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AXIALLY SYMMETRIC FLEXURAL VIBRATIONS OF A CIRCULAR DISK

by

H. DERESIEWICZ and R. D. MINDLIN

Office of Naval Research Project NR-064-388
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Technical Report No. 12
CU-13-53-ONR-266(09)-CE
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ABSTRACT

At high frequencies, the flexural vibrations of a plate are described very poorly by the classical (Lagrange) theory because of neglect of the influence of coupling with thickness-shear vibrations. The latter may be taken into account by inclusion of rotatory inertia and shear deformation terms in the equations. The resulting frequency spectrum is given, in this paper, for the case of axially symmetric vibrations of a circular disk with free edges and is compared with the spectrum predicted by the classical theory.

Introduction

The problem of free vibrations of a thin, isotropic, elastic, circular disk, of constant thickness, was first attacked by Poisson (1)[1] over a century ago. Basing his investigation on the classical (Lagrange) theory of plates, he obtained the lower frequencies of the axially symmetric flexural motions of such a disk with a traction-free boundary.

It is well known that the classical theory satisfactorily predicts actual behavior only for the first few flexural modes of motion of a plate whose thickness is small in comparison with its other dimensions. For the higher flexural modes the influence of coupling with the thickness-shear mode of motion becomes increasingly important. Hence, the classical theory, in which this effect is not taken into account, ceases to yield reliable information (2). In particular, at the frequency of pure thickness-shear vibration of an infinite plate (i.e., displacement constant in direction and parallel to the plane of the plate), a drastic change occurs in the frequency spectrum of a finite plate (3,4).

The influence of coupling between flexure and shear is taken into account by inclusion of rotatory inertia and shear deformation terms in the equations. The resulting change in the frequency spectrum has been given previously for the case of a free-free beam (3) and a class of antisymmetric modes of motion of a circular disk (4). The present paper contains a discussion of the axially symmetric, flexural vibrations of a free disk, with emphasis on behavior in the neighborhood of the thickness-shear frequency. The spectrum obtained with consideration of the effects of rotatory inertia and shear deformation is compared with that predicted by the classical theory.

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1 Numbers in parenthesis refer to Bibliography at the end of the paper.

Plate Equations

If account is taken of rotatory inertia and shear deformation, the plate stress-displacement relations in polar coordinates, for the axially symmetric case[2], become

$$M_r = D\left(\frac{\partial\psi}{\partial r} + \frac{\nu}{r}\psi\right)$$
$$M_\theta = D\left(\nu\frac{\partial\psi}{\partial r} + \frac{\psi}{r}\right)$$
$$Q_r = \kappa^2\mu h\left(\psi + \frac{\partial w}{\partial r}\right) \qquad [1]$$
$$M_{r\theta} = Q_\theta = 0$$

where $D = Eh^3/12(1-\nu^2)$, E, μ, ν, and h are Young's modulus, the shear modulus, Poisson's ratio, and the thickness, respectively, and $\kappa^2 = \pi^2/12$. The functions ψ and w are related to the radial and axial components of the displacement according to the approximations

$$u_r \approx z\,\psi(r,t)$$
$$u_z \approx w(r,t) \qquad [2]$$

u_θ , the circumferential component of the displacement, is zero.

The plate equations of motion[3], for the present problem, are

$$\frac{\partial M_r}{\partial r} + \frac{M_r - M_\theta}{r} - Q_r = \frac{\rho h^3}{12}\frac{\partial^2\psi}{\partial t^2}$$
$$\frac{\partial Q_r}{\partial r} + \frac{Q_r}{r} = \rho h\frac{\partial^2 w}{\partial t^2} \qquad [3]$$

where ρ is the density of the plate.

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2 These are given, in the general case, by Equations [3] of (4).
3 For the general case, see Equations [2] of (4).

If we now insert Equations [1] in [3] we obtain the plate displacement equations of motion. Omitting the time factor e^{ipt}, these become

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2} + \frac{\rho p^2 h^3}{12 D}\right)\psi - \frac{\kappa^2 \mu h}{D}\left(\psi + \frac{dw}{dr}\right) = 0$$

$$\left(\frac{d}{dr} + \frac{1}{r}\right)\psi + \left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} + \frac{\rho p^2}{\kappa^2\mu}\right) w = 0 \qquad [4]$$

Equations [4] may be uncoupled by differentiating the second equation once and subtracting the result from the first one. This procedure yields an expression for ψ in terms of w, which, when inserted into the second of Equations [4], gives a single equation on w only:

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} + \delta_1^2\right)\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} + \delta_2^2\right) w = 0 \qquad [5]$$

where

$$\delta_1^2, \delta_2^2 = \frac{\delta_0^4}{2}\left[R + S \pm \sqrt{(R-S)^2 + 4\delta_0^{-4}}\right]$$

$$S = D/\kappa^2\mu h, \quad \delta_0^4 = \rho p^2 h/D, \quad R = h^2/12$$

The expression for ψ in terms of w is given by

$$\psi = (R\delta_0^4 - S^{-1})^{-1}\frac{d}{dr}\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} + (S\delta_0^4 + S^{-1})\right] w \qquad [6]$$

Solution of Equations of Motion

Equation [5] may be solved for w by noting that

$$w = w_1 + w_2 \qquad [7]$$

where w_1 and w_2 satisfy, respectively, the equations

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} + \delta_i^2 \right) w_i = 0 \,, \quad i = 1,2 \qquad [8]$$

Hence, the shear displacement ψ is found to be

$$\psi = (\sigma_1 - 1)\frac{dw_1}{dr} + (\sigma_2 - 1)\frac{dw_2}{dr} \qquad [9]$$

where

$$\sigma_1 \,,\, \sigma_2 = (\delta_2^2 \,,\, \delta_1^2)(R\,\delta_0^4 - S^{-1})^{-1}$$

Both of Equations [8] are Bessel equations of zero order. In dealing with a solid disk, their appropriate solutions are

$$w_1 = A_1 J_0(\delta_1 r)$$
$$w_2 = A_2 J_0(\delta_2 r) \qquad [10]$$

where A_1, A_2 are arbitrary constants.

For a plate with a traction-free edge, the boundary conditions are

$$M_r = Q_r = 0 \qquad \text{at} \qquad r = a \qquad [11]$$

where a is the radius of the plate.

Inserting Equations [1], [9] and [10] in Equations [11], the boundary conditions become

$$(\sigma_1 - 1)\left[\delta_1^2 a^2 J_0''(\delta_1 a) + \nu \delta_1 a J_0'(\delta_1 a)\right] A_1$$
$$+ (\sigma_2 - 1)\left[\delta_2^2 a^2 J_0''(\delta_2 a) + \nu \delta_2 a J_0'(\delta_2 a)\right] A_2 = 0 \qquad [12]$$
$$A_1 \sigma_1 \delta_1 J_0'(\delta_1 a) + A_2 \sigma_2 \delta_2 J_0'(\delta_2 a) = 0$$

where primes indicate differentiation with respect to the argument.

Frequency Equation

The secular equation, governing the frequency, is obtained by setting the determinant of Equations [12] equal to zero. The resulting equation may be written in the form

$$\frac{g\beta^2}{1+g}\gamma\Gamma_1 + \frac{1-g\beta^2}{1+g}\beta\gamma\Gamma_2 - (1-\nu)(1-\beta^2) = 0 \qquad [13a]$$

where

$$\beta = \delta_2/\delta_1, \quad \gamma = \delta_1 a, \quad g = R/S,$$
$$\Gamma_1 = J_0(\gamma)/J_1(\gamma), \quad \Gamma_2 = J_0(\beta\gamma)/J_1(\beta\gamma)$$

It may be observed that g is a material constant, depending only on Poisson's ratio, while the remaining functions in Equation [13a] depend on the frequency.

Since δ_1 is real for all positive values of the frequency p, while δ_2 is real or imaginary, depending on whether p is greater or less than $\bar{p} = \pi(\mu/\rho)^{1/2}/h$, β will be real or imaginary according as $p \gtrless \bar{p}$.[4] For the range $p < \bar{p}$, if we let $\beta = i\beta_1$, the frequency equation may be transformed to the more convenient form

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[4] $\bar{p}$ is the frequency of the first thickness-shear mode of an infinite plate.

$$\frac{g+\beta_1^2}{1+g}\gamma\Gamma_1 + \frac{1+g\beta_1^2}{1+g}\beta_1\gamma G_2 - (1-\nu)(1+\beta_1^2) = 0 \qquad [13b]$$

where $G_2 = I_0(\beta_1\gamma)/I_1(\beta_1\gamma)$, and $I_0(x)$, $I_1(x)$ are modified Bessel functions of the first kind.

We may find an explicit formula for the frequency by means of the relation $\beta = \delta_2/\delta_1$ and the expressions for δ_1 and δ_2 which immediately follow Equation [5]. Thus,

$$p/\bar{p} = \left[1-\beta^2(1+g)^2/g(1+\beta^2)^2\right]^{-\frac{1}{2}}, \qquad p > \bar{p}$$
$$p/\bar{p} = \left[1+\beta_1^2(1+g)^2/g(1-\beta_1^2)^2\right]^{-\frac{1}{2}}, \qquad p < \bar{p} \qquad [14]$$

In addition, from the relation $\gamma = \delta_1 a$, we find

$$d/h = \gamma(\bar{p}/p)\left[(1+\beta^2)/3(1+g)\right]^{\frac{1}{2}}, \qquad p > \bar{p}$$
$$d/h = \gamma(\bar{p}/p)\left[(1-\beta_1^2)/3(1+g)\right]^{\frac{1}{2}}, \qquad p < \bar{p} \qquad [15]$$

where d is the diameter of the disk.

The complete solution of the problem is contained in Equations [13], [14], and [15]. With the value of ν specified for the material of the plate, a choice of β or β_1 determines $p/\bar{p}$ by Equations [14], and yields an infinite set of roots γ of Equations [13]. For the chosen β or β_1 each of these roots furnishes a ratio d/h through Equations [15], so that there results an infinite set of values of d/h corresponding to every value of $p/\bar{p}$.

The resulting curves, for $\nu = 0.312$ (corresponding to $g = 0.283$), are shown in Fig. 2.

The presence of the thickness-shear mode is represented, in the present theory, by the rotatory-inertia coefficient, R, and the shear-deformation coefficient, S. (In the sequel, this theory is referred to as RS and the classical theory as C). It may be verified that the secular equation of RS (Equation [13]) degenerates to that of C (Equation [17]) for the limiting case R = S = 0 (so that δ_1^2, $\delta_2^2 = \pm\delta_0^2$). Hence the curves of Fig. 1, when extended down to $p/\bar{p} = 0$, approach those of Fig. 2 asymptotically. The suppression of shear deformation in C serves as a constraint which raises the frequencies, in that theory, above those of RS. As a result, for a given plate, RS reveals that there are many more resonances, in a given frequency range, than are predicted by C. For example, for $d/h = 40$, there are 25 resonances in the range $0 < p/\bar{p} < 1$, whereas C predicts only 16.

Furthermore, in designing a plate to resonate at a certain frequency, in its fundamental mode, one would be led to choose too large a plate on the basis of C. For example, for $p/\bar{p} = 0.05$, the diameter would be in excess by 2.61%. This discrepancy increases with the ratio of plate thickness to wave length of mode and, hence, with the order of the mode. For example, if the second mode of the plate is to vibrate at $p/\bar{p} = 0.05$, the error in diameter chosen on the basis of C would be 2.86%. The discrepancies are more pronounced at higher frequencies. Thus, the error in diameter of a plate designed, according to C, to resonate at $p/\bar{p} = 0.1$, in the first mode, is 5.33% and, in the second mode, 5.79%.

The most striking difference between the two theories occurs at frequencies above that of the fundamental thickness-shear mode, as may be seen from Figs. 1 and 2.

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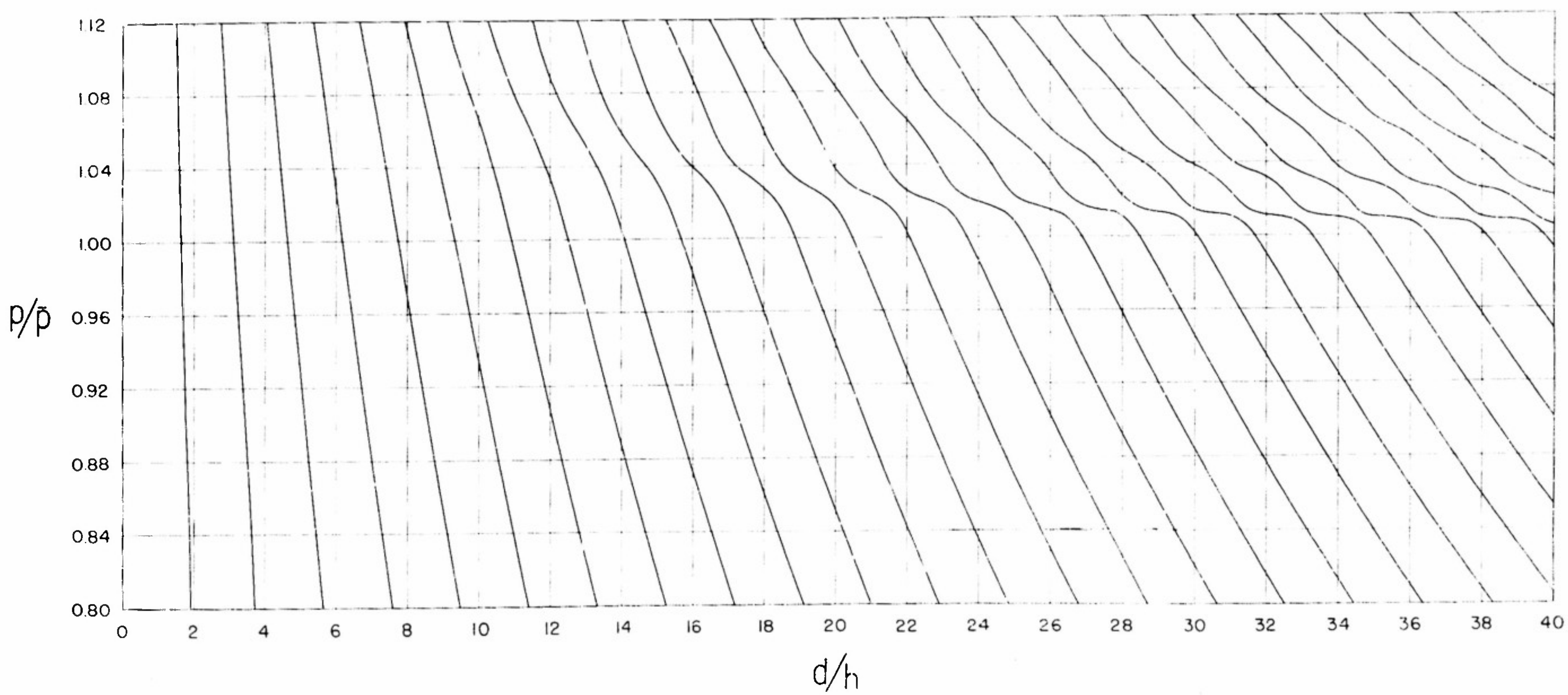

Fig. 1: Frequency spectrum of axially symmetric vibrations of a free, circular disk, illustrating results of coupling of flexure and thickness-shear. (d/h = ratio of diameter to thickness; $p/\bar{p}$ = ratio of resonant frequency to frequency of pure thickness-shear vibration of an infinite plate of thickness h. $\nu = 0.312$.)

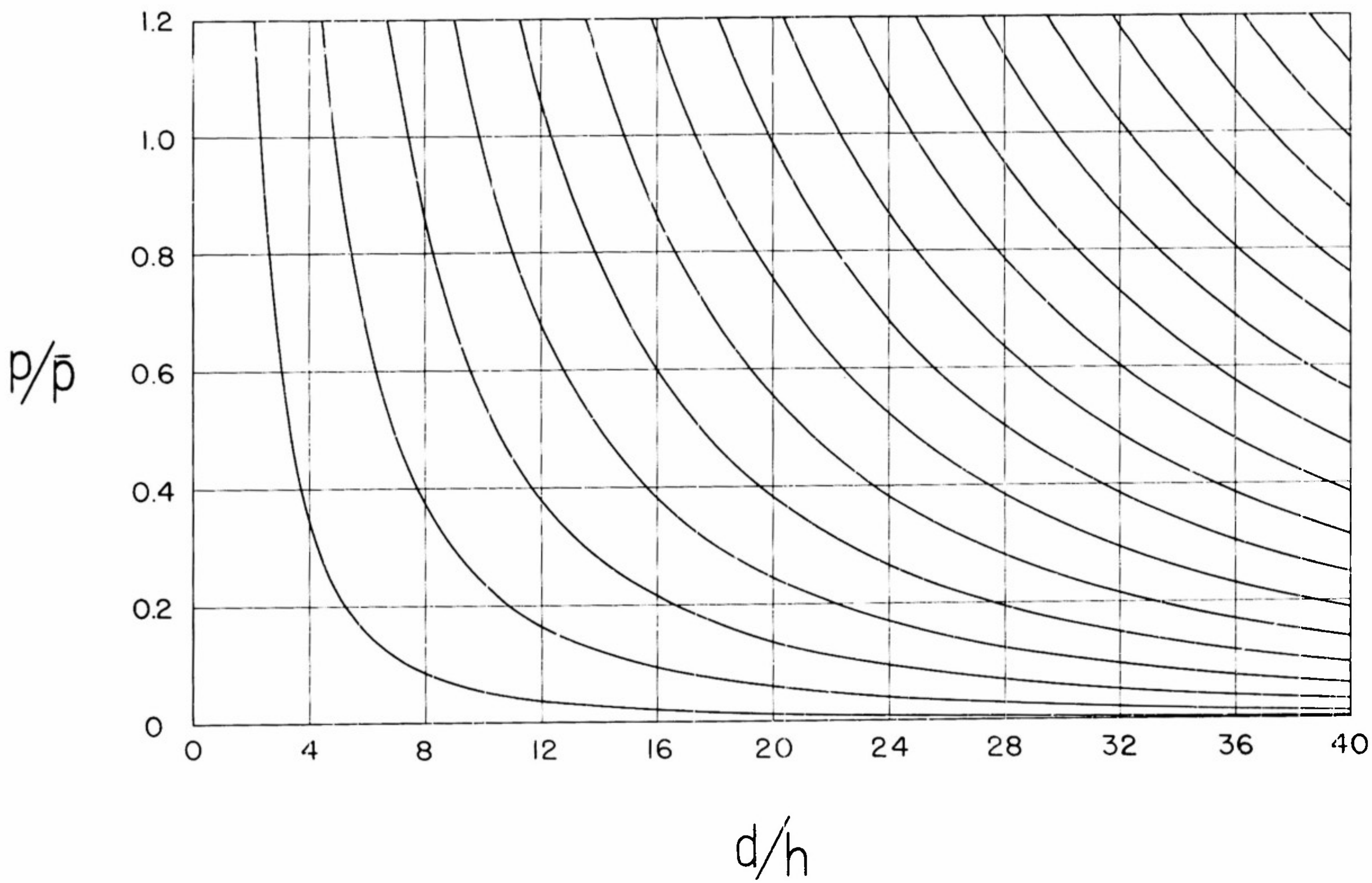

Fig. 2: Frequency spectrum of axially symmetric vibrations of a free, circular disk according to the classical (Lagrange) theory of flexure of plates. ($\nu = 0.312$)

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