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PROBLEMS IN SERVOMECHANISM SYNTHESIS

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REPORT NO. 1

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Progress Report on Work Conducted Under
Office of Naval Research Contract
N7ori - 30306 and 30308

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Progress

This is the first of a series of reports of an investigation of servomechanism synthesis under Navy Contracts N7ori30306 and N7ori30308. The investigation was initiated originally for the purpose of studying the application of magnetic amplifiers in servomechanism design. It rapidly became apparent, however, that (A) it has already been demonstrated that magnetic amplifiers are applicable to servomechanism problems in a wide variety of cases, (B) that such applications are commonly made on a rule-of-thumb basis and that, (C) a more refined application of magnetic amplifiers in servomechanism design cannot be separated from the general problems of servomechanism synthesis and that this project should start with the redefinition of the basic problem, that of an optimum criterion for synthesis on a realistic basis.

In the classical approach to servomechanism analysis it is assumed that (1) signals are of regular functional form, typically step functions, (2) such artificial criteria as phase margin or peak overshoot values are satisfactory as measures of synthesis success. More recently servomechanism approaches have been directed to (1) statistical properties of signals and, (2) realistic criteria both linear and nonlinear. This is the direction undertaken in this particular study.

The results of this study to date are as follows:

a) A method of evaluating a linear servomechanism system in the light of any type of signals that can be statistically described, and any type of error criteria for which a power series approximation can be written (with the common quadratic norm as one term of the series). This work is described herein as report number one.

b) A method of prescribing the optimum transfer function for systems having certain specified time-varying characteristics, or for signals having certain specified time-varying properties (including random variations), as well as the best adjustment for first and second order systems for a set of commonly assumed, statistically described signals which also may have time variations. In some respects this work is an extension of Wiener's methods. This work is described in a report which will be number two of the series.

c) A method of deriving a transfer function for a magnetic or a dielectric amplifier starting with the B-H or D-E characteristics of the non-linear material. The method permits determining in addition to the linear terms the harmonic distortion terms of both the envelope and the carrier functions. This material is being readied for report in the near future.

d) A review of some classical and graphical methods of treating non-linear systems which have been compiled but not yet prepared in report form.

The work contained herein was presented in preliminary form at the:
I.R.E. 1953 National Convention, "Generalized Servomechanism Evaluation", Kaufman and Caywood, Session 43: Remote Control Systems.
Symposium on Statistical Methods in Communication Engineering, Berkeley Campus, University of California, Aug. 17-18, 1953, "Generalized Servomechanism Evaluation", Caywood, Lyman, and Kaufman.

The first analyses made of servomechanism systems, the present day classical methods, allow first for the calculation of absolute system stability, and second for the calculation of system response to an input signal of known, geometrical shape. The calculation of performance usually comprises determining the amount of the first overshoot and the free resonant frequency in response to a Heaviside step function, or the steady-state amplitude and phase response to a sinusoidal signal. In instances when the system is of complexity greater than the most simple, the response calculated may be simply by rule-of-thumb inference from the Heaviside response of the close loop system, obtained from the sinusoidal relation which describes the system with its feedback path disconnected.

As a means of evaluating a servomechanism the classical analysis appears to have rather severe limitations in accuracy of a fundamental origin. First there can be no prior knowledge of the exact input signal that will be demanded of the servomechanism if it is to serve as a control device, rather than a mere generator of waveforms. Instead, there must be a certain repertory of possible messages, and the occurrences of the individual possibilities can be stated only in terms of probabilities. Additional uncertainty in the functional expressions for the input signal result from the inevitable contamination of the desired signal with an undesired one (i.e. noise). Such an undesired signal comprises another repertory of possible values, the occurrence of which is in part, if not wholly, unrelated to the occurrence of specific, possible messages of the signal and again it can be anticipated only in terms of probabilities. The response to the combination of signal and noise generally is the one of importance in performance evaluation or synthesis problems.

A second limitation on the applicability of the classical approach is imposed by the difficulty of interpreting the calculated system response in terms of a realistic criterion of error-weighting function. As an example consider the case of a second order system whose adjustment is limited to the damping factor alone. Consider further the rather artificial condition of a signal which is a step function of known magnitude, and that the performance desired in the specific application is such that the error is judged to be either satisfactorily small and is called zero, or is too large and is called unity. The integrated error due to each input step is shown in Figure 1. Obviously the best adjustment depends on both the permissible error as well as on the size of the input step; that is to say it depends on the application of the system, as well as on the detailed characteristics of the signal. The illustration just described is extended one step further in Figure 2. Here it is assumed that the input steps are of various sizes and such that they are statistically described by a gaussian distribution of standard deviation sigma and having a mean value of zero. The best adjustment is calculated to be as shown; again it is a function of the input signal and of the application. These examples are intended only to illustrate this dependence rather than to have value in themselves.

On the problem of a statistical evaluation of servomechanism systems, recognizing the necessity of a realistic criterion of integrated error, the progress to date made by the efforts of the authors has resulted in three separate approaches, two described herein.

(a) A computer which includes a system analogue and an error criterion and integrating circuit, and which accepts statistically typical signals from a signal storage or generating device. There results a single number describing the performance of the system.

(b) An analytical method, applicable only to linear systems, which makes use of the autocorrelation function of the signal and noise inputs, and also of the higher moment correlation functions of the two inputs.

A. The Computer

The solution selected involves a straight-forward approach to the general problem utilizing a model of the system under study to which signals of the correct statistical characteristics are supplied. An evaluation of the performance of the model is facilitated by the use of a network which evaluates an instantaneous error, and which integrates a weighted error to give a single number representative of performance. A block diagram of such a computer is shown in Figure 3.

The form of the computer constructed at Carnegie is rudimentary. The elements are signal sources, summing amplifiers, a frequency-response network comprising resistors, condensers and cathode follower stages for isolation, a criterion-simulating network of limited flexibility (but with provisions for external circuit connections) and an error-integrating circuit with an indicating meter that reads the average weighted error. The order of the system that can be analyzed is virtually unlimited; however, the types of nonlinearities permitted are quite restricted unless the electronic analog is replaced by a more realistic model or actual hardware.

The computer, as described, is believed to have considerable academic value in addition to its practical application. There are also two extensions of its present form that suggest themselves. One of them results from the realization that a realistic error criterion may dictate the inclusion of the duration of error, as well as its instantaneous amplitude, in order to establish a realistic measure of the expected performance under use. The other extension embodies the possibility of a controlling servomechanism to repeatedly readjust and test the servomechanism analogue and to either record the relative value of performance, including its minimum, or to actually seek the minimum weighted error as an answer to the proper system adjustment. No progress has been made on these extensions of the operation of the described computer.

B. Utilizing Correlation Functions

An analytical method has been developed for solving some of the problems that can be treated by the computer described. Certain restrictions must be tolerated, namely:

1. That the servomechanism system perform linear operations inveriant with respect to the choice of an origin in time.

2. That the convolution integral given by the expression:

$$\int_{-\infty}^{\infty} a(\tau)\, H(t-\tau)\, d\tau$$

possesses mean square or mean " n^{th} " integrability at least, when $a(\tau)$ and $H(t-\tau)$ and mean " n^{th} " integrability are as defined below.

This portion of this paper is divided into two parts. The first comprises a review of the use of correlation methods to evaluate the performance of servomechanism systems for cases in which the input signal is treated statistically. The second is an extension of the applicability of the methods reviewed in the first part and which is effected by considering numerical measures of error, or norms, that are more general than the quadratic. That is to say, the error that may exist between the output and the input at any instant is weighted according to an appropriate weighting function that is tailored the system application under consideration. For input signals that are statistically typical, the integrated (or average) value of the weighted error is calculated to form a single number that is a measure of the "poorness" of performance. A synthesis of the best system for any particular application is a matter of minimizing the mean weighted error by proper choice of the system parameters.

Error, as it applies to communications and servomechanism theory, is the difference between the actual output of a system and the desired or ideal output.

In symbols,

$$e = f_o - f_d \tag{2}$$

where e = error; f_o = actual output; f_d = desired output.

If, for example, f_o and f_d are voltages as functions of time, e is also a voltage as a function of time, and the mean square value of error is a measure of power developed by the error that must be overcome.

In any linear system, the output is the result of some linear operation on the input.

In symbols,

$$f_o(t) = A(t) f_i(t) \tag{3}$$

where A(t) is some linear operator; $f_i(t)$ is the input. This could also include a time delay, and it also can be written as the convolution integral as follows:

$$A(t) f_i(t) = \int_{-\infty}^{\infty} a(\tau) f_i(t-\tau)\, d\tau \tag{4}$$

$a(\tau)$ is the derivative of the system response to a unit step and upon integration becomes $A(t) f_i(t)$. The desired output can be written similarly

$$f_d(t) = B(t) f_i(t) \tag{5}$$

For the B operator there is the choice of desiring the output to be identical with the signal component of the input, in which case B is unity. On the other hand there may be conditions such as the desirability of limiting the power rating of motors that suggest other choices of the B operator.

If the actual input is the sum of a signal input function $f_i(t)$ and some random noise function N(t), both resulting from a stationary random process, then the output is

$$f_o(t) = A(t)\left[f_i(t) + N(t)\right] \tag{6}$$

The error is the difference between the desired and actual output, and can be expressed:

$$e(t) = A(t)f_i(t) + A(t)N(t) - B(t)f_i(t) \tag{7}$$

The quadratic norm can be further expanded:

$$e^2(t) = \left\{\left[A(t) - B(t)\right]f_i(t) + A(t)N(t)\right\}^2 \tag{8}$$

$$\begin{aligned} e^2(t) = {} & \left\{\left[A(t) - B(t)\right]f_i(t)\right\}^2 + \left\{A(t)N(t)\right\}^2 \\ & + 2A(t)\left[A(t) - B(t)\right]N(t)f_i(t) \end{aligned} \tag{9}$$

Employing the convolution integral, as in equation 4 above, and determining the average value:

$$\begin{aligned} \overline{e^2} = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\Bigg\{ & \int_{-\infty}^{\infty}\left[a(\tau_1) - b(\tau_2)\right]f_i(t-\tau_1)\,d\tau_1 \\ & \int_{-\infty}^{\infty}\left[a(\tau_2) - b(\tau_2)\right]f_i(t-\tau_2)\,d\tau_2 \\ & + \int_{-\infty}^{\infty}a(\tau_1)N(t_1-\tau_1)\,d\tau_1\int_{-\infty}^{\infty}a(\tau_2)N(t-\tau_2)\,d\tau_2 \\ & + \iint_{-\infty}^{\infty}a(\tau_1)N(t_1+\tau_1)\left[a(\tau_2)-b(\tau_2)\right]f_i(t-\tau_2)\,d\tau_1\,d\tau_2\Bigg\}\,dt \end{aligned} \tag{10}$$

Assuming mean square integrability (if uniform convergence is not complete) the order of integration may be interchanged and:

$$\overline{e^2} = \iint_{-\infty}^{\infty} [a(\tau_1) - b(\tau_2)][a(\tau_2) - b(\tau_2)] \left\{ \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f_i(t-\tau_1) f_i(t-\tau_2)\, dt \right\} dt_1\, dt_2$$

$$+ \iint_{-\infty}^{\infty} a(\tau_1)\, a(\tau_2) \left\{ \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} N(t-\tau_1) N(t-\tau_2)\, dt \right\} dt_1\, dt_2 \qquad (11)$$

$$+ 2 \iint_{-\infty}^{\infty} a(\tau_2) [a(\tau_1) - b(\tau_2)] \left\{ \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f_i(t-\tau_1) N(t-\tau_2)\, dt \right\} dt_1\, dt_2$$

The three limiting processes can be seen to define in equation 11 the two auto-correlation functions for the signal and the contaminating noise, and the cross correlation between signal and noise respectively. Expressing the auto correlations as:

$$\phi_{ii}(\tau_2 - \tau_1)\ ; \quad \phi_{NN}(\tau_2 - \tau_1)$$

and the cross correlation by: $\phi_{Ni}(\tau_2 - \tau_1)$

$$\overline{e^2} = \iint_{-\infty}^{\infty} [a(\tau_1) - b(\tau_1)][a(\tau_2) - b(\tau_2)]\, \phi_{ii}(\tau_2 - \tau_1)\, d\tau_1\, d\tau_2$$

$$+ \iint_{-\infty}^{\infty} a(\tau_1)\, a(\tau_2)\, \phi_{NN}(\tau_2 - \tau_1)\, d\tau_2\, d\tau_1 \qquad (12)$$

$$+ 2 \iint_{-\infty}^{\infty} a(\tau_2) [a(\tau_1) - b(\tau_1)]\, \phi_{Ni}\, d\tau_2\, d\tau_1$$

In many cases the noise is independent of the signal and the cross correlation between the two, ϕ_{Ni} , is zero. In this case the last term of equation 12 is removed.

Determining the best solution to the synthesis problem is done by finding a stationary value of mean squared error which represents a minimum. It is treated in the literature by Wiener, by Levinson and by others and will not be pursued further here.

The More General Criterion of Error

The quadratic norm has a certain realism in some applications, such as when the power or energy of the error is important. However, its mathematical convenience may not be supported by a very useful applicability in many situations. Consider fire control systems as an example. Rather than an error being "bad" in proportion to its size, the target may either be hit or not. And, if it is missed completely, it may make no difference by how much.

As a general means of treating the problem of the error criterion, let us assume that whatever the realistic relation is, it can be expanded as a power series:

$$e_w = C_0 + C_1 e + C_2 e^2 + C_3 e^3 + \cdots \qquad (13)$$

In cases of symmetry the odd terms are absent of course, and in many cases there will be no need to move the origin by use of C_0. The average value of weighted error:

$$\overline{e}_w = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} e_w \, dt \qquad (14)$$

or:

$$\overline{e}_w = \lim_{T \to \infty} \frac{1}{2T} \left\{ \int_{-T}^{T} C_0 \, dt + \int_{-T}^{T} C_1 e \, dt + \int_{-T}^{T} C_2 e^2 \, dt + \cdots \right\} \qquad (15)$$

Considering the separate integrals:

$$\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} C_0 \, dt = C_0 \qquad (16)$$

which contributes a shift in the origin. The second term:

$$\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} C_1 e(t)\,dt = C_1 \overline{e} \tag{17}$$

With e given as the difference of the system operator A operating on message plus noise and the idealistic operator B on message:

$$C_1\overline{e} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} C_1 \left\{ A(t)f_i(t) + A(t)N(t) - B(t)f_i(t) \right\} dt \tag{18}$$

Applying the convolution integral as in equation (1) above:

$$C_1\overline{e} = \lim_{T\to\infty} \frac{C_1}{2T}\int_{-T}^{T}\int_{-\infty}^{\infty} \left[a(\tau)f_i(t-\tau) + a(\tau)N(t-\tau) - b(\tau)f_i(t-\tau) \right] d\tau\, dt \tag{19}$$

Interchanging the orders of integration

$$\begin{aligned} C_1\overline{e} = {} & C_1\int_{-\infty}^{\infty}\left[a(\tau) - b(\tau)\right]\left[\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} f_i(t-\tau)\,dt\right] d\tau \\ & + C_1\int_{-\infty}^{\infty} a(\tau)\left[\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} N(t-\tau)\,dt\right] d\tau \end{aligned} \tag{20}$$

which is zero if the overall average of both $f_i(t)$ and N(t) is zero. The third term represents the quadratic norm that is described above. The fourth is zero in cases of criteria that are even functions. The next term:

$$\begin{aligned} C_4\overline{e^4} = C_4 \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} \Big\{ & \int_{-\infty}^{\infty}\left[a(\tau) - b(\tau)\right] f_i(t-\tau)\,dt \\ & + \int_{-\infty}^{\infty} a(\tau)N(t-\tau)\,d\tau \Big\}^4 \end{aligned} \tag{21}$$

To reduce the complexity of the expression consider cases in which there is no correlation between signal and noise. The changing the order of integration and proceeding as for the quadratic case above:

$$C_4\overline{e^4} = \iiiint_{-\infty}^{\infty}\Big\{[a(\tau_1)-b(\tau_2)-b(\tau_2)][a(\tau_3)-b(\tau_3)][a(\tau_4)-b(\tau_4)]\,\phi_{iiii} + 6[a(\tau_1)-b(\tau_1)][a(\tau_2)-b(\tau_2)]\,a(\tau_3)\,a(\tau_4)\,\phi_{iiNN} + a(\tau_1)\,a(\tau_2)\,a(\tau_3)\,a(\tau_4)\,\phi_{NNNN}\Big\}\,d\tau_1\,d\tau_2\,d\tau_3\,d\tau_4 \tag{22}$$

where:

$$\phi_{DDDD} = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} D(t-\tau_1)\,D(t-\tau_2)\,D(t-\tau_3)\,D(t-\tau_4)\,dt$$

is a function that depends on the third moment probabilities, and hence is termed the third moment correlation function. It can be calculated, or measured on a machine, in a manner analogous to the auto-correlation functions. One can proceed to the general case which is given by:

$$\overline{e^n} = \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\Big\{[a(\tau_1)-b(\tau_1)\cdots[a(\tau_n)-b(\tau_n)] \times \phi_{i\cdots i}(\tau_1, \tau_2, \cdots \tau_n) + a(\tau_1)\cdots a(\tau_n)\,\phi_{N\cdots N}(\tau_1, \tau_2, \cdots \tau_n)\Big\} \times d\tau_1\,d\tau_2\cdots d\tau_n \tag{23}$$

where:

$$\phi_{D \cdots D}(\tau_1, \tau_2, \cdots \tau_n)$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} D(t-\tau_1)\, D(t-\tau_2) \cdots D(t-\tau_n)\, d\tau$$

The correlation function $\phi_{D \ldots D}$ is again a higher moment one, in general of nth moment where n is also the degree of the term in the power series being considered.

To evaluate the higher moment correlation functions, three possibilities are being studied for convenience of application:

a) The direct approach, the construction of a machine that will perform the multiple multiplications and form the integral as a function of the various t's. For reasons of compactness and simplicity, magnetic tape devices are considered as most suitable.

b) The possibility of simplifying the multiplication process in the direct approach of a) by quantization of one or more of the members of the n'th product within the integral, using as a range of variation of the quantized number only the plus or minus sign of the function (i.e. infinite clipping at zero level). Considerable work yet remains in establishing regions of validity. The improvement in accuracy and "simplicity is expected to be quite considerable.

c) The digital computer approach. In many instances data is already of digital form, such as when recorded on frames of motion picture film, or on punched cards. While relative merits and disadvantages as yet have not been evaluated, the approach is quite straight forward and merits serious consideration.

- Integrated Error vs Step Size -

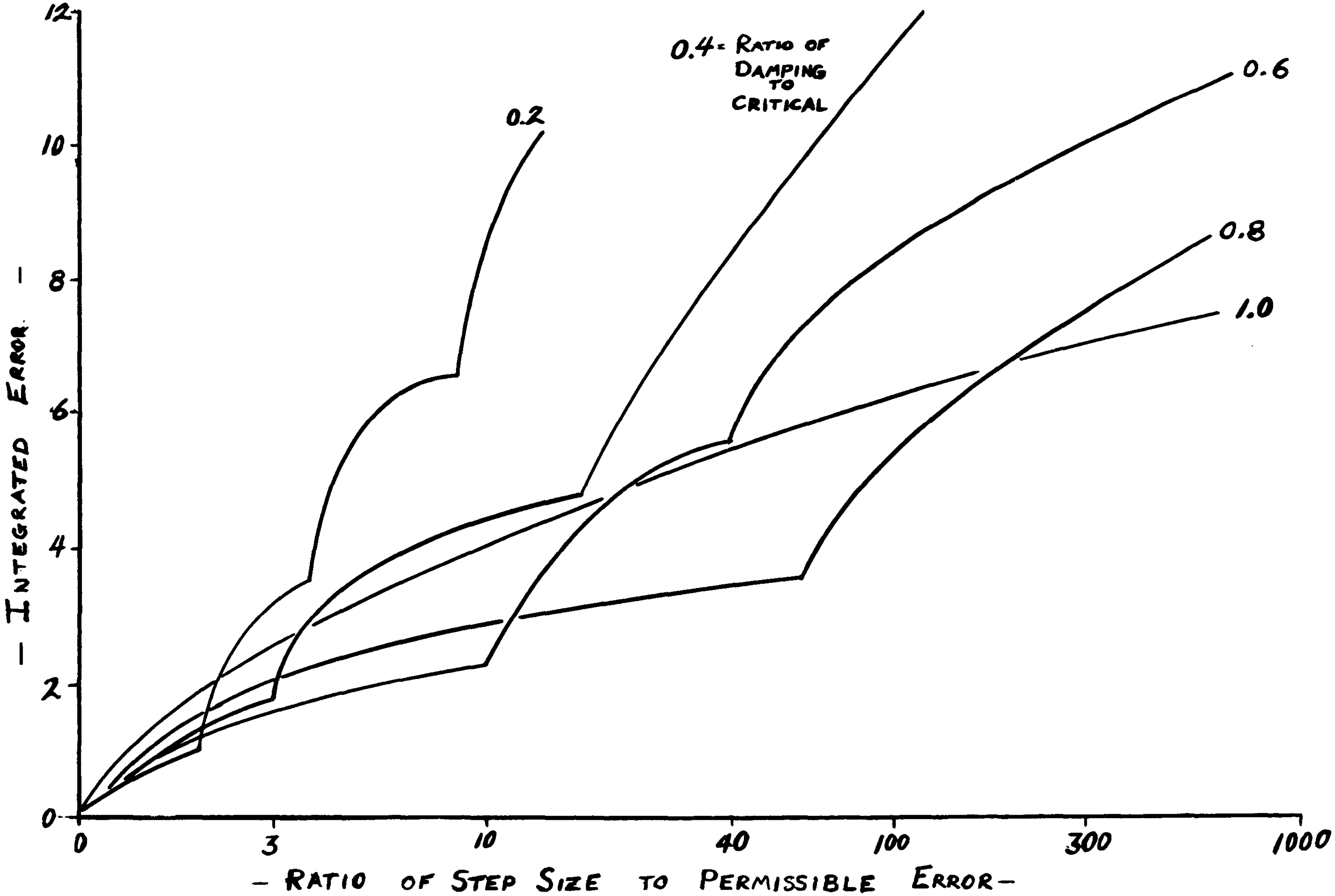


Figure 1

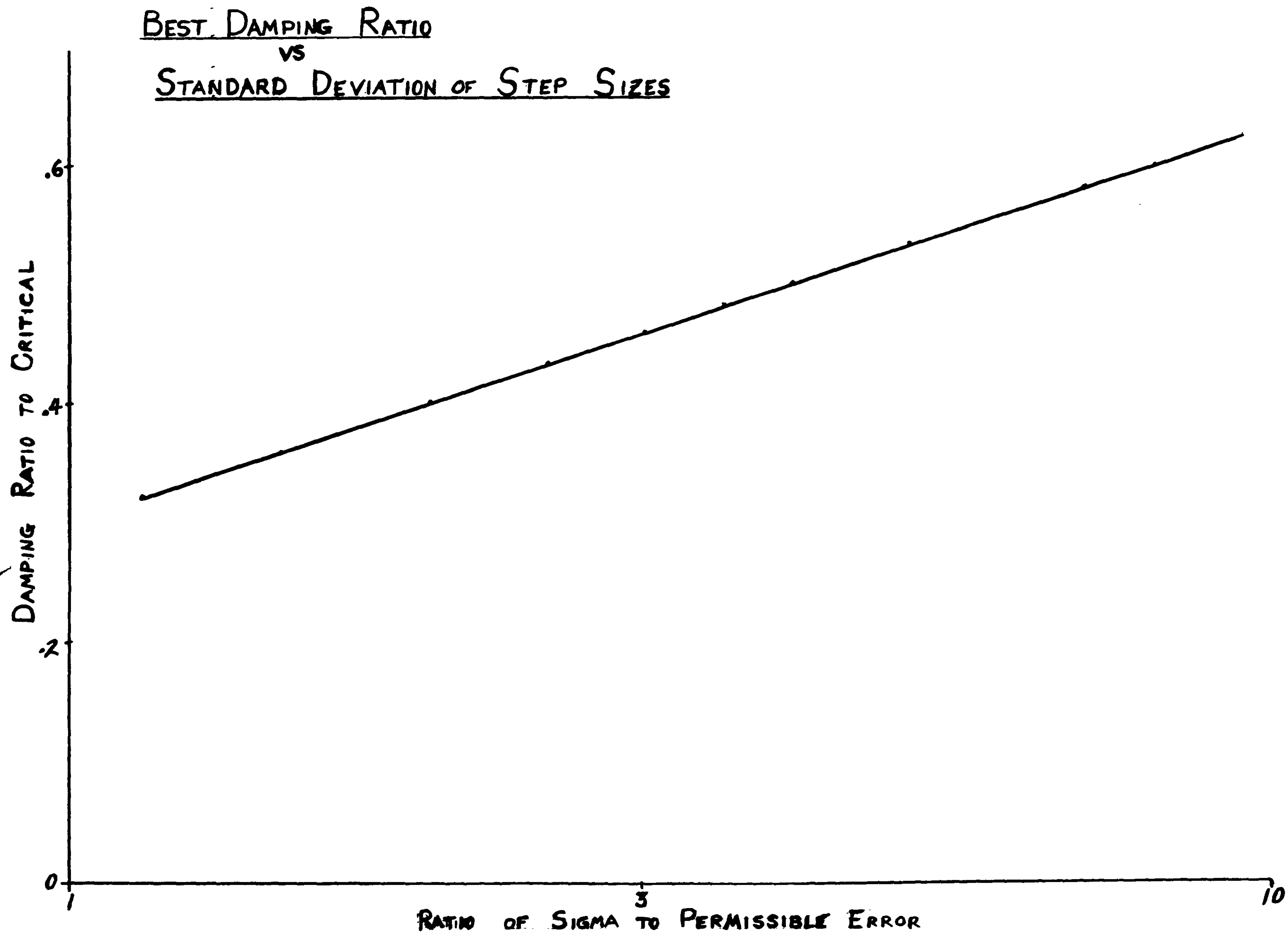
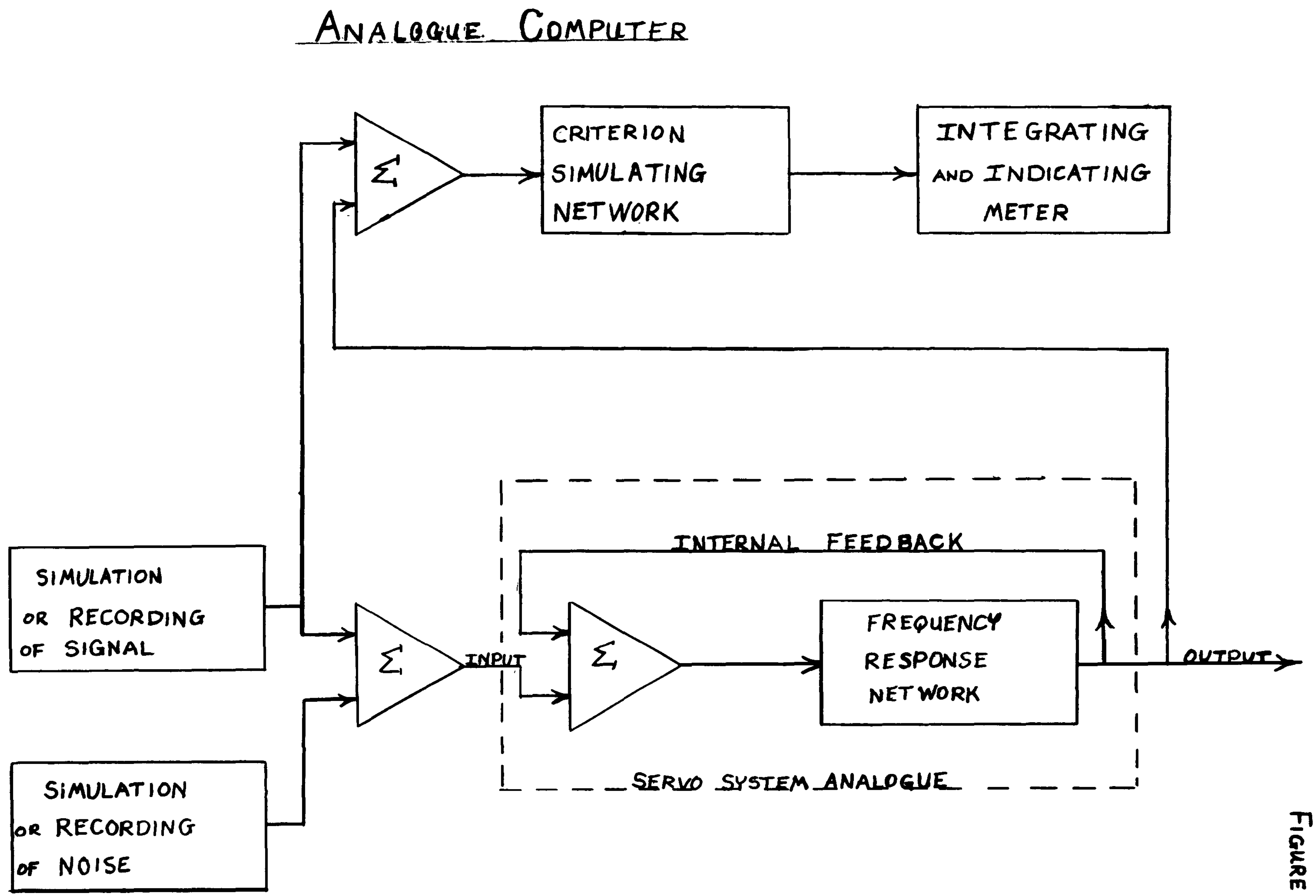

Best Damping Ratio
vs
Standard Deviation of Step Sizes
Damping Ratio to Critical
.6
.4
.2
0
1
3
10
Ratio of Sigma to Permissible Error

FIGURE 3


ANALOGUE COMPUTER
Σ
CRITERION SIMULATING NETWORK
INTEGRATING AND INDICATING METER
SIMULATION OR RECORDING OF SIGNAL
SIMULATION OR RECORDING OF NOISE
Σ
INPUT
INTERNAL FEEDBACK
Σ
FREQUENCY RESPONSE NETWORK
OUTPUT
SERVO SYSTEM ANALOGUE