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PROJECT MICHAEL

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Technical Report No. 17

Wave Generation by Point Sources

by

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Director

Research Sponsored by
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WAVE GENERATION BY POINT SOURCES

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I. Introduction

The problem of wave generation in water of arbitrary depth by complicated situations such as moving ships, surface disturbances, etc., has been discussed many times. However, the treatment of the simple model of a point source at a fixed circular frequency has not been published. The advantages of such a model are obvious. The judicious disposition of such sources in space can be used as a rough model for almost any source of disturbance. The Fourier synthesis of the solution can be used to interpret the disturbance by a source varying with the time in any manner.

In this report, the exact solution for all distances from the source for a point source in a linearized theory is obtained a la Lamb. Application is made to a dipole source whose dimension is short compared to the wavelength of the generated disturbance at large distances.

II. The Differential Equation and the Boundary Conditions

In this problem, the equation to be solved is the equation of continuity on the velocity potential Φ .

$$\nabla^2 \Phi = 0 \tag{1}$$

The solution to this equation will be made to satisfy the free surface boundary condition and the condition of a spherical source at a depth h from the surface. The z -axis coincides with the vertical, with z being measured as positive in the upward direction. The water depth is H. The usual technique of splitting the region into two parts using a horizontal plane through $z = z_1$ will be followed. The region $0 \geq z \geq z_1$ will be labeled with the subscript I, the region $z_1 \geq z \geq -H$ with the subscript II. The elementary solutions for the two regions are:

$$\Phi_I = e^{i\omega t}\left(Ae^{kz} + Be^{-kz}\right)J_0(kr) \qquad (2)$$

$$\Phi_{II} = e^{i\omega t}\left(Ce^{kz} + De^{-kz}\right)J_0(kr) \qquad (3)$$

ω is the circular frequency of the harmonic source, k is the separation constant, A, B, C, D are arbitrary constants, and r is the cylindrical radius from the origin. The boundary conditions to be satisfied are:

$$\frac{\partial^2 \Phi_I}{\partial t^2} + g\frac{\partial \Phi_I}{\partial z} = 0 \quad , z = 0 \qquad (4)$$

$$\frac{dZ_{II}}{dz} - \frac{dZ_I}{dz} = \frac{kV}{2\pi} , \quad z = z_1 \qquad (5)$$

$$\frac{\partial \Phi_I}{\partial t} = \frac{\partial \Phi_{II}}{\partial t} \quad , z = z_1 \qquad (6)$$

$$\frac{\partial \Phi_{II}}{\partial t} = 0 \quad , z = -H \qquad (7)$$

Equation 4 is the well known free surface condition. In Eq. 5 Z refers to the z-factor of Eqs. 2 and 3. The discontinuity in the z component of the velocity is illusory, for the final linear combination will involve an integral like "

$\int_0^\infty J_0(kr)\,k\,dk$ which is equivalent to $\delta(r)$. Equation 6 represents continuity in pressure. Equation 7 requires vanishing vertical motion at the bottom boundary. V is a constant which will be interpreted subsequently.

The indicated substitutions are performed. The constants are evaluated. Finally, the simple linear combination of summation over all k is chosen, yielding

$$\bar{\Phi}_I = \frac{V}{2\pi} e^{i\omega t} \int_0^\infty \frac{\cosh k(H+z_1)}{gk \sinh kH - \omega^2 \cosh kH} \left\{ gk \cosh kz + \omega^2 \sinh kz \right\} J_0(kr)\,dk \qquad (8)$$

$$\bar{\Phi}_{II} = \frac{V}{2\pi} e^{i\omega t} \int_0^\infty \frac{gk \cosh kz_1 + \omega^2 \sinh kz_1}{gk \sinh kH - \omega^2 \cosh kH} \cosh k(H+z) J_0(kr)\,dk \qquad (9)$$

There is still an ambiguity in the path of integration: the integrand has two poles on the real axis given by

$$\omega^2 = gk \tanh kH \qquad (10)$$

These roots are of the same absolute value but opposite in sign . We will call them $\pm k_0$. The path of integration is chosen to avoid the pole by a small semicircular excursion into the upper half of the complex plane. (Fig. 1) This will guarantee

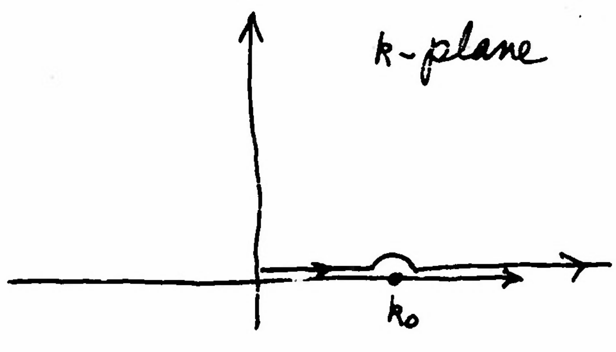

Figure 1

that the final solution will consist only of outgoing waves at infinity. We note that the real solutions of Eq. 10 give the correct phase velocity for the outgoing waves.

Finally, there is the question of verifying the singularity at the origin and obtaining the normalization of the source. Inspection of Eqs. 8 and 9 shows that the integrals go to infinity as both $r \to 0$ and $z \to z_1$. Furthermore, the infinite contributions come from the very large values of k in the integrand. Therefore, to the approximation that $r \to 0$ and $z \to z_1$ we may approximate the solution by

$$\Phi_I \Rightarrow \frac{V}{4\pi} e^{i\omega t} \int_0^\infty e^{-k(z-z_1)} J_0(kr)\,dk = \frac{V}{4\pi} \frac{e^{i\omega t}}{R} \qquad (11)$$

$$\Phi_{II} \Rightarrow \frac{V}{4\pi} e^{i\omega t} \int_0^\infty e^{k(z-z_1)} J_0(kr)\,dk = \frac{V}{4\pi} \frac{e^{i\omega t}}{R} \qquad (12)$$

$$R = \frac{1}{\sqrt{(z-z_1)^2 + r^2}}$$

It follows that, in the neighborhood of the singularity, the solution is represented by a source whose peak strength is V units of volume per second.

III. The Evaluation of the Integral

In addition to the two real poles, the integrands of Eqs. 8 and 9 have an infinite number of pure imaginary poles, symmetrical as to sign, given by Eq. 10. These roots will be denoted by k_n, $n = \pm 1, \pm 2, \pm 3$. There are two limiting cases of general interest.

Case 1. or the shallow water approximation.

In the limit we find:

$$|Hk_n| = n\pi \qquad n = 1,2,3\ldots\ldots \quad (13)$$

Case 2. There is no simple approximation for k_n but the early roots start as

$$|Hk_n| = (2n-1)\frac{\pi}{2} \qquad n = 1,2,3\ldots\ldots \quad (14)$$

For large values of n, the solution turns over to agree with (13) but the first roots are the only practical ones in the limiting solution. If more than a few roots are required, the series expansion from the origin is more useful. Since there are no branch points, the integral is evaluated by elementary technique yielding

$$\Phi_I = \Phi_{II} = \frac{Ve^{i\omega t}}{2iH} \sum_{n=0}^{\infty} \frac{\cosh k_n(H+z)\cosh k_n(H+z_1)}{1+\frac{1}{2k_nH}\sinh 2k_nH} H_0^2(k_n r) \qquad (15)$$

where $H_0^2(u)$ is the zero-order Hankel function of the second kind. This is the solution to the problem. It is to be noticed that the artificial division of the solution valid for two regions has disappeared in the final expression in Eq. 15. This must be so and the fact that this occurs is a check on the solution. It will be convenient to regard Eq. 15 as the Green's function for the problem.

In any practical application, this solution can be used to idealize the situation by appropriate combinations of sources and sinks. As an important example, we shall consider, in addition to the source at the origin, another source whose coordinate is

$z = z_2$, $r = d$, $\varphi = 0$ which is oscillating just π radians out of phase with the first. In this case, we have the difference of the two solutions. Since the various addition theorems are to be applied, the solution is manipulated in a preliminary way:

$$\Phi = \frac{Ve^{i\omega t}}{2iH}\sum_{n=0}^{\infty}\frac{\cosh k_n(H+z)\left[\cosh k_n(H+z_1)-\cosh k_n(H+z_2)\right]}{1+\frac{1}{2k_nH}\sinh 2k_nH}H_0^2(k_n r)$$

$$+\frac{Ve^{i\omega t}}{2iH}\sum_{n=0}^{\infty}\frac{\cosh k_n(H+z)\cosh k_n(H+z_2)}{1+\frac{1}{2k_nH}\sinh 2k_nH}\left[H_0^2(k_n r)-H_0^2(k_n r_1)\right] \tag{16}$$

where r_1 is the cylindrical coordinate measured from the second source.

The two terms of Eq. 16 may be written in a more familiar form through the use of addition theorems.

$$\Phi = \frac{Ve^{i\omega t}}{2iH}\sum_{n=0}^{\infty}\frac{2\cosh k_n(H+z)\sinh k_n\frac{z_1-z_2}{2}\sinh k_n\left(H+\frac{z_1+z_2}{2}\right)}{1+\frac{1}{2k_nH}\sinh 2k_nH}H_0^2(k_n r)$$

$$+\frac{Ve^{i\omega t}}{2iH}\sum_{n=0}^{\infty}\frac{\cosh k_n(H+z)\cosh k_n(H+z_2)}{1+\frac{1}{2k_nH}\sinh 2k_nH}\times \tag{17}$$

$$\left[\left\{1-J_0(k_n d)\right\}H_0^2(k_n r)-2\sum_{m=1}^{\infty}J_m(k_n d)\,H_m^2(k_n r)\cos m\varphi\right] \qquad d<r.$$

This solution is exact. In the next section various approximations are considered.

IV. The Various Approximations

1. The Asymptotic Approximation

In this approximation, the concern is with the wave amplitude at distances such that $k_0 r >> 1$. The customary technique is to employ the asymptotic expansions of the Bessel functions. In addition, we recognize that the terms for n = 0 yield the outgoing waves, whereas the terms n = -1, -2, etc., yield exponentially damped waves with distance. In practice, the damped terms may generally be neglected at about one wavelength from the source. This approximation gives

$$\Phi = \frac{V \cosh k_0 (H+z)}{2H\left(1+\frac{1}{2k_0H}\sinh 2k_0H\right)} \frac{e^{i\omega t - ik_0 r - \frac{i\pi}{4}}}{\sqrt{\frac{1}{2}\pi k_0 r}} \times$$

$$\Big[2\sinh k_0 \frac{z_1-z_2}{2} \sinh k_0 \left(H + \frac{z_1+z_2}{2}\right) + \cosh k_0(H+z_2)\Big\{1 - J_0(kd) - 2\sum_{m=1}^{\infty} e^{\frac{im\pi}{2}} J_m(k_0 d)\cos m\varphi\Big\}\Big] \tag{18}$$

2. The Shallow Water Asymptotic Approximation

In this approximation, the departure is Eq. 18 with the additional assumption that $k_0 H << 1$. The expression further simplifies to

$$\Phi = \frac{V e^{i\omega t - ik_0 r - \frac{i\pi}{4}}}{4H\sqrt{\frac{1}{2}\pi k_0 r}} \times$$

$$\left[k_0^2 (z_1 - z_2)\left(H + \frac{z_1+z_2}{2}\right) + 1 - J_0(kd) - 2\sum_{m=1}^{\infty} e^{\frac{im\pi}{2}} J_0(k_0 d)\cos m\varphi\right]. \tag{19}$$

This approximation is intentionally inconsistent in the terms of the second order. The effect of the vertical separation of sources vanishes in first order and the second order terms have been kept for the special case d = 0. It is also useful to have an expression for an isolated source to the same approximation. Reference to Eq. 16 yields

$$\Phi = \frac{e^{i\omega t - ik_0 r - \frac{i\pi}{4}}}{4H\sqrt{\frac{1}{2}\pi k_0 r}} V \qquad (20)$$

3. <u>The Dipole Approximation in Shallow Water</u>

In this approximation, the assumption is made that the two sources are considerably less than a wavelength apart or that both $k_0 r \ll 1$ and $|k_0(z_1 - z_2)| \ll 1$

To terms of the first order <u>only</u>,

$$\Phi = -\frac{k_0 d \cos\varphi \, e^{i\omega t - ik_0 r + \frac{i\pi}{4}}}{4H\sqrt{\frac{1}{2}\pi k_0 r}} V \qquad (21)$$

In effect, the single source is reduced by the fraction $k_0 \rho_1 \cos\varphi$. The effect of the vertical separation $z_1 - z_2$ in shallow water is of higher order and is evaluated in Eq. 19.

There are further useful approximations but they will not be listed here since they are of secondary importance.

V. Numerical Estimates

As an example of the order of magnitude of the effects involved, we consider the case $H = 100'$, $\omega = \pi/10 \ sec^{-1}$, single source.

Since $p = \rho \frac{\partial \varphi}{\partial t}$

$$p_{max} = \frac{\rho \omega V}{2H\sqrt{2\pi k_0 \Omega}} \tag{20a}$$

This corresponds to a wavelength of 1120 ft. If an overpressure of 1 inch of water (approx. 5 lbs/ft^2) is considered at a distance of 1000 ft, the peak flow volume in ft^3 per sec is

$V = 9.0 \times 10^3$ ft^3/sec or

$V = 5.4 \times 10^5$ c.f.m.

If the source is a horizontal doublet of spacing 20 ft the V required would be 6×10^6 ft^3/min which is considerable. For purposes of computation, it is convenient to write Eq. 20a in terms of ω and H alone.

$$p_{max} = \frac{g^{1/4} \rho \omega^{1/2}}{2H^{3/4} \sqrt{\Omega}} V \frac{1}{\sqrt{2\pi}} \tag{20a}$$

This indicates a rather slow dependance on ω .

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