MASSACHUSETTS
INSTITUTE OF
TECHNOLOGY

DEPARTMENT OF
MECHANICAL
ENGINEERING

CAMBRIDGE 39, MASS.

Energy Criteria of Fracture

E. OROWAN

Technical Report No. 3
Office of Naval Research
Contract Number N5ori-07870

July 1954

ENERGY CRITERIA OF FRACTURE

TECHNICAL REPORT NO. 3

E. OROWAN

Office of Naval Research
Contract Number N5ori-07870

Massachusetts Institute of Technology
Department of Mechanical Engineering

D. I. C. 6949

July 1954

ENERGY CRITERIA OF FRACTURE

1. The Griffith energy principle.

In the course of the last few years, it has become clear that the Griffith equation for the tensile strength of a brittle solid cannot be applied in its original form to brittle fracture in normally ductile steels. X-ray back reflection photographs show[1] that a thin layer at the surface of apparently quite brittle fractures of low carbon steels contains significant plastic distortion; the plastic work p in this layer amounts to roughly 2×10^6 ergs/cm^2 if the fracture has occurred not too far below room temperature. Compared with this value, the surface energy (a few times 10^3 ergs/cm^2) is negligible; consequently, if an expression of the Griffith type can be used at all in this case, the surface energy (representing the work for creating unit area of the surface of fracture) has to be replaced by the plastic surface work p. Thus the crack propagation condition would be[2]

$$\sigma \approx \sqrt{\frac{Ep}{c}} \qquad (1)$$

The presence of considerable plastic distortion at the surface of fracture raises the question under what conditions the Griffith principle of virtual work can be applied to fractures accompanied by plastic deformation. This principle can be started in the following manner: let dW be the free energy required for increasing

the length* of a crack from c to c + dc, and -dU the elastic energy released simultaneously in the specimen if this is held between rigidly fixed grips so that the external forces cannot do work. The critical length of the crack above which it can propagate spontaneously is then determined by the condition

$$dW = -dU. \tag{2}$$

It is easily seen that the assumption of rigidly fixed grips is not essential; the same result is obtained if the crack propagation is assumed to occur under constant load. Let M(c) be the elastic compliance, i.e., the reciprocal spring constant, of a specimen containing a crack of length c; thus,

$$x = MF \tag{3}$$

where F the tensile force acting upon the specimen and x its elastic elongation. The elastic energy of a specimen containing a crack of length c is

$$U = \int_{x=0}^{x=MF} F \cdot dx = \frac{M(c)F^2}{2} \tag{4}$$

and

$$dU = \frac{F^2}{2} dM + MF \cdot dF; \tag{5}$$

$dM = \frac{dM}{dc} \cdot dc$ is the increment of the elastic compliance due to

---

* As in the original work of Griffith, only two-dimensional cases (cracks in plate-specimens) will be considered here for simplicity. The general results can be easily extended to three-dimensional cases.

the increase by dc of the crack length.

If the crack length increases while the specimen is held between rigidly fixed grips, $x = MF = \text{const.}$ and

$$dx = MdF + FdM = 0; \tag{6}$$

substitution of $MdF = -FdM$ in eq. (5) gives

$$(dU)_x = \frac{-F^2 dM}{2} . \tag{7}$$

On the other hand, if the crack propagates while the load is kept constant ($dF = 0$), eq. (5) gives

$$(dU)_F = \frac{F^2 dM}{2} . \tag{8}$$

At the same time, the force F does the work

$$dL = F \cdot dx = F^2 dM, \tag{9}$$

since, at constant F, $dx = FdM$.

Eqs. (8) and (9) show that, if the crack propagates at constant load, half of the external work is stored as additional elastic energy of the specimen, and the other half is available for increasing the free energy of the crack. If the length of the crack exceeds the critical value at which eq. (2) is just satisfied, the work of the applied force is more than sufficient to provide the increment of its free energy; the balance creates kinetic energy and accelerates the rate of crack propagation.

If, on the other hand, the crack propagates between fixed grips, the elastic energy of the specimen decreases according to

eq. (7), and its decrement is available for increasing the free energy of the crack and the kinetic energy. Comparison of eqs. (7), (8), and (9) shows that the energy available for crack propagation at fixed load is the same as at fixed grips; in the former case, -dU in eq. (2) has to be replaced by $dL - (dU)_F$ which is numerically equal to $-(dU)_x$ for the same increment dc of the crack length.

In the present paper, two questions will be treated that have been widely discussed in connection with the brittle fracture of structural and ship steel, and on which a wide divergence of opinions has arisen. They are:

A) Does the Griffith equation

$$\sigma \approx \sqrt{\frac{E\alpha}{c}} \qquad (\alpha = \text{surface energy}) \qquad (10)$$

represent a necessary and sufficient condition of completely brittle fracture? And is the present writer's eq. (1) a necessary condition of brittle fracture in low carbon steels?

B) Under what conditions can the Griffith principle, eq. (2), be applied to fractures involving plastic deformation?

2. The Griffith equation as a necessary and sufficient condition of completely brittle fracture.

It is obvious that eq. (10) is a necessary condition of crack propagation in a completely brittle specimen under tension. If it is not satisfied, propagation of the crack with the accompanying

increase of its (free) surface energy would violate the first or the second law of thermodynamics. In particular, thermal fluctuations (disruption of atomic bonds at the tip of the crack by thermal activation) cannot propagate the crack if the Griffith equation is not satisfied, because any such process would result in the creation of free energy from thermal energy without heat flowing from one reservoir to another of a lower temperature. Of course, thermal fluctuations of free energy do occur; however, they cannot lead to any significant crack propagation because the greatest energy fluctuation that may arise with any probability amounts to a few electron volts which is equivalent to the disruption of a few individual atomic bonds at the tip of the crack.

From the fact that the Griffith equation is a necessary condition of completely brittle fracture, it does not follow that it is also a sufficient condition. However, it can be proved that once the condition is satisfied, crack propagation is not merely possible but is bound to follow. This can be shown by proving that, if the applied stress has the value given by the Griffith equation, the stress concentration at the tip of the crack reaches the value of the molecular cohesion (theoretical strength) at which fracture is bound to take place.

The molecular cohesion of a brittle material can be estimated in the following well known way. When a rod of unit cross sectional area breaks with a smooth surface of fracture perpendicular to the axis of the rod, two new surfaces of unit area are created; the work required for this is 2α (α = surface energy). This work

is done against the intermolecular attractive forces as the two fragments are pulled apart. Fig. 1 shows the variation of the molecular forces between the two fragments, per unit of cross sectional area, as a function of the distance d between the layers of molecules in the two fragments that are adjacent to the surface of separation. The force is zero when d = b = the molecular spacing in the absence of stress; it rises to a maximum σ_m which is the molecular cohesion and then falls to zero with increasing separation of the fragments. The area below the curve is the work of fracture per unit of the cross sectional area; i.e., it is equal to 2 α . At the maximum of the curve in Fig. 1, the amount of energy represented by the shaded area below the curve must be present between all neighboring pairs of molecular (or atomic) planes perpendicular to the tension; it is identical with the elastic energy stored in the material between two adjacent atomic planes. If, for an order-of-magnitude estimate, Hooke's law is assumed to be valid up to the theoretical maximum σ_m of the stress, the density of elastic energy between two atomic planes of unit area, spaced at b, is $b \cdot \sigma_m^2/2E$. If it is assumed that the shaded

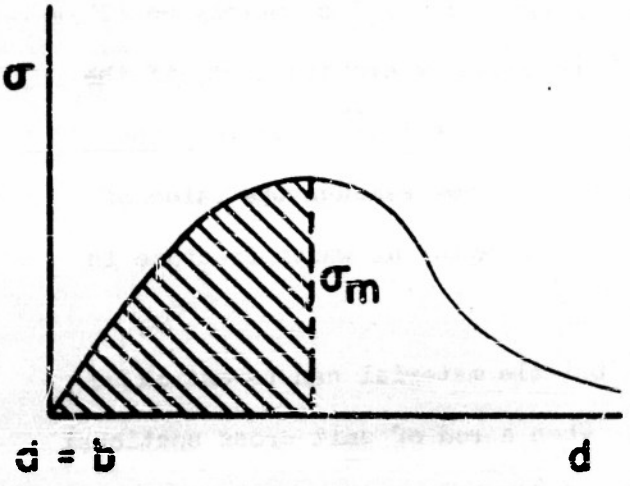

Fig. 1

area is about one-half of the total area below the curve and therefore approximately equal to α , the relationship

$$b \cdot \frac{\sigma_m^2}{2E} = \alpha \qquad (11)$$

gives the order of magnitude of the molecular strength as

$$\sigma_m \cong \sqrt{\frac{2E\alpha}{b}} \quad . \qquad (12)$$

The next question is: what is the value of the applied tensile stress at which the critical value σ_m is reached at the tip of the crack? The stress concentration factor of a surface crack of depth c and root radius ρ is[3]

$$q = 2\sqrt{\frac{c}{\rho}} \quad ; \qquad (13)$$

this relationship shows that the maximum stress would be infinitely high for any finite value of σ and c in an elastic continuum containing a perfectly sharp crack, and therefore the tensile strength would be zero. The reason why brittle solids have a finite strength lies in the atomic structure of matter. Fig. 1 shows that Hooke's law breaks down when the increment of the atomic spacing becomes comparable in magnitude with the atomic spacing itself: near the tip of the crack the stress versus strain curve levels out, and the situation can be regarded roughly as if a certain region at the tip, comparable in linear dimensions with the interatomic spacing, would be under the constant stress σ_m instead of obeying Hooke's law.

This case of the laws of elasticity ceasing to be valid in a region at the tip of the crack has been treated by L. Föppl[4] and,

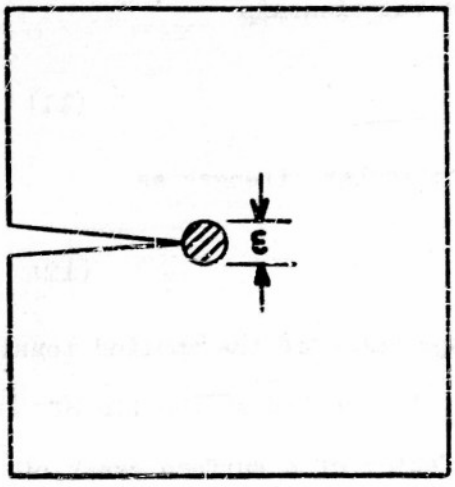

Fig. 2

in particular, by Neuber [5]. Neuber proved the following theorem: let there be a region of linear dimensions ε at the tip of the crack (Fig. 2), so that the specimen is Hookean elastic outside this region, whereas the stress in the region is approximately constant at the value existing at its boundary; the ratio of the stress in the region to the tensile stress applied to the specimen is then equal to the stress concentration factor of a crack of the same length and of the root radius $\varepsilon/2$ in a purely Hookean elastic material. (The quantity ε is assumed to be small compared with the length c of the crack which itself must be small compared with the dimensions of the specimen.)

In the present case, the diameter of the region in which Hooke's law breaks down and the stress levels out is obviously of the order of magnitude of the interatomic spacing b; if it is assumed to be approximately 2b, Neuber's theorem indicates that the effective stress concentration factor is that of a crack of tip radius b in a purely Hookean specimen. According to eq. (13), this is

$$q = 2\sqrt{\frac{c}{b}}. \qquad (13a)$$

Thus, the value of the applied tensile stress at which the molecular strength is reached at the tip of the crack is given by

$$\sigma_m = \sigma \cdot 2\sqrt{\frac{c}{b}}; \qquad (14)$$

if σ_m is replaced from (12), the tensile strength σ is obtained as

$$\sigma \approx \sqrt{\frac{E\alpha}{2c}} \qquad (15)$$

which, within the accuracy of the estimate, is identical with the Griffith equation (10).

This derivation of the Griffith equation directly from the stress concentration factor of the crack shows that, when the applied tensile stress has the value given by the equation, the stress at the tip of the crack reaches the highest value that can be withstood by the interatomic forces in the material. Any further straining is bound to produce crack propagation and fracture. In other words, the Griffith equation represents not only a necessary but also a sufficient condition of fracture in a completely brittle specimen.

3. Can the Griffith principle be applied to ductile fracture?

In recent years the view has been expressed that the Griffith energy principle eq. (2) may be applied to all types of fracture, not only to essentially brittle ones. In what follows, it should be pointed out that this is not so: the principle can only be applied if plastic deformation is either absent or confined to a thin layer at the crack walls so that the bulk of the specimen is still elastic.

Fig. 3 indicates the manner of crack propagation in a purely elastic material: owing to elastic strain release around the crack, its walls are pulled apart, and its length increases. Fig. 4, on the other hand, shows one of the simplest types of ductile fracture [6], such as is observed in aluminum single crystals or (polycrystalline) plates of ductile metals in tension. The crack (which in this case has

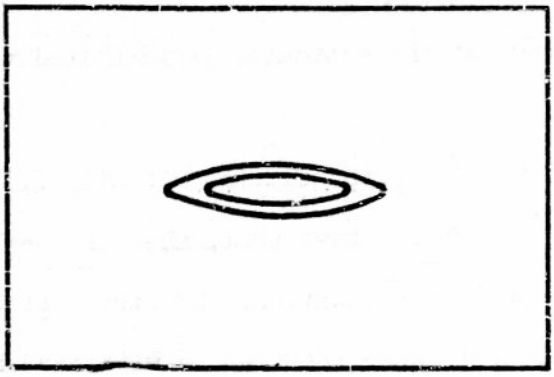

Fig. 3

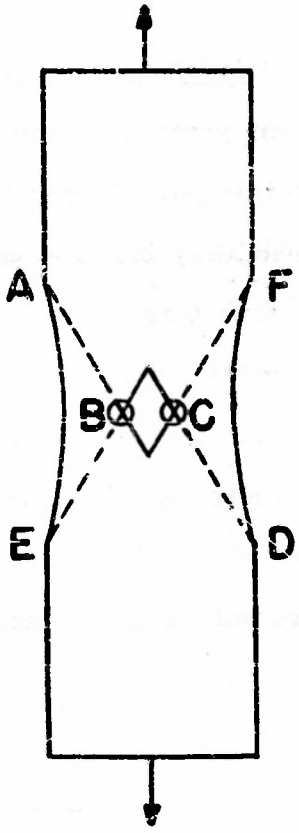

Fig. 4

a square cross section) is propagated by slip in the planes AB + CD and EB + CF, where B and C are lines perpendicular to the plane of the sketch; in the course of this process the cross section of the crack increases until fracture is complete.

The fundamental difference between the propagation of the brittle crack shown in Fig. 3 and the ductile mechanism of Fig. 4 is that the former is based essentially on the elasticity of the material, while the latter could work in the same way even if the elastic moduli were infinitely high. The Griffith equation (10) shows directly that the tensile strength of a brittle material would rise to infinity with an infinite increase of the value of Young's modulus: in such a material, the crack could not open up because there would be no elastic strains to release. On the other hand, the slip mechanism shown in Fig. 4 is quite independent of the elastic moduli.

It is immediately obvious that the force required for propagating the crack in Fig. 4 cannot be derived from the Griffith principle eq. (2). Its value is simply

$$F = Y \cdot A \qquad (16)$$

where Y is the yield stress of the material in tension and A the projection of the areas AB plus CD on the plane perpendicular to the direction of the tension; if F satisfies eq. (16), the plastic deformation that opens up the crack can progress, and the crack propagates. The elastic moduli do not appear in eq. (16); they could be infinitely high without any consequence to the propagation of the crack. On the other hand, infinitely high elastic moduli would make the right hand side of eq. (2) vanish: this shows that the tensile

strength obtained by any application of the Griffith energy principle would rise to infinity with the elastic moduli.

The conclusion is, then, that the Griffith energy principle can only be applied to fully or substantially brittle fractures; ductile fractures are quite outside its scope.

In arguing the applicability of the elastic energy release principle to ductile fractures, occasionally the point has been made that if the specimen is long enough, the elastic energy stored in it should be sufficient to produce rapid crack propagation even if the energy absorption of the crack is as high as it is in typically ductile fractures. The answer to this is that a fast fracture is not necessarily a brittle fracture (i.e., a fracture involving very low energy absorption). Any ductile fracture can be made to run fast, at least from a certain stage onwards, if the specimen is connected in series with a large enough spring (or, what is the same, if the specimen is long enough). It can be shown that the condition for a ductile fracture to become a fast fracture is not eq. (2) but equality of the second derivatives of W and U*.

4. The writer's crack propagation condition for brittle fracture in normally ductile steels.

As mentioned in the first Section, the present writer has suggested that brittle fracture in ductile steels may obey the crack

---

* To be published in a separate paper.

propagation condition

$$\sigma \approx \sqrt{\frac{Ep}{c}} \qquad (1)$$

which results if, in the Griffith equation (10), the surface energy is replaced by the surface plastic work p. It can be obtained by starting from the Griffith principle of elastic energy release eq. (2) and equating the free energy required for producing unit area of the crack wall to p instead of α.

The first question is: Can the Griffith energy principle be applied to a fracture process that involves plastic deformation? It was seen in Section 2 that the Griffith equation can be derived from the elastic stress concentration factor of the crack; however, can this be done if plastic deformation takes place and redistributes stresses at the tip of the crack? The Neuber principle, mentioned in Section 2, shows that the stress concentration factor can be calculated on the basis of the classical theory of elasticity if the plastically deformed region is small compared with the length of the crack. In that case it can be treated in the manner explained in connection with Fig. 2: the stress concentration factor is the same as that of a crack in a purely elastic body with a tip radius equal to half of the diameter of the plastically deformed region. In fact, this case is only quantitatively different from that of the completely brittle material in which, in order to take into account the atomic structure of matter, the same consideration had to be applied to the region at the tip of the crack in which the stress distribution flattens out owing to the maximum of the force-displacement curve, Fig. 1. The only difference is that in the Griffith case the diameter of the non-Hookean region is

of the order of the interatomic spacings, while in the brittle fracture of steel it is about twice the thickness t of the plastically deformed layer at the surface of the crack. According to the Inglis equation (13), the stress concentration factor is then

$$q = 2\sqrt{\frac{c}{t}} \qquad (17)$$

X-ray measurements indicate [1] that t is of the order of 0.2 to 0.4 mm in low carbon steels broken not too far above or below room temperature.

In the Griffith theory, the tensile strength of the specimen was obtained by dividing the molecular cohesion by the stress concentration factor. What is the quantity corresponding to the molecular cohesion in the brittle fracture of steels? The clue is given by the important observation [7] that in steels the crack does not propagate continuously: before it has broken through a grain boundary, unconnected small cracks arise in grains ahead of the tip of the main crack. This shows at once that the brittle strength of steel cannot have the order of magnitude of the theoretical strength (molecular cohesion); in fact, it must be quite low if independent fracture processes can start ahead of the main crack at points where the stress cannot be much above the yield stress. This may be due to the presence of numerous invisible cracks scattered in the material; or to the well known fact that plastic deformation can produce high microscopic internal stresses and subsequently crack formation. It seems that the cleavage strength of the material at the tip of the

crack is not, or not much, higher than the ordinary brittle strength of steel obtained experimentally as the stress at which brittle fracture occurs. Since the cleavage strength of steel depends on the plastic strain which is difficult to estimate in the small region around the tip of a crack, only a rough idea of its magnitude can be obtained; it is probably somewhere between 100,000 and 200,000 psi for a low carbon steel. For a tensile stress of, say, 20,000 psi, therefore, a stress concentration factor between 5 and 10 would be needed. If the thickness t of the cold worked layer in eq. (16) is taken as 1/100 inch, the necessary crack length

$$c = \frac{1}{4}t\left(\frac{\sigma_m}{\sigma}\right)^2 = -t \cdot q^2 \qquad (18)$$

is between 1/16 inch and 1/4 inch; for a tensile stress of 10,000 psi, the stress concentration factor is four times higher, and the necessary crack length is between 1/4 inch and 1 inch. These orders of magnitude appear quite reasonable in the light of experimental observations.

The last question to be discussed is whether eq. (1) represents a sufficient as well as a necessary condition of crack propagation. At this point a significant difference appears between the fracture, say, of glass and of low carbon steel. The stress concentration in glass is not limited by plastic deformation; in steel, however, the stress at the tip of the crack cannot exceed the yield stress multiplied by a plastic constraint factor which probably has a value between . and 3 [1]. If, therefore, the cleavage strength is higher than 2 or 3 times the yield stress Y in tension, the tensile stress at the tip of the crack cannot reach the fracture level no matter how high the stress

concentration factor (i.e., no matter how low the applied stress is that can produce the highest possible stress 2Y or 3Y at the tip of the crack). An additional point of great importance is that the yield stress of steel increases with the rate of deformation more rapidly than the yield stresses of most metals; between the usual rates of "static" tests and the fastest rates at which measurements could be carried out it seems to increase by a factor approaching 3. It seems that, in typical cases of brittle fracture in low carbon steels, the velocity increase of the yield stress is the salient feature of the phenomenon. Although cleavage fracture can arise at slow deformation rates, it then requires so much plastic deformation for producing the necessary plastic constraint that the resulting cleavage fracture is anything but brittle; its energy absorption may be almost equal to that of a ductile fracture. Typical brittle fracture in a low carbon steel, therefore, can occur usually only after the crack propagation has reached a sufficiently high velocity; in laboratory experiments, the fracture is almost always initiated by some ductile (fibrous) cracking, accompanied by considerable local plastic deformation.

It can be said, therefore, that a characteristic feature of brittle fracture in ductile steels is the enormous decrease of the crack propagation work with increasing velocity of the crack. The crack propagation condition eq. (1) may well be fulfilled for a rapidly running crack with its low value of p but not for a stationary crack, the propagation of which may require, per unit of crack length, an energy of a higher order of magnitude. In such cases, cleavage

fracture is initiated in laboratory experiments by large deformations producing strong plastic constraint and usually some fibrous cracking; the plastic deformation may have to extend across the entire specimen, so that the yield load has to be reached before cleavage cracking can start. After a cleavage crack has arisen, it may accelerate rapidly provided that the condition eq. (1) is satisfied, so that there is sufficient elastic energy released during the crack propagation to increase the kinetic energy around the running crack. In this sense, it may be assumed that eq. (1) represents the condition for the fast, and therefore, brittle, propagation of a cleavage crack. The initiation of the cleavage crack, however, may have to be done by ductile crack propagation not governed by eq. (1) or any other brittle crack propagation condition derived from the Griffith principle eq. (2).

It should be remarked that many service fractures seem to start without significant plastic deformation in spite of static loading. An interesting possibility for understanding this has recently arisen and should be discussed in a subsequent paper.

This paper represents an expanded version of remarks that were stimulated by the work done under Office of Naval Research Contract No. N5ori-07870, and contributed to the Conference on Brittle Fracture Mechanics held at the Massachusetts Institute of Technology on October 15 and 16, 1953, under the auspices of the Committee on Ship Structural Design, advisory to the Ship Structure Committee, National Research Council.

References

1. E. Orowan, Trans. Inst. Engrs. Shipbuild. in Scotland, v. 165, (1947).

2. E. Orowan, Proc. of a Symposium, Fatigue and Fracture of Metals, Massachusetts Institute of Technology; New York: John Wiley and Sons, p. 139, (1950).

3. C. E. Inglis, Trans. Inst. Naval Arch., Vol. 55, No. 1, p. 219.

4. L. Föppl, Ing.-Arch., Vol. 7, p. 229. (1936).

5. H. Neuber, "Kerbspannungslehre", Berlin: Julius Springer, p. 142 ff, (1937).

6. E. Orowan, "Fracture and Strength of Solids", in Report on Progress in Physics, Physical Society of London, Vol. 12, p. 185, (1949).

7. C. F. Tipper, Report of Conference on Brittle Fracture in Steel Plates, Cambridge, England, British Iron and Steel Research Association, p. 24, (1945).