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Technical Memorandum M-090

AN ANALYSIS OF A SYSTEM FOR THE DETERMINATION OF MICROWAVE REFLECTION COEFFICIENTS

30 April 1954

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Technical Memorandum M-090

AN ANALYSIS OF A SYSTEM FOR THE DETERMINATION OF MICROWAVE REFLECTION COEFFICIENTS

30 April 1954

W.L. Starr, R.D. Hitchcock, D.B. Wright, and A.W. Gosley

SUMMARY

A system for determining microwave reflection coefficients by multiple reflections is described and theory presented. This theory yields an expression giving electric field intensity, E, as a function of distance, x, and reflection coefficient, r. In order to attain the desired agreement between data and theory, attenuation and floor reflection had to be considered. Computation of the reflection coefficient of one material is given as an example of the application of this theory.

CONTENTS

INTRODUCTION

The development of a simple method for determining the reflection coefficients of various types of construction materials was undertaken by this Laboratory as one phase of Project Grubless (NY 340 004-1). The system developed by the Laboratory is described in NAVCERELAB Technical Note N-127, " Determining Reflection Coefficients by Multiple Reflection."

This report presents the theory on which the measurements are based and one example of the calculation of reflectance from the data and theory. In the previous report it was stated that the reflectance of a material might be computed simply by measuring the standing wave ratio. This statement was made with the assumption that floor reflections and attenuation would cause no appreciable modulation of the envelope of the measured field intensity. Study of the data revealed, however, that it was necessary to develop a rigorous mathematical theory for the computation of accurate reflection coefficients - that is, a theory which would not disregard attenuation and floor reflections.

EXPERIMENTAL METHOD

For the sake of easy reference the experimental set-up and procedure will be redescribed and refinements in the previously reported system noted.

The technique used to reduce the effect of spurious signals from the surroundings and to increase the magnitude of the signals measured makes use of a system having two large parallel reflecting panels and a horn antenna which radiates from the center of one toward the center of the other. These panels are space-adjusted so as to be an integral number of half-wavelengths apart. This insures that all of the waves traveling in the same direction between the panels will be in phase. By using two panels a signal of increased magnitude without substantial change in the voltage standing wave ratio is obtained. Figure 1 shows the basic test arrangement and is useful in pointing out some of the refinements in the test apparatus.

The tests were conducted in the large auditorium and with the same basic equipment as previously reported. A 1000-mc signal frequency was used throughout the experiments. Additional primary power regulation was used to compensate for the large fluctuations in line voltage. The equipment used to transport the pick-up antenna in a direction perpendicular to the plane of the test panel was modified so as to reduce the effect of the mast and cart on the signal. The former wooden mast was replaced by a mast constructed from four fiber-glass rods having one-half inch square cross section. These rods were held by means of polystyrene spacers to form a rigid tower-type mast. This reduced the distortion of the field in the vicinity of the pick-up antenna. The cart and guidance track were modified to provide better control over the direction and velocity of the antenna. These modifications were of minor structural nature but were primarily for the purpose of reducing friction losses in the antenna transporting system.

Numerous distances, from six wavelengths to 60 wavelengths, were used during these tests. The adjustment of the spacing between reflectors to integral half-wavelengths was very critical, especially on materials having a high reflection coefficient. Spacings from six to twelve wavelengths between reflectors resulted in data which was least affected by interference from spurious reflections. Operating at this distance caused attenuation of the transmitted wave to be an important factor to be considered since measurements were being made in the border regions of the near field.

The increased accuracy with which data could be transcribed by using the projection process described later made it possible to isolate the process of adjusting the panels to multiple half wavelength spacings as the primary source of experimental error in so far as obtaining repetitive data was concerned. Other possible sources of experimental error were distortion of the field by the pick-up antenna and irregularities in the test and secondary reflecting panels. Unless the floor reflectance and the degree of attenuation is known, these may also be possible sources of error. Though some of these experimental difficulties were encountered, the tests were adequate to show the feasibility of such a system and to indicate the additional refinement necessary for more reliable application of this technique at the test frequency and for extending the frequency range to more realistic radar detection frequencies.

DATA AND DATA HANDLING

The data obtained was in the form of recorded profiles of the electric field intensity in the direction of propagation and on a centerline with the transmitting antenna. The special transparent scales referred to in the previous report were used to transcribe the raw data into tabular form showing the magnitudes of successive maxima and minima. A refinement in data handling was employed which consisted of using an opaque projector for magnifying both the scale and the data by a factor of five, thus permitting more accurate interpolation between successive scale graduations. This was particularly important on the higher amplitude signals since the data was recorded on a logarithmic scale.

No test data, except that necessary to the explanation and theoretical analysis of the system, is included in this report, since the primary purpose of this phase of the project to date has been the development of a method of determining reflection coefficients of construction materials.

THEORY

Part A

One of the simplest techniques for measuring reflectance factors is to use two parallel reflecting surfaces, one of which is the material whose reflectance is to be determined. The antenna radiates plane waves from the center of one panel toward the center of the other. The distance between the panels is adjusted to an integral number of half wavelengths. The radiation is polarized perpendicular to the plane of incidence, i.e., perpendicular to the plane of the paper as seen in Figure 1. It will be shown below that a simple mathematical expression can describe the physical factors involved. This expression will of necessity involve the reflectance factors of both panels, thus by measuring the electric field intensity in the proper manner the reflectance factor of the test panel can be determined.

The electric field intensity in the space between the panels will be the summed contribution of an infinite number of waves due to an infinite number of reflections from the panels.

Let

x	=	distance from horn panel
c	=	velocity of electromagnetic radiation
r	=	reflection coefficient of test panel
ρ	=	reflection coefficient of secondary panel
ν	=	frequency of radiation ($\omega = 2\pi\nu$)
λ	=	wavelength of radiation
$\frac{n\lambda}{2}$	=	distance between panels, where n is an integer
y_1	=	instantaneous amplitude (electric field intensity or electric vector) of wave emitted by the horn and incident at x before suffering any reflection
y_2	=	electric vector of the wave incident at x after reflection from test panel
y_3	=	electric vector of wave incident at x after two reflections, one from each panel
y_4	=	etc.
E_0	=	maximum amplitude of emitted wave
t	=	time in seconds

The resultant electric field at x will then be,

$$y = y_1 + y_2 + y_3 + \dots \quad (1)$$

$$= \sum_{n=1}^{\infty} y_n$$

It is to be noted that atmospheric absorption is considered to be zero. Assuming a 180-deg phase shift upon reflection from both panels, then we have

$$y_1 = E_0 \sin \omega (t - \frac{x}{c}) \quad (2)$$

$$y_2 = -rE_o \sin \omega [t - (\frac{n\lambda - x}{c})]$$

$$= -rE_o \sin \omega [(t + \frac{x}{c}) - \frac{n\lambda}{c}] \quad (3)$$

$$= -rE_o [\sin \omega (t + \frac{x}{c}) \cos 2n\pi - \cos \omega (t + \frac{x}{c}) \sin 2n\pi]$$

$$= -rE_o \sin \omega (t + \frac{x}{c})$$

$$y_3 = \rho r E_0 \sin \omega [t - (\frac{n\lambda + x}{c})]$$

$$= \rho r E_0 \sin \omega [(t - \frac{x}{c}) - \frac{n\lambda}{c}]$$

$$= \rho r E_0 \sin \omega (t - \frac{x}{c}) \quad (4)$$

$$= \rho r y_1$$

$$y_4 = -\rho r^2 E_0 \sin \omega [t - (\frac{2n\lambda - x}{c})]$$

$$= -\rho r^2 E_0 \sin \omega (t + \frac{x}{c}) \quad (5)$$

$$= \rho r y_2$$

Similarly,

$$y_5 = \rho^2 r^2 y_1 \quad (6)$$

$$y_6 = \text{etc.,} \quad (7)$$

Thus

$$y = E_0 \sin \omega \ (t - \tfrac{x}{c})(1 + \rho r + \rho^2 r^2 + \ldots)$$
$$- rE_0 \sin \omega \ (t + \tfrac{x}{c})(1 + \rho r + \rho^2 r^2 + \ldots)$$

and

$$1 + \rho r + \rho^2 r^2 + \ldots = (1 - \rho r)^{-1}$$

thus

$$y = (1 - \rho r)^{-1} \ E_0 \left[\sin \omega \ (t - \tfrac{x}{c}) - r \sin \omega \ (t + \tfrac{x}{c})\right]$$

or

$$y = A \sin \omega \ (t - \tfrac{x}{c}) - rA \sin \omega \ (t + \tfrac{x}{c}) \qquad (8)$$

where

$$A = (1 - \rho r)^{-1} \ E_0$$

As can be seen from Equation 8, the infinite number of reflections can be considered to be the same as two waves, one traveling in the x direction with amplitude A and one traveling in the $-x$ direction with amplitude rA and differing in phase from the first wave by 180 deg. If the amplitudes are equal $(r = 1)$, the result of the two waves is the usual standing wave. With $r < 1$, the result is an apparent standing wave. In the apparent standing wave there are no real nodes since there are no values of x for which the amplitude of the wave is zero for all time.

Expanding Equation 8, we have

$$y = A \ (\sin \omega t \cos \tfrac{\omega x}{c} - \cos \omega t \sin \tfrac{\omega x}{c} - r \sin \omega t \cos \tfrac{\omega x}{c}$$
$$- r \cos \omega t \sin \tfrac{\omega x}{c})$$

$$= \sin \omega t \left[A(1 - r) \cos \tfrac{\omega x}{c}\right] - \cos \omega t \left[A(1 + r) \sin \tfrac{\omega x}{c}\right]$$

or $y = H \sin \omega t + J \cos \omega t$ (9)

where

$$H = A(1 - r) \cos \frac{\omega x}{c}$$

and

$$J = -A(1 + r) \sin \frac{\omega x}{c}$$

Since

$$k \sin(\omega t + \alpha) = k \sin \omega t \cos \alpha + k \cos \omega t \sin \alpha$$

Equation 9 may be written as

$$y = k \sin(\omega t + \alpha) \qquad (10)$$

where

$$H = k \cos \alpha \quad \text{and} \quad J = k \sin \alpha$$

and

$$H^2 + J^2 = k^2$$

For any x, y will have a maximum value when $\sin(\omega t + \alpha) = 1$. The absolute value of y for $\sin(\omega t + \alpha) = 1$ gives the upper curve of the apparent standing wave, i. e., the upper envelope of y.

Denoting this envelope by y_m, then we have

$$y_m = \left| (H^2 + J^2)^{1/2} \right|$$

or

$$y_m = \left| \left[A^2 (1 - r)^2 \cos^2 \frac{\omega x}{c} + A^2 (1 + r)^2 \sin^2 \frac{\omega x}{c} \right]^{1/2} \right|$$

Rewriting, we get

$$y_m = \left| A \left[1 + r^2 + 2r(\sin^2 \frac{\omega x}{c} - \cos^2 \frac{\omega x}{c}) \right]^{1/2} \right|$$

It can be readily seen that the expression in parentheses has a maximum value of 1 and a minimum of -1 . Thus the maximum of y_m is

$$y_m' = |A(1 + 2r + r^2)^{1/2}|$$
$$= A(1 + r)$$

and the minimum value of y_m is

$$y_m'' = |A(1 - 2r + r^2)^{1/2}|$$
$$= A(1 - r)$$

If a receiver is placed in the region between the panels and moved until a maxima and a minima are noted, these two signals will be y_m' and y_m''. This ratio is the well known voltage standing wave ratio (VSWR)

Let the ratio

$$y_m'/y_m'' = N$$

then

$$N = A(1 + r)/A(1 - r)$$

or

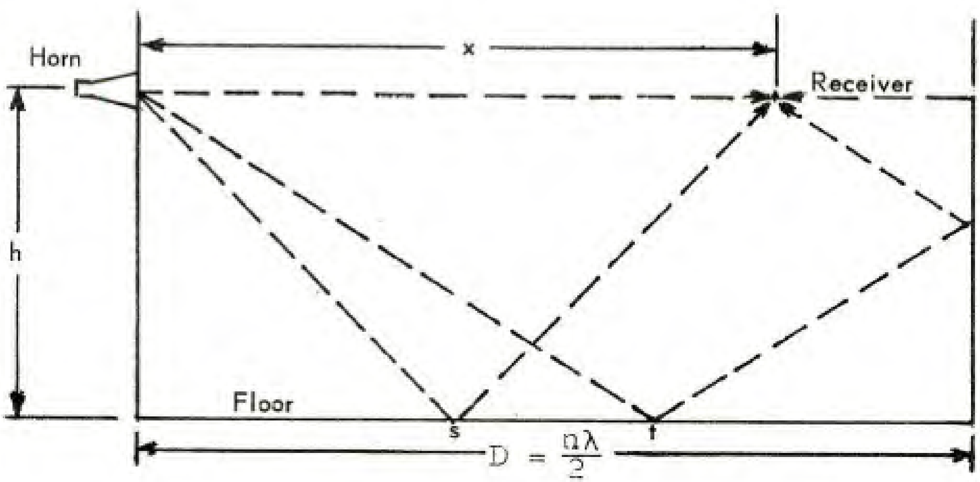
$$r = (N - 1)/(N + 1) \qquad (11)$$

This is the desired relationship from which the reflectance may be computed.

Part B

However, in the attempt to devise an apparatus which could easily make in situ measurements, these simple conditions were not realized. Specifically, the radiations emitted by the horn were not plane waves, thus lobe factors and attenuation resulting from the non-plane wave fronts had to be considered. In addition to the above, further attenuation could be expected by atmospheric absorption. Also, since the apparatus was close to a floor, two floor reflections were possible. (This assumes that reflections from the floor and test panel would be essentially specular). Finally, the apparent reflecting surfaces of many materials, e.g., the irregular surface of certain microwave absorbing materials, cannot be accurately determined and thus the separation between the panels cannot be readily set to an integer multiple of $\lambda/2$. This was partially overcome by setting the panels to an approximate spacing of $n\lambda/2$, moving the receiver to a maxima point, and then moving the panel in either direction so that a largest maximum signal is obtained. In the theoretical expression derived below, the separation distance is taken as the initial measured separation plus (or minus as the case may be) that distance which will give a separation of $n\lambda/2$.

Thus in the experimental arrangement employed the electric field at any point between the panels will be the summed contribution from the infinite number of waves reflected between the panels, the reflection from the floor and directly incident at the point, and the reflection from the floor incident on the test panel and then reflected again to the point. The geometry of the problem is shown below.



Let

E_o = amplitude of wave emitted in the direction normal to the horn panel and incident at $x = 1$ cm

R = reflection coefficient of floor

D = distance between panels

k_s = lobe factor in the direction s i.e., the ratio of the amplitude of the electric field in the direction s to the amplitude in the direction normal to the horn panel

k_t = lobe factor in the direction t

From examination of data concerning antenna radiation characteristics, the attenuation factor was found to be approximately $1/x$.

Using the same notation as previously, we have

$$y_1 = E_1 \sin\omega\left(t - \frac{x}{c}\right) \text{ and } \frac{E_1}{E_o} = \frac{1}{x}$$

$$\therefore\ y_1 = \frac{E_o}{x} \sin\omega\left(t - \frac{x}{c}\right)$$

$$y_2 = -rE_2 \sin\omega\left(t + \frac{x}{c}\right) \text{ and } \frac{E_2}{E_o} = \frac{1}{2D - x}$$

$$\therefore\ y_2 = \frac{-rE_o}{2D - x} \sin\omega\left(t + \frac{x}{c}\right)$$

$$y_3 = \frac{\rho r E_o}{2D + x} \sin\omega\left(t - \frac{x}{c}\right)$$

$$y_4 = \frac{-\rho r^2 E_o}{4D - x} \sin\omega\left(t + \frac{x}{c}\right)$$

$$y_5 = \frac{\rho^2 r^2 E_o}{4D + x} \sin\omega\left(t - \frac{x}{c}\right)$$

$$y_6 = \frac{-\rho^2 r^3 E_o}{6D - x} \sin\omega\left(t + \frac{x}{c}\right), \text{ etc.}$$

Thus the electric field resulting from the infinite number of reflections is

$$y_a = \left[E_o \sin\omega\left(t - \frac{x}{c}\right)\right]\left(\frac{1}{x} + \frac{\rho r}{2D + x} + \frac{\rho^2 r^2}{4D + x} + \ldots\right)$$

$$-\left[rE_o \sin\omega\left(t + \frac{x}{c}\right)\right]\left(\frac{1}{2D - x} + \frac{\rho r}{4D - x} + \frac{\rho^2 r^2}{6D - x} + \ldots\right)$$

The infinite series could not be expressed in closed form as previously; however, since $\rho \cong 1$ (copper mesh) and $r < 1$, terms of power greater than two can be neglected. Thus

$$y_a = \left(\frac{1}{x} + \frac{r}{2D + x} + \frac{r^2}{4D + x}\right) E_o \sin\omega\left(t - \frac{x}{c}\right)$$

$$- \left(\frac{1}{2D - x} + \frac{r}{4D - x} + \frac{r^2}{6D - x}\right) rE_o \sin\omega\left(t + \frac{x}{c}\right) \quad (12)$$

The wave emitted in the direction s after traveling a distance d is

$$y_s = -k_s RE_s \sin\omega\left(t - \frac{d}{c}\right) \quad \text{and} \quad \frac{E_s}{E_o} = \frac{1}{d}$$

The total distance the wave travels from the horn to the receiver is $(4h^2 + x^2)^{1/2}$, thus

$$y_s = \frac{-k_s RE_o}{(4h^2 + x^2)^{1/2}} \sin\omega\left[t - \frac{(4h^2 + x^2)^{1/2}}{c}\right] \quad (13)$$

Similarly, the wave emitted in the direction t and incident at the receiver travels a total distance of $[4h^2 + (2D - x)^2]^{1/2}$ but suffers two 180-deg phase change reflections, thus

$$y_t = \frac{k_t rRE_o}{[4h^2 + (2D - x)^2]^{1/2}} \sin\omega\left\{t - \frac{[4h^2 + (2D - x)^2]^{1/2}}{c}\right\} \quad (14)$$

The total field at the receiver is then the sum of Equations 12, 13, and 14, that is,

$$y = y_a + y_s + y_t \quad (15)$$

For convenience, let

$$S = E_o \left(\frac{1}{x} + \frac{r}{2D + x} + \frac{r^2}{4D + x}\right)$$

$$T = rE_o \left(\frac{1}{2D - x} + \frac{r}{4D - x} + \frac{r^2}{6D - x}\right)$$

$$B = \frac{k_s RE_o}{(4h^2 + x^2)^{1/2}} \; ; \quad C = \frac{k_t rRE_o}{\left[4h^2 + (2D - x)^2\right]^{1/2}}$$

$$a = \frac{x}{c}; \quad b = \frac{(4h^2 + x^2)^{1/2}}{c}; \quad d = \frac{\left[4h^2 + (2D-x)^2\right]^{1/2}}{c}$$

Using the above substitutions, Equation 15 then becomes

$$y = S \sin \omega\,(t - a) - T \sin \omega\,(t + a)$$
$$- B \sin \omega\,(t - b) + C \sin \omega\,(t - d) \qquad (16)$$

Expanding Equation 16 and rewriting, we have

$$y = \left[(S - T) \cos \omega a - B \cos \omega b + C \cos \omega d\right] \sin \omega t$$
$$- \left[(S + T) \sin \omega a - B \sin \omega b + C \sin \omega d\right] \cos \omega t$$

or

$$y = F \sin \omega t + G \cos \omega t \qquad (17)$$

where

$$F = \left[(S - T) \cos \omega a - B \cos \omega b + C \cos \omega d\right]$$
$$G = -\left[(S + T) \sin \omega a - B \sin \omega b + C \sin \omega d\right]$$

As shown previously, Equation 17 may be written as

$$y = P \sin(\omega t + \phi) \qquad (18)$$

where

$$F = P \cos\phi \qquad G = P \sin\phi$$

and

$$P^2 = F^2 + G^2$$

And as before the envelope of the curve is found by letting $\sin(\omega t + \phi) = 1$. Thus

$$y_m = \left| (F^2 + G^2)^{1/2} \right| \qquad (19)$$

Equation 19 is the peak value of the electric field read at any distance x. Again the value of the field at two points (a maximum and a minimum is convenient) is measured and Equation 19 is solved for r. Figure 3 is a plot of Equation 19. Figures 2A through 2D show similar plots of different sets of data. So that the agreement between theory and experiment may be readily seen, Equation 19 as plotted in Figure 3 has been normalized so that the first minima and maxima coincide with the corresponding points of the data expressed in Figure 2D.

To illustrate; from antenna radiation patterns and other data the following constants were determined to be:

$$k_s = k_t = 0.3$$

$$R = 0.2$$

$$D = 285.0 \text{ cm}$$

$$h = 223.52 \text{ cm}$$

$$\lambda = 30.0 \text{ cm}$$

Using a microwave absorbing material as the test panel the ratio of the electric field at the two points, $x = 277.5$ and $x = 270.0$, was measured as $19.2/11.0$.
Thus at $x = 277.5$

$$F_1^{\,2} + G_1^{\,2} = (1.280 + 3.215\, r + 3.353\, r^2)\, E_0 \cdot 10^{-5}$$

And at $x = 270.0$

$$F_2^{\,2} + G_2^{\,2} = (1.444 - 1.548\, r + 0.081\, r^2)\, E_0 \cdot 10^{-5}$$

Dividing $F_1^{\,2} + G_1^{\,2}$ by $F_2^{\,2} + G_2^{\,2}$ and equating to the square of the measured ratio, $19.2/11.0$, we get

$$r^2 + 2.553\, r - 1.004 = 0$$

$$\therefore r = 0.347$$

$$r^2 = 0.120$$

These are the amplitude and power reflectances, respectively, of the microwave absorbing material at the frequency under consideration.

CONCLUSIONS

Since in most cases, a small area of a few square inches is not representative of a sample, so far as microwave reflectances as applied to countermeasures is concerned, then the determination of reflectances of small samples is not valid. This eliminates the possibility of employing some of the simpler methods utilizing wave guides. Also since background effects cannot be tolerated if accurate reflectances are to be determined, pulse techniques are impossible since the distance between the receiver and the sample would have to be so great as to allow background influence. The method described in this report does not suffer from the above limitations.

However, as pointed out previously the arrangement employed in this experiment deviates considerably from that of the ideal experiment. Specifically, floor reflection and attenuation were present. Even though these deviations can be considered in the theory, an exact knowledge of their values is not always possible. It is of course possible to use a set of simultaneous equations and solve for all constants. However this would be extremely laborious and is not to be considered desirable.

It is thus recommended that for future measurements of reflectance, an attempt should be made to modify the apparatus so as to eliminate floor reflections and attenuation. In this way very rapid determinations of reflectances would be possible.

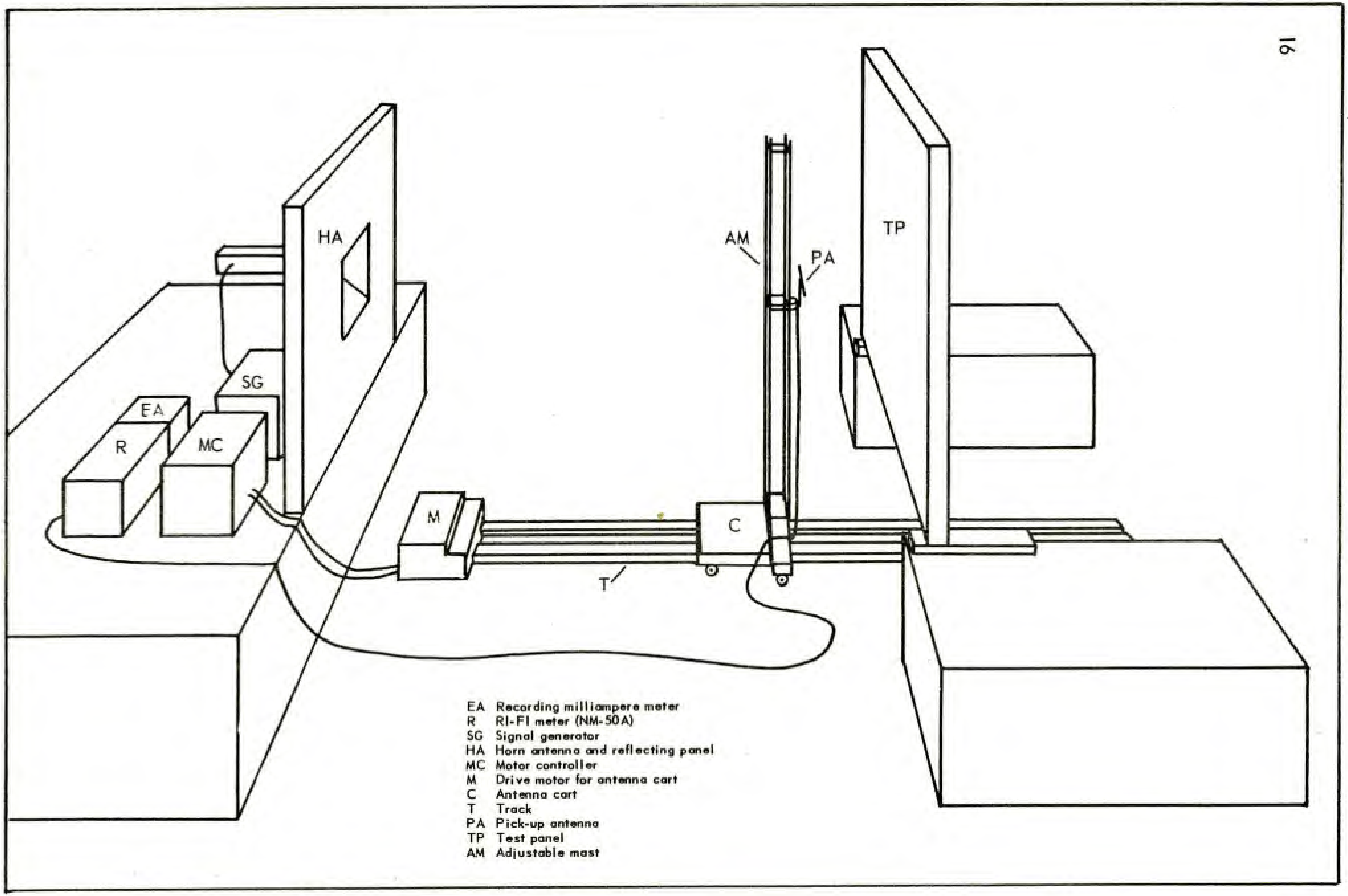
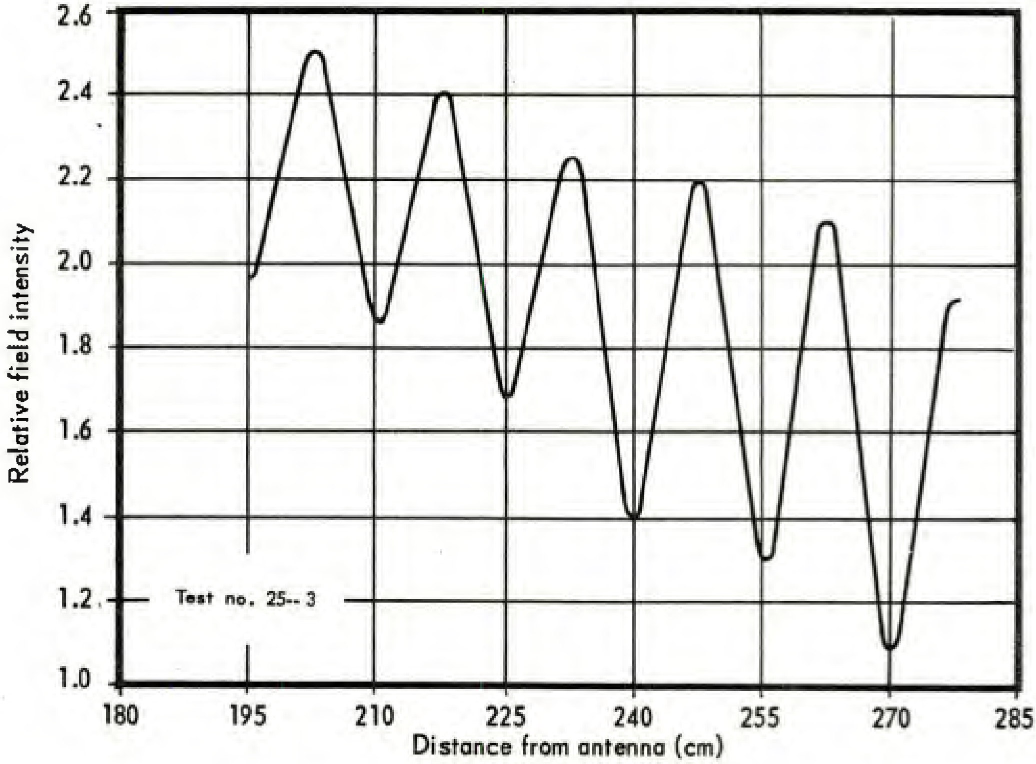
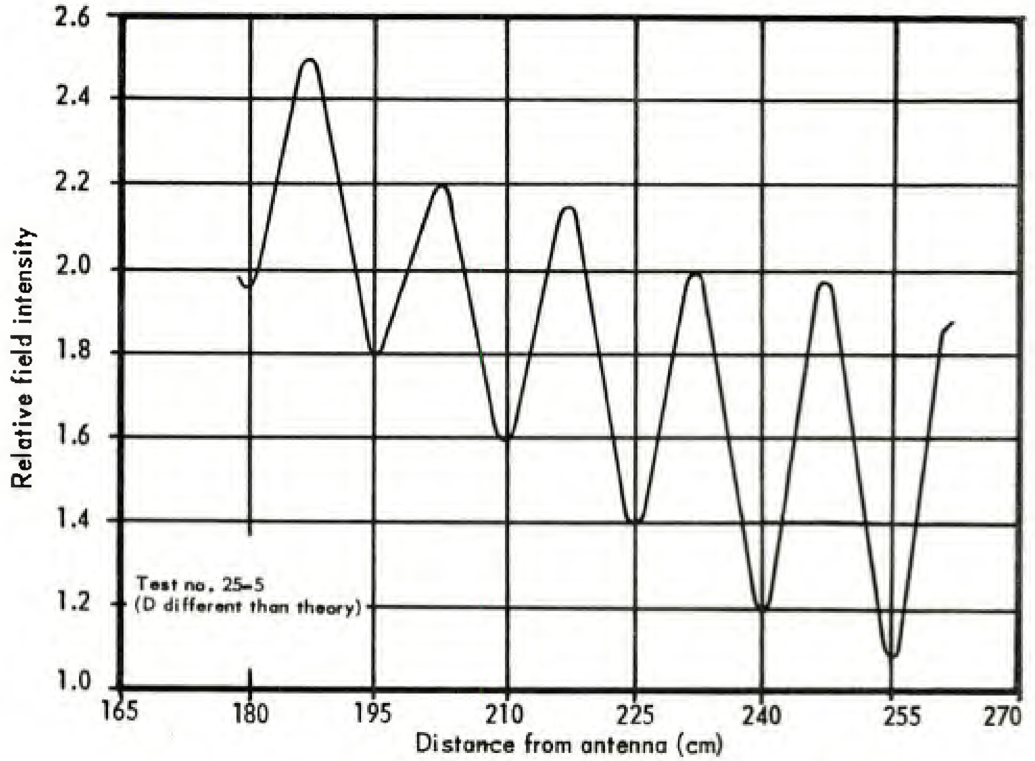
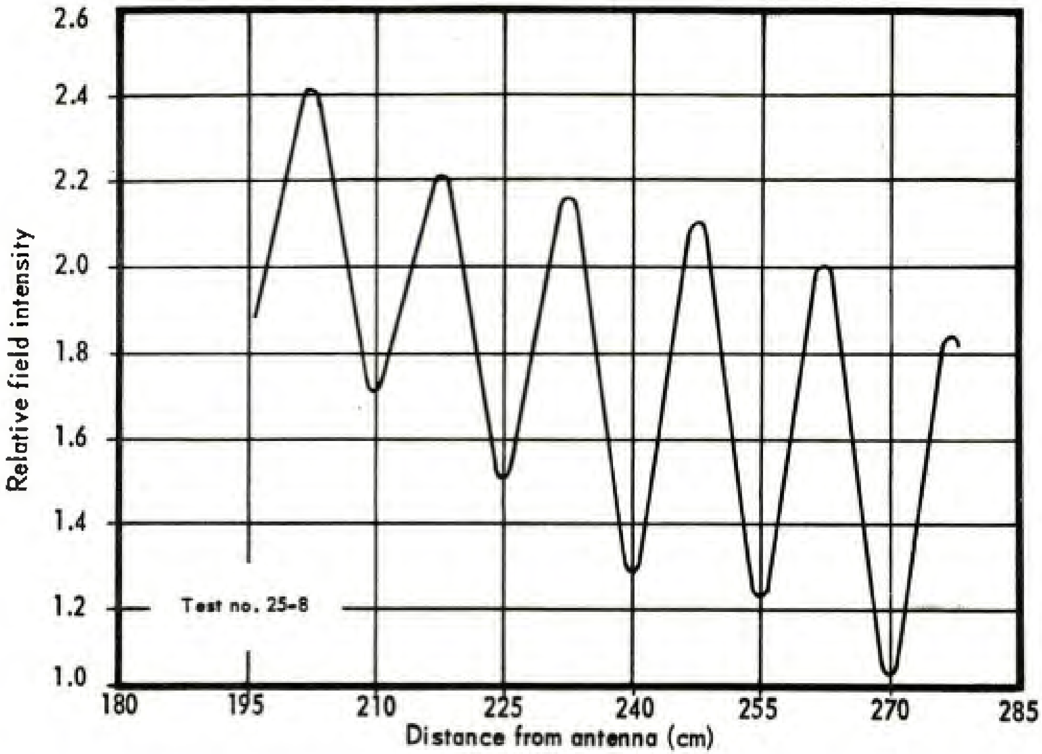


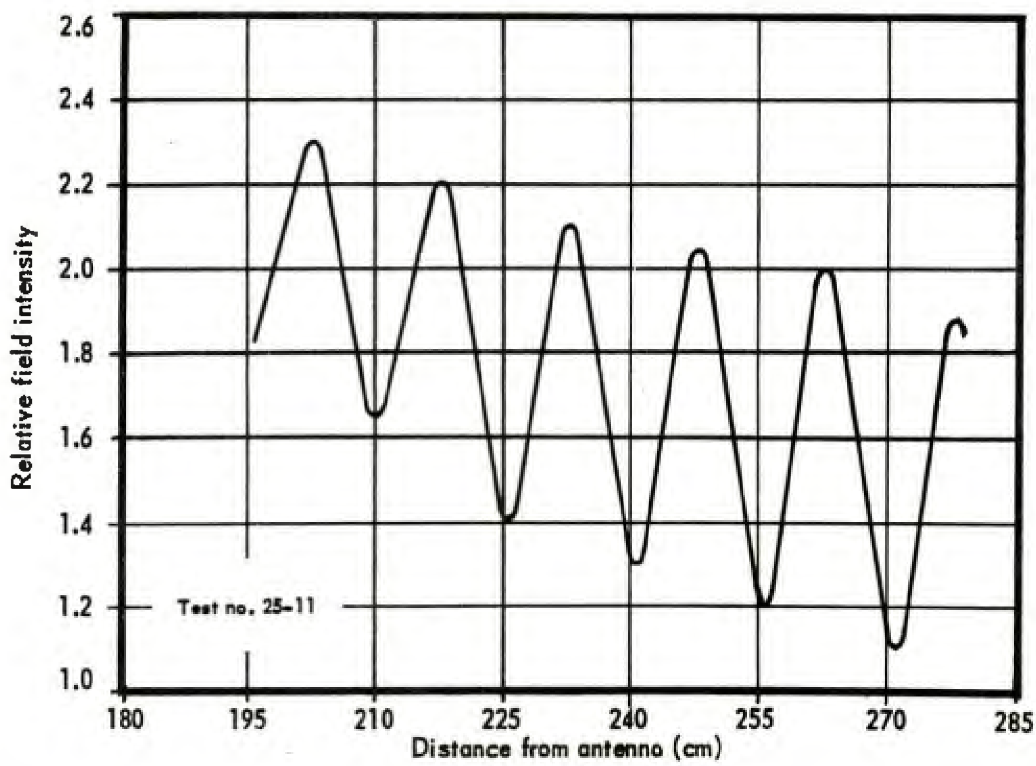
Figure 1. System for the measurement of microwave reflectances.





Figures 2A and 2B. Experimentally determined envelope of relative electric field intensity vs distance from horn antenna.





Figures 2C and 2D. Experimentally determined envelope of relative electric field intensity vs distance from horn antenna.

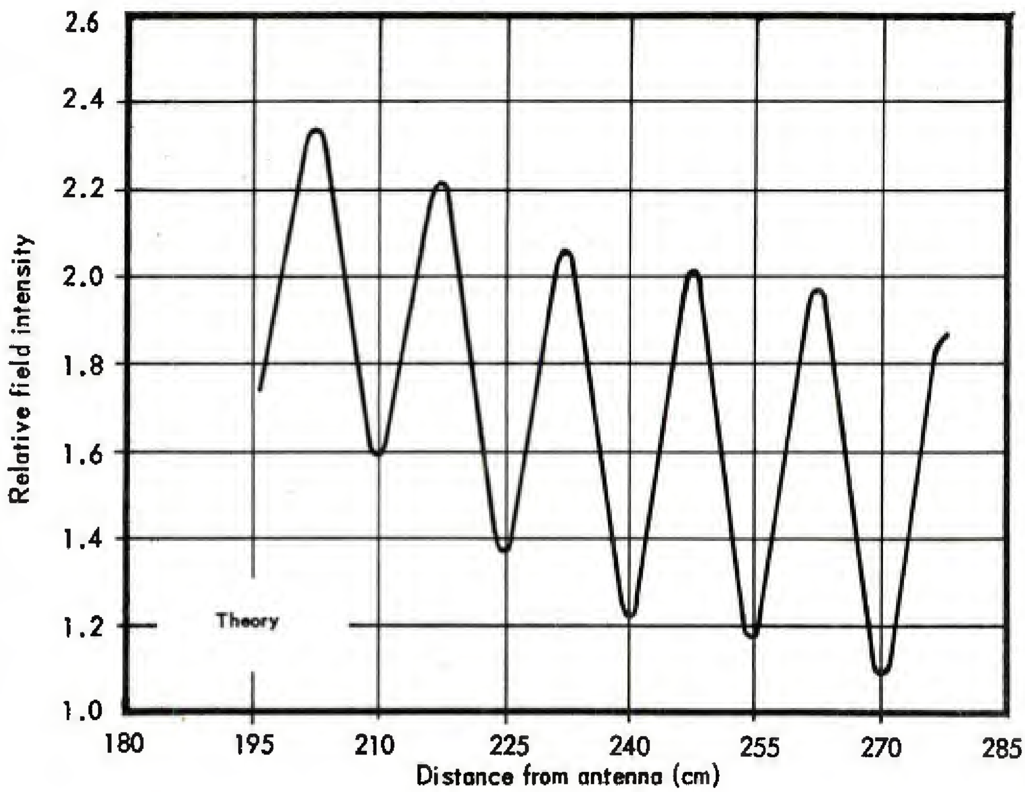


Figure 3. Theoretically determined envelope of relative electric field intensity vs distance from horn antenna.