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Influence of Moisture Conductivity during Electro-Osmosis

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In the studies of electro-osmotic drainage and in the interpretation of laboratory observations of electro-osmotic processes, the influence of moisture conductivity and of thermal moisture conductivity is usually not taken into consideration.[1] Often this can lead to mistaken conclusions, for example, an incorrect estimate of the dependence of the electro-osmotic coefficient on the moisture content. In the general case, the velocity of motion of moisture in a porous medium (soil) is determined from the equation[2-4]

$$\mathbf{v} = -k_p \operatorname{grad} h - k_e \operatorname{grad} \varphi - k_m \operatorname{grad} w - k_t \operatorname{grad} \theta, \qquad (1)$$

where h is the piezometric head, φ is the electric potential, w is the absolute moisture content, θ is the temperature, and k_p, k_e, k_m, and k_t are the coefficients of permeability, electro-osmosis, moisture conductivity, and thermal moisture conductivity, respectively.

Moisture conductivity is connected with the migration of the liquid phase of water under the action of capillary pressure, such as $2\sigma/r$, where σ is the surface tension of water on the air boundary in a model of the system of narrow capillaries of radius r, when there is no artificial hindrance to total wetting. Thermal conductivity of moisture is determined by the displacement of moisture menisci due to the dependence of σ (and therefore of $2\sigma/r$) on the temperature and by the vapor migration resulting from the dependence of the pressure of the saturated vapors on the temperature.[2]

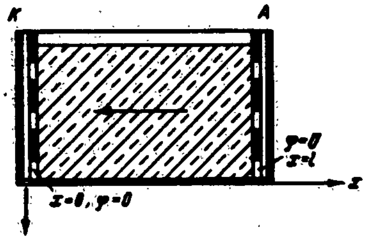

Fig. 1. Experimental diagram for determining electro-osmotic properties of soils. *A*, anode; *K*, cathode. The arrows indicate the direction of the movement of the moisture.

We shall examine the simplest laboratory apparatus for observing electro-osmosis under conditions of varying moisture (Fig. 1). Moist soil is placed between the anode and the cathode, which are flat, perforated plates; the soil is limited by the walls of the bath, which are at right angles to the surface of the electrodes. By the action of the electric field, the moisture in the soil moves from the anode to the cathode, where the excess moisture flows into a measuring vessel; the amount of water indicates the velocity of the motion of the moisture near the cathode. Water can run off only if the moisture content of the soil at the cathode is not less than a certain limiting value $w_{\lim}$.

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If there is no flow of moisture from the anode, the moisture content of the soil is changed and the reduction in the velocity of the displacement of the moisture which is observed in such a case is, to a considerable extent (and even mainly), determined by the presence of forces counteracting moisture conductivity. This fact is not usually taken into account, and the observed decrease in the amount of outflowing liquid is attributed to a decrease of the electro-osmotic effect accompanying the decrease in the average moisture of the soil.

We shall show how the velocity of the motion of moisture must change when the electro-osmotic coefficient is assumed to be constant but when moisture conductivity is taken into consideration.

It is easy to set up the differential equation of the problem by adding to Eq. (1) the equation relating the change in moisture content and the divergence of the flow:

$$-\gamma_0 \frac{\partial w}{\partial t} = \operatorname{div} \mathbf{v}, \tag{2}$$

where γ_0 is the specific weight of completely dry soil.

In our one-dimensional problem (Fig. 1), moisture content and velocity depend on one space coordinate x. In addition, assuming that

$$\frac{\partial h}{\partial x} = \frac{\partial \vartheta}{\partial x} = 0, \quad \operatorname{grad} \varphi = \text{const}, \tag{3}$$

we shall write (2) as

$$\frac{\partial w}{\partial t} = \frac{k_m}{\gamma_0} \frac{\partial^2 w}{\partial x^2}. \tag{4}$$

Its solution $w = w(x, t)$ must satisfy the boundary conditions that are formulated below.

I. We assume that at the initial moment the moisture content is the same everywhere and is equal to w_0, i.e.,

$$w(x, 0) = w_0. \tag{5}$$

II. According to our assumption, there is no flow of moisture to the anode; hence $v = 0$ for $x = l$, or, in view of (1) and (3),

$$\left(\frac{\partial w}{\partial x}\right)_{x=l} = -\frac{k_e}{k_m} E = -D, \tag{6}$$

where E is the intensity of the electric field.

III. Until the moisture content at the cathode ($x = 0$) reaches the limiting value, i.e., until $w(0, t) < w_{\text{lim}}$, the velocity at the cathode is also zero and

$$\left(\frac{\partial w}{\partial x}\right)_{x=0} = -D. \tag{7}$$

IV. If at the instant $t = t_1$ the moisture content of the soil near the cathode reaches the value w_{lim}, then, from this instant on, the boundary condition for $x = 0$ changes and, instead of (7), we have

$$w(0, t) = w_{\text{lim}} \quad \text{when } t \geqslant t_1. \tag{8}$$

The moisture content at the cathode does not exceed the limiting value because the moisture runs off.

If the initial moisture content w_0 is less than $w_{\lim}$, first one must find the solution for the conditions in (5), (6), and (7). It is easy to show the following function satisfies (4) and the conditions stated above:

$$w(x, t) = w_0 + D\left(\frac{l}{2} - x\right) - \frac{4}{\pi^2} Dl \sum_{\nu=1}^{\infty} \frac{\cos\left[(2\nu - 1)\frac{\pi}{l}x\right]}{(2\nu - 1)^2} \exp\left[-(2\nu - 1)^2\theta\right], \tag{9}$$

where

$$\theta = \frac{k_m}{\gamma_0}\frac{\pi^2}{l^2}t. \tag{9a}$$

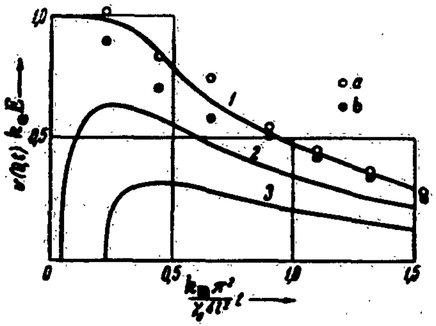

Fig. 2. Graphs showing the dependence of the velocity of water runoff on time. 1, theoretical dependence computed from (18) for $w_0 = w_{\lim}$; 2, theoretical dependence computed from (18) for $w_0 = 0.9w_{\lim}$ and $Dl = 0.6w_{\lim}$; 3, theoretical dependence computed from (18) for $w_0 = 0.8w_{\lim}$ and $Dl = 0.6w_{\lim}$. a, experimental points for $w_0 = w_{\lim}$, $E = 1$ v/cm, $\gamma_0 l^2/k_m = 1100$ min; b, experimental points for $w_0 = w_{\lim}$, $E = 2$ v/cm, $\gamma_0 l^2/k_m = 1100$ min.

Having determined the value $\theta = \theta_1$ $(t = t_1)$, for which

$$w(0, t_1) = w_0 + D\frac{l}{2} - \frac{4}{\pi^2} Dl \sum_{\nu=1}^{\infty} \frac{\exp\left[-(2\nu - 1)^2\theta_1\right]}{(2\nu - 1)^2} = w_{\lim}, \tag{10}$$

we look for a new solution of

$$\overline{w}(x, \overline{t}) \quad \text{when} \quad \overline{t} = t - t_1 \geqslant 0. \tag{11}$$

This solution must satisfy the same basic equation

$$\frac{\partial \overline{w}}{\partial \overline{t}} = \frac{k_m}{\gamma_0}\frac{\partial^2 \overline{w}}{\partial x^2}, \tag{12}$$

the boundary conditions (6) and (8), namely

$$\left(\frac{\partial \overline{w}}{\partial x}\right)_{x=l} = -D, \tag{13}$$

$$\overline{w}(0, \overline{t}) = w_{\lim}, \tag{14}$$

and the initial condition

$$\overline{w}(x, 0) = w(x, t_1). \tag{15}$$

This new solution has the form

$$\overline{w}(x, \overline{t}) = w_{lim} - Dx + \sum_{\mu=1}^{\infty} R_{\mu} \sin\left[(2\mu - 1)\frac{\pi}{2l}x\right] \exp\left[-(2\mu - 1)^2 \overline{\theta}\right], \quad (16)$$

where

$$\overline{\theta} = \frac{k_m}{\gamma_0} \frac{\pi^2}{4l^2} (t - t_1). \quad (16a)$$

The coefficients R_{μ} are determined by the equation

$$R_{\mu} = \frac{4(w_0 - w_{lim}) + 2Dl}{\pi(2\mu - 1)} +$$

$$+ \frac{16}{\pi^2} Dl(2\mu - 1) \sum_{\nu=1}^{\infty} \frac{\exp\left[-(2\nu - 1)^2 \theta_1\right]}{\left[4(2\nu - 1)^2 - (2\mu - 1)^2\right](2\nu - 1)^2}. \quad (17)$$

When we know $\overline{w}(x, t)$, it is easy to find from (1), under the conditions (3), the velocity of flow of moisture from the cathode:

$$v(0, t) = -v_x = k_e E + k_m \left(\frac{\partial \overline{w}}{\partial x}\right)_{x=0} =$$

$$= \frac{\pi}{2l} k_m \sum_{\mu=1}^{\infty} (2\mu - 1) R_{\mu} \exp\left[-(2\mu - 1)^2 \overline{\theta}\right]. \quad (18)$$

Fig. 2 shows the $v(0, t)$ curves for various values of w_0/w_{lim}.

We have disregarded above the thermal moisture conductivity ($-k_t \operatorname{grad} \theta$). However, this term can have a noticeable effect at high current densities. In some experiments, for instance, a maximum decrease of moisture is observed in the central region between the cathode and the anode; this effect is easily explained by the fact that in the same region there is observed a temperature maximum because of the increase in heat transmission through the surfaces of the electrodes.

In other cases (for instance, in reference 5) the heating of the soil near the cathode results in a greater drying of the soil near the cathode than in the soil farther from the electrodes, despite the action of electro-osmosis. Casagrande[5] does not give the correct interpretation of the phenomenon he observed.

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[1] *Iskusstvennoye zakrepleniye gruntov* [Artificial Stabilization of the Soil], Sbornik No. 17, 1952.

[2] A. V. Lykov, *Teoria sushki* [Theory of Drying], 1950.

[3] A. V. Netushil, Elektrichestvo, No. 8 (1952).

[4] Polivanov, Netushil, Burdak, and Kuzmenko, Elektrichestvo, No. 8 (1951).

[5] L. Casagrande, J. Boston Soc. Civil Engrs., 39, No. 1, 51 (1952).

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