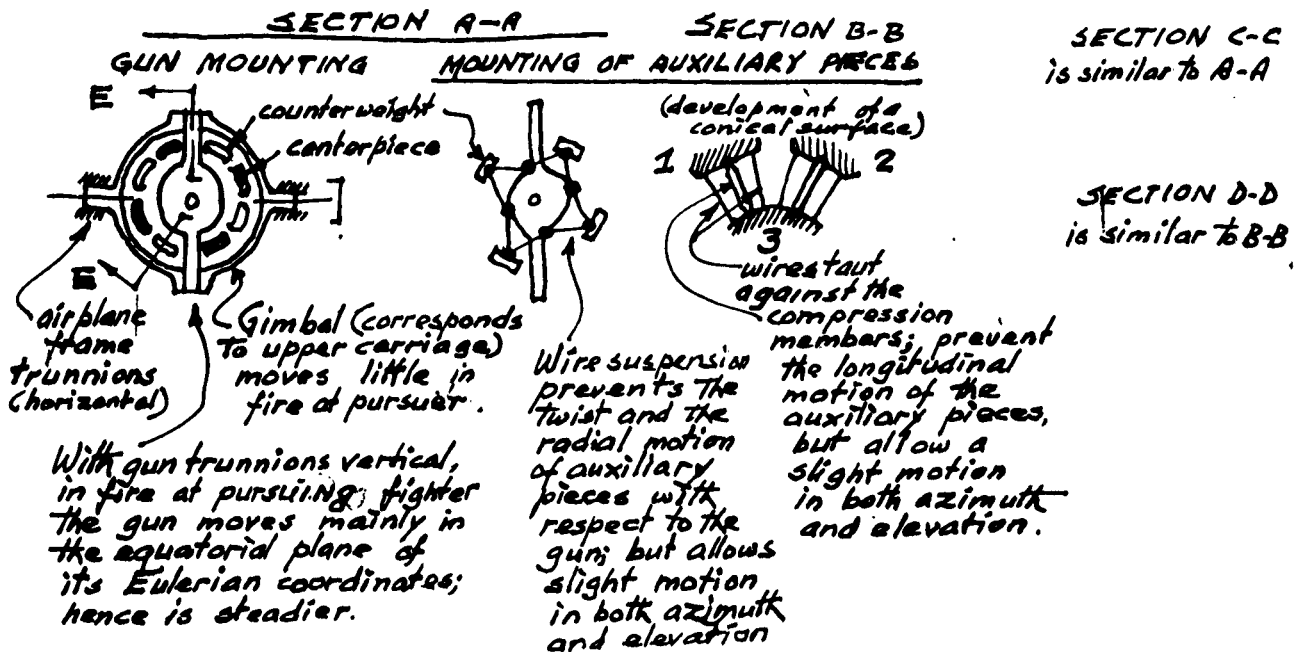




Connections to airplane frame for the equilibrating, scissors-closing and control mechanisms, belt feed, firing mechanism and sight are not shown.

Fig. 3

CRUDE SKETCH OF THE ELEMENTS OF INSTALLATION IN TWO DEGREES OF FREEDOM



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