CALIFORNIA INSTITUTE OF TECHNOLOGY

NORMAN BRIDGE LABORATORY OF PHYSICS

THE SPECTROSCOPY OF NUCLEAR GAMMA-RAYS BY DIRECT CRYSTAL DIFFRACTION METHODS

by

Jesse W. M. DuMond

Contract Supervisor

A REPORT ON RESEARCH CONDUCTED UNDER CONTRACTS, FIRST WITH THE OFFICE OF NAVAL RESEARCH WITH FINANCIAL ASSISTANCE FROM THE ATOMIC ENERGY COMMISSION AND SINCE OCTOBER 1952 UNDER DIRECT CONTRACT WITH THE ATOMIC ENERGY COMMISSION

No. 28

OFFICE OF NAVAL RESEARCH
N6onr-244, T.O. IV
(NR 017-602)

No. 8

ATOMIC ENERGY COMMISSION
AT(04-3)-8

March 1954

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A REPORT ON FUNDAMENTAL RESEARCH IN PRECISION SPECTROSCOPY OF X-RAYS AND GAMMA-RAYS CONDUCTED UNDER CONTRACT BEGINNING MARCH 1947 WITH THE OFFICE OF NAVAL RESEARCH, FINANCIALLY ASSISTED BY THE ATOMIC ENERGY COMMISION FUNDS AND SINCE OCTOBER 1952 UNDER DIRECT CONTRACT WITH THE ATOMIC ENERGY COMMISSION.

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MARCH 1954

FOREWORD AND ABSTRACT

Foreword

This report contains the first comprehensive account of a new technique developed at the California Institute of Technology (and to date duplicated nowhere else) for the measurement, with high absolute precision, by means of a focusing curved crystal diffraction spectrometer, of the wavelengths and energies of gamma-rays from natural and artificial emitters over a wide range of quantum energies from 1.3 Mev (wavelength, 9 x-units) to 25 kev (wavelength 500 x-units).

Design of the first instrument for this purpose (a spectrometer of 2-meter focal length herein referred to as Mark I) was started as early as the autumn of 1937 and the construction and assembly of the principal compoments was completed in December 1941. This earlier design and construction phase was financially supported directly by the California Institute of Technology. The work was necessarily deferred from 1941 to 1946 while the personnel and facilities of this Institute were engaged in war research problems. Work on the diffraction spectroscopy of nuclear gamma-rays was resumed in 1946 and as of March 1947 was financed through a contract with the Office of Naval Research (N6onr-244, T.O. IV, NR 017-602). During this period the problems (1) of precision profiling the crystal clamping blocks and (2) of the construction of D.A. Lind's first gamma-ray detecting and measuring equipment, the multicellular G.M. counter with 14 anticoincidence protective counters, were successfully solved and in March 1948 the instrument operated successfully for the first time. The possibilities opened up by this entirely new technique called for and justified considerable expansion in the scope and funds of the contract which till that time had been cautiously and conservatively restricted because the success of the method had been in doubt. From then on until October 1952 the further development of the Mark I instrument and its application to the precision measurement of gamma-ray wavelengths was supported by the above mentioned O.N.R. contract with generous financial assistance from the Atomic Energy Commission as the expanding requirements called for additional sources of funds. It was during this period that a number of important improvements in the technique were introduced: (1) The scintillation crystal detecting and measuring system was developed to replace the multicellular G.M. counter, (2) automatic robot programming and observing of spectral intensities was perfected, and (3) an extremely thorough study of all minute sources of error with calibration curves and auxiliary provisions for correcting the data was completed by D.A. Muller and coworkers.

A direct contract for the support of the general program of precision spectroscopy of x-rays and gamma-rays between the California Institute of Technology and the Atomic Energy Commission became effective for the support of this work in October 1952. The O.N.R. contract, with its scope contracted to cover only certain non-nuclear phases of the x-ray program, has also remained in force. Because no

comprehensive technical report covering all of the technique of focusing curved crystal spectrometry of short wavelength gamma-rays had been issued during the period of nearly six years while the Office of Naval Research sponsored this work, the present report is issued for the joint information and benefit of that agency as well as for the present sponsors, the Atomic Energy Commission.

Abstract

The early history of the development of the idea of using a curved (elastically bent) crystal for x-ray spectroscopy is reviewed and the basic principles of this type of instrument are discussed. Two ways are cited for adapting the transmission type of curved crystal focusing spectrometer to gamma-ray spectroscopy: (1) With an extended source furnishing converging radiation to the crystal which selectively focuses this in the form of spectral lines on a cylindrically bent photographic emulsion (the conventional method), (2) with a source of very restricted dimensions and high specific intensity placed on the focal circle and made to explore the different wavelength positions in successive closely spaced settings at each of which the intensity selectively reflected by the crystal is measured by a detector placed in the diverging beam beyond the crystal lamina. For the very short wavelength region the latter is shown to be the more suitable method and this is the one almost exclusively followed with the Mark I instrument. The design and use of a diverging "collimator", indispensable to suppress the direct beam and transmit the selectively reflected beam to the detector, is described.

The mechanical design of the Mark I spectrometer, insuring that the reflected beam always stays in exact alignment with the collimator baffle system, and giving, on a precision screw, a linear scale of wavelengths (about 1 x-unit per turn of the screw) is described in considerable detail. Source and source holder designs are described. The preparation of the quartz crystal and its clamping blocks, including the shop and laboratory techniques for the all-important precision profiling of the latter, are described in some detail as are also the scintillation crystal gamma-ray detecting and measuring system and the procedures followed in making precision wavelength measurements including the "method of profile superposition" devised by D. E. Muller. The robot system which operates the spectrometer according to a predeterminable schedule of setting and prints the observations is described and illustrated. The final intensive calibration and study of all minute sources of error is described in full. A section is devoted to the presentation of typical results obtained with the Mark I spectrometer, together with a list of references to all published work to date.

Beside the Mark I sspectrometer two others are discussed: (1) The Mark II, a projected improvement in which the sine of the angle of of rotation of the crystal relative to its "zero" position is

measured by a novel form of optical interferometer and (2) the Mark III now under construction and well advanced at the Argonne National Laboratory (Lemont, Illinois) in which the source is to be located in a region of high neutron flux density inside a chain reactor from which the gamma-rays issue through a tube to the crystal on the spectrometer with a focal distance of over 7-meters.

TABLE OF CONTENTS

THE SPECTROSCOPY OF NUCLEAR GAMMA-RAYS BY DIRECT CRYSTAL DIFFRACTION METHODS

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1. EARLY HISTORY AND ELEMENTARY IDEAS RELATIVE TO BOTH FLAT AND BENT CRYSTAL GAMMA AND X-RAY SPECTROMETERS.

The earliest studies of nuclear gamma-rays by direct crystalline diffraction were those of Rutherford and Andrade[1] utilizing flat crystalline lamina in the arrangement schematically illustrated in Fig. 1. The shortest wavelengths measured in this early work were about 70 milliangstroms.

A photographically recording reflection type spectrometer for nuclear gamma-rays employing Bragg surface reflection at very grazing angles from a flat crystal was successfully developed by Frilley[2] following a method apparently first used for such short wavelength work by Thibaud[3]. Frilley's shortest measured wavelengths were of order 16 milliangstroms. At such short wavelengths the chief difficulties in Frilley's instrument were (1) low resolving power and poor wavelength accuracy, (2) impossibility of shielding the direct beam and its diffuse scattering from masking the selectively reflected spectrum, (3) indeterminacy of the "zero" of wavelength scale. The last could indeed have been obviated by rotating the crystal through almost 180° and taking photographic spectra of the gamma-ray lines formed by reflection both to left and to right on the same negative. Strangely enough this was not done. At the shortest wavelengths Frilley's accuracy and resolving power were very low.

Y. Cauchois[4] has taken photographic spectra of x-rays emitted by

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1) Rutherford and Andrade, Phil. Mag. 27, 854; 28, 262 (1914). See also Rutherford, Chadwick and Ellis "Radiations from Radioactive Substances," Cambridge University Press (1930).

2) M. Frilley, Ann. phys. 11, 483 (1929).

3) Thibaud, Thesis, Paris (1925)

4) Y. Cauchois, Comptes Rendus 199, 857 (1934)

radioactive isotopes with her transmission type curved crystal focusing spectrometer, the shortest wavelengths reached in this case being stated by her to be in the region of 160 milliangstroms. A mica crystal was used. Here again the chief difficulty in reaching the shorter wavelength domain is that of shielding the film from the directly transmitted gamma ray beam so that it does not obscure the spectrum of selectively reflected lines.

Beginning in 1947 much shorter wavelengths than any previously referred to have been measured with considerably higher precision (from $\pm$ 1 part in 10^3 to $\pm$ 1 in 10^4) utilizing a transmission type curved crystal focusing spectrometer developed by the writer at the California Institute of Technology in which the gamma-ray source in very concentrated form is placed on the focal circle and the radiation propagates in the opposite direction to that in the Cauchois photographic instrument. In this relatively new instrument, which is in reality a monochromator of variable wavelength rather than a spectrograph, only one wavelength setting can be observed at one time. The source must explore the successive wavelength locations on the focal circle, either continuously or in small steps, and the intensity of the reflected gamma-rays must be recorded as a function of source position on the focal circle relative to the crystal to delineate the profiles of the different spectral lines. Nevertheless in the short wavelength region there are great advantages in this approach and by this method wavelengths as short as 9 milliangstroms have been measured with an absolute precision of the order of $\pm$ 1 part in 10^3, or perhaps slightly better. Because of its superior ability to yield high absolute precision in the short wavelength nuclear gamma-ray region we shall here confine our discussion of precision gamma-ray spectroscopy by direct crystal diffraction methods chiefly to this one type of instrument.

The direct spectroscopic study of gamma-rays by crystal diffraction has the following advantages and disadvantages relative to the more prevalent indirect studies by means of "converted" electrons with magnetic spectrometers. The disadvantages are: (1) Much more intense sources are required (100 to 1000 times as intense) even with the present high luminosity instrument. The development of the neutron reactor has greatly diminished this difficulty. (2) Relatively small solid angles from the source have been used so that coincidence work to date is infeasible. (3) Only information of one kind (wavelength or quantum energy values) is obtained whereas by the study of converted β-rays

questions
/such as the multipolarity of various lines can frequently be answered by the study of the relative conversion of electrons from different atomic levels. The advantages of this direct spectroscopic crystalline diffraction method are: (1) Interpretation of the direct γ-ray spectra is much easier since the difficulty of assigning to each conversion β-ray the atomic level from which it was ejected as well as the nuclear quantum energy responsible for the ejection is not present. (2) This multiplicity of atomic levels in β-ray spectroscopy frequently results in very complicated and closely juxtaposed patterns of lines in the magnetic spectrum which are therefore difficult to resolve and to interpret. This complexity is avoided in the direct crystal diffraction method. (3) In the lower energy range the resolution of the method is much better than in the magnetic method. (4) The lines in the direct method are very symmetrical and the difficulty of long trailing "tails" caused by retardation of the electrons within the source material is absent. (5) The problem of preparing suitable sources is much easier. (6) Over a considerable range of energies the precision of wavelength and energy determination is at least an order of magnitude better.* This permits elimination of many proposed level schemes by the more rigorous application of the Ritz combination principle as a test of their validity.

These advantages were not all self evident at the beginning of the developments here to be described. In developing this new technique and especially in the case of the first of the three gamma-ray crystal diffraction focusing spectrometer designs which we shall describe here, (the "Mark I") the primary objective was simply to furnish a precision method of linking the absolute wavelength scale of the x-ray spectrum to that of the gamma-ray spectrum. Since the absolute wavelengths of x-ray lines are now known in angstrom units to a precision of about three or four parts in 10^5, (through the work of numerous physicists, 5,6,7,8) who have

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5) J.A. Bearden, J.Appl. Phys. 12, 395 (1941)
6) Bäcklin, Z. Physik 93, 450 (1935)
7) Söderman, Nature 67, 135 (1935)
8) F. Tyrèn, Z. Physik 109, 722 (1938). This entire subject has also been well reviewed by R.T. Birge, Am. J. Phys. 13, 63 (1945)

* Some of the recent remarkable high precision magnetic β-spectroscopy of K. Siegbahn and his collaborators, in which very exceptional precautions as to field and source geometry were taken, is entirely comparable with the results of the crystal instrument even in the longer wavelength region. The statement in the text is true however of the great majority of magnetic β-ray spectroscopy.

made absolute determinations of the wavelengths of certain x-ray lines using optically calibrated ruled gratings at grazing incidence) this linkage of the gamma-ray spectrum to the x-ray spectrum is all that is required to place the entire range of gamma-ray quantum energies on the absolute scale of physical units. The conversion factor[9] from wavelengths to quantum energies in electron volts is now known with considerably more accuracy (about 4 part in 10^5) than necessary, i.e., than that so far attained in the measurement of gamma-ray wavelengths by crystal diffraction or in measurements of gamma-ray energies by means of converted β-rays.

In addition to the original objective of the focusing curved crystal gamma-ray spectrometer as an instrument to establish a reliable series of precision fixed points in the gamma-ray energy spectrum, the new device promises to be of more value as an exploratory instrument than was at first anticipated because of the above mentioned far greater ease of interpreting gamma-ray spectra than β-ray spectra in many cases. The gamma-ray instrument must however not be thought of as a superior substitute for the β-ray method, but as a very valuable complement to the latter. There are obviously questions in nuclear spectroscopy, such as the multipolarity of various gamma-ray transitions which are best determined by studies of converted β-rays.

The development of nuclear physics and in particular nuclear energy level spectroscopy in say the first four decades of this century had been, as is always the case in a new field of endeavor, largely exploratory and more or less qualitative with the object of discovering the broad outlines of the subject. Since then there has been increasing effort in the direction of better and better precision. It is this writer's hope that, with a sufficient accumulation of good reliable precise numerical data on the absolute values of nuclear energy levels, the time may arrive when regularities among these data may be discerned which will lead to the disclosure of important general principles regarding the internal mechanism of nuclei about which we are still very much in ignorance. This hope is of course little more than reasoning by analogy from the history of optical

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9) DuMond and Cohen, Rev.Mod.Phys. 25, 69 (1953) gives the least-squares adjusted "best" values of a large number of constants and conversion factors of atomic physics. The value there arrived at for the conversion factor from wavelengths in centimeters to quantum energy in electron volts is $(12\ 397.8 \pm 0.5) \times 10^{-8}$ ev-cm.

spectroscopy and the part it has played in disclosing the atomic mechanism. It seems logical however to expect that we must look for the principles that control the normal behavior of the nuclear structure, not by bombarding the nuclei with particles of such extremely high energy that the nucleus is completely destroyed or modified in the process but by using gentler methods which are searching because of their accuracy rather than because of their energy, (provided, of course, that the distinction violates no fundamental principle such as that of Heisenberg).

1.1 Invention and Principles of the Curved Crystal Focusing Spectrometer. Transmission and Reflection Types. Exact and Approximate Forms.

Between the epoch-making crystal diffraction discoveries of Laue and the Bragg's in 1912 and the first published enunciation[10] in 1930 of the principles of exact focusing with curved crystals for both reflection and transmission cases (which was rapidly followed by the first papers[11,12,13] describing the different approximate and exact realizations of that principle) there elapsed a period of no less than eighteen years. This would indeed seem strange if one fell into the error of thinking that the only mental barrier which had to be overcome was the simple idea of bending a crystal lamina into a curved cylindrical form. In fact during the eighteen year interval in question several suggestions coming very close to the focusing curved crystal idea had been published such as, for example, De Broglie's curved crystal lamina bent to a logarithmic spiral[14] (which however can have one real focus only for Bragg reflection of a single fixed wavelength), the axially focusing mica cylinder of M Gouy[15] or, still closer yet, Darbord's pile of superposed, cylindrically bent mica lamina[16]. In spite of these foreshadowings however,

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10) J.W.M. DuMond and H.A. Kirkpatrick, Rev. Sci. Instr. 1, 88 (1930)

11) H.H. Johann, Z. Physik 69, 185 (1931)

12) Y. Cauchois, Comptes Rendus 195, 1479 (1932); Ann. phys. 1, 215 (1934)

13) T. Johansson, Z. Physik 82, 507 (1933)

14) M. De Broglie, J. Physique 5, 265 (1914)

15) M. Gouy, Ann. phys. 5, 241 (1916)

16) R. Darbord, J. Physique 3, 218 (1922)

approximately two decades elapsed after the discovery of x-ray diffraction by crystals before the first curved crystal focusing spectrometers were actually built. The writer firmly believes that the real obstacle to creative thinking which occasioned this long delay was the fact that, for exact focusing, the Bragg law required that at every point of the curved crystal's reflecting boundary surface two apparently incompatible conditions had to be satisfied, one of which defined the position of each point of the surface while the other defined the direction of the surface at that point. So long as physicists had failed to perceive that these two conditions need not, and in fact do not, apply to the same surfaces the exact solution appeared to be insoluble.

Consider the two-dimensional case shown in Fig. (2) in which A is a point source of composite x-rays and B a point image of one narrow wavelength band, $\Delta\lambda$, of wavelength, λ, which we seek to select from the spectrum of source A by Bragg reflection at all points of the curved crystal surface. If we set ourselves the geometrical problem of finding a curve (say $y = f(x)$) such that the cleavage surface of a flexible crystal (mica, for example) conforming thereto would, by Bragg reflection over an extended area, take x-radiation from A and focus it selectively at B, we find the problem is in the strict sense insoluble because Bragg reflection imposes two conditions at every point of the surface PQ which cannot be met everywhere by any single equation $y = f(x)$. These two conditions are: (1) at all points on the surface, the angles of incidence and reflection, θ_1 and θ_2, referred to the reflecting atomic planes must be equal (condition for specular reflection), and (2) at all points on the surface, the angle of deviation, ϕ, of the reflected beam must be constant and equal to

$$\phi = 2 \sin^{-1}(n\lambda/2d)$$

It is easy to see that these are incompatible conditions by noting that (2) is satisfied if, and only if, the curve is a circular arc passing through A and B of radius R where

$$R = L(2 \sin \phi)^{-1}$$

L being the distance AB, whereas condition(1) is clearly exactly satisfied only at the midpoint of the arc AB and is seriously violated the nearer we approach its ends.

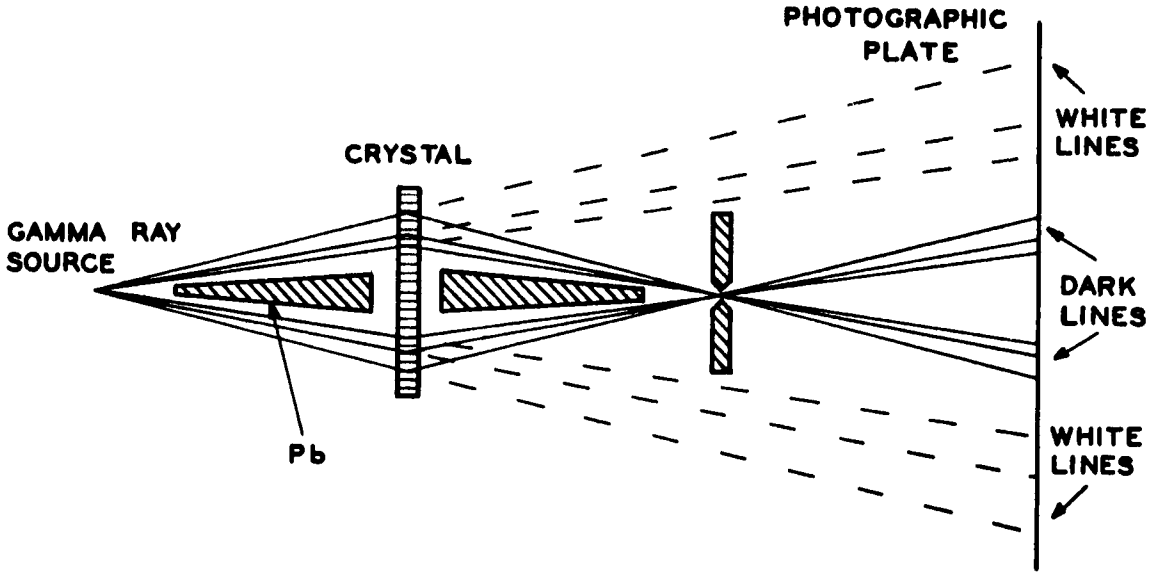

Fig. 1 Rutherford and Andrade's method for the study of nuclear gamma-ray spectroscopy utilizing a flat unstressed crystalline lamina in transmission.

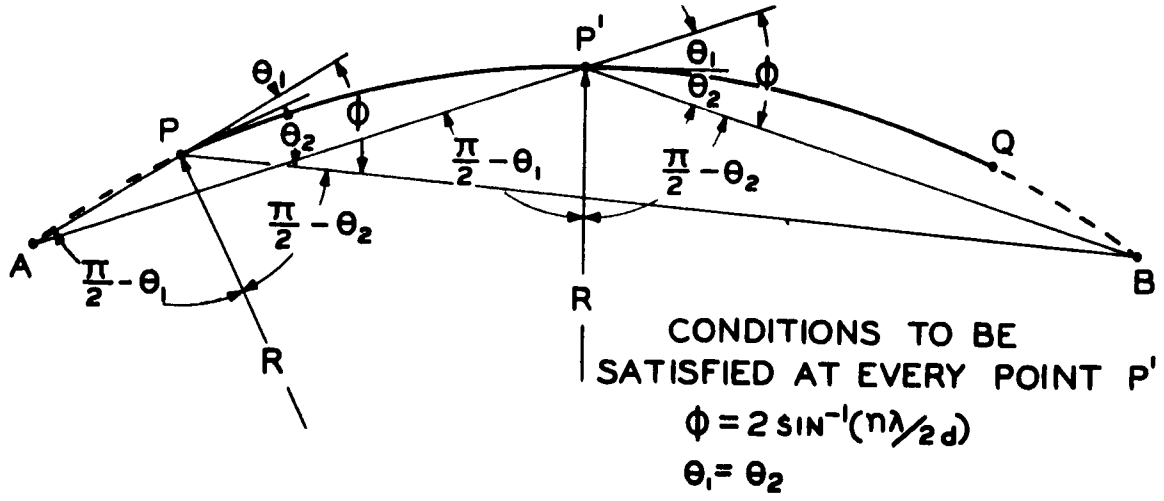

Fig. 2 To illustrate the impossibility of exactly satisfying both conditions required by Bragg reflection with a bent crystal over an extended surface if the atomic reflecting planes coincide with the reflecting boundary of the crystal and if two conjugate real foci are required.

The writer believes that he was the first to see the way out of this apparent impasse and thus to break down the psychological obstacle which had impeded the invention of the curved crystal focusing spectrometer. After much reflection he suddenly realized that the two conditions above mentioned are not in fact imposed on the same surface provided we do not insist on having the atomic reflecting planes everywhere parallel to the boundary surface of the crystal. Condition (2) dictates the position of every point on the boundary surface (the points must lie on the arc AB) but demands nothing regarding the direction of the reflecting planes at that surface. Condition (1) dictates the direction of the atomic reflecting planes (they must be everywhere normal to the bisector of the angle AP'B) but imposes no condition on the position of the boundary surface where the reflecting planes are situated. (One is forcibly reminded by this physical situation of some of the beautiful and subtle mathematical ideas of Weierstrass concerning non-differentiable analytic functions). If we arrange matters as in Fig.(3)[17] we have two cases, the reflection case, R, (first so-called because a "real image" was formed) or the transmission case, V (first so-called because a "virtual image" was formed). It will be evident that our ideal problem is exactly solved for either of these cases by the arrangement shown. Let us start by discussing the reflection case (case R), the one which occurred to the present writer first. A crystal is shown whose reflecting boundary is a circular arc with center at O. The atomic reflecting planes of this crystal, however, are bent so that they coincide with concentric circles centered on the circumference of the circle, O, at the point, C. For all points of the surface, condition (1) is fulfilled since the equal subtended arcs, SC, and IC how measure twice the angles of incidence and reflection, θ_1 and θ_2, with respect to the reflecting atomic planes whose normal is their radius of curvature, PC. Condition (2) is seen to be fulfilled since the arc SPI measures twice the angle of deviation of the beam for all points, P. It is also evident that the locus along which the spectrum is focused lies on the circle of center, O, and that the two conjugate foci for each wavelength lie at points on this circle symmetrically situated relative to the point C_2.

The transmission case, Fig. 3 V, was arrived at by the writer as

17) Taken from the original DuMond and Kirkpatrick article. Rev. Sci. Instr. 1, 90 (1930). See Fig. 2 of that article.

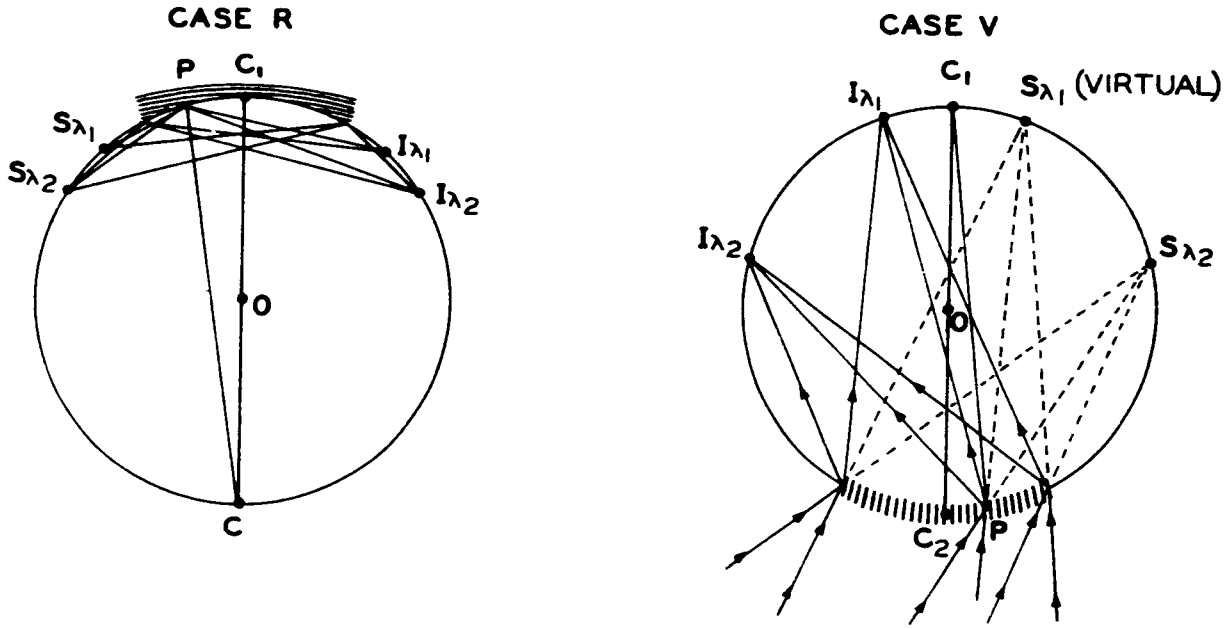

Fig. 3 Two solutions of the problem of selective x-ray focusing. This picture giving the simple geometrical principle of the exact focusing cases for both reflection and transmission curved crystal focusing spectrometers is taken directly from an article published by DuMond and Kirkpatrick in January 1930 (Fig. 2 of that article). The idea was conceived by DuMond in 1927.

a logical continuation of the reflection case, R. Essentially the requirements define a direction for the crystal planes at every point of the focal circle and if we inquire as to the directions in the region diametrically opposite to that occupied by the arc of crystal shown in case R, we automatically arrive at Case V. In this case the short reflecting planes traversing the thin crystal lamina may either be thought of as normal to a radius through C_2 (just as are the atomic planes in case R) or as planes which, if produced, would intersect in C_1, on the focal circle diametrically opposite to C_2. It is clear from the figure that the transmission case has one virtual and one real focus, while the reflection case has two real conjugate foci. For rather obvious reasons the reflection case is usually better adapted to long wavelengths and large Bragg angles, while the transmission case is usually better adapted to short wavelengths and small Bragg angles.

Johann and Cauchois realized that condition (2) fixing the position of the boundary or boundaries of the crystal, was less stringent than condition (1) fixing the direction of the atomic planes so that if the arc of the focal circle covered by the crystal is not too large, the aberration of focusing resulting from violation of (2) can be kept sufficiently small for practical purposes without requiring the difficult curved profiling of the boundary surface[18]. Secondly, Cauchois pointed out the important new observation that, in the transmission case there is also a focusing effect through the thickness of the curved sheet as well as from side to side. When the bending stress is applied so that the crystal lamina is curved, the strain (tension) near the outside (convex) surface of the plate results in a larger grating constant than the strain-free value obtaining on the neutral axis and hence there is a slightly smaller Bragg angle reflected from these regions than the one that applies to reflection

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18) Others however have succeeded in building curved crystal spectrometers of the exact focusing type (Case R of Fig. 3) by suitably profiling the crystal lamina (in its unstressed state) to the required radius of curvature, twice the radius of the focal circle, and then bending it until the spectra focus on the focal circle, i.e., to the same radius of curvature as the latter. The first to do this in the reflection case was T. Johannsson, Z. Physik 82, 507 (1933). See also, R. Bozorth and F.E. Haworth, Phys. Rev. 53, 538 (1938), and A. Guinier, Ann. phys. 12, 161 (1939).

by regions of the crystal in the vicinity of its elastic neutral axis. The opposite effect occurs for reflection by regions of the crystal near the inner (concave) surface. It is easy to show that this results in exactly the required convergence of all the reflected beams over the entire thickness of the slab to give perfect focusing from front to back. This leads to the conclusion that in the transmission spectrograph it is the neutral axis of the crystalline slab which should be placed in contact with the focal circle.

Cauchois and Johann have each made careful quantitative studies of the geometrical aberrations of the respective types of instrument they developed. For gamma-ray spectroscopy we shall here be exclusively concerned with the Cauchois approximate focusing transmission type instrument. In this instrument, for simplicity of construction the crystal in its unstressed state is a plane lamina. The chief geometrical aberration then comes from the fact that the neutral axis of this lamina, after it has been elastically curved to the required radius (equal to the diameter of the focal circle) deviates at its extremities from exact coincidence with the focal circle because the latter has only half as large a radius. Fig. 4 shows how to compute the relative instrumental line broadening, $\Delta\lambda/\lambda$, which results from this aberration. Simple geometrical considerations yield the following result,

$$\Delta\lambda/\lambda \cong \cos\theta\,(1-\cos\alpha)\sec(\alpha+\theta) \qquad (1)$$

wherein θ is the Bragg angle and α the half angle of aperture of the crystal. Clearly for small Bragg angles θ and small apertures, α , such that $\theta < \alpha << 1$, $\Delta\lambda/\lambda$ approximates the quantity, $\alpha^2/2$ and therefore rapidly becomes negligible.

There is also a slight geometrical aberration, which can be neglected in the gamma-ray spectrometer, coming from "crossfire", the obliquity of some of the radiation relative to the base plane of the focal cylinder.

An interesting result of the type of reflection taking place in the transmission-type curved crystal spectrometer is that no correction whatever need be made to the observed Bragg angle for the refractive index of the x-rays in the crystal used, provided the internal atomic

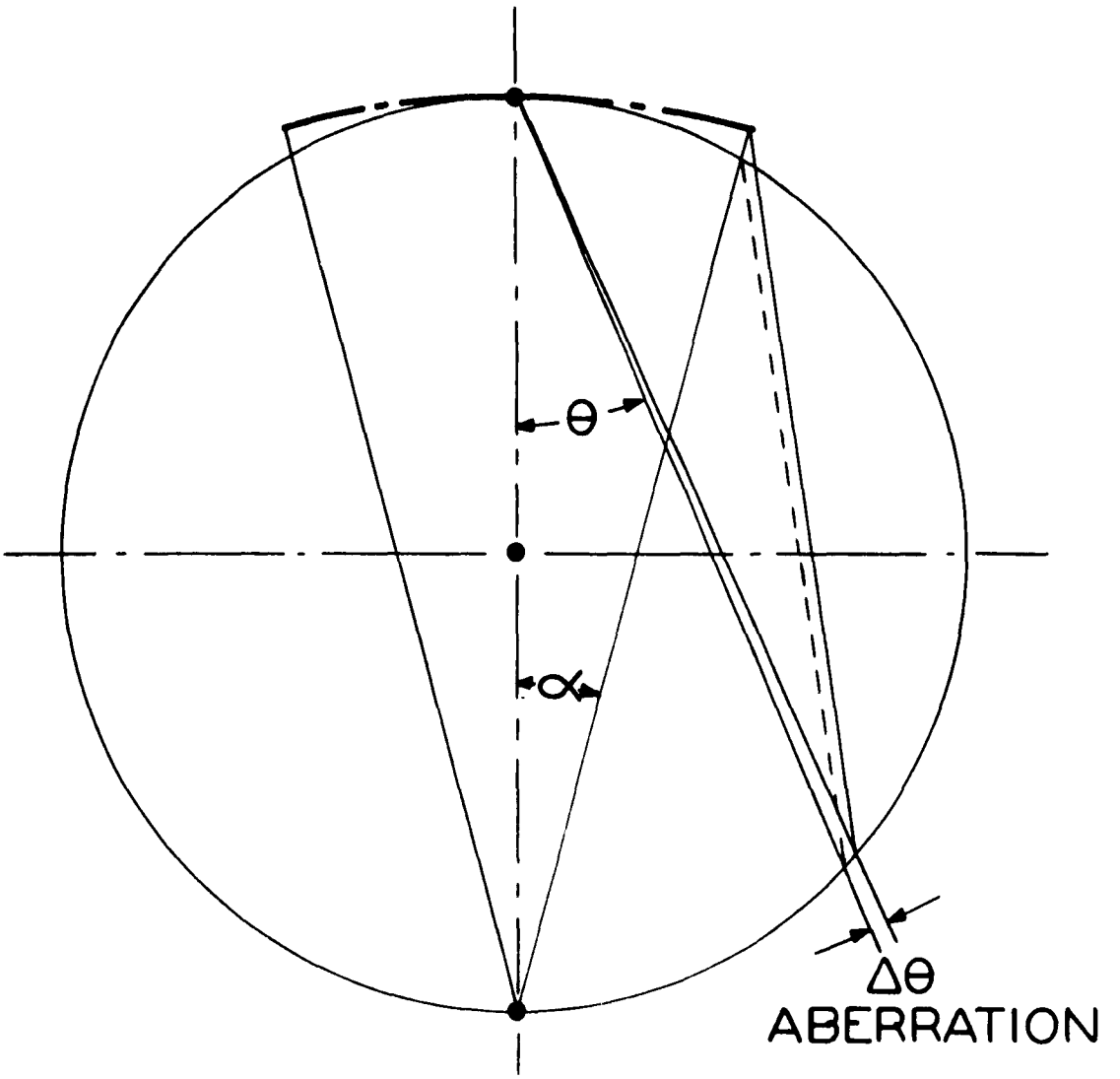

Fig. 4 Geometry determining the line broadening or aberration in the Cauchois approximate type of transmission spectrograph which results from the fact that near its extremities the neutral axis of the lamina fails to coincide with the focal circle. The aberration $\Delta\theta$ results in a corresponding aberration $\Delta\lambda$ given in formulae (1) and (2) of the text. For small θ and α this aberration is extremely small.

reflecting planes are normal to the crystal slab. This is not true of the reflection type.

Referring to Fig. 5, we see that the ratio of the wavelengths, λ_1, and λ_2, outside and inside the crystalline medium will be equal to μ, the refractive index of the crystal

$$\lambda_1/\lambda_2 = \mu \quad (\mu < 1) \tag{2}$$

while the ratio of the sines of the angles, θ_1 and θ_2, of incidence and reflection at both entrance and exit boundaries of the crystal will also be equal to

$$\sin \theta_1/\sin \theta_2 = \mu \tag{3}$$

If the reflecting planes are normal to the boundary surface, the Bragg equation inside the crystal is

$$n\lambda_2 = 2d \sin \theta_2 \tag{4}$$

and it follows from (2) and (3) that outside the crystal we shall also have

$$n\lambda_1 = 2d \sin \theta_1 \tag{5}$$

Furthermore, as the planes of reflection incline away from the normal to the boundary surfaces of the lamina, the refractive index correction at first increases very slowly indeed. The correction factor, F, for this case of oblique planes is

$$F = 1 - \delta \sin \epsilon \operatorname{cosec} \theta \sec(\theta - \epsilon) \tag{6}$$

wherein $\delta = 1 - \mu$ is the difference between unity and the refractive index of the crystal for the wavelength in question, θ is the Bragg angle, ϵ is the angle of obliquity between the reflecting planes and the normal to the face of the crystalline lamina and F is the correction factor in the Bragg equation when the latter is written in its rigorous form

$$n\lambda = 2d\, F \sin \theta \tag{7}$$

λ and θ being respectively the wavelength and corresponding Bragg reflection angle observed outside the crystal.

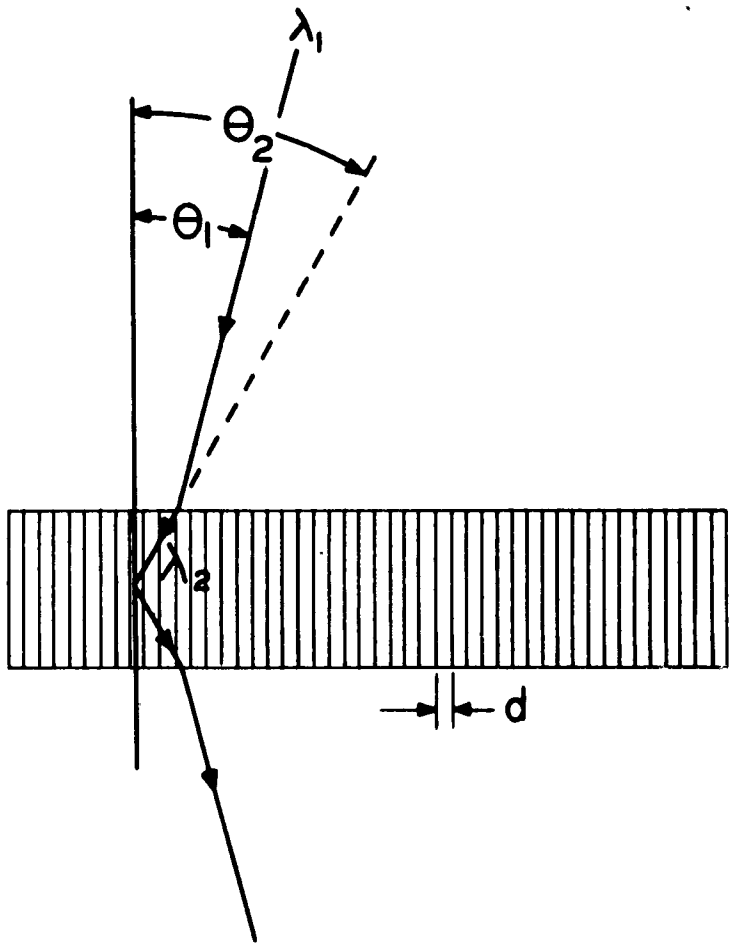

Fig. 5 Geometry demonstrating the absence of correction for refractive index when the atomic reflecting planes are normal to the exit and entry faces of the lamina in the transmission-type spectrograph.

The design of the first precision gamma-ray spectrometer, which we shall designate as Mark I and which is still the only instrument of its kind in operation, was started early in 1938 and its construction and assembly were practically completed in 1940 save for the all-important crystal and its clamping blocks and for the multicellular G.M. counter which was developed later in 1947 by D.A. Lind as the first detector of gamma-rays used in this instrument. Unfortunately, the progress on this entire technique was interrupted, and the work had to be abandoned for the long interval from 1940 to 1946. We shall describe the Mark I instrument in the next section.

Two other designs of focusing curved crystal gamma-ray spectrometer which we shall designate as Mark II and Mark III have been under very serious consideration but are not in operation at the time of writing this report. Mark II is a version in which the sine of the angle of rotation of the curved crystal (the Bragg angle)away from the position for zero gamma-ray wavelength is measured by an optical interferometric method of very high precision. Mark III is an instrument whose construction is at the date of writing well advanced at the Argonne National Laboratory (Lemont, Illinois). It is planned to have in this instrument a very large (probably 30 x 30 cm) bent quartz lamina some 6 mm thick and with a radius of curvature of 7.7 meters. The instrument is intended for the study of neutron-capture gamma-rays below 2 Mev quantum energy and is designed to be used for the study of radiation from an emitting source situated in a region of very high neutron flux density in a tube passing through or near the cneter of a large liquid chain reactor.

2. ADAPTATION OF THE TRANSMISSION TYPE BENT CRYSTAL SPECTROMETER PRINCIPLE TO THE PRECISION MEASUREMENT OF VERY SHORT WAVELENGTHS (NUCLEAR GAMMA-RAYS)

2.1 Two Possible Arrangements of Source and Crystal

The philosophy behind the design of the Mark I instrument about to be described, was first set forth in a paper[19] published in 1947.

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19) J.W.M. DuMond, Rev. Sci. Instr. 18, 626 (1947)

These considerations still apply at the time of writing and we shall briefly summarize them here.

The transmission type of curved crystal spectrometer, the one best adapted to the measurement of short wavelengths has, as we have pointed out above in Fig. 3, one real and one virtual focus, and hence it is not immaterial in which direction the radiation propagates. Let us designate two alternative arrangements by roman numerals I and II. Referring to Fig. 6, we may either place an extended source at A as shown in the left hand sketch (I) or a very concentrated source at the real focus R as shown in the right hand sketch II. Now if we are to work with gamma radiation from artificial or natural radioactive substances, it is clear that in order to obtain as much gamma-ray line intensity as possible of some specified wavelength with the spectrometer at a fixed setting and with a fixed amount of source activity, the real focus, R, is the best place to put the source (arrangement II in Fig. 6) for two reasons: (1) With the source concentrated at the appropriately chosen point on the focal circle for the given wavelength, every nucleus in the sample at R has a chance to radiate the specified gamma-ray wavelength into a solid angle subtended by the entire working aperture of the crystal. (2) By interposing a set of fan-shaped lead baffles as shown, between the crystal and the counter or scintillation detector, it is possible, with arrangement II, to shield the intense unresolved direct beam of gamma-rays from reaching the detector and completely masking the much weaker selectively diffracted monochromatic beam which it is the object of the instrument to measure. II is seen to be the arrangement in which the instrument is essentially a monochromator as we have already mentioned and the line profiles are to be delineated and the spectrum explored point by point. The arrangement I shown at the left in Fig. 6 which is essentially the transmission photographic spectrometer of Cauchois is better adapted to recording a whole region of the spectrum simultaneously on a film placed on the focal circle. Clearly an entire region between the points Q_1 and Q_2 could thus receive selectively diffracted radiation from the source in the geometry shown in the figure. Beyond Q in the clockwise direction the strong direct radiation from A transmitted through the crystal without reflectiom, would completely mask the spectrum and indeed it is found that long before the point

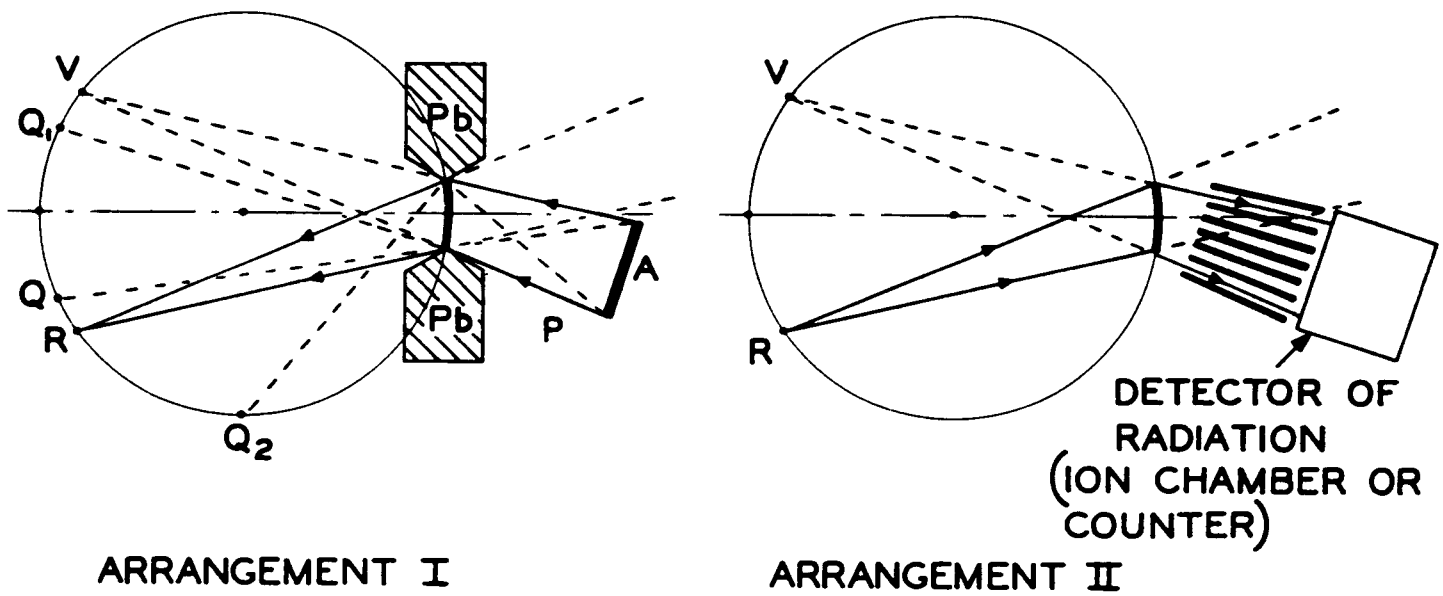

Fig. 6 Two ways of using the transmission-type, curved-crystal spectrometer. An extended source may be placed at A, in which case spectral lines will be focused as at R, Arrangement I. This case is appropriate for the study of fluorescence excited in a screen placed at A, but is uneconomical if the total intensity from the source is a limiting factor. For gamma-ray spectra the source may be placed at R, Arrangement II, and the intensity measured in an ion chamber or scintillation counter, is thus plotted as a function of the position of the source R on the focal circle.

Q is reached β-rays and diffusely scattered gamma-rays at small angles coming from the crystal completely mask the spectrum to be studied. Furthermore in the arrangement, I, in Fig. 6, for an emitted gamma-ray line of given wavelength, each nucleus at some point P in the extended source placed at A, can only make use of a very small solid angle subtended by a very tiny portion of the curved crystal where the incident radiation strikes the reflecting planes at just the right Bragg angle. This is a much more stringent restriction in short wavelength gamma-ray spectroscopy than in ordinary x-ray work. Because of the very small Bragg angles involved, in gamma-ray spectroscopy only the most nearly ideally perfect crystals can profitably be used. A study with the two-crystal spectrometer using unstressed flat quartz plates ("parallel" position) has permitted us to establish that, for the (310) planes of quartz the selective reflection of a given wavelength is restricted to an angular region of the order of only a few seconds of arc[20]. When this is compared with the available horizontal angular opening of 1.5 degrees, for the arrangement II, Fig. 6, in the case of the Mark I instrument, it is seen that the latter position should yield of the order of five thousand times as much intensity at a specified wavelength as the arrangement I.

At longer wavelengths where the problem of shielding the film from the direct beam is less severe and where the radiation is soft enough so that photographic spectra can be profitably used with reasonable exposures the use of the curved crystal spectrometer in the arrangement I as a photographic spectrograph does have advantages for exploratory work. The lines are all formed simultaneously over a considerable segment of spectrum in a single long exposure. To explore exhaustively point-by-point the same range of spectrum using the spectrometer in arrangement II would in many cases take as long or longer than by photography with arrangement I. This statement is however far less likely to be true <u>if one knows approximately where to look for the lines</u> using arrangement II so that it is unnecessary to make a search run in sufficiently fine detail everywhere to insure that no line shall be passed over. Here we shall be almost exclusively concerned with the

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20) Watson, West, Lind and DuMond, Phys. Rev. <u>75</u>, 505 (1949)

arrangement II of Fig. 6.

2.2 The Baffle or Collimator for Arresting the Heterogeneous Transmitted Beam

The hardest nuclear gamma radiation so far measured with the Mark I instrument at the time of writing is a line of quuatum energy approximately 1.3 Mev following the decay of Co^{60}. The wavelength of this line is about 9 milliangstroms. At a gamma-ray wavelength of 9 milliangstroms, reflection in the first order from the (310) planes of quartz (d = 1.2 angstroms) occurs at about 1/5 of one degree and the angle of deviation between the reflected and directly transmitted beams which pass through the crystal lamina is therefore about 0.4 of a degree. The directly transmitted beam may easily be two thousand times as intense as the diffracted beam which is the object of study. In the Mark I spectrometer, in which the crystal subtends an aperture angle at the radioactive source of about 1.3 degrees the geometry is such that the transmitted and reflected beams, after they have passed through the crystal, never cease to overlap on each other no matter how far they may be produced, if the wavelength setting is less than about 27 milliangstroms. The means provided to arrest the transmitted beam as completely as possible while still permitting about half of the diffracted beam to reach the detector, consists of a baffle system of twenty-four tapering, die-cast, lead-tin-antimony alloy partitions 31.5 inches long and 0.040 of an inch thick at the thin entry end, separated by tapering spacers clamped at top and bottom between the partitions so as to leave twenty-five open tapering channels each 31.5 inches long, 0.040 inch wide at their entry ends and 3 inches high. This assembly illustrated in Fig. 7, is clamped in a massive and very rigid steel enclosing box whose four sides are held together with large cap screws. It is of the utmost importance that the surfaces of the tapering partitions should be as straight and true as possible and should be aligned so that if produced these surfaces intersect as closely as possible in a common vertical line somewhat more than two meters in front of the entry end. Very slight departures from straightness in the partitions can obviously reduce the effective ratio of aperture to opaque partition very materially and the problem of correctly

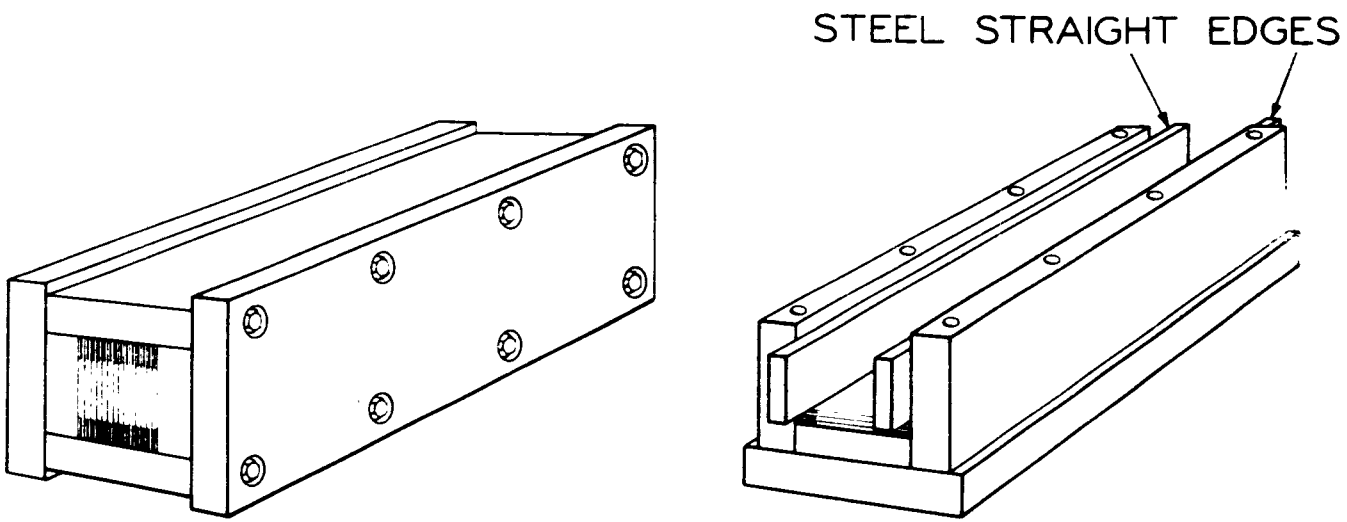


Fig. 7 Perspective line drawing of tapering "collimator" consisting of twenty-four die cast tapering lead-tin-antimony alloy partitions. There are twenty-five tapering spacers of the same material at the top and an equal number at the bottom of the openings between partitions. The method of testing the obliquity and straightness of the partitions with straight-edges during assembly is also shown.

assembling such a collimator is therefore a severe one.

In the case of Mark I this was done by removing one of the heavy side faces of the steel clamping box, turning the remaining three sides with the open face up and assembling the tapering partitions and tapering spacers in the U-shaped enclosure. As the assembly progressed, two heavy steel straight-edges were laid, after inserting each new pair of spacers and the next partition, one on each side of the pile, and micrometer measurements from these straight edges to a reference surface were taken at both the entry and exit ends for comparison with a calculated schedule to verify whether the obliquity of each successive partition was correct. A thin "feeler" gauge was also used to explore any possible irregularities in the contact between the straight-edges and the partition under them. It was found that the die-casting process did not insure perfectly plane surfaces on the tapering spacers and partitions so that it was necessary to correct these locally with a lead file and by scraping. In some instances thin shims also were resorted to as permanent parts of the assembly in order to keep the obliquity of the pile of partitions more nearly correct. A better result could probably have been obtained by using a hard material such as tungsten or uranium for the partitions and spacers which could then have been ground to give the correct taper to high precision in a surface grinder. Such a solution would have been much more expensive and was not followed for this reason. The tapering collimator here described was designed with partitions and spacers of identical thicknesses to have a theoretical transmission of 50 percent. Measurement after assembly with a radioactive source indicated a transmission of 35 percent, the difference being undoubtedly due to the residual irregularities in geometry.

The design of the collimator is such that the system of long tapering slots is completely surrounded on the top, bottom and sides with one or more inches of solid lead. The spacers supply this thickness on the top and bottom while solid lead alloy castings carefully machined to the correct taper fill out the steel box on either side.

The theoretical geometrical angular resolving power of this "collimator" (if we ignore scattering and penetration of the gamma radiation in the lead plates) is 8 minutes of arc. That is to say, were it not for scattering and penetration, the upper working limit of quantum energy of the gamma radiation which could be studied using the (310)

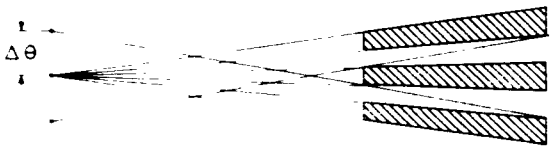
planes of quartz would be 3 Mev. In actual practice the background from scattering of the direct beam incident on the collimator partitions becomes rapidly more and more excessive in the region of 1.3 to 1.5 Mev.

Although we refer to this baffle system as the "collimator" it should be clearly understood that this element of the instrument has nothing to do with the spectral resolving power. It is only a baffle to discriminate between the reflected beam and the transmitted beam and its angular resolving power is some twenty times coarser than the angular resolving power of the focusing crystal in Bragg reflection, the true factor which determines the angular and spectral resolving power of the spectrometer.

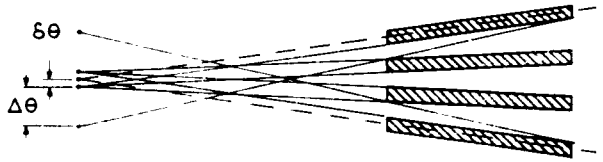
Clearly, in order that the collimator shall not unduly reduce the intensity of the reflected beam, and thus introduce spurious artificial deviations in the intensity readings, it must be kept accurately aligned with the reflected beam at all times while the spectrum is being explored, that is to say the convergence point of the surfaces of its partitions must align correctly with the virtual image point V, Fig. 6. The method of accomplishing this in the Mark I and in other instruments will be described presently. In the Mark III instrument for the Argonne Laboratory a collimator is planned in which only the spacers but not the partitions taper. This is illustrated in Fig. 8. The advantage of this design is that the transmission characteristic (intensity transmitted as a function of angle, in Fig. 8) of such a collimator has a flatter top than is the case of the collimator just previously described and the intensity readings are therefore even less sensitive to a slight misalignment of the collimator relative to the reflected beam.

2.3 Mechanical Design

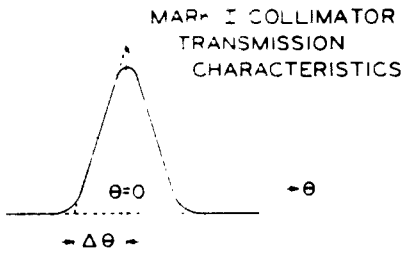
The Mark I instrument was designed before the feasibility of such a radically new device had been proven and at a time when economic factors placed severe limitations on fundamental research. For these reasons the choice regarding many features of the design was conditioned by costs either for materials or fabrication. The Mark I design, like every first attempt, has certain weaknesses which its use has revealed and which now could be avoided in later designs.



MARK I COLLIMATOR. PARTITIONS TAPER IN THICKNESS THE SAME AS OPENINGS.



MARK III COLLIMATOR. PARTITIONS OF UNIFORM WALL THICKNESS. ONLY THE OPENINGS TAPER.



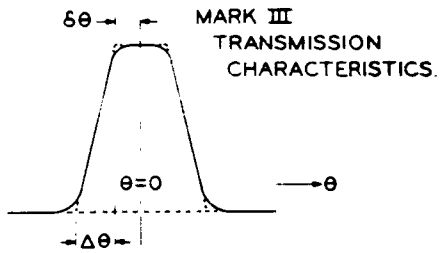


Fig. 8 Two different collimator designs and their characteristic curves plotted as function of the angle θ of obliquity of the radiation in the channels. The dotted lines give the ideal transmission characteristic for complete opacity and no scattering by the partitions.

The primary objective in the Mark I was, as already stated, to furnish a precision device which would bridge the gap between the known wavelengths of x-ray lines and the less accurately known wavelengths of gamma-ray lines so as to link these two regions of the spectrum together with high precision. The x-ray and gamma-ray wavelengths to be measured are, in the transmission type focusing instrument, strictly proportional to the sine of the grazing angle at which the radiation is incident on the atomic planes of the crystal. The cheapest and easiest way to build a device for measuring the sines of angles appeared to be a linkage simulating a right triangle with constant hypothenuse in which a precision screw would displace a carriage in such a way as to vary the length of that leg of the triangle which was opposite to the angle whose sine was desired. We might characterize this briefly as a sine-screw arrangement. A precision sine-screw device is one of the easiest and cheapest angle measuring devices to produce - far cheaper for the same angular precision than a worm wheel and worm for instance. This is partly because precision can actually be generated in the screw by the very simple process of lapping with a long appropriately designed nut.

It is important also to provide means that will maintain the lead collimator and the diffracted monochromatic beam from the crystal always in correct alignment while the spectrum is being explored, as we have explained above. In the Mark I instrument, to avoid the necessity of moving the heavy collimator and the still more heavily shielded gamma-ray detecting components behind the collimator the instrument was designed so that the direction of the reflected beam while the spectrum is being explored, is at all times sensibly invariable. The principle of the design is schematically illustrated in Fig. 9. To accomplish the desired result both the curved crystal and the focal circle, rigidly related thereto, must rotate about a point C situated at the center of the neutral axis of the bent crystal slab and at the same time the source, while always remaining on the moving focal circle, must explore along its circumference in such a way that a line from the source to the point C on the crystal rotates at twice the angular rate at which the crystal rotates. The collimator channels will then always remain sensibly pointed at the virtual image point V as shey should. The length of the

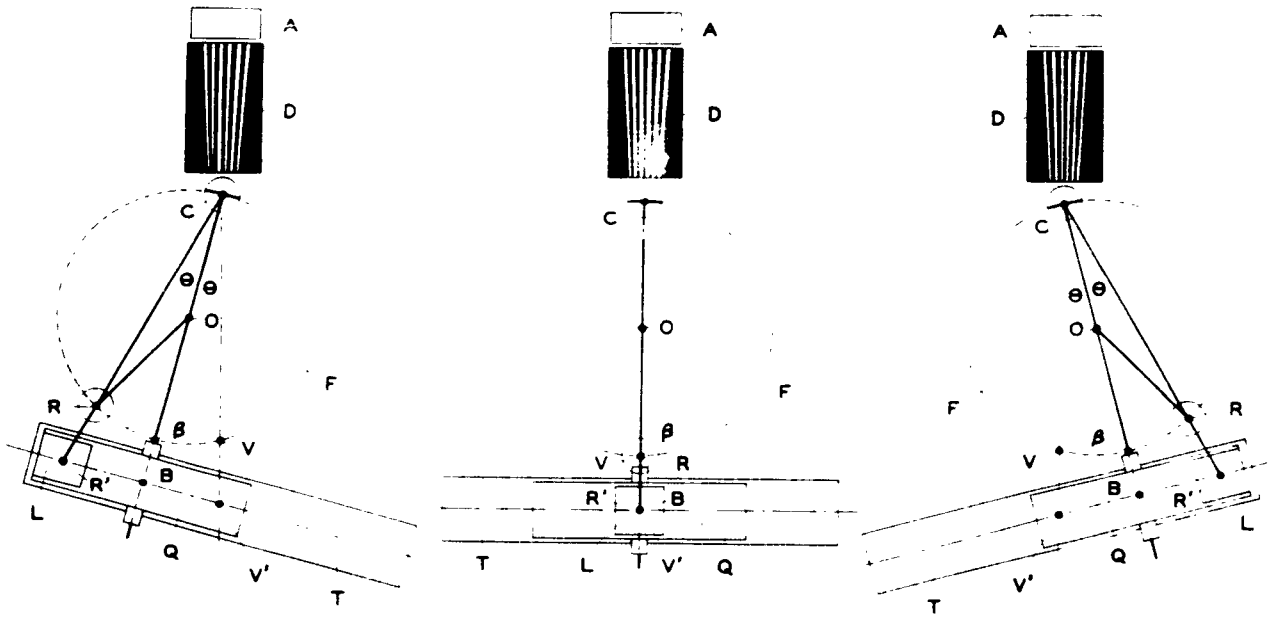


Fig. 9 To illustrate schematically the geometry of the gamma-ray spectrometer. The view at center shows the instrument at the zero wavelength position, while the views to left and right show different wavelength settings for reflections to left and to right of the atomic reflecting planes. The constant distances CR' and CV' are made rigorously equal. Displacements of the carriages L and Q, effected by means of precision screws, are proportional to the sine of the Bragg angle, θ, and, hence, proportional to wavelengths. The drawing is not to scale. The aperture of the crystal at C and the width of the collimator are exaggerated.

chord, CV, changes slightly so that at long wavelengths and large Bragg angles V is slightly closer to the collimator but this introduces a quite negligible change in transmission of the reflected beam through the latter.

It is also highly desirable to be able to interchange the position of R relative to V (Fig. 9) so that spectra resulting from reflection on both sides of the (310) planes of the quartz crystal can be conveniently studied. By measuring the distance (on the precision screw) between the spectra reflected from both sides of the planes, the exact position of the "zero" of the wavelength scale need not be known beforehand.

Some pains are taken, when the crystal lamina is sawed from the original quartz crystal, to orient the cut as nearly as possible at right angles to the desired reflecting planes. If this has been done perfectly then after the crystal is bent in the cylindrically profiled jaws of its clamp and mounted with its neutral axis accurately tangent to the focal circle at the point C, the reflecting planes produced will converge to a common point β exactly diametrically opposite on the focal circle to C. If however the crystal is cut so that the planes make a slight angle with the normal to the face of the lamina, the point β will not be quite diametrically opposite to C. In either case, clearly the point, β, is the "zero" of the wavelength scale on the focal circle.

In the Mark I instrument the focal circle is approximately 2 meters in diameter. It is defined merely by a radius bar OR which links the source carriage on the upper beam CR' to the pivot, O, defining the center of the focal circle on the lower beam.

Referring to Fig. 9 and also to Fig. 10 to which the same reference letters apply, Q is a carriage mounted on ball-bearing rollers so that it can be displaced along a track T. This track is pivoted so that it can swing in the horizontal plane about a fixed vertical ball bearing pivot V' on the base of the machine. The carriage, Q, is provided inside with two horizontal longitudinal precision screws (both right handed), one situated vertically above the other, which are geared together with equal gears so that they rotate at equal rates in opposite senses. (See also the cross section view Fig. 11 below.) The upper screw, by means of a nut, drives a smaller carriage, L, along a track on the top of carriage Q. The lower screw drives the carriage, Q, along the track, T, by means of a nut provided with a vertical pin at the point, V'. This pin is coaxial with

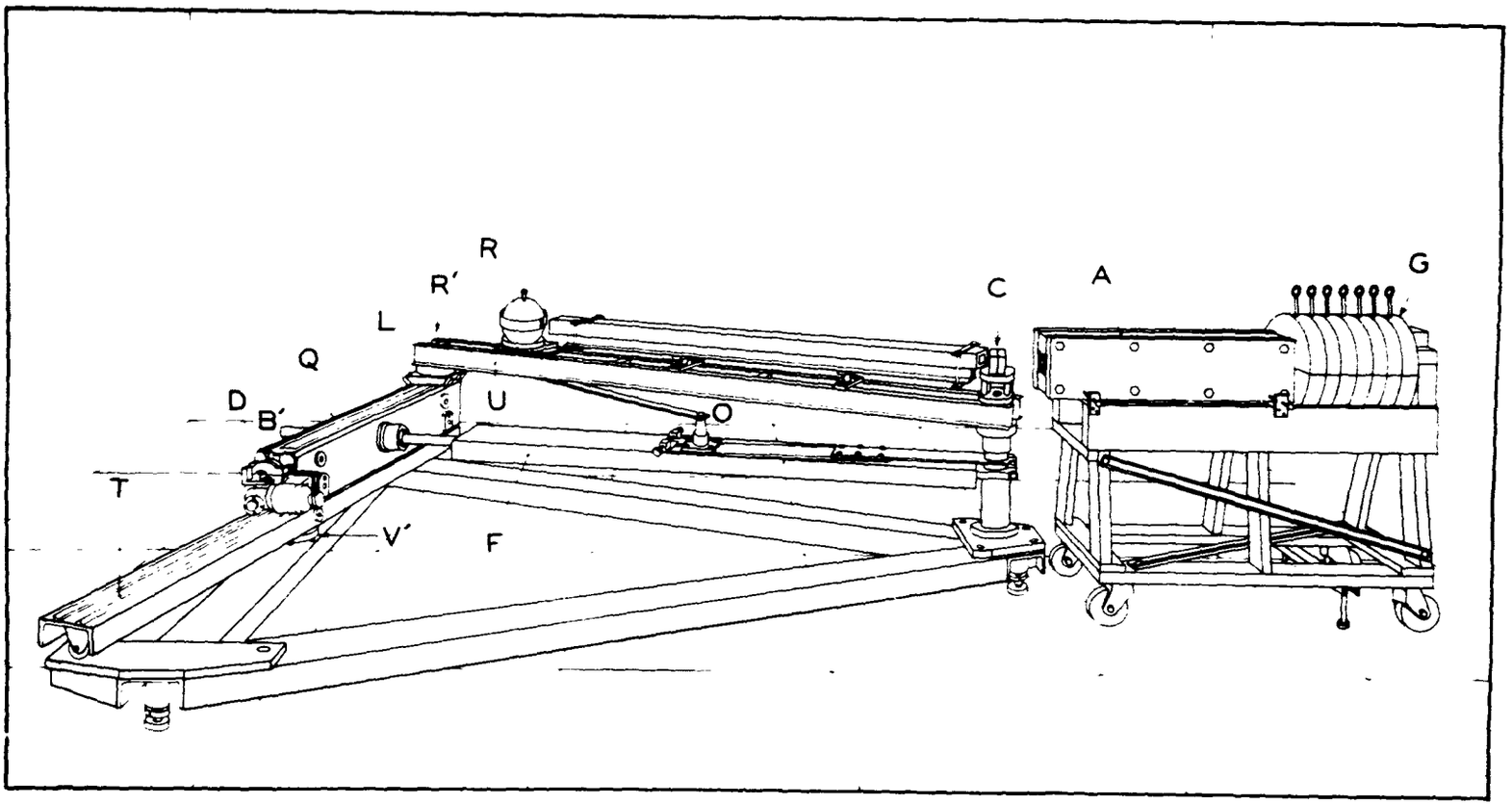


Fig. 10 Perspective line drawing of the gamma-ray spectrometer showing the curved crystal at C and the source in its shielded holder 2 meters from it at R.

the pivot for the track T. The points V' and R' are thus made to remain at all times equidistant on either side of the line Cβ. Two long radial steel beams are provided, each pivoted so as to swing in a horizontal plane about C at different heights above the base. The higher of these furnishes the physical realization of the line CRR' in Fig. 9. It constitutes a link of fixed length between the pivot C and a pivot R' on the small carriage L. This beam, constructed of two channel sections held apart by spacers, has therefore a slot-shaped vertical opening down between the channel irons. The source carriage is supported on top of this beam in such a way that the carriage can roll on hardened ball bearing ways over a short travel along the length of the beam and a pivot on this carriage passes down through the slot in the beam where said pivot is linked by the radius bar, OR, to an adjustable center pivot, O, (the center of the focal circle) supported on the lower beam. The upper radial beam pivots at C freely and independently of the crystal holder and lower beam. The crystal holder, although it is above the upper beam, is supported on a vertical shaft which passes down through the bearing for the upper beam and is rigidly clamped to the lower beam. (See also the cross section view of Fig. 12 below.) The lower beam furnishes the physical realization of the line Cβ in Fig. 9, the altitude of the isosceles triangle, CV'R', and since this triangle has legs, CV, and, CR', of constant length, the length of this altitude, that is to say the distance CB from the pivot C to the center line of carriage Q and track, T, must be capable of variation as the base of the triangle, V'R', varies and the focal circle is explored. To accomplish this the lower beam is terminated in a projecting cylindrical steel shaft which can slide axially in ball-bearing guides passing, as it does this, transversely through the long carriage, Q, just midway in height between the upper and lower precision screw shafts mounted therein. The ball bearing guides through which this cylindrical steel shaft on the lower beam slides are provided with adjustments to take up all play so that while the shaft slides longitudinally very easily there is no lateral shake or lost motion. (In subsection 2.8, which follows, the perspective line drawing, Fig. 24 shows this sliding shaft B' and its adjustable ball bearing guide along with a number of other clarifying details. The same reference letter system in Fig. 24 identifies the same parts as those on Figs. 9

and 10.) It will be noted that the displacement (by means of the precision screw) of the pivot, R', on carriage, L, relative to point, B, on carriage, Q, measures the sine of the Bragg angle, θ, for the incident beam, RC, relative to the crystal planes, CB. The distance from the pivot, V', to the point, B, on carriage Q is maintained rigorously equal to BR' by the two geared driving screws and thus the reflected gamma-ray beam is maintained always in strict alignment with the lead collimator.

The equal interpivotal distances, CV', and, CR', together with the pitch of the two master driving screws in the carriage, Q, have been so chosen that when the (310) planes of quartz are used, one revolution of the screws is almost exactly equivalent to one milliangstrom and, by means of a divided drum and vernier on the upper screw, motions corresponding to 0.001 milliangstrom unit can be recorded. Each screw advances its carriage about 1 mm per revolution.

Fig. 10 is a perspective line drawing of the spectrometer structure with the lead collimator in place at the extreme right and the spherical lead source holder on the upper beam clearly visible. A long tubular lead shield of rectangular cross section is carried on the upper beam to serve as a conduit for the gamma-radiation. (Comparison of Fig. 10 with Figs. 24 and 25 in subsection 2.7 will be found helpful.)

The adjustment provided to compensate for any slight departure of the (310) reflecting planes in the curved quartz plate from orientation normal to the faces is as follows. Referring again to Fig. 9, we have seen that if such an obliquity exists, the β point will not be quite diametrically opposite on the focal circle to the point, C, (of tangency of the neutral axis of the bent crystal with the focal circle). This then requires that the pivot, O, at the center of the focal circle (to which the radius bar, OR, is attached) should be slightly offset from the center line of the lower beam in the correct direction to compensate for the obliquity of the crystal planes so as to bring the β-point onto that center line. In rotating the focal circle through this small compensation the crystal should turn in unison with it so that the neutral axis of the latter always remains tangent to the focal circle. A subsidiary lever arm visible in Fig. 10 mounted on top of the lower beam, is therefore provided. This lever can be swung through a small angle relative to the lower beam by means of tangent screws and the pivot, O,

which is mounted on this lever can thus be displaced to suit the obliquity of the crystal planes. A clamp is provided so that the lever can be clamped to the pivot shaft which supports the curved crystal. In this way the crystal turns in unison with the lever to which the center of the focal circle is attached when this adjustment is being made.

The distance, CC, from the pivot for the center of the focal circle to the main crystal holder pivot and also the length of the radius bar, OR, are adjustable to suit the required diameter of the focal circle. This last must be determined experimentally by a method to be described (the "Hartmann" test).

Means are provided for an optical test to insure that the neutral axis of the bent crystal is indeed strictly tangent to the focal circle at the point C, or perhaps it would be more accurate to say that means are provided to insure that a normal to the bent crystal at the center of its arc passes accurately through the focal circle center. This is accomplished by a fitting with a conical socket which, after temporary removal of the radius bar, can be fitted over the pivot at O so as to define an axis truly vertically above the center of this pivot at the same height as the crystal. Light coming from a narrow slit placed exactly on this axis is allowed to fall in a diverging beam on a small region of the curved crystal surface exactly at its center as defined by a slot in a paper shield mounted on the crystal holder. The light reflected by the curved crystal will return to the slit as parallel light (because O is at the principle focus of the cylindrical curved crystal) and, if the crystal is correctly oriented, this narrow parallel beam will illuminate the two slit jaws equally on either side.

The entire design of the Mark I instrument was carefully planned so that it admitted of a rational procedure for measuring and adjusting all of the essential geometry of the instrument to insure its accuracy. Space limitations however preclude a detailed description of these many details. A simple but important general principle is worth mentioning however. This is that the position of a point in a horizontal plane or the position and orientation of a vertical axis in space can usually most conveniently be defined physically by means of the vertical axis of rotation of a pivot so designed as to be adjustable as to the position

and orientation of its axis, provided care is taken to arrange said pivot so that it can turn completely around through 360° unhindered and so that a sensitive level can be conveniently mounted upon it to permit the strict verticality of the axis of rotation to be tested and adjusted. The complete rotatibility of such a pivot also facilitates the easy establishment on the rotating system of crosshairs, slits, tight wires or any other desired physical object so that a desired feature of this object coincides accurately with the position and direction of the pivot axis. This is most conveniently done by observing the crosshair, slit, or other feature to be centered with a "travelling microscope" (a microscope mounted on a carriage adjustable with a micrometer screw) as the pivot is rotated and thus correcting the measured eccentricity. There are five vertical pivots in the Mark I instrument (C,O,R,R' and V' in Fig. 10) and these were all designed with the above principle in mind, a feature which has proved of great value in making many of the adjustments. Care has also been taken to provide true turned reference surfaces accurately concentric with the pivot axes at convenient locations on pivots and pivot housings in order to facilitate measuring interpivotal distances by means of long contact standards.

Fig. 11 is an assembly drawing showing transverse and longitudinal vertical section views through the long screw carriage Q while Fig. 12 is a vertical section through the main pivot at C which supports the curved crystal as well as the upper and lower beams.

Returning once more to Fig. 9 it should now be clear how, as carriage Q translates along track T and carriage L simultaneously translates in the same direction along C at equal speed, the point R (location of the source) and the point β simultaneously come into coincidence at V (as the instrument passes through the zero wavelength position) and cross over each other on the other side. The determination of the wavelength of a gamma-ray line then consists in delineating the profile of the line by observing the counting rate as a function of a series of closely spaced settings of the wavelength screw (the upper screw in carriage, Q, which drives carriage, L) first for reflection on the right hand side then for reflection on the left hand side. Ideally the number of turns (and fractions) of the screw in passing from one of

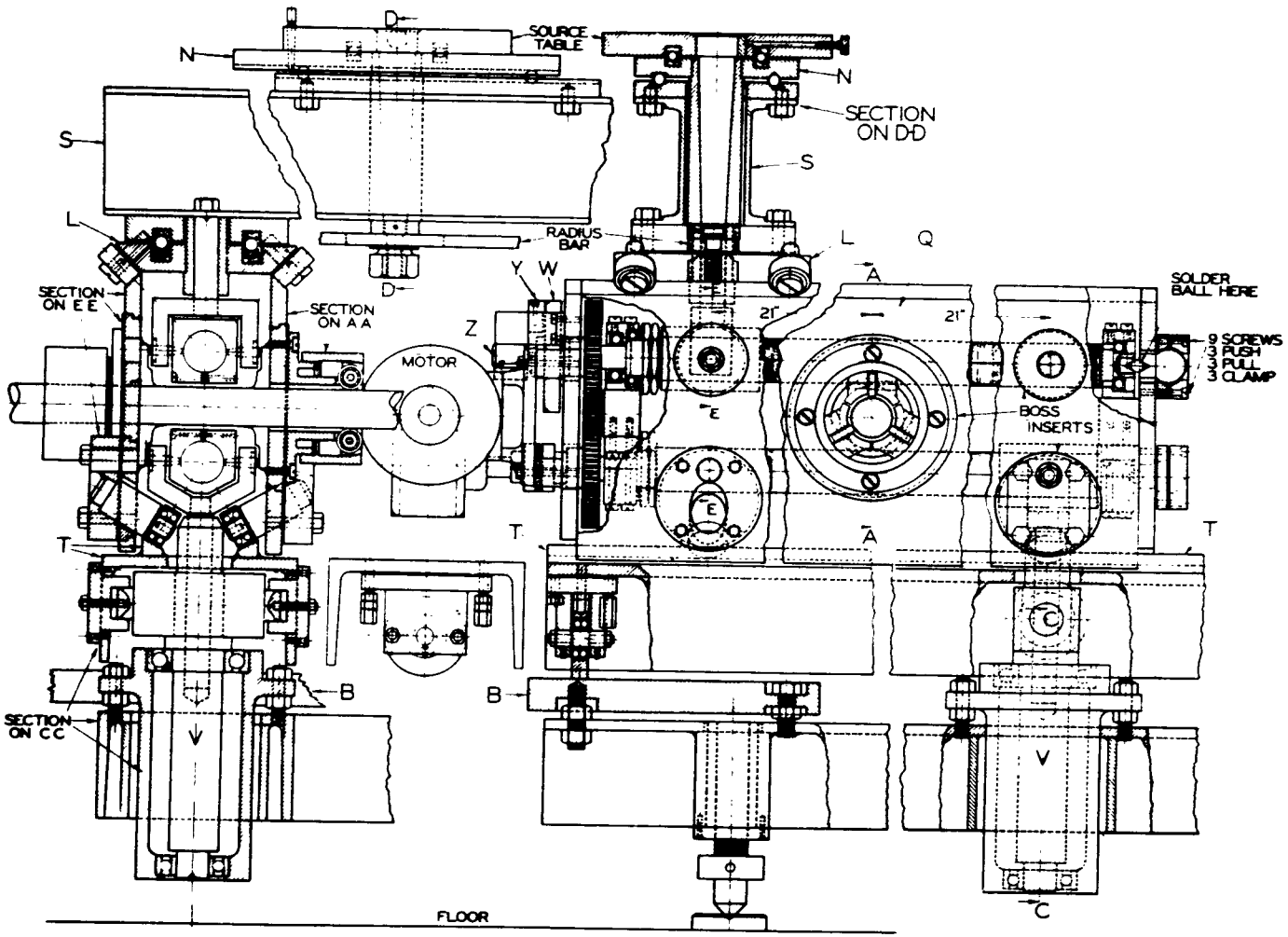


Fig. 11 Transverse and longitudinal vertical cross section views through the long screw-carriage, Q, of the Mark I gamma-ray spectrometer.

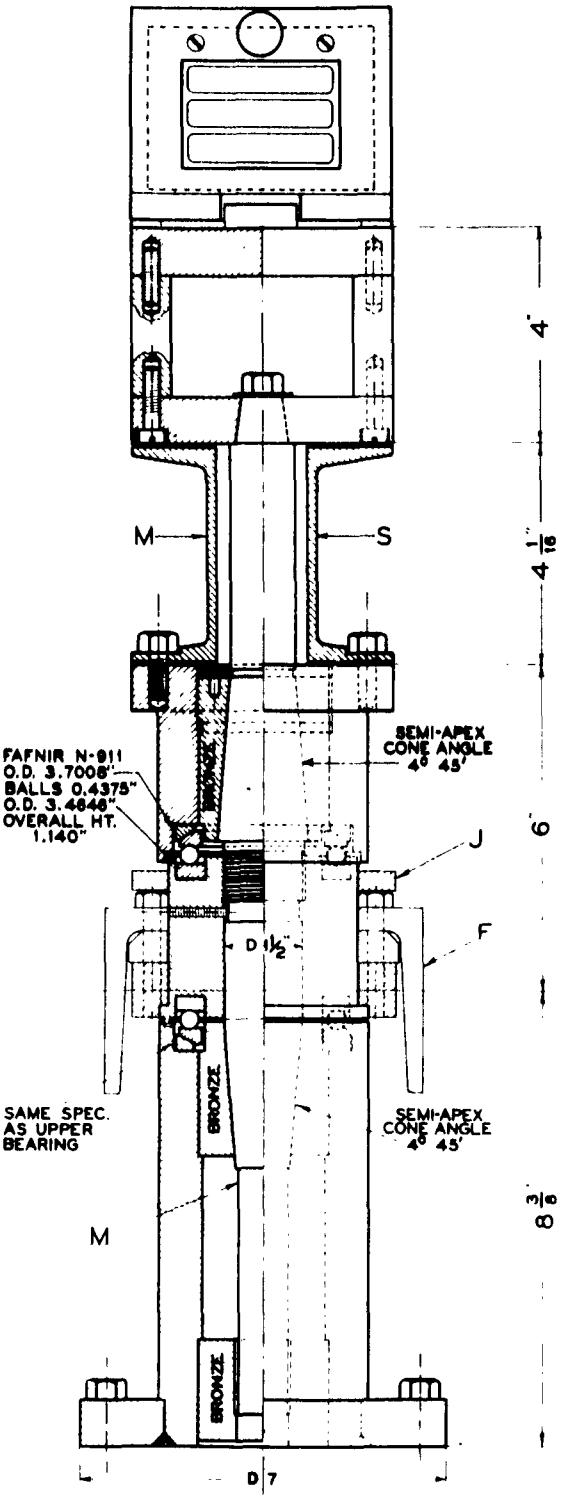


Fig. 12 Vertical half-section view of main pivot of Mark I gamma-ray spectrometer.

these line profiles to the other should be strictly proportional to the wavelength (after corrections for small departures from uniformity in the pitch of the screw). The constant of proportionality between screw readings and wavelength can be readily established by delineating the profiles of x-ray lines, such for example as the tungsten $K\alpha_1$ line, whose wavelengths have already been determined in angstrom units by other precision x-ray work already alluded to.

It should be noted that all that is required of this instrument in order to bridge the gap reliably between the known x-ray wavelengths and the unknown gamma-ray wavelengths, is that its wavelength scale (measured in turns on the screw after correction for its errors) shall be linear with wavelength. Therefore all sources of error which would require corrections to the wavelength of magnitude proportional everywhere to the wavelength itself (i.e., linear corrections) can be ignored since the calibration with x-rays automatically cares care of these. It is only non-linear corrections that are to be feared.

In contrast to the design of the Mark I just described, in both the Mark II and Mark III instruments the plan has been to keep the gamma-ray emitting source stationary and to explore the spectrum by turning the curved crystal through small angular displacements while at the same time turning the entire assembly of the collimator and the detecting agency through angular displacements just twice as large so as to satisfy the law of specular reflection. The feature permitting a stationary source is obviously highly desirable since it makes possible the study of sources situated inside chain reactors or large immobile particle accelerating machines. The plan in both these cases was to control the turning of the heavy collimator and detector system by a servo-mechanism and to utilize an optical light beam reflected from a mirror mounted on the crystal pivot as the controlling element.

When studies in different orders of reflection revealed that certain minute mechanical flexures in parts of the Mark I instrument were the causes of small systematic departures from linearity in the wavelength scale as read on the precision screw, D. E. Muller[21] conceived

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21) The objective here was to eliminate just as far as possible the errors incident to the use of mechanical parts of limited rigidity,

bearings and the like and to exploit as much as possible the precision and rigidity of light waves. A model of the Muller interferometer has been constructed and proven its feasibility. (This was described in the 22nd Quarterly Report to the Office of Naval Research, period July to September 1952, Contract No. N6onr-244, T.O. IV). Only one pivot (to support the curved crystal) is involved and the interferometer is so designed that minute irregularities and lost motion do not introduce first order errors in measuring the desired sine by counting interferometer fringes. The device has been briefly described in an article: J.W.M. DuMond, Physics Today 5, 13 (1952). After some work had been done on this it was discovered that U.S. patents have been granted to one E.R. Peck on an interferometric idea very close to this one. Continuation of work on the Mark II design is problematical at the date of writing and will depend on whether funds can be made available for this rather ambitious project.

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the idea of an optical interferometric method of measuring the sine of the Bragg angle which would eliminate to the greatest possible extent the defects and non-rigidities of a physical mechanism. This led to a proposed design which we shall here refer to as Mark II. In the Mark II, described in an appendix to this report, the all-important sine of the angle of rotation of the curved crystal away from the zero wavelength position was to be measured with high precision by means of an optical interferometer of very special design utilizing two internal trihedral mirrors.

In the very large (7.7 meter focal length) Mark III instrument for the neutron-capture gamma-ray studies at the Argonne N ational Laboratory, it is planned to measure the rotation of the crystal pivot (sine of the appropriate angle) with a Twyman interferometer whose moving carriage (which carries the moving optical flat) will be coupled to the crystal pivot by a simple sine lever mechanism. Less precision is aimed at in this instrument than in either the Mark I or the Mark II.

2.4 Source and Source Holder Designs

A typical neutron activated source for the Mark I instrument might consist of a thin rectangular strip of metal, such as tantalum or gold, of dimensions 30 mm x 5 mm and 0.05 mm thick. The strip is supported in the source holder on the beam with its long axis vertical and with its flat faces parallel to the direction of gamma-ray propagation from source to crystal. In such a case the geometry of the source itself plays the role

of a spectroscopic slit. To give the thin source mechanical rigidity, it is usual to compress it in an aluminum sheath (pure aluminum must be used, not alloy) which, depending on the hardness of the radiation to be studied and other considerations, may or may not be cut away on the edge where gamma-rays are to emerge. Fig. 13 shows cross section views of such sources, They may be made by flattening an aluminum tube in a vise so as to clamp the material tightly. The assembly of source strip and aluminum sheath is irradiated as a whole in the neutron flux in a chain reactor. The activities set-up in the aluminum if it is pure are very short lived and therefore do not constitute a difficulty. When the source material is a powder or a precipitate (as in the case of fission products such as Cs^{137}) the source may consist of two flat strips of aluminum held together with tiny screws. One of these may be provided with a shallow cavity to contain the evaporated precipitate or powder. This cavity is made of about the same dimensions as those given above for the metallic strip sources. The other aluminum piece then serves as a cover to seal in the active material.

The aluminum sheath is made with the same external dimensions in either of the two cases and these are so chosen as to fit smoothly in the 30 x 13 x 2 mm cavity in one of the source holders. The source holder may be of lead, tungsten, "heavimet" (a sintered composite of copper and tungsten powder) or uranium. Its shape is pictured in Fig. 14. It provides a means of holding the aluminum sheathed source in a vertical position and of defining the utilized outgoing beam of gamma-rays in the shape of a narrow wedge which just about fills the useful aperture of the crystal and no more. This is done by a long tapering opening visible in the figure. These holders are made in two separable halves for convenience, the division being along a vertical plane running longitudinally through the center. When the holder is of lead this material is cast in two stainless steel shells for rigidity, the lead being exposed only on the sides facing inward toward the source and the emerging beam of radiation. The outer steel surfaces are ground accurately plane and the inner lead surfaces are accurately machined in relation to these to insure correct positioning of the source. A set screw is provided to immobilize the source.

The source in this source holder is accurately centered over a

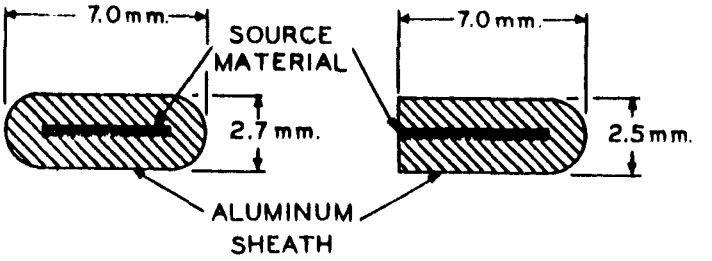

Fig. 13 Typical cross section of a source in its aluminum sheath. The closed form is preferable to avoid possible contamination of the source holder and other objects by minute amounts of active source material which might become detached. In the closed form it is therefore desirable, before irradiation, to mash the two ends of the aluminum sheath completely shut, the source strip being therefore made slightly shorter than the sheath.

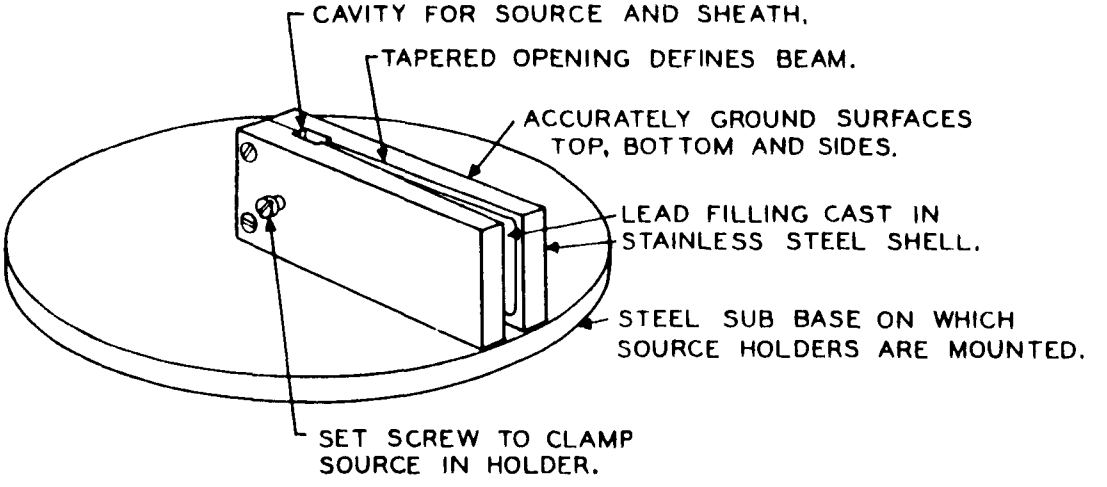

Fig. 14 Perspective line drawing of source holder of the lead-filled variety. Solid source holders of similar shape have been made of "heavimet" and also of uranium.

vertical pivot which connects with the radius bar defining the focal circle. This pivot is the center of rotation of a turn-table mounted on a carriage which can roll on ball bearings over a short longitudinal travel along the upper beam to accommodate the changing length from source to crystal as the spectrum is explored. The source holder is supported inside a spherical lead source bomb. In Fig. 24 of subsection 2.8 the source holder supported on the lower hemispherical half of this protective lead bomb can be clearly seen at R. In this figure the upper hemispherical cover of the bomb has been removed. The turn table and one of the centering screws for centering the source by rotation of the turn-table are clearly visible.

2.5 The Quartz Crystal and Its Clamping Blocks

Two general methods have been used by various experimenters to impose the requisite curvature on the focusing curved crystal lamina. In one of these which we might call the freely sprung lamina, equal and opposite torques are applied to the two ends of the lamina, (see Fig. 15), which is otherwise left free to assume its circular cylindrical curved shape by reason of the play of elastic forces[22] in it. The lamina must therefore be quite uniform in thickness and in elastic properties if the curvature is to be uniform and the all-important atomic reflecting planes correctly oriented. The second method, which has sometimes been called the "method of imprisonment," is the one utilized in the Mark I precision gamma-ray spectrometer and is the one which will be described here.

The quartz crystal slab, whose faces have been polished optically flat and closely parallel, is compressed between two hardened blocks of stainless steel, one with a concave, the other with a convex circular cylindrical profile, to hold the slab at the correct curvature. Only the

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22) J.B. Borovskii, Dokl. Akad. Nauk. SSSR 72, 485 (1950)
A.B. Gilvarg, Dokl. Akad. Nauk. SSSR 72, 489 (1950)
G. Brogen, Ark. Fys. 3, 515 (Paper 30, 1951)

Dr. Thomas C. Furnas, Jr. (see his Doctorate Thesis, M.I.T. (1952) has applied the method to a point focusing monochromator for low angle diffraction studies. A method somewhat similar has been applied in a commercially manufactured x-ray spectrometer based on patents assigned to the General Electric Co., by David Harker. In this case the crystal curvature varies as the spectrum is explored.

convex block actually determines the curvature of the quartz plate since a rubber gasket is placed between the concave steel surface and the quartz plate. The pressure is not applied directly by the screws which pull the two blocks together. Instead helical compression springs are used between the screws and the blocks to lessen the danger of too great a pressure which might break the crystal in the event of small dimensional variations from thermal expansion or accidents in adjustment. Fig. 16 shows the crystal clamping blocks and the ribbed window in the convex block through which the gamma-rays reach the crystal. The two horizontal ribs across this window not only serve the purpose of supporting the concave side of the quartz plate to insure a more accurate curved profile but they furnish an excellent opportunity to test the intimacy of the contact between the polished quartz surface and the very accurately ground and lapped convex surface of the steel by examination of the optical interference pattern formed in the interface.

The use of a special stainless steel[23] together with the procedure

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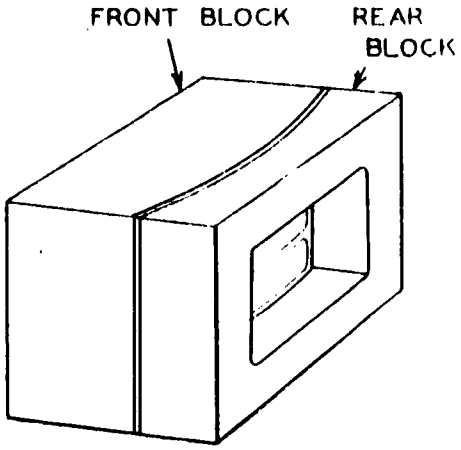
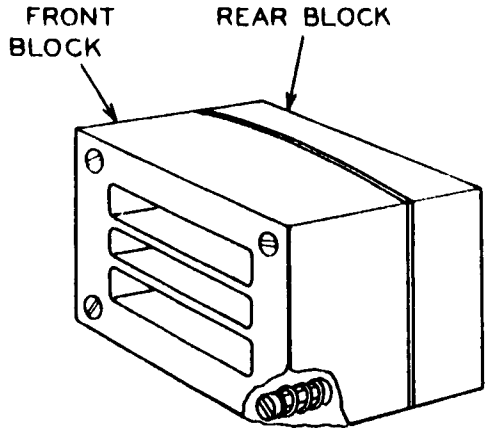
23) The material is a heat-treatable stainless steel having the composition, Cr 13.5 percent, C 0.35 percent, Mn 0.40 percent, Si 0.50 percent obtainable in America from Firth Sterling or Allegheny Ludlum Steel Companies. The recommended heat treatment is obtained by heating to 950° C and quenching in oil. Water must be avoided as it cracks the piece. This produces a hardness of about 75 measured on the Shore tests. The steel can then be "drawn" by heating in the furnace and cooling in the furnace or in air. Very little hardness is removed until 500°C is reached when it is about 66. Temperatures and corresponding hardnesses are 550° = 64, 650° = 46, 700° = 42, 750° = 36, 800° = 32. A good polish can be obtained under all these conditions. The higher drawing temperatures are recommended because they are believed to insure greater dimensional stability. This material, we were informed by C.G. Peters was extensively used at the U.S. National Bureau of Standards for precision gauges because of its excellent dimensional stability, when heat treated as prescribed. It takes a good optical polish. We have found it extremely stable and satisfactory as a crystal holder material. It has the fortunate property of possessing a thermal expansion coefficient which matches quite closely with that of quartz in the direction transverse to the optic axis.

Fig. 15 One method of bending the crystalline lamina: by application of equal and opposite torques or couples at its ends.



CRYSTAL CLAMPING BLOCKS VIEWED FROM EXIT SIDE.



CRYSTAL CLAMPING BLOCKS VIEWED FROM ENTRY SIDE.

Fig. 16 The stainless steel clamping blocks for bending the crystal. A rubber gasket is placed between the rear block and the crystal.

for hardening and tempering it was kindly recommended to us by C.G. Peters of the U. S. National Bureau of Standards.

We have used quartz slabs in the Mark I instrument some of which were 1 mm in thickness, others 2 mm in thickness. A valuable safe rule we have found for quartz of this order of thickness is that the quartz may be safely bent to a radius one thousand times its own thickness provided the surfaces and particularly the edges have been etched to remove the danger of tiny incipient cracks. The thicker lamina give higher reflected gamma-ray intensity but, for reasons not entirely clear to the writer, they also give somewhat broader lines even when the aperture of crystal used is stopped down to such a point that this broadening cannot be ascribed to imperfect focusing. It is difficult to see why the reasoning of Cauchois regarding focusing from front to back in the thickness of the lamina (for atomic reflecting planes normal to the slab) should not hold in a crystal which does not exhibit permanent plastic deformation. It is the impression of D.A. Lind[24] that, in spite of the complete absence of plastic yield or permanent dislocation in crystalline quartz when it is subjected to strain, quartz must be regarded as behaving like a mosaic rather than like a "perfect" crystal when it is bent and he ascribes this as probably the reason for the line broadening. The writer feels that the question is still not clearly settled.

The quartz is sawed with a diamond saw at the correct angle from perfect specimens of monocrystalline material known to be completely free from either optical or electrical twinning into slabs preferably about 5 mm or more thick and these are etched with H F to remove strains, and polished optically flat on one side[25]. It is well to prepare about

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24) D.A. Lind, W.W. West, and J.W.M. DuMond, Phys. Rev. 77, 475 (1950)

25) Companies engaged in the cutting of quartz for radiofrequency piezoelectric oscillators are usually well equipped to supply the raw monocrystalline quartz and to do the sawing. In the Southern California area we have had very satisfactory specimens prepared by the Monitor Piezo Products Co. The grinding, lapping, and polishing to optical flatness may be done by almost any firm engaged in precision optical glass work. We have been most grateful to the Penn Optical Co., of Pasadena, for their kind cooperation in work of this kind. The very large quartz lamina for the Argonne National Laboratory Mark III spectrometer is being procured from the firm of Hilger and Watts, London, who are furnishing the material and doing both the sawing and the finishing to optical flatness.

five slabs at once so that they can be conveniently arranged in an array which will, by addition of four triangular scraps of quartz, fill out a roughly circular area to be ground, lapped and polished. The optically flat faces are then "contacted" on a thick, rigid, optically flat backing and the pieces are milled down with a diamond cutter till the slabs have the desired 1 or 2 mm thickness. If these slabs are now removed from their backing, the polished faces may be found to have bowed 20, 30, or even 50 fringes out of plane, an effect which is probably due to strains introduced in milling off the opposite faces. This bowing can usually be readily removed by etching the milled surfaces. With the flat polished sides contacted on the flat backing support the back sides are then polished optically flat and parallel to the other sides. The plates should then be given a final etch to remove strains. This etching should be quite prolonged on the edges but may (in fact should) be quite light on the optically flat faces just enough so that very faintly visible marks, residues from the grinding and polishing, begin to appear. We believe these marks are residual scratches made by abrasion in grinding. The process of fine polishing is in part one of levelling off protuberances but also partly one of filling up minute scratches and furrows with submicroscopic material so small that it adheres rather tightly to the quartz and cannot be resolved with wavelengths of visible light. This detritus is removed, by etching, much more rapidly than the solid quartz.

The all-important procedure[26] for the precision profiling (to 2 meter radius in the Mark I) of the convex cylindrical stainless steel block, upon which the high resolving power of the instrument is entirely dependent, will now be briefly described. In this gamma-ray spectrometer, unlike the Cauchois photographic instrument, the entire crystal aperture is used simultaneously at each wavelength setting on the focal circle and there must therefore be the most perfect focusing to as nearly one and the same point as possible over the entire crystal aperture if the lines are not to be unduly broadened by this cause. We have ample evidence that the highest degree of precision isrequired in this profiled surface.

The method depends on the inverse of the familiar geometrical

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26) This procedure has been described in greater detail in an article by J.W.M. DuMond, D.A. Lind, and E.R. Cohen, Rev. Sci. Instr. 18, 617 (1947) which should be consulted by those planning to apply this method.

theorem about the constancy of the angle inscribed in a constant circular arc. Referring to Fig. 17, the shaded piece or template may be thought of as cut out of sheet material with two straight edges forming an obtuse angle whose vertex is A. If the straight edges are held always in contact with pins inserted at P_1 and P_2 in a table while the template is displaced laterally, the vertex, A, will describe a circular arc on the table whose radius of curvature R is given by the formula,

$$R = L/(2 \sin \theta) \qquad (8)$$

where θ is the supplement of the obtuse angle formed by the straight edges and L is the distance between the pins, P_1 and P_2.

This principle is applied by adding a few auxiliary elements, easily obtainable in any machine shop, to an ordinary flat surface grinder[27]. Such machines have a carriage on which a magnetic chuck usually carries the work to be ground and which executes a rapid transverse horizontal reciprocating motion carrying the work to and fro under the grinding wheel. This entire system is mounted on a second carriage which provides a slow horizontal feed motion in small steps at right angles to the first mentioned reciprocating motion. A third vertical motion permits the grinding wheel to be lowered in very small steps, 0.0001 inch at a time. The auxiliary elements to adapt the grinder to precision profiling of the convex or concave cylindrical surfaces can best be understood by reference to the somewhat idealized drawing, Fig. 18. In this figure the steel block to be profiled is shown at B mounted on a rigid bracket, 4, projecting under the grinding wheel and capable of swinging or rocking on a pivot, 6. The transverse horizontal carriage motion carries this entire assembly to and fro under the grinding wheel and the work must be planned in such a way that on the last cut taken by the grinding wheel the center of the latter is exactly at the same height (the position shown in the figure) as the line of travel of the pivot, 6, in its to and fro motion. The two extreme positions of the rocking pivoted bracket in its to and fro motion are shown with the full and dotted lines in Fig. 18. The appropriate rocking motion is communicated to the pivoted bracket, 4, as it reciprocates horizontally, by means of an arm

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27) A Brown and Sharpe No. 5 surface grinder was the one used for the Mark I crystal blocks.

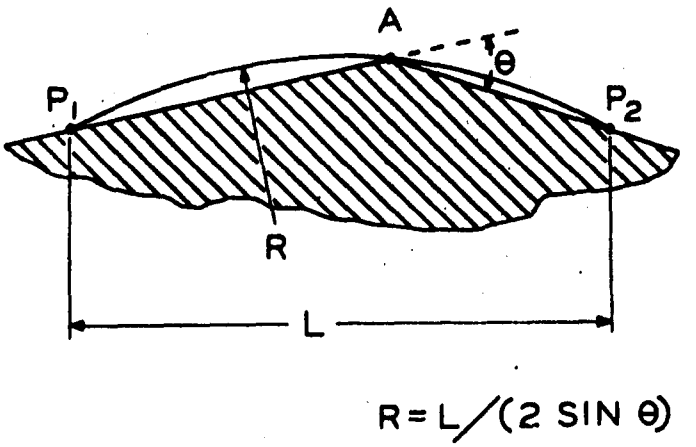

Fig. 17 Method of describing a circular arc of large radius of curvature R using a piece of thin rigid sheet material provided with two straight edges which form a rigid obtuse angle held in contact with two fixed points P_1 and P_2 separated a distance L apart. The vertex, A, of the angle describes the arc.

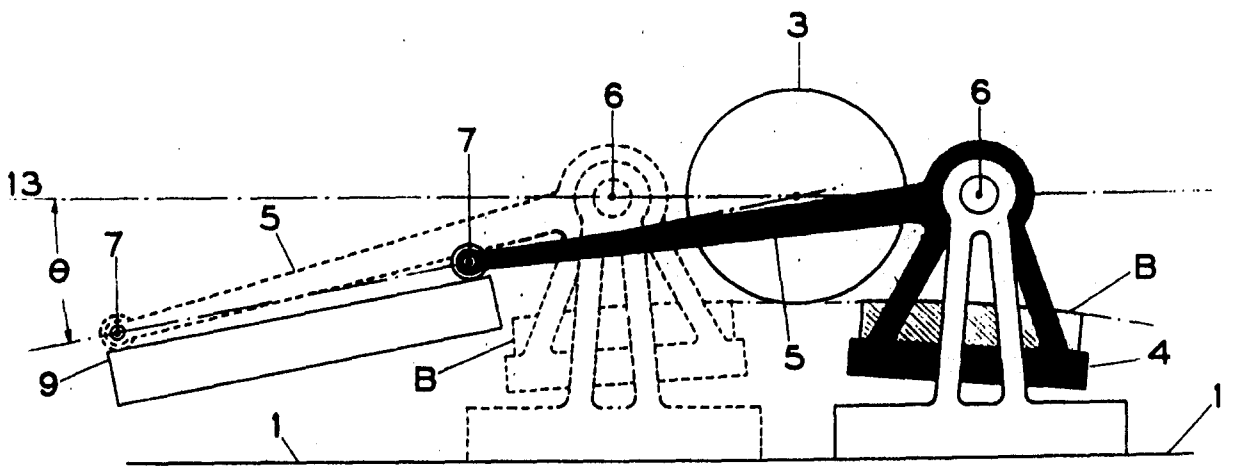

Fig. 18 Schematic view illustrating in principle the method of grinding a convex cylindrical surface of large radius on the surface grinder. For the actual set-up see Fig. 19. This view is not to scale and the curvature of the block is greatly exaggerated for clarity.

5, rigidly coupled to 4. The end of this arm is provided with a ball bearing roller at 7 which rolls up and down an inclined straight edge, 9 (of hardened steel) as the carriage, 1, on which the assembly is mounted executes its transverse travel. The straight edge 9 is rigidly supported relative to the second carriage, the one having the slow feed motion normal to the plane of the picture, by means of a bridge-like structure under which the carriage, 1, can travel freely. The position of the straight edge must be carefully adjusted so that the line of travel of the center, 7, of the ball bearing roller, if extended, would pass exactly through the center of the grinding wheel when the final cut is being made. Fig. 19 is a line drawing showing with less idealization a plan view (above) and an elevation view (below) of the entire arrangement on the surface grinder.

With a little consideration the reader can readily convince himself that this arrangement is equivalent to the simple geometry of Fig. 17. The reader must identify the system of the block, B, the rigid bracket, 4, and the arm 5 with the stationary table. The points 6 and 7 of Fig. 18 correspond to the two pins P_1 and P_2 in Fig. 17. The surface grinding machine with its carriages corresponds to the obtuse angled template. Thus the parts which move and the parts which are stationary are interchanged in going from Fig. 17 to Fig. 18, but this is the only slight difficulty calling for a little effort of imagination. The horizontal line of travel of the axis, 6, and the oblique line of travel of the center of the roller, 7, furnish the physical realization of the two straight edges of the template of Fig. 17. The angle of obliquity, θ, of Fig. 16 is the supplement of the obtuse angle of the template in Fig. 17. The center of the grinding wheel is the apex of the obtuse angle of the template. The fixed distance between centers 6 and 7 in Fig. 18 is the distance, L, of formula (8). The radius, R, of formula (8) corresponds to the radius described by the center of the grinding wheel relative to the system of the work, B, and the bracket, 4, and arm, 5. Clearly, if r is the radius of the grinding wheel, the radius generated on the work, B, by the cutting surface of the grinding wheel will be R-r in the case shown in Fig. 18 which is the case which gives a convex profile. In the case of a concave profile the radius generated on the work, B, will clearly be R+r. Thus in order to generate a concave profile of the same radius as the convex profile, without changing the distance between pivots 6 and 7, the angle, θ, must be slightly

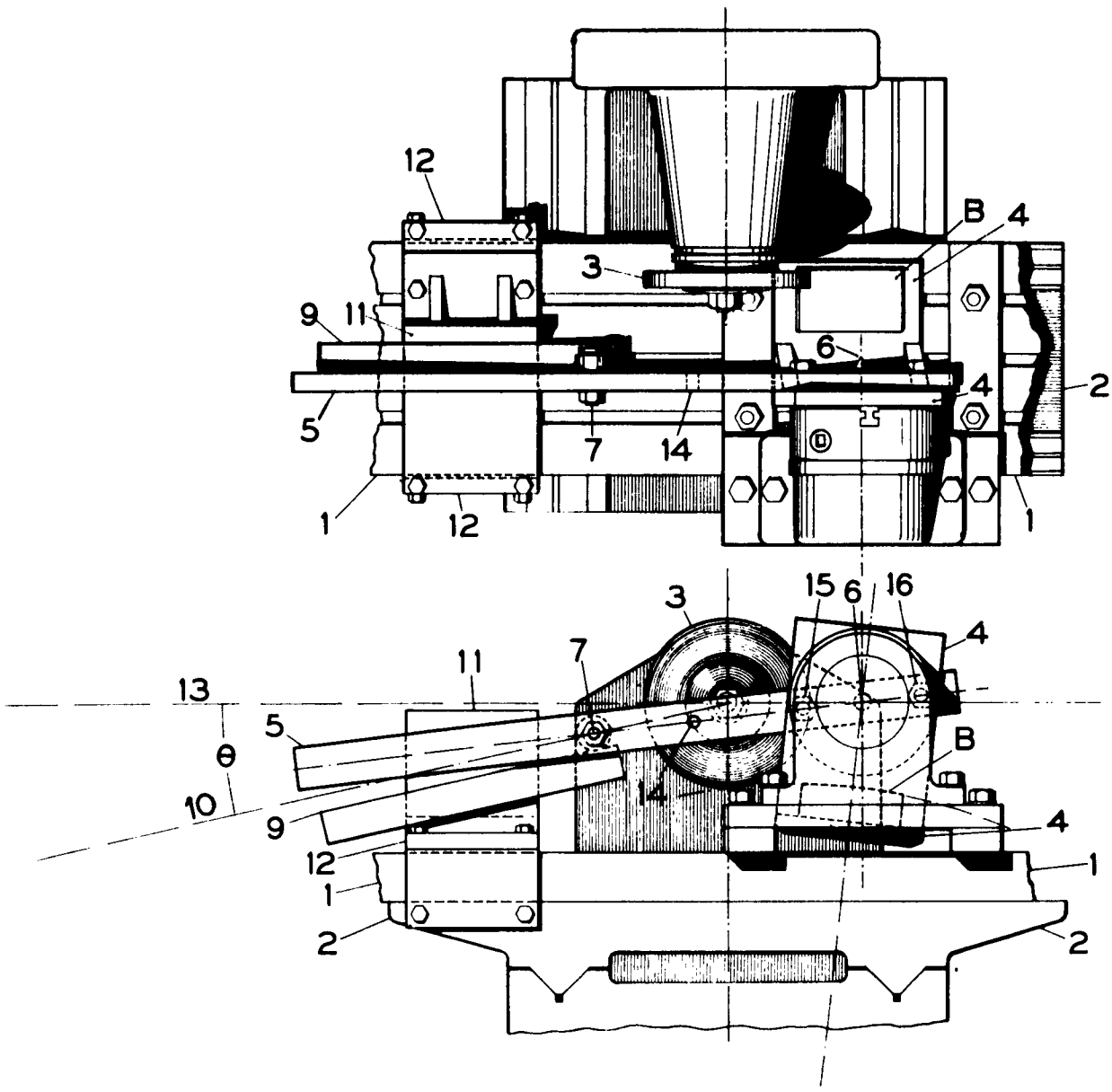

Fig. 19 Upper view is a plan and lower view an elevation of the set-up on a surface grinder for profiling a convex cylindrical surface (on block B). The curvature of the surface and the angle θ of h the inclined plane 9 are exaggerated for better clarity and the drawing is not to exact scale.

modified. The passage from the case of the convex to the case of the concave profile is easily accomplished by either one of two changes: (1) The straight edge 9 may be placed above the line of travel 13 so that it slopes downward at the requisite computed angle θ as one goes from left to right, or (2) if may be left in its general position shown in Fig. 18 (but with the above mentioned appropriate slight modification in the value of θ) and the bracket 4 may be turned so that the work, B, travels above the grinding wheel, the cutting point being then near the top of the wheel. For either of these changes the rigid relationship between the arm 5 and bracket, 4, which in Fig. 19 are seen to be clamped to each other with cap screws, must of course be remodified. The technical and practical details of all these procedures and adjustments are much more explicitly described in the article[26] above referred to.

The advantages of the method are numerous. (1) In rather compact compass with readily obtainable components, which because of their small dimensions can be very rigid, one can generate cylindrical surfaces, with all the precision a surface grinder is capable of, and the radius of such a surface can be made as large as we please up to infinity. (2) By observing the proper mechanical adjustments in setting up the work in the grinder, the generated cylinders can be made free from all conical or saddle-shaped errors (as regards the parallelism of the rectilinear generators of the cylindrical surface) to the same surface-grinder precision (about 0.0001 inch). (3) The method provides a procedure to insure that the generators of the cylindrical surface (the lines on that surface which are strictly parallel to the axis of the cylinder) shall be strictly normal to certain plane reference faces of the block B (or parallel to certain other reference faces) which have been ground on the block before the curved profiling is started. This is an extremely important point both to give reliable surfaces on the block to permit its correct alignment in the spectrometer and, more importantly still, to afford a means of aligning the convex block correctly in relation to the concave cast iron lap from which it receives its final surface finish.

The precision of 0.0001 inch in the generated cylindrical sufface obtainable with the surface grinder is not sufficient for the extreme demands of crystal spectroscopy of gamma-rays. The surfaces must be

corrected still further by a lapping procedure in which a concave cast iron lap of as closely as possible the same radius as the convex crystal clamp is rubbed against the latter with an extremely fine abrasive material of particles less than 2μ in size. If the grinding of the two surfaces has been carefully done in every respect, very little lapping will be required (perhaps 30 minutes net rubbing time all told) toobtain complete contact over the entire surface combined with a good mirror polish. It is best that the lapping should not need to be prolonged because lapping is a process resembling the convergence of the so-called "asymptotic infinite series" in mathematics - at the beginning, lapping improves the precision but the longer it must be continued the greater the chance that the point of minimum error will be passed and the errors will, from then on, get worse and worse in a way very difficult to correct.

Fig. 20 shows the simple hand-lapping machine which we have used to prepare several sets of blocks for Mark I. The two pieces to be lapped together are first carefully aligned with each other along two reference edges before the cap screws securing the blocks to the base or to the parallelogram motion are tightened. This insures that the generators of the two cylindrical surfaces are strictly parallel.

To insure better accuracy, one of the two blocks in the lapping machine is frequently turned through 180° relative to the other merely by disengaging the "self-aligning" ball bearings from the two projecting pins on the movable plate to which the upper block is bolted and subsequently reengaging them after the reversal.

The procedure is to grind not only the two (concave and convex) stainless steel crystal clamps but at the same time also to grind cylindrical surfaces on each of two cast iron blocks. Each block should be provided with five true plane-ground reference faces. Cast iron is a very desirable material for lapping the stainless steel since it is soft and becomes "charged" with the fine (2μ) abrasive agent[28] so that when the

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28) A very convenient lapping compound in which the abrasive particles are suspended in heavy grease is sold in small jars by the U.S. Products Corporation, 518 Melwood St., Pittsburgh, Pennsylvania. Two grades have proven useful. No. 305-A serves for faster cutting at the start. It should be followed by No. 38-1200 (the 2μ size). This product has proved extremely convenient and reliable for use in this lapping process.

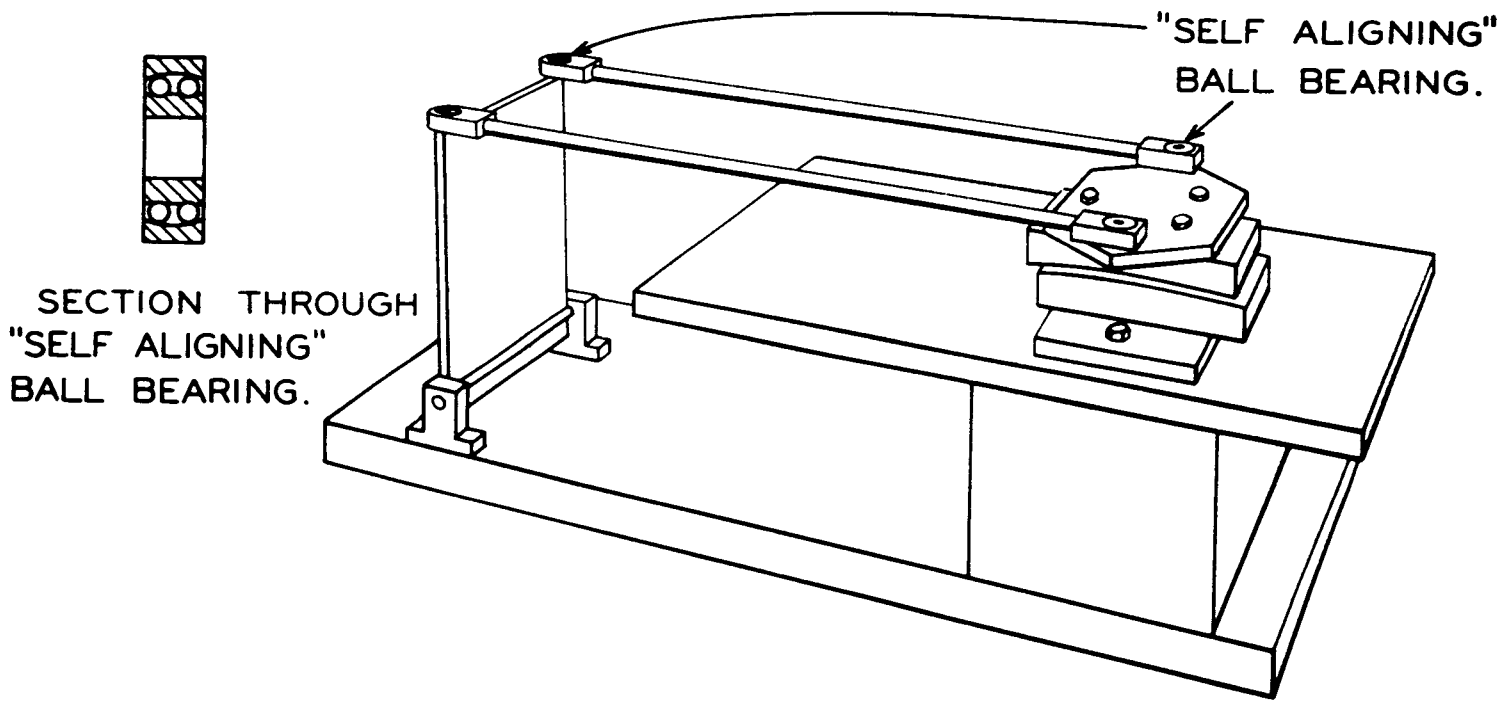


Fig. 20 Hand lapping machine.

cast iron lap is later rubbed on the hardened stainless steel surface the profile of the steel is corrected without much modification of the profile of the cast iron lap. The procedure is first to lap the two (concave and convex) cast iron blocks against each other till a good mirror surface is obtained. The concave cast iron lap is then tested optically by familiar methods similar to those of the Foucault "knife edge" test although the knife edge method is somewhat more rigorous than necessary and it really suffices very well merely to examine the image of a fine "straight-line filament" lamp (as formed by reflection in the concave surface) with a low power microscope while stopping off different regions of the mirror with paper shields. Usually no local correction of the lap will be necessary at all since the procedures described above, if correctly followed, almost guarantee a perfect result the first time.

The convex crystal block is next lapped against the concave cast-iron lap until perfect contact with a mirror finished surface has been obtained. The concave cast-iron lap may then be optically tested once more. If it is found to have lost its accuracy of figure somewhat it may be corrected once again by lapping the two cast iron blocks together till good contact is regained and the concave lap passes a good optical test. Then one returns to the convex crystal clamping block and gives it a final correction of figure with the corrected concave cast iron lap. Usually this second approximation is all that is needed.

The mounting of the crystalline lamina in the clamping blocks may require considerable patience to eliminate invisible particles of foreign matter from the interface between the quartz and the convex steel block. The work should be done under a transparent dust cover in as dust-free an atmosphere as possible. The quartz and the steel surfaces to be placed in contact are repeatedly washed with acetone and wiped with "lens paper". The lamina is clamped under pressure of the screws and springs and the colored interference fringes formed in the quartz--steel interface are observed. The central black fringe should occupy most of the width of the curved arc over the entire working aperture of the crystal. The first colored fringes may be permitted to show in the outer ten percent of the width on either side. The convex quartz surface is pressed with the eraser end of a lead pencil and if there is a particle of foreign matter

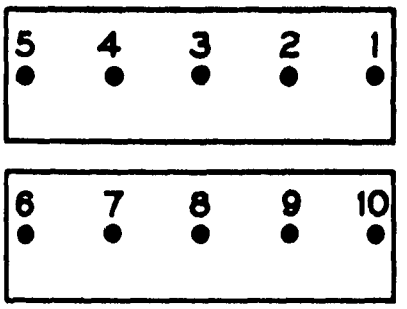
caught between steel and quartz the fringes will be observed to move as a result of such pressure, except when the pressure is applied in the region where the foreign matter is located.

The correct orientation of the quartz plate in the holder is less difficult if x-ray goniometer tests have been made and one reference edge has been ground on the quartz lamina strictly parallel (or perpendicular) to the (310) planes. This precaution, which is much to be recommended, can readily be specified as part of the contract with the firm which furnishes and saws the crystal slabs since manufacturers of piezo quartz crystals and the like are usually well equipped to do just this.

After the quartz plate has been mounted in its steel clamp, its concave cylindrical surface can be tested with optical light for correctness of curvature (focus). A still more significant test however, which the writer regards as indispensable in short wavelength gamma-ray spectrometers of the types here described, should be made using reflection of the x-rays or gamma-rays themselves from the atomic planes of the curved crystal slab. We shall refer to these as "Hartmann" tests, drawing the name from analogy with the well-known Hartmann method of testing the aberrations of focus of large astronomical mirrors. Formerly a fine slit was installed in place of the source holder and an x-ray tube mounted on the upper beam of the spectrometer behind the slit was used as the source of the characteristic line x-rays. Fortunately this is now done much more conveniently by using a neutron-activated radioisotope placed in the source holder. A strip of tantalum, say 0.05 mm x 5 mm x 30 mm, which has been "activated" in the strong neutron flux in a chain reactor to an activity of say 500 mc through the presence of the unstable β-emitting isotope Ta^{182} will emit, beside its rich series of nuclear gamma-ray lines copious tungsten and also tantalum K-series x-ray lines of ample strength for such a Hartmann test. Such a tantalum source is supported in the source holder with its long axis vertical and with its thin edge facing the crystal. The source itself then constitutes the resolving "slit". For the Hartmann test a lead shield is provided for the crystal aperture so that only one small portion of that aperture is used at one time. As many as 15 subdivisions of the total aperture have been made in this way. The Hartmann test consists in plotting for each of these isolated portions of the crystal the line profile of say the $K\alpha_1$ x-ray line of tungsten which might, in the Mark I

instrument have a total width at half maximum height of 0.12 mm displacement of the wavelength screw carriage L. Settings might be made every 0.02 mm with accumulation of counts for a chosen interval at each setting of say 2 minutes, to delineate these profiles. It might then be found that the positions of the centers of the line profiles as obtained using the several regions of the crystal might fail to coincide by some small fractions (say tenths) of a mm. (1 mm quite closely approximates 1 x.u. or 1 milliangstrom in the Mark I instrument.) A diagram of the kind shown in Fig. 21 is then plotted. The positions labelled 1, 10, 2, 9, 3, 8, 7, 5, 6 show to exaggerated scale the various positions at which the center of the line was focused by each portion of the curved crystal aperture bearing the corresponding number. The total aperture was in this test subdivided into ten regions, and the locations of the centers of these in the crystal aperture are shown in the upper part of the diagram. In the case illustrated in Fig. 21 we have chosen a scale of 100 to 1 to represent the lateral displacements of the different line profiles. Then one computes the tangents of the angles of obliquity of the different rays from each of the numbered regions of the crystal aperture to the focus and one multiplies these tangents by the lateral scale multiplication factor and with these obliquities one plots the rays associated with each region of the crystal to the point where it formed the center of the line profile from that region. The region where these intersecting lines give the sharpest focus with least aberration can then readily be found as shown in Fig. 21 and one learns from such a diagram just how much to increase or diminish the diamter of the focal circle in order to obtain the very best focus. One also learns from such a study the width of the pattern formed by the aberrational errors of focus of the curved crystal. To date the best we have been able to achieve in reducing this width due to aberrations of focus from all over an area of 20 cm^2 of crystal aperture at a focal distance of 2 meters has been an aberration width of about 0.05 mm.

Four sources of line breadth should be listed in the gamma-ray crystal spectrometer. These are: (1) The natural spectral breadths of the gamma-ray or x-ray lines. Most nuclear gamma-ray lines are far too narrow (frequently less than 1 part per million) to have detectable breadths with this crystal diffraction instrument. X-ray natural line



NUMBERING SCHEME GIVING
POSITIONS STUDIED IN CRYSTAL APERTURE

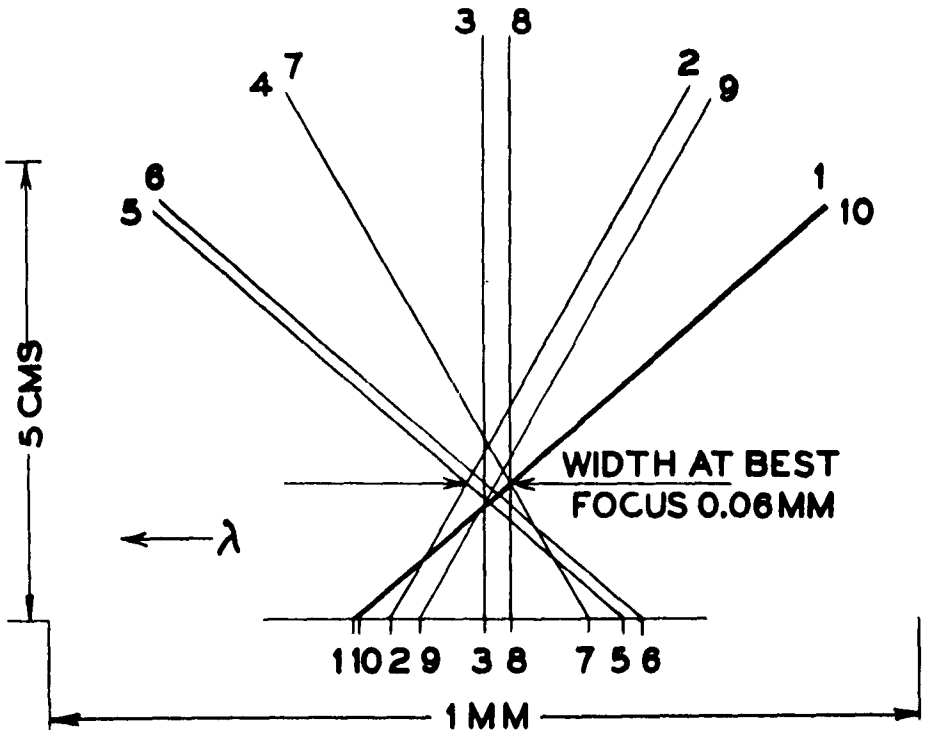


Fig. 21 The "Hartmann" optical test adapted to testing the aberrations of focus of the curved crystal with an x-ray or gamma-ray line. The diagram of intersecting rays, plotted from data on the minute displacements of the line obtained using the 10 different numbered regions of the crystal, is used to determine the best focal position and the width of the aberration pattern at that position. The transverse (horizontal) scale of this diagram is 100 times the vertical scale. The ten numbered positions in the crystal aperture are indicated above.

breadths are easily detected however, giving noticeably broader line profiles than gamma-rays, and this serves as one valuable way of discriminating between the two types of radiation when an unknown spectrum is being examined. (2) The geometrical width of the emitting source or the limiting slit on the focal circle. The distribution may or may not be uniform across this width depending on the preparation and nature of the source. (3) The aberrations of focus from residual imperfections of the crystal and of the curvature of its neutral axis. This is the component contributory to the line breadth which the Hartmann test determines. (4) The Darwin or Ewald diffraction pattern width intrinsic to any small region of the crystal itself including any mosaic or other effect due to crystalline structure. The observed line breadths and structures are to be regarded as the "fold" ("faltung") of all four of these sources combined. Fig. 22 gives a rough idea of typical relative sizes and shapes of these contributing factors.

D. A. Lind as his doctorate thesis[29] has studied the integrated reflection coefficient for the (310) planes of quartz as a function of wavelength for curved quartz plates over the large range from 700 x-units to 9 x-units and has found that the integrated reflection coefficient is proportional to the square of the wavelength. Tests made by William W. West, on the contrary, show that for unstressed quartz plates the integrated reflection coefficient falls off with diminishing wavelength, λ, even less rapidly than the first power of λ, (roughly as $\lambda^{1/2}$ in fact). These results are interesting from the point of view of purely empirical facts. Their theoretical interpretation is less clear however since no "dynamical theory" of x-ray reflection in elastically curved crystals has yet been worked out either for the mosaic or perfect crystal cases.

It is worth noting that, thanks to the design in the Mark I which permits delineation of line profiles for reflection on both sides of the crystal planes, it is possible to infer as to the cause of any asymmetry in line profile if such is observed. If the asymmetric profiles on the two sides stand in mirror image relation to each other the asymmetry is attributable to the spectral shape (close unresolved companion lines, for example). If the asymmetry of the line profile faces in the same spatial

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29) A published account has been given of these results. D.A. Lind, W.W. West, and J.W.M. DuMond, Phys. Rev. 77, 475 (1950)

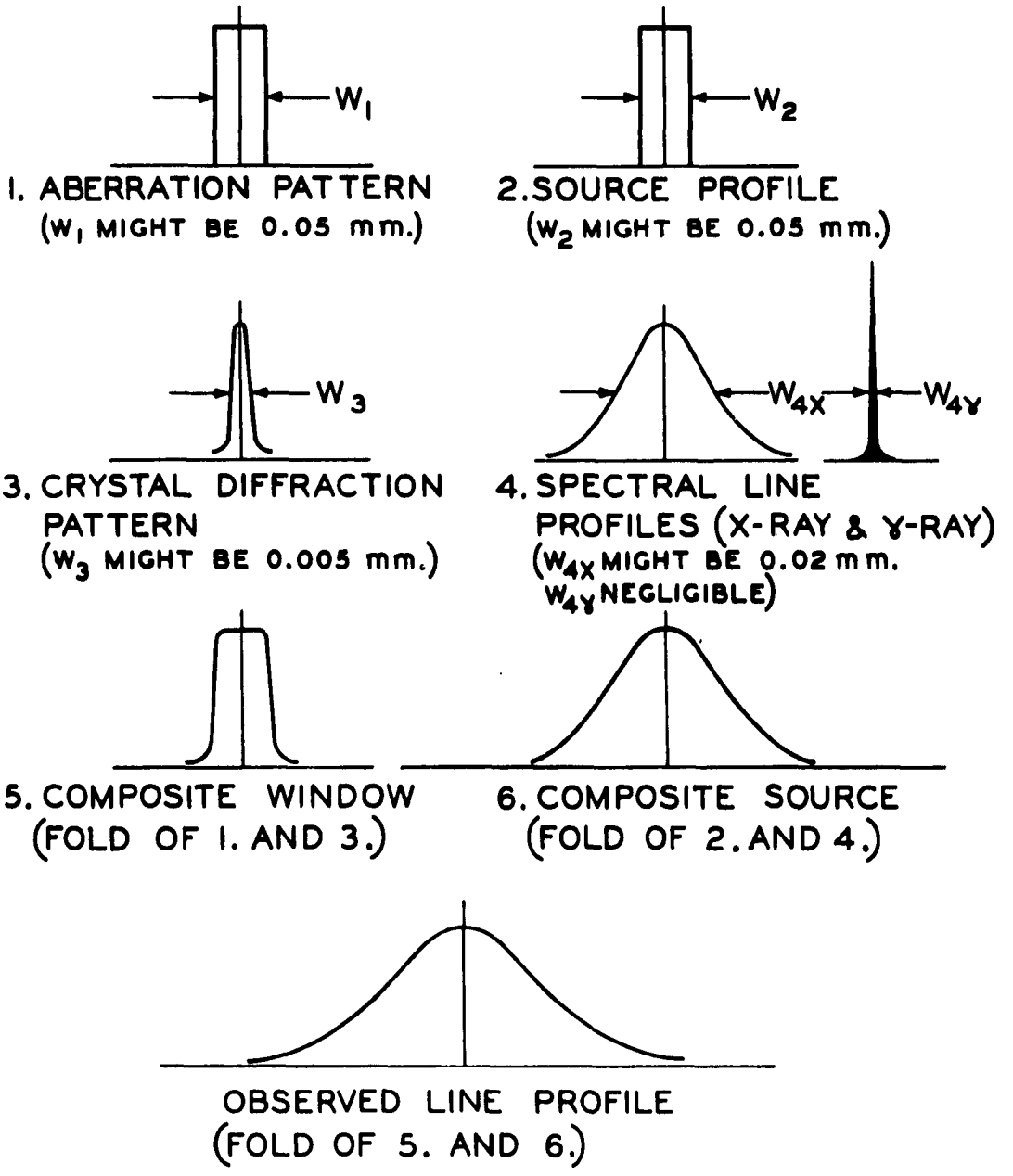

Fig. 22 To illustrate the four contributing sources of line broadening in the gamma-ray spectrometer whose fold yields the observed line profile. 2 and 4 folded together give the source profile, 6. 1 and 3 folded together yield the composite "window", 5, with which 6 may be conceived to be explored. The resulting "line" is shown at bottom.

direction on both sides (i.e., toward increasing wavelengths on one side and toward decreasing wavelengths on the other) then it is a characteristic of the source distribution or of the aberrations of focusing of the crystal (i.e., it is an instrumental artifact).

2.6 Procedure in Making Precision Wavelength Measurements. D.E. Muller's Method of Superposition of Profiles.

Asymmetric profiles of the last above mentioned type, though they are infrequent, do not interfere seriously with precision wavelength determinations. This is because to compute the wavelength, we determine the interval between the right-hand and left-hand orders of reflection of a given line by ascertaining just how far (in turns and fractions of turns of the precision wavelength screw) a standard master "composite" profile must be displaced in order that it shall superpose in best register throughout its entire profile structure upon the two profiles in question. (If any difference in intensity exists in the two profiles their ordinates are first normalized to equality before effecting the superposition.) The standard master "composite" profile, the registering tool to be used in determining displacements, is prepared by taking an average of carefully selected line profiles made with the same source sample. Its shape is thus characteristic not only of the slight aberrations of focus of the crystal but of the particular distribution of activity which obtains in that source sample. If there is no reason to expect any appreciable contribution to any of the observed line profiles from the natural spectral line structure (this is practically always the case for γ-ray lines) then all of these profiles may be averaged together to form the master composite tool. In the case of x-ray lines somewhat more circumspection is needed since the contribution of the natural spectral line breadth is detectable Usually x-ray lines of the same series have quite closely the same natural breadth and one master profile will serve for all. The use of a master profile is also of value since, by its occasional failure to superpose in good register on a given profile, it may reveal that the line is multiple in character or that some unsuspected weaker line (perhaps from an impurity) is closely superposed upon its structure. It will be noted

that any slight asymmetry introduced into the line profile by aberrations of curved crystal focusing and by non-uniform distribution of source activity interferes not at all with the above described method of matching profiles to determine wavelengths with high precision because both asymmetric profiles to be matched face the same way.

A quantitative analysis of the factors governing the precision with which wavelengths can be determined by the profile matching method is given[30] in Appendix II of this report.

2.7 The Gamma-Ray Detecting and Intensity Measuring System

The first method tried in the Mark I instrument for measuring the intensity of the reflected beam was by means of the "multicellular G.M. counter" developed by D.A. Lind[31] in which the gamma-rays passed successively through a series of thin absorbing septa from which electrons were ejected into the intervening gas filled spaces where they initiated "counts" by reason of various forms of fine wire anodes at high potential. This counter which presented a large cross section for cosmic ray counting had to be provided with a battery of 14 anti-coincidence counters surrounding it to reduce the resultant background. The maintenance of all this complicated counting equipment in stable operation was very troublesome indeed.

The present system, which is far more stable, sensitive, and satisfactory, makes use of a scintillation crystal[32] of thallium-activated

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30) This analysis due to D.E. Muller was first published in a paper by D.E. Muller, H.C. Hoyt, D.J. Klein, and J.W.M. DuMond, Phys. Rev. 88 775 (1952)

31) D.A. Lind, Rev. Sci. Instr. 20, 233 (1949). Many variants on the instrument described in this article were tried over a period of several years in an effort to develop a stable and easily constructed multicellular counter for gamma-rays, all to no avail. The results were very erratic and unpredictable. One or two examples were constructed which for some unknown reason operated rather well for months and then without warning even these became inoperable. The advent of the scintillation crystal detector has come as a very welcome solution to all such troubles.

32) We are much indebted to Prof. Robert Hofstadter of Stanford for generously supplying us with a large fund of extremely valuable information which has been of great help in the development of our scintillation detector. See J.A. McIntyre and R. Hofstadter, Phys.Rev.78, 617 (1950)

sodium iodide. The sensitive element is a monocrystal, 3 x 3 x 1 inches in dimensions, hermetically sealed in a rectangular thin walled glass box fitting closely around the crystal with mineral oil filling the space completely between crystal surfaces and glass. The crystal is placed so that its 3 x 3 inch face intercepts the diffracted gamma-ray beam from the quartz crystal immediately after it issues from the collimator. Scintillations from this crystal are detected by two R.C.A. 5819 photomultiplier tubes[33] making optical contact with two opposite 1 x 3 inch faces of the crystal through Plexiglass light conductors. These two tubes are operated in coincidence with a coincidence resolving time of about 1 microsecond, to exclude a large number of random shot noise signals from the photomultipliers. Such spurious signals would otherwise produce pulses in the counting circuit indistinguishable from those produced by low energy gamma-rays and thus cause a very troublesome background. Fig. 23 is a line drawing showing the scintillation-crystal-photomultiplier detecting system with its lead shielding removed.

No anti-coincidence shielding counters are found necessary with this system. Instead, pulses from the coincidence circuits whose magnitudes fall within a chosen range are selected by means of adjustable upper and lower pulse-height discriminators, thus providing a means of rejecting pulses due to cosmic rays (which are generally very large) and some unwanted gamma-rays. The lower bound of the pulse height discriminator eliminates many smaller pulses from tube noise and other accidental sources.

The monochromatic diffracted beam from the crystal, which it is the object of the scintillation detecting system to measure, is of course far more strictly monochromatic than the pulse-height discrimination method of Hofstadter is capable of resolving. About the best that has been accomplished with the scintillation method is an energy resolution of 10 percent, and in the present Mark I instrument with a rather large crystal and only two phototubes "looking into" a rather limited fraction of its surface the resolution by scintillation pulse heights is considerably lower still, of order 20 percent. The energy resolution of the

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33) We have recently acquired tubes manufactured by the Dumont Co., which appear to be superior.

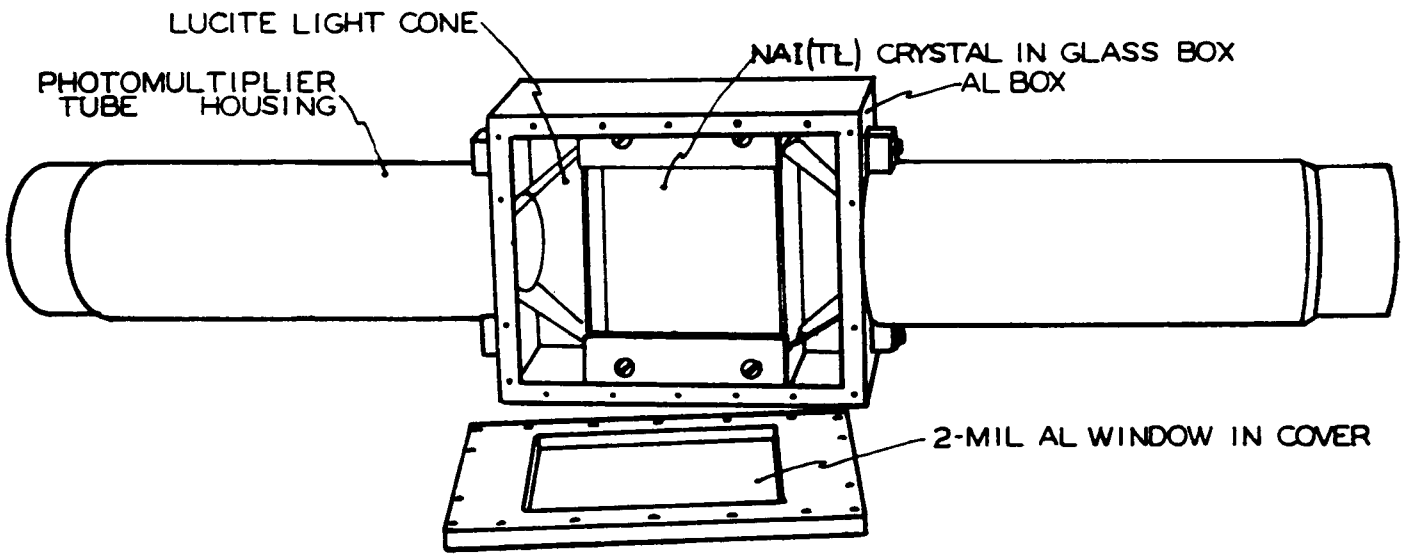

Fig. 23 The scintillation counter with front cover removed, exposing the crystal.

gamma-ray crystal diffraction spectrometer at 1 Mev corresponds to a line width at half maximum of about 1 percent only. (The precision of wavelength determination is much higher than this, however, since the profiles are very regular and the line positions can be measured to a small fraction of the line width). Because of its much lower resolution the differential pulse height discriminator setting need only be changed occasionally as the diffracted spectrum is progressively explored. However, if a given gamma-ray or x-ray line profile is suspected to be the result of reflection by the quartz crystal planes in second or third order instead of the first, the pulse height discriminator has amply sufficient spectral resolving power to reveal this immediately by running a pulse-height spectral exploration while the quartz crystal diffraction angle setting is left constant at the value for the peak of the line in question. This is clearly a very valuable property for the interpretation of the spectra.

2.8 Detection of Minute Sources of Error from Imperfect Rigidity of Mechanism in Mark I. Calibration of Screw Errors

The improvement in stability, sensitivity, and more reliable counting statistics which has been gained, thanks to the adoption of the scintillation crystal detection system, has made it possible to detect and correct certain faults in the linearity of the wavelength scale of the Mark I instrument which, it turns out, were due chiefly to minute flexures in different parts of the mechanism. Figs. 24 and 25 show the means adopted for a study and correction of these and other errors. The chief source of these troubles turned out to be flexures in the 1 1/4 inch round bar, B', Fig. 24, terminating the lower beam; although flexures in the base of the instrument also played a part. The reader is referred to two published articles[34, 35] the first of which describes the causes of these inaccuracies and the second gives a brief account of the optical accessories which have been added to the Mark I instrument and the other precautions

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34) J.W.M. DuMond, Physics Today 5, 10 (1952)

35) D.E. Muller, H.C. Hoyt, D.J. Klein, and J.W.M. DuMond, Phys. Rev. 88, 775 (1952)

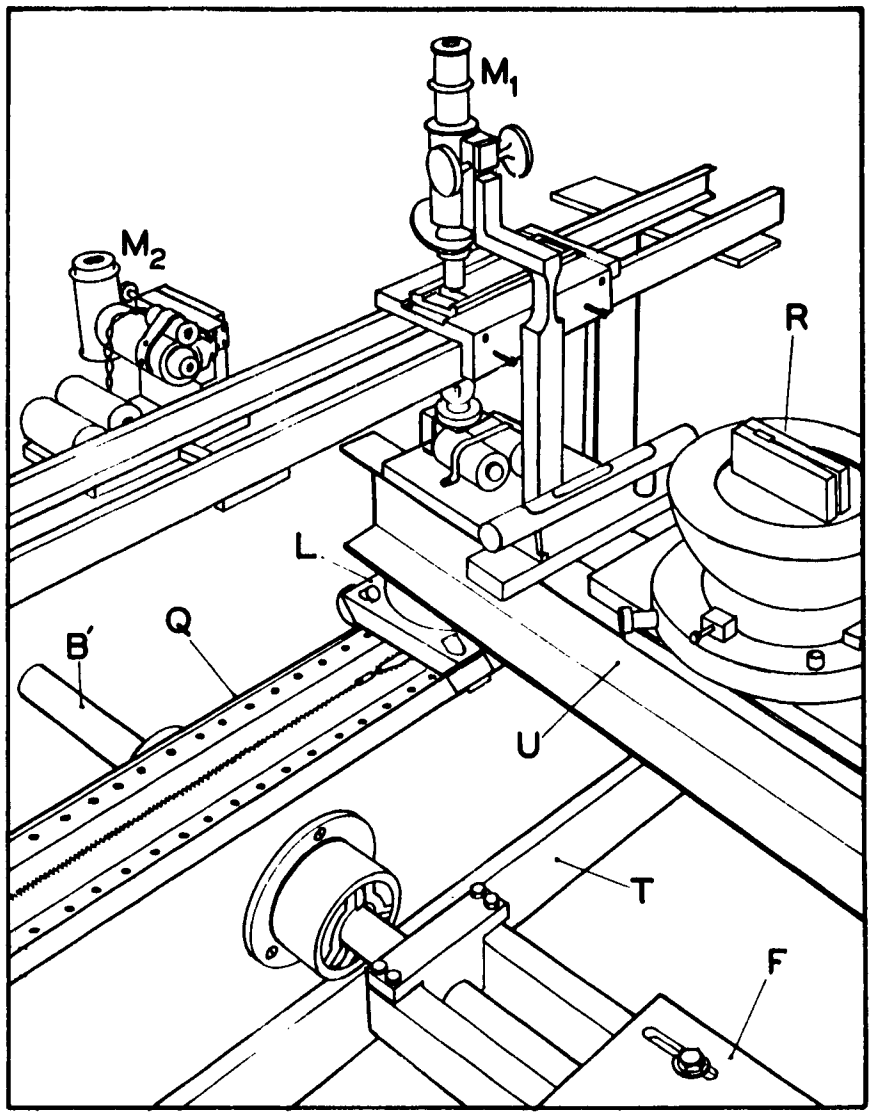

Fig. 24 Arrangements for the calibration of the spectrometer. A pair of rails rigidly supported on the long screw carriage carry a small carriage provided with a Bureau of Standards calibrated precision glass scale. A vertical microscope M_1, rigidly attached to the upper (source supporting) beam U, views the glass scale. The focus of the microscope is at the same height as the source R and is directly above the pivot of the small carriage L, supporting the upper beam. The horizontal elbow microscope M_2 and tiny light source used in conjunction with the 6-inch concave mirror to detect any anomalies in the motion of the crystal pivot are mounted at the center of the rails. The source holder R, standing on the lower half of the lead bomb used for shielding, can be seen at the right center.

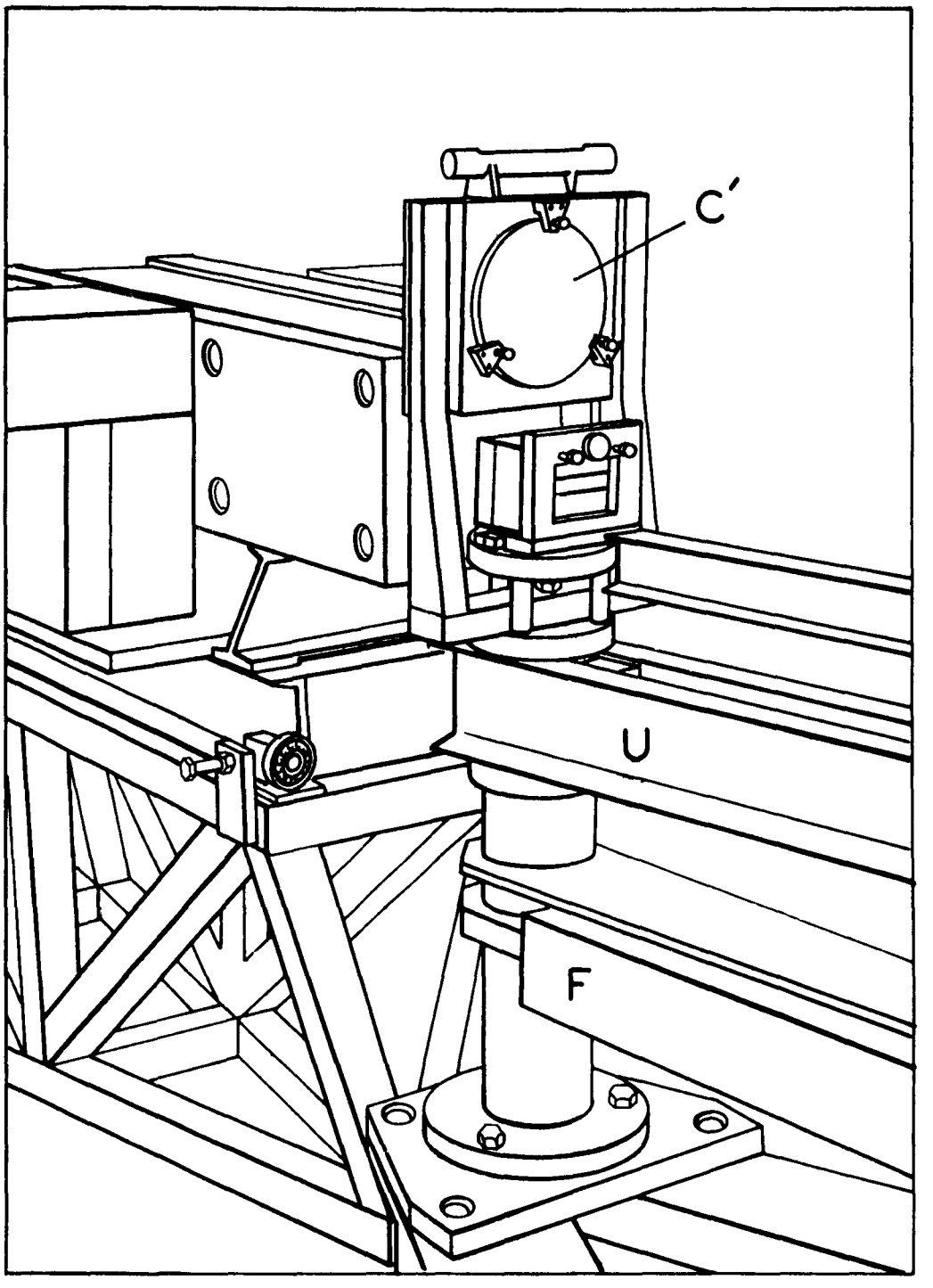

Fig. 25 The 6-inch precision-figured concave mirror C', mounted on the crystal pivot directly above the crystal, where it served, along with the elbow microscope M_2 and light source of Fig. 24, to check the fidelity with which the turning of the crystal pivot followed the motion of the long screw carriage.

taken to measure the small corrections to the screw readings which must be introduced to compensate for these flexural errors.

Appendix III in this report gives an account of how these flexural errors were detected and the chief sources from which they arose as well as the tests which have shown how satisfactorily these errors have been eliminated. We here describe briefly the means adopted for their correction.

A small elbow microscope M_2 and a pair of very fine illuminated cross hairs just above it are supported near the center of the system of two horizontal rails which has been added as a superstructure to the long carriage, Q, visible in Fig. 24. Just above the curved crystal and rigidly attached to its pivot is a 6-inch concave mirror, seen in Fig. 25, which forms an image of the crosshairs in the focal plane of the microscope. In passing from right hand to left hand reflections of a given spectral line, if no flexures occurred in rod B' the crystal and the mirror turning in perfect unison with the motion of carriage Q would leave the image of the illuminated crosshairs immovable in the field of the microscope. Any small motion which does occur can be read off on an eyepiece scale and correction can thus be made for this when reducing the data. This is called the "mirror correction".

Clearly visible also in Fig. 24 are the microscope M_1 rigidly attached to the upper beam U and the precision calibrated Bureau of Standards glass scale supported on the superstructure rails of carriage Q. These are the provisions for calibrating the errors of uniformity of the precision wavelength screw. The non-linearity in the instrumental wavelength scale was first detected by comparing the apparent wavelengths of different gamma-ray and x-ray lines as computed from the positions of their profiles reflected in the first, second and third orders. It was found also[35] that when means were taken to correct for all these flexural effects, a slight apparent discrepancy[36] between the measured value of the annihilation radiation wavelength and its value calculated from very accurate recent measurements[9] of the natural constants h, m and c vanished completely[35, 37] so that agreement to 1 part in 10^4 was obtained.

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36) J.W.M. DuMond, D.A. Lind, W.W. Watson, Phys. Rev. 75, 1226 (1949); Arne Hedgran and D.A. Lind, Phys. Rev. 82, 126 (1951); J.W.M. DuMond, Phys. Rev. 81, 468 (1951)

37) G. Lindstrom, Phys. Rev. 83, 465 (1951); L.A. Page, P. Stehle and S.B. Gunst, Phys. Rev. 1273 (1953)

2.9 Automatic Robot System for Operating the Mark I Gamma-Ray Spectrometer

The Mark I instrument is provided with a robot observing and recording system which replaces a human operator for the tedious and time-consuming task of observing scintillation counts as a function of spectrometer wavelength screw setting. The wavelength settings can be scheduled in advance for a period of many hours and need not in the present scheduling device[38] be uniformly spaced but may be made at less frequent intervals in background regions and more densely where lines are expected. On the shaft of the master wavelength screw is a disc whose periphery is provided with fifty conductin and fifty alternate insulating segments in accurately uniform spacing to 0.001 revolution. A gold contact brush on these segments then alternately makes and breaks an electrical circuit. The punched tape merely selects which of the possible interruptions of this circuit will stop a tiny driving motor which advances the wavelength screw when the robot is performing its function of selecting a new wavelength setting of the instrument. The precision of the setting thus is determined by the sector disc (not by the punched tape) to an accuracy of about $\pm$ 0.001 x-units. Once the wavelength setting has been made the counting interval starts automatically. Counts are accumulated over a predetermined interval (anything from a fraction of a minute to 30 minutes may be chosen) and the accumulated number is recorded by a printing recorder on a paper strip, not only at the end of the total counting interval for each wavelength setting but also at a number of sub-intervals of equal duration inside the total interval. The object of this is to check the reliability of statistical behavior of the counter. At each printing, on the same line across the paper strip there is also printed the time in minutes since the spectral run was started (thus supplying data which may be used to correct for the decay of the radioactive source if this is required) and the wavelength setting of the master screw to the nearest hundredth of an x-unit (transmitted from the spectrometer to the printer

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38) It is planned however, to simplify the mechanism and improve its reliability by dispensing with the punched tape and providing a simple choice of a few different uniform spacings which can be changed at will by the operator.

by a "selsyn" drive). Thus the printed paper strip supplies all the necessary information which can be reduced at leisure to plot the spectral line profiles. At predeterminable intervals a rotary lead shutter is introduced in the primary gamma-ray beam and the printing recorder then records the background counting level. It is not uncommon to observe at from 20 to 25 wavelength settings (for several minutes at each point) to delineate completely a single line profile. In the case of an unknown source, to explore a spectral range say of 540 x-units (150 kev to 20 kev) with settings every 0.1 of an x-unit (so as not to miss any lines) would clearly require 5400 settings and occupy a period of 90 hours. Thus the value of robot observing as a labor saving device should be clear. It has the further advantage of avoiding the necessity for long continued exposure of human operators to a certain level of stray gamma-rays which, though not above tolerance level, is not especially to be desired.

Fig. 26 is a photograph in which can be seen the wavelength screw drum and vernier and the sector wheel, the small slow motion setting motor which drives through a friction disc, the selsyn, and the large rapid slewing motor. Fig. 27 is a view of the printing recorder.

3. TYPICAL RESULTS OBTAINED WITH THE MARK I CRYSTAL DIFFRACTION GAMMA-RAY SPECTROMETER

The list of artificial and natural radioisotopes and the number of gamma-ray lines from each which have to the date of this writing been measured with the Mark I instrument together with the references giving a description of the latest work is given in Table I. In the case of the γ-ray lines the isotope named in each item is the parent rather than the daughter product after β-decay or K-capture although in reality the γ-ray is properly to be associated with the daughter product. For the x-ray lines, on the contrary, the element of which each is characteristic is named.

As an illustrative example we show Fig. 28 which is a "bird's-eye-view" of the entire spectrum of x-ray and gamma-ray lines (for reflection from one side of the crystal planes only) obtained with a source of Ta^{182} (the parent isotope). The spectrum for reflection from the other side

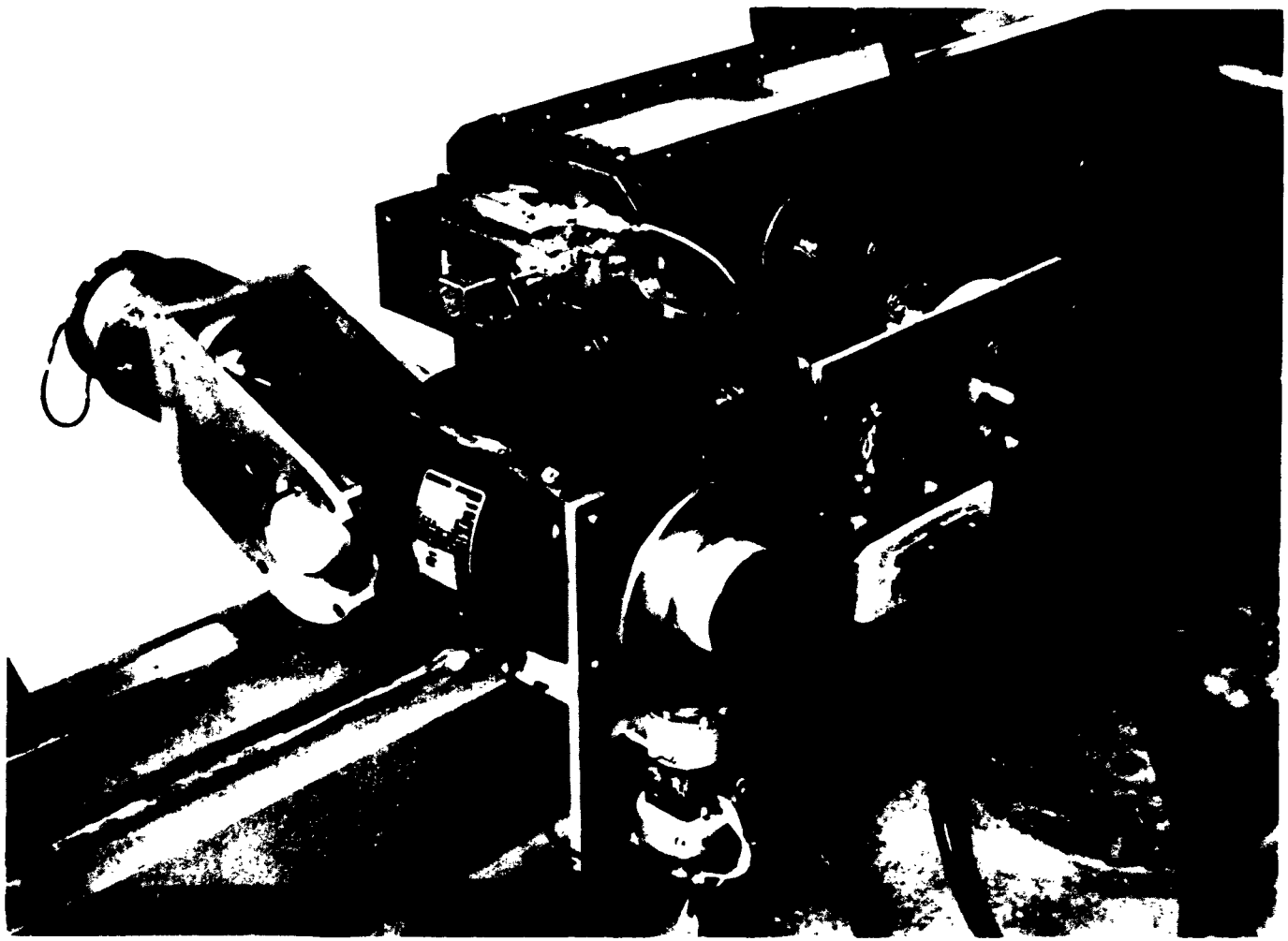


Fig. 26 Photograph showing motors and "Selsyn" coupled to end of precision wavelength screw. The small motor with vertical shaft drives the spectrometer carriage screws through a friction-disc drive when settings are being automatically made by the robot. This drive is held in contact with the driving disc by means of a solenoid magnet only while the small motor is making the change in setting.

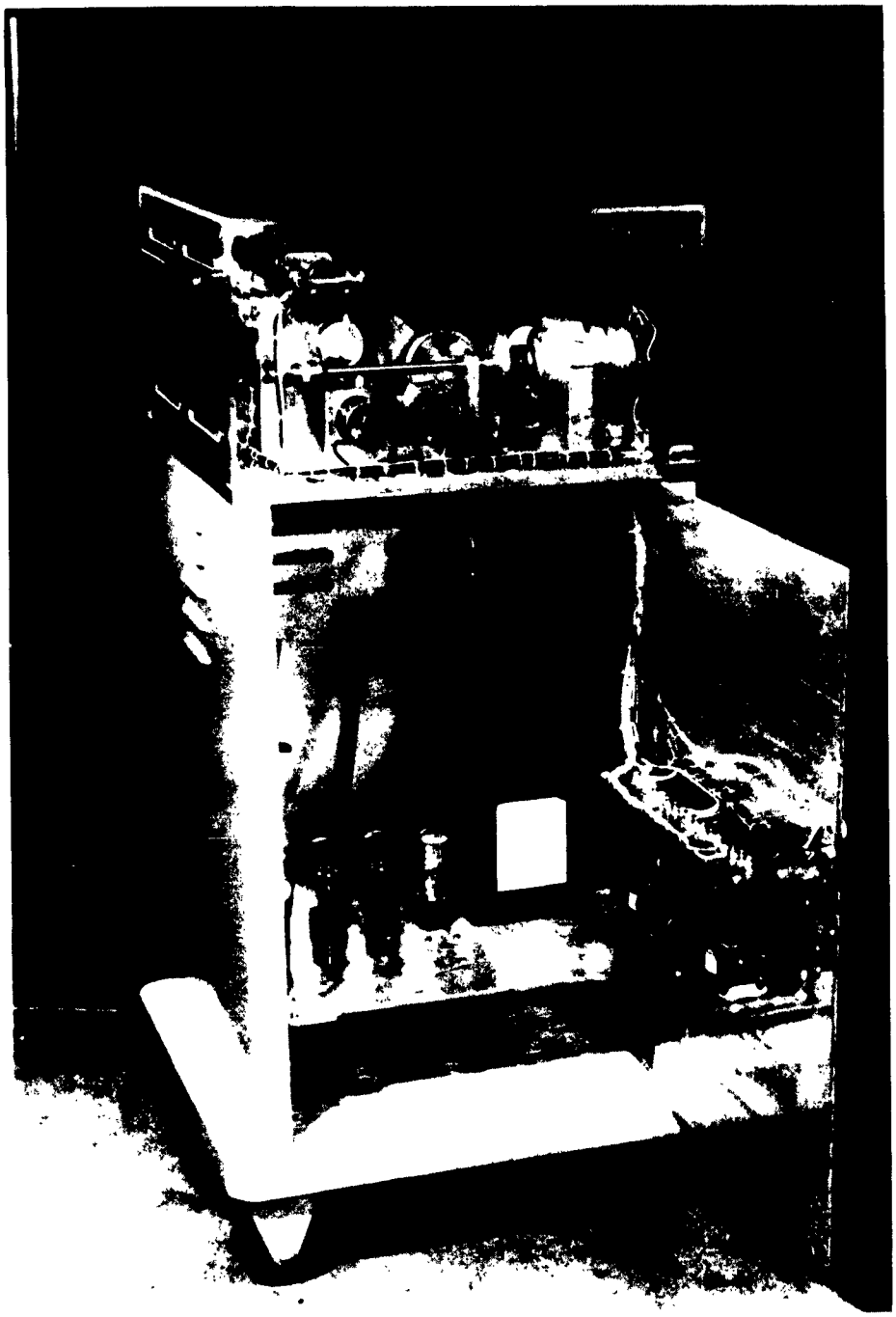


Fig. 27 Photograph of printing recorder.

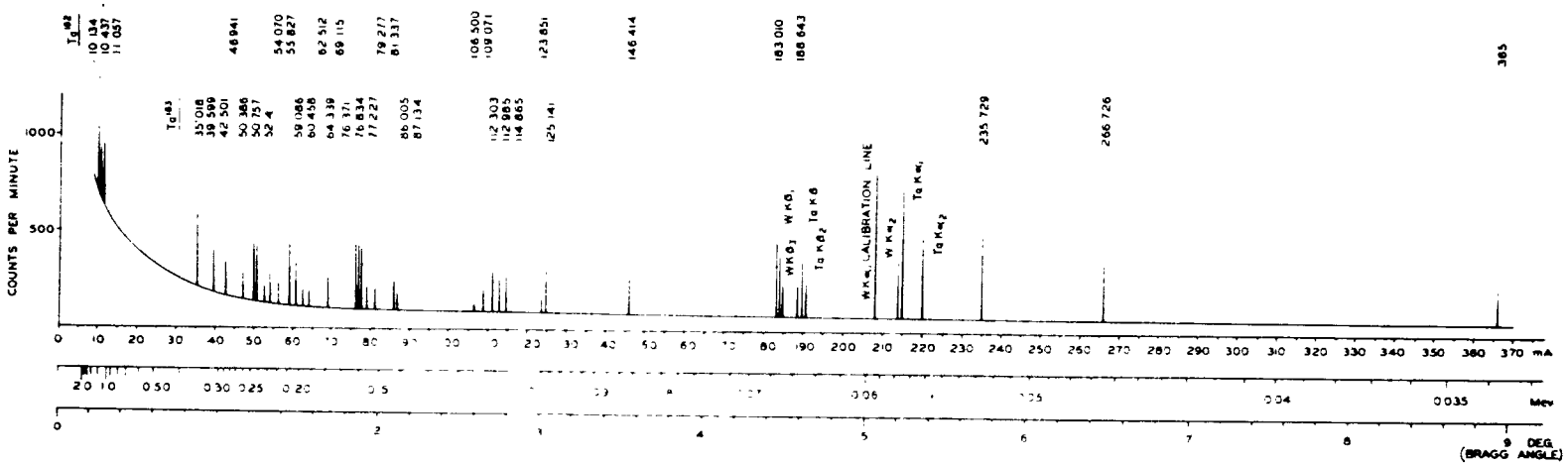


Fig. 28 "Bird's-eye-view" of complete spectrum of gamma-ray and x-ray lines observed with a neutron activated source of $Ta^{182,183}$. X-ray lines both of Ta and of the daughter product, W, (following β-decay) are obtained. All gamma-ray lines are from excited states of the nuclei, W^{182} or W^{183}. The lines appear superposed on a continuous background of scattering from the walls of the collimator, which can be seen to rise rapidly at the smaller angles.

Two such spectra, mirror images of each other - one for reflection from each side of the crystal planes, - are required for precision determination of the wavelengths of the lines. The heights of the lines in all cases depict the observed intensities corrected for decay to a time 5.5 days after the tantalum source was removed from the chain reactor but without correction for crystal reflectivity or self-absorption in the source. (The decay periods of Ta^{182} and Ta^{183} are respectively 112 days and 5.2 days). Three abscissa scales are shown giving wavelengths in milliangstroms, line energy in Mev and Bragg angles of observation in degrees.

TABLE I

Measurements Made With Mark I Crystal Diffraction Gamma-Ray Spectrometer

Isotope or Source	Kind of Radiation	Number of Lines Measured	References
Gamma-Ray Lines: (The isotope named is the parent although the γ-ray lines are of course emitted by the daughter).			
Au^{198}	γ	1	b
Cu^{64}	Annihil. Rad.	1	b
I^{131}	γ	3	c
Co^{60}	γ	2	a, g
Ta^{182}	γ	17	b
Ta^{183}	γ	20	d
W^{187}	γ	5	b
Ir^{192}	γ	11	b
Cs^{137}	γ	1	b
Radon	γ	5	b
Radio-Th.	γ	6	b
X-Ray Lines:			
Ta	K x-ray	4	f
W	K x-ray	4	e
Ir	K x-ray	5	b
Po	K x-ray	1	b
Os	K x-ray	4	b
Pt	K x-ray	6	b

a. D.A. Lind, J.R. Brown, J.W.M. DuMond, Phys. Rev. 76, 1838 (1949)

b. D.E. Muller, H.C. Hoyt, D.J. Klein, J.W.M. DuMond, Phys. Rev. 88, 775 (1952)

c. H.C. Hoyt, and J.W.M. DuMond, Phys. Rev. 91, 1027 (1953)

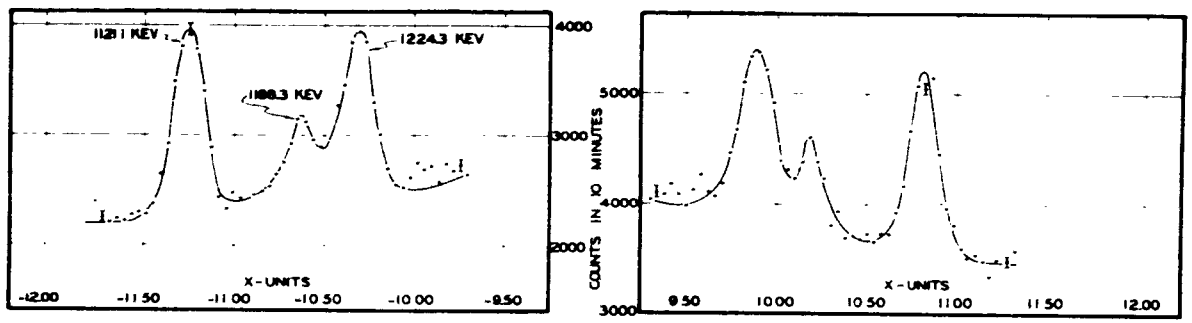
d. J.W.M. DuMond, H.C. Hoyt, P. Marmier, J.J. Murray, Phys. Rev. 92, 202 (1953)

e,f,g To be published

of the planes is simply a mirror image of this plot, save that the center of symmetry of the two halves may be slightly shifted from the nominal zero of the wavelength scale. (Such small displacements usually occur as a result of small variations in the positioning of the source in its holder. The wavelength is determined by the distance in screw turns between each positive line position and its negative counterpart and such small asymmetries of the screw scale can therefore readily be shown to have negligible influence on the measured wavelength values). The abscissa coordinate is a linear scale of wavelength and the energy scale is therefore highly compressed toward the short wavelength end. The rising background at short wavelengths coming from the increasing scattering on the partitions of the collimator can be clearly seen. On the scale of this "bird's-eye-view", individually observed points are too close together to be depicted. Fig. 29 shows a detailed view of the line profiles for the three hardest lines in this spectrum (at 10.13, 10.44, and 11.06 milliangstroms) in which the individually observed points appear. The statistical uncertainty (square root of the total number of counts observed) on one of these points is indicated by a vertical stroke.

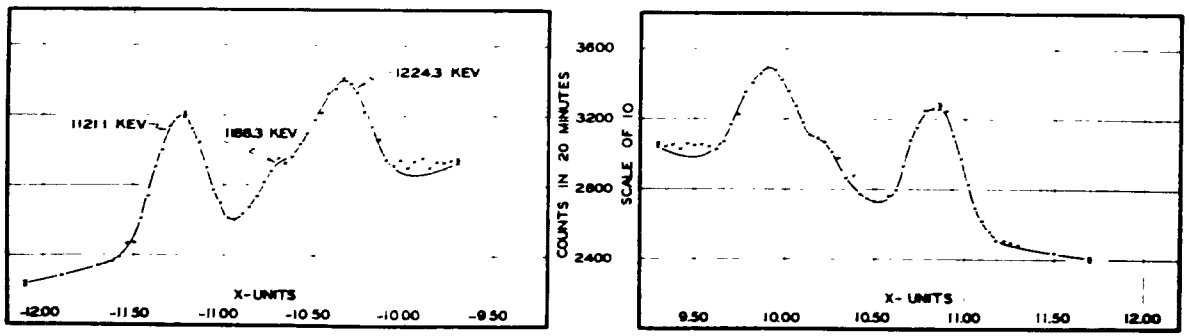
When lines are as closely adjacent (relative to their instrumental width) as the three of Fig. 29 and with as unfavorable a background as these (from the point of view both of its large value and its steep slope) an absolute wavelength determination by the method of superposition of profiles to better than about $\pm$ 8 or 10 percent of the width of the lines at half maximum height is about the best that should be claimed. The lines of Fig. 29 depict about the worst example of this sort. To give an idea of about the best that can be done we show in Fig. 30 a typical pair of line profiles for the 412 kev gamma radiation following decay of Au^{198}. The size of the dots represents approximately the statistical uncertainty of the counting and these can be seen to be entirely consistent with the minute fluctuations in the otherwise level background. The method of superposition of profiles is reliable here to 2 or 3 percent of the width of the lines at half maximum height i.e., to from $\pm$ 0.008 to $\pm$ 0.01 milliangstrom. We give in Table II the results of 13 repeated measurements on this line.

The annihilation radiation resulting from the recombination with negative electrons of positrons emitted by the nuclei, Cu^{64}, in a small



SOURCE WIDTH = 0.004 IN

$_{73}Ta^{182}$ – HIGH ENERGY GAMMA RADIATION



SOURCE WIDTH = 0.011 IN

Fig. 29 Detailed spectrum of the three shortest wavelength lines in the bird's eye-view of Fig. 28. The statistical error (square root of the total number of counts observed at a given point) is shown for a few points.

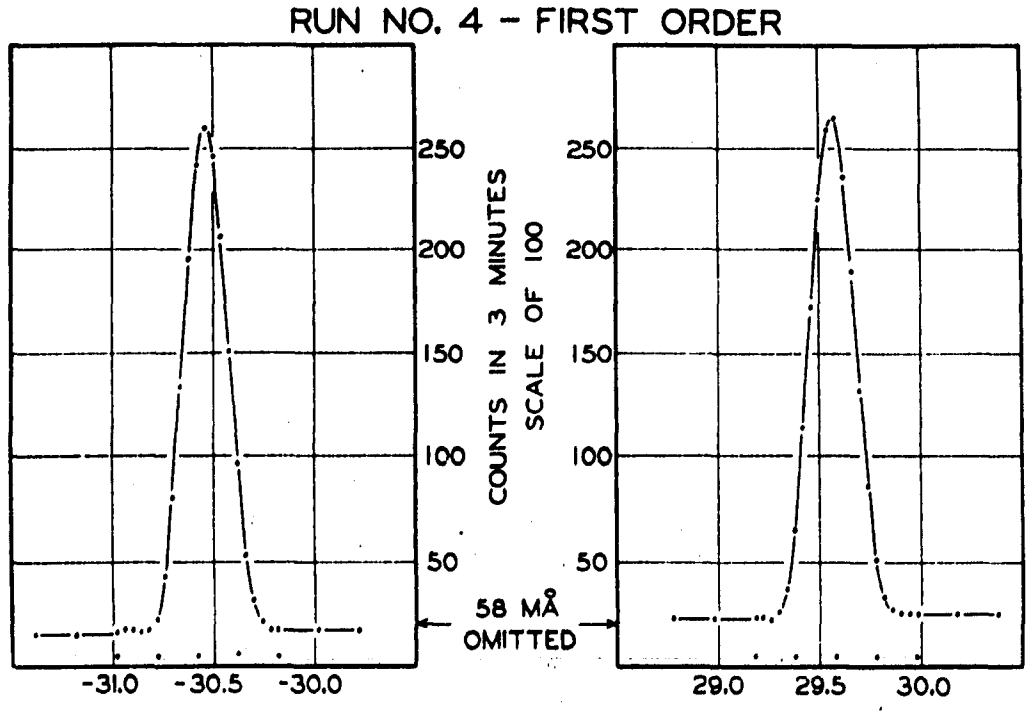


Fig. 30 A typical pair of line profiles. This pair of profiles was obtained for the 412-kev gamma-radiation following decay of gold 198. The wavelength is found by first forming a composite profile by superposition of all profiles (plotted to the same scale) obtained with the same source and then matching each profile individually to the composite profile. The difference in screw readings for the two members of a pair corresponding to a fiducial mark on the composite profile gives twice the wavelength, before corrections.

TABLE II

The wavelength and energy of the 412 kev gamma-ray line following decay of Au^{198}, as obtained by D.E. Muller and H.C. Hoyt in measurements of first, second and third order reflections. The uncertainties given are standard deviations. They are given to one extra place to avoid incorrect weighting when combining these values with those of other workers. All corrections including the "mirror corrections" have been made.

Order	Number of Independent Runs over Profile	Corrected Wavelength in Screw Divisions	Wavelength in Milliangstroms	Energy in kev
1	10	30.016 ± 0.0028		
2	1	30.019 ± 0.0023		
3	2	30.021 ± 0.0016		
Weighted Mean		30.019 ± 0.0012	30.105 ± 0.0026	411.770 ± 0.036

block of ordinary copper which has been exposed to a strong slow neutron flux in a chain reactor has been studied repeatedly and with considerable attention to high precision. Accurately ground uranium slit jaws of special design, whose cross section is indicated in Fig. 31, were used to give sharpest definition to the line. The use of a relatively large block of activated copper behind a fine slit (0.2 mm wide) which defines the effective instrumental source width is indicated in this case because the range of the positrons emitted by Cu^{64} before these are decelerated to thermal velocities is about 0.3 mm. The cross section for annihilation is very low except for thermal positrons. The slit thus utilizes the annihilation radiation formed by positrons which have been supplied from a layer of activated copper 0.3 mm thick on both sides of the central region and this affords a very considerable gain in intensity as compared to a source of total thickness say 0.2 mm which might be used instead of the defining slit.

Nine pairs of annihilation line profiles were delineated with the measured wavelengths from each pair as listed in Table III.

Table IV lists the results of the annihilation wavelength measurements by two different methods, (1) and (2), and gives also for comparison the value, $\lambda_C = h/m_o c$, computed from the November 1952 least-squares analysis of the values of the atomic constants by DuMond and Cohen based on entirely independent sources of data of many kinds and of very much higher precision[9]. The table also gives the annihilation quantum energies corresponding to each of the three wavelengths and the inferred electron-positron mass difference, which last is seen to be less than its own standard deviation and hence zero to the precision of these measurements. The items labelled (1) are obtained by direct measurement of the annihilation wavelength from the Cu^{64} source with the Mark I spectrometer. The items labelled (2) are obtained indirectly from the crystal diffraction measurement of the Au^{198} wavelength combined with Hedgran and Lind's[36] determination by magnetic β-ray spectroscopy of the energy difference between the annihilation radiation and the gold radiation. In Hedgran and Lind's work the energy levels from which the β-rays were

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36) Arne Hedgran and D.A. Lind, Phys. Rev. 82, 126 (1951)

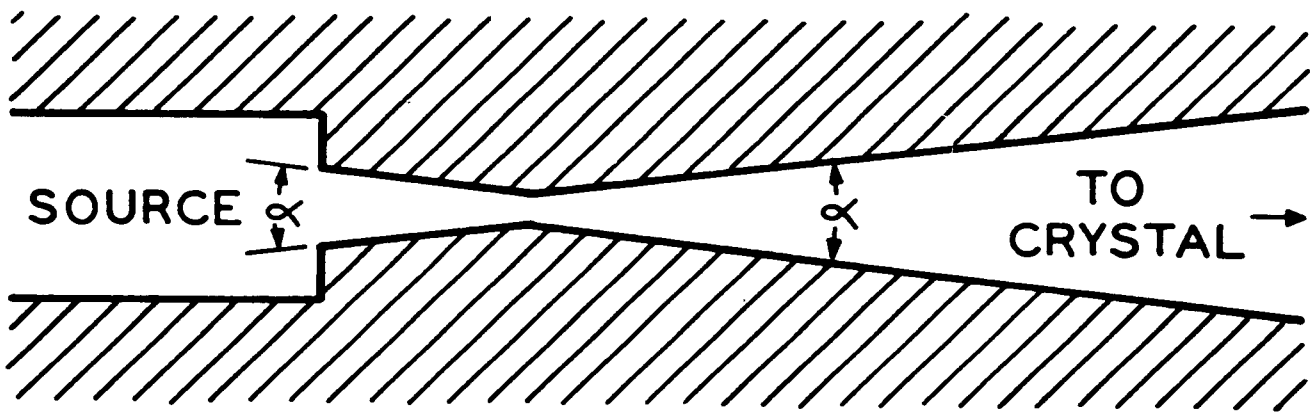

Fig. 31 Design for uranium slit jaws used to define Cu^{64} source in annihilation radiation experiment. $\propto = 1.45^{\circ}$. Width at narrowest point = 0.2 mm. Distance between source and narrowest point = 1 cm. The sketch is not to scale.

Table III

The values of 2λ (twice the wavelength), in screw divisions for the annihilation radiation from Cu^{64} are given. The first column gives the direct values as read off by superposition of the master profile on the line profiles. The se cond column gives the values after correction for the elastic flexure in the 1 1/4 inch bar terminating the lower beam of the instrument. This correction, since it is made by observing the motion of the image of illuminated cross hairs formed by a six inch mirror mounted above the crystal pivot, is called the "mirror correction".

Run No.	2λ Before mirror correction	2λ After mirror correction
1	48.328	48.383
2	48.352	48.390
3	48.338	48.388
4	48.330	48.381
5	48.332	48.385
6	48.328	48.378
7	48.326	48.384
8	48.336	48.390
9	48.344	48.392

Table IV

Results on annihilation radiaton. The uncertainties given after each ± are standard deviations. These are given to one extra place to avoid incorrect weighting when combining these values with those of other experimenters.

Wavelength:	
1. $\lambda_A = 24.262 \pm 0.0033$ mA	Cu^{64} direct measurement of λ_A
2. $\lambda_A = 24.263 \pm 0.0033$ mA	Au^{198} expt. + Hedgran and Lind
3. $\lambda_c = 24.26067 \pm 0.00048$ mA	DuMond and Cohen least squares analysis
Energy:	
1. $E_A = 510.941 \pm 0.067$ kev	Cu^{64} direct measurement of λ_A
2. $E_A = 510.920 \pm 0.070$ kev	Au^{198} expt. + Hedgran and Lind
3. $m_- c^2 = 510.969 \pm 0.015$ kev	DuMond and Cohen least squares analysis
Positron mass:	
1. $(m_- - m_+)/m_- = (1.01 \pm 1.85) \times 10^{-4}$	Cu^{64} direct measurement of λ_A
2. $(m_- - m_+)/m_- = (1.92 \pm 2.80) \times 10^{-4}$	Au^{198} expt. + Hedgran and Lind

ejected were so chosen that there was very little difference indeed as to $H\rho$ in the two cases so that errors in calibration of the magnetic spectrometer were very unimportant.

Fig. 32 shows the effect of Doppler broadening on the line profile of the annihilation radiation obtained from the Cu^{64} source. This broadening is occasioned by the fact that the annihilating pairs are in motion and the momentum inferred from the broadening $p = 9.8 \times 10^{-3} m_e c$, indicates that, for the most part, only the conduction electrons of the copper take part in the annihilation process.

The need for irradiation with high neutron flux densities to obtain sources of high specific radioactivity for crystal gamma-ray spectrometry has led to the discovery and study of a new isotope Ta^{183}. This nuclide is formed from natural Ta^{181} by two successive neutron captures during irradiation with neutrons and the amount produced is therefore proportional to the square of the neutron flux density. The presence of a short lived gamma-line at 250 kev (half life 5 days) in studying the spectrum of Ta^{182} was first noticed by H. Hoyt in 1951 and reported in his doctorate thesis (1952). The suggestion that this line is emitted by Ta^{183} was first made by W. Mihelich in 1952 and this is now verified. Ta^{183} emits a very rich spectrum of lines of which we list 20 in Table V. See Fig. 28. The same process of double neutron capture has been observed earlier in the study of the spectrum of Au^{198} where some lines ascribable to Au^{199} have been observed. There seems little doubt that many other similar cases remain to be found and studied.

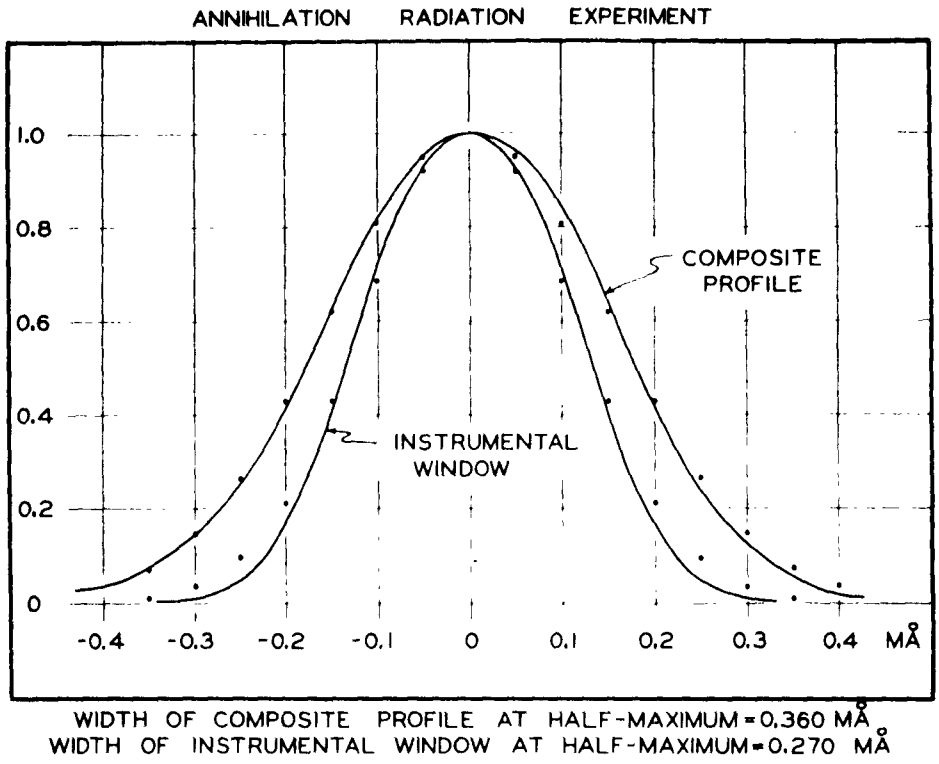

Fig. 32 Showing the effect of Doppler broadening on the observed line profile for the annihilation radiation from copper 64. The composite profile formed by superposition of all observed annihilation line profiles is compared with the calculated instrumental window. The dots represent Gaussian functions, $e^{-h^2x^2}$, having the same widths at half-maximum as the solid curves. The composite profile has not been corrected for decay of the source during the time required to obtain a single profile, since such corrections are only slight.

Table V

γ-Rays emitted by a Ta^{183} source

Line	Wavelength (in Å)	Energy (kev)	γ-ray relative intensity (internal conversion not taken into account)
A	266.726 ± 0.025	46.481 ± 0.005	-
B	235.729 ± 0.022	52.593 ± 0.005	-
C	-	83 ± 1	Weak
D	125.141 ± 0.013	99.070 ± 0.010	1.4
E	114.865 ± 0.013	107.933 ± 0.012	2.4
F	112.985 ± 0.013	109.729 ± 0.013	0.4
G	112.303 ± 0.15	110.395 ± 0.150	0.1
H	87.134 ± 0.025	143.773 ± 0.050	0.4
I	86.005 ± 0.025	144.151 ± 0.050	2.7
J	77.229 ± 0.015	160.532 ± 0.030	1.1
K	76.834 ± 0.015	161.458 ± 0.025	3.3
L	76.371 ± 0.015	162.335 ± 0.030	1.8
M	64.339 ± 0.022	192.692 ± 0.065	0.6
N	60.458 ± 0.015	205.062 ± 0.050	1.8
O	59.086 ± 0.100	209.823 ± 0.035	1.9
P	-	236 ± 1	Weak
Q	50.757 ± 0.010	244.259 ± 0.040	3.0
R	50.386 ± 0.006	246.051 ± 0.030	10
S	42.501 ± 0.010	291.708 ± 0.060	2.5
T	39.599 ± 0.010	313.078 ± 0.065	2.6
U	35.018 ± 0.010	354.038 ± 0.090	3.9

APPENDIX I

DESCRIPTION OF THE SINE-MEASURING INTERFEROMETER FOR THE MARK II GAMMA-RAY SPECTROMETER

The original conception of this interferometer, whose purpose is to measure with high precision the sine of the Bragg angle of rotation of the curved crystal relative to the gamma-ray source was due entirely to Dr. David E. Muller[1].

Fig. AI-1 illustrates schematically the geometrical principle of this proposed instrument. Its purpose is to measure with high precision the sine of the angle of rotation of the curved crystal relative to a fixed gamma-ray source. The zero position to which the angle of rotation is referred is of course the position of the curved crystal where its reflecting atomic planes (produced) converge to a point coincident with the gamma-ray source. Fig. AI-1 is schematized in some of its features to simplify the explanation and the actual design, for reasons to be made clear below, will be slightly different.

Referring to Fig. AI-1, a beam from a monochromatic Hg^{198} light source is split by a half-silvered mirror surface so that half the light goes to each of two front-silvered trihedral mirrors on the pivoted disc which supports the curved crystal shown at O. Each trihedral mirror, when accurately adjusted can be shown to be optically equivalent to an optical flat coinciding with the trihedral corner and oriented so as to return the beam incident upon it into exactly the reverse direction from whence it came. (The trihedral mirror is thus the optical substitute for a mechanical universal joint but with far greater reliability and freedom from lost motion than any mechanism could have). In the zero setting of the interferometer the light paths PA and PB from the interference point P to the trihedral corners A and B are exactly equal and make an angle of exactly 90° with the radii OA and OB. As the crystal table rotates away from this setting through some angle, θ, each of these trihedral mirrors continues to return the beam it receives always parallel to the same direction as

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1) Now in the Department of Mathematics, University of Illinois.

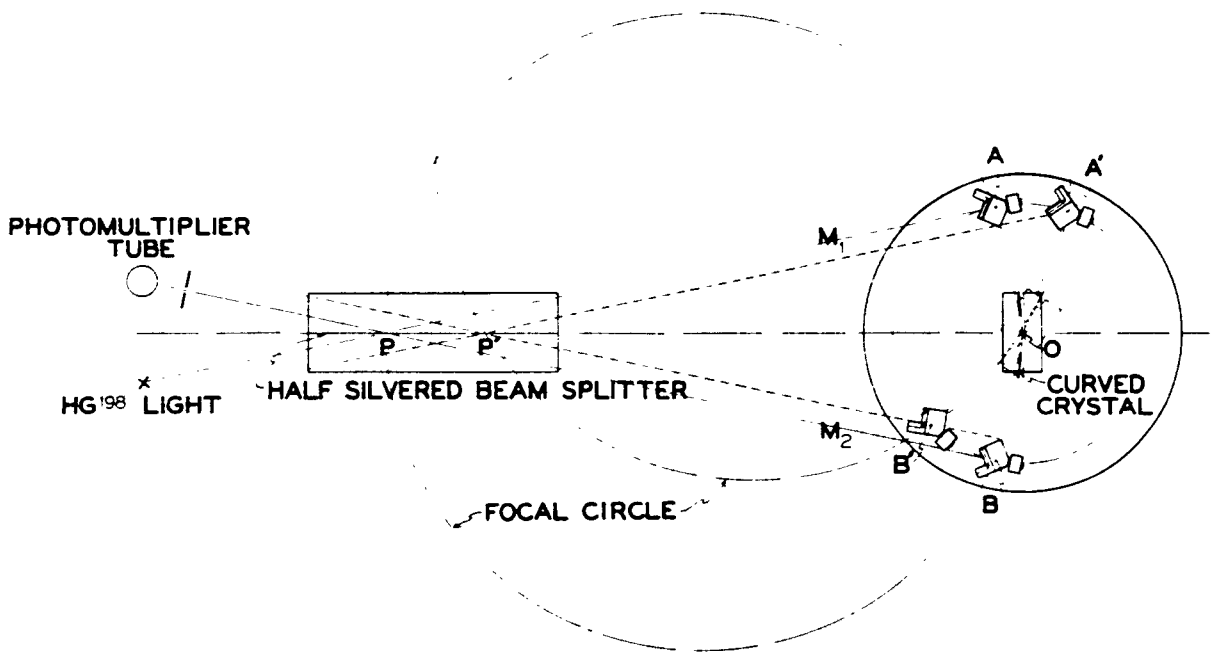

Fig. AI-1 Schematic layout to explain the virtual geometry of the new interferometric γ-ray spectrometer. The actual instrument has mirrors situated at M_1 and M_2 which bend the beams to and from the trihedral mirrors A and B out of the plane of the picture so that they interfere on the beam splitting mirror below that plane and at an angle to that mirror of about 45°. The γ-ray source is placed at the virtual point P (or P' if larger angles are being studied) and in virtue of this the fringe count measures the sine of the angle of rotation of the crystal (away from the position of zero light path difference) relative to the position of the γ-ray source independent of errors due to slight shake or inaccuracy in pivot O.

described for the zero position, for a reason which will soon become clear, and it is easy to see, given this parallelism of P'A' to PA and P'B' to PB, that the difference of the two paths, 2P'A' and 2P'B' (as measured by counting the fringes as the crystal rotates) will be a reliable measure of the sine of the angle rotated through. If R is the radial distance $R = OA = OB$ the difference of path $2P'A' - 2P'B'$ will clearly be $4R \sin \theta$. The reason why parallelism of P'A' and PA is insured becomes clear if one notes that the image of the trihedral corner B' mirrored in the half-silvered beam-splitting surface always must occur at the point of intersection of the line P'A' with the dot-and-dash circle through the trihedral mirror corners. The direction of the beam which forms the center of the circular fringe system of the interference pattern is determined by the line through the corner A' and this image of B' at the aforementioned intersection because this is the direction which yields an extremum (in this instance a maximum) for the difference of path. Thus that part of the beam belonging to the center of the fringe system will always remain parallel to PA. A completely analogous argument holds to insure parallelism of the path P'B' with PB.

We may conceive of the trihedral mirror at A' and the image of the trihedral mirror B' with its corner where the line PA' intersects the dot-and-dash circle as each replaced by equivalent optical flats passing through the trihedral corners. If these equivalent flats are kept parallel to each other the actual fringe pattern may be conceived as the result of interference over the difference of path between them and it is clear that this normal separation between the two mirrors will be a maximum when they stand normal to the line joining the two fixed points through which the mirror surfaces pass.

Now it can easily be shown that if the gamma-ray source can be made to coincide with the optical interference point P, first order errors due to shake or looseness in the pivot at O become unimportant because small transverse displacements of the pivot which correspond to rotation around P (or P', depending on the setting) change neither the path difference nor the angle of the crystal planes relative to the source at P to be measured. Displacements of the pivot along the axis PO also leave both these quantities unchanged. In the actual instrument, in order to

get the beam splitting mirror out of the way of the gamma-ray source and still enjoy the benefits of freedom from errors caused by minute looseness or inaccuracy in pivot O, it was planned to introduce two diverting mirrors at M_1 and M_2 so that the light paths PM_1 and PM_2 are bent out of the plane of the drawing in such a way as to make them intersect the beam splitting mirror at a much more convenient and economical angle (from the point of view of the required size and cost of this expensive component).

It was planned to count the fringes at rather high speed with photomultiplier tubes, keeping a record of the total on a scaling circuit, and by means of a second fringe system in quadrature phase arrangement with the first to cause the scaling circuit to count backward (deduct the count) with reversal of the direction of rotation of the crystal table so that the scaler always indicates correctly in the algebraic sense the sine of the angle of rotation on either side of the zero position.

Because of the involved 3-dimensional geometrical optics of the system, a half-scale model of the Muller sine measuring interferometer has been constructed of Balsa wood and provided with rough mirrors and transparent plastic optical parts to convey a clear idea of its structure and operation and to insure against gross errors in planning the lay-out. Two photographs of this model are here reproduced in Figs. AI-2 and AI-3. The diverting mirrors and the two trihedral mirrors on their horizontal rocking arm are clearly visible as is also the beam splitting mirror on a somewhat lower level below and between the diverting mirrors. The supporting structure for these optical parts was planned to be cast out of a semi-steel such as Meehanite. The hole in this casting between the diverting mirrors through which the gamma rays pass from the source to the curved crystal can be clearly seen. Neither the source nor the curved crystal in its clamp (to be mounted directly above the main pivot) are visible in these two views.

In order to test the feasibility of such an interferometer, the character of the fringe systems it forms and the problems of its correct adjustment we have constructed a static version consisting of the two trihedral mirror assemblies with the beam splitting mirror between them situated so that the beams are incident upon it at 45°. These components are mounted on a fixed base but adjustments are provided so that the path

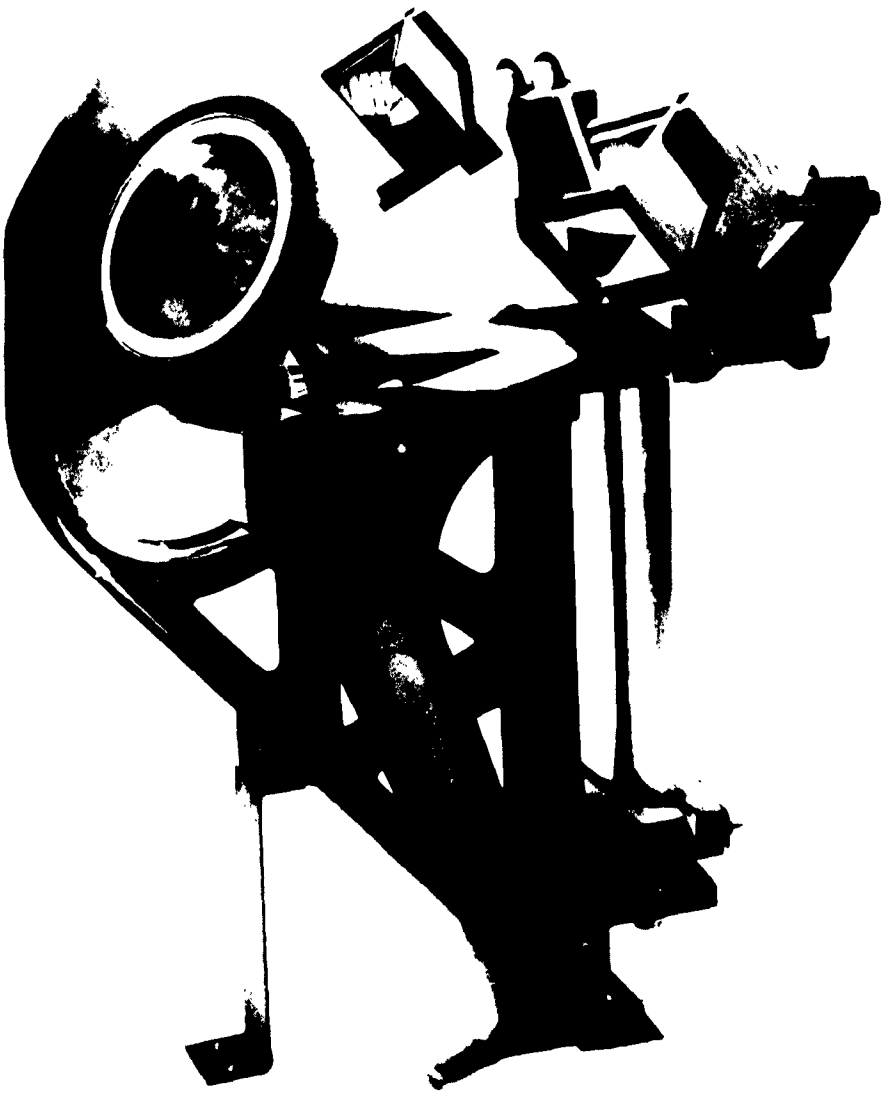

Fig. AI-2 Photograph (side view) of half scale model of Muller's sine-measuring interferometer for new high precision curved crystal gamma-ray spectrometer. This is not a working model and was constructed chiefly to assist in the design layout of the full scale instrument. The two diverting mirrors are visible with the beam splitting mirror between and somewhat below them. The round hole in the casting between the diverting mirrors is for passage of the gamma-rays from the source to the curved crystal which latter will be mounted above the pivot axle.

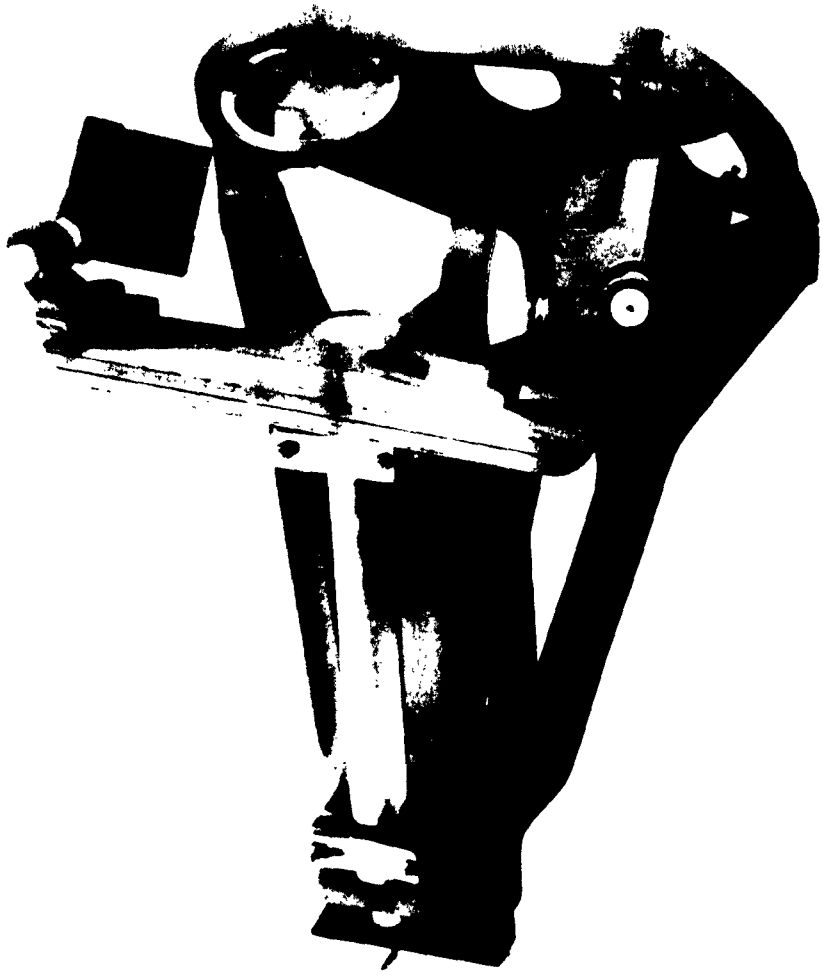

Fig. AI-3 Another view of the half scale model of Muller's sine-measuring interferometer.

difference can be varied. The trihedral mirror assemblies have been very carefully designed with coarse and fine adjustments for the orientation of the three optical flats each of which is 4 x 4 inches square silvered on the front and flat to 1/10 fringe. The beam splitting mirror of glass whose optical quality is excellent is 10 inches in diameter and flat on both sides to 1/10 fringe. A telescope with a well corrected 4 inch objective has been used with this interferometer and the entire assembly can be seen in the three views Figs. AI-4, AI-5, and AI-6.

Four major questions have been tested and answered satisfactorily with this static version of the interferometer in a way to encourage hope of the final success of the proposed instrument. They are:

(1) The feasibility of building up and adjusting the two trihedral front surfaced mirrors in holders adjustable with sufficient stability, fineness and accuracy to insure the permanent optical accuracy of the three dihedral right angles.

(2) The shape of the fringes to be expected at different path differences from such a complicated many-reflection system. Dr. Muller's prediction that the fringes would be circular has proven correct.

(3) The "visibility" or contrast of the fringes over the longest path differences which will be required (about 200,000 fringes) utilizing a Hg^{198} light source.

(4) Stability and sensitivity of the system to extraneous sources of mechanical vibration.

It has been found possible to adjust the trihedrals much more easily than was anticipated by using a telescope with a well corrected 4 inch objective as an autocollimator. Diverging light from a single brilliant but fine point source rendered parallel by the autocollimator and thrown back by the trihedral mirror is reimaged at the focus of the autocollimator as six points if the trihedral mirror is out of adjustment. It is a fairly simple matter with a little practice to bring all six of these images into coincidence by means of the coarse and fine adjusting screws provided in the trihedral mirror supports. These adjusting screws are clearly visible in Figs. AI-4, AI-5, and AI-6. They have been so designed that each coarse adjusting screw controls one dihedral angle independently. This is also true of the fine adjustment screws. The

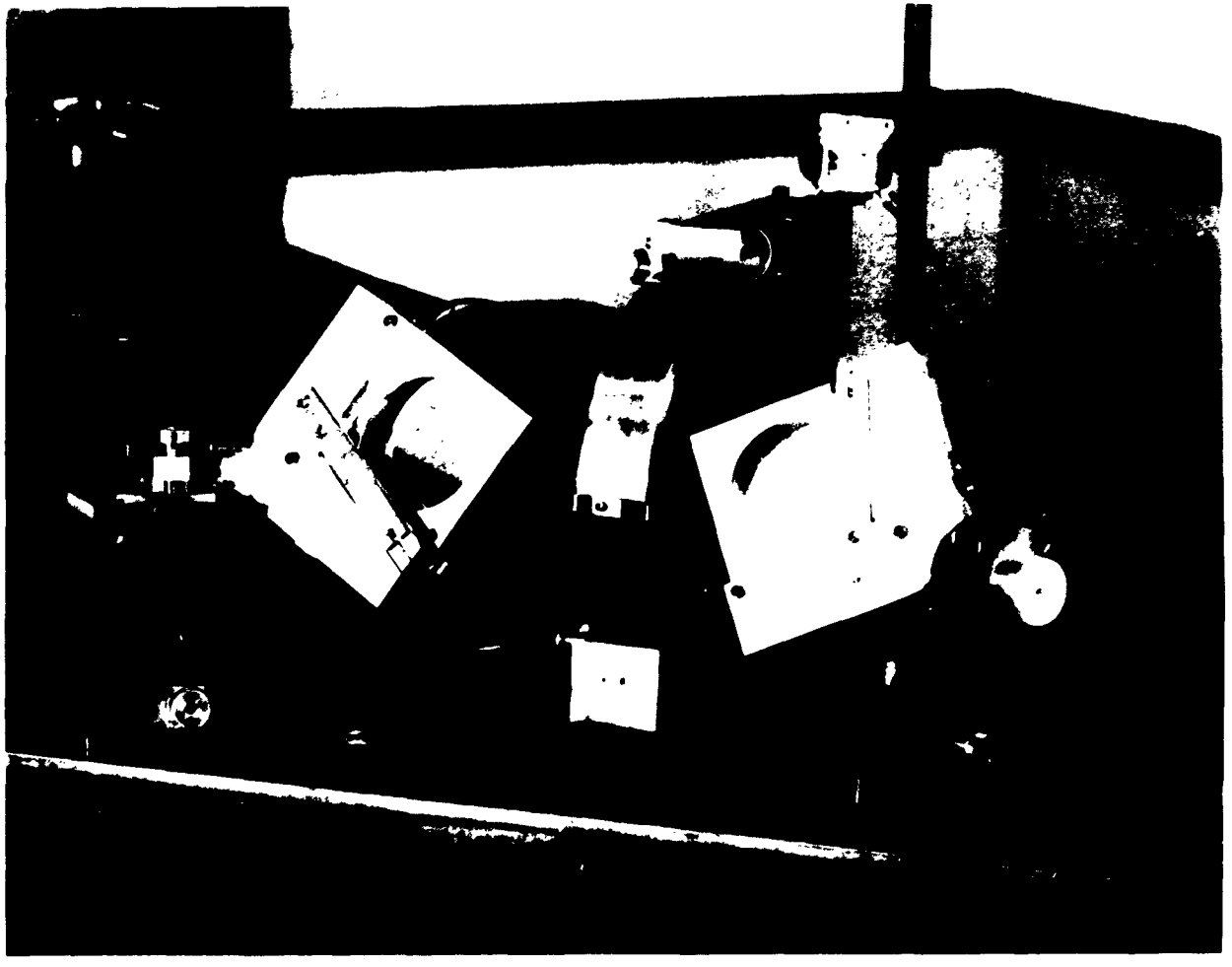

Fig. AI-4 Photograph of the static version of the Muller sine-measuring interferometer showing the coarse and fine adjustments (the latter by the differential leaf spring) for the dihedral angles between the trihedral mirrors. The large beam-splitting mirror and the telescope for focusing the fringes can be seen. The Hg^{198} light source is out of the picture to the right.

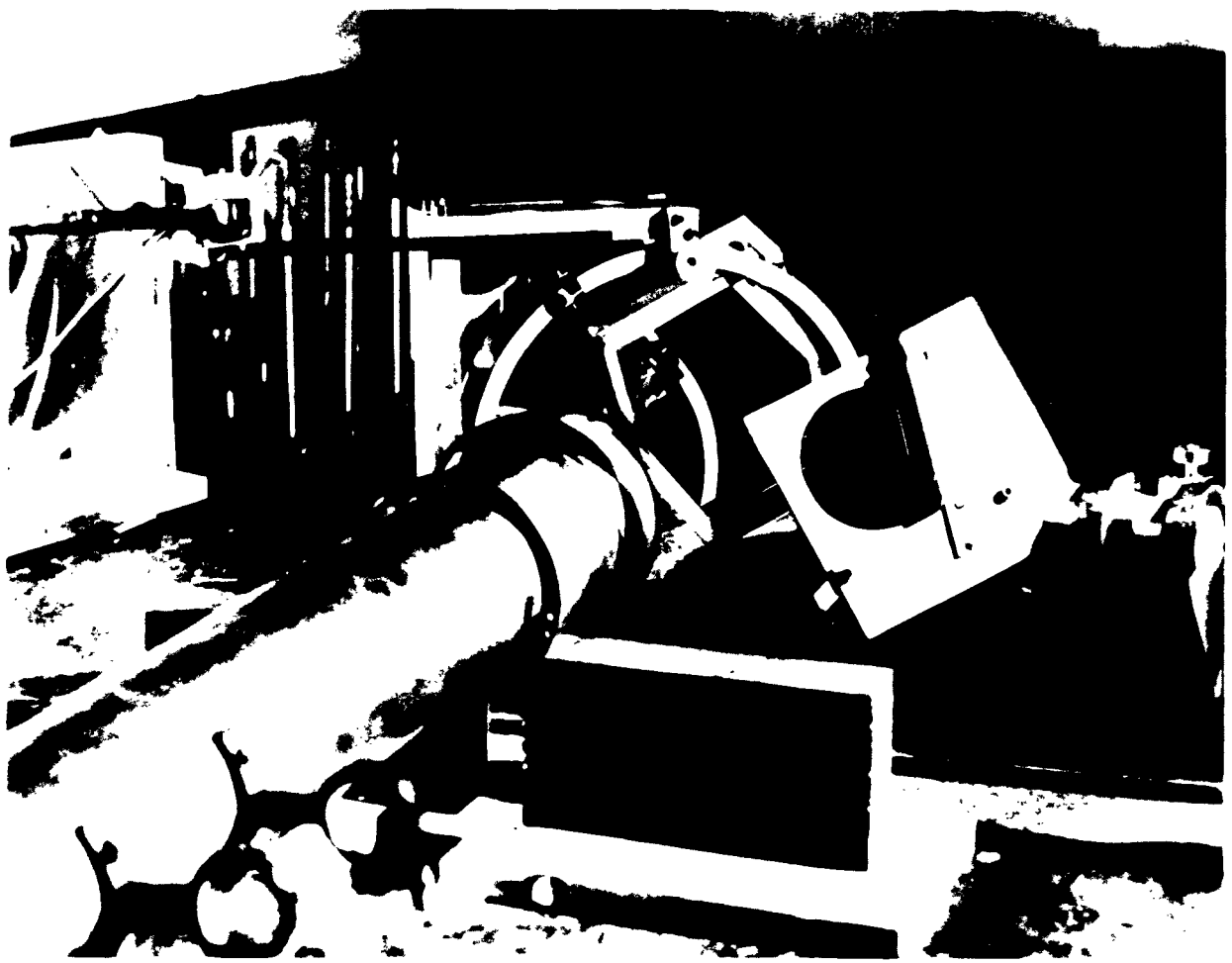

Fig. AI-5 Another view of the static version of the Muller sine-measuring interferometer. Here one looks through the beam splitting mirror at the interior of one of the trihedral mirror assemblies. The condensing lens for the Hg^{198} light source is visible at the extreme left of the picture.

Fig. AI-6 A third view of the static version of the Muller sine-measuring interferometer, set up in the sub-basement of the West Norman Bridge Laboratory in which the precautions found necessary for vibration insulation under ordinary room conditions are shown. The table can be seen standing on a plywood board which is in its turn supported on four partially inflated inner tubes. These precautions have been shown to be unnecessary if mounting is made on the vibrationless pier in the bottom of the spectrographic pit of the Astrophysics Department.

fine adjustment is accomplished by the minute elastic yield of stiff tongues of cast steel (1/2 x 1/2 inch thick) formed by appropriate saw cuts in the casting of the trihedral mirror support which flex ever so slightly under a bending force exerted on them by light phosphor bronze springs whose deflections are controlled by the fine adjusting screws. This differential spring arrangement gives a demultiplication in the order of 1 : 1000. Once the six images of the point source in the auto-collimator have been made to coincide it has been found that the trihedral mirror assemblies are sufficiently accurately adjusted to yield approximately circular fringes in the interferometer and the remaining refined adjustments on the trihedral mirrors can then be made using the shapes of fringes as a criterion.

In the static version of the interferometer illustrated here clearly visible circular fringes with ample contrast were obtained out to the maximum path difference of 200,000 fringes using a Hg^{198} source. It was found impossible however to get good circular fringes at nearly equal path differences without the use of a compensating plate in the beam splitting assembly so as to yield an optically symmetric system.

The severest problem encountered was that of mechanical vibration. It was found necessary in the preliminary studies to place the interferometer on a heavy wooden table whose legs stood on a large plywood panel which was in its turn supported on four partially inflated automobile inner tubes visible in Fig. AI-6. Even with this precaution in the sub-basement of the West wing of the Norman Bridge Laboratory the effect of vibration from ventilating equipment and rotating machinery was far from being totally suppressed. A vibrationless pier, originally installed to support a spectrographic grating in the bottom of the solar spectrographic pit in the Astro-Physics building, was found however to be remarkably free of vibration and the interferometer was finally set up on this pier. With no vibration insulation whatever the fringes in this location were perfectly steady even when a man jumped from a height of two feet on to the adjacent concrete floor.

The final interferometer design is planned so that with it a count of one fringe will be equivalent to a rotation a little less than 0.1 second of arc. When the (310) planes of a curved quartz crystal are used each fringe will correspond quite closely to a microangstrom change in gamma-ray wavelength.

APPENDIX II

THEORY OF ERRORS IN WAVELENGTH MEASUREMENT RESULTING FROM RANDOM VARIATIONS IN COUNTING RATE ALONE WHEN THE METHOD OF SUPERPOSITION OF COMPLETE LINE PROFILES IS USED.

For wavelength measurement, plots are made of the two line profiles obtained by internal reflection from opposite sides of the chosen atomic planes of the curved crystal. A typical pair of such plots is shown in Fig. 30 of the main body of this report. Since the wavelength of the line corresponds to the number of screw divisions of displacement between such opposite plots, the following system has been devised for determining such displacement with high precision. A composite line profile is drawn from the superposition of several such plots. This composite line profile is then matched as carefully as possible to the two opposite plots of a line. By measuring the displacement of this composite profile in going from one plot to the opposite one, a measurement of wavelength can be made with an error which is a small part of the actual breadth of a line.

In order to analyze the error[1] resulting from the above procedure it is necessary to derive an expression for the precision with which a composite profile can be fitted to the plot of a line, assuming that a least-squares fit is obtained[2]. Let L(u) be the average counting rate at

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1) This analysis only aims to give that component of the total error in the wavelength determination which comes from one cause alone, the statistical fluctuations of counting.

2) In actual practice the composite profile is fitted to the observed points on the profile of a given line, not by the laborious numerical method of least-squares, but by a visual estimate of the "best" fitting position. It is found by actual test that this can be judged with remarkable accuracy in accord with the least-squares criterion. Hence for this analysis a least squares fit is assumed. Errors resulting from imperfect visual fitting of the composite profile have been studied and have been found experimentally to be of the same order or less than those due to statistical fluctuations of counting.

spectrometer setting u, $L(u)$, therefore, represents a line profile if u covers a range of settings which include a line. If a series of measurements of counting rate is made lasting τ minutes each at a set of positions u_i, $i = 1, \cdots, n$ the expected number of counts at each point will then be given by $\tau L(u_i)$, and the standard variation of each such point from its expected number of counts will be $[\tau L(u_i)]^{1/2}$ if the normal distribution is used to approximate to the Poisson distribution. Each counting rate measurement at a point u_i may be thought of as constituting an independent measurement of the position of the composite profile, since the profile, once obtained, could conceivably be fitted to a single point. As such, the standard error of the position measurement corresponding to a run at u_i lasting τ minutes is $[\tau L(u_i)]^{1/2}/\tau L'(u_i)$, provided that the slope $L'(u)$ of the curve $L(u)$ does not change appreciably over the vertical interval $[\tau L(u_i)]^{1/2}$, so that the error in fitting the composite profile horizontally may also be considered normal. The combined effect of the measurements of counting rate at all n points can be regarded as a combination of independent measurements, each contributing with a weight inversely proportional to the square of its standard error. Hence, if σ is the standard error of the fitting of the composite profile to all points we obtain

$$1/\sigma^2 = P = \sum_{i=1}^{n} \tau [L'(u_i)]^2/L(u_i). \tag{1}$$

$P = 1/\sigma^2$ is a convenient measure of the precision of fit.

Regarding the setting u as a function of time t, we may let n increase indefinitely, and replacing τ by dt we obtain another representation of Eq. (1),

$$1/\sigma^2 = P = \int \{[L'(u)]^2/L(u)\}\, dt, \tag{2}$$

which is somewhat more general than Eq. (1) since it holds regardless of how the interval is traversed. It should be reiterated that Eqs. (1) and (2) were based on the following approximations:

(1) All counting intervals contain enough counts so that the Poisson distribution may be approximated by a normal distribution.

(2) All errors are small enough so that, within their range, the curve $L(u)$ does not change slope appreciably.

Of these two approximations the second one is the more serious since it tends to break down when one considers extremely weak lines. Nevertheless, this assumption is necessary for a systematic analysis of these errors.

In order to obtain an explicit expression for the errors in fitting a particular profile we make the following additional approximations:

(3) Points on the profile are uniformly spaced and sufficiently closely spaced that a continuous distribution of points may be used to approximate the actual discrete distribution.

(4) The line profile has the shape of an isosceles triangle, its apex representing the line peak, and the extended background representing its base.

This simple profile seems to be very nearly the shape of the actual one in many cases (see Fig.30) and certainly is not so far wrong as to cause a greatly incorrect estimate of P.

Assigning a base width W to the line and a peak height H above background B, we obtain $[L'(u)]^2 = 4H^2/W^2$ over the profile so that if a total time T is required to cross the line, then

$$P = \frac{2T}{W}\int_0^{W/2} \frac{4H^2/W^2}{(2H\ell/W)+B}\, d\ell ,$$

giving

$$P = 1/\sigma^2 = (4HT/W^2)\log\left[(H+B)/B\right] . \qquad (3)$$

Actual lines which have been investigated have σ's as calculated by Eq. (3) which are less than 0.001 milliangstrom in almost all cases, and for this reason we feel that errors due to random variations in counting rate are considerably smaller than instrumental errors.

APPENDIX III

RECALIBRATION OF THE MARK I SPECTROMETER. DETECTION AND EXPLANATION OF ERRORS FROM IMPERFECT RIGIDITY. TESTS OF THEIR ELIMINATION.

In 1948, the 2-meter curved crystal spectrometer was used to make a direct precision measurement[1] of the annihilation radiation wavelength from the positron emitter Cu^{64}. This measurement exhibited a slight discrepancy of about 6 microangstroms (in 24,000) from the expected value, $\lambda_c = h/m_o c = (2.4266 \pm 0.0003) \times 10^{-10}$ cms computed from the at that time most probable values of these constants, determined by least-squares adjustment[2] from all precision sources of information. (These sources yielded m_o the mass of the negative electron.) Or the wavelength screw carriage of our 2-meter curved crystal spectrometer this discrepancy (of 6 microangstroms) represented a displacement of only 12 microns and though it was about eight times the probable error of the mean of 7 independent runs computed from their reproducibility, we were not prepared to guarantee that systematic errors in our wavelength scale of this magnitude might not be present. At the time, just because of this uncertainty, we multiplied our probable error (based on reproducibility) by four to take account of such possible systematic errors. Shortly after this measurement, Dr. David E. Muller introduced a very considerable improvement in the sensitivity of the 2-meter instrument by perfecting the present sodium iodide scintillation crystal detector to take the place of the earlier multicellular Geiger-Muller counters. Thanks to the resulting great improvement in sensitivity, early in 1951, Muller was able to measure, in first, second, and third orders of reflection, four different gamma-ray lines from Ir^{192}, and in first and second orders the Ir $K\alpha_1$ x-ray line.

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1) J.W.M. DuMond, D.A. Lind, and B.B. Watson, Phys. Rev. 75, 1226 (1949)

2) The adjustment referred to is that of DuMond and Cohen, Rev. Mod. Phys. 20, 82 (1948). The values obtained at that time have been revised in more recent adjustments using recent atomic beam and microwave data to a state of still higher accuracy but the changes make no material difference in the present argument. For the most recent review of the subject see DuMond and Cohen, Rev. Mod. Phys. 25, 691 (1953).

TABLE I

Apparent Wavelengths of Four Strong γ-Ray Lines and One X-Ray Line Measured in Three Different Orders With 2-Meter Curved Crystal Spectrometer. (Lines are those following decay of Ir^{192})

	Wavelengths in Milliangstroms		
Energy	1st Order	2nd Order	3rd Order
295.79 Kev	41.909	41.894	41.898
308.26 Kev	40.214	40.205	40.191
316.28 Kev	39.197	39.183	39.174
467.53 Kev	26.515	26.503	26.496
Ir $K\alpha_1$	191.025	190.993	

Table I shows the results of these measurements in which it will be noted that, in all but one case, higher order reflections indicated slightly shorter wavelengths in disagreement with lower order reflections. We concluded that a hitherto unsuspected nonlinearity in the wavelength scale of our spectrometer must exist such that, in the wavelength region below the calibration point for our screw, the wavelengths indicated by the spectrometer were a little too long. (This calibration point is at 208 milliangstroms, the wavelength of the $WK\alpha_1$ x-ray line.)

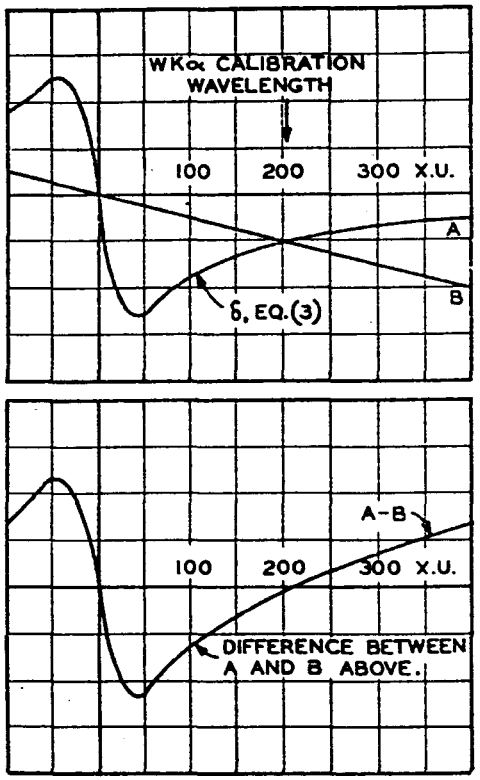
Because of this clear internal evidence of a nonlinearity in our wavelength scale, we undertook in 1951 a very careful intensive study of all possible sources of error in the instrument and an extremely careful recalibration as regards the linearity of the wavelength scale. The chief causes of the trouble we have found come from mechanical flexures in some of the parts of the instrument for which we have devised optical means of detection and correction, briefly described in the main body of this report.

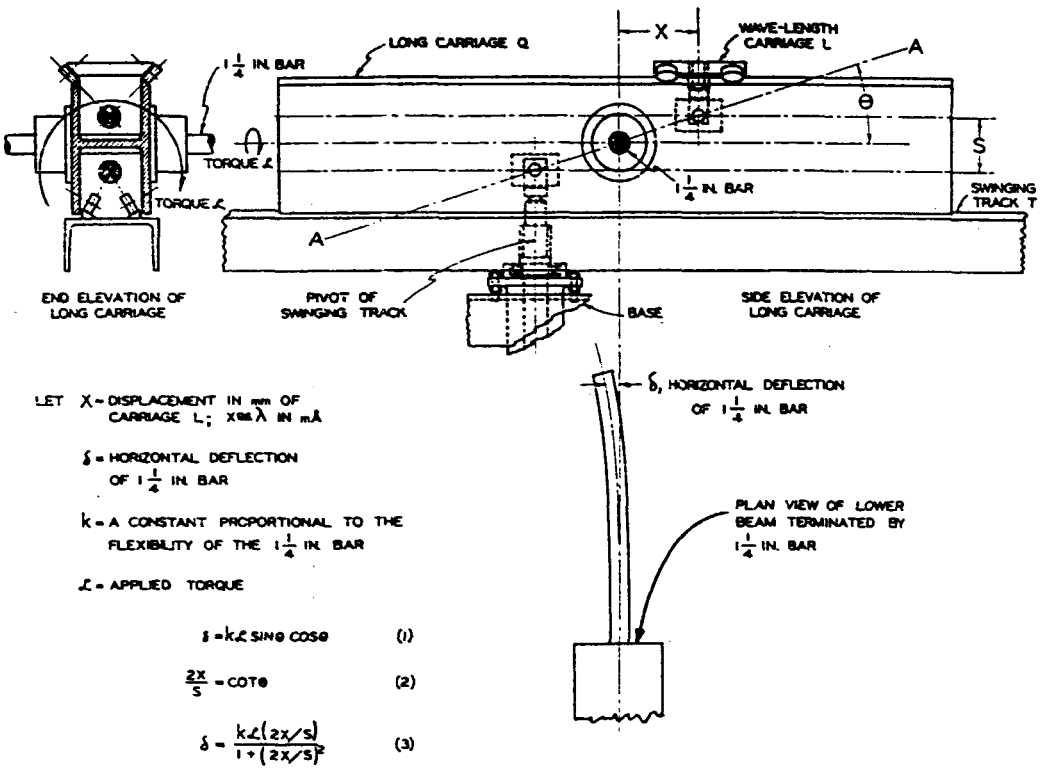
The chief source of nonlinearity we have found to come from the minute mechanical flexures in the 1 1/4-inch bar, the extension of the lower beam CO β passing through the ball bearing guides in the carriage Q.

(See Figs. 9, 10, 11 and especially 24 in the main body of this report.)

An analysis of the forces which impose a systematic bending on this bar because of the overhanging weight of the driving motor which drives the precision screw on carriage Q (see Fig. 26 in the main body of this report) was made in 1951 by Dr. D.E. Muller. He showed that the torque exerted on carriage Q by this overhanging weight tends to bend the 1 1/4-inch bar not in a vertical plane but around an oblique axis AA shown in Fig. AIII-1 because of the constraints placed upon carriage Q by the two driving nuts on the two precision screws. It is the horizontal component of this bending of the 1 1/4-inch bar which introduces a nonlinearity in the motion of the crystal. By this argument Muller was able to deduce a theoretically predicted correction curve for the nonlinearity which bears a striking resemblance to our experimentally measured nonlinearity correction. However, our measurements have shown that in addition to this effect, there are other sources of nonlinearity which cannot be neglected. These result from (1) the effort which must be exerted by the 1 1/4-inch bar to swing the track T against the frictional resistance in the ball bearings supporting the track, which depends to some extent on the past history of setting, and (2) certain flexures in the base of the machine which tilt the crystal pivot to varying degrees as the weight of the wavelength carriage is shifted along the track. The optical means we have provided for studying and correcting the above mentioned faults are shown and explained. Fig. 24 and 25 in the main body of this report. We call this for brevity the "mirror correction". We have found this procedure to be distinctly superior to the use of a permanently prepared calibration curve of the mirror correction because our studies have clearly shown that this correction involves a rather large nonreproducible component, part of which is random and part dependent on the past history of wavelength settings which have been made. Such nonreproducibility in the mirror correction undoubtedly results from elastic hysteresis, friction and minute mechanical lost motion chiefly in the parts linking the motion of the crystal with the precision screw carriage.

Fig. AIII-2 shows the average curve for the reproducible component of the mirror correction as observed by means of the elbow microscope and the 6-inch concave mirror described in Section 2.8 of this report





LET X = DISPLACEMENT IN mm OF CARRIAGE L; $X \equiv \lambda$ IN mÅ

δ = HORIZONTAL DEFLECTION OF $1\frac{1}{4}$ IN. BAR

k = A CONSTANT PROPORTIONAL TO THE FLEXIBILITY OF THE $1\frac{1}{4}$ IN. BAR

$\mathcal{L}$ = APPLIED TORQUE

$$\delta = k\mathcal{L} \sin\theta \cos\theta \qquad (1)$$

$$\frac{2x}{s} = \cot\theta \qquad (2)$$

$$\delta = \frac{k\mathcal{L}(2x/s)}{1+(2x/s)^2} \qquad (3)$$

Fig. AIII-1

Illustrating Muller's explanation of the effect of flexure in the 1 1/4 inch bar and the nonlinear correction it introduces in the wavelength scale of the Mark I spectrometer. The torque $\mathcal{L}$ (upper left hand corner of drawing) exerted on the long screw carriage by the overhanging weight of the driving motor (see Fig 26 of report) exerts forces on the 1 1/4 inch bar which bend it in a plane oblique to the vertical because of the constraints imposed on the long screw carriage by the driving nuts on the two precision screws one of which is rigidly attached to the base, the other to the upper beam. Because the track T is free, save for the constraint placed upon it by the 1 1/4 inch bar, to swing in the horizontal plane round the vertical axis of its pivot, it should be clear that line A-A is the axis around which the long screw carriage is free to rotate, except for the constraint of the 1 1/4 inch bar. Thus only a component of the horizontal torque of the unbalanced weight of the motor, its projection on A-A, namely $\mathcal{L}\cos\theta$, is operative to bend the bar and it does so in such a way that the curved axis of the bar lies in a plane normal to A-A. The total deflection of the bar at its end is thus $k\mathcal{L}\cos\theta$, where the constant k is inversely as the stiffness of the bar. Only the horizontal component of this flexure of the bar is in its turn operative to introduce the error in orientation of the crystal. This horizontal component is $\delta = k\mathcal{L}\cos\theta \sin\theta$. The obliquity θ of the line A-A changes with the wavelength setting of the instrument and this is what makes the error, δ, a nonlinear function of the wavelength setting. Equation (3) in the above figure gives the deflection of the 1 1/4 inch bar as a function of the setting, X. Since the constant of proportionality between screw turns and wavelengths has been calibrated by a setting on the $K\alpha_1$ line of tungsten at 208 x-units we must subtract a linear function from δ such that the difference curve will pass through zero at that wavelength. This will give the predicted curve of nonlinear correction. This is the second of the two curves, the one labelled A-B, and its general similarity to our final observed curve, Fig. AIII-2 for the reproducible component of the mirror correction is striking. The flexure here referred to is in fact very minute. Its maximum value which occurs at 60 or 70 x-units is only about 20 microns:

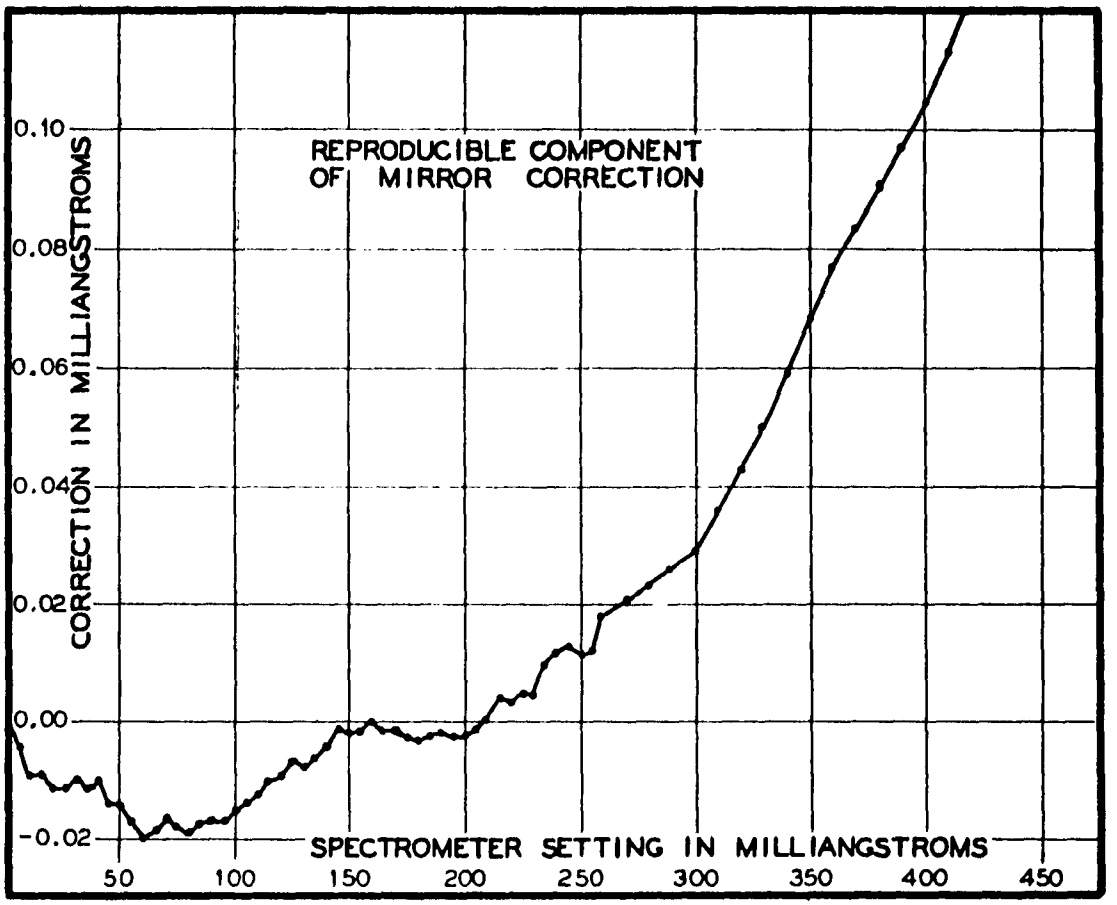


Fig. AIII-2 The wavelength correction necessitated by misalignment of the crystal with the long screw carriage. This is the average curve representing the reproducible component of the mirror correction as observed by the elbow microscope, M_2 Fig. 24, and the six-inch concave mirror mounted above the crystal, Fig. 25.

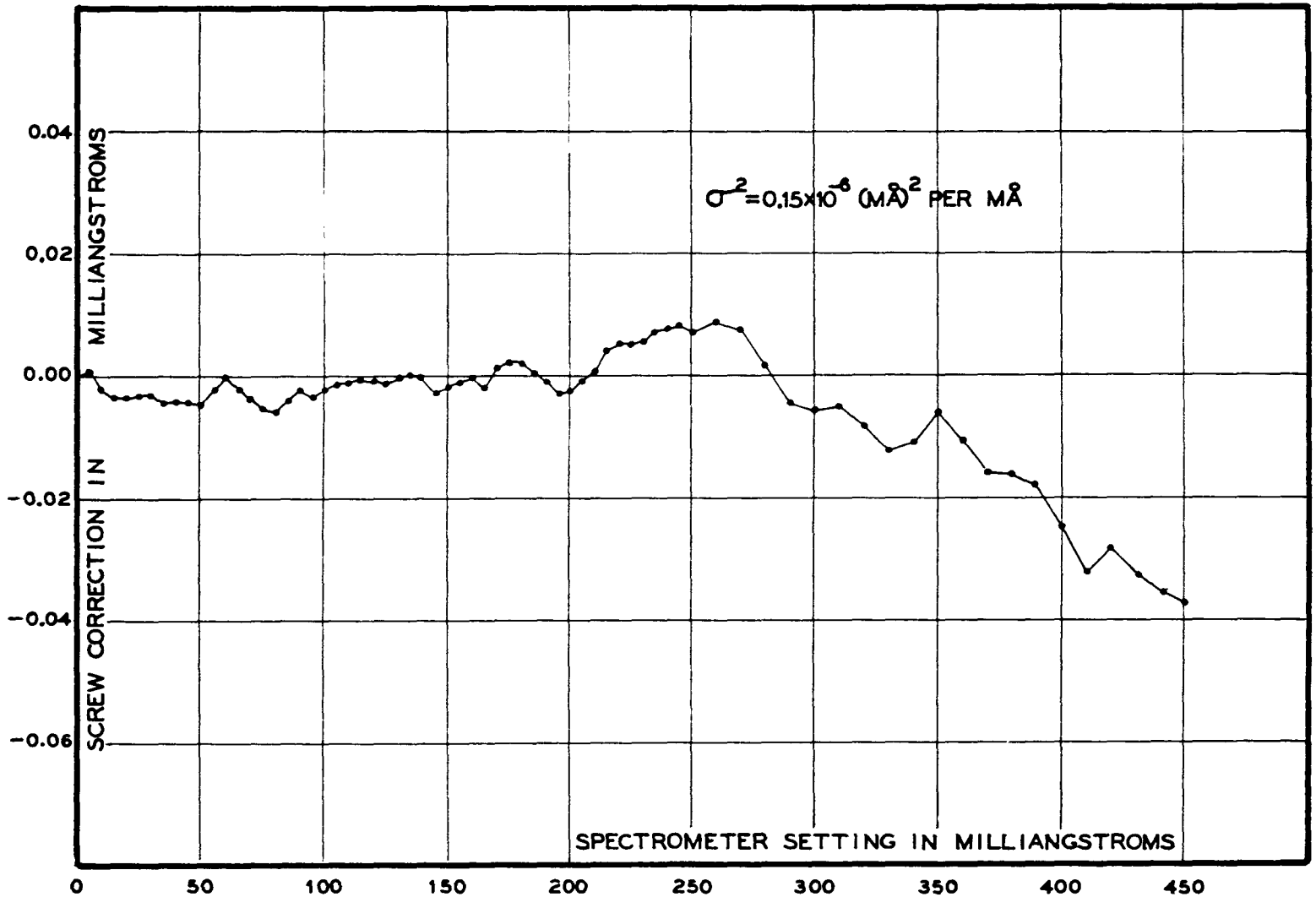


Fig. AIII-3 The wavelength correction curve necessitated by nonlinearity of the screw.

(see Figs. 24 and 25). Because of its lack of reproducibility we prefer to make this correction directly with the mirror and microscope each time a measurement is being made but we have also prepared this curve to give an idea of the magnitude of the correction and also for use on old data of which we have amassed a considerable quantity. Such corrected old data is less reliable than new data in which the mirror correction is observed directly each time and we therefor plan, as part of our program, to repeat all measurements made prior to our discovery of the need for the mirror correction. The general shape of this mirror correction curve bears a remarkable general resemblance to the predictions of Muller's simple explanation given in Fig. AIII-1. Other factors are certainly also present, such as flexures in the base of the instrument which we have measured with sensitive levels, and these distort the curve from Muller's predictions somewhat.

With a carefully calibrated Bureau of Standards glass scale mounted on the frame above the screw carriage, clearly visible in Fig. 24, we have also calibrated the departures from linearity in the travel of the source, measured in turns of the screw, by means of the microscope, M_1, which is mounted on the upper source-supporting beam. These errors are not only due to periodic and secular errors in the screw but also to minute departures from rectilinearity in the upper ways on which the small carriage L rolls. These impart a slight rocking motion to that carriage which in turn is transmitted to the source through torsional flexure of the upper source supporting beam to a degree clearly detectable with a sensitive level. It is for this reason that the microscope used in this calibration is rigidly attached to the source-supporting beam with its focal plane at the same height as the radioactive source so that microscope and source will partake of identical errors of this type.

Fig. AIII-3 shows the wavelength correction curve necessitated by nonlinearity of the screw as obtained by a long and very tedious series of oft-respected observations with the microscope M_1 and the Bureau of Standards glass scale. A quasi-periodic correction (periodic with one turn of the screw) which varies slowly in character as one progresses along the entire length of the screw cannot be shown on this

curve because of the great detail it would involve).

The complete screw calibration was accomplished in several stages involving determination of the errors (departures from linearity) at intervals of different sizes at each stage. To accomplish the calibration over the entire useful length of the screw the glass decimeter scale had to be reset many times just as a surveyor's 100 foot tape might be used repeatedly to measure off a mile. The net result of this was that the absolute precision of the calibration was less for long than for short stretches (a well known phenomenon in surveying) and in fact, just as statistical theory predicts, it is the square of the standard deviation which increases linearly with distance. As indicated on the calibration curve of Fig. AIII-3, the precision of the calibration was given by $\sigma^2 = 0.15 \times 10^{-6}$ (milliangstroms)^2 per milliangstrom of scale length. This was determined by a "random walk" analysis from the reproducibility of the observations themselves.

In our earlier work with this instrument up to the spring of 1951, we had indeed corrected for periodic and aperiodic errors in our screw, but these calibration curves were much less carefully observed and also took no account of the then unsuspected "mirror correction." The scale in this earlier calibration was, furthermore, not supported at the same height as the source so that the calibration did not correct for minute rocking of the source beam.

The first definite internal evidence of slight residual nonlinearity in our supposed linear wavelength scale came, as already explained above when, thanks to the greatly improved sensitivity secured by replacing our earlier "multicellular" Geiger counters with a sodium iodide scintillation crystal, we were able to observe one and the same wavelength in three different orders of reflection from the curved quartz crystal for four different intense nuclear gamma-ray lines emitted by an exceptionally strong source of iridium 192. As stated earlier, we were also able to observe the intense $K\alpha_1$ x-ray line from iridium in first- and second-order reflection. These results (see Table I of this Appendix) exhibited a very definite systematic trend: Wavelengths computed from higher order reflections were (in every case but one) slightly shorter. These disagreements, however, were not sufficient to establish a very reliable

correction curve by themselves. Nevertheless, it was because of them that the intensive recalibration and study of the minute causes of nonlinearity in the instrument (lasting over a period of six months) was undertaken.

An idea of the greatly enhanced reliability of the new calibration may be gleaned from a comparison of the intense lines of iridium 192 which were measured in first, second, and third orders. These results, which are compared in Table II after making the new calibration correction and the mirror correction, agree well within their assigned errors and correspond to measurements made at one, two, and three times the wavelength settings of these lines. It is interesting to note that even the measurements on the intense iridium $K\alpha_1$ line, which was obtained in the second as well as the first order, agree quite well in spite of the fact that corrections in this case were relatively large.

TABLE II

Wavelengths (in milliangstroms) of five intense lines from iridium 192 as measured in various orders after being corrected using the latest calibration.

Energy, kev	3rd order	2nd order	1st order
295.94	41.891	41.885	41.887
308.45	40.187	40.190	40.191
316.46	39.172	39.172	39.175
467.98	26.486	26.491	26.495
Iridium $K\alpha_1$ x-ray		191.041	191.031

At present the calibration equipment is to be used to provide still higher precision than was possible before. By permanent installation of the 6-inch mirror above the curved crystal, the alignment of the crystal with the long carriage is now and henceforth checked at the time and every time that the gamma-rays are actually being measured.

In this way, any possible hysteresis or nonreproducibility present in this motion is automatically taken into account. Similar methods are used for checking the behavior of the screw.

With the use of these improvements a further exceedingly satisfactory check has been made on the reliability of the calibration by observing in three different orders of reflection the 412-kev line from a strong source of gold 198. Wavelength measurements in the first, second and third orders of reflection are 30.102 $\pm$ 0.0038, 30.105 $\pm$ 0.0033, and 30.107 $\pm$ 0.0028 milliangstroms, respectively. Disagreement between these measurements lies well within the assigned errors, thus confirming even more satisfactorily the reliability of the calibration. This is definitely not the case in the data obtained before the screw calibration and mirror corrections were applied.

By making multiple calibrations of the spectrometer, it was possible to estimate the errors which arose during the calibration. This was done by determining the mean square deviations from the mean and dividing by the number of degrees of freedom. A random walk analysis of the over-all calibration of the screw was used to determine the error of the calibration. For this part of the calibration $\sigma^2 = 0.15 \times 10^{-6}$ $(mA)^2$ per mA as stated above. A determination of the errors in the quasi-periodic calibration was accomplished by repeating the calibration of single turns of the screw. Random variations in screw readings and in the "mirror correction" were treated in a similar fashion. Results of these calculations indicate that the errors to be expected in wavelength measurements made subsequent to the calibration are less than 0.005 milliangstrom because "mirror" readings are taken at the time the gamma-rays are measured. Measurements made prior to the calibration are subject to the somewhat larger uncertainty of about 0.01 milliangstrom because no check had been made at that time of the alignment of the crystal with the long carriage. In order to correct these older measurements the curve shown in Fig.AIII-2 (which was obtained during the calibration) had to be used.