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Self-Energy Effects on Meson - Nucleon
Scattering According to the
Tamm-Dancoff Method[*] [**]

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Abstract

The lowest order Tamm-Dancoff equations for the meson-nucleon system
are derived by the method of Cini, using the wave-functions defined
by Dyson. The contributions of the meson and nucleon self-energies
to the kernel of the momentum-space integral equation are renormalized.
Their effects are absorbed into the coupling G^2, making it momentum
and energy dependent. The effective coupling turns out to exhibit an
anomalous behavior, having a pole for a space-like momentum, to which
it is difficult to attach a sensible physical interpretation.

I. Introduction

Calculations of the meson-nucleon scattering phase shifts have
recently been performed by the Tamm-Dancoff Method.[1] This paper is
essentially a continuation of one in which that undertaking is re-
ported. The lowest-order Tamm-Dancoff equations were there derived
and solved approximately by numerical methods, with several of the
terms in the kernel, namely, those which correspond to the nucleon,
meson, and vacuum self-energies, and some which occur only in the
$I = 1/2$, $J = 1/2$ state, simply dropped. It is the purpose of the
present paper to ascertain the effects of the former omission on the
scattering phase shifts; in other words to find the renormalization
corrections to the scattering in states in which either I or J, or
both, are different from $1/2$.

One of the omitted terms, the vacuum self-energy, may be eliminated
by a simple redefinition of the Tamm-Dancoff amplitudes, as was shown by
Dyson.[2] In addition, this redefinition makes possible a consistent
relativistic renormalization of mass and charge.[3] In order to perform

(1) M. Fubini Nuovo Cimento 10, 564; F. Dyson, S. Schweber, and
W. Visscher Phys. Rev. 90 372.
(2) F. J. Dyson Phys. Rev. 90 994 (1953). We shall call Dyson's
modification of the Tamm-Dancoff theory the DTD theory.
(3) F. J. Dyson Phys. Rev. 91 421, 1543 (1953)

a reliably unambiguous separation of the divergent parts of the self-energy integrals, it is essential to write them in a covariant form. For this purpose we use the method of Cini [4], applied to the DTD theory.

In part III we reduce the equations derived in part II to a form similar to that obtained formerly [1]. They turn out to be, in fact, identical, except for some of the self-energy terms, and one of the terms that occurrs only in the $I = 1/2$, $J = 1/2$ state. Thus the self-energy corrections which will be derived in Part IV may be applied to the phase shifts calculated previously. [1] The results are discussed in part V.

II. The Cini method applied to the DTD equations.

We define a two-particle amplitude

$$\phi_{\rho\alpha}(x, y; t) = \left(\Psi_0^*(t)\, \psi_\rho(x)\, \phi_\alpha(y)\, \Psi(t) \right) \tag{1}$$

where $\Psi_0(t)$ is the interacting vacuum state in the interaction representation, $\Psi(t)$ is the real state-vector in the interaction representation, and $\psi_\rho(x)$ and $\phi_\alpha(y)$ are interaction representation operators for the nucleon and meson fields, respectively. x and y need not be on the space-like surface t. $\Psi(t)$ and $\Psi_0(t)$ satisfy the Schrödinger equation

$$i \frac{d\Psi(t)}{dt} = H_I(t)\, \Psi(t) \tag{2}$$

It is a consequence of equations 1 and 2 that

$$\phi_{\rho\alpha}(x, y; t) - \phi_{\rho\alpha}(x, y; -\infty) = - \int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt''$$

$$\left(\Psi_0^*(t'')\, \left[H_I(t''), \left[H_I(t'), \psi_\rho(x)\, \phi_\alpha(y) \right] \right]\, \Psi(t'') \right) \tag{3}$$

[4] M. Cini. Nuovo Cimento 10 526 (1953)

if one requires that the initial state $\underline{\psi}(-\infty)$ be a state with one meson present.

We choose equation (3) as the fundamental equation for our system, because it will yield results in a simple way which are equivalent to the lowest-order TD approximation. The Tamm-Dancoff approximation as applied to equation (3) may be stated in the following manner. One arranges the operators occurring in the integrand in the normal order, (absorption operators to the right of emission operators) yielding a sum of terms of the general form of ϕ (x,y;t), possibly containing more than two field operators. We then drop all the terms except those having exactly the form of ϕ (x,y;t). That this procedure is equivalent to a Tamm-Dancoff approximation may be seen in the following way. A diagram is drawn in figure 1 which is a convenient way to enumerate the non-zero matrix elements of the interaction Hamiltonian H_1. Since H_1 contains two nucleon field operators, and one meson operator, the non-zero elements are only those between states which differ by zero or two nucleons, and by one meson. The lines in Fig. 1 represent matrix elements which connect the "ground state" (1 meson, 1 nucleon in state ψ in excess of those in ψ_0) to other states. It is clear that our approximation is equivalent to the Tamm-Dancoff approximation which restricts the allowed states of the system to the solid circles in Fig. 1.

When the above procedure has been carried out, we shall be left with an integral equation for $\phi_{\rho\alpha}$ (x,y,t) similar in appearance to the lowest order Bethe-Salpeter equation for the meson-nucleon system, with the important difference that our field operators are in the interaction representation, and their time-dependence is known.

Inserting $H_I(x_o) = \int d^3x \ H_I(x)$

$$H_I(x) = i G \ \bar{\psi}(x) \gamma_5 \tau_\alpha \phi_\alpha(x) \psi(x)$$

(4)

into Eq. (3), we obtain

$$\phi_{\rho\alpha}(x, y; t) - \phi_{\rho\alpha}(x, y, -\infty) = G^2 \int_{-\infty}^{t} dz \int_{-\infty}^{z_o} dw$$

(5)

$$\left(\psi_\sigma^*(w_o) \left[\bar{\psi}(w) \gamma_5 \tau_\delta \phi_\delta(w) \psi(w), \left[\bar{\psi}(z) \gamma_5 \tau_\delta \phi_\delta(z) \psi(z), \psi_\rho(x) \phi_\alpha(y) \right] \right] \psi(w_o) \right)$$

(here $\int_{-\infty}^{t} dz$ means $\int_{-\infty}^{t} dz_o \int d^3z$)

The double commutator in (5) may, without much difficulty, be put into normal order; if one retains only those terms which correspond to a one-meson, one-nucleon amplitude, it becomes

$$-3i \ S(x-z) \gamma_5 (1 + \varepsilon(z-w)P) \left[\langle \phi(w)\phi(z) \rangle_0 \langle \bar{\psi}(w) \psi(z) \rangle_0 \right] \gamma_5 \phi_\alpha(y) \psi(w) \quad (6(a))$$

$$-2i \ \Delta(y-z) (1-P) S_\rho \left\{ \gamma_5 \langle \bar{\psi}(w) \psi(z) \rangle_0 \gamma_5 \langle \psi(w) \bar{\psi}(z) \rangle_0 \right\} \phi_\alpha(w') \psi_\rho(x) \quad (6(b))$$

$$\left\{ \langle \phi(z) \phi(y) \rangle_0 \langle \bar{\psi}(z) \psi(x) \rangle_0 + \langle \phi(y) \phi(z) \rangle_0 \langle \psi(x) \bar{\psi}(z) \rangle_0 \right\}$$

$$\times \gamma_5 (1 + \varepsilon(z-w)P) \langle \bar{\psi}(w) \psi(z) \rangle_0 \gamma_5 \tau_\alpha \tau_\delta \phi_\beta(w) \psi(w) \quad (6(c))$$

$$i \ \Delta(y-z) \left\{ \langle \bar{\psi}(w) \psi(z) \rangle_0 \gamma_5 \langle \psi(w) \bar{\psi}(z) \rangle - \langle \psi(z) \bar{\psi}(w) \rangle_0 \gamma_5 \langle \bar{\psi}(z) \psi(w) \rangle_0 \right\}$$

$$\times \gamma_5 \tau_\delta \tau_\alpha \phi_\beta(w) \psi(z) \quad (6(d))$$

$$-i \ S(x-z) \left\{ \langle \phi(w) \phi(y) \rangle_0 \gamma_5 \langle \bar{\psi}(w) \psi(z) \rangle_0 + \langle \phi(y) \phi(w) \rangle_0 \gamma_5 \langle \psi(z) \bar{\psi}(w) \rangle_0 \right\}$$

$$\times \gamma_5 \tau_\delta \tau_\alpha \phi_\beta(z) \psi(w) \quad (6(e))$$

By application of the usual rules for drawing the Feynman diagram corresponding to a given S-matrix element, the terms (6(a)) -(6(e)) may be seen to correspond to the time-ordered graphs drawn in Fig. 2.

\

It will be noticed that the graphs (a), (b), and (c) occur alone, without their negative-energy intermediate state analogs, and that the vacuum self-energy fails to appear at all, in contrast to what one finds with the old Tamm-Dancoff method. The reason that these terms do not appear is that the structure of the commutator in Eq. (5) is such that any Feynman diagram representing parts of it must have a line connecting z with x or y, or both.

If we next insert the propagation functions defined by[5]

$$P\langle \phi(x) \phi(y) \rangle_0 = \frac{1}{2} \Delta_F (x-y)$$

$$\varepsilon(x-y) P\langle \psi(x) \overline{\psi}(y) \rangle_0 = -\frac{1}{2} S_F (x-y)$$

$$\langle \psi(z) \overline{\psi}(y) \rangle_0 = -i S^T (z-y)$$
$$= -i S_A^T (z-y) \qquad z_0 < y_0$$
$$= +i S_R^T (z-y) \qquad z_0 > y_0$$

$$\langle \overline{\psi}(y) \psi(z) \rangle_0 = -i S_A^- (z-y) \qquad z_0 < y_0$$
$$= +i S_R^- (z-y) \qquad z_0 > y_0$$

$$\langle \phi(z) \phi(y) \rangle_0 = i \Delta_A^+ (z-y) \qquad z_0 < y_0$$
$$= -i \Delta_R^+ (z-y) \qquad z_0 > y_0$$

$$\langle \phi(y) \phi(z) \rangle_0 = -i \Delta_A^- (z-y) \qquad z_0 < y_0$$
$$= +i \Delta_R^- (z-y) \qquad z_0 > y_0$$

(7)

for the vacuum expectation values in 6(d) and 6(e) which depend on field operators at w and z, we may extend the integration over w in (5) to all space-time, because 6(a), 6(b), and 6(c) are already zero when $z_0 < w_0$. The result

of this substitution is that (5) becomes

$$\phi_\alpha(x,y;t) - \phi_\alpha(x,y;-\infty) = i G^2 \int_{-\infty}^{t} dz \int_{-\infty}^{\infty} dw$$

$$\left\{ \left[3\, S(x-z)\, \gamma_s\, \Delta^-(z-w)\, S(z-w) + \frac{3}{4}\, S(x-z)\, \gamma_s\, \Delta_F(z-w)\, S_F(z-w)\, \gamma_s \right] \phi_\alpha(w,y,w_0) \right.$$

$$+ \Delta(y-z)\, S_P \left\{ 2\gamma_s\, S(z-w)\, \gamma_s\, S^+(w-z) \right.$$

$$\left. + \frac{1}{2}\, \gamma_s\, S_F(z-w)\, \gamma_s\, S_F(w-z) \right\} \phi_\alpha(x,w;w_0)$$

$$+ \frac{i}{2}\left\{ -\Delta(y-z)\, S(x-z) + \Delta^+(y-z)\, S^+(x-z) \right\} \gamma_s$$

$$\times \left\{ 2i\, S^-(z-w) + S_F(z-w) \right\} \gamma_s\, \tau_\alpha \tau_\beta\, \phi_\beta(w,w,w_0) \tag{8}$$

$$+ \Delta(y-z)\left\{ -S(x-w)\, \gamma_s\, S_A^+(w-z) + S^+(x-w)\, \gamma_s\, S_A^-(w-z) \right\}$$

$$\times \gamma_s\, \tau_\beta \tau_\alpha\, \phi_\beta(z,w,w_0)$$

$$+ S(x-z)\left\{ -\Delta(y-w)\, \gamma_s\, S_R^-(z-w) + \Delta^+(y-w)\, \gamma_s\, S_R^+(z-w) \right\}$$

$$\left. \times \gamma_s\, \tau_\beta \tau_\alpha\, \phi_\beta(w,z,w_0) \right\}$$

III. Reduction to the non-covariant form.

It will be noticed from the form of $\phi(x,y,t)$ that it is non-zero in four different physical situations[3]. Specifically, decomposing ψ and ϕ into their positive and negative energy parts,

$$\phi_{\rho\alpha}(x,y;t) = \phi^{++} + \phi^{+-} + \phi^{-+} + \phi_{\rho\alpha}^{--}(x,y;t) , \tag{9}$$

where, for example,

$$\phi_{o\alpha}^{-+}(x, y; t) = \left(\bar{\Psi}_0^*(t) \, \phi_\alpha(x) \, \psi_\beta^+(y) \, \Psi^+(t) \right).$$

This decomposition may be done explicitly here because the time dependence of the interaction representation operators ϕ and ψ is known. A non-zero ϕ^{++} corresponds to having a meson and a nucleon in the state Ψ in excess of those in Ψ_0; ϕ^{-+} to one more nucleon and one less meson; ϕ^{+-} to one more meson and one less nucleon, ϕ^{--} to one less meson and one less nucleon in Ψ than Ψ_0. We will now operate from the left on Eq. (9) with a positive energy projection operator for both nucleons and mesons, yielding a somewhat simplified equation:

$$\phi_\alpha^{++}(x, y; t) - \phi_\alpha^{++}(x, y, -\infty) = i G^2 \int_{-\infty}^{t} dz \int d\xi$$

$$\left\{ \left[3 S^+(x - z) \gamma_5 \Delta^-(\xi) S(\xi) \gamma_5 + \frac{5}{4} S^+(x - z) \Delta_F(\xi) \gamma_5 S_F(\xi) \gamma_5 \right] \right.$$

$$\times \left[\phi_\alpha^{++} + \phi_\alpha^{-+} \right](z - \xi, y; z_0 - \xi_0) + \Delta^+(y - z) S_P \left\{ 2 \gamma_5 S(\xi) \gamma_5 S(-\xi) \right.$$

$$\left. - \frac{1}{2} \gamma_5 S_F(\xi) \gamma_5 S_F(-\xi) \right\} \left[\phi_\alpha^{++} + \phi_\alpha^{+-} \right](x, z - \xi, z_0 - \xi_0)$$

$$+ \frac{i}{2} \Delta^+(y - z) S^+(x - z) \gamma_5 \left(2 \, S(\xi) + S_F(\xi) \right) \gamma_5 \tau_\alpha \tau_\beta \, \phi_\beta(z - \xi, z - \xi, z_0 - \xi_0)$$

$$+ \Delta^+(y - z) S^+(x - z + \xi) \gamma_5 S_A(-\xi) \gamma_5 \tau_\beta \tau_\alpha \, \phi_\beta(z, z - \xi, z_0 - \xi_0)$$

$$+ S^+(x - z) \Delta^+(y - z + \xi) \gamma_5 S_R^+(\xi) \gamma_5 \tau_\beta \tau_\alpha \, \phi_\beta(z - \xi, z, z_0 - \xi_0) \right\}$$

(10)

This is exactly the equation which would have been obtained if we had started with ϕ^{++} in equation 3. We will next drop all terms on the right-hand side of eq. (10) which contain either meson or nucleon negative-energy amplitudes. This is consistent with the Tamm-Dancoff approximation; the dropped components of the wave-function are not among those connected on figure 1 to the ground state.

The terms in eq. (10) containing $\Delta(\vec{q}) S(\vec{q})$, $S(\vec{q}) S(-\vec{q})$ and $S(\vec{q})$ will drop out in the integration over $\vec{q}_0$.

After dropping the negative-energy amplitudes, we take the Fourier transform of equation (10). The expansion of ϕ^{++} is found to be

$$\phi^{++}(x, y ; t) = \int \frac{d^3q \, d^3r}{(\omega_q \, E_r)^{1/2}} e^{i r \cdot x + i \vec{q} \cdot \vec{y}}$$

$$\times \sum_{u_+} u \, a(r u, q \alpha ; t) \tag{11}$$

where $r \cdot x = \vec{r} \cdot \vec{x} - r_0 x_0$, $E_r = (M^2 + r^2)^{1/2}$, $\omega_q = (\mu^2 + q^2)^{1/2}$

(M and μ are the masses of the nucleon and meson), and

$a(r u, q \alpha ; t) = (\overline{\psi}_0(t) b_{r u} a_{q \alpha} \psi(t))$. Here

$b_{r u}$ and $a_{q \alpha}$ are annihilation operators for the nucleon and meson, respectively. Inserting the time dependence of the state vectors,

$$\overline{\psi}(t) = e^{-i(E - H_0)t} \psi(0)$$
$$\overline{\psi}_0(t) = e^{-i(E_0 - H_0)t} \psi_0(0) \tag{13}$$

where H_0 is the no-interaction Hamiltonian, E is the total energy of the system, and E_0 is the vacuum state energy, equation 12 becomes ($\varepsilon = E - E_0$)

$$a(r u, q \alpha, t) = e^{-i(\varepsilon - E_r - \omega_q)t} a(r u, q \alpha) \tag{14}$$

where $a(r u, q \alpha) = (\psi_0^*(0) b_{r u} a_{q \alpha} \psi(0))$.

With this notation, equation (10), written in momentum space in the center-of-mass system becomes ($p =$ momentum of nucleon ; $\Delta = E - E_p - \omega_r$)

$$\Delta o(p u_i, p\alpha) = -\frac{3}{4} i G^2 \frac{M}{E_p} \int d^4\xi \left(\overline{u}(p) \gamma_5 S_F(\xi) \gamma_5 u(p)\right) \Delta_F(\xi)$$
$$\times e^{-i\vec{p}\cdot\vec{\xi} + i(\Delta + E_p)\xi_0} \quad a(p u_i, p\alpha)$$

$$-\frac{1}{4} i G^2 \frac{1}{\omega_p} \int d^4\xi \, S_p \left\{ \gamma_5 S_F(\xi) \gamma_5 S_F(-\xi) \right\} e^{i\vec{p}\cdot\vec{\xi} + i(\Delta + \omega_p)\xi_0} \quad a(p u_i, -$$

$$+ \frac{M^2 G^2}{16\pi^3} \int \frac{d^3 k}{(E_p \omega_p E_k \omega_k)^{1/2}} \left\{ \left[\frac{(\overline{u}(p) \gamma_5 \Lambda_+(0) \gamma_5 v(k))}{M(M - \varepsilon)} \right. \right.$$

$$+ \frac{(\overline{u}(p) \gamma_5 \Lambda_-(0) \gamma_5 v(k))}{M(M + \varepsilon)} \right] \tau_\alpha \tau_\beta + \left[\frac{(\overline{u}(p) \gamma_5 \Lambda(-p-k) \gamma_5 v(k))}{E_{p+k}(E_p + E_{p+k} + E_k - \varepsilon)} \right. \qquad (15)$$

$$+ \frac{(\overline{u}(p) \gamma_5 \Lambda_-(p+k) \gamma_5 v(k))}{E_{p+k}(E_{p+k} + \omega_k + \omega_p - \varepsilon)} \right] \tau_\beta \tau_\alpha \right\} a(k v_i, -k\beta)$$

Here we have, for convenience in the next section, left the self-energy parts in terms of invariant functions. Except for these terms, and for the terms involving the energy denominator $1/M + \varepsilon$, which contributes only to the states for which $I = J = \frac{1}{2}$, this equation is identical to that derived by the old Tamm-Dancoff method [1]. We may therefore evaluate these self-energy parts, and use them to find corrections to the phase shifts previously calculated, except for the $I = J = \frac{1}{2}$ states.

IV. Renormalization.

Let
$$\Omega(\xi) = \gamma_5 S_F(\xi) \gamma_5 \Delta_F(\xi)$$
$$\qquad (16)$$

and
$$\sum(\xi) = S_p_{ur} \left\{ \gamma_5 S_F(\xi) \gamma_5 S_F(-\xi) \right\}$$

These are the well-known nucleon and meson self-energy propagation factors from second-order perturbation theory whose strong singularity at $\xi = 0$ causes the divergence of their momentum-space integrals. We shall follow the usual relativistic subtraction procedures to extract their finite parts,

We take the 4-dimensional fourier transform of Ω and Σ :

$$\Omega(\rho) = \int e^{-i\rho\xi} \, \Omega(\xi) \, d^4\xi$$

$$\Sigma(\kappa) = \int e^{-i\kappa\cdot\xi} \, \Sigma(\xi) \, d^4\xi \tag{17}$$

These are now relativistically invariant functions of the 4-vector momenta ρ and κ. More specifically, $\Omega(\rho)$ is a function of $\rho_\mu\gamma_\mu$ only, since $\rho_\mu\gamma_\mu$ is the simplest invariant matrix function of ρ. Similarly $\Sigma(\kappa)$ is a function of κ^2 only. These facts may be easily verified by explicit calculation.

In the usual way, [6] we set the observable (renormalized) parts of the self-energy operators equal to the following:

$$\Omega_R(\rho\cdot\gamma) = \Omega(\rho\cdot\gamma) - \Omega(iM) - (\rho\cdot\gamma - iM)\left[\frac{\partial\Omega}{\partial(\rho\cdot\gamma)}\right]_{\rho\cdot\gamma = iM} \tag{18}$$

$$\Sigma_R(\kappa^2) = \Sigma(\kappa^2) - \Sigma(-\mu^2) - (\kappa^2 + \mu^2)\left[\frac{\partial\Sigma}{\partial\kappa^2}\right]_{\kappa^2 = -\mu^2} \tag{19}$$

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(6) P.T. Matthews Phil. Mag. 41 185 (1950)

NUCLEON SELF-ENERGY RENORMALIZATION

The subtractions indicated in (19) may be carried out by following exactly the procedure of Karplus and Kroll[7] for the electron self-energy.

$$\Omega(p\cdot\gamma) = \int e^{-ip\cdot\xi}\, \gamma_5\, S_F(\xi)\, \gamma_5\, \Delta_F(\xi)\, d^4\xi$$

$$= -\frac{4i}{(2\pi)^4}\int d^4s\,\gamma_5\,\frac{s\cdot\gamma + p\cdot\gamma + iM}{(p+s)^2 + M^2}\,\gamma_5\,\frac{1}{s^2+\mu^2} \tag{20}$$

We use

$$\frac{1}{ab} = \int_0^1 \frac{du}{[av - b(1-u)]^2} \tag{21}$$

to get

$$\Omega(p\cdot\gamma) = \frac{4i}{(2\pi)^4}\int d^4s\int_0^1 du\,\frac{s\cdot\gamma + p\cdot\gamma - iM}{\{[(p+s)^2+M^2]u + [s^2+\mu^2](1-u)\}^2} \tag{22}$$

The denominator of the integrand may be written $\left[(s+pu)^2 + \Lambda^2\right]^2$ where $\Lambda^2 = p^2(u-u^2) + M^2u + \mu^2(1-u)$.

We expand the integrand in a power series in pu; keeping only the first two terms, because the higher ones may be transformed into vanishing surface integrals. This casts (22) into the form

$$\Omega(p\cdot\gamma) = \frac{4i}{(2\pi)^4}\int d^4s\int_0^1 du\left[\frac{s\cdot\gamma + p\cdot\gamma(1-u) - iM}{[s^2+\Lambda^2]^2}\right.$$
$$\left. + pu\cdot\frac{\partial}{\partial s}\frac{s\cdot\gamma}{[s^2-\Lambda^2]^2}\right] \tag{23}$$

(7) R. Karplus and N.M. Kroll Phys Rev 77 536 (1950)

Upon carrying out the differentiation in (23), and making use of the facts that

$$\int f(t^2)\, t\cdot\gamma\, d^4 t = 0$$

$$\int f(t^2)(a\cdot t)(b\cdot t)\, d^4 t = \frac{1}{4}(a\cdot b)\int f(t^2)\, t^2 d^4 t \qquad (24)$$

we find the following

$$\Omega(\rho\cdot\gamma) = \frac{4i}{(2\pi)^4}\int d^4 t \int_0^1 du \left[\frac{\rho\cdot\gamma - iM}{(t^2+\Lambda^2)^2} - \frac{u t^2}{(t^2+\Lambda^2)^3}\right] \qquad (25)$$

The subtraction procedures (18) may be applied to this equation, and, making use of (8)

$$\int d^4 t \left\{\frac{1}{(t^2+\Lambda^2)^2} - \frac{1}{(t^2+\Lambda_0^2)^2}\right\} = -i\pi^2 \log\left|\frac{\Lambda^2}{\Lambda_0^2}\right|$$

$$= -i\pi^2 \int_0^1 dv\, \frac{\Lambda^2 - \Lambda_0^2}{\Lambda_0^2 + (\Lambda^2 - \Lambda_0^2)\, v} \qquad (26)$$

one obtains

$$\Omega_R(\rho\cdot\gamma) = \frac{1}{(2\pi)^2}(\rho\cdot\gamma - iM)^2 \int_0^1 du \int_0^1 dv$$

$$\qquad (27)$$

$$\times \frac{u - u^2}{\Lambda_0^2 + (\rho^2 + M^2)(u - u^2)\, v}$$

$$\times \left\{(\rho\cdot\gamma + iM)\left(1 - u - \frac{2 M^2 u^2(1-u)\, v}{\Lambda_0^2}\right) - iMu\right\}$$

(8) R. P. Feynman Phys. Rev. 76 769 (1949)

where $\quad \Lambda_0^2 = M^2 u^2 + \mu^2 (1 - u)$

It is worthwhile noting that, although it would greatly simplify the approximate evaluation of (27) to put $\mu^2 = 0$, it may not be done, because Ω_R is not an analytic function of μ^2 at $\mu^2 = 0$.

MESON SELF-ENERGY RENORMALIZATION

$$\sum (x) = \int e^{-i x \cdot \xi} \, S_p \left\{ \gamma_5 \, S_F(\xi) \, \gamma_5 \, S_F(-\xi) \right\} d^4 \xi$$

$$= \frac{1}{4\pi^4} \int d^4 p \, S_p \left\{ \gamma_5 \frac{p \cdot \gamma + i M}{p^2 + M^2} \gamma_5 \frac{(p - x) \cdot \gamma + i M}{(p - x)^2 + M^2} \right\} \tag{28}$$

By evaluating the spur here, one may write

$$\sum (x) = - \frac{1}{2\pi^4} \int \frac{d^4 p}{(p - x)^2 + M^2} - \frac{1}{2\pi^4} \int \frac{d^4 p}{p^2 + M^2}$$

$$+ \frac{x^2}{8\pi^4} \int \frac{d^4 p}{[(p - x)^2 + M^2][p^2 + M^2]} \tag{29}$$

The second term clearly gives no contribution to $\sum_R$, since it will drop out in the first subtraction in (19). This is also true of the first term, as may be seen by expanding the integrand, as was done in (23). We find

$$\int \frac{d^4 p}{(p - x)^2 + M^2} = \int \frac{d^4 p}{p^2 + M^2} - x^2 M^2 \int \frac{d^4 p}{(p^2 + M^2)^3}$$

This will completely drop out in the subtraction (19). Therefore the observable parts of $\sum$ are all contained in the third term of (29), $\sum_3$.

Application of (21) yields

$$\sum_3 = \frac{\gamma^2}{8\pi^4} \int_0^1 du \int d^4 s \frac{1}{\left[s^2 + u(1-u) \kappa^2 + M^2 \right]^2} \tag{30}$$

after a change of variable $s = p - u\kappa$. We define

$$J(\Lambda) = \frac{\kappa^2}{8\pi^4} \int_0^1 du \int d^4 s \left\{ \frac{1}{\left[s^2 + u(1-u)\kappa^2 + M^2 \right]^2} \right.$$
$$\left. - \frac{1}{\left[s^2 + u(1-u)\kappa^2 + \Lambda^2 \right]^2} \right\} \tag{31}$$

where Λ is a very large mass.

Using (26), (31) becomes

$$J(\Lambda) = -\frac{1}{8\pi^2} \kappa^2 (M^2 - \Lambda^2) \int_0^1 du \int_0^1 dw \frac{1}{\Lambda^2 + \kappa^2(1-u)u + (M^2 - \Lambda^2)w} \tag{32}$$

The subtractions (19) may be conveniently performed on this expression; when one does them, and lets $\Lambda^2 \to \infty$, one obtains

$$\sum_R (\kappa^2) = \frac{1}{8\pi^2} (\kappa^2 + \mu^2)^2 \int_0^1 du \int_0^1 dw \frac{u(1-u)}{M^2 - u(1-u)\mu^2}$$

$$\times \left\{ \frac{\kappa^2 u(1-u)w}{u(1-u)w(\kappa^2 + \mu^2) + M^2 - u(1-u)\mu^2} - 1 \right\}, \tag{33}$$

which may be quite accurately approximated by letting $\mu^2 = 0$ under the integral sign. One then obtains

$$\sum_R = -\frac{1}{8\pi^2} (\kappa^2 + \mu^2)^2 \int_0^1 du \frac{1}{\kappa^2} \log \left| 1 + u(1-u) \frac{\kappa^2}{M^2} \right| \tag{34}$$

V. Conclusion

The integrals in (27), (34) are elementary, and may be carried out without difficulty. The arguments of Ω and $\sum$ are replaced by the following functions of momentum and energy in the center-of-mass system p and ε ;

$$\rho_\mu = (\vec{P}, \Delta + E_\rho) \quad , \quad and$$

$$\chi_\mu = (-\vec{P}, \Delta + \omega_\rho) \quad .$$

Thus $(\rho \cdot \gamma - iM)^2$ becomes $-\Delta^2$ and $(\rho \cdot \gamma - iM)^2 (\rho \cdot \gamma + iM)$ becomes $-i\Delta \frac{E_\rho}{M}(\Delta + 2E_\rho)$, since what occurs in (15) is $(\bar{u}(\rho) \Omega(\xi) u(\rho))$ and $u(\rho)$ satisfies

$$(\vec{P} \cdot \vec{\gamma} + i \gamma_4 E_\rho - iM) u(\rho) = 0$$
$$(\bar{u}(\rho) u(\rho)) = 1 \quad . \tag{35}$$

Eq. (15) may now be rewritten in the form

$$\Delta \quad a(\rho u, -\rho d) = \left(\frac{G^2}{4\pi}\right)_{effective} \times (\text{interaction terms}) \quad , \tag{36}$$

where the interaction terms include all those on the right-hand-side of (15) except the self-energy terms, and

$$\left(\frac{G^2}{4\pi}\right)_{effective} = \frac{G^2}{4\pi}\left\{1 + \frac{3iG^2}{4}\frac{M}{E_\rho}\Omega + \frac{iG^2}{4\omega_\rho}\sum\right\}^{-1} \quad . \tag{37}$$

The effect of the self-energy terms may therefore be considered as causing the coupling constant to become a function of momentum and energy. When one calculates the asymptotic expressions for the integrals (34) and (27) for large p and inserts them into (37), the latter becomes

$$\left(\frac{G^2}{4\pi}\right)_{effective} = \frac{G^2}{4\pi}\left\{1 - \frac{3G^2}{32\pi^2}\log{^P/_M} - \frac{G^2}{32\pi^2}\frac{\varepsilon}{P}\log{^P/_M}\right\} \quad . \tag{38}$$

Here the first term within the brackets arises from the nucleon self-energy, the second from the meson. It is seen that for large p (38) is negative.[9] For $\Delta = 0$ it is by definition positive and equal to $G^2/4\pi$. Therefore for some $p = K$ for which $\Delta < 0 \left(\frac{G^2}{4\pi}\right)_{eff,}$ has a pole. A pole in the wave function $\alpha(pu, -p\alpha)$ would imply that an outgoing wave of momentum K was present, if one assumed an ingoing wave for which $\Delta = 0$. This K, however, with the given energy of the system $\mathcal{E}$, would form a space-like energy-momentum four-vector, which is highly unphysical, corresponding to something like production of particles of imaginary mass. No reconciliation of this nonsensical prediction with reality has yet been made, and at present it must be considered as raising serious doubt about the validity or consistency of the Tamm-Dancoff approximation.

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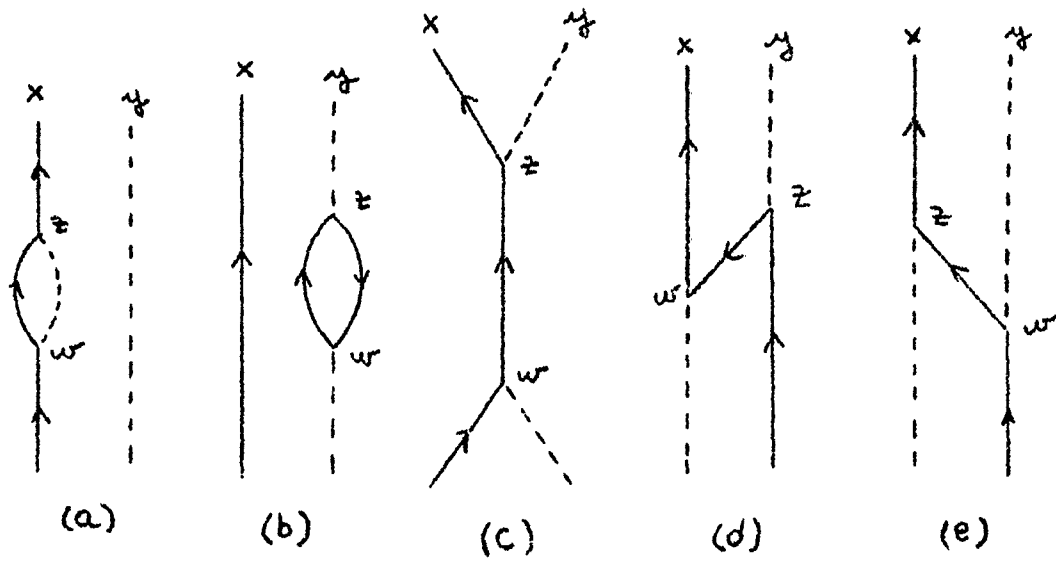
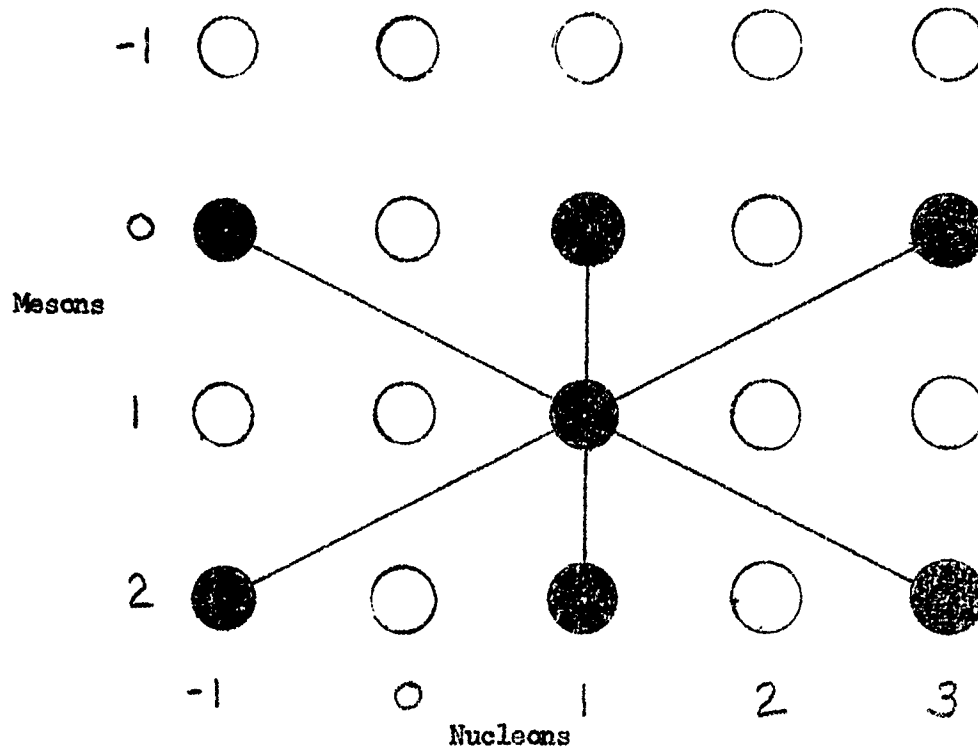Figure 2. Diagrams corresponding to the terms of equation 6. Time increases vertically.

Figure 1. Intermediate states of the meson - nucleon scattering system which are allowed in the Tamm-Dancoff approximation. Vertically is plotted the number of mesons present in the real state in excess of those present in the vacuum, horizontally the number of nucleons.