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# UNCLASSIFIED

# PRINCETON UNIVERSITY

DEPARTMENT OF AERONAUTICAL ENGINEERING

# COMBUSTION INSTABILITY

# IN

# LIQUID PROPELLANT ROCKET MOTORS

## Fifth Quarterly Progress Report

For the Period 1 May 1953 to 31 July 1953

Aeronautical Engineering Report No. 216e

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1 September 1953

PRINCETON UNIVERSITY
Department of Aeronautical Engineering

CONTENTS

## LIST OF FIGURES

# I. SUMMARY

Operation of the monopropellant rocket motor has been performed with artificial flow modulation at frequencies up to 200 cycles per second at several chamber pressures, and some satisfactory measurements of the pressure lag between injector and chamber have been made. Eventually, this data should demonstrate the suitability of the technique employed to make measurements and at the same time provide a rough check on the fundamental precepts of Crocco's theoretical analyses as presented in earlier reports. Improvements in the accuracy of the measurements and both theoretical and experimental corrections to the instantaneous flow rate determinations must be completed before the data can be fully analyzed.

Several methods of measuring the pressure lag have been evolved utilizing DC amplifiers, constant-phase AC amplifiers, and direct recording of pressure traces on a four-channel oscilloscope. Amplitude calibrations have not yet achieved the desired degree of accuracy, but it is expected to be only a matter of time until additional refinement of circuit components will permit obtaining the required precision.

The Mittelmann electromagnetic flowmeter proved unsatisfactory due to equipment malfunctions, despite the many hours devoted to its operation and repair. It will be returned to the vendor for additional repairs. The Li mass flowmeter has been received, and preliminary checks indicate that the leakage has been reduced to the point at which it can be used in the flow calibration rig. Pulsing flow tests utilizing this meter and the hot wire in conjunction with pressure pickups will be made after concluding the current

series of monopropellant rocket motor runs.

Detailed information on the performance of the improved Li-Liu pressure pickup under high-temperature conditions has been obtained at our request by the NACA Lewis Flight Propulsion Laboratory. Heat transfer rates up to about 7.6 BTU per square inch per second (refered to the pickup diaphragm area) have been experienced under "screaming" conditions when the pickup was flush-mounted in an acid-gasoline motor, and the pickup drift was shown to be small at temperatures up to 500°F. All available data on this pickup design are now being collected, and will be presented at the Rocket Combustion Instability Symposium scheduled for 28 October 1953.

Construction of the bipropellant test stand is about one-third completed, and is proceeding according to schedule. Details of the system are presented later in the report.

## II INTRODUCTION

### A. Object

BuAer Contract Noas 52-713-c was undertaken as a part of the jet propulsion research program of the Department of Aeronautical Engineering at Princeton to "conduct an investigation of the general problem of combustion instability in liquid propellant rocket engines. This program shall consist of theoretical analyses and experimental verification of theory. The ultimate objective shall be the collection of sufficient data that shall permit the rocket engine designer to produce power plants which are relatively free of the phenomena of instability. Interest shall center in that form of unstable operation which is characterized by high frequency vibrations and is commonly known as 'screaming'".

### B. HISTORY

Interest at Princeton in the problem of combustion instability in liquid propellant rocket motors was given impetus by a Bureau of Aeronautics symposium held at the Naval Research Laboratory on the 7th and 8th of December 1950. This interest resulted in theoretical analyses by Professors M. Summerfield and L. Crocco of this Center.

Professor Summerfield's work, "Theory of Unstable Combustion in Liquid Propellant Rocket Systems" (JARS, Sept. 1951), considers the effects of both inertia in the liquid propellant feed lines and combustion chamber capacitance with a constant combustion time lag, and applies to the case of low (up to about 200 cycles per second) fre-

II INTRODUCTION (Cont'd.)

B. History (Cont'd.)

quency oscillations sometimes called "chugging".

Professor Crocco advanced the concept of the pressure dependence of the time lag in mid-1951; his paper, "Aspects of Combustion Stability in Liquid Propellant Rocket Motors" (JARS, Nov. 1951 and Jan.-Feb.1952), presents the fundamentals resulting from this concept, and analyzes the cases of low frequency instability with monopropellants, low frequency instability with bipropellants and high frequency instability, with combustion concentrated at the end of the combustion chamber.

Desiring to submit the concept of a pressure dependent time lag to experimental test a preliminary proposal was made by the University to the Bureau of Aeronautics in the summer of 1951 and, following a formal request, a revised proposal was submitted which resulted in Contract Noas 52-713-c.

Analytical studies with concentrated and distributed combustion had been carried on in the meantime under Professor Crocco's direction and within the sponsorship of the Guggenheim Jet Propulsion Center by S. I. Cheng and were issued as his Ph.D. thesis, "Intrinsic High Frequency Combustion Instability in a Liquid Propellant Rocket Motor", dated April 1952.

Time was devoted, in anticipation of the contract, during the first third of 1952, to constructing facilities, securing personnel, and planning the experimental approach.

II INTRODUCTION (Cont'd.)

B. History (Cont'd.)

During the first three month period of the contract year, personnel and facilities at the new James Forrestal Research Center were assigned, and the initial phases of the experimental program were planned in some detail.

A constant rate monopropellant feed system was designed and preliminary designs of the ethylene oxide rocket motor and the instrumentation systems were worked out. Special features of the projected systems included a pulsing unit to cause oscillations in propellant flow rate, a water-cooled strain-gauge pressure pickup designed for flush mounting in the rocket chamber, and several possible methods for dynamic measurement of an oscillating propellant flow rate.

Searches were made of the literature for sources of information on combustion instability and ethylene oxide, and visits to a number of activities working on liquid propellant rocket combustion instability problems were made for purposes of familiarization with equipment and results.

The basic precepts of Crocco's theory for combustion instability were reviewed, and detailed analyses made for specific patterns of combustion distribution.

Operational tests and calibration of the Princeton-MIT pressure pickup proved the value of the design, although failure of the pickup under "screaming" rocket conditions showed the necessity for modification

II INFORMATION (Cont'd.)

B. History (Cont'd.)

of the cooling system.

Construction of the monopropellant test stand and rocket motor was completed. Modifications were made to the Princeton-MIT pressure pickup to provide for higher permissible heat-transfer rates in order that it be satisfactory for use under "screaming" conditions in a bipropellant rocket motor. Construction and preliminary testing of the hot-wire flow phasemeter and its associated equipment were completed.

A new contract, NOas 53-817-c, dated 1 March 1953, was granted by the Bureau of Aeronautics to continue the program originally started under NOas 52-713-c. Operation of the monopropellant rocket motor was begun under this new contract, and shakedown operations were completed. It was found that decomposition of ethylene oxide could not be attained with the original motor design despite many configuration changes, and it was decided to avoid a long and costly development program by operating the "monopropellant" motor with small amounts of gaseous oxygen. The required limits of oxygen flow rate were determined at several chamber pressures, and it was demonstrated that the oxygen would probably have a negligible effect on performance when compared to the effect of ethylene oxide flow rate modulation. Preliminary tests with flow rate modulation up to 100 cps were performed for the purposes of system checkout, using interim AC amplifiers in lieu of the necessary DC instruments.

The time constant of the hot-wire liquid flow phasemeter was found to be 0.15 milliseconds and preparations for instantaneous flow calibrations were made. A test rig was constructed for this purpose.

II INFORMATION (Cont'd.)

B. History (Cont'd.)

A bipropellant rocket system using liquid oxygen and 100% ethyl alcohol was designed on the basis of monopropellant operational experience incorporating an adjustable-phase flow modulating unit operating on both propellants. Injector design was based on a configuration used extensively by Reaction Motors, Inc.

Subsequent efforts to date are presented in detail in this report.

## III APPARATUS

### A. Monopropellant Rocket Motor

Operation of all components of the motor proper continued satisfactorily during this report period, using the oxygen flow rates indicated in the previous report.

The slowdown of the flow modulating unit experienced on earlier runs was traced to the extremely high bearing friction which occurred when fluid pressure was applied to the piston, rather than to the use of an underpowered drive motor as was first suspected. Addition of a pressurized lubrication system to the main connecting-rod sleeve bearing did not correct the situation, so a new split shaft was manufactured which permitted the use of a roller bearing in this application. Furthermore, the two 80-pound flywheels were removed from the unit. This was originally done in order to reduce the excessive time required for the shaft to attain constant speed (over one minute at 6,000 RPM, and well over three minutes at 12,000 RPM). No visible effect on the resulting pressure-wave shape was observed with this configuration, and since the reduction in speed-up time (10 seconds at 12,000 RPM) resulted in appreciable reduction in wear of critical parts, the "flywheel-less" unit was used to obtain all subsequent test data.

### B. Bipropellant Rocket Motor

Overall design of the bipropellant feed system was based partly on experience obtained with the monopropellant system and partly on general experience gained in the rocket industry. Although the first propellant

III APPARATUS (Cont'd.)

B. Bipropellant Rocket Motor

combination to be used will be liquid oxygen-100% ethyl alcohol, the system was designed with a view toward future use of any of the propellants now in common usage throughout the country, with the possible exception of liquid fluorine.

Figure 1 shows the overall schematic layout of the feed system. The fuel tank is a surplus stainless steel vessel of approximately 20 gallon capacity, hydrostatically tested to 2500 psi. A surplus 40-gallon Monel tank with a vacuum-insulated jacket was also obtained and hydrostatically tested to 1500 psi. for use with liquid oxygen at 1000 psi maximum pressure. This latter tank will be suitable for the first few months of operation, but testing at the higher chamber pressures will require a vessel of greater strength. The vent-relief valves are of the same design as the one used so satisfactorily on the monopropellant tank, but are constructed of stainless steel, with the liquid oxygen valve protected from malfunction due to frosting. A pressure-actuated emergency propellant valve from the RMI 400-pound thrust Lark motor has been modified for use as the main propellant valve. Cavitating venturis of three different flow ratings will be used for the three chamber pressure levels anticipated, and Potter flowmeters will again be used to record steady-state flow rates.

A flow modulating unit for operation at frequencies up to 200 cycles per second has been constructed for use in this system. The unit consists of two easily replaceable piston-cylinder assemblies, with the pistons driven from a common shaft (see Figure 2). In order to avoid variation in

III APPARATUS (Cont'd.)

B. Bipropellant Rocket Motor (Cont'd)

propellant mixture ratio, the crank angle between the two piston strokes is continuously adjustable from zero to 360° so that simultaneous modulation of the two propellant flow rates at the injector orifices can be obtained. Design of the unit was based directly on experience with the monopropellant flow modulator, and incorporates several improvements additional to the use of pressure-lubricated roller bearings on the connecting rods. A 5 HP U. S. Motors Vari-drive has been procured to drive the unit at speeds from 3,000 to 12,000 RPM.

The thrust stand and thrust-measurement devices are identical to those used in the monopropellant system. All piping, fittings, valving, etc. is of type 304 or 316 stainless steel in order to provide for general usage with nearly all of the common rocket propellants. When using liquid oxygen, or any other low-temperature propellant, provision has been made for precooling the entire feed line right up to the rocket motor injector in order to avoid the hard starts sometimes occasioned by vapor in the feed line.

The firing diagram for the bipropellant motor is shown in Figure 3. Special features include pyrotechnic igniters utilizing a safety fuse to prevent propellant valve operation in case of igniter misfiring, a safety chamber pressure switch which will shut off the propellant valves in case of a rocket motor misfire, and automatic pressurization of fuel and oxidant tanks to a preset value. No special provision is necessary to obtain oxidant "lead" at start or "lag" at shutdown on the liquid-oxygen system, since oxidant is automatically introduced into the rocket motor at these

III APPARATUS (Cont'd.)

B. Bipropellant Rocket Motor (Cont'd.)

times via the precooling arrangement mentioned earlier.

Figure 4 illustrates the initial rocket motor design. The chamber is of 1½-inch thick copper, three inches in inner diameter and four inches long. It will operate uncooled for the first series of tests, in order to facilitate convenient instrumentation placement. The nozzle is also of copper, but is provided with water passages designed for cooling at uniform heat transfer rates. The injector is of the one-on-one impinging jet type, using 12 pairs of holes located on a concentric circle. Sizes of the fuel and oxidant orifices are identical, the proper mixture ratio being obtained by variation in feed pressures. The basic design of this injector is patterned after a model used extensively by Reaction Motors, Inc. for performance evaluation and instability studies.

Additional instrumentation components consisting of a 4-channel Electronic Tube Corporation oscilloscope, a 4-channel modification of the present 2-channel Ampex tape recorder, and additional pressure pickups and intermediate equipment have been ordered. The basic instrumentation scheme will be similar to that used on the monopropellant motor, with the exception of the heat transfer instrumentation for use on later bipropellant test operations.

C. Instrumentation

1. D.C. Amplifiers

The problem of obtaining D. C. amplification conforming to project requirements has been only partially solved. Three channels of a chopper-feedback design have been developed and tested and appear to be satisfact-

III APPARATUS (Cont'd.)

C. Instrumentation (Cont'd.)

1. D. C. Amplifiers (Cont'd.)

ory from linearity, drift, and differential amplification standpoints, but suffer from an excessive noise level. These instruments were used to record the D. C. pressure levels described later, but are apparently unsuited for use with low-level A. C. signals such as are characteristic of the pressure pcikup during runs with modulated flow rate. The pressure-lag data were thus recorded without benefit of D. C. differential amplification.

It is believed that the high noise level is not an inherent feature of this design of amplifier, but rather is a function of the detailed circuit development. During the last month, two commercial amplifiers which use the chopper-feedback principle and which are claimed to be suitable for this application have appeared on the market, and will be investigated as possible solutions to the problem. Meanwhile, the instrumentation system of Figure 5 was calibrated and used to record the pressure time-lag data.

2. Overall System for Monopropellant Pressure Time Lag Measurement

Figure 5 shows the instrumentation used to measure the time lag between a change in injector pressure and the resulting variation in chamber pressure. Two differential pressure pickups were used, one mounted in one of the injectors, and one in the chamber just adjacent to that injector. The signal from each pickup was then split four ways. One branch

III APPARATUS (Cont'd.)

C. Instrumentation (Cont'd.)

2. Overall System for Monopropellant Pressure Time Lag Measurement (Cont'd.)

led directly to the four-channel oscilloscope, upon one channel of which a 200 cps timing trace was impressed by an audio oscillator calibrated with a Berkeley precision electronic counter. The second signal branch led to a cathode follower and thence to a Leeds and Northrup recording potentiometer. The third branch was passed through the chopper-stabilized D. C. amplifier and was recorded on the Hathaway oscillograph. The final branch of the signal from each pickup was amplified by a constant-phase A. C. amplifier and recorded on magnetic tape.

The input impedance of each of the instruments described above was well into the megohm range, and hence feedback between instruments was negligible compared with the output from the 2500-ohm pickup. The Leeds and Northrup potentiometer records were used to obtain the D. C. level of the two pressures, while the D. C. Hathaway oscillograph trace was used primarily to provide a record of run history and to show transients or other unlooked-for events. Accurate time values were taken from the A. C. Ampex tape recordings, and were checked with oscilloscope readings (as recorded by the General Radio strip-film camera). The oscilloscope and Ampex records also supplied pressure amplitude measurements. Heise bourdon-tube gauge readings were used to check the steady-state component of each pressure recording.

3. Pressure Pickups

Three differential pressure pickups incorporating the improved coolant

III APPARATUS (Cont'd.)

C. Instrumentation (Cont'd.)

3. Pressure Pickups (Cont'd.)

water passage design, extended linear range, and back-pressure chamber sealing modifications have been delivered, and have been used to obtain all test data included in this report. Calibration tests show what first appeared to be a large amount of hysteresis, but which was demonstrated to occur only when high back pressures were applied without any test pressure. Calibrations made under simulated operating conditions were entirely satisfactory, as will be illustrated later in the report. Four additional pickups of this type have been ordered, and a slight modification in diaphragm spacer design is expected to eliminate even the apparent hysteresis which occurs under the artificial calibration condition just described.

Thermal drift and "screaming" heat transfer motor tests on an absolute pressure pickup with the improved coolant passage design were made at our request by the NACA Lewis Flight Propulsion Laboratory. A differential pickup was also sent to the Bell Aircraft Corporation for evaluation of thermal drift, vibration sensitivity, and ruggedness under typical rocket motor test conditions. The NACA data are included later in the present report, while the Bell tests will be reported upon when complete.

4. Flowmeters

The Mittelmann electromagnetic flowmeter was operated for an extended period without achieving so much as a steady-state calibration. Multitudes of circuitry difficulties ranging from poor contacts to resonance in the

III APPARATUS (Cont'd.)

C. Instrumentation (Cont'd.)

4. Flowmeters (Cont'd.)

cables were encountered, and the instrument is to be returned to the vendor for revision and repair. Due to its inability to measure even a steady flow rate, no evaluation of the flowmeter's dynamic performance could be made, and the only recommendation that can be forwarded at this time is that a good deal of electronic circuit cleanup is necessary before the meter becomes suitable for use in any flow measurement application.

The Li mass flowmeter has been received after revision by the vendor to correct its excessive leakage. Preliminary checks indicate its suitability for use in calibration tests, but the leakage rate, although much smaller, still precludes the instrument's use on direct rocket motor tests. Full dynamic water calibrations in conjunction with the hot-wire liquid flow phase-meter discussed in the previous Quarterly Report will be made on the monopropellant flow stand at the conclusion of the present series of rocket motor tests.

## IV. INFORMATION AND DATA

### A. Monopropellant Rocket Tests

The data taken during the current series of test runs are not yet considered to be of sufficient accuracy for comparison with theory. Although no direct measure of instantaneous flow is being taken on these tests, it is expected that subsequent theoretical corrections checked by flow-stand calibrations will permit use of the instantaneous pressure drop readings for flow-rate determinations, following the theory outlined in Appendix A of the Third Quarterly Progress Report.

The difficulty of obtaining a direct record of the instantaneous injector pressure drop through a suitable D. C. amplifier makes it preferable to obtain the corresponding data analytically from more elementary measurements. In fact, determination of the injector pressure drop can be made using only the mean levels, amplitudes of oscillation, and phase difference between injector pressure and chamber pressure. If the oscillating portions of the injector and chamber pressures are sinusoidal with respect to time, the injector pressure $P_i$ may be written

$$P_i = \overline{P}_i + \widetilde{P}_i \sin \omega t$$

The chamber pressure $P_c$ then becomes

$$P_c = \overline{P}_c + \widetilde{P}_c \sin(\omega t + \alpha)$$

where $\alpha$ = measured phase difference between injector and chamber pressure oscillations

$\overline{P}_i, \overline{P}_c$ = mean values of injector and chamber pressures respectively

$\widetilde{P}_i, \widetilde{P}_c$ = amplitudes of oscillation of injector and chamber pressures, respectively.

IV. INFORMATION AND DATA (Cont'd.)

A. Monopropellant Rocket Tests (Cont'd.)

The instantaneous pressure drop $\Delta P$ across the injector may be written in the form

$$\Delta P = P_i - P_c = \overline{\Delta P} + \widetilde{\Delta P} \sin(\omega t + \gamma)$$

where $\gamma$ = phase difference between $P_i$ and $\Delta P$.

But, we are ultimately interested in determining the pressure perturbation as a function of the mass flow perturbation. Assuming for the moment that the relationship between flow rate and $\Delta P$ were known, this means that we meed the relation between $P_c$' and the quantity

$$\frac{\Delta P - \overline{\Delta P}}{\overline{\Delta P}}$$

Hence, we are interested in the ratio

$$\frac{P_c - \overline{P}_c}{\overline{P}_c} \Big/ \frac{\Delta P - \overline{\Delta P}}{\overline{\Delta P}} = \frac{\widetilde{P}_c \sin(\omega t + \alpha)}{\overline{P}_c} \Big/ \frac{\widetilde{\Delta P} \sin(\omega t + \gamma)}{\overline{\Delta P}}$$

This can be written in the form

$$\frac{P_c - \overline{P}_c}{\overline{P}_c} \Big/ \frac{\Delta P - \overline{\Delta P}}{\overline{\Delta P}} = R e^{i(\alpha - \gamma)} = R e^{i\beta}$$

where the amplitude of R of the required ratio is given by

$$R = \frac{\left(\frac{\overline{P}_i}{\overline{P}_c} - 1\right)}{\sqrt{1 + \left(\widetilde{P}_i/\widetilde{P}_c\right)^2 - 2\left(\widetilde{P}_i/\widetilde{P}_c\right)\cos\alpha}}$$

IV. INSTRUMENTATION AND DATA (Cont'd.)

A. Monopropellant Rocket Tests (Cont'd.)

and the phase difference $\beta$ between $\frac{p_c}{\bar{P}_c}$ and $\frac{\Delta P}{\overline{\Delta P}}$ is

$$\tan \beta = \frac{\sin \alpha}{\cos \alpha - (\tilde{P}_c / \tilde{P}_i)}$$

Thus, the necessary relationship between chamber pressure perturbation and injector pressure drop perturbation (which can be correlated to the flow perturbation) is obtained directly from the mean values, amplitudes of oscillations, and phase difference between injector and chamber pressure. This determination can be made without the use of a high-response D. C. amplifier, requiring only that the pressure oscillations be sinusoidal in nature.

Typical oscilloscope recordings of injector and chamber pressure are shown in Figure 6. The uppermost trace is a 200 cps timing trace, the middle one is injector pressure, and the lower one is chamber pressure.

Forthcoming cold flow stand tests will be used to determine whether the high-frequency hash (about 4,000 cps) which appears on the injector pressure trace is due to combustion or to fluid line oscillations. In any case, before entirely adequate phase and amplitude measurements can

IV INFORMATION AND DATA (Cont'd.)

A. Monopropellant Rocket Tests (Cont'd.)

be drawn from this data, it will be necessary to remove the hash by means of a suitable low-pass filter.

The excessively high amplitude of the injector pressure oscillation in the current tests results from the fact that the largest-size piston is being used in the flow modulating unit, i.e., the one designed for 900 psi chamber pressure operation. This one piston has been used at all chamber pressures (the run illustrated in Figure 6 is at 300 psi) only as a matter of convenience, but when the final series of tests is made, the proper piston size for each value of the mean propellant flow-rate will be used.

B. Pressure Pickups

Results of tests made at the NACA on heat-transfer and temperature characteristics of the Li-Liu double-diaphragm pickups are included in Figures 7, 8, and 9. Figures 7 and 8 show the coolant water flow, coolant temperature, and heat transfer rate for a non-screaming and screaming run, respectively. The data of Figure 8 are incomplete due to burnout of the rocket motor during screaming. No damage to the pickup was observed. (The heat transfer rate is based on the overall exposed area of the pickup. If only the diaphragm area is considered, the heat transfer rates shown in the figures should be multiplied by about 2.2. The actual heat transfer rate probably falls somewhere between the two.)

Figure 9 shows the temperature performance of an absolute pickup up to

IV. INFORMATION AND DATA (Cont'd.)

B. Pressure Pickups (Cont'd.)

500°F. These data are somewhat questionable, due to both the long-term exposure to furnace temperature and the low-accuracy pressure gauges used on the tests, but illustrate the general nature of the pickup's performance. Actually, it is not possible to determine from these data whether the drift is due to temperature or change in sensitivity, and additional temperature tests will be made both to clarify this point and to supply a more satisfactory temperature characteristic. During the temperature drift tests, the signal dropped suddenly to zero, and the pickup (absolute pickup #4) was returned to the vendor for internal examination. No exterior damage was observed. Another live pickup (#9) was sent to NACA for evaluation on a screaming rocket motor.

All available information on this pickup design will be collected and brought up to date for presentation at the Second Symposium on Liquid Propellant Rocket Combustion Instability and will appear as an appendix to the next Quarterly Report.

C. Theory

A theoretical analysis of the effect of nozzle impedance on low-frequency combustion instability in liquid-propellant rockets is included as Appendix A to the present report. This analysis indicates that the low-frequency stability of a rocket motor is improved by decreasing the nozzle angle of convergence, irrespective of whether this is done by decreasing the contraction ratio

IV. INFORMATION AND DATA (Cont'd.)

C. Theory (Cont'd.)

or increasing the length of theconvergent section.

V. DISCUSSION

Detailed discussion of the monopropellant test data will be postponed until analysis of the current series of runs has been completed. Evaluation of results to date on the pressure pickup development program will appear in the next quarterly.

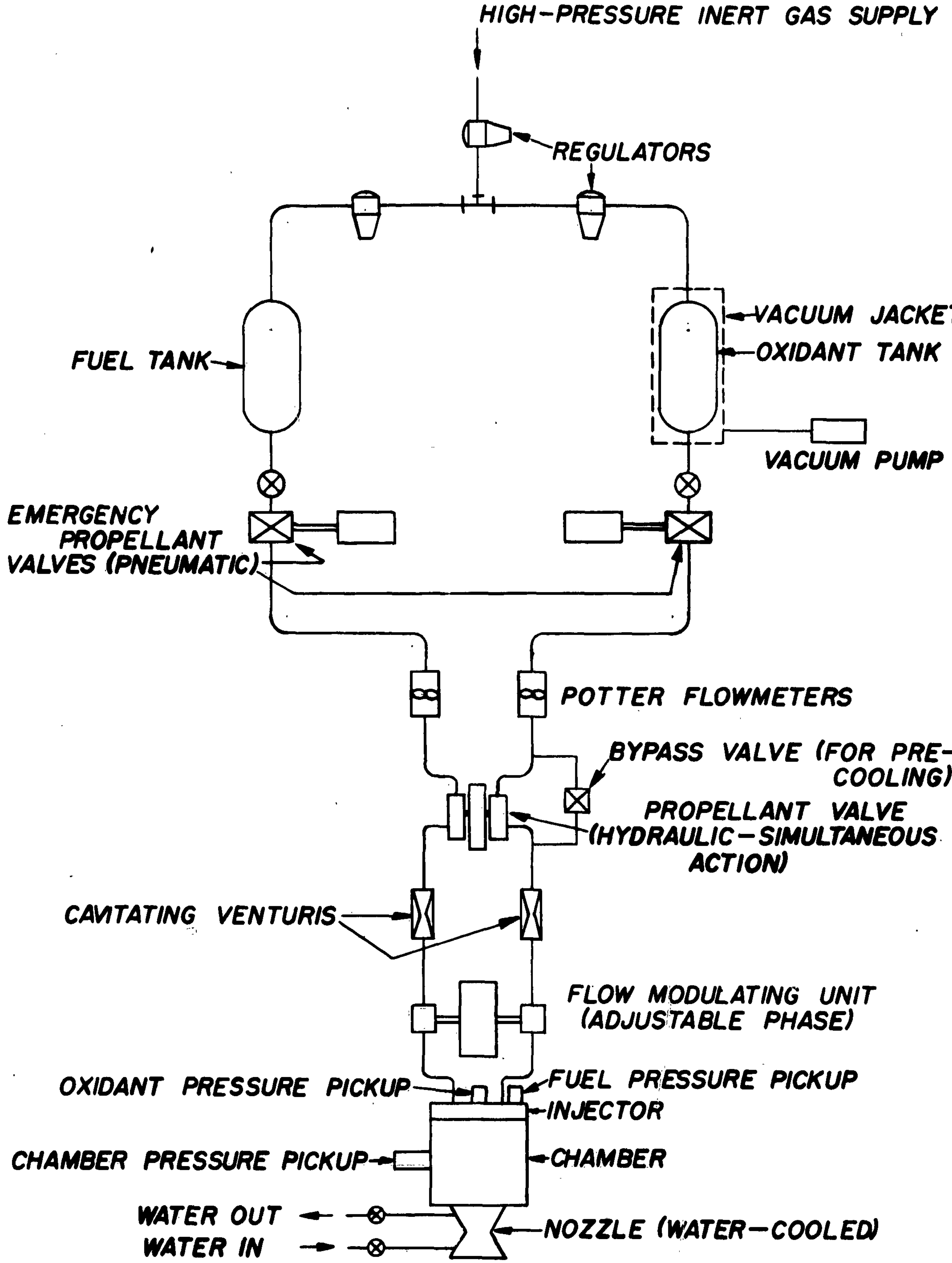



FIGURE 1

SCHEMATIC: BIPROPELLANT FEED SYSTEM

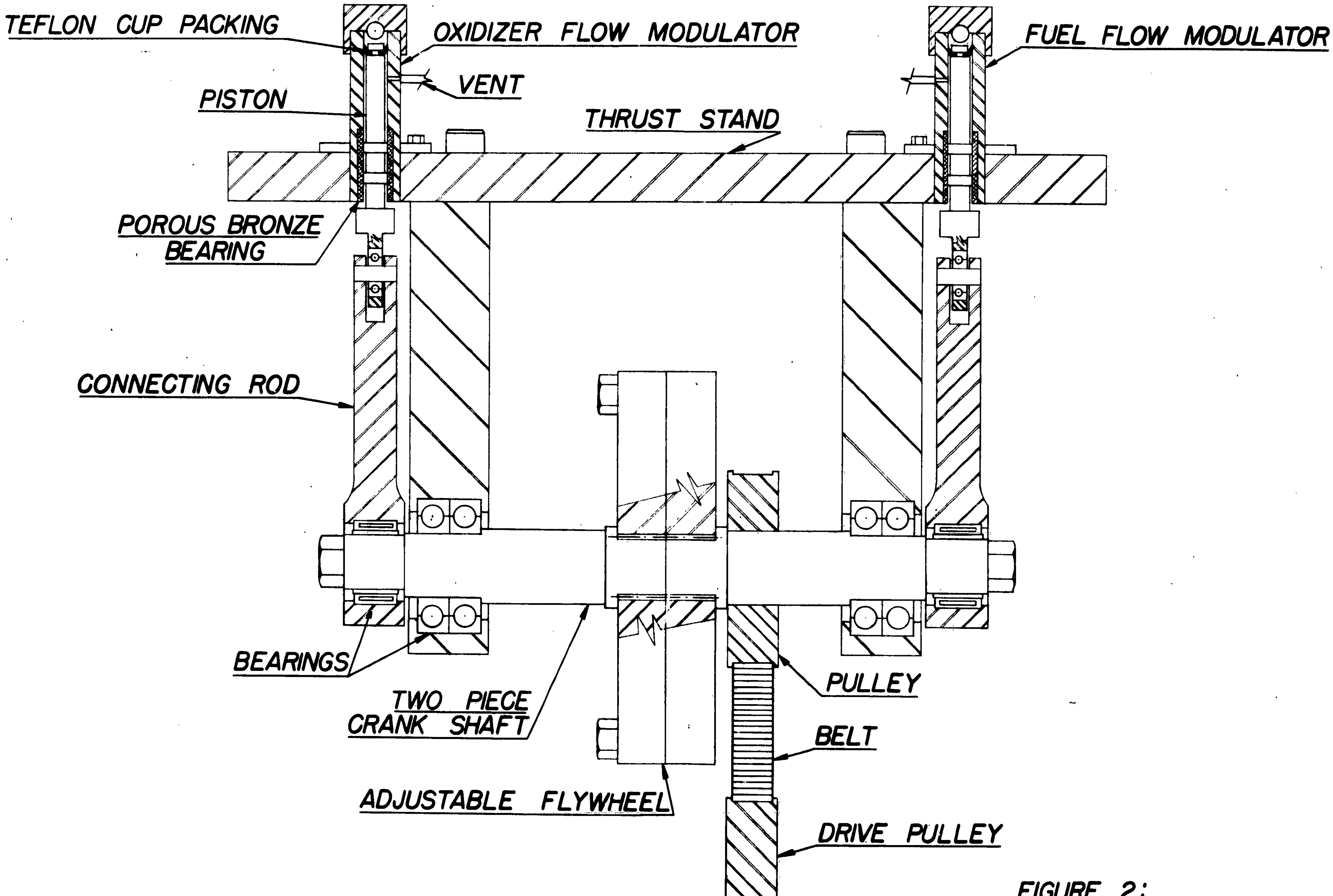
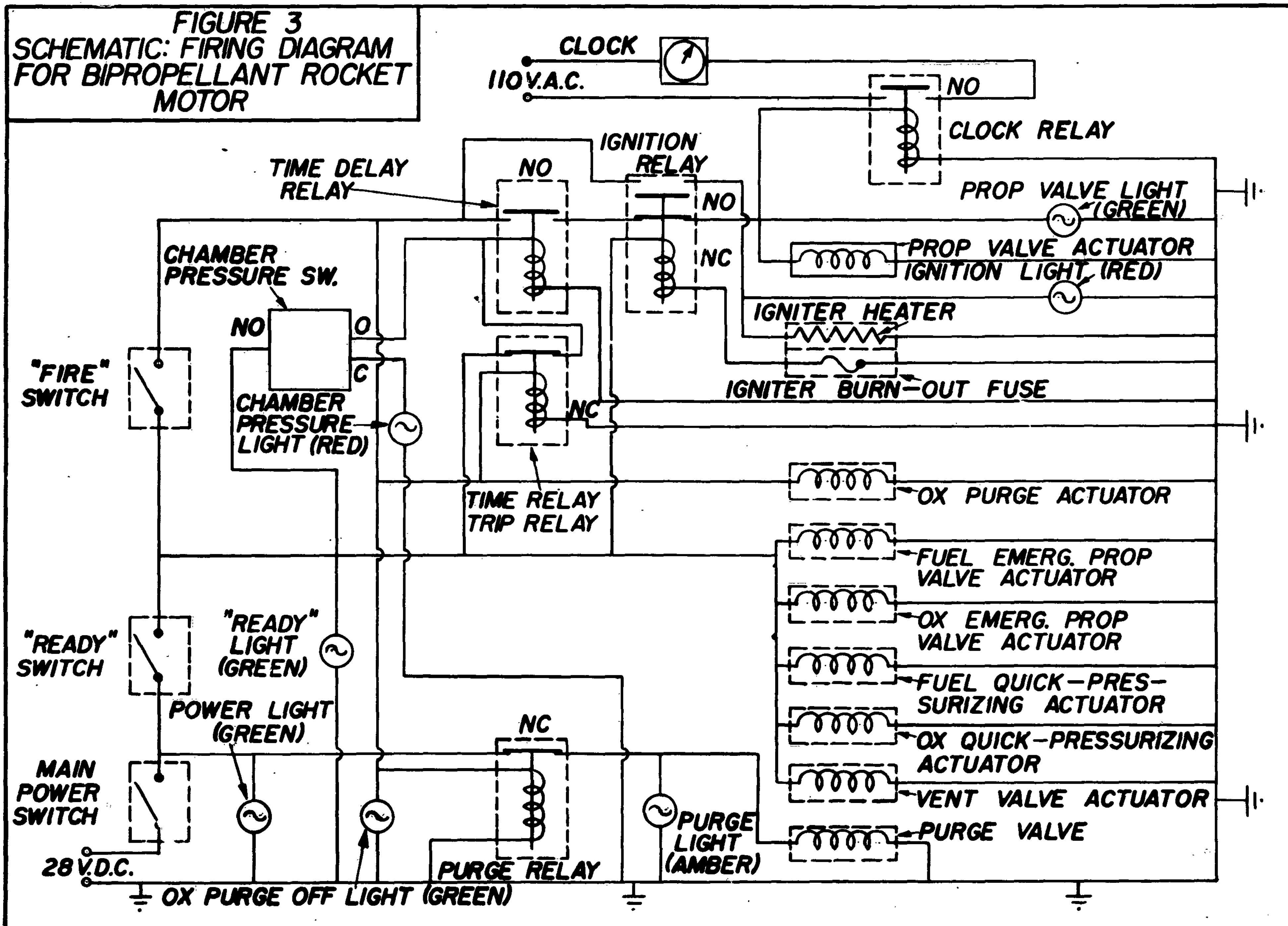



FIGURE 2:
BIPROPELLANT FLOW MODULATING UNIT

FIGURE 3
SCHEMATIC: FIRING DIAGRAM FOR BIPROPELLANT ROCKET MOTOR



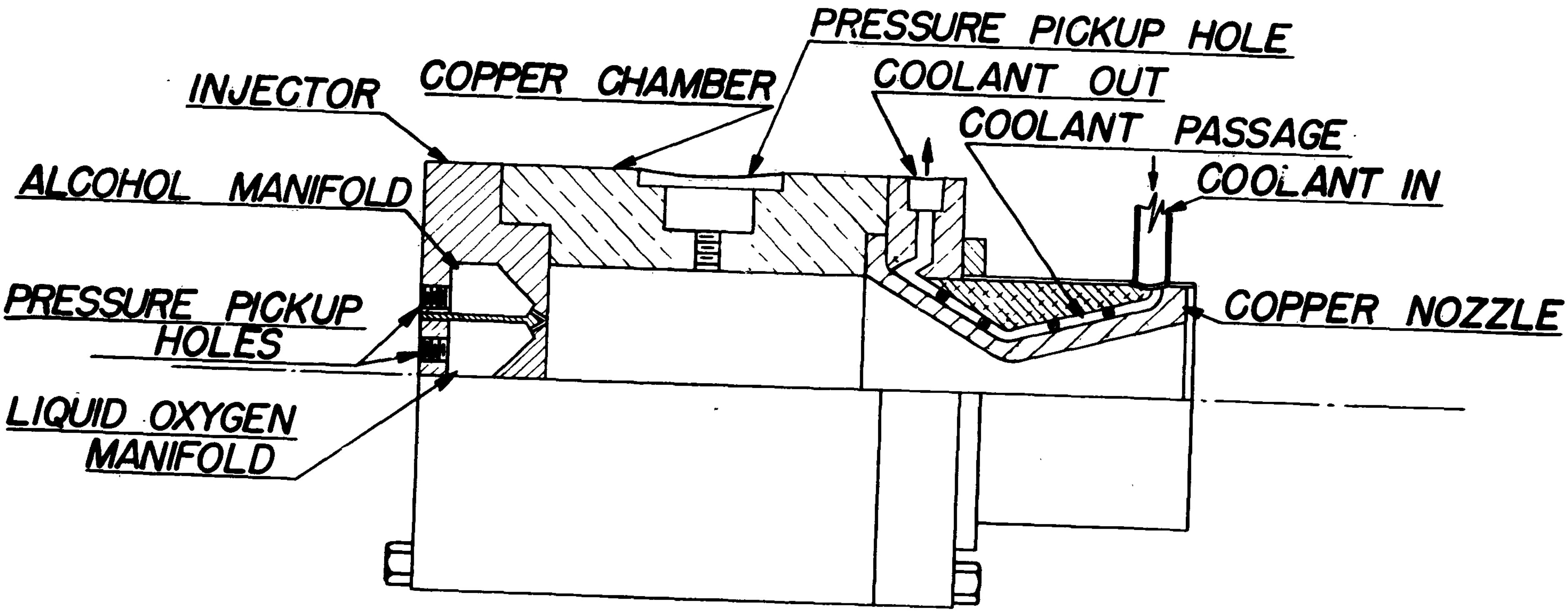



FIGURE 4
BIPROPELLANT ROCKET MOTOR № 1
(LIQUID OXYGEN-ALCOHOL)

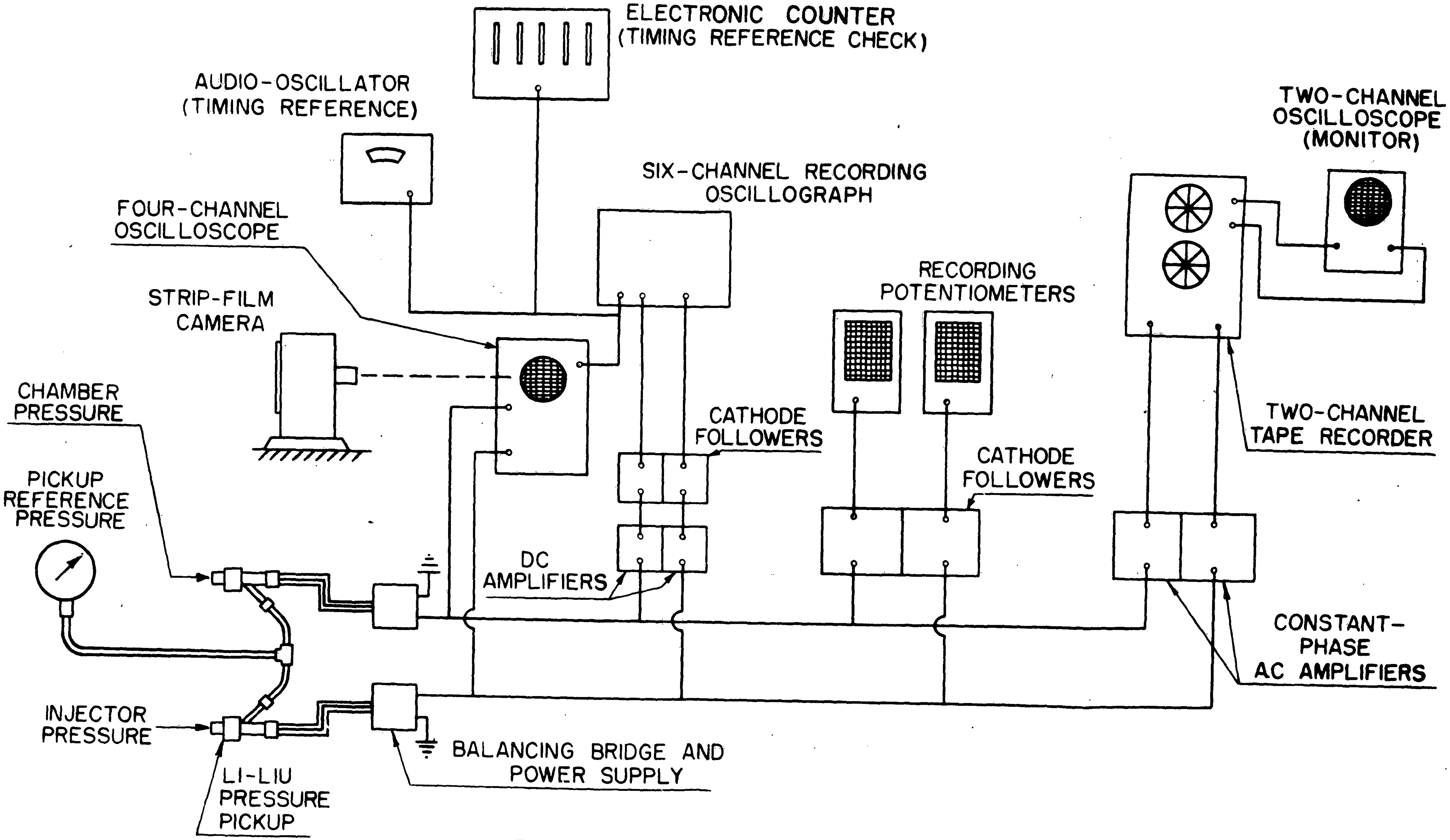



FIGURE 5

SCHEMATIC: MONOPROPELLANT PRESSURE-LAG INSTRUMENTATION

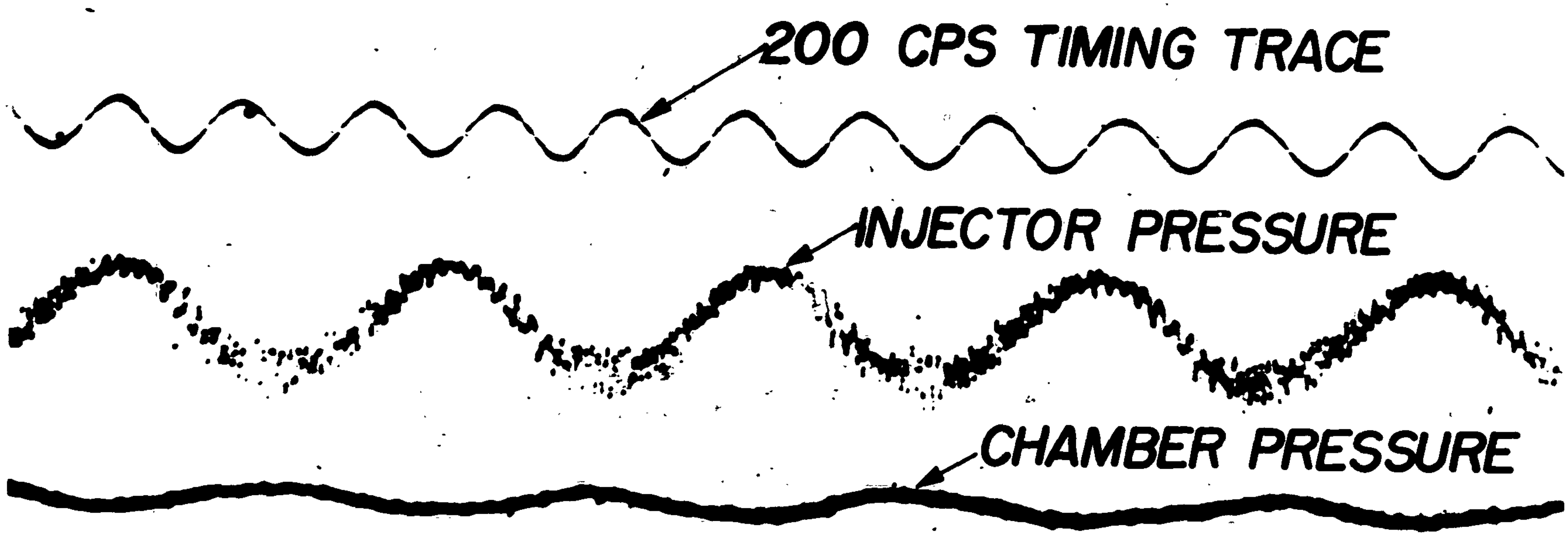



FIGURE 6

TYPICAL OSCILLOSCOPE RECORDINGS OF INJECTOR AND CHAMBER PRESSURE OSCILLATIONS

## HEAT TRANSFER TO LI-LIU DUMMY PRESSURE PICKUP (NON-SCREAMING)

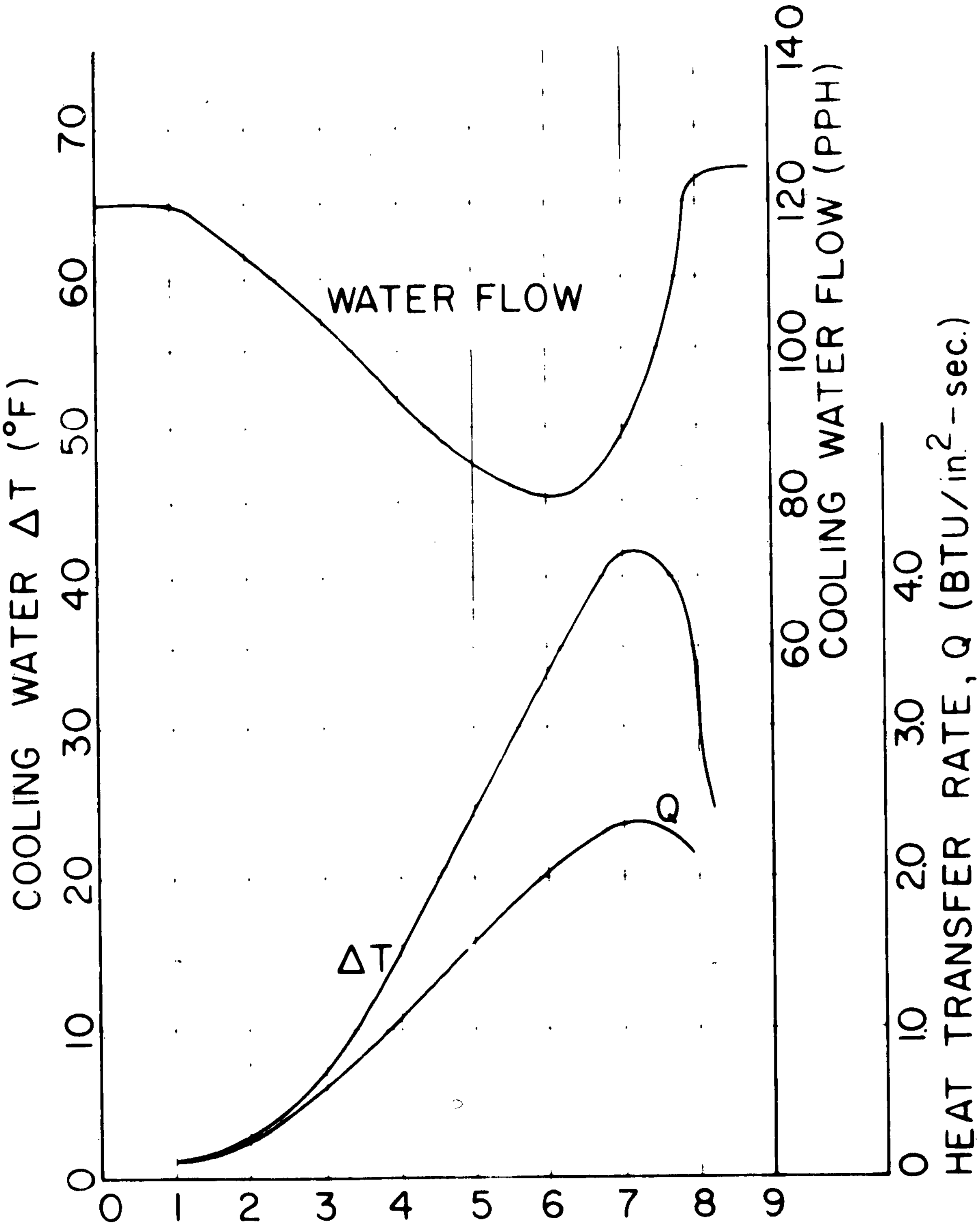



FIGURE 7

## HEAT TRANSFER TO LI-LIU DUMMY PRESSURE PICKUP (SCREAMING)

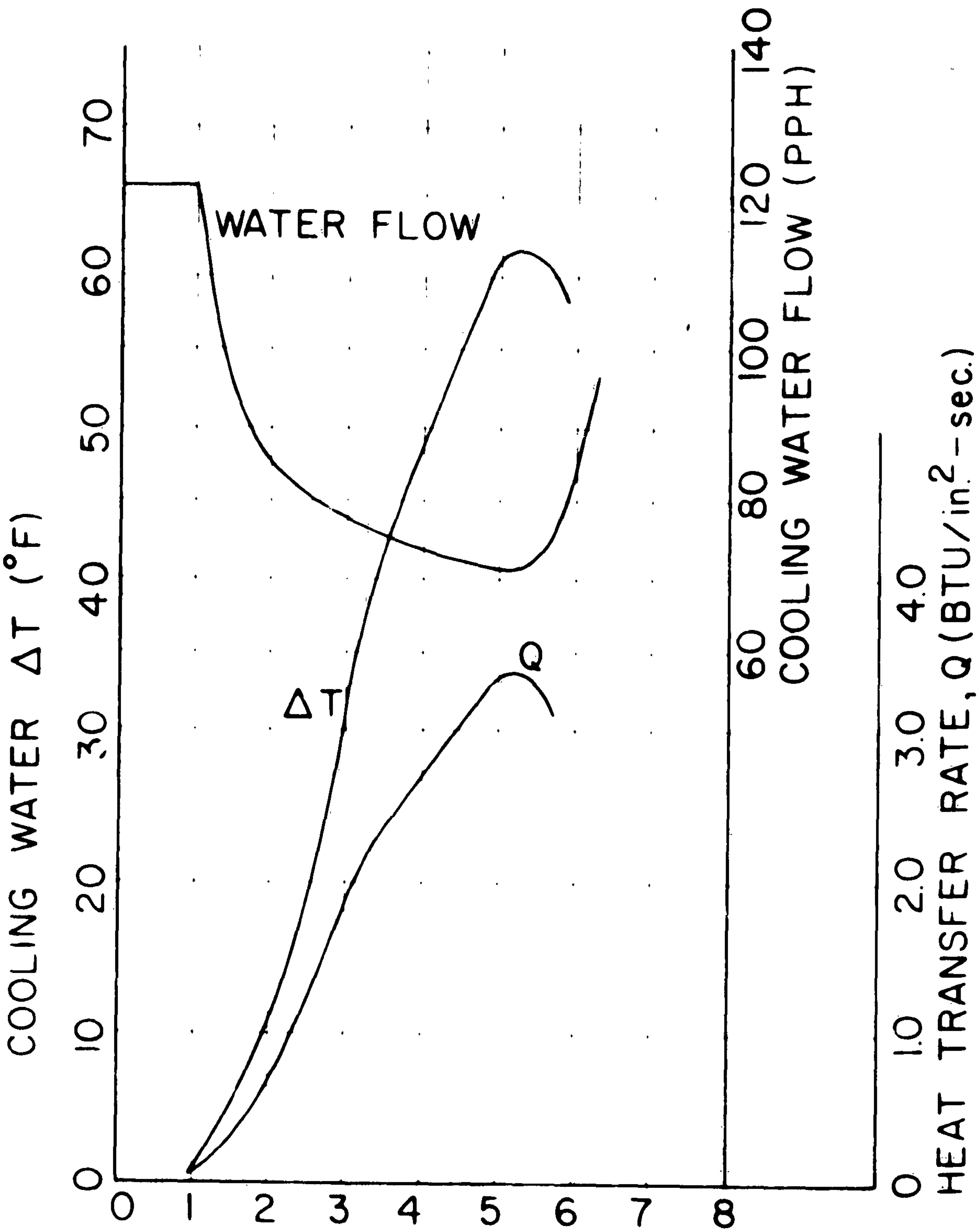



FIGURE 8

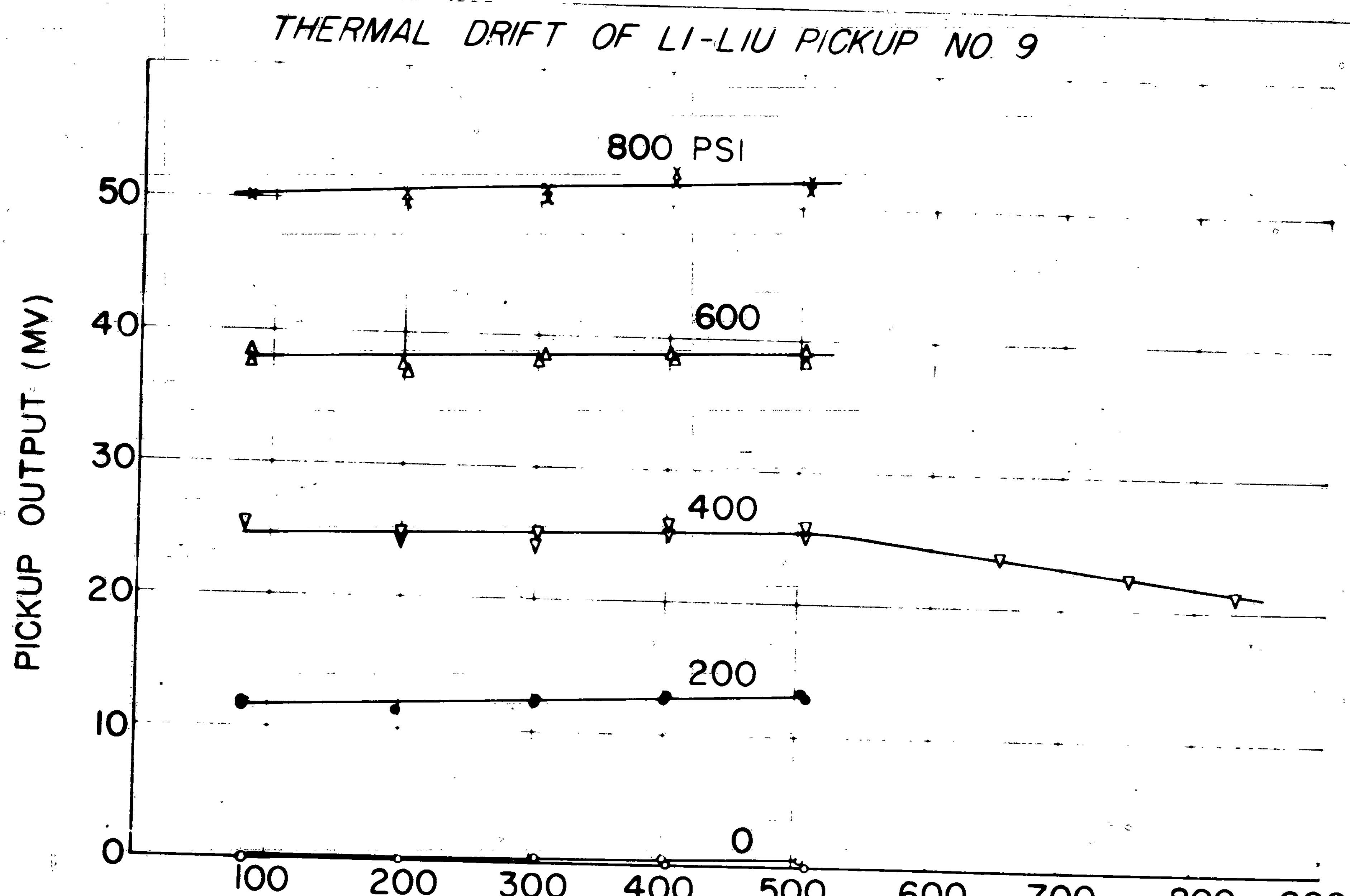


FIGURE 9 FURNACE TEMPERATURE (°F)

APPENDIX A

"LOW FREQUENCY COMBUSTION STABILITY
OF LIQUID ROCKET MOTOR WITH DIFFERENT
NOZZLES"

# LOW FREQUENCY COMBUSTION STABILITY OF LIQUID ROCKET MOTOR WITH DIFFERENT NOZZLES

## I. INTRODUCTION

Previous analyses of low frequency oscillations (1) to (7)* are made on the assumption of quasi-steady gas flow in deLaval nozzle through which combustion gases are accelerated and ejected at supersonic speeds. The boundary condition as deduced from this quasi-steady flow assumption is that the Mach number of the gas flow entering the deLaval nozzle is constant so that the rate of flow of burned gas out of the combustion chamber into the deLaval nozzle is directly proportional to the stagnation pressure and inversely proportional to the square root of the stagnation temperature of the burned gas entering the nozzle. Tsien (8) questioned the validity of this assumption and analyzed the transfer function of the nozzle for several special cases. Crocco (9) extended Tsien's treatment with linear steady state velocity distribution in subsonic portion of the nozzle and determined the specific acoustical admittance ratio of the nozzle flow for considerable range of the reduced frequency parameter $\beta$. The results show that for small values of $\beta$ the real part of the admittance ratio is essentially constant and is equal to the value corresponding to that of quasi-steady nozzle flow. However, the imaginary part of the admittance ratio is directly proportional to the reduced frequency $\beta$, and the rate of increase of the imaginary part with $\beta$ is quite considerable especially when the entering Mach number is small as is for the practical case. Furthermore, the magnitude of $\beta$ for chugging frequencies with a conventional nozzle is not really very small but is of the order of one tenth and can be considerably bigger if the deLaval nozzle is exceptionally long. The object of the present paper is to investigate the effect of the deviation of the nozzle flow from being quasi-steady on the stability of low frequency oscillation on liquid propellant rockets.

* Number in parenthesis refers to references.

## II. Fundamental Equations

The status in the combustion chamber of a liquid propellant rocket is extremely complicated. For simplicity, we shall first consider the monopropellant case and adopt the following assumptions in accordance with previous authors without further discussion.

a. The gas pressure inside the combustion chamber is practically uniform everywhere in the combustion chamber and at any instant. In unsteady state operation, the chamber pressure oscillates about the mean or steady state value as a whole.

b. The adiabatic flame temperature of the burned gas is assumed to be constant and independent of the small variations of pressure under which combustion takes place. The temperature of the burned gas is further assumed to remain unchanged throughout its motion in the combustion chamber irrespective of the chamber pressure oscillation.

c. The time lag of all the propellant elements are assumed to be the same.

The first two assumptions enable us to neglect the equations of conservation of momentum and of energy for the flow of the burned gas in the combustion chamber. The dynamics of the burned gas flow is governed only by the equation of conservation of mass, that is, the rate of burned gas generation $\dot{m}_b$ (t) must be equal to the sum of the rate of ejection $\dot{m}_e$ (t) of burned gas out of the combustion chamber into the deLaval nozzle and the rate of accumulation $\frac{dM_g}{dt}$ of burned gas in the combustion chamber. All notations are adopted from Crocco (4) except where new symbols are necessary. The equation of mass balance is reduced to the following dimensionless form:

$$\frac{d\varphi}{dz} + \mu_e(z) = \mu_b(z) \qquad (2.1)$$

where $z = \frac{t}{\theta_g}$ is the reduced time with $\theta_g = \frac{g}{\gamma}\left(\frac{2}{\gamma+1}\right)^{-\frac{\gamma+1}{\gamma-1}} \frac{L^*}{c^*}$ = the average gas residence time in the combustion chamber.

We shall assume that the entire time lag is pressure sensitive and the dependence of $\tau$ on pressure is defined as

$$\int_{t-\tau}^{t} p^{n}(t')\,dt' = \text{Constant}$$

where n is the index of interaction in terms of instantaneous pressure including the effect of temperature variations of the burned gas. It has been shown (4) that

$$\mu_b(z) = \mu_i(z-\delta) + n\left[\varphi(z) - \varphi(z-\delta)\right] \qquad (2.2)$$

The fractional variation of the mass flow rate out of the combustion chamber depends on the transfer function $N_n$ of the deLaval nozzle with supersonic exit velocity. For quasi-steady nozzle flow and isentropic oscillations of the burned gas, we have

$$\mu_e = \frac{p'}{p} - \frac{1}{2}\frac{T'}{T} = \frac{\gamma+1}{2\gamma}\varphi_e$$

with the nozzle transfer function based upon fractional pressure variation given as $\frac{\gamma+1}{2\gamma}$ which is equal to unity when the temperature variation of the gas is neglected. The specific admittance ratio $\alpha_n$ is defined as the ratio of the fractional variation of gas velocity to that of gas density at the entrance of the deLaval nozzle and can be represented for low frequency oscillations as

$$\alpha_n = \frac{\gamma-1}{2} + ik\cdot\frac{\Omega}{\bar{u}_x} \qquad (2.3)$$

where $\Omega$ is the dimensional angular frequency of neutral oscillation and $\bar{u}_x$ is the constant velocity gradient of the gas flow in the subsonic part of the nozzle. The constant k for isentropic oscillations can be obtained from reference (9) as

$$k = -\left( \tfrac{1}{2} \frac{\ln z_e}{1 - z_e} + \frac{\gamma - 1}{4} \right) \qquad (2.4)$$

in which $\frac{\gamma-1}{4}$ can be neglected in accordance with assumption b/. The quantity $z_e$ is defined as

$$z_e = \frac{\frac{\gamma+1}{2} M_e^2}{1 + \frac{\gamma-1}{2} M_e^2}$$

with $M_e$ representing the Mach number of the burned gas entering the nozzle. The quantity $z_e$ is usually less than 0.1 for conventional liquid rockets. The magnitude of $\frac{\sigma b}{\bar{u}_x}$ for a deLaval nozzle with converging portion of 1 foot long and for an oscillation of 100 cycles per second is of the order of 0.2 for example. Thus we see that the imaginary part of the specific admittance ratio $\alpha_n$ is quite significant as compared to its real part. The transfer function of the mass flow based on fractional pressure variation at the entrance is

$$N_n = \frac{1}{\gamma} \cdot \left[ 1 + \alpha_n \right] = \frac{\gamma + 1}{2\gamma} + i \frac{k}{\gamma} \frac{\sigma b}{\bar{u}_x}$$

which reduces to

$$N_n = 1 + i k \frac{\sigma b}{\bar{u}_x} = 1 + i b \cdot \omega$$

when $\gamma = 1$ under the assumption of negligible temperature variation. The proportionality constant b is dimensionless and is equal to $k / (\sigma g \cdot \bar{u}_x)$. When the steady state velocity in the subsonic portion of the nozzle is linear, the value of k is given by equation (2.4) and the factor b is given as

$$b = \frac{k}{\sigma g \, \bar{u}_x} = -\tfrac{1}{2} \frac{\ln z_e}{1 - z_e} \cdot \frac{1}{\bar{u}_x} \cdot \frac{1}{\sigma g} \qquad (2.5)$$

It is conjectured that for practical nozzle shape where the velocity distribution is not linear, this constant b will still be independent of the frequency of oscillation and of the same order of magnitude as that with linear velocity distribution in nozzle. It is thus seen that the absolute magnitude of the transfer function will not be significantly different from unity, which is the quasi-steady value, but the phase lead of the mass flow oscillation is quite considerable even for low frequency oscillations.

If we write the nozzle transfer function in operational form $N_n = 1 + b\frac{d}{dz}$ while investigating the stability of the solutions of exponential type, then

$$\mu_e = \left(1 + b\frac{d}{dz}\right)\varphi_e = \left(1 + b\frac{d}{dz}\right)\varphi \qquad (2.6)$$

where $\varphi_e$ is replaced by $\varphi$ since we assumed that the fractional variation of pressure at any instant is the same everywhere in the combustion chamber. Introducing equations (2.2) and (2.6) into equation (2.1), we obtained the equation of mass balance as:

$$\underbrace{\frac{d\varphi}{dz}}_{\text{Accumulation}} + \underbrace{\left(1 + b\frac{d}{dz}\right)\varphi}_{\text{Ejection}} = \underbrace{\mu_i(z-\delta) + n\left[\varphi(z) - \varphi(z-\delta)\right]}_{\text{Generation}} \qquad (2.7)$$

The formulation for the analysis of the oscillation problem will require a relation between the fractional variation of the injection rate $\mu_i$ and the fractional variation of chamber pressure. This relation depends upon the dynamic characteristics of the feeding system in response to the chamber pressure variation. The dynamics of the feeding system will be formulated following Crocco (4) and Tsien (5), with the following assumptions:

(a). The pressure drop due to wall friction in feed line is negligibly small compared to the overall drop of the feeding system.

(b). The effect of the elasticity of the feeding line can be represented by an equivalent concentrated spring capacitance $C_\ell$ located at a distance $y \cdot \ell$

downstream of the pump delivery, where $\ell$ is the equivalent length of the feeding line. The spring constant $\chi$ of the capacitance is defined as the change in volume of the feed line produced by unit pressure rise over the entire feed line.

(c). The feed pump responds to oscillations of delivery pressure instantaneously and the fractional variation of mass delivery rate is directly proportional to the fractional variation of the delivery pressure with the proportionality constant D defined as

$$\frac{\Delta p}{p} = -D\,\frac{\Delta \dot{m}}{\dot{m}} \tag{2.8}$$

and D is assumed real and therefore can be evauated from the steady state performance curve of the pump.

(d). A control capacitance $C_c$ excited by a feed back circuit is introduced just upstream of the injector. The characteristics of the feed back circuit is described by the transfer function $F\left(\frac{d}{dz}\right)$ as

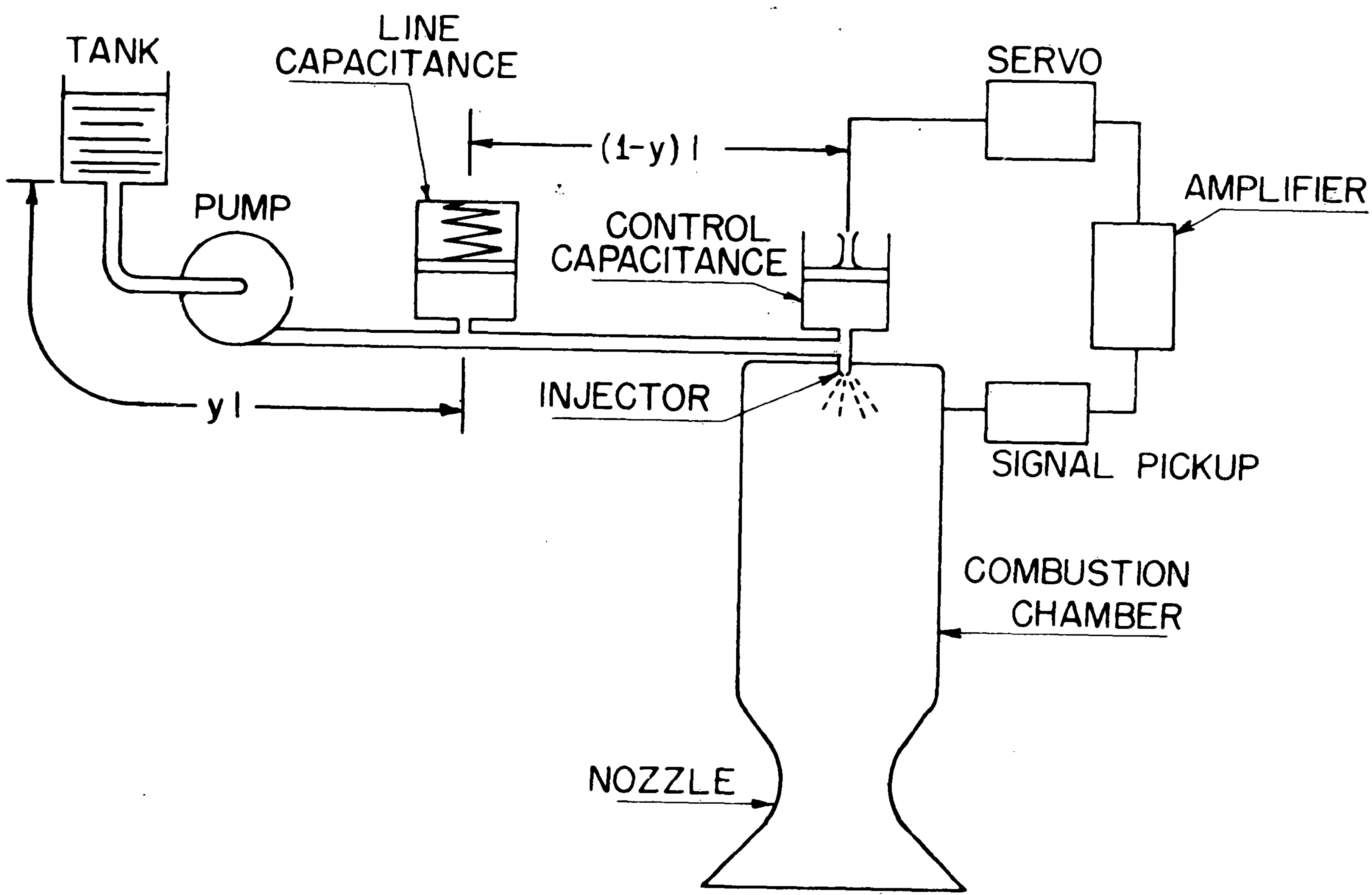
$$F\left(\frac{d}{dz}\right)\varphi = \frac{C_c}{\bar{\dot{m}}\,\theta_g} \tag{2.9}$$

A schematic diagram of the liquid rocket is given in figure 1. A straight forward process of elimination leads to the following dimensionless equation of the feeding system dynamics relating $\varphi$ and $\mu_i$ as

$$\Big\{P\Big[1+DE\left(P+\tfrac{1}{2}\right)\frac{d}{dz}+JEy\frac{d^2}{dz^2}\Big]+\Big[D\left(P+\tfrac{1}{2}\right)\frac{d}{dz}+J\frac{d^2}{dz^2}+DJE(1-y)\left(P+\tfrac{1}{2}\right)\frac{d^3}{dz^3}$$
$$+J^2Ey(1-y)\frac{d^4}{dz^4}\Big]F\left(\frac{d}{dz}\right)\Big\}\varphi$$

$$+\Big\{\Big[1+D\left(P+\tfrac{1}{2}\right)\Big]+\Big[DE\left(P+\tfrac{1}{2}\right)+J\Big]\frac{d}{dz}+\Big[DJE(1-y)\left(P+\tfrac{1}{2}\right)+JEy\Big]\frac{d^2}{dz^2} \tag{2.10}$$
$$+J^2Ey(1-y)\frac{d^3}{dz^3}\Big]\mu_i = 0.$$

where $P = \frac{\bar{p}}{2\Delta\bar{p}}$ is the pressure drop parameter, a relative measure of the pressure drop across the injector. $E = \frac{2\Delta\bar{p}\,\rho_0\,\chi}{\bar{\dot{m}}\,\theta_g}$ is the elasticity


TANK
LINE
CAPACITANCE
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(1-y) l
AMPLIFIER
PUMP
CONTROL
CAPACITANCE
INJECTOR
y l
SIGNAL PICKUP
COMBUSTION
CHAMBER
NOZZLE

parameter, a ratio of the rate of mass accumulation in the line capacitance due to a rate of pressure increase of $2\Delta p / \theta_g$ to the mean mass flow rate of the system. $J = \frac{\ell \bar{m}}{2 \Delta p \cdot A_t \cdot \theta_g}$ is the inertia parameter, a ratio of the time required to accelerate a given element from rest to the state of motion in the feed line under the pressure drop of $2\Delta \bar{p}$ as compared to the gas residence time $\theta_g$ .

Equation (2.10) essentially defines the ratio $\mu_i / \varphi$ in operational form which is the transfer function of the entire feeding system in response to the chamber pressure variation. Substituting $\mu_i$ from equation (2.10) into (2.7), we obtain a single hystero-ordinary differential equation with constant coefficients for the fractional pressure perturbation $\varphi$ as a function of the reduced time $z$. The characteristic equation for the investigation of the stability of the solutions of exponential type is obtained by replacing the operator $\frac{d}{dz}$ with the characteristic value s. It is obvious that the dynamics of the feeding system which creates the variation of the injection rate is responsible for most of the algebraic complications for the solution of the problem.

## III. TRANSFORMATION OF CHARACTERISTIC EQUATIONS WITH DIFFERENT ROCKET NOZZLES

It is interesting to note that the correction factor b for the phase lead component of the nozzle transfer function which represents the deviation of the nozzle flow from being quasi-steady can be absorbed by defining a new characteristic time $(1+b)\theta_g = \theta_g'$ instead of the average gas residence time $\theta_g$ as defined by Crocco. The dimensionless parameters which involve the characteristic time should of course be correspondingly modified. Thus

$$z' = t/\theta_g' = z/(1+b)$$
$$\delta' = \delta/\theta_g' = \delta/(1+b)$$
$$E' = 2\Delta\bar{p}\rho_0 \mathcal{K}/\bar{\dot{m}}\,\theta_g' = E/(1+b)$$
$$J' = \ell\,\bar{\dot{m}}/2\Delta\bar{p}\cdot A\cdot\theta_g' = J/(1+b)$$
$$F'\left(\frac{d}{dz'}\right) = C_c/\bar{\dot{m}}\,\theta_g' = F\left(\frac{d}{dz}\right)/(1+b)$$

and $$\omega' = \Omega\cdot\theta_g' = (1+b)\,\omega \qquad (3.1)$$

Those dimensionless parameters P, D, and n which do not involve characteristic time remain unchanged.

With this similarity transformation, equations (2.7) and (2.10) can be easily verified to be completely independent of b and are identical with the form of these equations with b = o corresponding to a "very short nozzle" where the quasi-steady nozzle flow assumption is approximately valid. For different deLaval nozzles the values of b will be different, but its effect can be accounted for simply by changing the characteristic time according to $\theta_g$ (1+b). The solution of the characteristic equation remains identical in terms of the dimensionless quantities defined in equations (3.1). Therefore the solutions and the stability boundaries given by previous authors for the case b = 0 in terms of these dimensionless quantities represent the similar or the universal solutions for rockets with given feeding system and combustion chamber but different deLaval nozzles.

The characteristic equation becomes,

$$(s'+1-n+ne^{-\delta' s'})\left\{[1+D(P+1/2)]+[DE(P+1/2)+J]s' + [DJ'E'(1-y)(P+1/2)+J'E'y]s'^2+J'^2E'y(1-y)s'^3\right\}$$

$$= P\left[1+DE'(P+1/2)s'+J'E'y\,s'^2\right]$$

$$+\left[D(P+1/2)+J's'+DJ'E'(1-y)(P+1/2)s'^2+J'^2E'y(1-y)s'^3\right]s'F'(s') \qquad (3.2)$$

From equation (3.2), we observe that the dimensionless quantities $E'$ and $J'$ occur only as $E'\omega'_*$ and $J'\omega'_*$ when $s' = i\,\omega'_*$ for the determination of the stability boundary. Equations (3.1) show that these products $E'\omega'$ and $J'\omega'$ are independent of b and equal to $E\omega$ and $J\omega$ respectively. The quantity $\omega'\delta'$ which represents $2\pi$ times the ratio of time lag to the period of oscillation is also independent b with $\omega'_*\ \delta'_* = \omega_*\ \delta_*$. Consequently, the effect of non-zero values of b enters only through $s' = i\omega'_*$ and $s'F'(s') = i\omega'_*F'(i\omega'_*)$ in equation (3.2). The critical value of $\delta'_*\ \omega'_*$ for neutral oscillations of frequency $\omega'_*$ is a function n, P, D, y, $E'\omega'_*$, $J'\omega'_*$ and the coefficients of $F'(s')$. If the feeding system of the rocket is not servo-controlled by feed back link, then $F' = F = 0$. The critical values of $\delta'_*\ \omega'_*$ and $\omega'_*$ are therefore functions of dimensionless quantities which are all independent of b. For such uncontrolled systems, the effect of the phase lead component $ib\omega$ can be directly obtained without determining the similar or the universal solution of the characteristic equation. Since $\omega'_*$ and $\delta'_*\ \omega'_*$ are constants for any values of b for a given uncontrolled system, and since

$$\begin{cases} \omega'_* = \Omega_* \cdot \theta_g\,(1+b) \\ \omega'_*\ \delta'_* = 2\pi\dfrac{\bar{\tau}_*}{T} = \Omega_*\cdot\bar{\tau}_* = \omega'_*\cdot\dfrac{\bar{\tau}_*}{\theta_g}\Big/(1+b) \end{cases}$$

we see that for increasing b or larger phase lead, the dimensional frequency $\Omega_*$

in cycles per second of neutral oscillation is decreased and the dimensional critical time lag $\bar{\tau}_*$ is increased. It can therefore be concluded that for rockets with feed system not servo-controlled the phase lead component of the nozzle transfer function tends to stabilize the system towards low frequency oscillations, and larger phase lead component results in larger stabilizing effect in increasing the critical time lag.

For rockets with feed system controlled by a given feed bac': servo-mechanism, the coefficients in the transfer function F(s) are different from those in F'(s'). The qualitative effect of the nozzle constant b cannot be drawn without further investigation.

## IV. BIPROPELLANT ROCKETS

In bipropellant rocket systems the dynamic behaviors of the feeding systems of the oxidizer and the fuel are in general different. The variations of the injection rates of oxidizer and of fuels in response to the same chamber pressure variation are different with a consequent variation in the oxidizer-fuel or mixture ratio $r = \dot{m}_o / \dot{m}_f$ . The adiabatic flame temperature of a given propellant combination depends to a certain extent on the mixture ratio. The stagnation temperature of the burned gas at a given position in the combustion chamber therefore varies with time as the chamber pressure oscillates and the stagnation temperature of the burned gas at different positions in the chamber at a given instant are also different. This variation of stagnation temperature of burned gas, which is absent in monopropellant rockets, must be taken into account in formulating the equation of mass balance in the combustion chamber for bipropellant rockets.

$$\frac{d}{d\bar{z}}\left(\frac{M_g}{\bar{M}_g}\right) + \mu_e = \mu_b \qquad (4.1)$$

By assuming that all combustion takes place practically at the injector end, and that all propellant elements have the same gas residence time $\Theta_g$ during which all these particles travel from the injector end to the combustion chamber exit and preserve their respective temperature at the instant when they were generated, Crocco (4) found the following expressions for the fractional burning rate perturbation $\mu_b$

$$\mu_b = \mu_i(\bar{z} - \delta) + n\left[\varphi(\bar{z}) - \varphi(\bar{z} - \delta)\right] \qquad (4.2)$$

with

$$\mu_i = (\tfrac{1}{2} + H)\mu_o + (\tfrac{1}{2} - H)\mu_f \qquad (4.3)$$

and $\psi = \frac{1}{2} \frac{\bar{r}-1}{\bar{r}+1}$

For the rate of mass accumulation,

$$\frac{d}{dz}\left(\frac{M_g(z)}{\bar{M}_g}\right) = \frac{d\varphi}{dz} - 2K\left[\mu_o(z-\delta-1) - \mu_f(z-\delta-1)\right]$$

$$+ 2K\left[\mu_o(z-\delta) - \mu_f(z-\delta)\right] \tag{4.4}$$

For the fractional variation of ejection rate evaluated under the assumption of quasi-steady nozzle flow,

$$\mu_e = \varphi(z) - K\left[\mu_o(z-\delta-1) - \mu_f(z-\delta-1)\right] \tag{4.5}$$

where $2K = \frac{\bar{r}}{\bar{T}_g} \frac{d\bar{T}_g}{d\bar{r}}$ represents the dimensionless slope of the adiabatic flame temperature curve for different mixture ratio.

Just like the monopropellant case, the fact that the flow of burned gas in the nozzle is not quasi-steady only modifies $\mu_e$, not the other terms. But owing to the variation of the stagnation temperature of the burned gas, there is also an entropy oscillation of the burned gas entering the nozzle. The specific admittance ratio $\nu_e/\delta_e$ is given by Crocco (9) as

$$\frac{\nu_e}{\delta_e} = \frac{\gamma-1}{2}\frac{\theta}{\theta-\sigma} + i\frac{\omega b}{\bar{u}_x}\left\{-\frac{\theta}{\theta-\sigma}\left[\frac{1}{2}\frac{\ln z_e}{1-z_e} + \frac{\gamma-1}{4}\right] + \frac{\sigma}{\theta-\sigma}\left[\frac{\gamma+1}{4\gamma}\frac{\ln z_e}{1-z_e} + \frac{\gamma-1}{4\gamma}\right]\right\} + \left(i\frac{\omega b}{\bar{u}_x}\right)^2 \cdots \tag{4.6}$$

for a deLaval nozzle with linear steady state velocity in the converging portion where $\theta$ and $\sigma$ are the amplitudes of the temperature and the entropy oscillations at the entrance of the nozzle respectively. In accordance with the assumption that the temperature oscillation of the burned gas in the combustion chamber is neglected when the oscillation is isentropic, $\gamma-1$ is neglected as compared to unity. Thus the first approximation of phase lead component of the admittance ratio becomes $-\frac{1}{2}\frac{\ln z_e}{1-z_e}\frac{\omega b}{\bar{u}_x}$ which is identical with the value given in equation (2.4) for monopropellant case. That the phase lead component is independent of the ratio $\sigma/\theta$ of the amplitude of entropy to temperature oscillation is correct only when $\gamma = 1$. Since $\gamma - 1$ for practical combustion gases are usually 0.2 or 0.3, the ratio of $\sigma/\theta$ may be of importance, and the magnitude of this phase lead component will depend on many thermodynamic properties of the combustion gases even for an idealized nozzle with linear steady state velocity distribution.

The transfer function for the mass flow through the nozzle is, for the bipropellant case,

$$\frac{\mu_e}{\varphi_e} = N_n = (1+\alpha_n)\left(1 - \frac{\sigma_e}{\varphi_e}\right)$$

Thus in accordance with the assumption of $\gamma \cong 1$, we have

$$\mu_e = \left(1 + b\frac{d}{dz}\right)(\varphi_e - \sigma_e) \tag{4.8}$$

where $b = -\frac{1}{2}\frac{\ln z_e}{1-z_e}\frac{1}{\sigma_g \bar{u}_x}$ which is the same for monopropellant case. The expression for $\mu_e = \varphi_e - \sigma_e$, when $b = 0$ is given by equation (4.5). If the steady state gas velocity in the nozzle is not linear, we would of course expect a slight change in the expression and the value of b. The equation of mass balance becomes

$$\begin{aligned}(1+b)\frac{d\varphi}{dz} + (1-n)\varphi + n\varphi(z-\delta) \\ = (-2K+\tfrac{1}{2}+H)\mu_o(z-\delta) - (-2K-\tfrac{1}{2}+H)\mu_f(z-\delta) \\ + K\left(3+b\frac{d}{dz}\right)[\mu_o(z-\delta-1) - \mu_f(z-\delta-1)]\end{aligned} \tag{4.9}^*$$

It is no longer possible to reduce equation (4.9) to the form with b = 0 by changing the characteristic time as was done in previous sections. The effect of a phase lead component of the nozzle transfer function on the stability of a bipropellant motor cannot be determined in a simple manner without further

* The author is indebted to Prof. L. Crocco for calling his attention to the sign mistake in equation (10.1) in reference 4 which is used in deducing equation (4.9).

approximation. Observing the fact that the dimensionless slope K of the adiabatic temperature curve is usually a rather small fractional quantity except when the liquid rocket is designed to operate at very lean or very rich mixture ratio. Thus, as a rough approximation, the term

$$b\,\frac{d\theta_e}{dz} = K\,b\,\frac{d}{dz}\left[\mu_o\,(z-\delta-1) - \mu_f\,(z-\delta-1)\right]$$

may be neglected as compared to other terms in equation (4.9) without affecting the qualitative trend of the effect of the phase lead component of the nozzle transfer function. After dropping this term from equation (4.9), it can be seen that the new definition of the characteristic time $\theta_g$ (1+b) will reduce the equation of mass balance to the form by putting b = 0. The two equations of the dynamics of the fuel and the oxidizer feeding system, each of them of the form of equation (2.10), can also be reduced with the new dimensionless parameters as defined in equation (3.1).

With this additional approximation of neglecting $b\,\frac{d\theta_e}{dz}$, the qualitative conclusion with regard to the increasing stabilizing effect of larger phase lead component of the nozzle transfer function, as deduced in previous sections for the monopropellant case, holds good for the bipropellant case as well provided that the feeding systems are not servo-controlled. The stabilizing effect manifests itself as the increase of the critical time lag for neutral oscillation in the liquid rockets. For the bipropellant case, however, the factor b of the phase lead component will in general not only be a function of the geometry of the nozzle and the specific heat ratio $\gamma$ of the combustion gases as for the monopropellant rocket, but may also depend on the magnitudes of K and $\sigma/\theta$. The entropy variation of the burned gas due to the variation of the stagnation temperature can no longer be neglected as was in the monopropellant case where the entropy variation is essentially due to different dissipative agents. Furthermore the neglect of the term $b\,\frac{d\theta_e}{dz}$ which involves all the feeding system parameters, may result in a small dependence of the apparent value of b on feeding system constants.

## V. PHYSICAL ASPECTS AND DISCUSSION OF RESULTS

The fact that the existence of a phase lead component of the nozzle transfer function is in effect an increase of the gas residence time need some clarification. If we examine the analyses (8) and (9) for the nozzle admittance ratio, we see that the phase lead component arises because of the inertia of the gas. The inertia of the gas in the subsonic portion of the nozzle leads to an accumulation of burned gas when the pressure in the chamber is increased in addition to the accumulation of gas in the combustion chamber. Therefore this constitutes an increase of the effective capacity of the chamber in storaging the gas and a corresponding increase of the characteristic time of the system. In the dimensionless formulation of Crocco (4) which is followed in the present treatment, the characteristic time is the gas residence time $\theta_g$ defined as the ratio of the mass $\bar{M}_g$ of the burned gas in the combustion chamber and the rate $\bar{m}$ of flow of gas out of the chamber in steady state operation,

$$\theta_g = \bar{M}_g / \bar{m} \sim \bar{V}_c / (A_t \cdot c_*)$$

where $\bar{V}_c$ is the volume of the combustion chamber. $A_t$ is the throat area of the nozzle and c* is the characteristic exhaust velocity. With $A_t$ and c* of a given rocket fixed, an increase in the effective volume of the combustion chamber corresponds to an increase in the gas residence time $\theta_g$. It is therefore natural, though it may not be quite right, to think that the increase of the characteristic time due to the phase lead component simply means that the volume $\bar{V}_n$ of the subsonic portion of the nozzle should be added to the volume $\bar{V}_c$ of the combustion chamber in determining the characteristic time. It should, however, be noted that the pressure and temperature level of the gas in the nozzle is decreasing continuously toward the throat and while the chamber pressure varies, the pressure in the nozzle also varies but in a different manner. Therefore the capability of the volume $\bar{V}_n$ of the subsonic portion of the nozzle in accumulating burned gas in response to an increase in the chamber pressure is different from the capability of an equal volume $\bar{V}_n$ in the combustion chamber. Knowing the nozzle constant b of the phase lead component, the coefficient $\mathcal{E}$ of the effectiveness of the nozzle volume as compared to chamber volume can be determined from

$$\theta_g' = \theta_g \left( \frac{\bar{V}_c + \bar{V}_n \, \mathcal{E}}{\bar{V}_c} \right) = \theta_g \, (1 + b)$$

as

$$\mathcal{E} = \frac{b \, \bar{V}_c}{\bar{V}_n} = \frac{b \, \theta_g \cdot \bar{m}}{\bar{V}_n \cdot \rho_c} \qquad (5.1)$$

For the case of a nozzle with linear steady state velocity distribution in the subsonic portion and for isentropic oscillations, the coefficient $\mathcal{E}$ is obtained from equations (2.4), (2.5) and (5.1) as

$$\mathcal{E} = \left( - \frac{\ln z_e}{1 - z_e} - \frac{\gamma - 1}{2} \right) \Big/ \int_{z_e}^{1} \left( 1 - \frac{\gamma - 1}{\gamma + 1} z \right)^{\frac{-1}{\gamma - 1}} z^{-1} \, dz$$

$$\cong \left[ - \frac{\ln z_e}{1 - z_e} - \frac{\gamma - 1}{2} \right] \Big/ \left[ - \ln z_e + \frac{1}{\gamma + 1} \left\{ (1 - z_e) + \frac{\gamma}{\gamma + 1} \frac{1 - z_e^2}{2 \cdot 2!} \right. \right.$$

$$\left. \left. + \frac{\gamma (2\gamma - 1)}{(\gamma + 1)^2} \frac{1 - z_e^3}{3 \cdot 3!} + \frac{\gamma (2\gamma - 1)(3\gamma - 2)}{(\gamma + 1)^3} \frac{1 - z_e^4}{4 \cdot 4!} + \cdots \right\} \right] \qquad (5.2)$$

which is a function of contraction ratio $\frac{A^*}{A_e}$ of the nozzle alone. Sample calculations for $\gamma = 1.20$ with $z_e = 0.20$, 0.10, and 0.05 corresponding to Mach number $M_e \cong 0.4$, 0.3 and 0.2 respectively at the entrance section, the coefficient $\mathcal{E}$ is equal to 0.93, 0.89, and 0.87 for each of the three cases. The variation of $\mathcal{E}$ for such a wide range of entrance Mach number variation is rather insignificant and the magnitude of $\mathcal{E}$ can be approximately taken as 0.9.

Crocco* has shown that the admittance ratio of a nozzle with velocity substantially linear upstream of the sonic throat but not linear in the neighborhood of the entrance, can best be approximated by the ratio of the nozzle where the

* Unpublished work of Professor L. Crocco. An outline of this work will be included in the appendices of the volume on "Combustion Instability in Liquid Propellant Rocket Motors" of the Monograph series of AGARD, NATO.

velocity distribution is linear with velocity gradient equal to that at the sonic throat of the given nozzle. Under such circumstances, the magnitude of $\mathcal{E}$ would be expected to be approximately 0.9. However, for nozzles whose velocity distribution is far from being linear, we don't have any idea as to the relation of b with the volume $\bar{V}_n$ , and the value of $\mathcal{E}$ may be very much different from 0.9. It cannot be over-emphasized here to point out that while the concept of increasing available chamber volume leads to a simple and convenient interpretation of the stabilizing effect of the nozzle through the increase of characteristic time; this concept, however, is not established on sound basis but simply a conjecture even for low frequency oscillations. The stabilizing effect of the nozzle toward high frequency oscillation is known to be based on an entirely different mechanism which has little to do with the volume; and the stabilizing effect of the nozzle toward low frequency oscillations is not clearly connected with the volume of the converging portion of the nozzle. The complicated state of affairs is simply superficially avoided by using the coefficient $\mathcal{E}$ which cannot be determined without the precise knowledge of the nozzle constant b of the phase lead component of the nozzle transfer function.

Having concluded the stabilizing effect of the factor b that determines the phase lead component of the nozzle transfer function or the deviation of the flow in the nozzle from being quasi-steady, we can deduce several qualitative conclusions concerning the effect of nozzle geometry on the low frequency combustion stability of liquid rockets. Analytical expression for b is known only for the special nozzle with linear steady state velocity distribution in the subsonic portion of the nozzle. It is believed, however, that the change of the velocity distribution near the entrance is not going to change radically the qualitative dependence of the magnitude of b on the two parameters $\bar{z}_e$ and $\bar{u}_x$ . $\bar{z}_e$ is determined by the Mach number of the gas entering the nozzle and is therefore a parameter of the contraction ratio $\frac{A_*}{A_e}$ , the ratio of the throat area to the area of entrance section of the nozzle. $\bar{u}_x$ is defined only when there is a predominating slope in the profile of the velocity distribution. For a given entrance Mach number and a given type of velocity distribution, $\bar{u}_x$ is essentially a parameter representing the "effective length" of the converging section of the nozzle. Since b is increased by decreasing $z_e$ and $\bar{u}_x$ , it is increased by decreasing the contraction ratio $\frac{A_*}{A_e}$ or by increasing the length of the converging section of the nozzle. Consequently, we have:

1. Low frequency stability of a liquid rocket is improved by decreasing the contraction ratio of a nozzle while keeping the length of the converging section constant and the profiles of velocity of similar shape.

2. Low frequency stability of a liquid rocket is improved by increasing the length of the converging section of the nozzle while keeping the contraction ratio of the converging section unchanged and the profiles of velocity of similar shape.

The stabilizing effect manifests itself analytically as an increase of the critical time lag for neutral oscillation and can be observed experimentally by the decrease of the intensity of chugging and also by the decrease of the frequency of marginally unstable oscillations in the liquid rockets.

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