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AN ANALYSIS OF THE SIDE-OUTLET TEE FOR IMPEDANCE MEASUREMENTS

by

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AN ANALYSIS OF A SIDE-OUTLET TEE FOR IMPEDANCE MEASUREMENTS

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LIST OF SYMBOLS

E_{in} is the complex amplitude of a wave incident on arm n of the tee.

E_{on} is the complex amplitude of a wave emergent from arm n of the tee.

S_{nk} is the complex amplitude transfer coefficient, which relates the amplitude of a wave incident on arm n of the tee to that emergent from arm k.

R_n is the complex reflection coefficient of a terminating load on arm n.

P is power.

* An asterisk on any of the complex symbols denotes a complex conjugate. Thus, for example, S^*_{nk} is the complex conjugate of S_{nk}.

INTRODUCTION

The waveguide side-outlet tee consists of an E-plane tee and an H-plane tee arranged as shown in Fig. 1. If all four arms of such a tee were coupled to matched generators or matched loads, then, by virtue of the symmetrical construction, a wave coupled into arm 3 will divide between arms 1 and 2, but will not cross-couple into arm 4. This property enables use of the tee as an impedance bridge.

When the tee is used as an impedance bridge, power is coupled into arm 3 or 4 from the r-f source, and out of arm 4 or 3 to the detector. A matched termination is placed on one of the two side arms and the test piece on the other. Since there is no cross-coupling between arms 3 and 4, and since the reference termination is a reflectionless matched load, the only power coupled to detector is that reflected from the test piece. Hence, the power delivered to the detector is a measure of the test piece reflection coefficient. If the test piece were a matched load, then the bridge would be balanced and no power would be delivered to the detector.

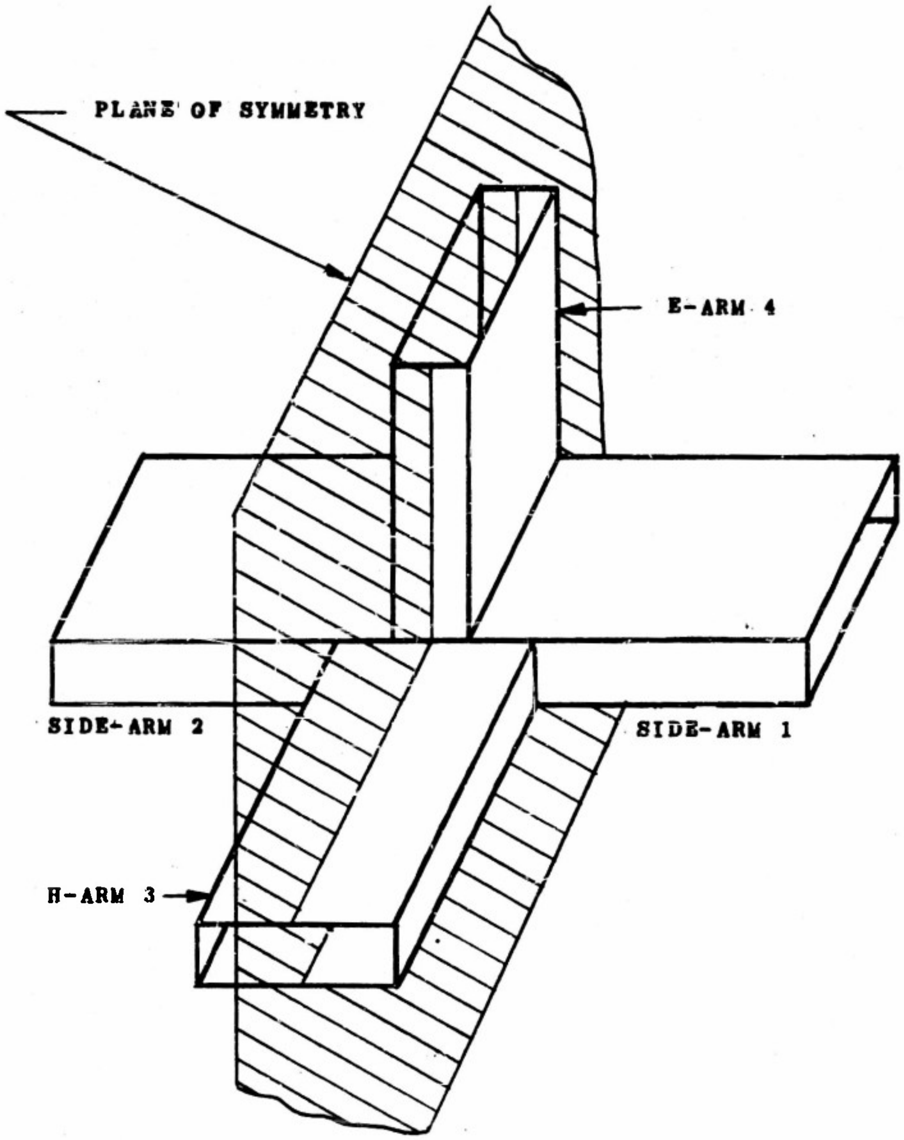
It should be emphasized that the side-outlet tee is not a bridge like the Wheatstone's bridge, which obeys the equation

$$AB = CD. \qquad (1)$$

Here A, B, C and D are four impedances, three of which are known and the fourth is unknown. The side-outlet tee, on the other hand, obeys the equation

$$A = B, \qquad (2)$$

where one of the two impedances, A and B, is known and the other unknown. The important difference between the two bridges is that for a bridge of the type (1) it is possible to match any impedance by using a fixed resistance plus variable reactances, while in the side-outlet tee



THE SYMMETRICAL SIDE-OUTLET TEE

FIG. 1

it is necessary to provide an adjustable reference impedance which will exactly equal the unknown. The latter requirement demands that the reference load should have a variable reactance as well as a variable resistance. This condition is difficult to fulfil at microwave frequencies. Therefore, in general, the side-outlet tee is used to measure merely the magnitude of the reflection coefficient of an unknown load. Thus it is desirable that the power delivered to the detector of a side-outlet tee bridge be mainly dependent on the magnitude of the reflection coefficient of the load.

However, even with matched terminations on all arms of a side-outlet tee, the impedance seen looking into the junction through the test arm will not be a match. Therefore, when the test piece is not a matched load, there will be interaction between the tee and the test piece. This makes the power delivered to the detector dependent on the phase, as well as magnitude, of reflection from the test piece. The dependence on phase introduces errors in the measurement of the magnitude of the reflection coefficient, and is, therefore, undesirable. In the symmetrical side-outlet tee, this trouble can be alleviated by certain procedures which change the simple tee into a so-called "magic" tee.

Such a bridge was being used at the University of California to measure the back-scattering from obstacles, and it was found that the tee was sensitive to the phase of reflection from the test piece in spite of the fact that the conditions for making the tee "magic" were complied with. Consequently, this problem of analysis was undertaken to understand, and eliminate, if possible, the causes of errors in measurements made with the tee.

The power delivered to the detector of the bridge can be dependent on the following five factors:

1. Frequency.
2. The degree of symmetry in the tee construction.
3. Magnitude of the test load reflection coefficient.
4. Phase of the test load reflection coefficient.
5. Impedance seen looking into the test arm with matched loads on the other arms.

The first two of these factors are independent variables on which the other three factors depend; and for any particular tee the constructional symmetry is non-varying.

As mentioned before, it is desirable that the power delivered to the detector be dependent solely on the magnitude of the load reflection. Therefore the dependence on the other four factors introduces errors in measurements. The analysis of the ideally symmetrical tee has been ably presented by Dr. Louis B. Young[1]* and others[2,3,4,5]. That analysis, however, does not hold for a tee which is not perfectly symmetrical. Hence, in this report, an analysis of a general four-arm junction is made and the asymmetrical side-outlet tee is then treated as a special case.

The scattering matrix representation is used to describe the fields associated with the junction. For a detailed explanation of this representation the reader is referred to "Principles of Microwave Circuits" by Montgomery, Dicke, and Purcell, M.I.T. Rad. Lab. Series, Vol. 8, McGraw-Hill, New York, 1948, and to "Waveguide Handbook" by N. Marcuvitz, M.I.T. Rad. Lab. Series, Vol. 10, McGraw-Hill, New York, 1951.

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* All references are listed at the end of this report.

The writer is greatly indebted to Dr. Leonard J. Black for suggesting this problem and helping in its analysis. Dr. John Whinnery, Dr. Nathan Marcuvitz of the Polytechnic Institute of Brooklyn, and Dr. D. J. Angelakos also helped in the theoretical aspects of this problem. Mr. Fred Clapp gave invaluable technical assistance in the experimental aspects. Thanks are also due to Mr. J. D. Axtell, Jr. and the other members of the staff of the Electronics Research Laboratory at the University of California, where this research was carried out, for their criticisms and suggestions.

THEORY

General

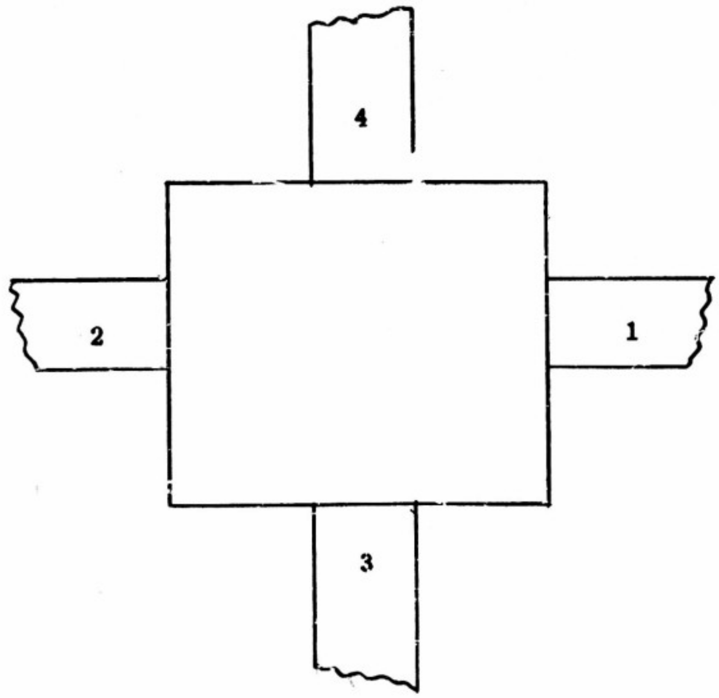
Consider a lossless junction made up of four waveguides having the same characteristic impedances. Nothing else will be assumed of this junction. The junction arms are labelled as shown in Fig. 2. Coupling the arms of this junction to generators and loads would result in waves incident on the junction through each arm and waves coming out of the junction through each arm. The amplitude of the wave out of any arm may be related to the amplitudes of the waves incident on all arms by a simple linear combination of the input amplitudes as follows:

$$E_{on} = \sum_{k=1}^{4} E_{ik} S_{nk} ,$$

where E_{on} is proportional to the amplitude of the wave out of the n^{th} arm, E_{ik} is proportional to the amplitude of the wave incident on the k^{th} arm, and S_{nk} is a constant, complex, amplitude transfer coefficient which relates the wave amplitude out of arm n to the one incident on arm k. This constant is determined by the junction's physical structure and the frequency of its operation. For k equal to n the coefficient S_{nn} is the complex reflection coefficient seen looking into the junction through the n^{th} arm. Thus, for example, the amplitude of the wave out of arm 4, that is, n = 4, is

$$E_{o4} = E_{i1}S_{41} + E_{i2}S_{42} + E_{i3}S_{43} + E_{i4}S_{44} .$$

Similarly, equations can be written for the other three arms. These equations can also be expressed in a matrix notation wherein the constants S_{nk} will form the following matrix:



A GENERAL FOUR-ARM JUNCTION

FIG. 2

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

This is the scattering matrix of a four-arm junction. Since the junction is made up of guides having identical characteristic impedances, reciprocity conditions will be satisfied by this matrix. Therefore

$$S_{nk} = S_{kn} \ .$$

Hence

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix}$$

For all further considerations in this paper, this form of representation of S will be used.

The scattering matrix of any lossless junction is a unitary matrix. A unitary matrix is defined as that matrix which when multiplied by its complex conjugate gives the product unity. In other words, a matrix S is unitary when

$$SS^* = 1 \ ,$$

where S^* is the complex conjugate of S. The "magic" tee is defined by the scattering matrix:

$$S = \begin{bmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix}$$

If a generator is now connected to arm 3, a detector having complex reflection coefficient R_4 to arm 4, and two loads having complex reflection coefficients R_1 and R_2 on arms 1 and 2 respectively, then the amplitude of the wave delivered to the detector will be

$$E_{o4} = E_{i1}S_{14} + E_{i2}S_{24} + E_{i3}S_{34} + E_{i4}S_{44}$$

where

$$E_{i1} = E_{o1}R_1$$

$$E_{i2} = E_{o2}R_2$$

and

$$E_{i4} = E_{o4}R_4$$

Normalize the terms so that

$$E_{i3} = 1$$

Therefore

$$E_{o4} = E_{o1}R_1S_{14} + E_{o2}R_2S_{24} + S_{34} + E_{o4}R_4S_{44}$$

But

$$E_{o1} = E_{i1}S_{11} + E_{i2}S_{12} + E_{i3}S_{13} + E_{i4}S_{14}$$

$$= E_{o1}R_1S_{11} + E_{o2}R_2S_{12} + S_{13} + E_{o4}R_4S_{14}$$

Hence

$$E_{o1}(1-R_1S_{11}) = S_{13} + E_{o2}R_2S_{12} + E_{o4}R_4S_{14}$$

Similarly

$$E_{o2}(1-R_2S_{22}) = S_{23} + E_{o1}R_1S_{12} + E_{o4}R_4S_{24}$$

and

$$E_{o4}(1-R_4S_{44}) = S_{34} + E_{o1}R_1S_{14} + E_{o2}R_2S_{24}$$

The solution of the preceding three equations results in

$$E_{o4} = \frac{A}{B}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where

$$A = \left\{\begin{array}{l} R_1S_{13}S_{14} + R_2S_{23}S_{24} + S_{34}(1-R_1S_{11})(1-R_2S_{22}) \\ + R_1R_2\,[S_{14}(S_{12}S_{23}-S_{22}S_{13}) + S_{24}(S_{12}S_{13}-S_{11}S_{23})] \\ - R_1R_2S_{12}^2S_{34} \end{array}\right\}$$

and

$$B = \left\{\begin{array}{l} (1-R_1S_{11})(1-R_2S_{22})(1-R_4S_{44}) - S_{12}^2R_1R_2 \\ - R_1R_2R_4[S_{14}(S_{12}S_{24} - S_{22}S_{14}) + S_{24}(S_{12}S_{14}-S_{11}S_{24})] \\ + R_1R_2R_4S_{12}^2S_{44} - R_4(R_1S_{14}^2+R_2S_{24}^2) \end{array}\right\}$$

If R_1 is the reflection coefficient seen looking into an unknown load and $R_2=R_4=0$ (that is, the reference load and detector are matched), then equation (1) becomes

$$E_{o4} = \frac{S_{13}S_{14}R_1 + S_{34}(1-S_{11}R_1)}{1-S_{11}R_1}$$

$$= S_{34} + \frac{S_{13}S_{14}R_1}{1-S_{11}R_1}. \quad . \quad . \quad . \quad . \quad (2)$$

The power delivered to the detector is proportional to the square of the magnitude of E_{o4}, and the power incident on the tee through arm 3 is proportional to the square of the magnitude of E_{13}. Now, if the same matched detector were used to measure the input and the output power, the constants of proportionality in the two cases would be identical. In the preceding derivations, all the wave amplitude terms have been normalized with respect to E_{13}. The power output will be similarly normalized with respect to input. Thus the normalized power delivered to the detector on arm 4 is

$$P = \left| E_{04} \right|^2 = \left| S_{34} + \frac{S_{13} S_{14} R_1}{1 - S_{11} R_1} \right|^2$$

and is thus dependent both on the magnitude and phase of R_1. The phase-sensitivity is, of course, undesirable from the practical point of view. Therefore, the possibility of eliminating the phase-sensitivity by reducing the values of S_{34} and $S_{11}R_1$ to zero, must be considered. Before going further, it is worthwhile now to examine an ideally symmetrical side-outlet tee.

The Ideally Symmetrical Side-outlet Tee

The defining characteristics of the symmetrical side-outlet tee are that the coupling between arms 3 and 2 is identical to that between arms 3 and 1; and the coupling between arms 1 and 4 is equal in magnitude to that between arms 2 and 4, but they are 180° out of phase with each other. And, of course, the reflection coefficient S_{11} seen looking into arm 1 with matched loads on the other three arms is equal to its counterpart S_{22}. Thus, for the symmetrical side-outlet tee

$$S_{11} = S_{22}$$

$$S_{23} = S_{13}$$

and

$$S_{24} = -S_{14}$$

Therefore the scattering matrix,

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{11} & S_{13} & -S_{14} \\ S_{13} & S_{13} & S_{33} & S_{34} \\ S_{14} & -S_{14} & S_{34} & S_{44} \end{bmatrix} \quad \cdots\cdots \text{I}$$

It has been proved by Dr. Louis B. Young[1] that for a symmetrical, lossless side-outlet tee the following six statements are true:

Statement I: If a tee is matched looking into arms 3 and 4, there is an equal power split between arms 3 and 4 for power reflected toward the tee from a load on arm 1. That is, if

$$S_{33} = S_{44} = 0$$

$$|S_{14}| = |S_{13}|$$

Statement II: Even if the E- and H- arms (that is, arms 3 and 4) do not appear matched looking into the tee, there is no cross-coupling between the E- and H- arms. That is, even if

$$S_{33} \neq 0$$

and

$$S_{44} \neq 0$$

$$S_{34} = 0$$

Statement III: If the E- and H- arms appear matched looking into the tee, the side-arms appear matched looking into the tee, and there is no cross-coupling between them. That is, if

$$S_{33} = S_{44} = 0$$

then

$$S_{11} = S_{12} = S_{22} = 0$$

Statement IV: If a tee is matched looking into the side arms, there is an even power split between arms 3 and 4 for power reflected from a load on arm 1. That is, if

$$S_{11} = S_{22} = 0$$

then $$\left| S_{13} \right| = \left| S_{14} \right|$$

Statement V: If a tee is matched looking into the side arms, the magnitudes of the reflection coefficients seen looking into the E- and H-arms are equal, but not necessarily zero. That is, if

$$S_{11} = S_{22} = 0$$

then $$S_{33} = S_{44}$$

Statement VI: If a tee is matched looking into the side arms but not matched looking into the E- and H- arms, there can be cross-coupling between the side arms. That is, if

$$S_{11} = S_{22} = 0$$

but $$S_{33} = S_{44} \neq 0$$

then $$S_{12} \neq 0$$

Applying the conditions of symmetry and Statement II to equation (1) results in the amplitude of the wave delivered to an unmatched detector on arm 4 of the tee with unmatched loads on arms 1 and 2 being

$$E_{o4} = \frac{S_{13} \; S_{14} \; (R_1 - R_2)}{\left[\begin{array}{l} (1-S_{11}R_1)(1-S_{11}R_2)(1-S_{44}R_4) - S_{12}^2 R_1 R_2 + \\ R_1 R_2 R_4 \; (S_{12}^2 S_{44} + 2S_{12} S_{14}^2 + 2S_{11} S_{14}^2) - R_4 S_{14}^2 (R_1 + R_2) \end{array} \right]} \qquad (3)$$

The matrix I becomes

$$S = \begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{11} & S_{13} & -S_{14} \\ S_{13} & S_{13} & S_{33} & 0 \\ S_{14} & -S_{14} & 0 & S_{44} \end{pmatrix} \quad \text{---------- II}$$

If the detector were matched so that $R_4 = 0$, then equation (2) would become

$$E_{o4} = \frac{S_{13} S_{14} (R_1 - R_2)}{(1-S_{11}R_1)(1-S_{11}R_2) - S_{12}^2 R_1 R_2}$$

This equation would be greatly simplified if

$$S_{11} = S_{12} = 0$$

By Statement III, this would be possible only if

$$S_{33} = S_{44} = 0$$

whereby $S_{11} = S_{22} = S_{12} = 0$

Thus, for $R_4 = 0$

and $S_{33} = S_{44} = 0$

$$E_{o4} = S_{13} S_{14} (R_1 - R_2)$$

and the matrix

$$S = \begin{pmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{pmatrix}$$

This, by definition, is the matrix of a "magic" tee.

Dr. Young[1] has also shown that for a lossless and symmetrical side-outlet tee

$$\left| S_{13} \right| = \left(\frac{1 - \left| S_{33} \right|^2}{2} \right)^{\frac{1}{2}}$$

and

$$\left| S_{14} \right| = \left(\frac{1 - \left| S_{44} \right|^2}{2} \right)^{\frac{1}{2}}$$

Thus, for $S_{33} = S_{44} = 0,$ and $R_4 = 0$

$$\left| S_{13} \right| = \frac{1}{\sqrt{2}}$$

$$\left| S_{14} \right| = \frac{1}{\sqrt{2}}$$

and

$$\left| E_{o4} \right| = \frac{1}{2} \left| R_1 - R_2 \right|$$

Hence the normalized power

$$P = \left| E_{o4} \right|^2 = \frac{1}{4} \left| R_1 - R_2 \right|^2$$

If the reference load were a matched termination

$$R_2 = 0$$

and

$$P = \frac{1}{4} \left| R_1 \right|^2$$

Thus it is evident that for a perfectly symmetrical and lossless side-outlet tee, it is possible to match the tee so that it is "magic" and the normalized power delivered to the detector is a simple function of the load reflection coefficient. Moreover, the power is not dependent on the phase of the load reflection coefficient. For an inherently asymmetrical tee, however, it is not possible to obtain the "magic" match. But subsequent analysis of an asymmetrical tee will show how and when it is possible to obtain matching conditions which will enable the power out of arm 4 to be dependent only on the magnitude of the load reflection coefficient.

The Asymmetrically Constructed Side-outlet Tee

It has already been shown that for a four-arm junction with arm 3 coupled to a generator, arm 4 to a matched detector, and arm 2 to a matched load, the amplitude of the wave delivered to the detector is

$$E_{o4} = S_{34} + \frac{S_{13}\, S_{14}\, R_1}{1 - S_{11}\, R_1}$$

where R_1 = complex reflection coefficient of load on arm 1. The evident sensitivity to the phase of R_1 could be removed if S_{34} and S_{11} could be reduced to zero.

Supposing arm 1 is coupled to a matched load, and arm 4 to a matched detector. That is,

$$R_1 = R_4 = 0$$

but

$$R_2 \neq 0$$

Then

$$E_{o4} = S_{34} + \frac{S_{23}\, S_{24}\, R_2}{1 - S_{22}\, R_2}$$

The load on arm 2 can be chosen so that

$$E_{o4} = S_{34} + \frac{S_{23}\, S_{24}\, R_2}{1 - S_{22}\, R_2} = 0$$

That is

$$R_2 = \frac{S_{34}}{S_{22}S_{34}-S_{23}S_{24}}$$

If this load were maintained on arm 2, a matched load coupled to arm 3, a certain desired detector-load to arm 4, and the tee coupled to a generator through arm 1, then the load on arm 4 could be chosen so that there would be no reflections seen looking into arm 1. Thus, with

$$R_2 = \frac{S_{34}}{S_{22}S_{34}-S_{23}S_{24}}$$

and

$$R_3 = 0$$

it can be shown that for no reflections seen looking into arm 1, the reflection coefficient of the detector-load on arm 4 should be

$$R_4 = \frac{S_{34}S_{12}^2-S_{11}S_{23}S_{24}}{\left[\begin{array}{l} S_{34}S_{44}S_{12}^2-2S_{12}S_{34}S_{14}S_{24}+S_{23}S_{24}S_{14}^2 \\ -S_{11}S_{23}S_{24}S_{44}+S_{11}S_{34}S_{24}^2 \end{array}\right]}$$

If, with these two predetermined loads on arms 2 and 4, arm 3 were coupled to a matched generator, and arm 1 to the unknown load, the magnitude of the wave delivered to the detector would be

$$E_{o4} = \frac{R_1\alpha}{\beta}$$

where

$$\alpha = S_{12}S_{34}(S_{14}S_{23}+S_{13}S_{24}-S_{12}S_{34}) - S_{13}S_{14}S_{23}S_{24}$$

and

$$\beta = -S_{23}S_{24} + \frac{S_{24}(S_{23}S_{44}-S_{24}S_{34})(S_{34}S_{12}^2-S_{11}S_{23}S_{24})}{\left[\begin{array}{l} S_{12}S_{34}(S_{12}S_{44}-S_{14}S_{24})+S_{14}S_{24}(S_{14}S_{23}-S_{12}S_{34}) \\ +S_{11}S_{24}(S_{24}S_{34}-S_{23}S_{44}) \end{array}\right]}.$$

It should be emphasized that all S_{nk} are dependent only on the frequency and physical characteristics of the tee. Therefore they are all complex constants. Therefore α and β are also constants. Thus, power delivered to the detector is

$$P = \left|E_{o4}\right|^2 = \frac{\left|T_1\right|^2 \left|\alpha\right|^2}{\left|\beta\right|^2}$$

and it is no longer dependent on the phase of reflection from the unknown load; thus the power measured would be a correct indication of the magnitude of the reflection coefficient (therefore, VSWR) of the load.

In general, this elimination of phase-sensitivity is sufficient for measurements. However, sometimes it is desirable to use a calibrated attenuator between the generator and the input arm 3 of the bridge. Since the attenuators are usually calibrated for matched terminations, it becomes necessary to match the tee looking into arm 3. This can be accomplished by tuning arm 3. However, if this is done after the phase-sensitivity of the tee is removed by the foregoing procedure, due to the coupling between arms 3 and 1, the tuning of arm 3 will result in reflections seen looking into arm 1. Therefore another procedure must be devised.

It can be shown that for any lossless junction of four waveguide arms whose characteristic impedances are identical, it is possible to match two arms for no reflection seen looking into them, and to de-couple one of these from one of the other two. However, if the junction were not a directional coupler or side-outlet tee, the process of such matching would reduce the four-arm junction to a two-arm one. In the case of the side-outlet tee, however, the elimination of the coupling between arms 3 and 4 and the matching of arms 3 and 2 will result in matching all arms and also decoupling arms 1 and 2. This does not mean that a "magic" tee can be accomplished from an asymmetrical tee. There will still be a difference between the two. For, in a "magic" tee

$$|S_{13}| = |S_{14}| = |S_{23}| = |S_{24}| = \frac{1}{\sqrt{2}}$$

but for a matched asymmetrical tee

$$|S_{13}| = |S_{14}| = \sqrt{1 - |S_{24}|^2} = \sqrt{1 - |S_{23}|^2}$$

The experimental procedure for accomplishing this desirable condition would be as follows (See Fig. 3):

1) Couple arm 3 to a slide-screw tuner and then to a standing-wave detector followed by the power source.

2) Couple arm 1 to a matched load, arm 2 to a tuner followed by a matched load, and arm 4 to a tuner followed by a matched detector.

3) Tune arm 2 to eliminate output out of arm 4. This is essentially choosing R_2 to decouple arms 3 and 4, as described heretofore.

4) Tune arm 3 to eliminate reflections seen looking into arm 3. Since this arm is already decoupled from arm 4, this tuning will in no way affect the accomplished decoupling.

5) Remove arm 3 with its tuner from the standing-wave detector and couple a matched load to it. Then remove the matched load from arm 1 and couple this arm to the standing-wave detector. Thus, in effect, arm 3 with its tuner will have exchanged positions with arm 1.

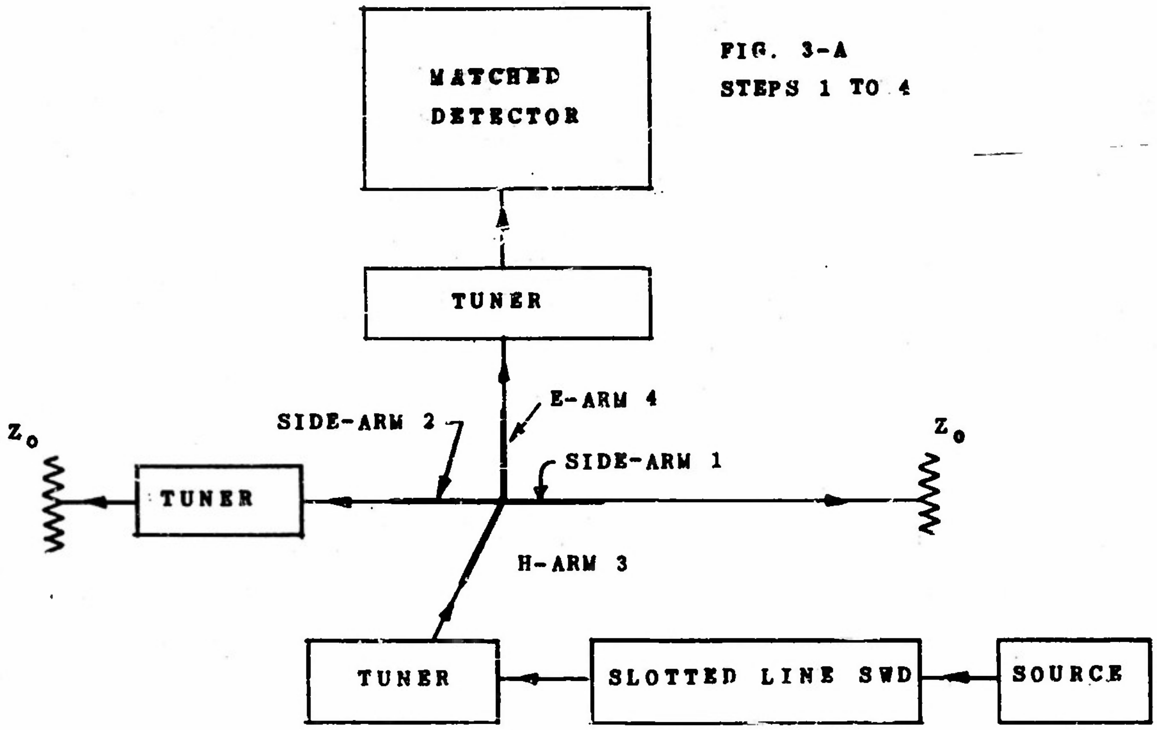


FIG. 3-A
STEPS 1 TO 4

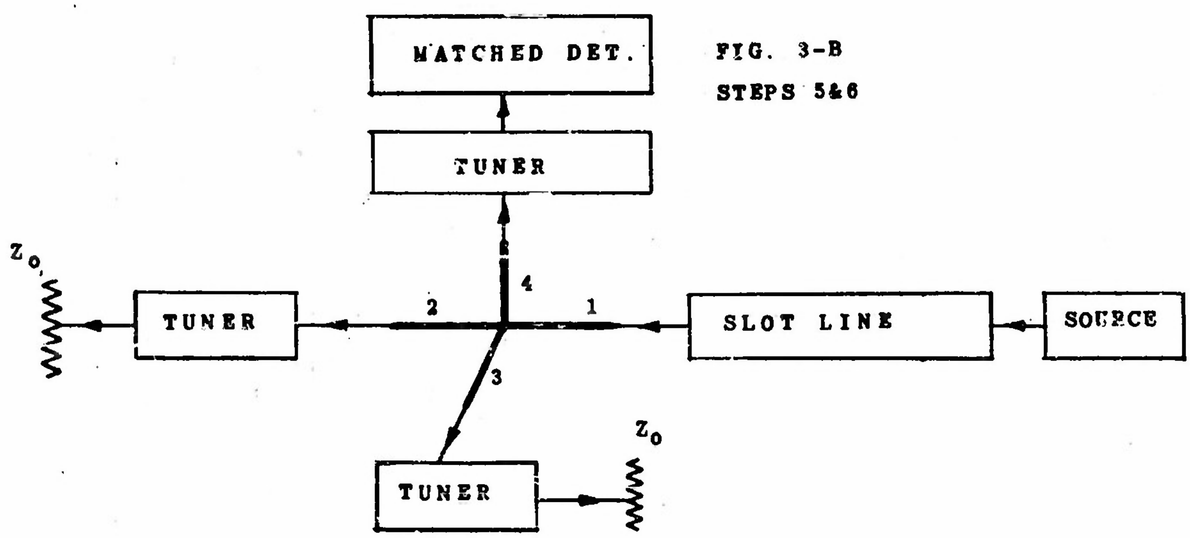


FIG. 3-B
STEPS 5&6

6) Tune arm 4 so that a match is seen looking into arm 1. Again, since arms 3 and 4 are already decoupled, this tuning of arm 4 will have no effect on the decoupling.

Through the above procedure, essentially the following is accomplished:

A tee with a scattering matrix

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix}$$

is first transformed to one with matrix

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & 0 \\ S_{14} & S_{24} & 0 & S_{44} \end{bmatrix}$$

then to

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & 0 \\ S_{14} & S_{24} & 0 & S_{44} \end{bmatrix}$$

and finally to

$$S = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & 0 \\ S_{14} & S_{24} & 0 & S_{44} \end{bmatrix}$$

Since the junction is lossless, power incident on arm 3 is equal to the sum of the powers coupled to the other three arms. In matrix notation this is expressed by a unitary scattering matrix which, as defined before, is such that its product with its own conjugate is unity. Due to this property, the following two theorems hold true for a unitary matrix:

Theorem I - The sum of the squares of the absolute magnitudes of the terms in any row or column is equal to unity. That is,

$$\sum_{k=1}^{4} \left| S_{nk} \right|^2 = 1$$

Theorem II - For any pair of rows (or columns), the sum of the products of each term in one row (or column), with conjugate of the corresponding term in the other row (or column), is equal to zero. That is,

$$\sum_{k=1}^{4} S_{n_1 k} S^*_{n_2 k} = 0$$

where n_1 and n_2 represent two separate rows (or columns), and $S^*_{n_2 k}$ is the conjugate of $S_{n_2 k}$.

Applying these two theorems to the last of the preceding matrices results in

$$S = \begin{bmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & 0 \\ S_{14} & S_{24} & 0 & 0 \end{bmatrix}$$

It should be pointed out that in order to use this tee for correct impedance measurements, like the "magic" tee, it should be coupled to a matched generator. For, during the procedure of tuning S_{11} to zero by Step 5, the H- arm was coupled to a matched load. Therefore, in order to maintain $S_{11} = 0$, arm 3 should be coupled to a matched generator.

To summarize, an asymmetrical side-outlet tee can be tuned to eliminate the sensitivity of the power delivered to the detector to the phase of reflection from the test load. Furthermore, the tee can also be tuned for a match looking into the input arm (H-arm 3), thereby achieving a totally matched but not a "magic" tee.

It should be emphasized again that the foregoing theoretical analysis applies only to junctions which are lossless and whose arms have the same characteristic impedance. Moreover, the matching accomplished by the foregoing procedure will hold only at constant frequencies; therefore, with a change in frequency, the matching procedures will have to be repeated.

EXPERIMENT

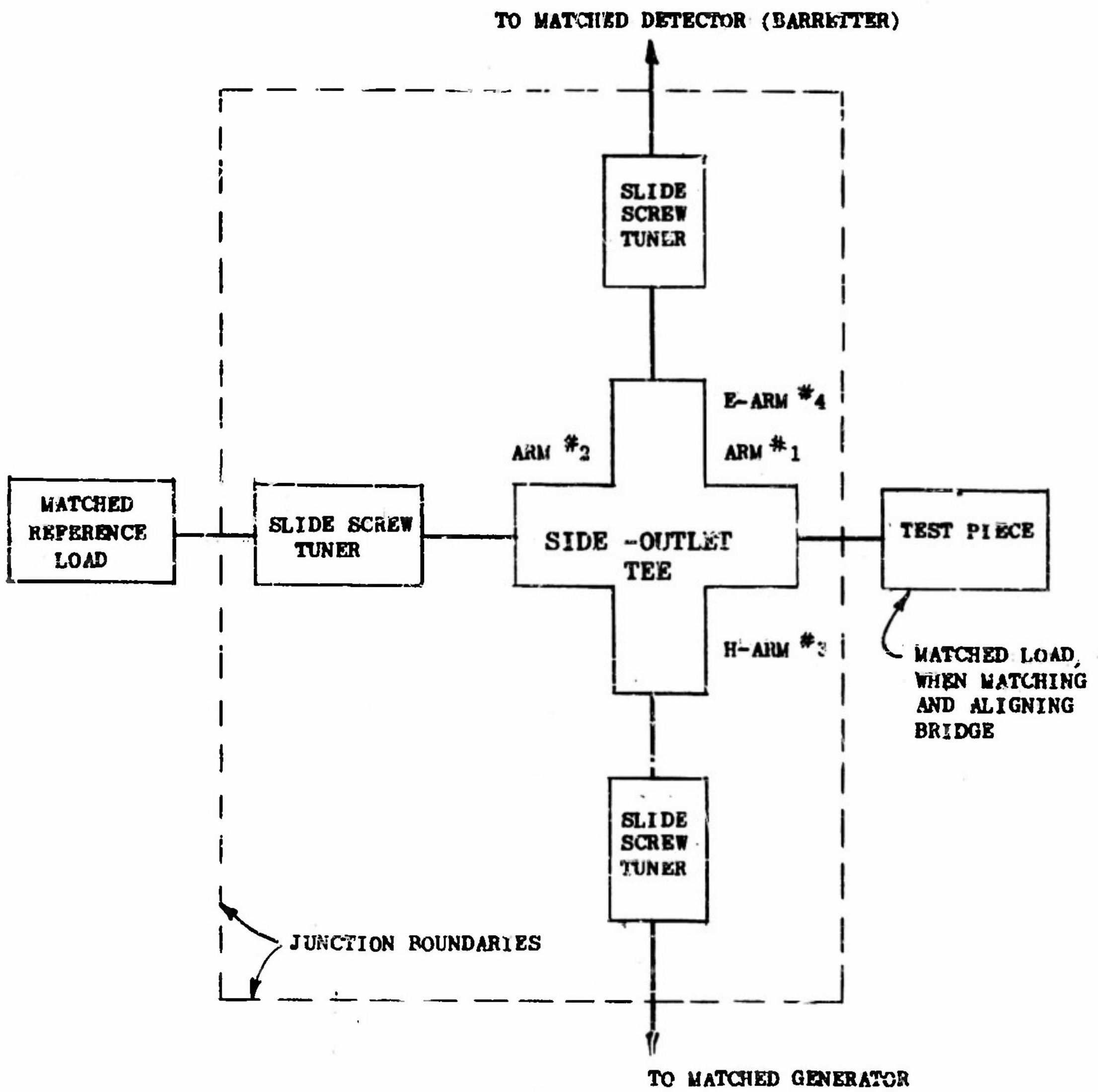
Procedure

Three arms (arms 1,3, and 4) of a side-outlet tee were coupled to slide-screw tuners. Then, with arm 3 as the input arm, matched loads on arms 1 and 2, and a matched detector on arm 4, as shown in Fig. 1, the tuner on arm 2 was tuned to eliminate the power delivered to the detector. This, in effect, decouples arm 3 from arm 4, and vice versa. It was not possible to decouple arms 3 and 4 beyond 66 db. After this was done, it was found -- as per theoretical expectations - that any adjustment of the tuners in arms 3 or 4 did not in any way alter the decoupling which was already accomplished. Next, the tuner in arm 3 was tuned to obtain a reflectionless match looking into arm 3. It was not possible to obtain a VSWR of less than 1.02.

Then, arm 3 with its tuner was decoupled from the generator and arm 1 was coupled to the generator. With a matched load used as termination for arm 3, the tuner in arm 4 was tuned to obtain a match looking into arm 1. Here again it was not possible to obtain a VSWR of less than 1.02.

Thus the coupling between arms 3 and 4 - in other words, S_{34} - was eliminated by the first process, S_{33} by the second, and S_{11} by the third. The resulting junction, shown in Fig. 1, included the three tuned tuners in arms 2, 3, and 4, and has a scattering matrix

$$S = \begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{pmatrix}$$



EXPERIMENTAL SET-UP FOR IMPEDANCE MEASUREMENTS USING A SIDE-OUTLET TEE.

where S_{11}, S_{33}, and S_{34} are almost zero. The magnitude of the wave delivered to the detector is

$$E_{04} = S_{13} S_{14} R_1$$

where R_1 = Test load reflection coefficient.

Therefore the normalized power delivered to the detector is

$$P = \left| E_{04} \right|^2 = \left\{ \left| S_{13} \right| \left| S_{14} \right| \left| R_1 \right| \right\}^2$$

It should be noted that in the theoretical analysis which resulted in this equation, all the terms were normalized, such that the amplitude of the wave incident on arm 3 was considered unity. Therefore, this equation gives the ratio of power delivered to the detector on arm 4 to that incident from the generator on arm 3. Therefore the db. separation between the two is given by

$$N = 20 \log \left| S_{13} \right| + 20 \log \left| S_{14} \right| + 20 \log \left| R_1 \right|$$

This db. separation is equivalent to attenuation through the tee and can easily be measured by first, coupling a matched detector to a matched generator and recording the db. output; then, coupling the bridge arm 3 to the same generator and, for any load on arm 1 and matched loads on the others, the db. output from arm 4 can be noted. The difference between the two will be the db. separation required. It should be emphasized that in order to maintain the matching which makes S_{11} zero, the bridge should be coupled to a matched generator.

By this method, $|S_{13}|$ and $|S_{14}|$ were measured in dbs.; that is, the db. differences between the power into arm 1 and the outputs from arms 3 and 4 respectively, were measured. Similarly, $|S_{34}|$ was also measured. After the matching of the junction as described, and after the measurement of $|S_{13}|$, $|S_{14}|$, and $|S_{34}|$, arm 3 was coupled to a matched generator, arm 2 to a reference match load, and arm 4 to a matched and calibrated barretter-detector. Arm 1 was coupled consecutively to two types of loads.

Of one type were loads having reflection coefficients of constant magnitude but varying phase. Two loads of this type were used; one load with a constant VSWR of 1.68, and the other with a constant VSWR of 1.17. The second type of load had a reflection coefficient of constant phase but varying magnitudes. Only one load of this type was used. These loads were calibrated by means of a slotted line. The crystal-detector used for making VSWR measurements, with the slotted line, was also calibrated for accuracy.

Having measured the generator output, which is identically equal to the input through arm 3 of the bridge, and the power delivered to the detector on arm 4, the db. separation between the two was recorded. This db. separation is

$$N = 20 \log |R_1| + 20 \log |S_{13}| + 20 \log |S_{14}|$$

From this equation, $|R_1|$ (the only unknown quantity), was computed. The values of

$$VSWR = \frac{|R_1| + 1}{|R_1| - 1}$$

were then computed and compared with the values measured with the slotted line.

Data and Results

Frequency maintained constant at 9440 mcs.

Generator output reading = 5.1 dbs.

With arm 3 coupled to generator, and arms 1 and 2 to matched loads, output reading from arm 4 = 71 dbs.

With arm 1 coupled to generator and arms 2 and 4 to matched loads, output reading from arm 3 = 8.3 dbs.

With arm 1 coupled to generator and arms 2 and 3 to matched loads, output reading from arm 4 = 8.6 dbs.

Therefore, measured values of

$$20 \log |S_{34}| = 71-5.1 = 65.9 \text{ dbs.}$$

$$20 \log |S_{13}| = 8.3-5.1 = 3.2 \text{ dbs.}$$

and

$$20 \log |S_{14}| = 8.6-5.1 = 3.5 \text{ dbs.}$$

Using the calibration curves for the 8 ma. barretter-detector (which was used for power measurements), the correct values of

$$20 \log |S_{34}| = 67 \text{ dbs.}$$

$$20 \log |S_{13}| = 3.6 \text{ dbs.}$$

and

$$20 \log |S_{14}| = 3.9 \text{ dbs.}$$

Therefore,

$$20 \log |R_1| = N(\text{corrected}) - 3.6 - 3.9 \text{ dbs.}$$

With matched loads on the other arms

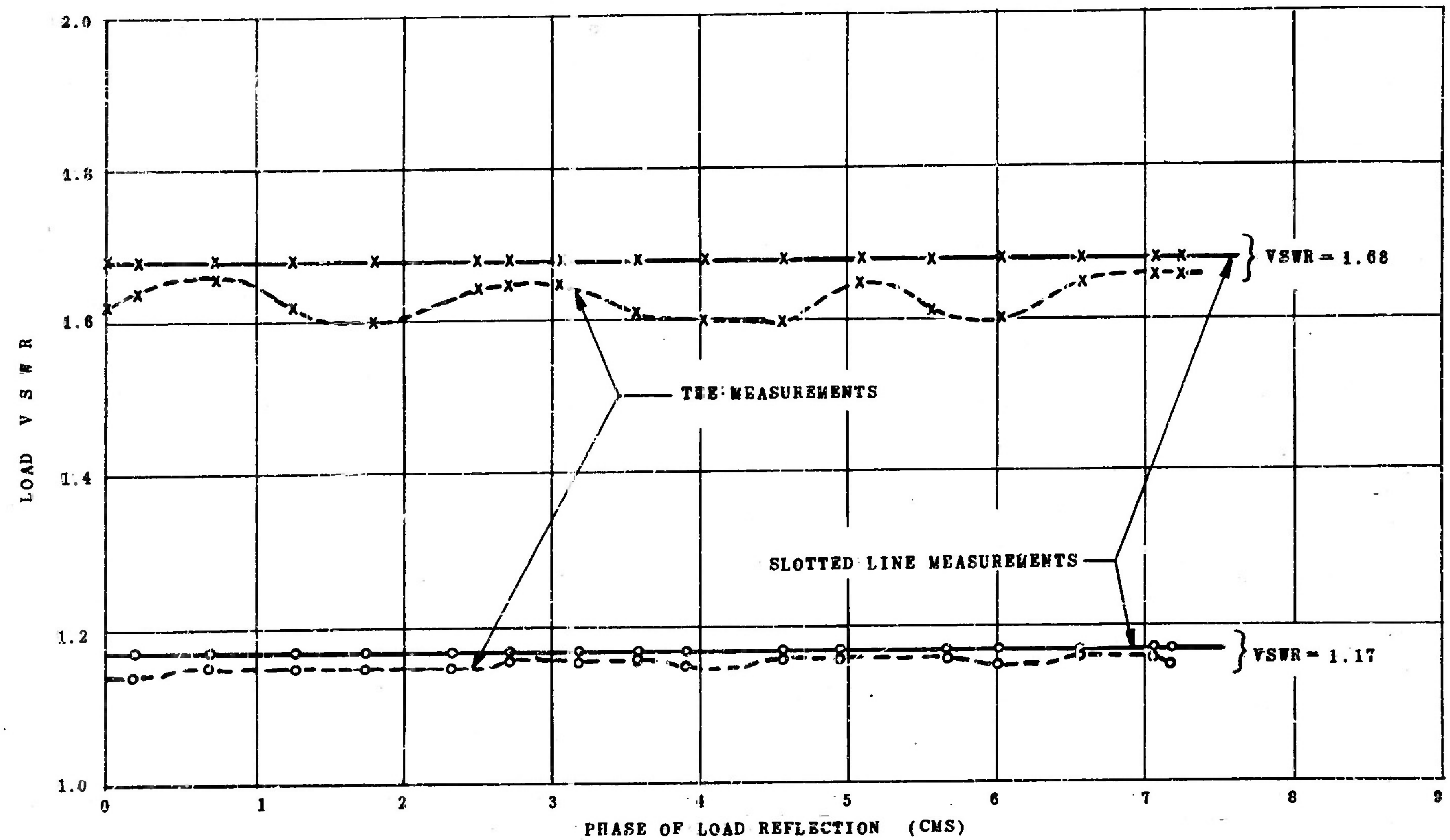
~~VSWR looking into arm 3 < 1.02 ;~~

VSWR looking into arm 1 < 1.02 .

The VSWR of the barretter-detector varies from less than 1.01 to 1.06.

The results of measurements on loads of the first type (having reflection coefficients of constant magnitude but varying phase), for two values of VSWR, are shown by the curves on page 30.

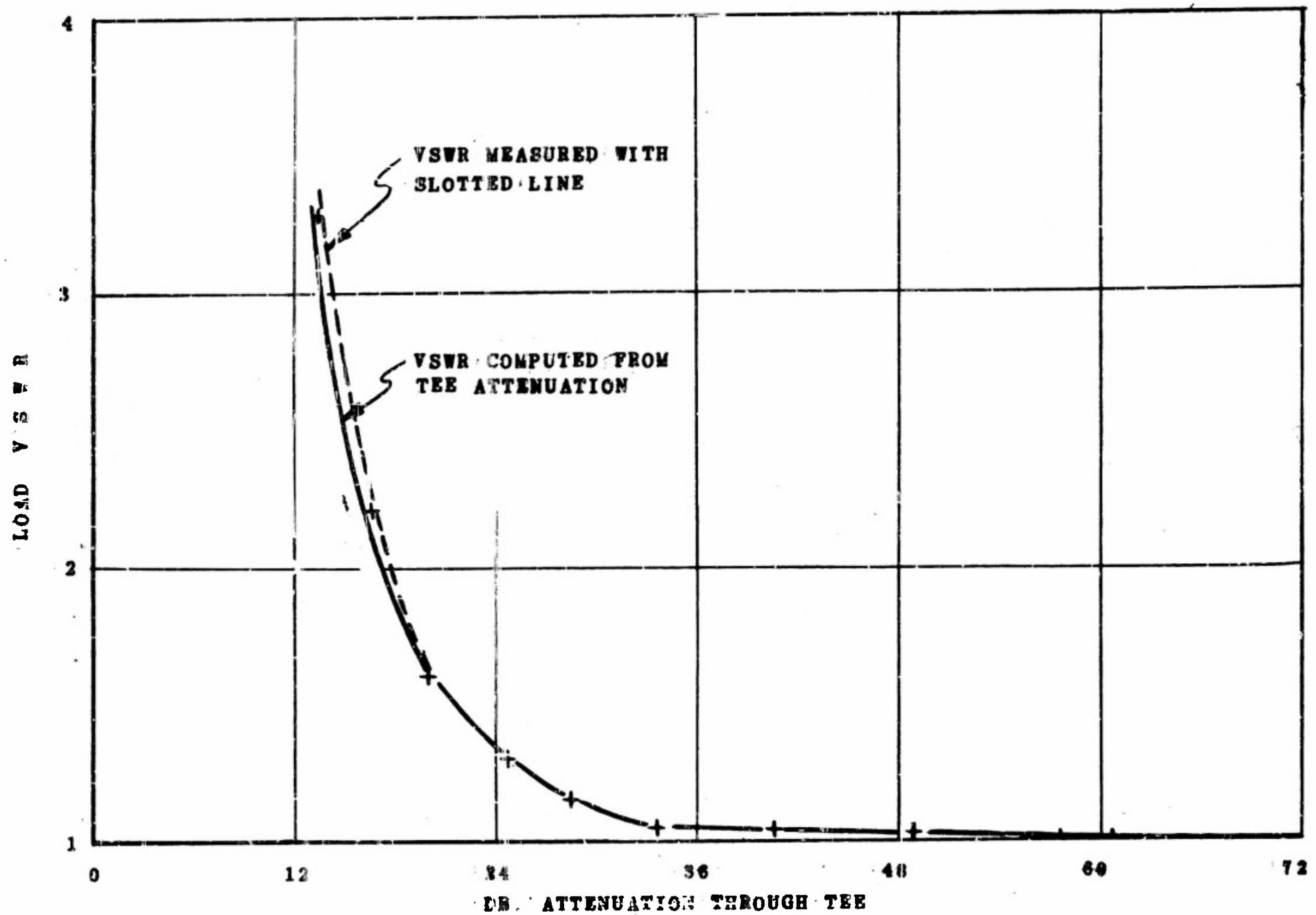
The results of measurements on a load that had a reflection coefficient of constant phase but varying magnitude are shown by the curves on page 31.



COMPARISON OF VSWR MEASUREMENTS MADE WITH TEE WITH THOSE MADE ON THE SLOTTED LINE.

(FREQUENCY = 9440 MCS.)

(TWO LOADS WITH REFLECTIONS OF CONSTANT MAGNITUDE)



VSWR OF LOAD (WITH CONSTANT PHASE OF REFLECTION)
VS
ATTENUATION THROUGH THE TEE

Discussion

The results show that the phase-sensitivity of the tee was almost removed by the matching procedures followed. For a load of constant VSWR equal to 1.68, the error in VSWR, due to phase sensitivity, was ± 2%; for the load of constant VSWR equal to 1.2 the same error was less than ± 0.05%. This relationship of the percentage error to the magnitude of the load VSWR shows that the major source of this negligible phase sensitivity was the term $S_{11} R_1$ in the equation

$$P = \left| E_{04} \right|^2 = \left| S_{34} + \frac{S_{13} S_{14} R_1}{1 - S_{11} R_1} \right|^2$$

Moreover, $\left| S_{34} \right|$ had already been measured to be 65.9 db. below the input power, and therefore could safely be neglected for even the smallest value of the load reflection coefficient, R_1. An experimental check further showed that with the tee coupled to the generator, S_{11} was not actually equal to zero (that is, the VSWR looking into arm 1 was not unity). The procedure of this experiment was to couple a good sliding short, as a load, to arm 1. Thus, $\left| R_1 \right|$ was almost equal to unity, and therefore S_{34} was negligible. This resulted in the equation

$$P = \left| \frac{S_{13} S_{14} R_1}{1 - S_{11} R_1} \right|^2 = \left| \frac{K_1 R_1}{1 - S_{11} R_1} \right|^2$$

where K_1 and S_{11} are complex constants and $\left| R_1 \right|$ is unity. The sliding short was then moved along the load arm. This was equivalent to varying the phase of R_1 while keeping the magnitude $\left| R_1 \right|$ constant at unity. This variation in the phase of the shorted load varied the power P, delivered to the detector, from a minimum to a maximum, and vice versa.

$$P_{max.} = \left\{ \frac{|K_1|\ |R_1|}{1 - |S_{11}|\ |R_1|} \right\}^2 = \left\{ \frac{|K_1|}{1 - |S_{11}|} \right\}^2$$

and

$$P_{min.} = \left\{ \frac{|K_1|\ |R_1|}{1 + |S_{11}|\ |R_1|} \right\}^2 = \left\{ \frac{|K_1|}{1 + |S_{11}|} \right\}^2$$

The separation between $P_{max.}$ and $P_{min.}$ was found to be equal to 1.2 db.

Therefore

$$10 \log \frac{P_{max.}}{P_{min.}} = 20 \log \left\{ \frac{1 + |S_{11}|}{1 - |S_{11}|} \right\} = 1.2 \text{ db.}$$

From this equation the VSWR looking into arm 1 was found to be 1.148. This discrepancy is mainly due to the fact that it was not experimentally possible to achieve a perfectly matched generator. Therefore, though S_{11} was made equal to zero with matched loads on arms 2, 3, and 4, when arm 3 is coupled to an imperfectly matched generator, S_{11} is no longer equal to zero. Another contribution to this error is from the detector (on arm 4), which does not stay well matched for all amounts of power delivered to it.

For the load having a reflection coefficient of constant phase it was found, as expected, that the power delivered to the detector was proportional to the square of the magnitude of load reflection coefficient. The measurements made with the tee compared very favorably with the slotted line measurements within the limits of experimental error.

In conclusion, it can be stated that the matching procedures suggested by theory and followed in the experiments, proved successful in eliminating the phase sensitivity which is the major cause of errors in impedance measurements (at one frequency) using the side-outlet tee.

<u>Frequency Sensitivity</u>

Until now, the effect of frequency on a side-outlet has been neglected. It has been mentioned heretofore that the scattering matrix of a junction is a function of the physical characteristics of the junction and the frequency at which it is used. Thus, for the same junction, a frequency shift necessarily means a change in the values of the terms in the scattering matrix.

The perfectly symmetrical tee does not need alignment to decouple the E- and H- arms, and therefore any frequency shift will not affect this decoupling. However, the VSWR's, looking into the four arms, do change with frequency. Calibration curves of such changes in a symmetrically constructed tee have been presented by Young,[1] who has also shown that, to obtain a match which would not appreciably be affected by frequency changes, the matching should be accomplished by elements placed as close to the junction as possible. Thus, different ways of matching have been devised[1] for different sizes of side-outlet tees.

For a non-symmetrical tee, however, the decoupling of the E- and H- arms is also necessary; and in this investigation it was accomplished, at one frequency, by means of slide-screw tuners placed on the junction arms. Since these tuning elements were situated away from the junction point, the decoupling and matching were found to be frequency sensitive. The following experiment was performed to determine this sensitivity.

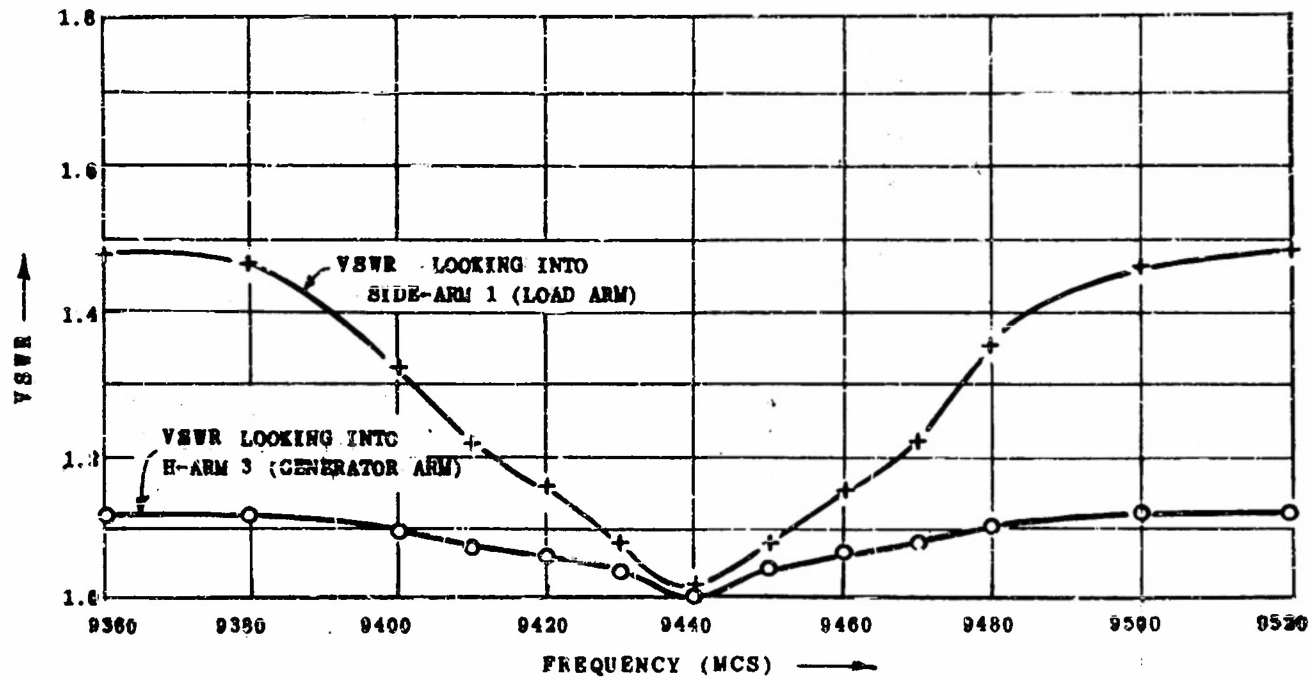
The tee was matched, as previously described, at a frequency of 9440 mcs. Then, for every change in frequency, the detector and the loads were matched for no reflections seen looking into them. With the matched loads on the side arms 1 and 2, the matched detector on arm 4, the VSWR looking into arm 3, and the coupling $|S_{34}|$ between arms 3 and 4, were measured. Similarly, with arm 1 coupled to the generator, arm 4 to the matched detector, and the other arms to matched loads, the VSWR looking into arm 1, $|S_{13}|$ (the coupling between 1 and 3), and $|S_{14}|$ (the coupling between 1 and 2) were measured. Their variations with frequency are shown by the curves on page 36.

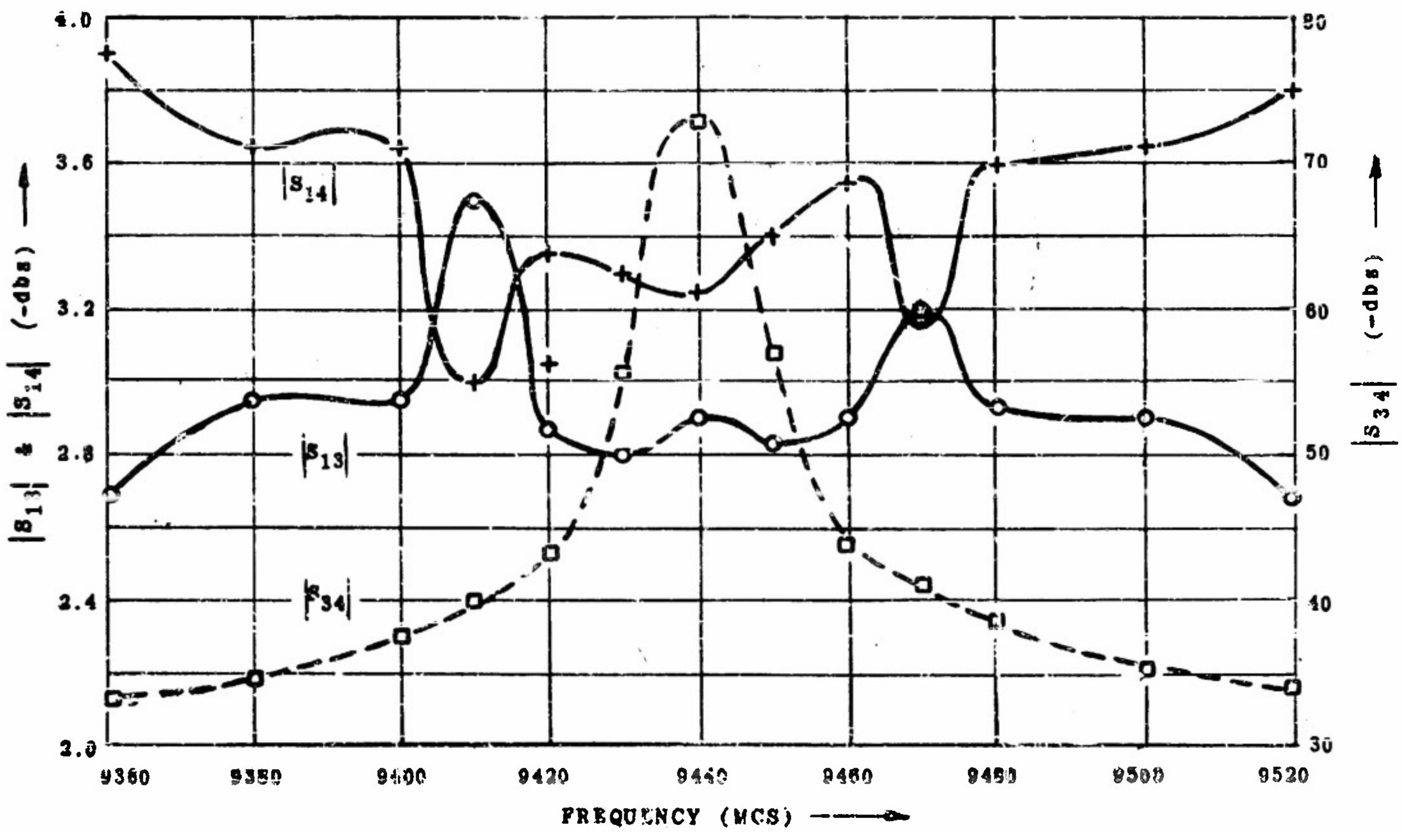
The results of the experiment show that a tee, matched by means of slide-screw tuners on its arms, is extremely frequency sensitive. With a frequency shift of less than ± 1%, the decoupling between arms 3 and 4 is changed by a factor of + 40 dbs., and the VSWR, looking into the load arm 1, is affected by almost 50%. The couplings between arms 1 and 3, and between arms 1 and 4, are also appreciably changed. It should be noted that the VSWR, looking into the input arm 3, was changed by a relatively small fraction. This was due to the fact that arm 3 of the tee under experiment was already partially matched by a post situated at the junction. This denotes the desirability of affecting all matching by means of elements situated as close to the junction as possible.

In order to examine the effects of these changes on load measurements, consider the equation

$$P = |E_{c4}|^2 = \left| S_{34} + \frac{S_{13}S_{14}R_1}{1-S_{11}R_1} \right|^2$$

where P is the power delivered to the detector for a unit power input to the tee. For a load of small VSWR (therefore small reflection coefficient R_1), $S_{11}R_1$ would be negligible, but S_{34} would introduce errors due to phase sensitivity. On the other hand, for a load of large VSWR, S_{34} may be negligible compared to $\frac{S_{13}S_{14}R_1}{1-S_{11}R_1}$, but it will not be possible to neglect $S_{11}R_1$. Therefore, again the tee output will be sensitive to the phase of R_1. One may think that since it is possible to measure the





FREQUENCY SENSITIVITY OF A SIDE-OUTLET TEE

(TEE WAS MATCHED AND ALIGNED AT 9440 MCS) magnitudes of S_{13}, S_{14}, S_{11}, and S_{34}, it would be simple to compensate for the errors due to them. However, the magnitudes of these terms are not the only ones involved in the equation. Their phases must also be known. Thus it becomes an extremely cumbersome experiment to measure an impedance with such a bridge at more than one frequency. Thus, while it is desirable to use the foregoing method of matching an asymmetrically constructed side-outlet tee at one frequency, it is not feasible to use such a method for matching such a tee for multi-frequency measurements. It should be emphasized that the foregoing data apply only to the side-outlet tee on which the experiment was performed. For different tees with different constructional properties, similar data could be obtained. However, these data do give an indication as to the behaviour of asymmetrical side-outlet tees under conditions of varying frequency.

CONCLUSIONS

The theoretical analysis and experimental investigation have shown that a side-outlet tee which is asymmetrical in construction cannot always be matched to make a "magic" tee. Moreover, unlike the symmetrical tee (which may be sensitive to the phase of load reflection, only due to a mismatch seen looking into the load arm), the asymmetrical tee is inherently phase-sensitive, because of asymmetric coupling between the E- and H-arms as well as a mismatched load arm. However, this phase-sensitivity can be eliminated (within the limits of experimental feasibility), so that the power delivered to the detector is proportional only to the magnitude of the square of the load reflection coefficient R_1. Thus, in general, with a matched generator coupled to the H-arm, a matched detector to the E-arm, a reference matched load to side-arm 2, and the testload to side-arm 1, the power delivered to the detector is given by

$$P = \left| S_{34} + \frac{S_{13} S_{14} R_1}{1 - S_{11} R_1} \right|^2$$

where R_1 is the complex reflection coefficient of the test load, and S_{nk} are complex constants. For a symmetrical tee, S_{34} is equal to zero. For a "magic" tee, both S_{34} and S_{11} are zero and

$$|S_{13}| = |S_{14}| = \frac{1}{\sqrt{2}}$$

For a matched asymmetrical tee, S_{34} and S_{11} are zero, but $|S_{13}|$ and $|S_{14}|$ are not necessarily equal, nor are they theoretically predetermined. Thus, for the tee which was investigated, $|S_{13}| = 0.66$, and $|S_{14}| = 0.638$. However, for both the "magic" tee and the matched asymmetrical tee,

$$P = K|R_1|^2$$

where

$$\bar{K} = |S_{13}|^2 |S_{14}|^2$$

For the "magic" tee, K is equal to 1/4, but for the asymmetrical tee, K must be experimentally determined.

For one frequency of operation, the matching and alignment can always be accomplished by means of slide-screw tuners or other such reactive elements placed in the junction arms. This method, however, will only be feasible for a single-frequency bridge. Slight changes in frequency will appreciably upset the bridge balance to reintroduce phase-sensitivity and its errors. Therefore, a matched generator with frequency stability is necessary for a successful operation of such a bridge.

In short, it is possible to make a "magic" tee out of a symmetrical side-outlet tee, and it is just as possible to make an "almost magic" tee out of an asymmetrical side-outlet tee.

APPENDIX A

For a lossless side-outlet tee (not necessarily symmetrical) whose arms have the same characteristic impedance, if arm 3 were coupled to a generator, arm 4 to an unmatched detector, and the side-arms 1 and 2 to unmatched loads, the amplitude of the wave delivered to the detector is given by:

$$E_{o4} = E_{i1}S_{14}+E_{i2}S_{24}+E_{i3}S_{34}+E_{i4}S_{44}$$

where

E_{ok} = Amplitude of wave out of arm k

E_{ik} = Amplitude of wave incident on arm k

and

S_{nk} = Complex coefficients of amplitude transfer between arms n and k.

Similarly,

$$E_{o1} = E_{i1}S_{11}+E_{i2}S_{12}+E_{i3}S_{13}+E_{i4}S_{14}$$

and

$$E_{o2} = E_{i2}S_{12}+E_{i2}S_{22}+E_{i3}S_{23}+E_{i4}S_{24}$$

But

$$E_{ik} = E_{ok}R_k$$

where

R_k = Complex reflection coefficient of load on arm k.

Therefore,

$$E_{i1} = E_{o1}R_1$$

$$E_{i2} = E_{o2}R_2$$

and

$$E_{i4} = E_{o4}R_4$$

where R_1, R_2, and R_4 are complex reflection coefficients of loads on arms 1, 2, and 4 respectively.

Normalize the equations so that

$$E_{13} = 1 \; .$$

Substituting these values into the three equations for E_{ok} gives

$$E_{o1}(1-S_{11}R_1) = S_{13}+E_{o2}R_2S_{12}+E_{o4}R_4S_{14} \; , \qquad \text{A}$$

$$E_{o2}(1-S_{22}R_2) = S_{23}+E_{o1}R_1S_{12}+E_{o4}R_4S_{24} \; , \qquad \text{B}$$

and

$$E_{o4}(1-S_{44}R_4) = S_{34}+E_{o1}R_1S_{14}+E_{o2}R_2S_{24} \; . \qquad \text{C}$$

The solution of equations A and B gives

$$E_{o1} = \frac{S_{13}+R_2(S_{12}S_{23}-S_{22}S_{13})+E_{o4}R_4\{S_{14}+R_2(S_{12}S_{24}-S_{22}S_{14})\}}{(1-R_1S_{11})(1-R_2S_{22}) - S_{12}^2R_1R_2}$$

and

$$E_{o2} = \frac{S_{23}+R_1(S_{12}S_{13}-S_{11}S_{23})+E_{o4}R_4\{S_{24}+R_1(S_{12}S_{14}-S_{11}S_{24})\}}{(1-R_1S_{11})(1-R_2S_{22}) - S_{12}^2R_1R_2}$$

Substituting these two values in Equation C gives

$$E_{o4} = \frac{\left[\begin{array}{l} R_1S_{13}S_{14}+R_2S_{23}S_{24}+S_{34}(1-R_1S_{11})(1-R_2S_{22}) \\ +R_1R_2\{S_{14}(S_{12}S_{23}-S_{22}S_{13})+S_{24}(S_{12}S_{13}-S_{11}S_{23})-S_{12}^2S_{34}\} \end{array}\right]}{\left[\begin{array}{l} (1-R_1S_{11})(1-R_2S_{22})(1-R_4S_{44}) - S_{12}^2R_1R_2 \\ +R_1R_2R_4\{S_{12}^2S_{44}-S_{14}(S_{12}S_{24}-S_{22}S_{14})-S_{24}(S_{12}S_{14}-S_{11}S_{24})\} \\ -R_4(R_1S_{14}^2+R_2S_{24}^2) \end{array}\right]}$$

APPENDIX B

If, with the tee coupled as described in Appendix A,

$$R_1 = R_4 = 0$$

Then from the final equation in Appendix A,

$$E_{o4} = S_{34} + \frac{S_{23}S_{24}R_2}{1-S_{22}R_2}$$

Therefore, for

$$E_{o4} = 0$$

$$R_2 = \frac{S_{34}}{S_{22}S_{34}-S_{23}S_{24}} \quad \text{- - - - - - - - - - A}$$

If arm 1 were coupled to the generator, arm 3 to a matched load, arm 4 to an unmatched detector, and arm 2 to a load of reflection coefficient given by Equation A, then

$$E_{o1} = E_{i1}S_{11}+E_{i2}S_{12}+E_{i3}S_{13}+E_{i4}S_{14}$$

where

$$E_{i2} = E_{o2}R_2 = \frac{E_{o2}S_{34}}{S_{22}S_{34}-S_{23}S_{24}}$$

$$E_{i4} = E_{o4}R_4$$

and

$$E_{i1} = E_{11}$$

Since $R_3 = 0$

$$E_{i3} = E_{o3}R_3 = 0$$

Therefore

$$E_{o1} = E_{11}S_{11}+E_{o2}R_2S_{12}+E_{o4}R_4S_{14} \quad \text{- - - - - - - - B}$$

Similarly

$$E_{o2} = E_{11}S_{12}+E_{o2}R_2S_{22}+E_{o4}R_4S_{24} \quad \text{- - - - - - - - - C}$$

and

$$E_{o4} = E_{11}S_{14}+E_{o2}R_2S_{24}+E_{o4}R_4S_{44} \quad \text{- - - - - - - - - D}$$

Solution of Equations C and D gives

$$E_{o2} = \frac{E_{11}\{S_{12}(1-R_4S_{44}) + R_4S_{14}S_{24}\}}{(1-R_2S_{22})(1-R_4S_{44}) - S_{24}^2R_2R_4}$$

and

$$E_{o4} = \frac{E_{11}\{S_{14}(1-R_2S_{22}) + R_2S_{12}S_{24}\}}{(1-R_2S_{22})(1-R_4S_{44}) - S_{24}^2R_2R_4}$$

Substituting these values in Equation B gives

$$E_{o1} = E_{11}\left\{ S_{11} + \frac{\begin{array}{c} R_2S_{12}[S_{12}(1-R_{44}S_{44}) + R_4S_{14}S_{24}] \\ + R_4S_{14}[S_{14}(1-R_2S_{22}) + R_2S_{12}S_{24}] \end{array}}{(1-R_2S_{22})(1-R_4S_{44}) - S_{24}^2R_2R_4} \right\}$$

Substituting for

$$R_2 = \frac{S_{34}}{S_{22}S_{34}+S_{23}S_{24}}$$

gives

$$\frac{E_{o1}}{E_{11}} = \frac{\left[\begin{array}{l} R_4(2S_{12}S_{34}S_{14}S_{24}+S_{11}S_{23}S_{24}S_{44}-S_{34}S_{44}S_{12}^2) \\ -R_4(S_{23}S_{24}S_{14}^2+S_{11}S_{34}S_{24}^2)+S_{34}S_{12}^2-S_{11}S_{23}S_{24} \end{array}\right]}{R_4S_{24}(S_{23}S_{44}-S_{24}S_{34}) - S_{23}S_{24}} \quad \text{- - - E}$$

In order that there be no reflections seen looking into arm 1, it must be that

$$\frac{E_{o1}}{E_{11}} = 0 .$$

Therefore the num. tor of the right-hand side of Equation E must be zero, for the denominator is finite. From this, the reflection coefficient seen looking into the detector is

$$R_4 = \frac{S_{34}S_{12}^2 - S_{11}S_{23}S_{24}}{\left[\begin{array}{l} S_{34}S_{44}S_{12}^2 - 2S_{12}S_{34}S_{14}S_{24} + S_{23}S_{24}S_{14}^2 \\ -S_{11}S_{23}S_{24}S_{44} + S_{11}S_{34}S_{24}^2 \end{array}\right]}$$

With R_2 and R_4 as prescribed above, if arm 3 were again coupled to the generator and arm 1 to a test load of reflection coefficient R_1, then substituting for R_2 and R_4 in the final equation for E_{04} in Appendix A will give

$$E_{04} = \frac{R_1\alpha}{\beta}$$

where the complex constants

$$\alpha = S_{12}S_{34}(S_{14}S_{23} + S_{13}S_{24} - S_{12}S_{34}) - S_{13}S_{14}S_{23}S_{24}$$

and

$$\beta = -S_{23}S_{24} + \frac{S_{24}(S_{23}S_{44} - S_{24}S_{34})(S_{34}S_{12}^2 - S_{11}S_{23}S_{24})}{\left[\begin{array}{l} S_{12}S_{34}(S_{12}S_{44} - S_{14}S_{24}) + S_{14}S_{24}(S_{14}S_{23} - S_{12}S_{34}) \\ + S_{11}S_{24}(S_{24}S_{34} - S_{23}S_{44}) \end{array}\right]}$$

It can be seen that for $R_1 = 0$, there is no output out of arm 4; that is to say, arms 3 and 4 are decoupled. Moreover, the magnitude of E_{04} is not a function of the phase R_1. The scattering matrix of such a decoupled junction would be

$$S = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & 0 \\ S_{14} & S_{24} & 0 & S_{44} \end{bmatrix}$$

Once arms 3 and 4 are decoupled it is always possible to accomplish matches looking into arms 2 or 3 or 4. But the physical possibility of this decoupling must first be considered. To do that, consider the impedance matrix of a 4-arm junction:

$$Z = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{12} & Z_{22} & Z_{23} & Z_{24} \\ Z_{13} & Z_{23} & Z_{33} & Z_{34} \\ Z_{14} & Z_{24} & Z_{34} & Z_{44} \end{bmatrix}$$

Decoupling arms 3 and 4 involves adding an impedance in arm 1 or 2 such that in the resultant matrix Z', Z'_{34} would be zero. Evidently that impedance would be $-Z_{34}$ added to all Z_{ik}. Then

$$Z' = \begin{bmatrix} Z_{11} - Z_{34} & Z_{12} - Z_{34} & Z_{13} - Z_{34} & Z_{14} - Z_{34} \\ Z_{12} - Z_{34} & Z_{22} - Z_{34} & Z_{23} - Z_{34} & Z_{24} - Z_{34} \\ Z_{13} - Z_{34} & Z_{23} - Z_{34} & Z_{33} - Z_{34} & 0 \\ Z_{14} - Z_{34} & Z_{24} - Z_{34} & 0 & Z_{44} - Z_{34} \end{bmatrix}$$

So the question now remains if $-Z_{34}$ is physically possible as a load. Since the junction is lossless, all the terms in the initial impedance matrix are imaginary. Therefore Z_{34} is imaginary and

$$0 \leq |Z_{34}| \leq \infty$$

Such a reactive load is always possible. If the junction is a degenerate junction, then it is possible that

$$|Z_{34}| = 0$$

or

$$|Z_{34}| = \infty$$

In this case a sliding short would accomplish the decoupling. However, for a side-outlet tee,

$$0 < |Z_{34}| < \infty$$

Therefore a slide-screw tuner or any other such reactive element would suffice.

It should be noted that if Z_{34} were complex, the junction would not be lossless, and reactive elements such as sliding shorts or slide-screw tuners would not enable decoupling of arms 3 and 4. Similarly, if the reactive elements had slight losses, then also complete decoupling of two arms of a lossless junction would not be possible.

Thus it is shown that it is possible to decouple two arms of a lossless four-arm junction.

Once S_{34} is made zero, then arm 3 can be tuned, without affecting the decoupling, to see a match looking into arm 3. Then, by similar procedure, arm 4 could be tuned so that a match is seen looking into arm 1. The resultant scattering matrix would be

$$S = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & 0 \\ S_{14} & S_{24} & 0 & S_{44} \end{bmatrix}$$

APPENDIX C

Consider a lossless four-arm junction whose arms have identical characteristic impedances and whose scattering matrix is

$$S = \begin{bmatrix} 0 & s_{12} & s_{13} & s_{14} \\ s_{12} & s_{22} & s_{23} & s_{24} \\ s_{13} & s_{23} & 0 & 0 \\ s_{14} & s_{24} & 0 & s_{44} \end{bmatrix} .$$

Since the junction is lossless, the above matrix is a unitary matrix. Therefore, there will be six equations of the type

$$\sum_{k=1}^{4} s_{n_1 k} s^*_{n_2 k} = 0$$

and four equations of the type

$$\sum_{k=1}^{4} \left| s_{nk} \right|^2 = 1$$

Substituting for

$$s_{11} = s_{33} = s_{34} = 0$$

and solving the above ten equations will result in the following five matrices which would express the junction:

1) $$S = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} .$$

2) $$S = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

3) $$S = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

4) $$S = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & S_{22} & 0 & S_{24} \\ 1 & 0 & 0 & 0 \\ 0 & S_{24} & 0 & S_{44} \end{bmatrix}$$

where $$|S_{22}| = |S_{44}|$$

and

5) $$S = \begin{bmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & 0 \\ S_{14} & S_{24} & 0 & 0 \end{bmatrix}$$

Of these five solutions, only the last matrix defines a side-outlet tee. The other four apply to degenerate junctions which cannot fulfil the primarily imposed conditions without the complete decoupling, from the junction, of two of the four arms.

Thus, for a side-outlet tee, from Solution 5,

$$S_{13} S_{14}^* + S_{23} S_{24}^* = 0$$

and

$$|S_{13}|^2 + |S_{23}|^2 = |S_{14}|^2 + |S_{24}|^2 = 1$$

Therefore

$$|S_{23}| = |S_{14}| = \sqrt{1 - |S_{13}|^2} = \sqrt{1 - |S_{24}|^2}$$

For the symmetrical tee (i.e., "magic" tee), since

$$S_{13} = S_{23} \text{ and } S_{14} = -S_{24}$$

$$|S_{13}|^2 = |S_{23}|^2 = |S_{14}|^2 = |S_{24}|^2 = \frac{1}{2}$$

APPENDIX D

In the experimental measurements which were made, a crystal was used for measuring VSWR and an 8 ma. barretter was used for power detection. For accuracy, it was found necessary to calibrate the crystal and 8 ma. barretter. They were calibrated with respect to a 4 ma. barretter and an attenuator which, in turn, were calibrated against the sine wave distribution of the standing wave inside a shorted line.

First, the 4 ma. barretter was used for detection in a PRD slotted line which was connected to a short as shown in Fig. 1. Assuming the slotted line to be lossless, there would be a sinusoidal standing wave set up in the line. With the attenuator set for zero attenuation, the positions along the slotted line of two adjacent minima were measured. Since the barretter rectifies the sine wave, distance between these two measured positions was identically half the guide wavelength, as shown in Fig. 2. The position of the maximum lies halfway between these two minima and the half-power points lie one-twelfth of a wavelength on either side of a minimum. With the slotted line probe at a maximum position, the barretter reading was noted. The probe was then shifted to a half-power point, and again the barretter reading noted. For a sine-wave distribution, which was assumed, the actual difference between these two power levels should be 6.02 dbs. This was compared with the measured value. The probe was then re-shifted to the position of maximum and the attenuator was adjusted to decrease the barretter reading by an amount equal to the db. difference between the preceding two barretter readings. Again the probe was placed on the same half-power point and the difference between these two readings recorded. These steps were repeated until the attenuation reduced the signal to noise level. This procedure essentially calibrated the 4 ma. barretter and the attenuator in steps of 6 dbs. each.

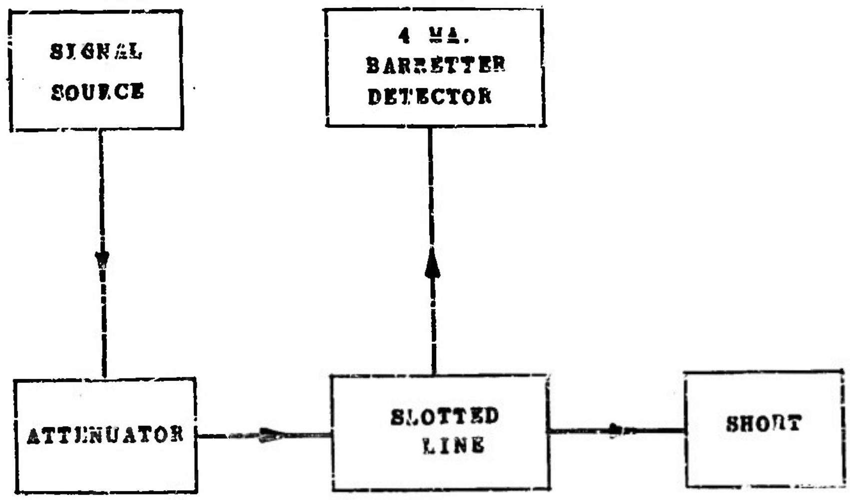

SIGNAL SOURCE
4 MA. BARRETTER DETECTOR
ATTENUATOR
SLOTTED LINE
SHORT

FIG. 1

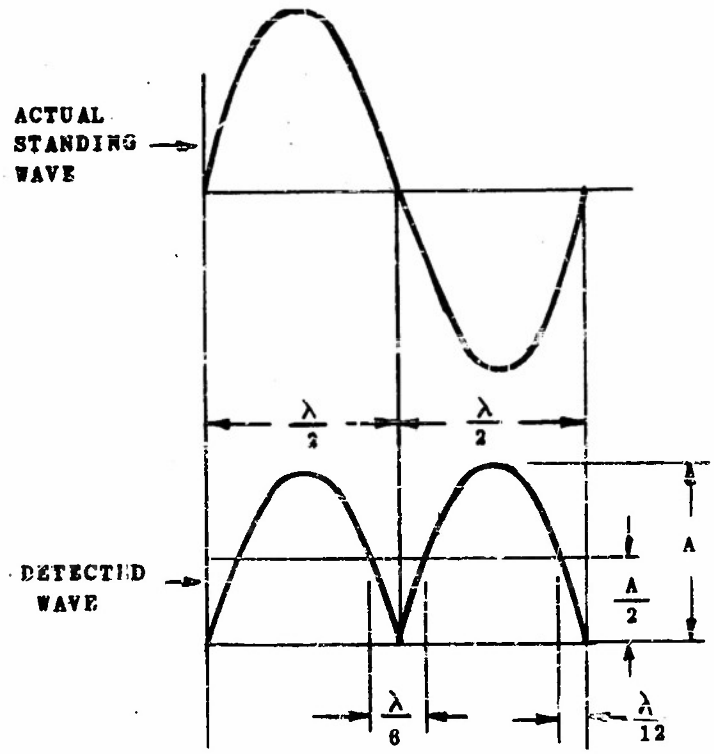


FIG. 2

The results are tabulated in Table I and the calibration curves plotted in Fig. 3. The barretter curve shows that in range of meter readings between -23.7 dbs. to -60 dbs., the barretter-detector reads accurately. It is for higher attenuation readings that the measured values deviate from theoretical values.

Results

Positions of adjacent minima are 9.55 cms. and 11.77 cms.

$$\text{Therefore position of enclosed maximum} = 9.55 + \frac{11.77 - 9.55}{2} = 10.66 \text{ cms.}$$

$$\text{Probe position for half-maximum reading} = 11.77 - \frac{11.77 - 9.55}{6} = 11.40 \text{ cms.}$$

With zero attenuation and probe at maximum position barretter-detector reading = 23.7 dbs.

Table I

Slotted Line Probe Position (cms.)	Actual Attenuation (dbs.)	Attenuation Measured by Barretter-Detector		Attenuator Reading
		Meter Reading (-dbs.)	Attenuation (dbs.)	
10.66	0	23.7	0	0
11.40	6.02	29.7	6	0
10.66	6.02	29.7	6	24.8
11.40	12.04	35.7	12	24.8
10.66	12.04	35.7	12	34.7
11.40	18.06	41.7	18	34.7
10.66	18.06	41.7	18	42.3
11.40	24.08	47.5	23.80	42.3
10.66	24.08	47.5	23.80	50.2
11.40	30.10	53.5	29.80	50.2
10.66	30.10	53.5	29.80	56.1
11.40	36.12	59.5	35.80	56.1
10.66	36.12	59.5	35.80	61.5

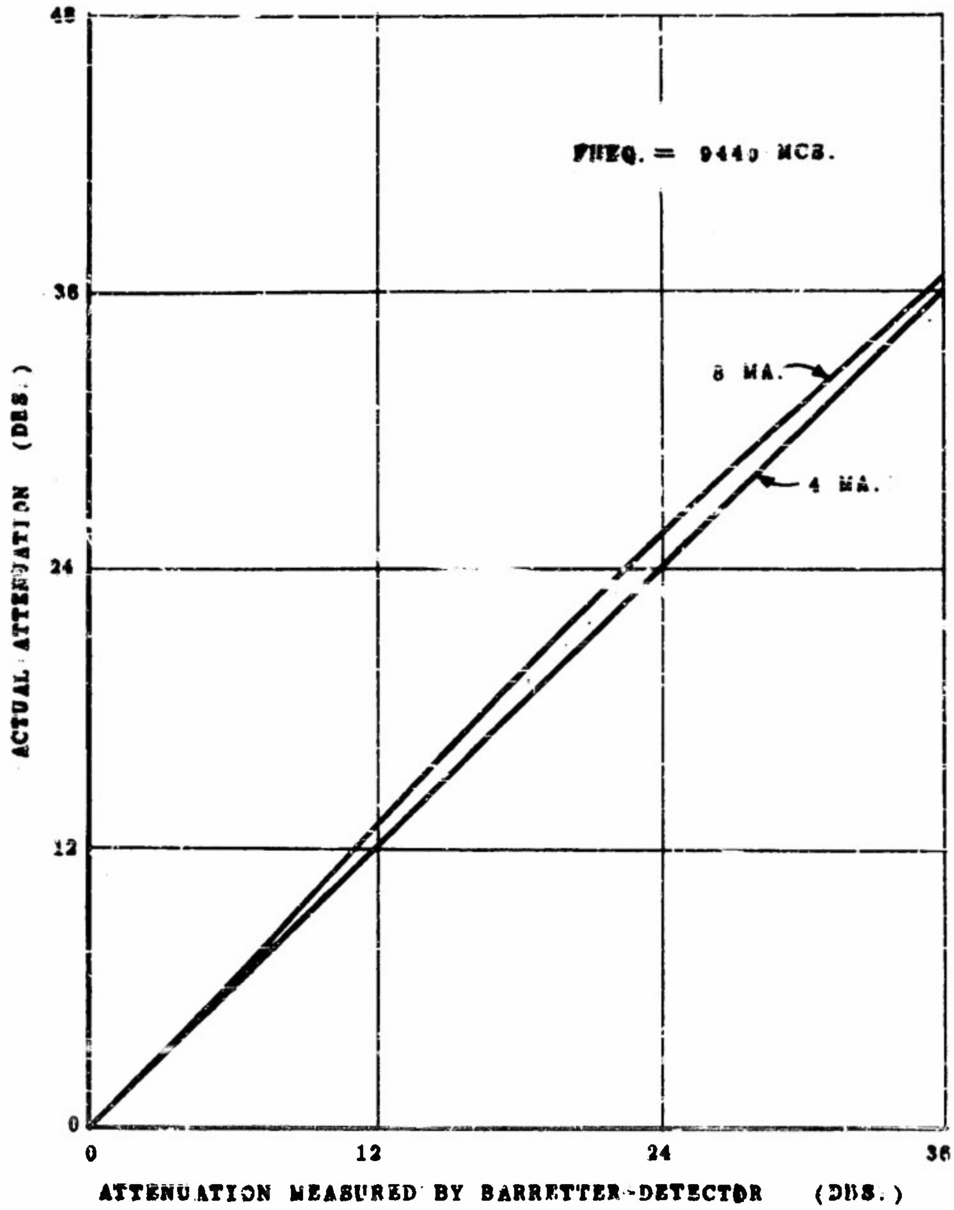


FIG. 3

CALIBRATION OF 4 & 8 MA. BARRETTERS.

For the calibration of the 8 ma. barretter the experiment was set up as shown in Fig. 4. The attenuation was varied in steps of 6 dbs. each, as determined by the attenuator calibration. And the attenuations read by the 8 ma. barretter-detector were recorded for each step. The following data were obtained:

Table II

Attenuator Reading	Attenuation (db.)	Attenuation Measured by Barretter-Detector	
		Meter Reading (-dbs.)	Attenuation (dbs.)
0	0	7.0	0
24.8	6.02	12.4	5.4
34.7	12.04	18.4	11.4
42.3	18.06	23.8	16.8
50.2	24.08	29.9	22.9
56.1	30.10	35.9	28.9
61.5	36.12	42.5	35.5

The calibration curve for the 8 ma. barretter is given in Fig. 3.

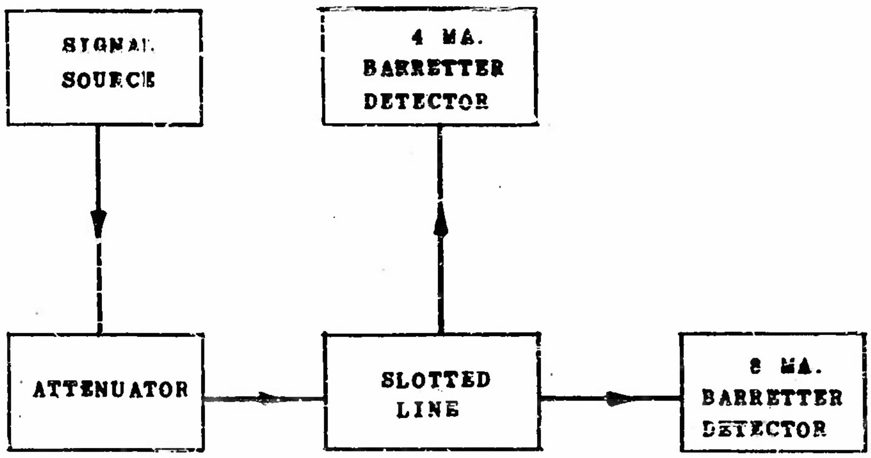

SIGNAL
SOURCE
4 MA.
BARRETTER
DETECTOR
ATTENUATOR
SLOTTED
LINE
8 MA.
BARRETTER
DETECTOR

FIG. 4

The experimental set-up for the calibration of the crystal is shown in Fig. 5. The load consisted of a 2-slug transformer (Sperry Gyroscope Co.), followed by a slide-screw tuner (Hewlett-Packard), and then a resistive load. The voltage standing-wave ratio (VSWR) of this composite load was varied by varying the penetration of the slide-screw in the tuner and the VSWR was measured, using the slotted line, first with the 4 ma. barretter, and then the crystal. The power level at which these measurements were made was within the range of accurate barretter readings. The results are tabulated in Table III, and the calibration curve plotted in Fig. 6.

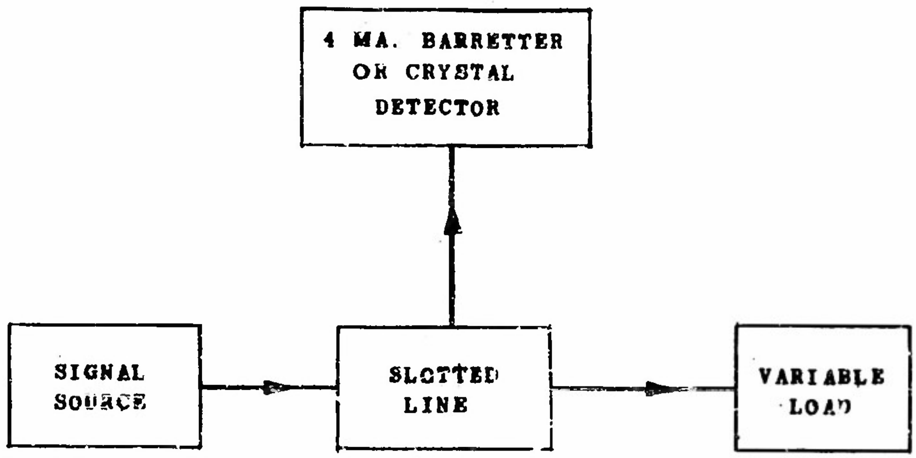
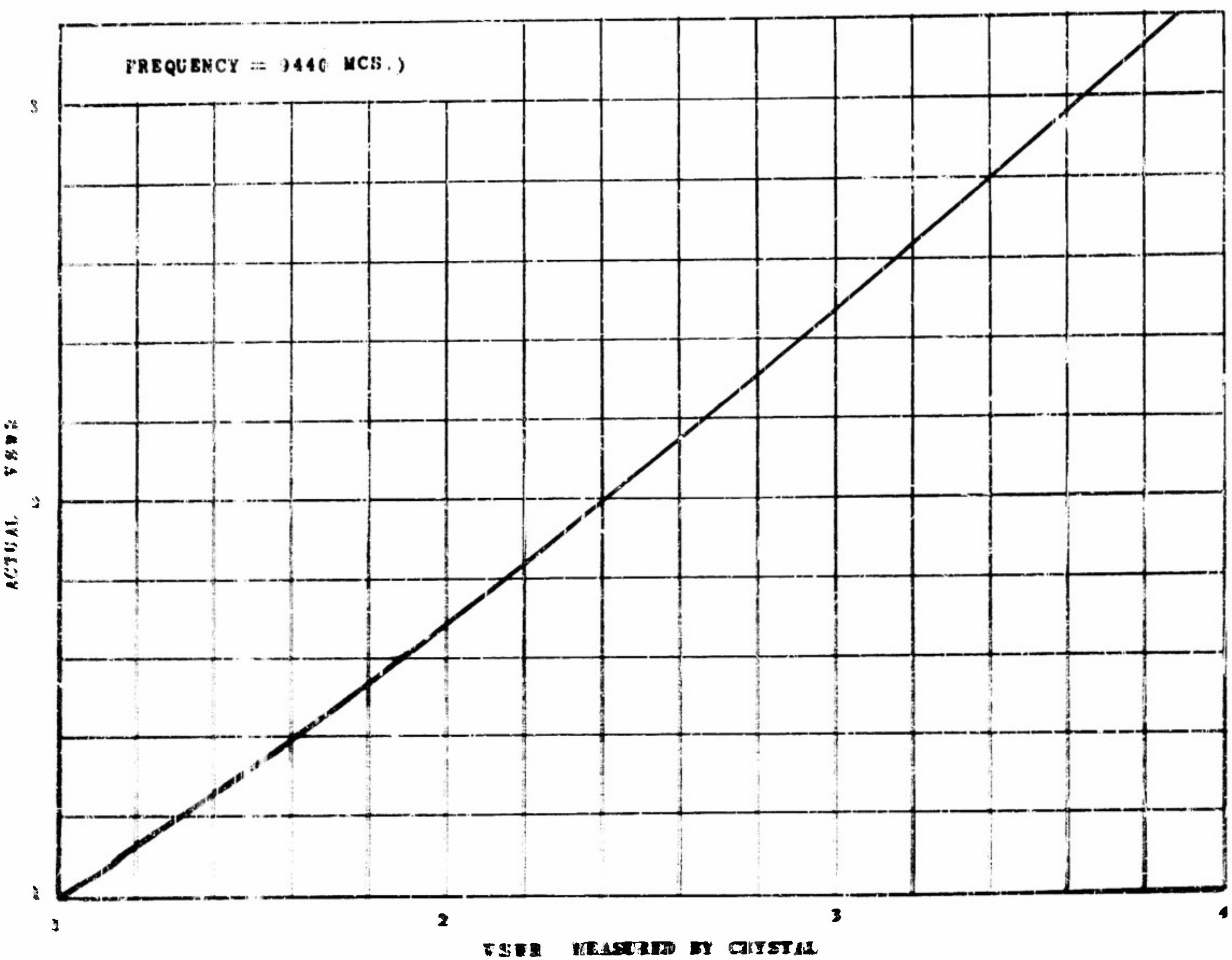

4 MA. BARRETTER
OR CRYSTAL
DETECTOR
SIGNAL
SOURCE
SLOTTED
LINE
VARIABLE
LOAD

FIG. 5

The calibration curve shows that the crystal deviates from accuracy as the VSWR is increased.

Table III

VSWR measured correctly with ¼ mm. barretter in slotted line	VSWR measured with crystal in slotted line
<< 1.01	<< 1.01
<< 1.01	<< 1.01
< 1.01	< 1.01
1.015	1.02
1.04	1.045
1.07	1.095
1.17	1.20
1.31	1.42
1.57	1.82
2.00	2.43
2.90	3.52
3.20	3.90



CRYSTAL CALIBRATION

FIG. 5

APPENDIX E

The procedures for obtaining (1) a load having reflections of constant magnitude but varying phase, and (2) a load having reflections of constant phase but varying magnitude, are as follows:

(1) A "Microline" two-slug transformer, a Hewlett-Packard slide-screw tuner, and a resistive load coupled as shown in Fig. 1 constituted the load. First, a match was obtained looking into the tuner and load. Then, with the separation between the slugs of the transformer maintained constant, the position of the slugs was varied to shift the phase of reflection from the load. However, it was observed that with the shift in position of the slugs, the VSWR of the load also varied. This was compensated for by varying the penetration of the slide-screw; the position of the screw was maintained constant. Thus, with the slugs' separation and slide-screw position held constant, the slugs' position and slide-screw penetration were varied to give a load of constant reflection magnitude, but varying phase.

A PRD slotted line with a calibrated* crystal-detector was used to measure the VSWR and the positions of minimum voltage. The latter was a measure of the phase. The following data were obtained:

Frequency = 9440 mcs.

Zero error on slug separation vernier = + 0.02 cms.

Constant slug separation reading = 2.33 cms.

Constant slug separation (actual) = 2.31 cms.

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* See Appendix D for calibration curves.

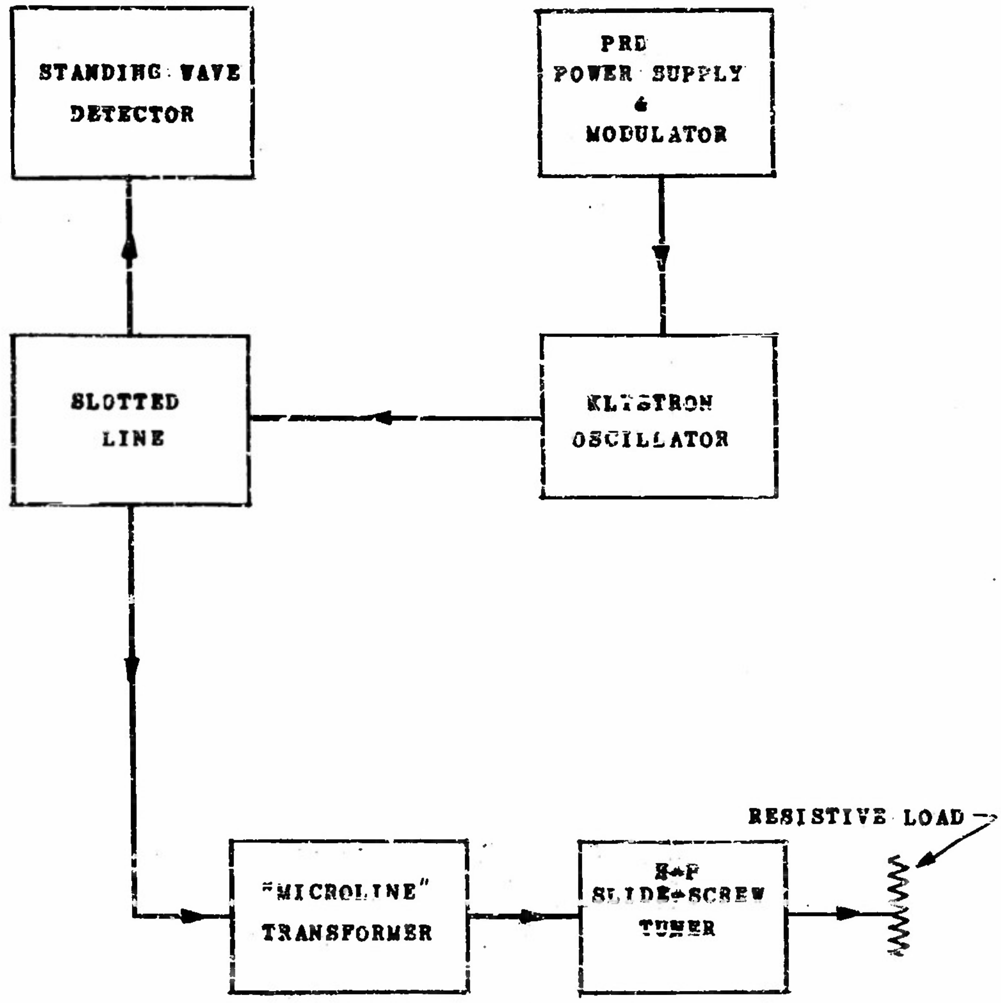

STANDING WAVE
DETECTOR
PRD
POWER SUPPLY
&
MODULATOR
SLOTTED
LINE
KLYSTRON
OSCILLATOR
RESISTIVE LOAD
"MICROLINE"
TRANSFORMER
H-P
SLIDE-SCREW
TUNER

FIG. 1

Slug Position Reading (cms.)	Tuner Screw Penetration Reading	VSWR (Corrected)	V_{min} Position Reading (cms.)	Phase Shift (cms.)
1.30	3.45	1.68	10.23	0
1.50	3.38	1.68	10.45	0.22
2.00	3.38	1.68	10.97	0.74
2.50	3.31	1.68	11.49	1.26
3.00	3.38	1.68	11.93	1.70
3.50	3.217	1.68	12.48	2.25
4.00	3.342	1.68	12.95	2.72
4.50	3.342	1.68	13.28	3.05
5.00	3.395	1.68	13.80	3.57
5.50	3.580	1.68	14.27	4.04
6.00	3.40	1.68	14.79	4.56
6.50	3.35	1.68	15.31	5.08
7.00	3.35	1.68	15.78	5.55
7.50	3.42	1.68	16.27	6.04
8.00	3.21	1.68	16.81	6.58
8.50	3.322	1.68	17.30	7.07
8.70	3.322	1.68	17.47	7.24

N.B. - Due to lack of mechanical rigidity in the slugs, the slug separation and position were always approached in the direction of increased readings.

A second load of constant VSWR equal to 1.2 was also calibrated by the same procedure, giving the following data:

Frequency = 9440 mcs.

Constant slug separation reading = 2.41

Actual slug separation = 2.39 cms.

Slug Position Reading (cms.)	Tuner Screw Penetration Reading	VSWR (Corrected)	V_{min} Position Reading (cms.)	Phase Shift (cms.)
1.30	3.45	1.17	10.21	0
1.50	3.40	1.17	10.40	0.19
2.00	3.30	1.17	10.90	0.69
2.50	3.41	1.17	11.48	1.27
3.00	3.49	1.17	11.94	1.73
3.50	3.22	1.17	12.53	2.31
4.00	3.25	1.17	12.93	2.72
4.50	3.76	1.17	13.40	3.19
5.00	3.51	1.17	13.78	3.57
5.50	5.17	1.17	14.11	3.90
6.00	3.245	1.17	14.75	4.54
6.50	3.155	1.17	15.15	4.94
7.00	3.485	1.17	15.80	5.69
7.50	3.535	1.17	16.20	5.99
8.00	3.205	1.17	16.78	6.57
8.50	3.26	1.17	17.26	7.05
8.70	3.235	1.17	17.38	7.17

(2) The second load was made up of the same components as in Fig. 1. With the slide-screw of the tuner completely removed, the load was matched by means of the two-slug transformer. Then the screw was replaced, and for different penetration positions of the screw, the VSWR and the position of minimum voltage were recorded. It was observed that this method provided a good variation in VSWR from less than 1.01 to 2 without much change in phase of the reflection coefficient. The following data were obtained:

Frequency = 9440 cms.

Slug separation reading (constant) = 2.45 cms.

Actual slug separation (constant) = 2.43 cms.

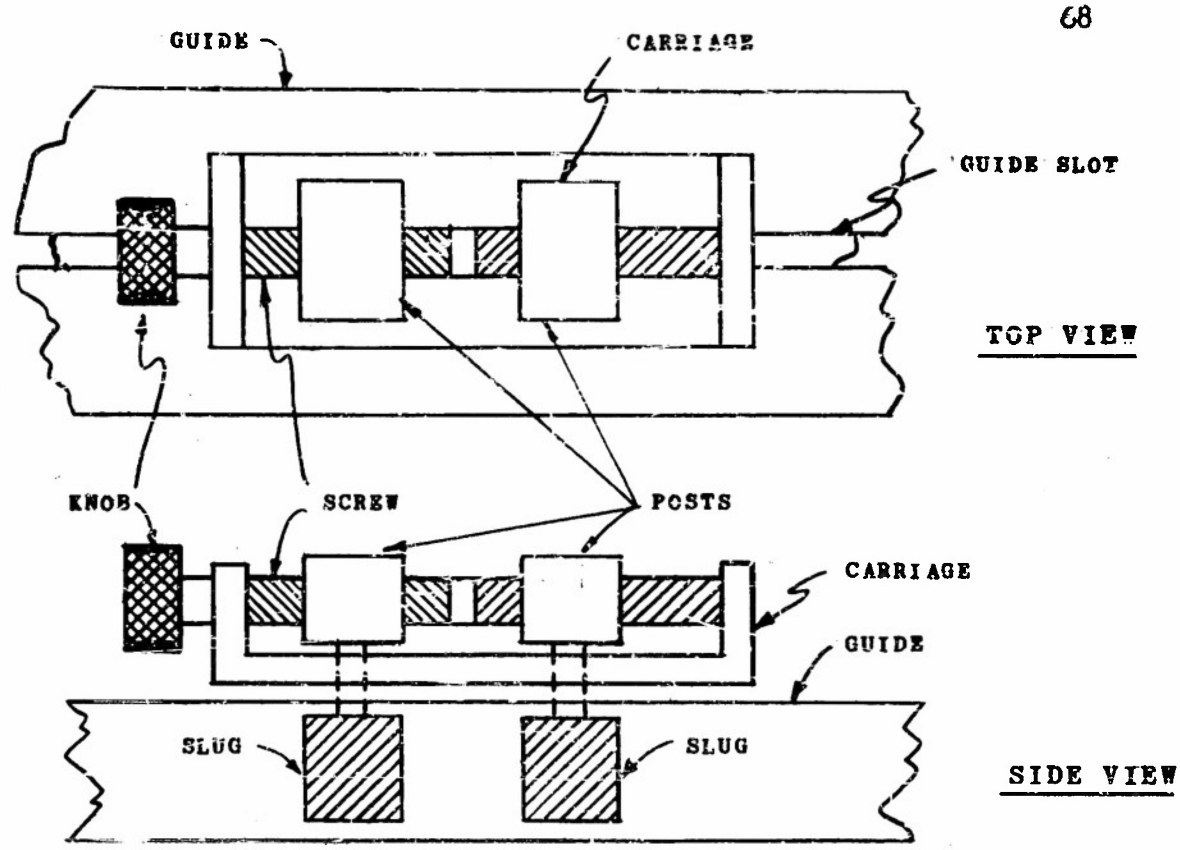
Slug position reading = 2.77 cms.

Slide-screw Penetration Position 1	VSWR (Corrected)	V_{min} Position Reading (cms.)
4.50	<< 1.01	-
4.25	<< 1.01	-
4.00	1.01	-
3.75	1.015	10.43
3.50	1.04	10.43
3.25	1.08	10.43
3.00	1.16	10.50
2.75	1.33	10.49
2.50	1.60	10.46
2.25	2.20	10.41
2.00	3.25	10.35

Some clarification is necessary in the readings on the two-slug transformer and the slide-screw tuner. Therefore a description of the two is in order.

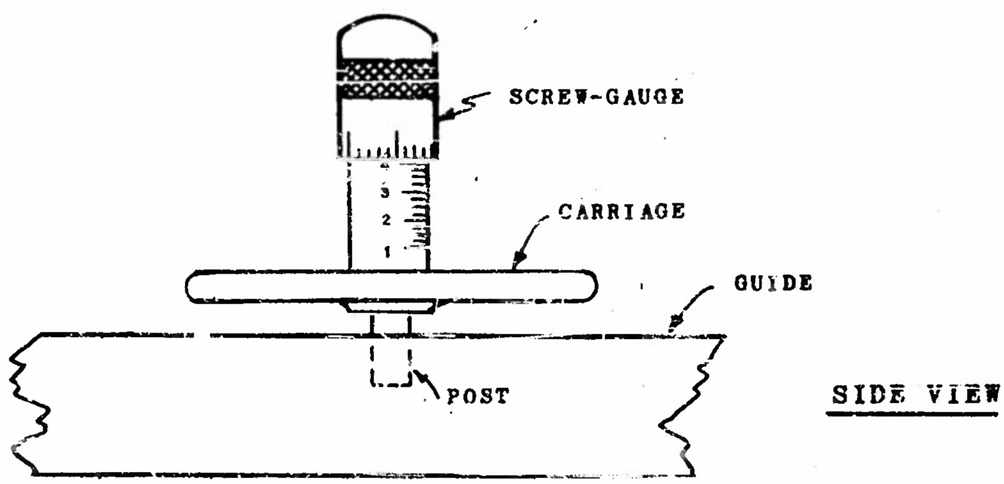
The Microline transformer, Fig. 2, consists of a slotted waveguide section in which two dielectric slugs are inserted. These slugs are free to slide along inside the guide and are attached through the slot to two posts which are mechanically coupled to each other by means of a screw. Turning this screw enables the alteration in the separation between the posts and therefore between the slugs. A scale and vernier are mounted on the posts to measure this separation. This whole system is mounted on a carriage which slides along the outside of the guide, thus varying the position of the two slugs inside the guide without affecting the separation between them. A scale and vernier are mounted - the scale on the guide, and the vernier on the carriage - to indicate the position of the carriage and slugs with respect to the guide. The readings on this scale and vernier merely indicate relative positions and therefore no absolute position reference is assumed in the measurements.

The Hewlett-Packard slide-screw tuner, Fig. 3, consists of a slotted waveguide section on which a carriage, carrying a conducting post, is mounted with the post inserted into the guide slot. This carriage can slide along the guide, thus varying the position of the post inside the guide. The post is coupled to the carriage through a screw-gauge, the scale on which reads positive in the direction away from the guide. Therefore an increment of scale reading signifies a decrease in the penetration of the screw into the guide.



"MICROLINE" TRANSFORMER

FIG. 2



HEWLETT-PACKARD SLIDE-SCREW TUNER

FIG. 3

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