# AD 35657



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RF IMPULSE SPECTRUM ANALYZER

SECOND QUARTERLY PROGRESS REPORT

AUGUST 15, 1953 TO NOVEMBER 15, 1953

SIGNAL CORPS CONTRACT NO. DA-36-039 SC-52629

DEPT. OF THE ARMY PROJECT NO. 3-54-03-042

SIGNAL CORPS PROJECT NO. 2224B

PLACED BY

U.S. ARMY SIGNAL CORPS ENGINEERING LABORATORIES,

FORT MONMOUTH, N. J.

Object: The design, development and construction of one (1) experimental RF impulse spectrum analyzer.

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CONTRACT NO: DA36-039 sc-52629 PROJECT NO: 2224B

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This contract is supervised by Mr. J. F. Chappell, Suppression Development Section, Suppression & General Engineering Branch, SCEL, Coles Signal Laboratory, Fort Monmouth, New Jersey, Telephone Eatontown 3-1060, Extension: 87-519.

## TABLE OF CONTENTS

## PURPOSE

The purpose of this contract is the design, development and construction of an impulse spectrum analyzer continuous over the frequency range of 150 KC to 1 kilomegacycle, for use in the measurement of the absolute spectral intensity of very short, separated, repetitious pulses. Work by Lynmar to date would indicate that one possible solution of the problem is the use of a series of cascaded and synchronously tuned RLC shunt circuits with vacuum tube isolation between stages (hereinafter known as CASTAS).

# ABSTRACT

In the Abstract Section of the First Quarterly report some of the problems involved in the proposed Lynmar method for determining impulse strength were discussed. A method for determining impulse strength was introduced involving the measurement of the envelope area of a CASTA response. It was shown in a preliminary fashion that the area so determined was directly proportional to impulse strength. In Appendix B an experimental verification for the area scheme is presented and in Appendix C a mathematical justification for this scheme is derived.

The overall physical equipment is now beginning to crystalize. For the present it is intended to utilize the Signal Corps Radio Receiver BE-344-D to cover the frequency range of 150 to 1500 KC. The IF frequency in this receiver is 92.5 KC and we are now in the process of building a CASTA for supplanting the present IF system. It is intended that this CASTA have a bandwidth sufficient to resolve a maximum repetition rate of approximately 1000 PPS. If possible, this will be extended to 2000 PPS.

The Radio Receiver R-274C/FRR will be used for the frequency range of 0.54 to 54.0 MC. The IF frequency of this receiver is 455 KC and an experimental CASTA has been built for the purpose of confirming its use in supplanting the IF system.

The CASTAS for use in the two radio receivers mentioned above are both designed for 5 tuned circuits. The complete description of

these two CASTAS will follow in our next quarterly report.

It is intended that the frequency range of 20 to 200 MC will be covered by means of the tuning unit T-1/NF-105 manufactured by Empire Devices. The IF frequency of this tuning unit is 10.7 MC. We expect to rectify this IF output and apply this pulse to either the 92.5 KC CASTA or the 455 KC CASTA depending on the designed relative bandwidths. This setup will then be in effect equivalent to that described in the First Quarterly Report Abstract.

For covering the range of 200 to 400 MC it is expected to use the tuning unit T-2/NF-105 also manufactured by Empire Devices. This tuning unit has an IF frequency of 30 MC. In this case it is expected that either of the aforementioned CASTAS will be used and in addition it will also be possible to confirm the operation of the lower frequency receivers by driving them with the rectified output of this unit.

As mentioned earlier Empire Devices is now developing a tuning unit for the Signal Corps covering the range of 400 to 1000 MC. Pending delivery of this unit an investigation will be made of the possibility of driving one of the low frequency CASTAS by means of a simplified tuning unit covering the 400 to 1000 MC range.

## CONFERENCES

Some of the conferences held in this report period were described in the first quarterly report. For completeness the conferences earlier described will be repeated in this report.

1. On 17 August 1953 a conference was held between Lynmar engineers and Signal Corps engineers in which Lynmar engineers presented the results of a theoretical investigation of measurement techniques that might conceivably simplify the desired end equipment. This scheme involved the integration of the pulse envelope thereby, at least theoretically, eliminating the effects of any intervening tuned circuitry. It was deemed of such importance that it was decided to hold another conference to review the results of further study.

2. On 25 August 1953, another conference was held between Signal Corps engineers and Lynmar engineers for the purpose of reviewing and discussing the scheme for impulse measurement suggested by Lynmar engineers in the conference of 17 August 1953. Both Lynmar engineers and the Signal Corps engineers presented the results of their investigations and discussions brought forth the following items.

a. Detailed investigation of the effect of noise on the area method for measuring impulse strength.

b. Further study on the results of repeated pulses on the accuracy of measurement.

3. Several short conferences were held thereafter in which equipment design was discussed. Further theoretical aspects of the area scheme for measuring impulse strength were also discussed.

Equipment requested of the Signal Corps comprised the following:

(a) Signal Corps Radio Receiver BC-344-D.

(b) Tuning Unit T-1/NF-105, 20-200 mc. Serial #44.

(c) Tuning Unit T-2/NF-105, 200-400 mc. Serial #65.

Item (a) above was to be supplied as part of the contract while items (b) and (c) are to be returned to the Signal Corps in kind. It was also decided to utilize a tuning unit now in development covering the range of 400 mc. to 1000 mc. This tuning unit is not yet fully developed but it is anticipated that it will be ready for use by January 1954. It was decided that pending delivery of this tuning unit, other schemes would be investigated for covering this frequency range.

On December 1, 1953 Signal Corps engineers made an inspection of the progress and plant of Lynmar engineers. The operation of the equipment at hand was demonstrated and pulse measuring techniques based on these demonstrations were discussed.

## FACTUAL DATA

### LIST OF SYMBOLS

The following is a list of symbols with definitions, that will be used throughout this report.

| | |
|---|---|
| CASTA | CASCADED AND SYNCHRONOUSLY TUNED AMPLIFIER OF n STAGES |
| $\alpha$ | $\frac{1}{2RC}$ |
| $\alpha T$ | Period damping time factor |
| Q | Quality factor $= \frac{f_o}{\Delta f}$ |
| $\beta$ | $\sqrt{\frac{1}{LC} - \frac{1}{4R^2C^2}}$ |
| $E_n$ | Detected envelope of output of a CASTA when impulse excited |
| x | $\beta t$ |
| K, C | Constants (except where otherwise defined) |
| $\omega_o$ | Resonant angular velocity, radians/seconds |
| $\gamma$ | $\frac{\alpha}{\beta}$ |
| y | $(n-\frac{1}{2})$ |
| n | Number of tuned circuits |
| S | Spectral intensity |
| $\Delta_a$ | Error in area by considering the approximate solution for the impulse response of a CASTA |
| R | Total parallel resistance of tank circuit in ohms (unless otherwise defined) |
| L | Total parallel inductance of tank circuit in henries |
| C | Total parallel capacitance of tank circuit in farads (unless otherwise defined.) |
| G | Gain at resonant frequency |
| $g_m$ | Mutual conductance |

The following is a concise discussion of the material in Appendices A to D inclusive.

Appendix A

"The response of a n stage CASTA to an impulse."

Part (a) of this appendix gives the formal Bessel function solution for $n = 1$ to $n = 7$. This material is exact in the sense that no approximations have been made in its derivations. Standard formulas for differentiation for Bessel functions were used in this derivation, The solutions themselves are too lengthy to be repeated here. However, they may be observed by turning to pages 8-2 and 8-3.

In Part (b) the quadrature solution for $n = 1$ to $n = 5$ is given. These formulas are not exact in the sense that the formulas in Part (a) are, since the approximation that $\gamma^2 \ll 1$ has been used throughout in order to reduce the formulas to a less cumbersome size. Since it would serve no purpose to repeat the formulas in Part (b) they are merely being referred to here. They may, however, be seen by turning to pages 8-4 and 8-5.

Appendix B

"13.7 KC CASTA."

The material in Appendix B is a continuation of the material given in Appendix F and Appendix G in the First Quarterly report. The actual purpose of this appendix is to show the electrical form of the circuit and more important to give a means of experimental comparison between the area method and the peak method of measuring spectral intensity. It is shown in this appendix that the agreement is within approximately 5%.

## Appendix C

"Mathematic justification for area method."

The establishment of a criterion for the validity of the approximate response as given in the First Quarterly Report, Appendix A, formula 19, page 8-8. It is shown quite clearly in Appendix C of this report by means of definition in equation (8) and the results of equation (9) that the error is quite small for easily realizable values of Q. For instance, the error for Q equals 10 is of the order of 0.5%.

## Appendix D

"Miscellaneous derivations."

Since we have dealt with the quantity $\gamma$ to some length in this report it was deemed wise here to set it down in terms of a more tangible quantity. Formula 7 in this appendix gives a value for $\gamma$ equal to $\frac{1}{2KQ}$ very closely equal to $\frac{1}{2Q}$ for $Q \geq 10$. Equation (13) is a combination of equation (7) (just referred to), equation (17) Appendix A First Quarterly Report, and information given in Appendix C, page 10-2 First Quarterly Report. The relationship given is one that fixes the minimum and maximum value of Q for the peak method of measuring spectral intensity when repeated pulses are considered.

$$\frac{10n}{4\sqrt{2}\,K} < Q \leq \frac{\omega_0 \tau}{2C}$$

## CONCLUSIONS

In this period a physical determination of the required components has been tentatively determined. This arrangement has been previously described in the Abstract section of this report (section 2-1).

In Appendix A the CASTA response to an impulse for n = 1 to n = 7 has been completely determined. An attempt was made to find a formal closed relationship for any value of n, but without success. This effort will be continued since it is deemed of value to find such an expression.

In Appendix B the circuitry and parts list are given for the 13.7 KC CASTA described in the First Quarterly Report. In addition an enlarged trace of this CASTA response is shown and was utilized for the purpose of comparison between the area method and the peak method of measuring spectral intensity.

In Appendix C there is shown a mathematical justification for the area method of determining impulse spectral intensity.

Appendix D contains miscellaneous derivations that appear from time to time.

As yet no clear cut decision has been made for favoring either the peak method or the area method of measurement. Intensive work is now being initiated for the purpose of making such a decision. It may be, however, that either or both methods will be utilized depending on the surrounding circumstances.

PROGRAM FOR NEXT INTERVAL

Our plan for the next interval will be to definitely crystalize and procure the necessary equipment for satisfying the requirements of our contract. To date experimental equipment has been constructed without any attempt to utilize Jan parts and components. As far as is possible such components will be used in our final experimental model of this equipment.

It is intended to consult with Signal Corps engineers as to the best means for accomplishing our purpose. In the event, naturally, it becomes impossible to procure certain required items the Signal Corps will be so advised.

Theoretical investigation, both confirmitory and experimental, will continue to take place and new avenues for the purpose of this contract will be investigated. This will be done even though these new schemes and methods may not be used in this end equipment.

## IDENTIFICATION OF KEY PERSONNEL

The personnel working on this project are:

Edward M. Beeler

Dr. Arsene N. Lucian

Samuel Sabaroff

Chico G. DeCoatsworth

A biographical sketch was included in the first quarterly report, Section 7, Page 1.

## APPENDIX A

## RESPONSE OF AN N STAGE CASTA TO AN IMPULSE

(a) Formal Bessel Function form for "n" equal 1 (one) to n equal 7 (seven).

Consider equation 10, Section 8, page 3 first quarterly Report

$$V_n(t) = \frac{A g_m^n}{C^n \beta^n 2^{n-1} \Gamma(n)} \frac{d^n}{dt^n}\left[t^{n-1} \epsilon^{-\alpha t} \sqrt{\frac{\pi \beta t}{2}}\, J_{n-\frac{1}{2}}(\beta t)\right] \tag{1}$$

where

$\alpha = \frac{1}{2RC}$

$\beta = \sqrt{\frac{1}{LC} - \frac{1}{4R^2C^2}}$

$J_{n-\frac{1}{2}}(\beta t)$ = Bessel Function of the first kind of order $(n-\frac{1}{2})$ and argument $(\beta t)$.

$\Gamma(n) = \lfloor n-1 = (n-1)!$

Let $(\beta t) = x$ and $\frac{\alpha}{\beta} = \gamma$ and $y = (n - \frac{1}{2})$

then

$$v_n(t) = \frac{A g_m^n \sqrt{\pi}}{C^n \beta^{y^2} 2^y \lfloor n-1} \frac{d^n}{dt^n}\left[x^y \epsilon^{-\gamma x} J_y(x)\right] \tag{2}$$

For simplicity let $J_y = J_y(x)$, and $K = \frac{A g_m^n \sqrt{\pi}}{C^n \beta^{y^2} 2^y \lfloor n-1}$

Also observing that $\frac{d[f(\beta,t)]}{dt} = \frac{\beta d[f(\beta,t)]}{d(\beta t)} = \frac{\beta d[f(x)]}{dx}$

We now proceed to operate on equation (2) as indicated and arrive at the following for $v_n(t)$ for n = 1 to n = 7

$$V_1(t) = K\beta\varepsilon^{-\gamma x}\left\{x x^{y-1}J_{y-1} - \gamma x^y J_y\right\}$$

$$V_2(t) = K\beta^2\varepsilon^{-\gamma x}\left\{\gamma^2 x^y J_y + (1 - 2\gamma x)x^{y-1}J_{y-1} + x^2 x^{y-2}J_{y-2}\right\}$$

$$V_3(t) = K\beta^3\varepsilon^{-\gamma x}\left\{-\gamma^3 x^y J_y + (3\gamma^2 x - 3\gamma)x^{y-1}J_{y-1} + (3x - 3\gamma x^2)\right.$$
$$\left. x^{y-2}J_{y-2} + x^3 x^{y-3}J_{y-3}\right\}$$

$$V_4(t) = K\beta^4\varepsilon^{-\gamma x}\left\{\gamma^4 x^y J_y + (6\gamma^2 - 4\gamma^3 x)x^{y-1}J_{y-1} + (6\gamma^2 x^2 - 12\gamma x + 3)\right.$$
$$\left. x^{y-2}J_{y-2} + (6x^2 - 4\gamma x^3)x^{y-3}J_{y-3} + x^4 x^{y-4}J_{y-4}\right\}$$

$$V_5(t) = K\beta^5\varepsilon^{-\gamma x}\left\{-\gamma^5 x^y J_y + (5\gamma^4 x - 10\gamma^3)x^{y-1}J_{y-1} + (30\gamma^2 x - 10\gamma^3 x^2 + \right.$$
$$-15\gamma)x^{y-2}J_{y-2} + (10\gamma^2 x^3 - 30\gamma x^2 + 15x)x^{y-3}J_{y-3} +$$
$$\left. + (10x^3 - 5\gamma x^4)x^{y-4}J_{y-4} + x^5 x^{y-5}J_{y-5}\right\}$$

$$V_6(t) = K\beta^6\varepsilon^{-\gamma x}\left\{\gamma^6 x^y J_y + (15\gamma^4 - 6\gamma^5 x)x^{y-1}J_{y-1} + (15\gamma^4 x^2 + 45\gamma^2 + \right.$$
$$-60\gamma^3 x)x^{y-2}J_{y-2} + (90\gamma^2 x^2 - 20\gamma^3 x^3 - 90\gamma x + 15)x^{y-3}J_{y-3} +$$
$$+ (15\gamma^2 x^4 + 45x^2 - 60\gamma x^3)x^{y-4}J_{y-4} + (15x^4 - 6\gamma x^5)x^{y-5}J_{y-5} +$$
$$\left. + x^6 x^{y-6}J_{y-6}\right\}$$

$$V_7(t) = K\beta^7\varepsilon^{-\gamma x}\left\{-\gamma^7 x^y J_y + (7\gamma^6 x - 21\gamma^5)x^{y-1}J_{y-1} + (105\gamma^4 x - 105\gamma^3 + \right.$$
$$-21\gamma^5 x^2)x^{y-2}J_{y-2} + (35\gamma^4 x^3 - 210\gamma^3 x^2 + 315\gamma^2 x - 105\gamma)x^{y-3}J_{y-3} +$$
$$+(-35\gamma^3 x^4 + 210\gamma^2 x^3 - 315\gamma x^2 + 105x)x^{y-4}J_{y-4} + (.05x^3 - 105\gamma x^4 +$$

$$+21\gamma^2 x^5)x^{y-5}J_{y-5} + (21x^5 - 7\gamma x^6)x^{y-6}J_{y-6} + x^7 J_{y-7}\Big\}$$

where

$$J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\ \sin x \tag{3}$$

$$J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\ \cos x \tag{4}$$

$$J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}}\ \left[\frac{\sin x}{x} - \cos x\right] \tag{5}$$

$$J_{\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}}\ \left[\left(\frac{3}{x^2} - 1\right) \sin x - \frac{3}{x} \cos x\right] \tag{6}$$

$$J_{\frac{7}{2}}(x) = \sqrt{\frac{2}{\pi x}}\ \left[\left(1 - \frac{15}{x^2}\right) \cos x + \left(\frac{15}{x^3} - \frac{6}{x}\right) \sin x\right] \tag{7}$$

$$J_{\frac{9}{2}}(x) = \sqrt{\frac{2}{\pi x}}\ \left[\left(1 - \frac{45}{x^2} + \frac{105}{x^4}\right) \sin x + \left(\frac{10}{x} - \frac{105}{x^3}\right) \cos x\right] \tag{8}$$

(b) Quadrature Solution for n equal 1 (one) to n equal 5 (five). The following is the quadrature form for $V_1(t)$, $V_2(t)$, $V_3(t)$, $V_4(t)$ and $V_5(t)$.

For the sake of simplicity we have neglected terms small in gamma. $\gamma < 1$ has been a premise of this work and therefore this attitude represents a consistent point of view. Further, it removes some of the obscurities that would naturally arise when expressions contain a myriad of terms. The absolute equations may be derived from the formal Bessel function form with the aid of equations (3) to (8). The order of the approximation will be demonstrated for $V_2(t)$ which we submit as a representative illustration.

$$V_1(t) = \sqrt{\frac{2}{\pi}} K\beta \varepsilon^{-\gamma x} (\cos x - \gamma \sin x) \qquad \text{exact} \qquad (9)$$

$$V_2(t) = \sqrt{\frac{2}{\pi}} K\beta^2 \varepsilon^{-\gamma x} \left[x(1-\gamma^2) \cos x + \left[(1+\gamma^2) - 2\gamma x\right] \sin x\right] \qquad \text{exact} \qquad (10)$$

let $1 >> \gamma^2$

$$V_2(t) = \sqrt{\frac{2}{\pi}} K\beta^2 \varepsilon^{-\gamma x} \left[x \cos x + (1-2\gamma x) \sin x\right] \qquad \text{approximate} \qquad (11)$$

$$V_3(t) = \sqrt{\frac{2}{\pi}} K\beta^3 \varepsilon^{-\gamma x} \left[(x^2-3\gamma x) \cos x + (-3\gamma x^2 + 3x - 3\gamma) \sin x\right] \qquad \text{approximate} \qquad (12)$$

$$V_4(t) = \sqrt{\frac{2}{\pi}} K\beta^4 \varepsilon^{-\gamma x} \left[(x^3 + 12\gamma x^2 - 3x) \cos x + (-4\gamma x^3 + 6x^2 + - 24\gamma x + 3) \sin x\right] \qquad \text{approximate} \qquad (13)$$

$$V_5(t) = \sqrt{\frac{2}{\pi}} K\beta^5 \varepsilon^{-\gamma x} \left[(x^4 + 30\gamma x^3 - 15x^2 + 45\gamma x) \cos x + + (-5\gamma x^4 + 10x^3 - 15\gamma x^2 + 15x - 45\gamma) \sin x\right] \qquad \text{approximate} \qquad (14)$$

Since we shall use $V_5(t)$ in Appendix C we will rewrite it here in another form

$$V_5 = \sqrt{\frac{2}{\pi}}\, K \beta^5 \varepsilon^{-\gamma x} \sqrt{x^8 - 40\gamma x^7 + 70x^6 - 1260\gamma x^5 + 525x^4 - 2700\gamma x^3 + 225x^2 - 1350\gamma x + 2025\gamma^2} \left[\cos (x - \phi)\right]$$

$$\text{where } K = \frac{A g^{n}_{m}}{C^n \beta^{y^2} 2^y \lfloor n-1}$$

$$\phi = \tan^{-1} \frac{(-5\gamma^4 + 10x^3 - 15\gamma x^2 + 15x - 45\gamma)}{(x^4 + 30\gamma x^3 - 15x^2 + 45\gamma x)}$$

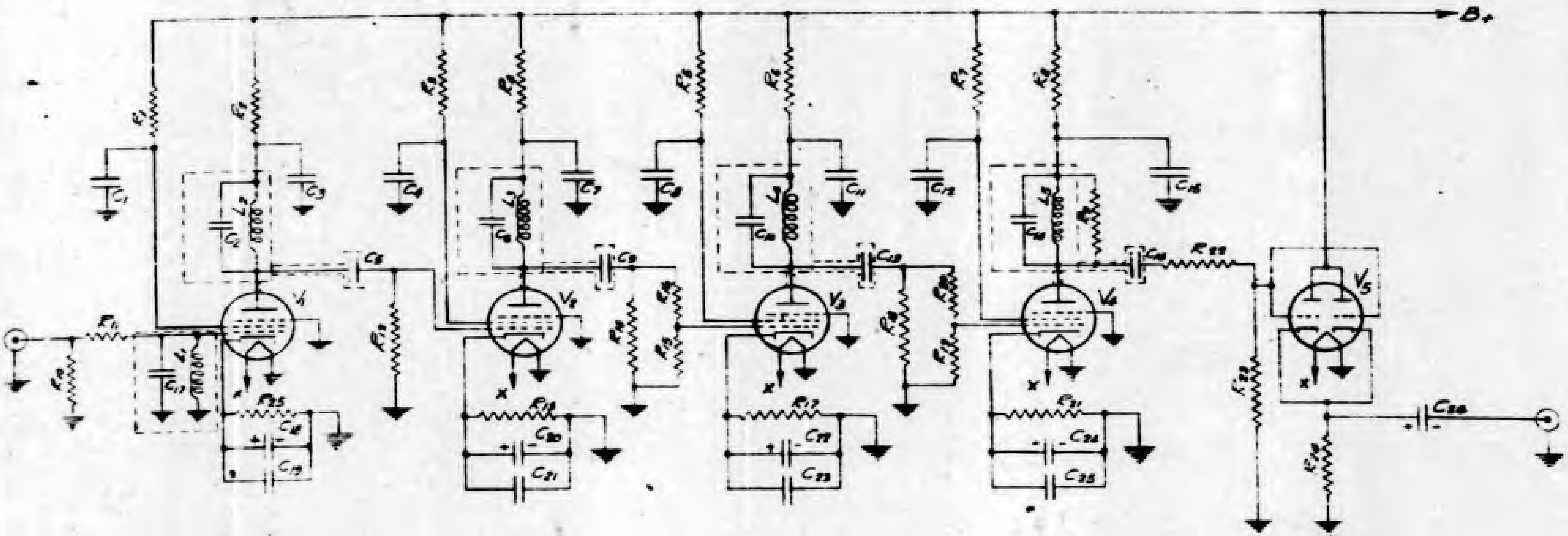
## APPENDIX B

13.7KC CASTA

In the first quarterly progress report Section 13, Pages 1 and 2 and Section 14, Pages 1 and 2 the data, frequency response and some wave forms with their explanations were given. The following pages of this section report give the schematic and component values for this CASTA. In addition an enlarged reproduction of Figure 2, Appendix G, Section 14 of the first quarterly report is being included here for the purpose of comparison between the area method and the peak method of measuring spectral intensity. The ensuing computations will point out that these methods check to within a very small degree.

It may be said here that although great care was taken in the reproduction of the cathode ray tube trace some distortion may have been produced in this representation of the original.

The material in Appendix C, Section 10 gives mathematical justification for this area method. This analytical work has not been intended to be a rigorous mathematical proof. However, it does show within its limitations that accuracies of the required amount may be achieved.




NOTE.
X = 6.3 VOLT FILAMENT
B+ = 300 VOLT

LYNMAR ENGINEERS
ELECTRONIC MANUFACTURERS & CONSULTANTS
PHILADELPHIA 3 PENNA.

13.7 KC SYNCHRONOUS AMPLIFIER
SCHEMATIC CIRCUIT
(CASTA)

| | | |
|---|---|---|
| DRAWN Shoor | DATE 10-26-5[illegible] | DRAWING NO. |
| CHECKED S. Salvaro[illegible] | | B-1006 |
| APPROVED [illegible] | | |

Fig. 1

13.7KC CASTA

(COMPONENT LIST)

| Component | Value |
|---|---|
| $R_1$, $R_3$, $R_5$, $R_7$ | = 33K ± 10% $\frac{1}{2}$ w |
| $R_2$, $R_4$, $R_6$, $R_8$ | = 4.7K ± 10% $\frac{1}{2}$ w |
| $R_9$, $R_{11}$, $R_{12}$, $R_{14}$, $R_{18}$ | = 7K ± 1% $\frac{1}{2}$ w |
| $R_{10}$ | = 50Ω ± 5% $\frac{1}{2}$ w |
| $R_{13}$, $R_{17}$, $R_{21}$, $R_{25}$ | = 220Ω ± 10% $\frac{1}{2}$ w |
| $L_1$, $L_2$, $L_3$, $L_4$, $L_5$ | = 19.4 mh (torroidal) ± 1% |
| $C_2$, $C_6$, $C_{10}$, $C_{14}$, $C_{17}$ | = 0.0069 mfd (400v) ± 1% |
| $C_1$, $C_3$, $C_4$, $C_5$, $C_7$, $C_8$, $C_9$, $C_{11}$, $C_{12}$, $C_{13}$, $C_{15}$, $C_{16}$, $C_{26}$ | = 0.5 mfd 600V |
| $V_1$, $V_2$, $V_3$, $V_4$ | = 6AC7 |
| $V_5$ | = 6J6 |
| $C_{18}$, $C_{20}$, $C_{22}$, $C_{24}$ | = 4.0 mfd 250V Electrolytic |
| $C_{19}$, $C_{21}$, $C_{23}$, $C_{25}$ | = 1.0 mfd 200 V Paper |
| $R_{15}$, $R_{19}$ | = 22K ± 10% $\frac{1}{2}$ w |
| $R_{16}$ | = 220K ± 1% $\frac{1}{2}$ w |
| $R_{20}$ | = 240K ± 10% $\frac{1}{2}$ w |
| $R_{24}$ | = 150Ω ± 10% $\frac{1}{2}$ w |
| $R_{23}$ | = 10K ± 10% $\frac{1}{2}$ w |
| $R_{22}$ | = 100K ± 20% $\frac{1}{2}$ w |
| B+ | = 300V |
| E+ | = 6.3V |

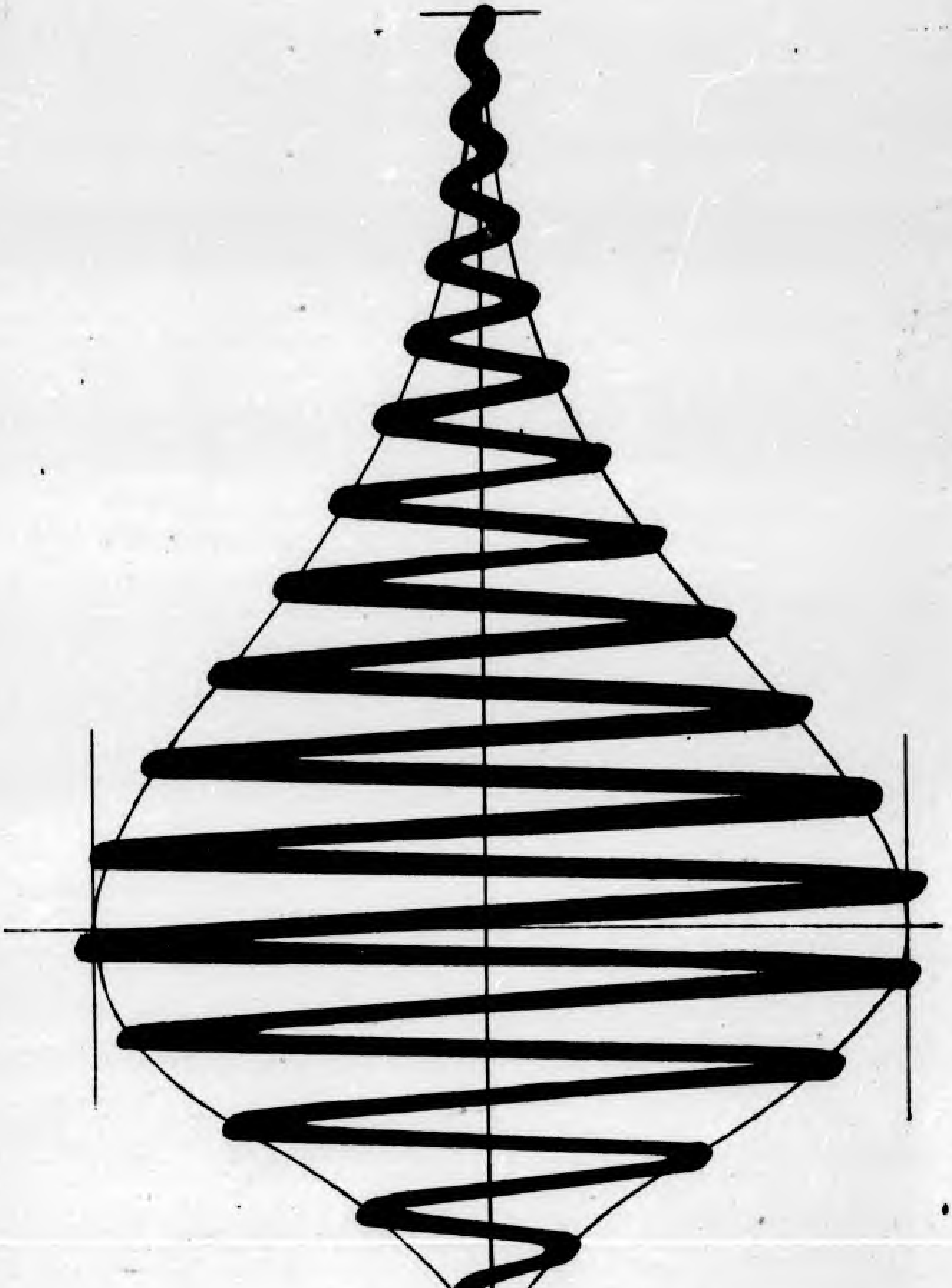


Fig. [illegible]

## Experimental Comparison Between the Area Method and the Peak Method of Measuring Spectral Intensity

Figure 2 in this appendix is a reproduction of an actual cathode ray tube trace of the response of the 13.7 KC CASTA to an impulse. For the purpose of comparison between the two spectral intensity measuring techniques we submit the following data and arithmetic as a means for comparative evaluation.

The envelope that was considered is clearly outlined along with the base line and the amplitude peak. The area of this figure was planimetered. The length of time under consideration was determined from the known center frequency of the CASTA and was determined more specifically by considering the period for times after the maximum - measuring this distance and prorating it back to $t_o$ insuring that error would not be accrued at times before the maximum due to the fact that the curve does not have a period equal to $2\pi$ for time close to $t = o$.

It should be pointed out, however, that the actual area was taken to be one-half the total area enclosed by the trace and that the actual amplitude was taken to be one-half the peak to peak amplitude. The data and arithmetic follow.

<u>Data and Arithmetic</u>

Arbitrary Conditions

1 inch = 1 volt

Gain = 1

By planimeter area = 11.085 in.$^2$ (1)

$$\text{Time} = 16.39 \text{ periods of } \omega_0 = 16.39 \left[\frac{1}{13.7 \times 10^3}\right] = 1.195 \times 10^{-3} \text{ sec.} \quad (2)$$

$$\text{Base length} = 8.88'' = 1.195 \times 10^{-3} \text{ sec.} \quad (3)$$

$$\frac{\text{Area}}{\text{Base}} = \frac{11.048 \text{ in.}^2}{8.88 \text{ in.}} = 1.255 \text{ in.} \quad (4)$$

$$S = \frac{\text{Area}}{\text{Base}} \times \text{time} = 1.255 \times 1.195 \times 10^{-3} \text{ sec.} = 1.5 \times 10^{-3} \text{ in secs.} \quad (5)$$

$$= 1.5 \times 10^{-3} \text{ volt seconds.}$$

$$\text{Peak ordinate} = 2.9 \text{ in.} = 2.9 \text{ volts} \quad (6)$$

$$S = \frac{\text{Peak ordinate}}{\text{Impulse noise bandwidth}} = \frac{2.9}{2.01 \times 10^3 \text{ cycles}} = 1.441 \times 10^{-3} \text{ in.sec.}$$

$$= 1.441 \times 10^{-3} \text{ volt seconds.} \quad (7)$$

$$\text{By area, } S = 1.5 \times 10^{-3} \text{ volt secs.} \quad (8)$$

$$\text{By Peak ordinate, } S = 1.441 \times 10^{-3} \text{ volt seconds} \quad (9)$$

$$\text{Agreement} = \frac{1.441}{1.5} = \underline{96.1\%}$$

The correlation between the area method and the peak voltage method as determined from the oscillascope trace is better than 5%. This accuracy was somewhat surprising in view of the difficulties involved in this kind of graphical analysis. It would point up, however, to the advisability of further investigation of the area scheme for determining impulse strength. A more detailed mathematical investigation of the area method was launched and will be described in Appendix C.

## APPENDIX C

### MATHEMATICAL JUSTIFICATION FOR AREA METHOD

We shall here establish a criterion for the validity of the approximate response of a 5 stage CASTA to an impulse.

Consider the modulus of $V_5(t)$

$$\text{Modulus} = E_5 = K\varepsilon^{-\gamma x}\sqrt{(-5\gamma x^4 + 10x^3 - 15\gamma x^2 + 15x - 45\gamma)^2 + (x^4 + 30\gamma x^3 - 15x^2 + 45\gamma x)^2} \quad (1)$$

$$= K\varepsilon^{-\gamma x}\sqrt{x^8 - 40\gamma x^7 + 70x^6 - 1260\gamma x^5 + 525x^4 - 2700\gamma x^3 + 225x^2 - 1350\gamma x + 2025\gamma^2} \quad (2)$$

This is exact with the exception of terms negligible in $\gamma$ since Appendix D equation (7) shows that in general $\gamma \ll 1$.

Let us assume that

$$x^4 + ax^3 + bx^2 + cx + d \quad (3)$$

is a solution for the square root in (2)

Dividing (2) by (3) and evaluating the constants we arrive

$$a = -20\gamma \quad (4)$$

$$b = (35 - 200\gamma^2) \quad (5)$$

$$c = (70\gamma - 4000\gamma^3) \quad (6)$$

∴ as a first approximation equation (2) is

$$x^4 - 20\gamma x^3 + (35 - 200\gamma^2)x^2 + (70\gamma - 4000\gamma^3)x + \ldots\ldots R \quad (7)$$

where R = remainder

more will be said about R later in this appendix

Let error $\equiv \Delta_a \equiv \dfrac{\int_0^\infty \varepsilon^{-\gamma x}(ax^3 + bx^2 + cx)\,dx}{\int_0^\infty \varepsilon^{-\gamma x} x^4 dx}$ (8)

Evaluation of (8) leads directly to

$$\Delta_a = \frac{\dfrac{35\underline{/2} - 20\underline{/3}}{\gamma^3} + \dfrac{70 - 200\underline{/2}}{\gamma} - 4000\,\gamma}{\dfrac{\underline{/4}}{\gamma^5}} \qquad (9)$$

$$\Delta_a = -2.08\gamma^2 + 13.75\gamma^4 - 167\gamma^6$$

Equation (7) Appendix D

$$\gamma = \frac{\alpha}{\beta} = \frac{1}{2K.}$$

for $Q = 10 \quad K \approx 1$

$$\gamma = .05$$

$$\Delta_a = -2.08\,(.05)^2 + 13.75\,(.05)^4 - 167\,(.05)^6$$

$= -2.08\,(25\text{x}10^{-4}) + 13.75\,(12\text{x}10^{-5}) - 167\,(15.6\text{x}10^{-9})$

$= -0.52\text{x}10^{-2} + 0.165\text{x}10^{-2} \; -0.261 \times 10^{-5}$

$= -0.355\text{x}10^{-2}$ or approximately <u>0.4% error</u>

It may be seen from the above that the coefficients of the smaller powers of x produce succeedingly smaller contributions to the error. The remainder term therefore will add but a very small amount to this error since its area contribution must be less than any of the terms considered in order for (7) to be valid. However the remainder term will be investigated and any pertinent information relating it to the work given here will be tabulated at a later date.

A comment here about the sign of $\Delta_a$ is in order - a simple plot of equation (2) and $K\varepsilon^{-\gamma x} x^4$ for reasonable $\gamma$ reveals that these curves become coincident at $x \approx 6\pi$ or approximately 3 cycles.

For times before this

$$K\varepsilon^{-\gamma x} x^4 < \text{equation } 2$$

Consequently the sign of $\Delta_a$ should be negative. We are, however, only interested in $|\Delta_a|$.

## APPENDIX D

### MISCELLANEOUS DERIVATIONS

(a) $\frac{\alpha}{\beta}$ RATIO = $\gamma$

In order that we may have an approximate idea of the value of ($\frac{\alpha}{\beta} = \gamma$) we will here set it down in terms of the Q of the circuit.

$$\text{Let } \beta = \sqrt{\omega_o^2 - \alpha^2} = K\omega_o \qquad (1)$$

$$\text{then } \frac{\alpha^2}{\omega_o^2} = (1 - K^2) = \frac{L}{4R^2C} \qquad (2)$$

$$Q \text{ (by definition)} = R\sqrt{\frac{C}{L}} \qquad (3)$$

$$\therefore Q = \frac{1}{2\sqrt{1 - K^2}} \qquad (4)$$

$$\text{or } K = \sqrt{1 - \frac{1}{4Q^2}} \qquad (5)$$

also from (1) and (2)

$$K\frac{\alpha}{\beta} = \frac{1}{2R}\sqrt{\frac{L}{C}} \qquad (6)$$

$$\therefore \frac{\alpha}{\beta} = \frac{1}{2KQ} \qquad (7)$$

From equations (4) and (5) we may derive some rather useful idea of the value of Q that we must have in order to let $\beta$ approach $\omega_o$ as closely as we desire.

The value that the ($\frac{\alpha}{\beta} = \gamma$) ratio takes for a particular value of Q and K may be seen from equation (7).

(b) CASTA DESIGN CRITERION

Consider equation (17) First Quarterly report

$$\frac{n^2(n-1)^2}{8(\omega t_o)^2} < \frac{1}{1000} \qquad (8)$$

which may be written

$$(\omega_o t_o)^2 > \frac{1000\, n^2(n-1)^2}{8} \qquad (9)$$

Let us assume that this shall be valid at time, $t_o = \left(\frac{n-1}{\alpha}\right)$, putting $\omega_o = 2KQ$ (where K is identified in equation (1) we get

$$Q > \frac{10n}{4\sqrt{2}\,K} \qquad (10)$$

This equation is valid for any n stage CASTA

For $n = 5$, $K \approx 1$ we get $Q > 9$ (11)

From page (10-2) Appendix C First Quarterly report we have for $n = 5$, $\alpha\tau = 10$ for approximately 0.65% maximum error.

In general $\alpha\tau$ = some number = C

If $\alpha\tau > C$ less error will exist

$\alpha\tau < C$ more error will exist

$\therefore$ let $\alpha\tau \geq C$

Then

$$\frac{\omega_o}{Q} \geq \frac{2C}{\tau} \quad \text{or} \quad Q \leq \frac{\omega_o \tau}{2C} \qquad (12)$$

Obviously the higher value of Q will be required for the lowest repetition rate all other parameters constant.

Equation (6) is then a design criterion when the peak method of spectral intensity measurement is used and repeated pulses are considered. Of course the above fails when pulses are considered individually.

Equations (10) and (13) may be combined viz.

$$\frac{10n}{4\sqrt{2}\,K} < Q \leq \frac{\omega_o \tau}{2C} \qquad (13)$$