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AN EXTENSION OF THE ALGEBRA OF CLASSES FOR
THE ASSOCIATION OF IDEAS

TECHNICAL REPORT NO. 4

PREPARED UNDER
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APRIL
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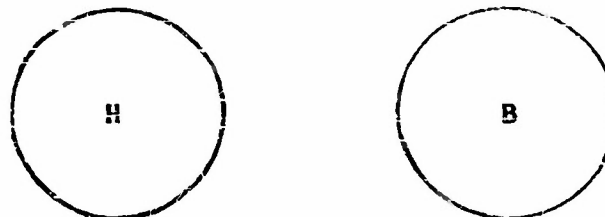
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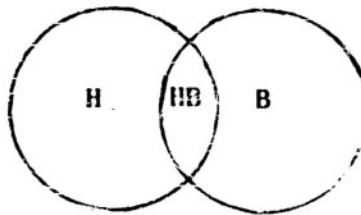
AN EXTENSION OF THE ALGEBRA OF CLASSES FOR
THE ASSOCIATION OF IDEAS

In the previous report we distinguished between the possible and actual associations in any system of information. Whether or not any association is actual in any system at a particular time is a material fact. An association may be a possible one at time t_1 , but at a later time t_2 it may become an actual association in the system. When an association which was at most a possible one at t_1 becomes actual at t_2 , we say in logic that the class has no members at t_1 , and has members at t_2 .

For example, at t_1 the class "Hydrogen Bomb" might have no members if t_1 were the 1930's. That is, the logical product of "Hydrogen" and "Bomb" is null.



But at t_2 , we have an actual association if t_2 were 1954, in that "Hydrogen" and "Bomb" have a logical product which is not the null class.



The concept of the null class is necessary in the logic of classes, since the product of any two classes is a class. Thus, there is a class "Hydrogen Bombs" at t_1 (1930's) but it is the null class, which has no members. To talk of the "Class of hydrogen bombs in 1930" as the null class may seem queer; but in logic there can be only one null class, which contains as members all round squares, all square circles, all griffins and all unicorns as well as all hydrogen bombs which existed in 1930. A proof that there can be only one null class is given below:

Assume the existence of two null classes O_1 and O_2 such that

$$(1) a + O_1 = a; \text{ and}$$

$$(2) a + O_2 = a$$

In each of these equations "a" can represent any class we choose, including O_1 or O_2 . By substituting O_2 for a in (1) and substituting O_1 for a in (2) we get

$$(3) O_2 + O_1 = O_2; \text{ and}$$

$$(4) O_1 + O_2 = O_1$$

But $O_2 + O_1 = O_1 + O_2$ since "+" is commutative.

Hence, $O_2 = O_1$ by substitution from (3) and (4), or the classes we assumed to be different are really a single class.

Unfortunately for all our sakes the class "HB" differs in one important respect from the class of unicorns or round squares. We are safe in regarding round squares as members of the null class, since we know that no changes in the world can change the fact that round squares are members of the null class. But changes in material facts, with time, change the status of hydrogen bombs. Round squares cannot exist but hydrogen bombs might happen to exist. It is this property of happening to have members that distinguishes the association of two classes from the ordinary logical product.

The logical product of two classes A and B is defined as the class which has as members the common members of A and B and is designated by the abbreviated symbolism "A x B" or simply "AB".

When we say that A is associated with B, therefore, we mean simply that $AB \neq 0$. For convenience in expressing this symbolically, we shall introduce a new symbol to represent the predicate "is associated with". For this purpose "*" will be used. It is defined by the relation

$$A * B \equiv AB \neq 0$$

which means that the statement "A is associated with B" is equivalent to the statement "the logical product of A and B exists".

Whereas "x" is a binary connective which joins terms to form other terms (that is, the product of two classes is a class), "*" is a connective which joins terms to form statements.

We shall also use " ," as indicating the enumeration of a set of terms or classes. Thus the expression "A, B" is read simply as "the class A and the class B". The "and" here should not be confused with the conjunction of two statements to make another statement "P·Q"; nor should it be confused with "A + B" (logical sum) which is understood as the class whose members are A or B or both.

The product operation "x" is distributive into summation "+", $A \times (B + C) = A \times B + A \times C$. Similarly, "·" is distributive into the relationship of "and"

$$A \cdot (B, C) \equiv A \cdot B \cdot A \cdot C$$

The properties of "x" are associativity, $(A \times B) \times C = A \times (B \times C)$; commutativity, $A \times B = B \times A$; and idempotence, $A \times A = A$. The properties of "·" are commutativity, $A \cdot B \equiv B \cdot A$; and reflexivity, $A \cdot A$ is true for any A; but not transitivity (it is false that $A \cdot B \cdot B \cdot C \supset A \cdot C$).

By analogy with "+", we shall occasionally use the symbol "·-·-" as follows: $A \cdot B \cdot C$ means $A \times B \times C \neq 0$, or A, B, and C are all associated with each other.

Using the relations defined above, the fact that A is associated with C and D can be expressed as

$$A \cdot (C, D)$$

which signifies $A \cdot C \cdot A \cdot D$. If A and B are associated with

each other and with C and D, we have

$$(A * B) * (C, D)$$

which is equivalent to $A \neq \emptyset \neq C$ and $A * B * D$. If, however, A and B are not associated with each other but are both associated with C and D, we have

$$(A, B) * (C, D)$$

which is equivalent to the set of statements

$$A * C$$

$$A * D$$

$$B * C$$

$$B * D$$

It should be noted that "*" is defined as a relation between non-empty classes; for if $A = \emptyset$, then $A \times B = \emptyset$, but we say $A \neq \emptyset$ only if $A \times B \neq \emptyset$.

To sum up, "*" can be applied only to non-empty classes, whose product is also non-empty, and when placed between the names of such classes simply states these conditions. We write $A * B$ for $A \neq \emptyset$, $B \neq \emptyset$, and $AB \neq \emptyset$.

As we shall see in the next report, the distribution of "*" over a set of enumerated classes $A * (B, C, D, \dots)$ occurs as a necessary consequence of: (1) the manner in which the association of each term is derived from a system of information; and (2) the way in which associations are recorded for mechanical manipulation.