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VECTOR-PHASE RADIO DIRECTION FINDERS

N6-ori-71 Task XV
ONR Project No. 076 161

TECHNICAL REPORT No. 11

ELECTRICAL ENGINEERING RESEARCH LABORATORY
ENGINEERING EXPERIMENT STATION
UNIVERSITY OF ILLINOIS
URBANA, ILLINOIS

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VECTOR-PHASE RADIO DIRECTION FINDERS

Technical Report No. 11
1 November 1950

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* Appendices B through I are available on request at any time within four months following the date of publication of this report.

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I. ABSTRACT

Three systems for using the vector-phase principle in instantaneous radio direction finders are proposed as being of potential merit.

~~These~~ Small diameter systems with three or four antenna elements are analyzed and shown to have no advantage over existing narrow-diameter systems so far as interfering signal discrimination is concerned.

It is shown how a collection of small diameter systems could be distributed over a region large in wavelengths with a scheme for automatically combining the bearing information from each small diameter system to give a resultant bearing with a greater discrimination against interfering signal than a single small diameter system would have.

A large diameter circular system is analyzed to show that it has the same interfering signal discrimination as a rectangular switched-Doppler system.

Wave interference bearing-error curves for all three types of systems are given and the practical difficulties of all three types of systems are discussed.

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II. INTRODUCTION

It has been recognized in the Radio Direction Finding Section at the University of Illinois as elsewhere[1] that one of the major problems to be faced in direction finding on "flash" or "squash" transmissions is the wave interference or "Heiligtag Error" effect. For observation periods of less than a few minutes there is no certainty that most existing narrow-aperture systems will not give bearings in serious error as a result of wave interference effects. Investigations of wide aperture systems such as Switched-Doppler, Wullenwebber, and WADONAS* have shown that these systems can minimize wave interference errors either by discriminating against the weaker signals or by resolving under ideal circumstances the separate directions[2] of arrival of many signals.

There is still another problem to be overcome in direction finding on flash transmissions in the case of directional or beam systems. The "beam" of the directional system must scan the entire azimuth preferably several times, in the presence of noise, for the duration of one transmission pulse. With the Wullenwebber and other arrays even slow-speed scanning represents a considerable commutation problem, and with any "beam" system a sufficiently high scanning rate for flash transmission requires an excessively large receiver bandwidth[1,3].

Although the switched Doppler system is not a "beam" system, it does require scanning in the sense that it should preferably complete several switching cycles during the duration of one transmission pulse. This deficiency led to a search for analogous systems that might eliminate switching and resulted in the conception of wide-aperture vector-phase systems.

A vector-phase RDF system is defined to be a direction finder which gives an indicated bearing resulting from the vector sum of two or more component vectors having the following characteristics. The magnitude is proportional to the phase difference (or some function of the phase difference) between the induced voltages in a pair of antennas of the system, the direction is parallel to a line joining these two antennas, and the sense is in the direction-sense of the antenna whose induced voltage leads in phase. A vector-phase direction finder is of a type which falls in the general calssification of phase-front direction finders. A phase-front direction finder is defined as being any system which, when small in aperture, tends to give a bearing normal to the curve of constant phase (in an electromagnetic field) passing through the center of the direction finder.**

A vector-phase direction finder having a small diameter is described first because of its relative simplicity, although the vector-phase systems which offer substantial potential advantages over existing systems are those having large wavelength dimensions.

Figure 1 of Plate 1 shows the schematic diagram of a possible form of a four-element narrow diameter vector-phase system using an oscilloscope as a vector summing device. This plate illustrates the

1. Numerical superscrips refer to the corresponding numbers in the bibliography.
* Wide-Aperture Distribution of Narrow-Aperture Systems.
** A Doppler system is another example of a phase-front direction finder.

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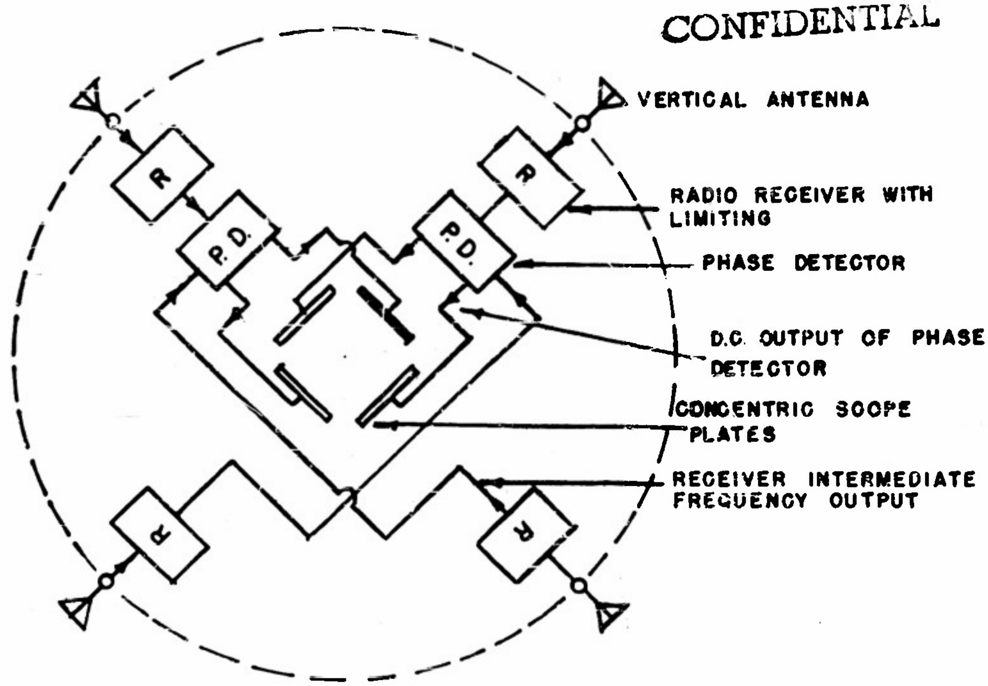

FIG. I FOUR ELEMENT VECTOR-PHASE SYSTEM WITH PHASE COMPARISON BETWEEN DIAGONALLY OPPOSITE ANTENNAS

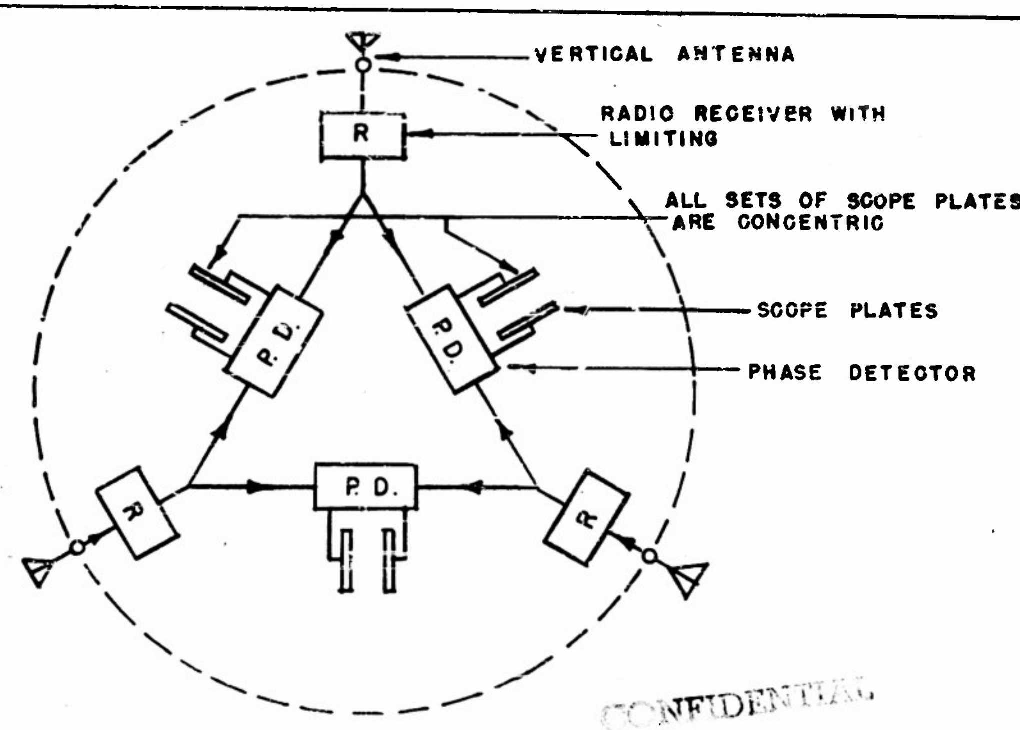

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FIG. 2 THREE ELEMENT CIRCULAR VECTOR-PHASE SYSTEM

NARROW DIAMETER VECTOR-PHASE SYSTEMS

PLATE I

DEPT. ELECTRICAL ENGR. – UNIVERSITY OF ILLINOIS method of connecting four antennas by amplitude limiting* receivers to phase detectors in such a manner that the direct voltage outputs of the phase detectors are proportional to the phase differences (or the sine of the phase differences) between diagonally opposite antennas.** These direct voltages are applied to concentric oscilloscope plates, the faces being perpendicular to a line joining the center of the antenna elements whose phase is being compared. Hence, the force vectors causing the beam to deflect are proportional to the direct voltages and, thus, the phase differences between antennas. If this system has parameters and circuits such that its phase detectors operate linearly (or nearly linearly) then the indicated bearing will be that (or nearly that) of the direction of arrival of a single signal. This is proven in the next section. If a single interfering signal is also arriving, this system may have large errors in the indicated bearing as discussed in Section V. Figure 2 of Plate 1 shows another type of vector phase system which utilizes only three antennas and three receivers, and is suitable for use as a narrow diameter system. This system is the simplest example of a general class of continuous circular systems which are discussed later in this section.

In all of these systems phase comparison is made directly between the intermediate frequency outputs of the receivers. The use of phase balanced receivers with a common high frequency oscillator has been assumed.

An alternative procedure is to transmit a local signal from a centrally located antenna (see Plate 2). This injection signal would differ from the incoming signal frequency by a small audio rate (for example one k.c.) and would arrive at all antennas in phase. When the two signals are mixed in the last detector of the receiver, an audio signal is produced which is at the same relative phase compared to the other antennas. Now the phase of the audio signals can be compared whereas before the phase of the intermediate frequency outputs was compared. The bearing presentation mechanism can be the same. Some of the difficulty in phase balancing of receivers should be eliminated by this procedure. Figures 1 and 2 of Plate 2 show systems of this type comparable to the equivalent systems of Plate 1 Fig. 1 and 2 respectively. The systems of Plate 2 are essentially equivalent to the "phase-ratio" systems as described by E. N. Dingley, Jr. in U. S. Patent No. 2,415,088, Feb. 4, 1947. They are more particularly "Analog Phase Ratio Systems".[1] One objection to an injection system, where the phase of audio rather than radio frequency signals is compared, is the relative slowness of system response. To give a bearing, an incoming pulse could be no shorter in duration than several cycles of the audio frequency. This may be a serious objection in a potentially instantaneous system. This objection and others are discussed more fully on page 21 and 22 of reference No. 3.

It is possible to obtain good discrimination against an interfering signal with a vector-phase system which has a large wave-length diameter and is made up of a relatively large number of antennas. This type of system is discussed in detail in Section IV.

* Only the phase information is desired in a vector phase system.
** The "phase of an antenna" refers to the phase of the induced voltage of that antenna.

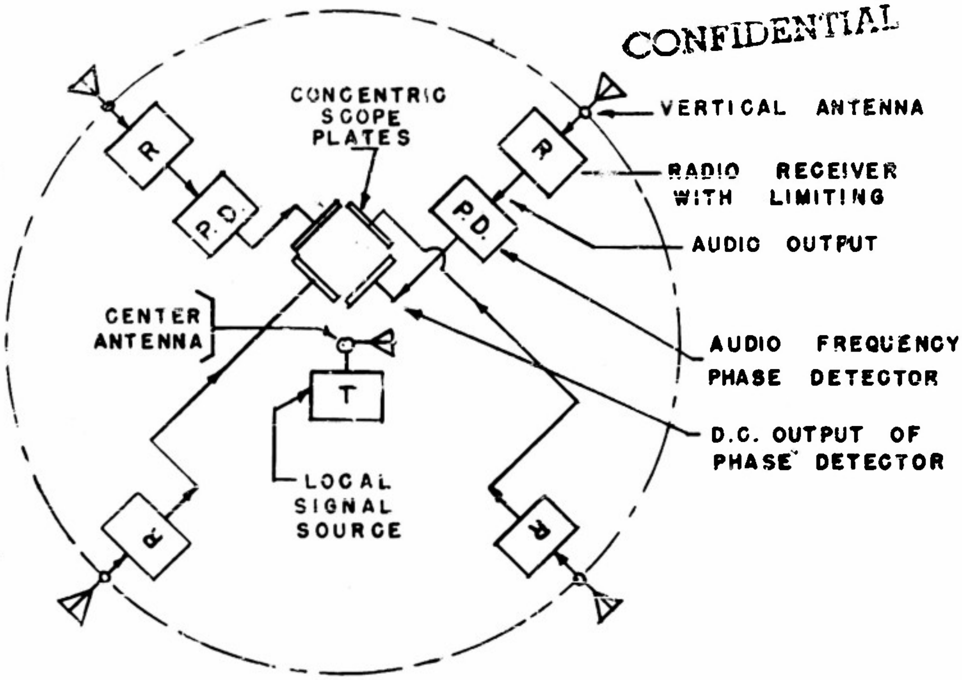

FIG. I FOUR ELEMENT VECTOR-PHASE SYSTEM WITH PHASE COMPARISON BETWEEN DIAGONALLY OPPOSITE ANTENNAS

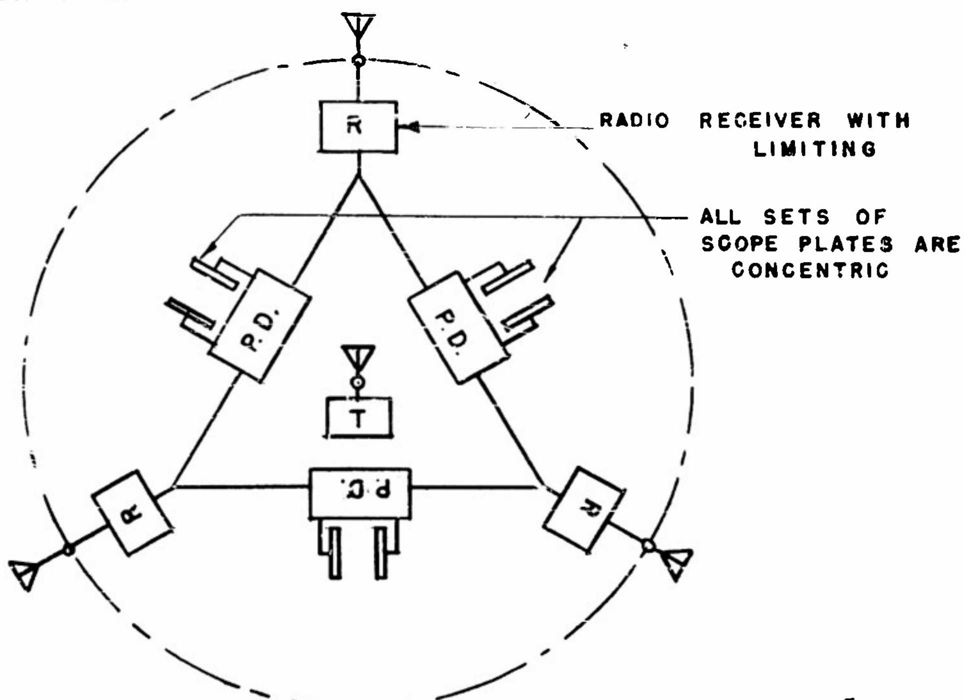

FIG 2 THREE ELEMENT CIRCULAR VECTOR - PHASE SYSTEM

NARROW DIAMETER VECTOR PHASE SYSTEMS WITH AUDIO PHASE. COMPARISON

PLATE 2

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If a number of small-diameter unit systems of the type shown in Plate 1, Fig. 1 were placed side by side, the system shown in Plate 3, Fig. 1 could result. It is assumed that all the phase-detectors are connected to the antennas by means of receivers having amplitude limiting, although for schematic simplicity these receivers are not shown. It is further assumed that, in some way, all of the deflection plates are made to act uniformly on a single beam in a cathode ray tube. Such a system is obviously too uneconomical of receivers and antennas to be practical, but if some of the control antennas and receivers are omitted as in Plate 3, Fig. 2, the system becomes feasible. Again all vector-summing scope plates are theoretically concentric, acting on a single beam, although they are shown in the diagrams in the most convenient illustrative positions. The orientation or direction of the plates is always shown correctly.

The system shown in Plate 3, Fig. 1 is classified as a continuous system while that of Plate 3, Fig. 2 is called a discontinuous or distributed system. A continuous system is one that compares the phase between every pair of adjacent (or diagonally opposite) antennas. A distributed system is a system made up of a group of "unit" systems with no phase comparison between any antennas of one unit system and those of another. Another example of a distributed system, a triangular system, is given by Plate 4, Fig. 1 where the same symbolic representation as that of Plate 3 is used, and the unit systems are the same as shown in Plate 1, Fig. 2.

Another example of a continuous system, a circular system*, is given by Plate 4, Fig. 2. This type of system is probably the most practical of the large diameter continuous systems and many of its characteristics are identical with those of switched-Doppler systems which have similar antenna configurations, (see Plate 10, Fig. 2). The multiple sets of oscilloscope plates again form a vector-summing device acting on a single oscilloscope beam, although in this case, the plates are not all at right angles. Later, it will be shown how a suitably designed resistor ring network can replace all except two sets of plates so that a conventional oscilloscope tube can be used. It will be shown in Section IV that the indicated direction of arrival given by any circular continuous system is equal to the angle of arrival of the single signal causing the phase differences between antennas, assuming linear phase detectors. In Section VII the errors of this system due to non-linear phase detection are discussed, and in Section V the errors due to a single interfering signal of the same frequency are analyzed.

Again it is possible to use a centrally located antenna to transmit a local signal to all antennas, producing audio signals at the receiver outputs. The phase of the audio signals can be compared where the phase of the intermediate frequency outputs was compared before.

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* This is by no means the only feasible type of circular vector-phase system. For example the phase difference could be measured between every antenna element on the periphery of a circle and an element at the center of the circle.

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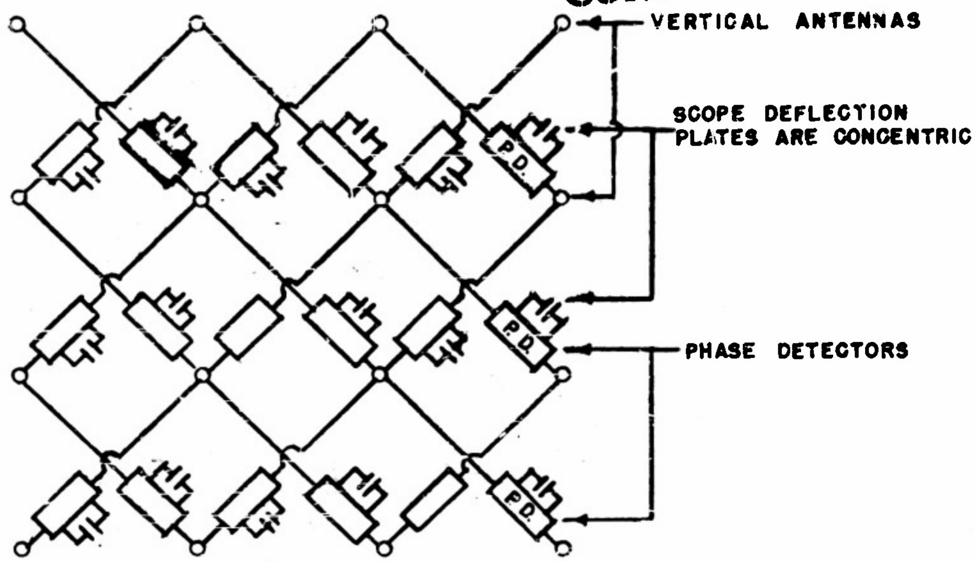

FIG. I CONTINUOUS VECTOR-PHASE SYSTEM

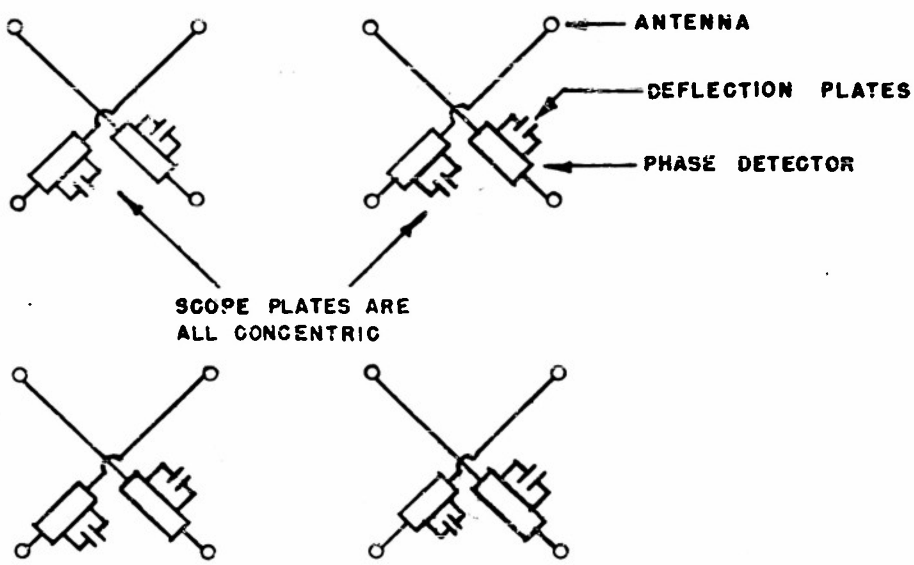

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FIG. 2 DISTRIBUTED VECTOR-PHASE SYSTEM

LARGE DIAMETER VECTOR-PHASE SYSTEMS '

PLATE 3

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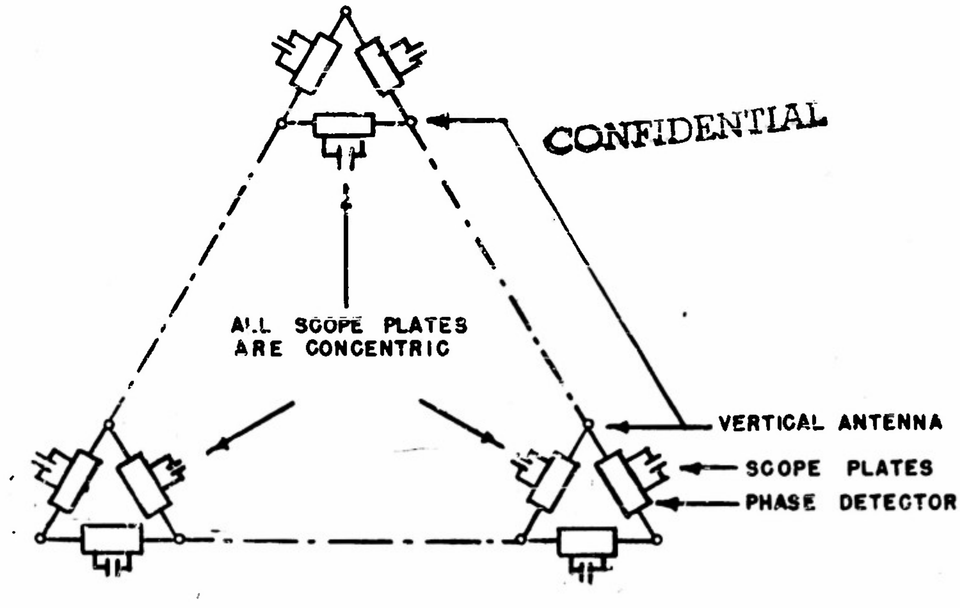

FIG. 1 TRIANGULAR DISTRIBUTED VECTOR-PHASE SYSTEM

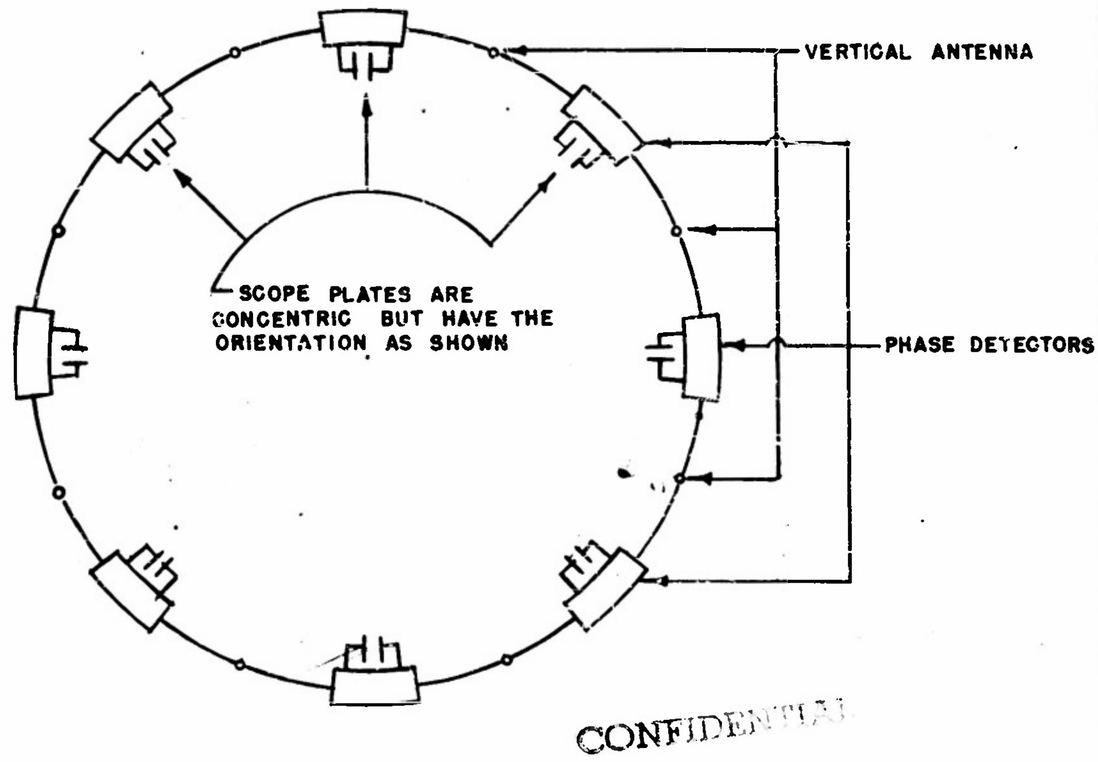

FIG. 2 CIRCULAR CONTINUOUS VECTOR-PHASE SYSTEM

LARGE DIAMETER VECTOR-PHASE SYSTEMS

PLATE 4

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III. DEFINITION OF SYMBOLS

h	relative magnitude of the weaker of two incoming plane waves referred to the stronger
$\Gamma = \gamma$	time phase difference between the strongest and the next strongest wave at the center of a complete system
Φ_n	azimuthal angle of the nth strongest wave measured from the reference counter-clockwise
Φ	azimuthal angular separation between two incoming plane waves
θ_n	polar angle of arrival of the nth strongest wave measured clockwise from the vertical axis
θ	polar angular separation between incoming plane waves
N	total number of incoming waves
n	nth wave
E_n	electric field strength of the nth wave
M_s	electric field strength at the center of narrow-aperture systems
A	total number of systems
a	ath antenna
S	total number of small-aperture systems
s	sth system
β	aperture or system radius in electrical radians
D	system diameter in wavelengths
C	spacing between antennas in electrical radians
ξ	bearing error
B_S	indicated bearing of narrow-aperture systems
B_I	indicated bearing of a direction finding system
B_{VA}	composite bearing resulting from vector-average bearing combination technique
$\Psi_{a,a+L}$	time phase difference between the lead angle of the resultant waves (or resultant induced voltages) at antenna (a + L) and at antenna a
$E_{a,a+L}$	d.c. output of a phase detector with two equal voltages of time phase difference $\Psi_{a,a+L}$ applied to the detector
P_n	angle of the normal to the curve of constant phase with reference to the stronger of two interfering signals as the reference axis
ρ_a	argument of the sum of two vectors of relative magnitude h
τ_a	difference of the arguments of two vectors
LPD	linear phase detection
SPD	sinusoidal phase detection
TSPD	triple angle vector-phase system, with sinusoidal phase detection
U	differential phase shift preceding detector adders (Plate 19)
J_n	Bessel function of nth order

IV. INDICATED DIRECTION OF ARRIVAL WITH A SINGLE SIGNAL

A. Small-Diameter Four-Element System with Diagonal Phase Comparison

The method of determining the indicated bearing as a function of the direction of arrival of a single signal for a small-diameter system (shown in Plate 1) is illustrated in Plate 5. Figure 1 of Plate 5 shows the method of obtaining the phase difference $\psi_{P,Q}$ between antennas as a function of angle of arrival of a single signal.

Let $\psi_{P,Q}$ be the time phase difference between the lead angle of voltages induced in antennas P and Q (lead angle of antenna Q subtracted from lead angle of antenna P). Then, referring to Plate 5, Fig. 1,

$$\psi_{4,2} = 2\beta \sin \varphi_1 \tag{4.1}$$

and

$$\psi_{3,1} = 2\beta \cos \varphi_1, \tag{4.2}$$

where φ_1 is the angle of arrival and β is the electrical radius of the system. Now

$$\beta = \frac{D}{2} \times \frac{2\pi}{\lambda}$$

where D is the system diameter in wavelengths and λ is the wavelength of the incoming signal.

Assuming sinusoidal phase detection, the d.c. outputs of the phase detectors and the magnitudes of the vectors are given by

$$E_{4,2} = K \sin \psi_{4,2} \qquad \text{for the east-west vector} \tag{4.3}$$

and

$$E_{3,1} = K \sin \psi_{3,1} \qquad \text{for the north-south vector} \tag{4.4}$$

Figure 2 of Plate 5 shows the resultant of these two vectors. The angle of the resultant gives the indicated bearing (B_I) which can be expressed as

$$B_I = \tan^{-1}\left[\frac{E_{4,2}}{E_{3,1}}\right] = \tan^{-1}\left[\frac{\sin \psi_{4,2}}{\sin \psi_{3,1}}\right] \tag{4.5}$$

or

$$B_I = \tan^{-1}\left[\frac{\sin (2\beta \sin \varphi_1)}{\sin (2\beta \cos \varphi_1)}\right]. \tag{4.6}$$

If 2β is very small, approximately linear phase detection results. For linear phase detection

$$B_I = \tan^{-1}\left[\frac{2\beta \sin \varphi_1}{2\beta \cos \varphi_1}\right] = \varphi_1. \tag{4.7}$$

Therefore, for linear phase detection this system gives a true bearing. The deviation that exists between the indicated bearing and the direction of arrival, with sinusoidal phase detection, is discussed in a later section.

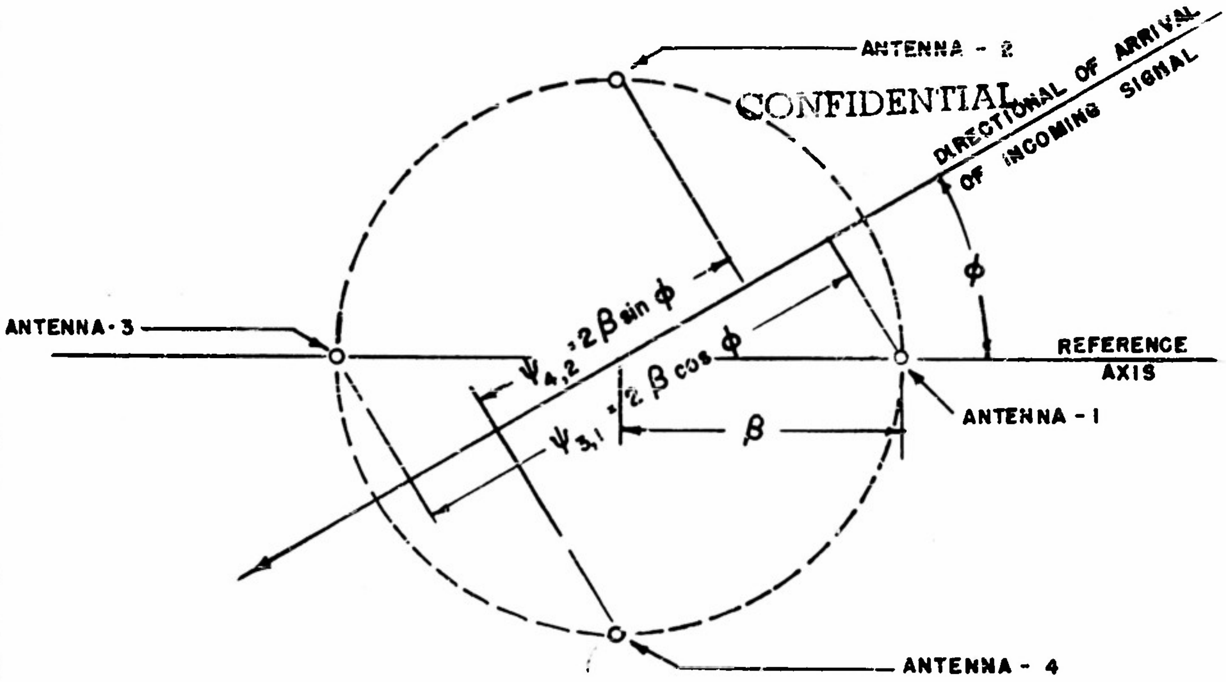

FIG. 1 PHASE DIFFERENCES BETWEEN ANTENNAS FOR A FOUR ELEMENT SYSTEM WITH DIAGONAL PHASE COMPARISON.

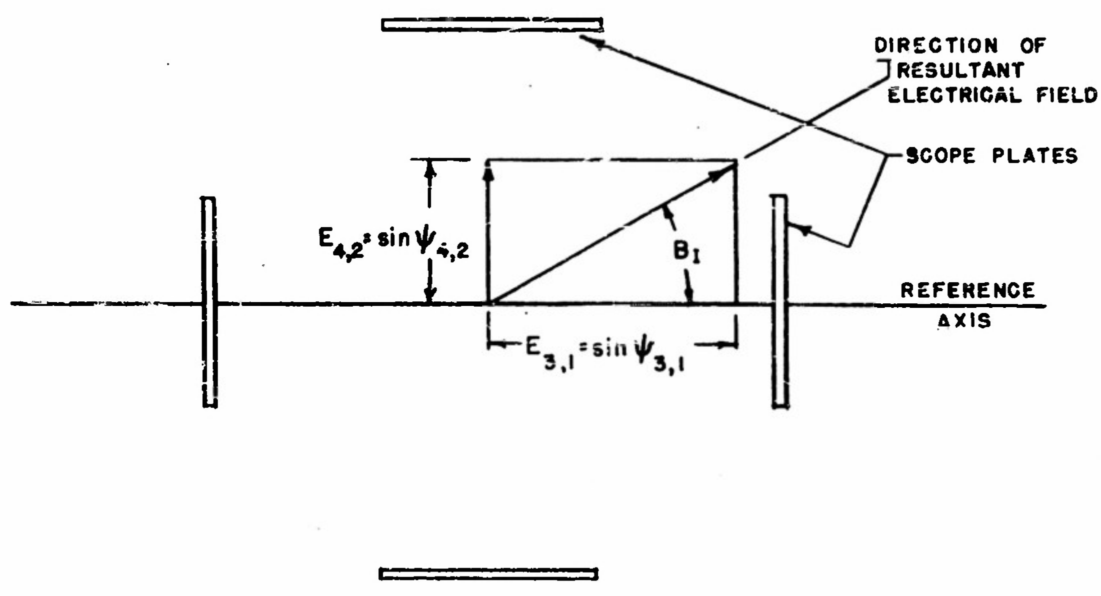

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FIG. 2 INDICATED BEARING FOR A FOUR ELEMENT SYSTEM WITH DIAGONAL PHASE COMPARISON

PHASE DIFFERENCES AND RESULTANT INDICATED BEARING FOR A FOUR ELEMENT SYSTEM WITH DIAGONAL PHASE COMPARISON

PLATE 5

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B. Distributed System

Assuming that each of the unit systems making up the resultant distributed system would give an indicated bearing equal to the direction of arrival of a single signal, then the complete system would give a true bearing. This follows from taking the vector resultant of vectors all at the same angle. If the unit systems all have the same orientation, as do those of Plates 2 and 3, then the error resulting from sinusoidal phase detection for the distributed system will be the same as the error for each of the unit systems.

C. Continuous Circular System

The method for determining the indicated bearing of a circular continuous system, similar to the system shown in Plate 4, as a function of the direction of arrival of a single signal is illustrated by Plate 6. Figure 1 of Plate 6 illustrates the method of obtaining the phase difference $\psi_{P,Q}$ between antennas as a function of angle of arrival of a single signal. $\psi_{P,Q}$ has the same definition as given in IVa. Referring to Fig. 1 $\psi_{a,a+1}$ is the phase difference between the $(a+1)$th and the (a)th antenna.

$$\psi_{a,a+1} = C \cos \left[a \frac{2\pi}{8} - \varphi_1\right] \tag{4.8}$$

for eight antennas, equally spaced on a circle of electrical radius β. Generalizing for A antennas, equally spaced on a circle of electrical radius β

$$\psi_{a,a+1} = C \cos \left[a \frac{2\pi}{8} - \varphi_1\right], \tag{4.9}$$

where

$$C = 2\beta \sin \frac{\pi}{A} \tag{4.10}$$

$$\beta = \frac{D}{2} \times \frac{2\pi}{\lambda}$$

D = the system diameter in wavelengths.

Referring to Fig. 2 of Plate 6 let the magnitude of the vector resulting from the phase difference $\psi_{a,a+1}$ be called $E_{a,a+1}$. Assuming sinusoidal phase detection

$$E_{a,a+1} = K \sin (\psi_{a,a+1}). \tag{4.11}$$

The vector of magnitude $E_{a,a+1}$ is at an angle $a(\frac{2\pi}{8})$ for eight antennas, or generalizing at an angle $a(\frac{2\pi}{A})$ for A antennas.

The vector sum of all eight vectors $E_{a,a+1}$ shown in Fig. 2 of Plate 6, is a resultant vector at an angle B_I which is the indicated bearing of this system. For the case of eight antennas, B_I can be expressed as

$$B_I = \tan^{-1}\left[\frac{\sum_{a=1}^{8} E_{a,a+1} \sin\left(a\,\frac{2\pi}{8}\right)}{\sum_{a=1}^{8} E_{a,a+1} \cos\left(a\,\frac{2\pi}{8}\right)}\right] \tag{4.12}$$

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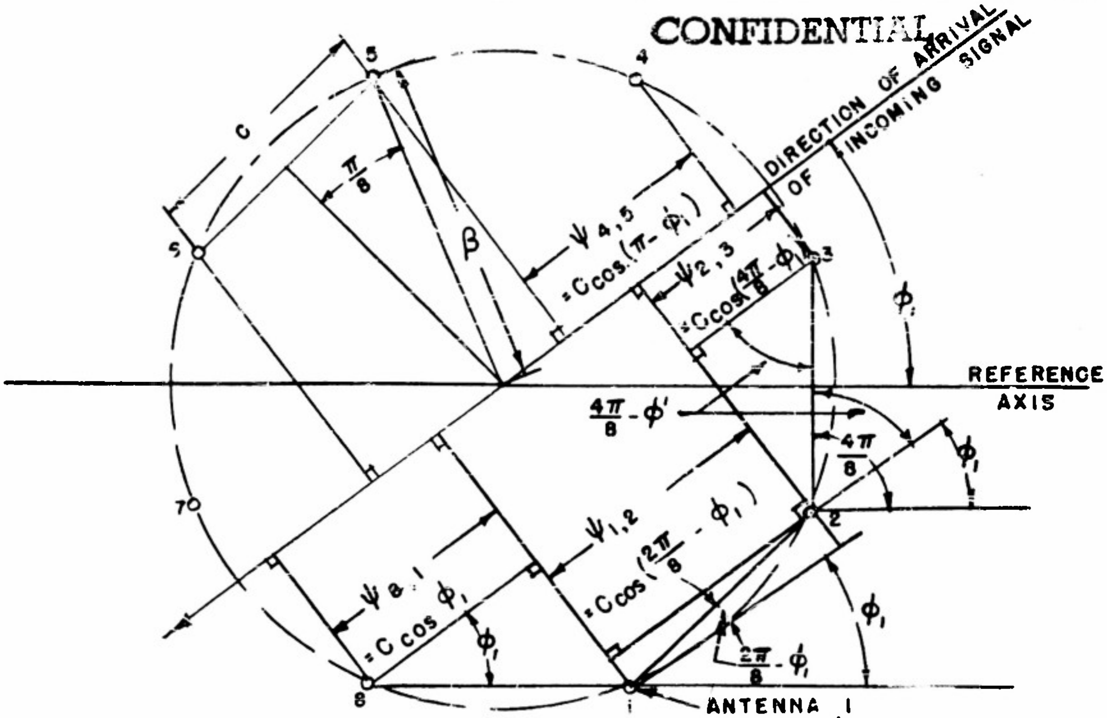

FIG. 1 PHASE DIFFERENCES BETWEEN ANTENNAS FOR AN 8 ELEMENT CIRCULAR CONTINUOUS SYSTEM

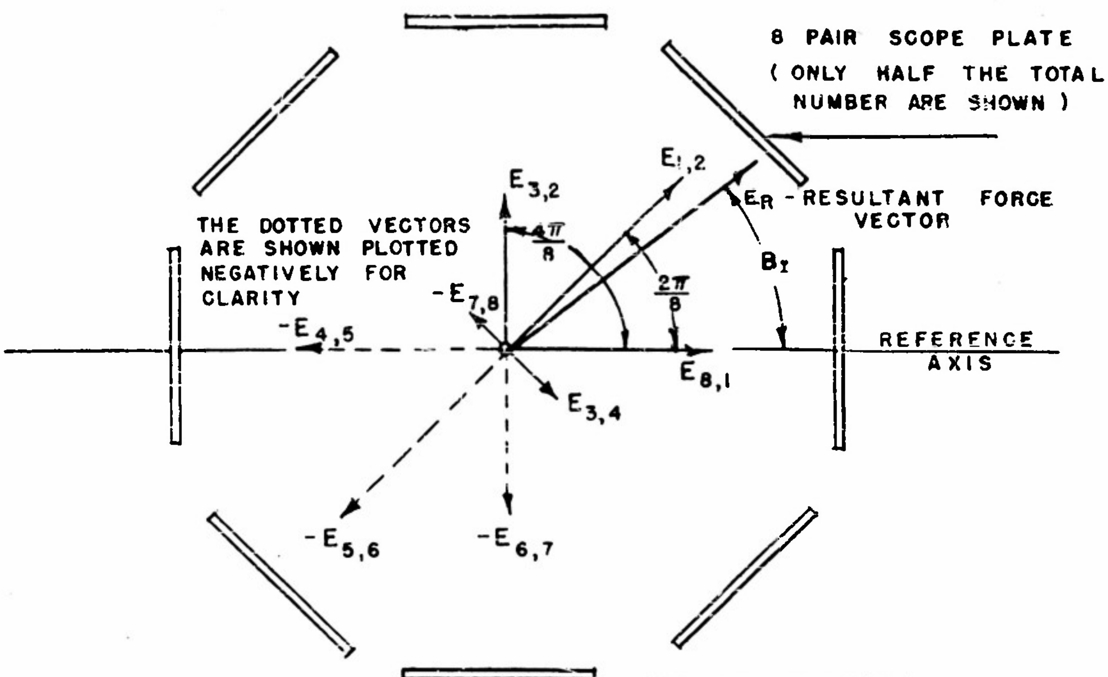

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FIG. 2 INDICATED BEARING FOR AN 8 ELEMENT CIRCULAR CONTINUOUS SYSTEM

PHASE DIFFERENCES AND INDICATED BEARING FOR AN EIGHT ELEMENT CIRCULAR CONTINUOUS SYSTEM

PLATE 6

DEPT. ELECTRICAL ENGINEERING – UNIVERSITY OF ILLINOIS where $E_{0,0}$ is assumed to signify $E_{a,1}$. Generalizing, for A equally spaced antennas

$$B_I = \tan^{-1}\left[\frac{\sum_{a=1}^{A} E_{a,a+1} \sin\left(a\frac{2\pi}{A}\right)}{\sum_{a=1}^{A} E_{a,a+1} \cos\left(a\frac{2\pi}{A}\right)}\right]. \tag{4.13}$$

In terms of the phase differences between antennas

$$B_I = \tan^{-1}\left[\frac{\sum_{a=1}^{A} \sin(\psi_{a,a+1}) \sin\left(a\frac{2\pi}{A}\right)}{\sum_{a=1}^{A} \sin(\psi_{a,a+1}) \cos\left(a\frac{2\pi}{A}\right)}\right]. \tag{4.14}$$

In terms of the direction of arrival of the single signal causing the phase differences

$$B_I = \tan^{-1}\left\{\frac{\sum_{a=1}^{A} \sin\left[c\cos\left(a\frac{2\pi}{A} - \varphi_1\right)\right]\sin\left(a\frac{2\pi}{A}\right)}{\sum_{a=1}^{A} \sin\left[c\cos\left(a\frac{2\pi}{A} - \varphi_1\right)\right]\cos\left(a\frac{2\pi}{A}\right)}\right\}. \tag{4.15}$$

If β and C are very small, approximate linear phase detection results. For linear phase detection

$$B_I = \tan^{-1}\left[\frac{\sum_{a=1}^{A} \cos\left(a\frac{2\pi}{A} - \varphi_1\right) \sin\left(a\frac{2\pi}{A}\right)}{\sum_{a=1}^{A} \cos\left(a\frac{2\pi}{A} - \varphi_1\right) \cos\left(a\frac{2\pi}{A}\right)}\right]. \tag{4.16}$$

In Appendix B it is shown that

$$\sum_{a=1}^{A} \cos\left[K\left(\frac{a2\pi}{A} - \varphi_1\right)\right] \sin\left[L\left(\frac{a2\pi}{A}\right)\right] = \frac{A}{2}\sin(K\varphi_1) \tag{4.17}$$

and

$$\sum_{a=1}^{A} \cos\left[K\left(\frac{a2\pi}{A} - \varphi_1\right)\right] \cos\left[L\left(\frac{a2\pi}{A}\right)\right] = \frac{A}{2}\cos(K\varphi_1) \tag{4.18}$$

if $K - L = NA$, $K + L \neq NA$ and $N = 0, 1, 2 \cdots$. If $K = L = 1$ then the above expressions 4.17 and 4.18 become identical to the numerator and denominator respectively of equation 4.16. After these substitutions the indicated bearing becomes

$$B_I = \tan^{-1}\left[\frac{\frac{A}{2}\sin\varphi_1}{\frac{A}{2}\cos\varphi_1}\right] = \varphi_1. \tag{4.19}$$

Hence for linear phase detection, this system gives a true bearing. The deviation that exists between the indicated bearing and the direction of arrival, with sinusoidal phase detection, is discussed in Section V.

V. DISCRIMINATION AGAINST A SINGLE INTERFERING SIGNAL

A. Small-Diameter Four-Element System with Diagonal Phase Comparison

As the vector-phase system shown in Fig. 1 of Plate 1 is a true phase-front system and the diameter is very small, it would be expected that a good indication of its behavior while receiving a desired (stronger) and an undesired (weaker) signal could be predicted from a knowledge of the phase front (lines of constant phase) in an interference field resulting from these two signals. Typical phase-front curves for such an interference field are shown in Plate 7, Page 16. It is seen that at some points of the phase-front curve, the normal to the phase front deviates greatly from the direction of arrival of the stronger signal (the assumed true bearing). If the relative magnitude of the weaker to the stronger signal, h, approaches unity, if the angle of separation between the weaker and stronger signal, φ, is small, and if the relative time phase difference between the weaker and stronger signal is nearly 180°, then the bearing error can approach 90°, and it would be expected that the system of Plate 1 could have bearing errors of nearly 90°.

The equation for the normal to the phase front in terms of the parameters h, r and φ defined in the preceding paragraph is derived in Appendix C. Assuming that the stronger signal (the desired signal) is arriving along the axis (zero degrees) this equation is

$$P_N = \tan^{-1}\left[\frac{\sin\varphi}{\left(\dfrac{\frac{1}{h} + \cos r}{h + \cos r}\right) + \cos\varphi}\right]. \tag{5.1}$$

The equation for the actual indicated bearing of the system of Plate 1, Fig. 1 can be determined from Equation 4.5 if the phase differences $\psi_{4,2}$ and $\psi_{3,1}$ between antennas can be determined when both a desired and an undesired signal are present. It is shown in Appendix C that in terms of the parameters h, φ, and r, $\psi_{3,1}$ and $\psi_{4,2}$ can be expressed as

$$\psi_{3,1} = 2\,\beta\cos\varphi_1 + \rho_1 - \rho_3 \tag{5.2}$$

and

$$\psi_{4,2} = 2\,\beta\sin\varphi_1 + \rho_2 - \rho_4 \tag{5.3}$$

where

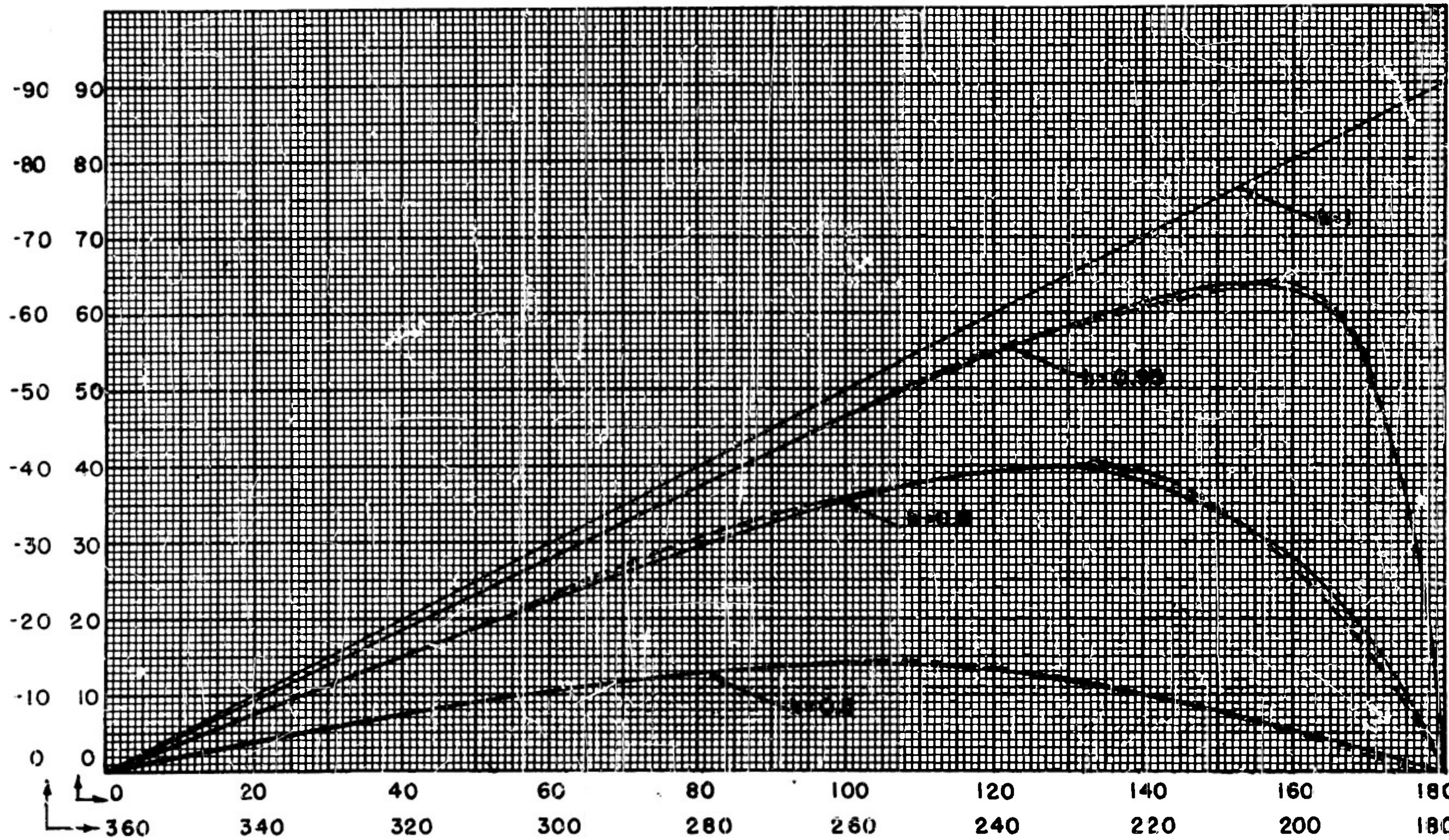
$$\rho_a = \tan^{-1}\left[\frac{h\sin\tau_a}{1 + h\cos\tau_a}\right],\quad (a = 1,2,3,4) \tag{5.4}$$

$$\tau_a = 2\beta\sin\left[(a-1)\left(\frac{\pi}{2}\right) - \frac{\varphi}{2} - \varphi_1\right]\sin\left(\frac{\varphi}{2}\right)+r,\quad (a=1,2,3,4) \tag{5.5}$$

β = the electrical radius of the system.

Substituting equations 5.2 and 5.3 in the equation for indicated bearing and substracting the angle of arrival of the stronger signal, φ_1 gives the bearing error ξ. For sinusoidal phase detection,

$$\xi = \tan^{-1}\left[\frac{\sin(2\,\beta\sin\varphi_1 + \rho_2 - \rho_4)}{\cos(2\,\beta\cos\varphi_1 + \rho_1 - \rho_3)}\right] - \varphi_1. \tag{5.6}$$



Φ = AZIMUTHAL SEPARATION IN DEGREES

COMPARISON BETWEEN THE DEVIATION OF THE PHASE FRONT NORMAL (SOLID CURVES) AND A FOUR ELEMENT VECTOR PHASE SYSTEM WITH A DIAMETER OF $\lambda/18$ (DOTTED CURVES)

$\Gamma = 90°$ $h = 0.5, 0.8, 0.95, 1.0$

PLATE 7

In Appendix D the limit of ξ as β approaches zero is developed. It is seen that this limit is exactly the amount by which the normal to the phase front deviates from the direction of arrival of the stronger signal (Equation 5.1). Curves of bearing error ξ for an actual system of 1/18 wavelength diameter, calculated from Equation 5.6, are shown plotted in Plate 7 together with the corresponding curves for the deviation of the phase-front normal calculated from Equation 5.1. It is seen that the phase-front normal curves give a very good approximation to the behavior of the actual system.

B. Distributed Systems

Referring to the interference field of Plate 8, Fig. 2, it is seen that at various locations in the field, corresponding to various DF sited, the normal to the phase front is to the left (System 1), to the right (System 2) or in line with the direction of arrival of the stronger signal. This corresponds to positive, negative, and zero bearing error for a very small-diameter vector-phase system centered at these points. It is not possible to know in advance where the locations corresponding to zero error would be, but it would be expected that if a collection of many small-diameter systems were dispersed over several wavelengths distance that some of the systems would fall at or near points of zero error and others would fall at points of both negative and positive error. If the bearings of this collection of systems were combined by some sort of averaging or limited vector-averaging* techniques, as in the distributed system of Plate 3, Fig. 2, then it would be expected that the positive and negative errors in general would tend to cancel. Thus the resultant error of the composite system would be much less than the maximum error of the unit systems comprising the composite distributed system. Hence, the guaranteed accuracy of the composite system would be much greater than the guaranteed accuracy of a narrow-diameter system located at random in the interference field.

This general technique for automatically combining bearings of narrow-diameter systems distributed over a region large in wavelengths is applicable to systems other than vector phase systems and the topic has been discussed in a previous report[5] which includes bearing error curves for various types of systems. In general, the larger the number of unit systems and the larger the diameter of the composite distributed system the greater will be the reduction in bearing error.

A curve of bearing error as a function of composite system aperture for a circular distribution of eight narrow-aperture vector-phase systems is shown in Plate 9. For comparison purposes the bearing curve for a true-Doppler system under the same field conditions is also shown. It is seen that, like most wide-aperture distributions of narrow-aperture systems with electrical combination of bearing information, the guaranteed improvement in bearing error increases rather slowly with increasing aperture as compared to a true-Doppler System, at least for small relative magnitudes and large angular separations.

---

* The amplitude-limited vector-average bearing - B_{LVA} of a collection of bearings $B_1, B_2 \cdots B_S$ will be taken to mean

$$B_{LVA} = \tan^{-1}\left[\frac{\sum_{s=1}^{S} \sin B_s}{\sum_{s=1}^{S} \sin B_s}\right].$$

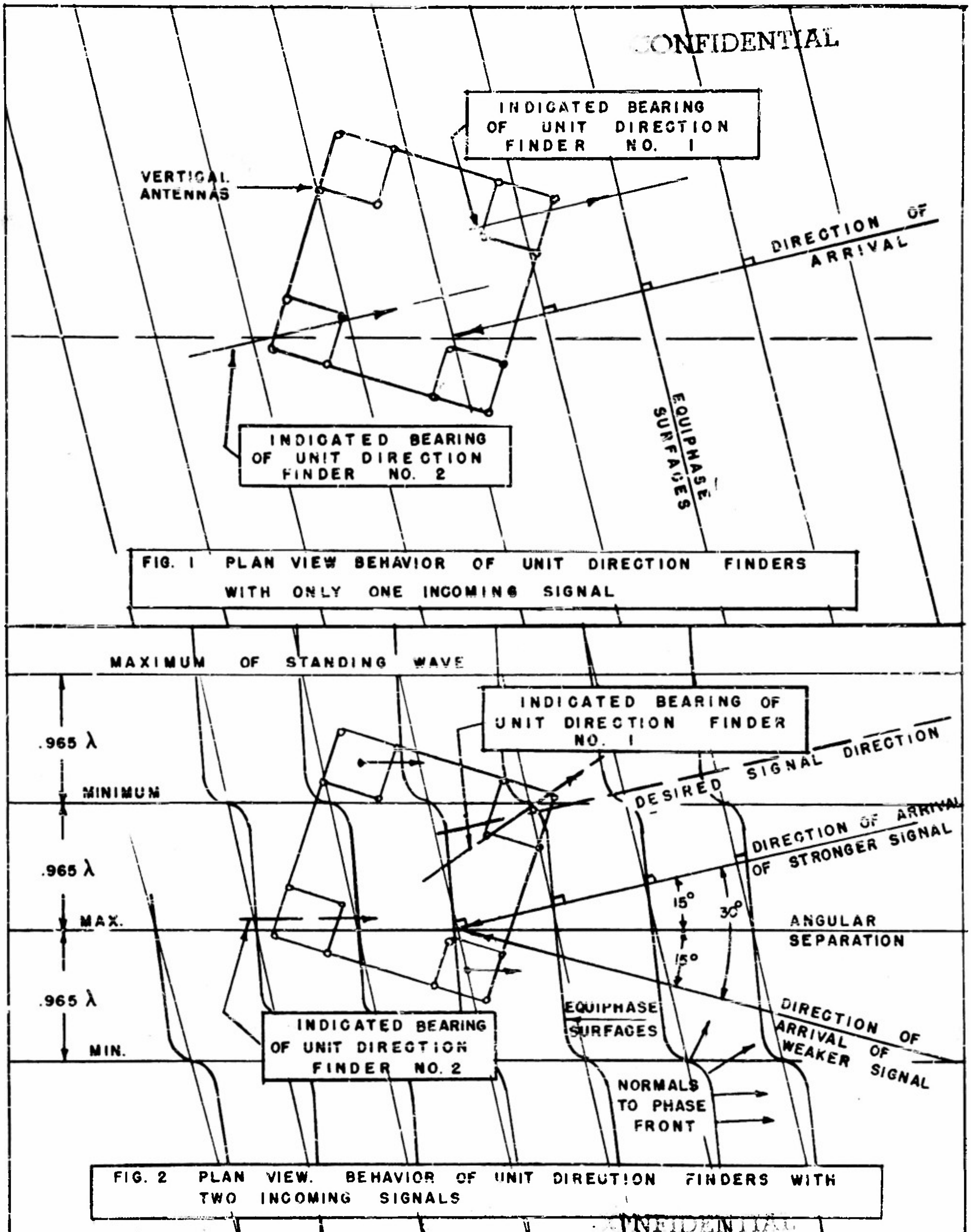


FIG. 1 PLAN VIEW BEHAVIOR OF UNIT DIRECTION FINDERS WITH ONLY ONE INCOMING SIGNAL

FIG. 2 PLAN VIEW. BEHAVIOR OF UNIT DIRECTION FINDERS WITH TWO INCOMING SIGNALS

BEHAVIOR OF THE BEARINGS OF A SERIES OF UNIT PHASE-FRONT DIRECTION FINDERS UNDER THE ACTIONS OF 1 AND 2 INCOMING SIGNALS

PLATE 8

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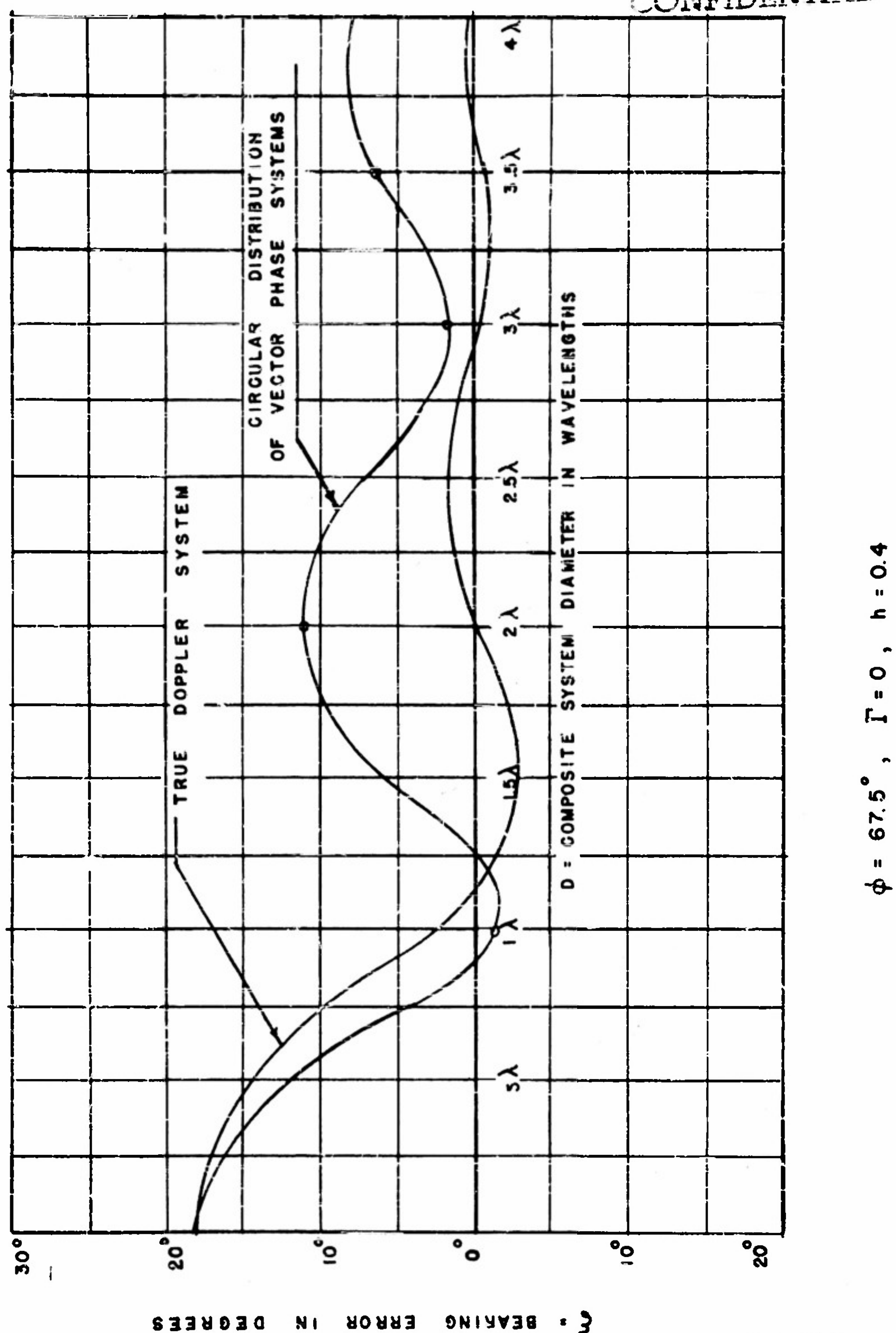

BEARING ERROR FOR A CIRCULAR DISTRIBUTION OF EIGHT NARROW APERTURE VECTOR-PHASE SYSTEMS AND A TRUE DOPPLER SYSTEM.

PLATE 9

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In order to reduce the errors caused by multipath ionospheric propagation, where large relative magnitudes and small angles of separation exist, quite large apertures are often required when using a wide-aperture distribution of narrow-aperture systems similar to those shown in Plate 3, Fig. 2. Diameters of five to ten wavelengths would be required together with a relatively large number of distributed systems. This is a fundamental difficulty resulting from the basic nature of interference field patterns which must be faced with nearly all wide-aperture systems. Under conditions of small angular separation the configurations in the phase front tend to spread out in wavelength separation so that in order to "average" these phase front variations a system must be quite large in wavelength to span them.

One basic technique for overcoming this problem is suggested in Technical Report No. 5. Under conditions of small angular separation and large relative magnitude it is recognized from consideration of Plate 8 that with general direction finders, which would be located at points of stronger field strength in the interference field, the tendency would be to give better bearings than with those located at points of weaker field strength. In particular, a direction finder located at the maximum in the interference field would give a bearing in error by less than half the angular separation. Although a distributed system making use of this amplitude information is not strictly speaking a vector-phase system, a wide-aperture system which weighted the individual bearings could be built out of narrow-aperture vector-phase systems.

The sine and cosine components represented by d.c. outputs from a unit narrow-aperture vector phase system would have to be weighted by a factor dependent on the field strength at the center of the unit system in question. Then the weighted components would be added and the resultant used for bearing indication. This is an example of a general bearing-combining technique referred to as a vector-average combination of bearing information in Report No. 9. The resultant bearing can be expressed as

$$B_{VA} = \tan^{-1}\left[\frac{\sum_{s=1}^{S} M_s \sin B_s}{\sum_{s=1}^{S} M_s \cos B_s}\right]$$

where B_s is the bearing of the sth system and M_s is the relative field strength at the center of the sth system.

The weighting might be done by simultaneously gating the sine and cosine components in accordance with an AVC voltage from the receivers. In most vector-phase systems, because of the amplitude limiting in all receiver channels, the determination of a weighting factor would be somewhat awkward but not basically difficult. For flash transmissions the gating voltage would have to be faster acting than most AVC voltages.

The lessening of the aperture required to obtain a given reduction in bearing error with a vector-average combination of bearing information, as compared to a limited vector-average combination, is illustrated by Plate XVII of Technical Report No. 9. Although these curves are for unit Adcock systems they should be reasonably typical of narrow-aperture vector-phase systems as well.

It should be emphasized that for large angular separations, such as might be encountered with site re-radiations, the vector-average combination technique often gives worse results than the limited vector-average combination technique.

C. Circular Continuous System

If a circular continuous system (Plate 4) is dispersed over a region large in wavelengths in an interference field it would be expected that the phase differences between some antennas would be increased and between other antennas decreased due to the effect of an interfering signal. The net effect of increased diameter would be to make these increases and decreases more random, resulting in less effect on the indicated bearing by the phase-front variations. The phase-front mechanism is not as clear cut here as for a distribution of narrow-diameter systems, however.

Another method of determining the effect of a single interfering signal on the indicated bearing is by comparing the continuous circular vector-phase system of Plate 10, Fig. 1 to a rectangular switched-Doppler system of the type shown in Plate 10, Fig. 2. It is shown in Appendix A that in any interference field resulting from two or more signals, a circular vector-phase system and a rectangular double-gate switched-Doppler system (Plate 10), having identical antenna systems, should give identical bearings, assuming that the phase detectors of both systems have identical characteristics. In other words a switched-Doppler system and a vector-phase system provide alternate equivalent techniques for presenting the same antenna information to a bearing indication scope. The bearing error for the two systems is identical. The switched-Doppler system and hence the vector-phase system would, in the case of a large number of antennas, be expected to give roughly the same discrimination against an interfering signal as a true-Doppler system for which extensive error curves have been drawn[5,6].

The actual indicated bearing and the bearing error for any continuous circular system can be found by determining the phase differences ($\psi_{a,a+1}$) which exist between antennas when a single interfering signal is present and then substituting these phase differences in Equation 4.14. This technique is analogous to that used in Section IVa for narrow-diameter four-element systems. The derivation is carried out in Appendix F. The resultant equation for the bearing error $\mathcal{E}$ is given by

$$\mathcal{E} = \tan^{-1}\left[\frac{\sum_{a=1}^{A} \sin\psi_{a,a+1}\, \sin\left(\frac{a2\pi}{A}\right)}{\sum_{a=1}^{A} \sin\psi_{a,a+1}\, \cos\left(\frac{a2\pi}{A}\right)}\right] - \varphi_1 \qquad (5.7)$$

where $\psi_{a,a+1}$, A, and φ have the same meanings given in Section IVa of this report

$$\psi_{a,a+1} = 2\beta \sin\left(\frac{\pi}{A}\right) \cos\left(\frac{2\pi a}{A} - \varphi_1\right) + \rho_{a+1} - \rho_a \qquad (5.8)$$

where

$$\rho_a = \tan^{-1}\left[\frac{h \sin \tau_a}{1 + h \cos \tau_a}\right] \qquad (5.9)$$

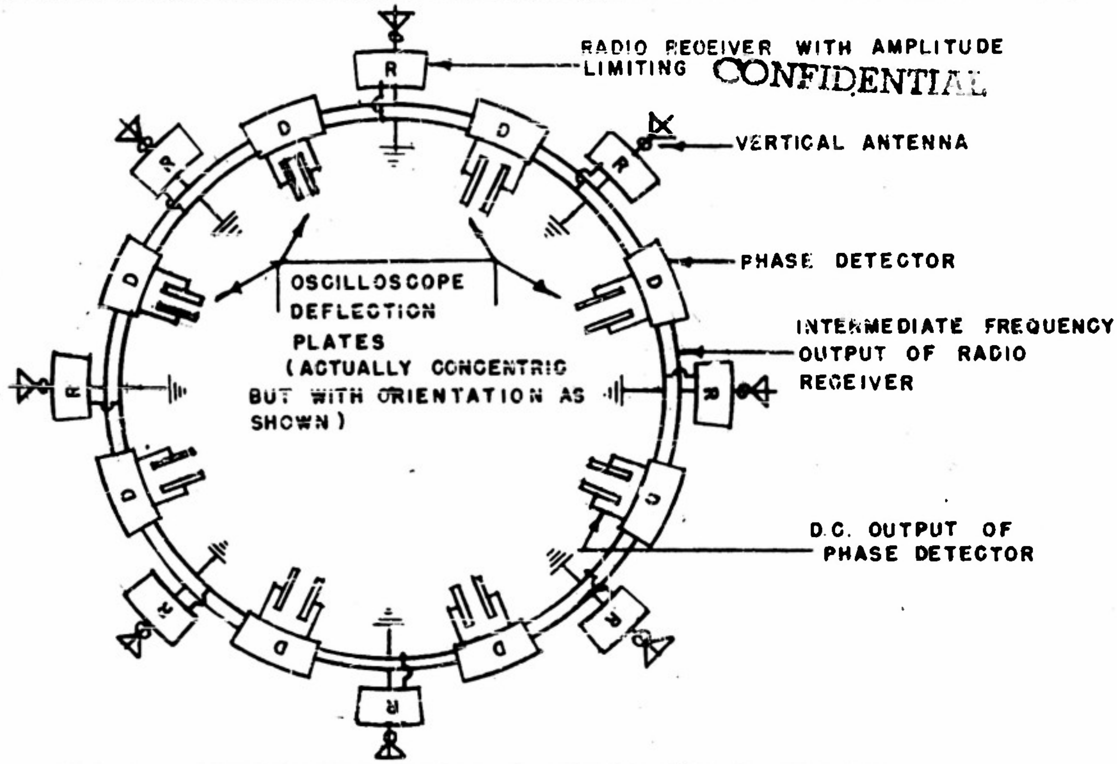

FIG. I CONTINUOUS CIRCULAR VECTOR PHASE SYSTEM

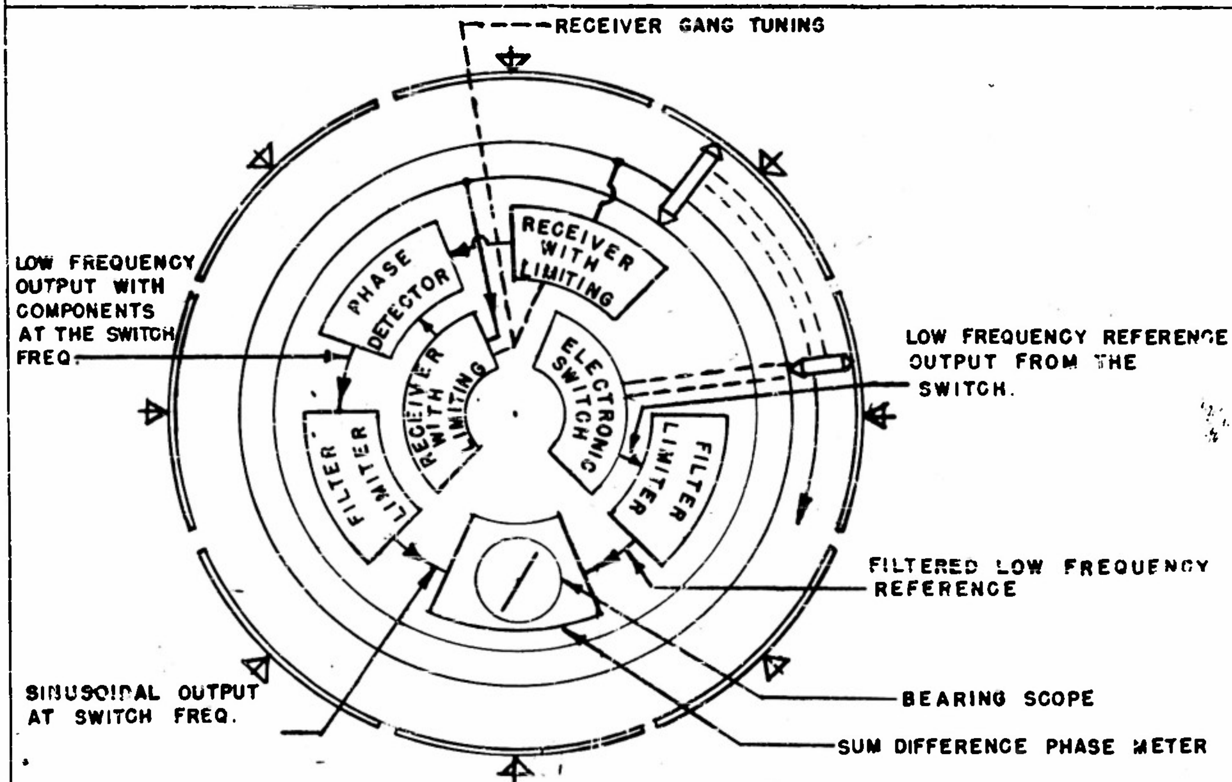

FIG. 2 RECTANGULAR SWITCHED "DOPPLER" SYSTEM

EQUIVALENT VECTOR-PHASE AND RECTANGULAR SWITCHED "DOPPLER" SYSTEMS

PLATE 10

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and

$$\tau_a = -2\beta \cos\left[\frac{\pi}{A}(2a-1) - \frac{1}{2}(\varphi + 2\varphi_1)\right] \sin\left(\frac{\varphi}{2}\right) + r \qquad (5.10)$$

and where h, β, φ, φ_1 and r also have the definitions given in Section IVa.

If the antennas are spaced so closely that $\psi_{a,a+1}$ is reasonably small, the phase detection is nearly linear. For linear phase detection Equation 5.7 becomes

$$\xi = \tan^{-1}\left[\frac{\sum_{a=1}^{A} \psi_{a,a+1} \sin\left(\frac{a2\pi}{A}\right)}{\sum_{a=1}^{A} \psi_{a,a+1} \cos\left(\frac{a2\pi}{A}\right)}\right] - \varphi_1 \qquad (5.11)$$

with $\psi_{a,a+1}$ still expressed by Equation 5.8. It is shown in Appendix F that Equation 5.11 reduced to

$$\xi = \tan^{-1}\left[\frac{\sum_{a=1}^{A} (\rho_{a+1}-\rho_a) \sin\left(\frac{2\pi a}{A} - \varphi_1\right)}{A\beta \sin\left(\frac{\pi}{A}\right) + \sum_{a=1}^{A} (\rho_{a+1}-\rho_a) \cos\left(\frac{2\pi a}{A} - \varphi_1\right)}\right]. \qquad (5.12)$$

Equation 5.12 shows that the bearing error can be made arbitrarily small by using a sufficiently large electrical radius. The number of antennas must naturally be increased as β increases if approximately linear phase detection is to result.

Two special cases of Equation 5.7 are of interest; one, as the electrical radius β of the system approaches zero, and two, as the number of antennas approaches infinity. It is shown in Appendix G that the limit of Equation 5.11 as β approaches zero is given by the equation of the phase-front normal (Equation 4.1) as would be expected if the circular system were a true phase-front system. It is shown in Appendix H that limit of Equation 5.11 as the number of antennas approaches infinity is given by

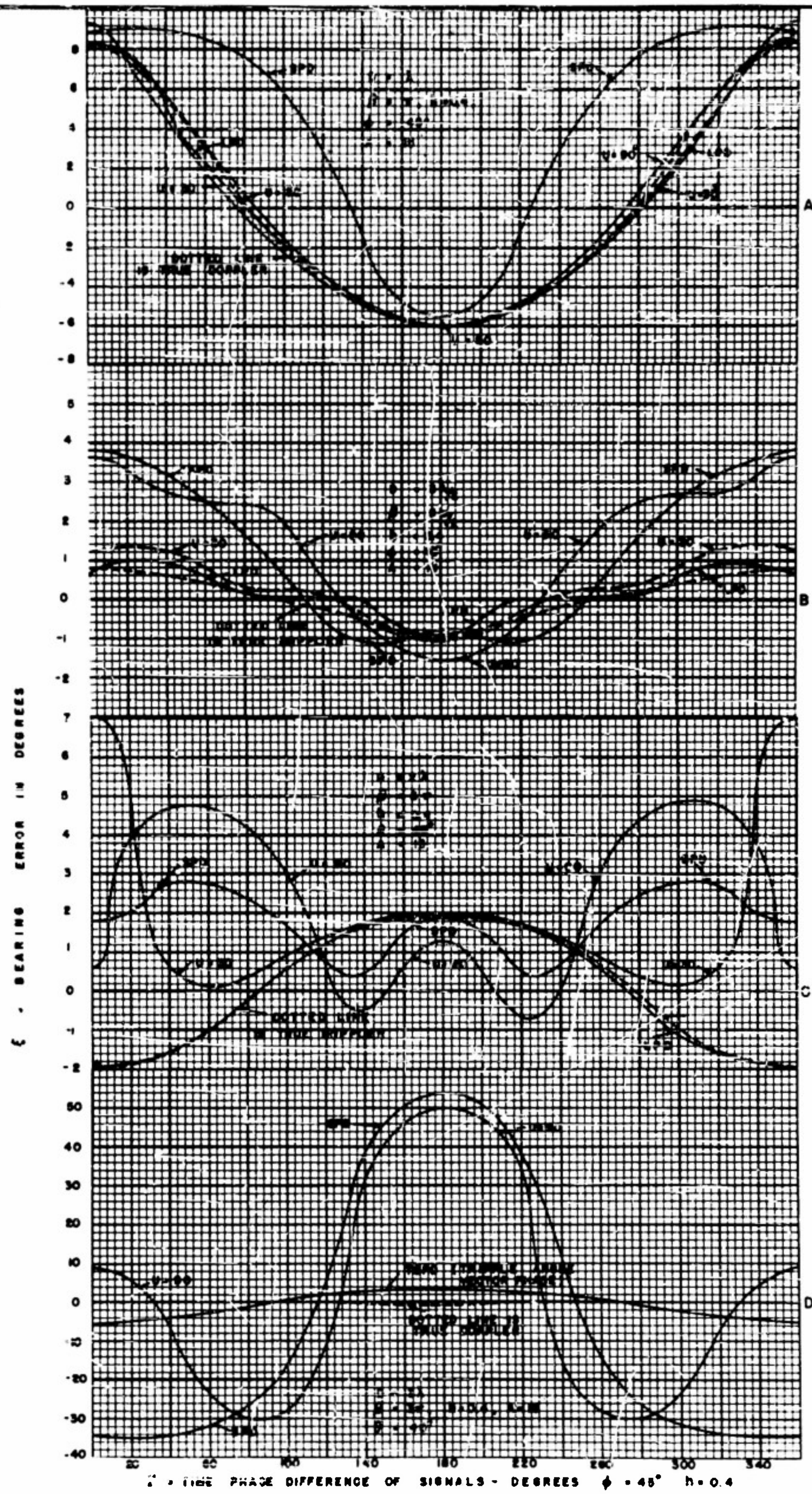
$$\xi = \tan^{-1}\left[\frac{D \cos\left(\frac{\varphi}{2}\right)}{B - D \sin\left(\frac{\varphi}{2}\right)}\right], \qquad (5.13)$$

where

$$D = \sum_{K=1}^{\infty} \frac{2h^K}{K} (-1)^{K+1} \cos(Kr)\, J_1\left[K\beta\, 2 \sin\left(\frac{\varphi}{2}\right)\right] \qquad (5.14)$$

and where h, r, β, and φ have the same definitions given in Section IVa. This is the equation for the bearing error of a true-Doppler system as would be expected once the equivalence between a switched-Doppler and a vector-phase system is recognized. The limiting case of a switched-Doppler system should be true-Doppler system.

Bearing error curves for a vector-phase (or a switched-Doppler) system are shown plotted against system diameter in Plate 13 and against time phase r in Plates 11 and 12, for systems with 16 antenna



WAVE INTERFERENCE BEARING ERROR FOR A TRUE DOPPLER SYSTEM AND A 16 ELEMENT VECTOR-PHASE (OR SWITCHED DOPPLER SYSTEM) WITH VARIOUS TYPES OF PHASE DETECTION

PLATE 11

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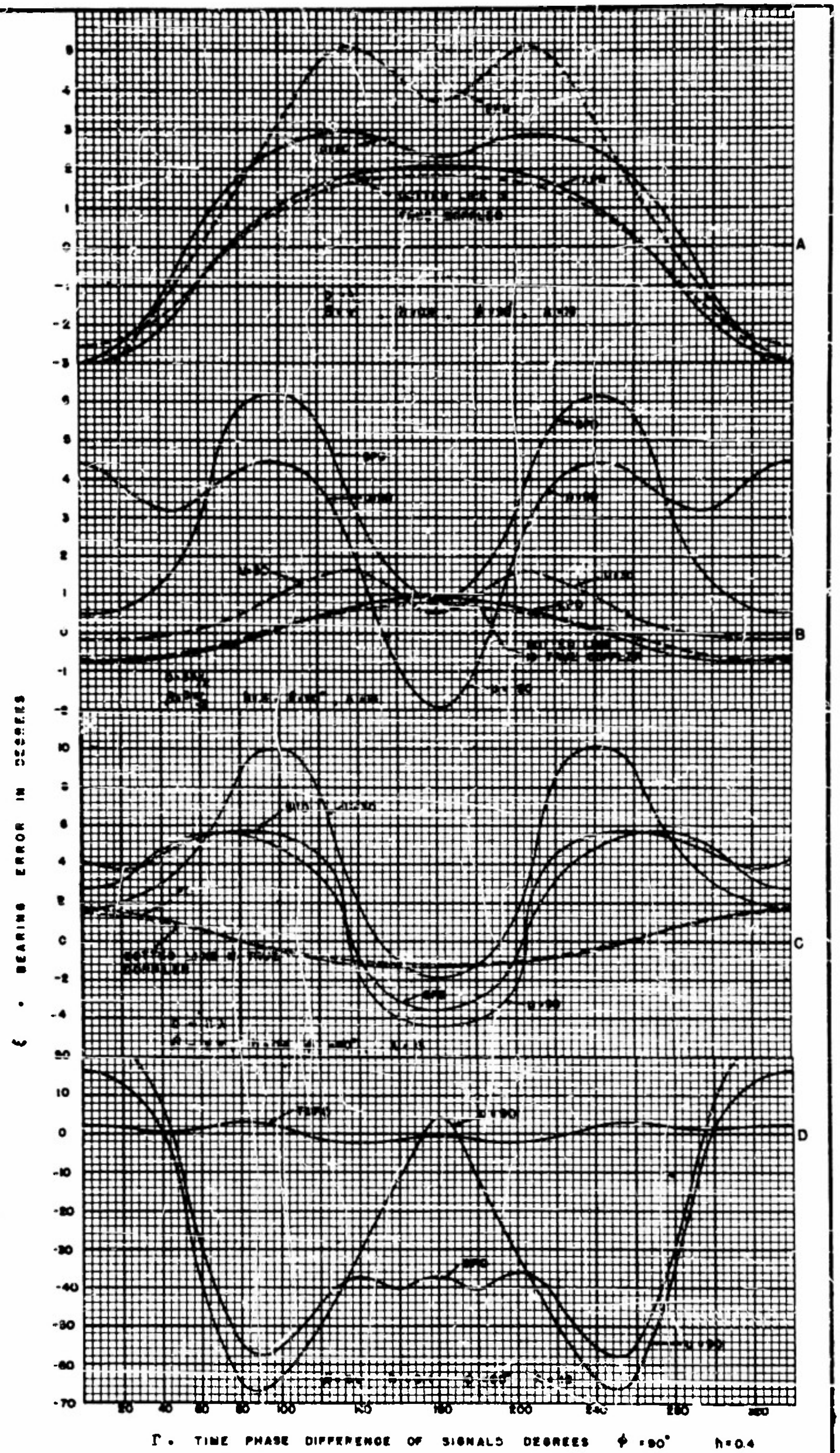
TECH. RPT. NO. 11



WAVE INTERFERENCE BEARING ERROR FOR A TRUE DOPPLER SYSTEM AND A 16 ELEMENT VECTOR-PHASE (OR SWITCHED DOPPLER SYSTEM) WITH VARIOUS TYPES OF PHASE DETECTION

PLATE 12

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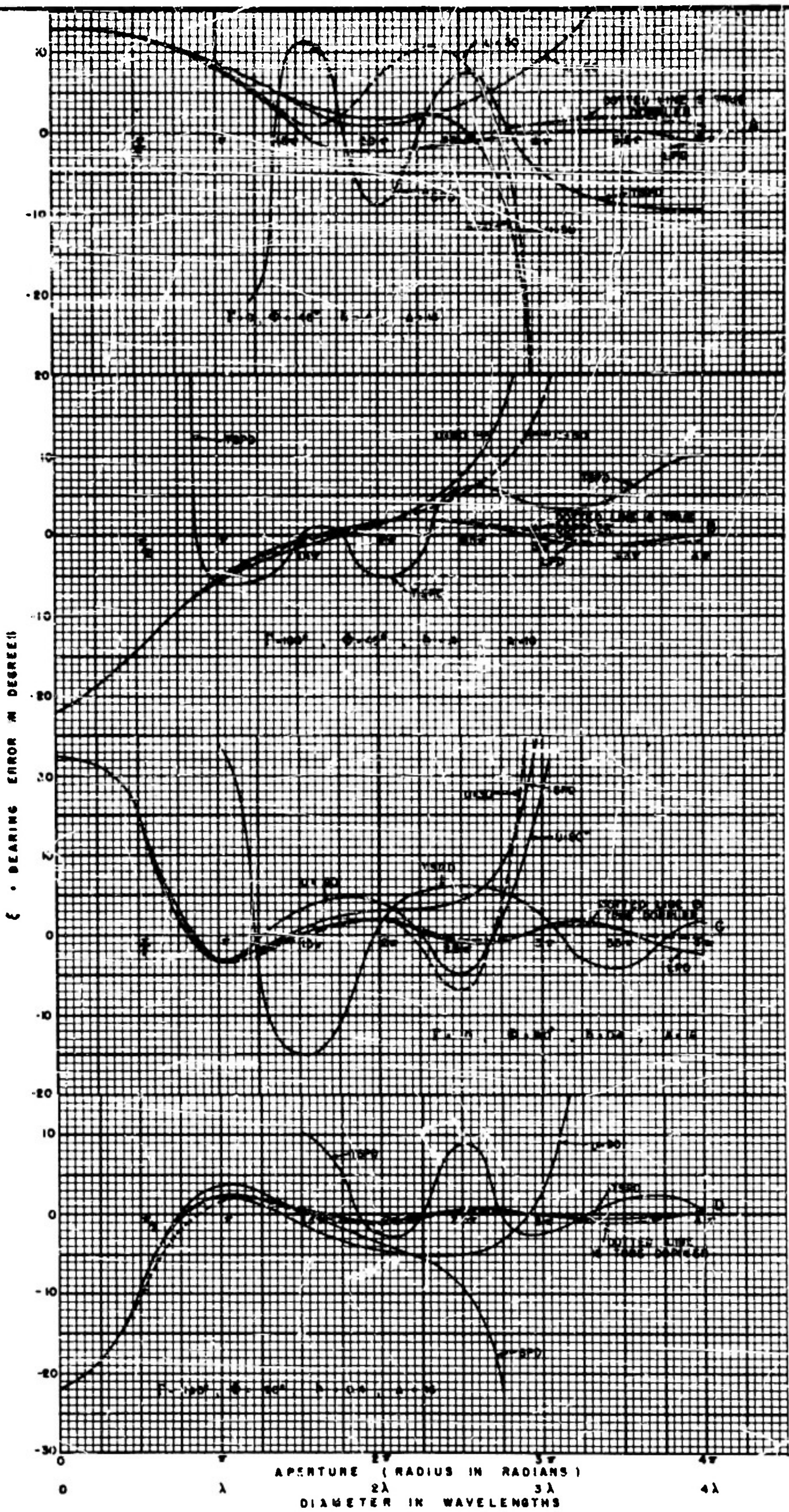

WAVE INTERFERENCE BEARING ERROR FOR A TRUE DOPPLER SYSTEM AND FOR VECTOR PHASE (OR SWITCHED DOPPLER) SYSTEMS WITH VARIOUS TYPES OF PHASE DETECTORS

PLATE 13

elements. For comparison purposes curves of the corresponding true-Doppler systems are also shown on these plates. Several types of phase detection have been assumed for the error curves of Plates 11, 12, and 13, while linear phase detection is assumed for the true-Doppler system. It is seen that the type of phase detector used has considerable effect upon the interfering signal discrimination of any of these systems and this is discussed in Section VIII. If the phase detection could be made linear for all diameters up to three wavelengths ($\beta = 3\pi$) it is seen that the bearing error of the 16-element vector-phase system would closely resemble that of the true-Doppler system. For diameters less than a wavelength ($\beta = \pi$) several of the practical phase detectors operate nearly linearly and the bearing error of the 16-element vector-phase system is nearly that of the true-Doppler. These curves are discussed further after the operation of the various types of phase detectors is explained in Section VIIIa.

Although the curves shown on Plates 11, 12, and 13 indicate that a circular continuous system can offer a large amount of protection against site error or other conditions where angular separation is large and relative magnitude is small, they do not give much indication about the severe error conditions resulting from multipath propagation conditions. If these conditions result in small angular separation, then large relative magnitudes, time phase differences near 180° and large diameters of approximately 5 to 10 wavelengths are required to reduce errors to reasonable values. The basic reasons for these conditions have been discussed in Section Vb. At best a circular continuous vector-phase system can do no better a job of suppressing these errors than a true-Doppler System as shown in Plate XVI of Technical Report No. 9.

VI. VERTICAL ANGLE OF ARRIVAL

A. Measuring the Vertical Angle

If a single incoming signal arrived at any vertical angle* φ_1 other than 90°, the time phase difference $\psi_{a,a+1}$ between two antennas is reduced by the amount sin φ_1 compared to the $\theta = 90°$ conditions assumed up to this point. It can be seen by an examination of Plate 14 that if the incoming signal is at an angle θ_1 with respect to a vertical axis and an angle φ_1 with respect to a horizontal axis the phase difference between antennas a and a + 1 can be expressed as

$$\psi_{a,a+1} = C \sin \theta_1 \cos \left(\frac{a2\pi}{A} - \varphi_1\right),$$

where $\frac{a2\pi}{A}$ is the angle which the line joining the two antennas makes with the horizontal reference axis as shown in Plate 5, Fig. 1, and C, the electrical spacing between antennas is

$$C = S \times \frac{2\pi}{\lambda},$$

S being the physical spacing between antennas measured in the same units as the wavelength λ of the incoming signal.

If θ_1 is other than 90° every phase detector will experience a smaller phase difference by the amount sin θ_1, and the magnitude of every vector including the resultant will be changed by some amount dependent on the phase detector characteristics. Hence it should be possible to determine the vertical angle of arrival, if the wavelength of the incoming signal is known, from the length of the resultant vector E_R. For example, if the phase detectors were nearly linear, sin θ_1 would be nearly proportional to E_R.

B. Effect of the Vertical Angle on Interfering Signal Discrimination

Case (1): If a single interfering signal and the desired signal both arrive at the same vertical angle θ_1, then the effective electrical radius, β, of the system is reduced by an amount sin θ_1. For example, if the vertical angle of both signals is 30°, then the error for a system of two wavelengths diameter is the same as that for a system with one wavelength diameter where the vertical angle is 90°.

Case (2): If the single interfering signal and the desired signal arrive at different vertical angles the action is more complicated. In general the effect of vertical angle is to permit discrimination in favor of the signal at the lowest vertical angle (greatest θ). In other words, the relative magnitude of the weaker signal to the desired signal is effectively decreased if the weaker signal is at a smaller vertical angle than the desired signal and effectively increased if the weaker signal is at a greater vertical angle. A more exact behavior of a circular continuous vector-phase system under the influence of desired and undesired signals at different vertical angles could be determined by comparison with a true-Doppler system. In reference No. 5, the exact equation including the vertical angle of arrival as a parameter is given for the bearing error of a true-Doppler system.

* measured from the vertical (Z) axis

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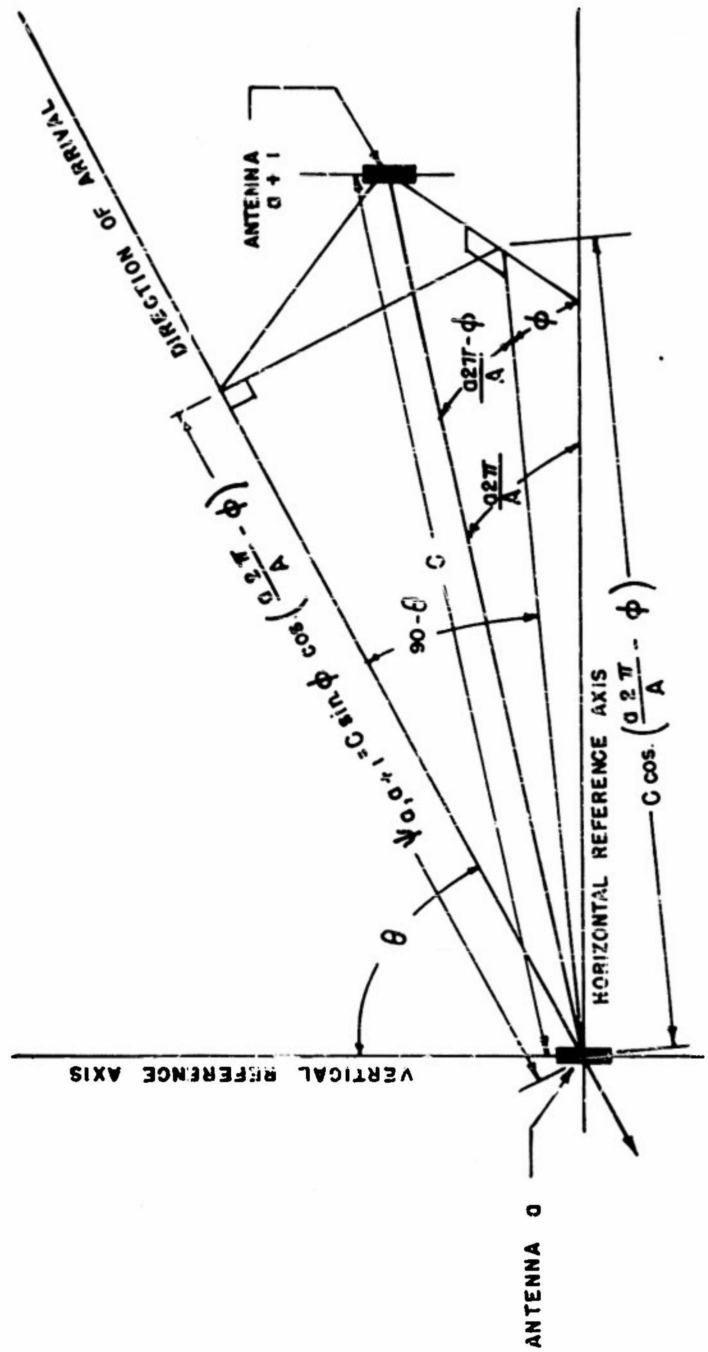

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PHASE DIFFERENCES BETWEEN ANTENNAS AS A FUNCTION OF THE VERTICAL ANGLE OF ARRIVAL

PLATE 14

VII. NONLINEAR PHASE DETECTION ERROR

Ideally it would be desirable to use phase detectors that are as linear as possible over a practical operating range. Characteristics of such phase detectors are expressed by

$$E_{a,a+1} = K\,\psi_{a,a+1}$$

where $E_{a,a+1}$ is the direct output voltage and $\psi_{a,a+1}$ is the time phase difference between the two radio frequency voltages applied to the phase detector.

For an operating range of less than $\pm\pi$ radians, linearity can be approximated with some types of phase detectors, but for operating ranges greater than $\pm\pi$ radians no practical detectors exist which are even approximately linear and suitable for use in a vector-phase system.

If the phase detector has non-linear characteristics, then the indicated bearing will deviate from the direction of arrival of a single signal for all except specific angles of arrival of this signal. As an illustration, consider the action illustrated in Plate 15 for the four-element vector phase system of Plate 1, Fig. 1.

Figure 1 of Plate 15 shows the case where the direction of arrival is halfway between two antennas. In this case the two phase differences $\psi_{4,2}$ and $\psi_{3,1}$ are both equal and the two resultant vectors must be of equal magnitude (whatever function of the phase differences they may be) so that the indicated bearing must be either the true bearing or 180° away from it. Figure 2 of Plate 15 illustrates several cases of the vectors resulting from $\psi_{4,2}$ and $\psi_{3,1}$. The solid vectors of magnitude $E_{4,2}$ and $E_{3,1}$ result from linear phase detectors, while the dotted vectors of magnitude $E'_{4,2}$, $E''_{4,2}$, $E'_{3,1}$ and $E''_{3,1}$ represent two cases which could occur with sinusoidal phase detection.[1] The first case with vectors of magnitude $E'_{4,2}$ and $E'_{3,1}$ gives a resultant magnitude E_R representing a true bearing, for the case of an electrical radius β between 0 and π radians, while the vectors of magnitude $E''_{4,2}$ and $E''_{3,1}$ give a 180° bearing error and demonstrate the action where β is between π and 2π radians.

If the direction of arrival passes through any antenna and the center of the four-element system, then only one vector can result and again the resultant bearing can be in error only by 0° or 180° regardless of the type of phase detection.

For any other case, (illustrated in general by Fig. 3 and Fig. 4 of Plate 15) the indicated bearing with non-linear phase detection will deviate from the direction of arrival by an amount dependent on the direction of arrival, the electrical radius of the system, the number of antennas of the system, and the characteristics of the phase detectors. In general, the more linear the phase detector over the operating range, the less is the phase detection error. Figure 3 illustrates the phase differences and Fig. 4 the resultant vectors for the general case with a four-element system. The solid vectors are those resulting from linear phase detection and the dotted vectors represent those resulting from sinusoidal phase detection. If the electrical radius β is under π radians than the resultant vector of

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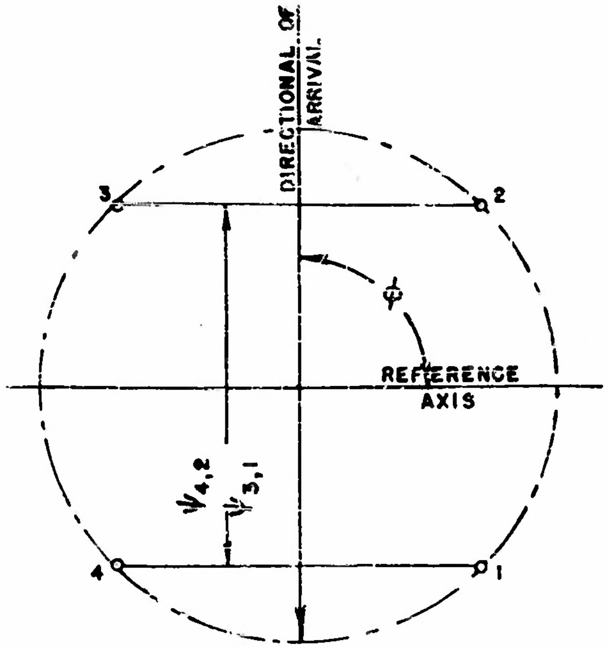

FIG. 1

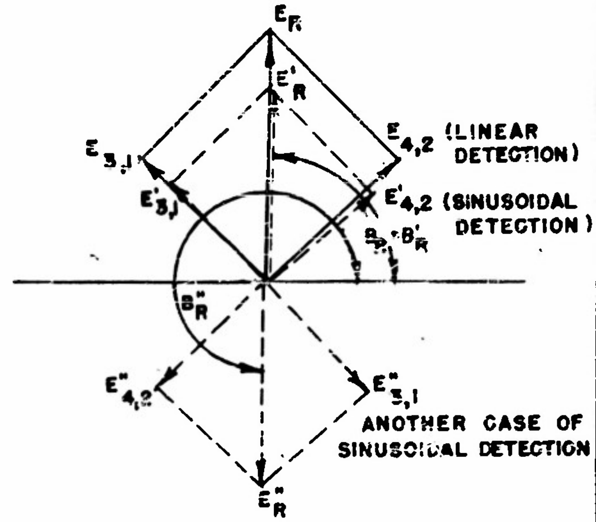

FIG. 2

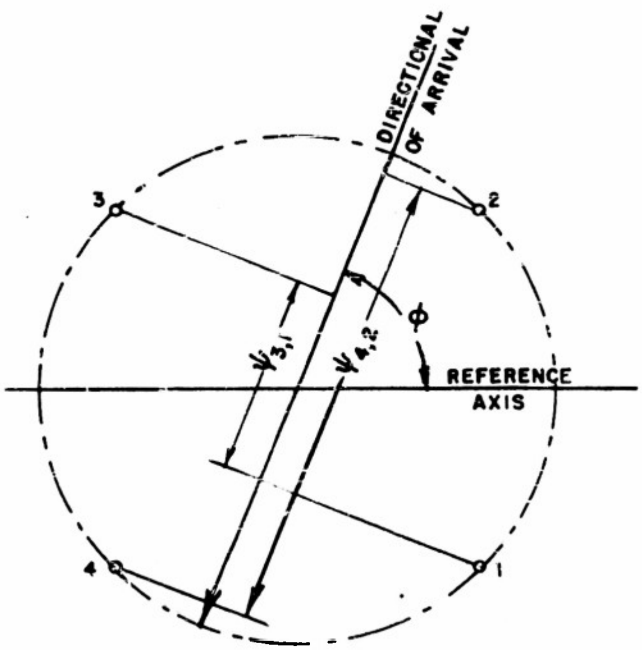

FIG. 3

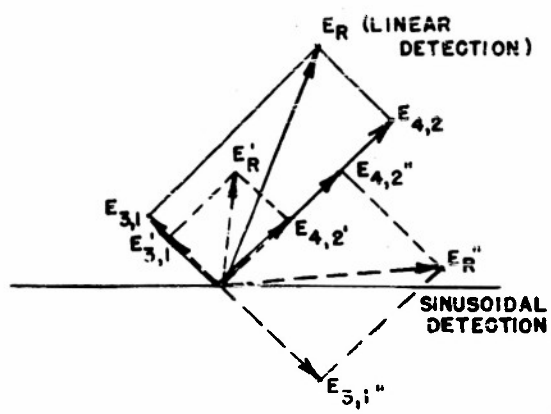

FIG. 4

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NON-LINEAR PHASE DETECTION ERROR FOR A FOUR ELEMENT SYSTEM WITH DIAGONAL PHASE COMPARISON

PLATE 15

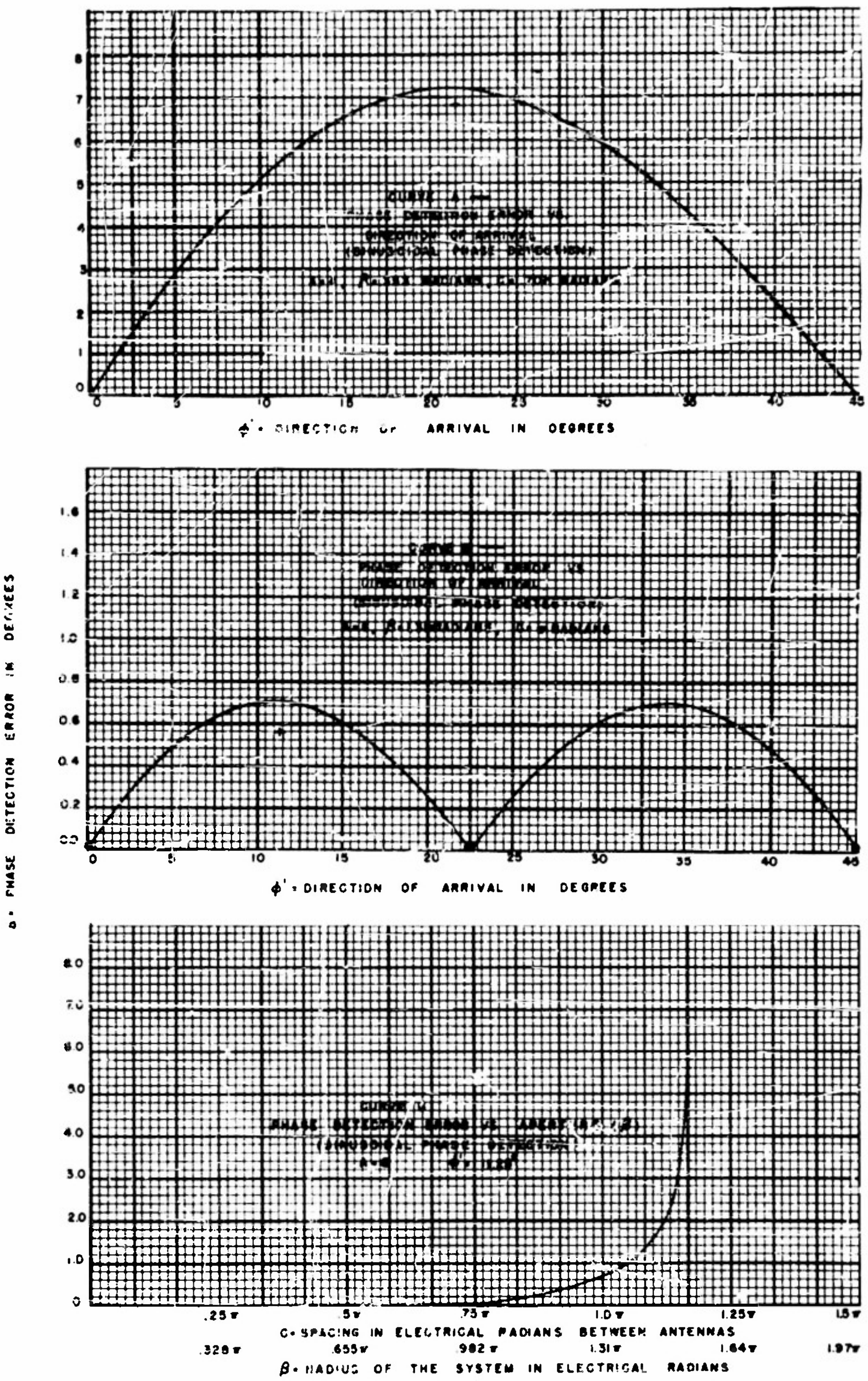
DEPT. OF ELECTRICAL ENGR. - UNIVERSITY OF ILLINOIS magnitude E'_R must point between the vectors whose magnitudes are $E_{4,2}$ and $E_{3,1}$ and the error cannot exceed the angular separation between antennas (in this case 90°) for sinusoidal phase detection. If the electrical radius is greater than π radians, the resultant vector of magnitude E''_R can be in any direction, and any bearing can result.

A curve of phase detection error as a function of angle of arrival of the signal with respect to the reference axis shown in Plate 15 is given in Plate 16 for the four-element system discussed up to this point. The error is a repetitive function of angle of arrival, becoming zero whenever the direction of arrival is in line with any antennas or half way between two antennas although only one cycle is shown for $A = 4$. This error is analogous to the error for switched-Doppler systems which C. W. Earp[7] calls repetitive error, and similarly it is dependent on the "approach planes of symmetry" of the system.

As the number of elements of a circular continuous system is increased the number of times the error curve vs. angular separation goes to zero also increases. As this zero will occur at every angle of arrival passing through an antenna or half way between two antennas, the number of zeros is given by twice the total number of antennas (if the number of antennas is even). The error, for a phase shift between antennas less than $\pm\pi$ radians, is less than the angular separation between two antennas. Therefore increasing the number of antennas decreases the phase detection error. Increasing the phase shift between antennas tends to make the detectors depart further from linear characteristics and increases the error. Curves of error for systems with four and eight antenna elements are shown in Plate 16.

For a system with an odd number of antennas as compared to a system with an even number of antennas, the number of "approach planes of symmetry" is increased (per antenna element), the number of zeros of error as a function of angle of arrival is increased, and the magnitude of the phase detection error is decreased. This action is illustrated in Plate 17 with a three-element circular continuous system. It is seen that not only is the error zero for an angle of arrival half way between two antennas (illustrated by Fig. 1 and Fig. 2) but also at an angle one quarter of the distance between any two antennas (illustrated by Fig. 3 and Fig. 4 of Plate 17). Curves of phase detection error for systems with 3 and 15 antenna elements are shown in Plate 18. The number of zeros is seen to be four times the number of antenna elements for a system with an odd number of antennas.

With a given antenna system and a given frequency of incoming signals it would be possible to use a correction calibration to compensate for phase detection error of a single signal, but with two or more signals present the value of such procedure is doubtful. Phase detection error makes it advisable to hold the maximum phase shift between antennas to less than $\pm\pi$ radians and to use an odd rather than an even number of antennas.



NON-LINEAR PHASE DETECTION FOR VECTOR-PHASE SYSTEMS WITH AN EVEN NUMBER OF ANTENNA ELEMENTS.

	PLATE 16
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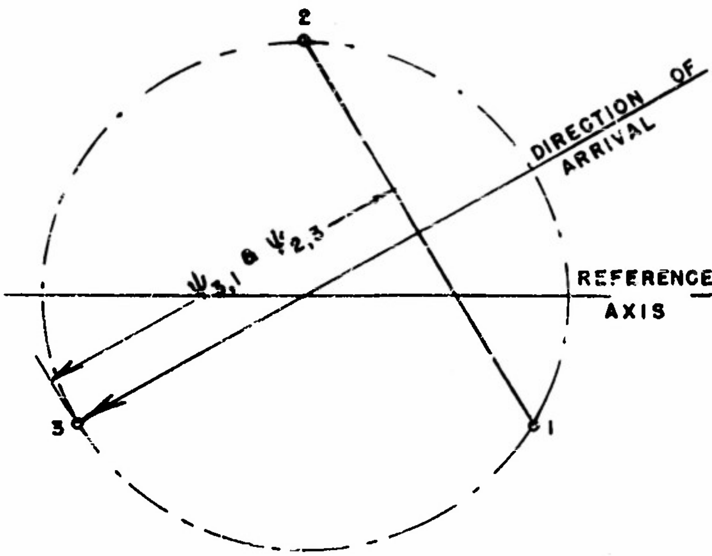

FIG. 1

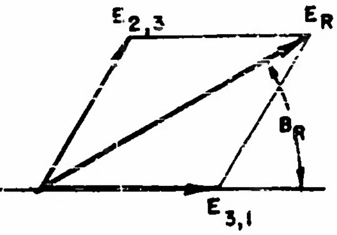

FIG. 2

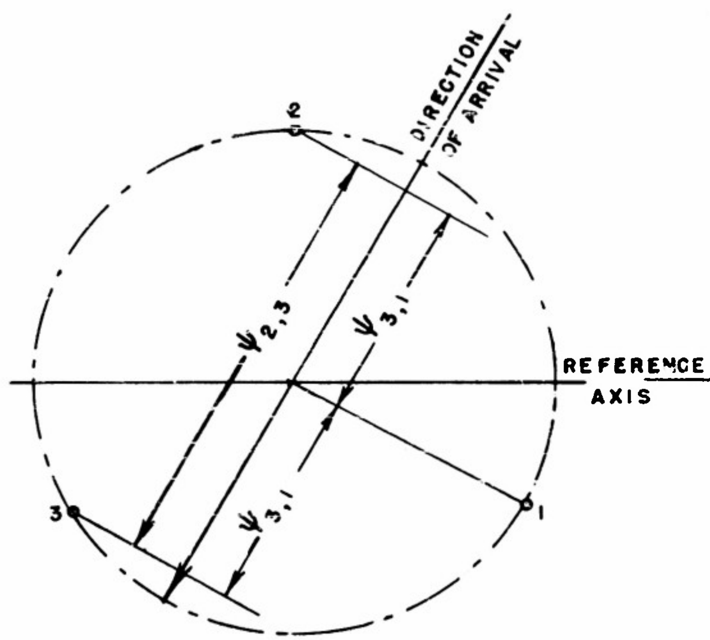

FIG. 3

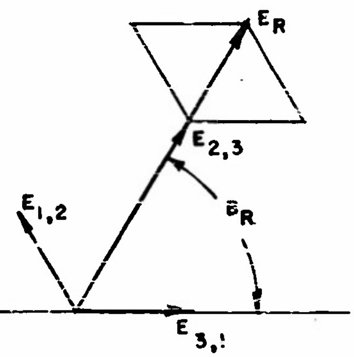

FIG 4

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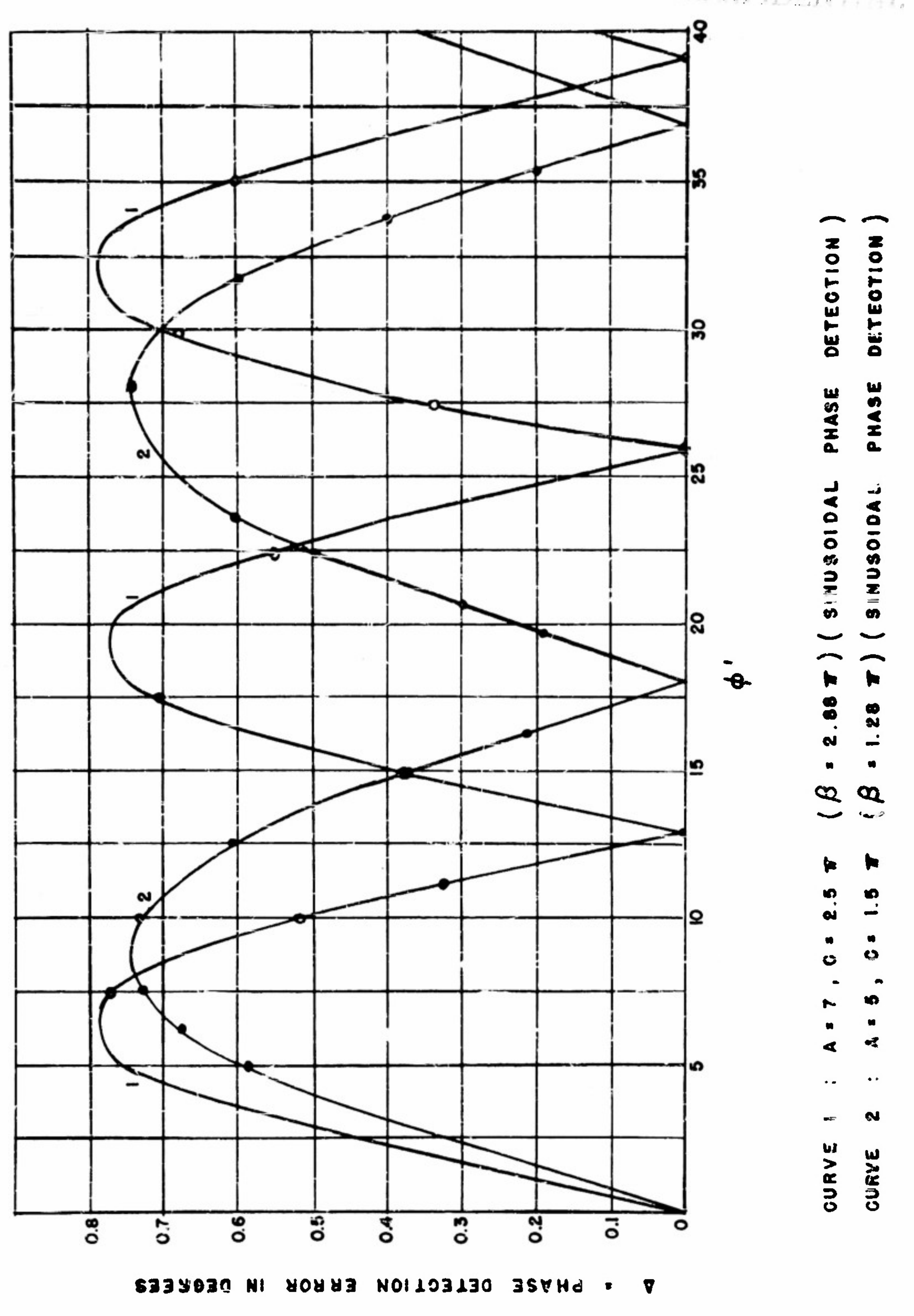
NON-LINEAR PHASE DETECTION ERROR FOR A THREE ELEMENT CIRCULAR SYSTEM

PLATE 17

DEPT. OF ELECTRICAL ENGR. - UNIVERSITY OF ILLINOIS



NONLINEAR PHASE DETECTION ERROR FOR SYSTEMS WITH AN ODD NUMBER OF ANTENNA ELEMENTS

PLATE 18

VIII. PRACTICAL CONSIDERATIONS

A. Phase Detectors

Up to this point, the main emphasis has been placed upon linear and sinusoidal phase detection. These two types lend themselves most conveniently to theoretical discussion and calculation. However, the phase detectors to be encountered in practice would never be linear over an operating range exceeding $\pm 180°$, nor would they be apt to have exactly sinusoidal characteristics($E_{a,a+1} = K \sin \psi_{a,a+1}$).

Probably the most practical phase detectors for use with a vector-phase system would be the phase detectors of the types shown in Plate 19. Figure 1 illustrates the block circuit diagram for such a system. Two equal voltages whose relative phase ψ is being compared are applied to two adder circuits with differential phase shifts of +U to one adder and -U to the other adder. Amplitude detectors, such as rectifiers, are connected to give direct voltage outputs proportional to the envelopes of the adder outputs and these envelope voltages are substracted (connected differentially). The magnitudes of the difference as a function of ψ are plotted in Figures 2 and 3 of Plate 19 for values of U = 90° and U = 30° respectively. A more thorough explanation of the operation of these detectors together with some details of their practical operation is given in reference No. 8. It is observed that the smaller the U the more nearly linear the detector characteristic curve becomes, but the detector output becomes smaller. In the limiting case, as U approaches zero, the detector characteristic become very linear over a range approaching $\pm 180°$ and the detector output approaches zero.

It would also be feasible to use "digital" or counting-type phase detectors, as discussed in reference No. 1, with vector-phase systems. However, the "standard phase detector" with nearly linear characteristics should give about the same theoretical operation as the counting type, consequently no special consideration was given to the counting-type detector.

With a knowledge of the available detector characteristic, it would be well to re-examine the wave interference bearing error curves of Plates 11, 12 and 13 to explain the effects of various types of detector operation. The first thing to be observed is that the curves resulting from linear phase detection agree fairly well with true-Doppler curves, as would be expected from the discussion in Section Vc, and represent a sort of theoretical minimum error. The second observation to be made from Plate 13 is that for small apertures ($\beta \leq \frac{\pi}{2}$) all detectors give nearly the same results. In this range the maximum phase excursions between antennas are small enough so that all of the detectors under consideration operate over a nearly linear portion of their characteristic curves. The third observation comes from the A curves of Plates 11 and 12. For an aperture of $\beta = \pi$ the operation of the U = 90° phase detector is more nearly linear, thus more nearly ideal, than a sinusoidal detector. This condition exists only as long as the β is small enough to prevent operation over the "break" in the U = 90° phase

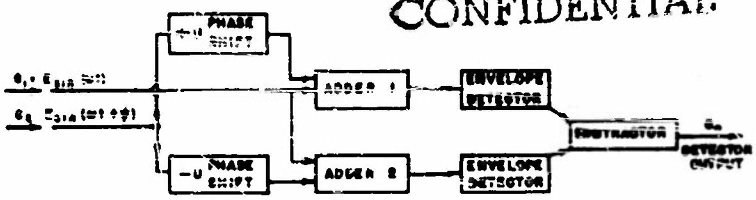
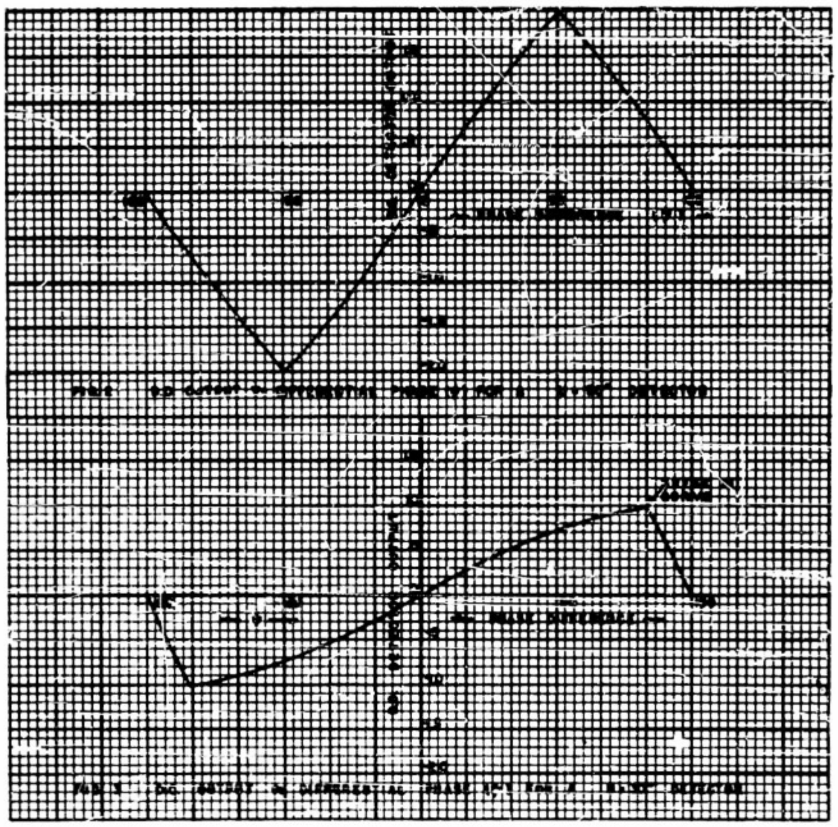


FIG. 1 BLOCK DIAGRAM OF GENERAL PHASE DETECTOR



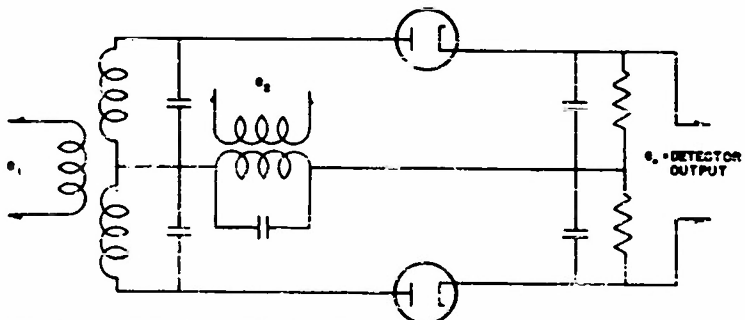


FIG. 4 PRACTICAL PHASE DETECTOR WITH U = 90°

PHASE DETECTORS

DEPARTMENT OF ELECTRICAL ENGINEERING-UNIVERSITY OF ILLINOIS	PLATE 19 TECH. RPT. NO. 11

detector curve. The B curves of Plates 11 and 12 illustrate that for a diameter of one and one-half wavelengths some of the phase excursions do exceed the "break" of the detector curve and linear operation is no longer approximated. There is little to choose between the sinusoidal and the U = 90° standard detector in this case. With the U = 30° phase detector however, operation is still over the linear region and the results again approximate the theoretical linear case. The C curves of Plates 11 and 12 show that for $\beta > 2\pi$ operation takes place over the break of even the U = 30° detector and all curves for systems with practical detectors deviate considerably from the theoretical curves for linear phase detection. To try to extend the linear (positive slope) operating range of the detector beyond $\pm 150°$ results in a rapidly decreasing detector sensitivity (output) over only a small extension of the linear range. The curves of Plates 11, 12 and 13 which remain to be discussed are called triple-angle vector-phase systems and for these systems sinusoidal phase detection alone has been considered. Performance of these systems results from the different way the detector information is used rather than from the detector action.

B. Wide Band Operation and Harmonic-Vector Summing Techniques

It would be desirable to operate a large diameter vector-phase system over a frequency range possibly as wide as a decade. This requires that a large diameter circular continuous system operate with a continuous variation in electrical radius β over the same range. As a desirable β for a practical system with 16 or fewer antenna elements lies within a very restricted range, roughly $\pi < \beta < 2.5\pi$ for A = 16, wide band operation is a serious problem. For a β much smaller than optimum, the magnitude of the interfering signal error is greatly increased, because of an insufficient spread over the interference field to "average out" the phase front variations. Also for very large values of β, phase detection error becomes prohibitive.

Of more importance, for certain critical values of β there will be no indicated bearing at all for a single signal, assuming a practical type of phase detector. The lack of indicated bearing, results from the magnitude of the resultant vector going to zero for these critical values of β. Some conception of the magnitude of the resultant vector as a function of β can be obtained for a circular continuous vector phase system by an expansion of the sine and cosine components of this vector as expressed in the numerator and denominator of Equation 4.15.

sine component:

$$\sum_{a=1}^{A} \sin \left[2 \beta \sin \frac{\pi}{A} \cos \left(\frac{a2\pi}{A} - \varphi_1\right) \sin \left(\frac{a2\pi}{A}\right)\right] \quad (8.1)$$

cosine component:

$$\sum_{a=1}^{A} \sin \left[2 \beta \sin \frac{\pi}{A} \cos \left(\frac{a2\pi}{A} - \varphi_1\right) \cos \left(\frac{a2\pi}{A}\right)\right]. \quad (8.2)$$

The sine and cosine components can be expanded in a series of harmonics with Bessel function coefficients as shown in Appendix I. Equations

I.7 and I.9 from Appendix I give the expansions of Equation 8.1 and 8.2 respectively. The sine component becomes

$$A J_1 (C) \sin \varphi_1 + A \sum_{n=1}^{\infty} (-1)^{\frac{NA}{2}} J_{NA+1} (C) \sin \left(\frac{a2\pi}{A}\right) \sin (NA + 1) \varphi_1$$
$$- A \sum_{n=1}^{\infty} (-1)^{\frac{NA-2}{2}} J_{NA-1} (C) \sin \left(\frac{a2\pi}{A}\right) \sin (NA - 1) \varphi_1 \qquad (8.3)$$

where

$$C = 2\beta \sin \frac{\pi}{A} ,$$

and the cosine component becomes

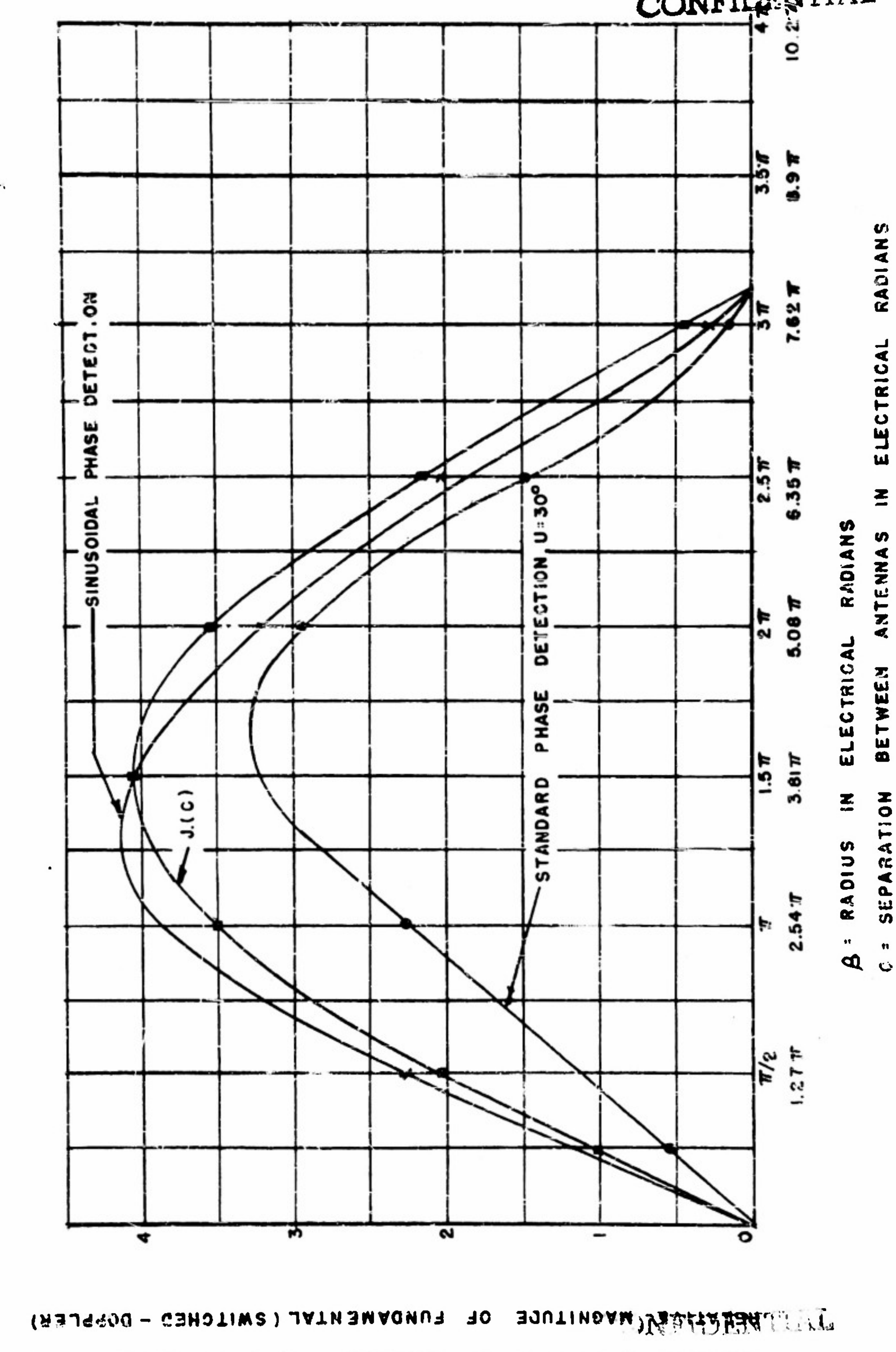
$$A J_1 (C) \cos \varphi_1 + A \sum_{n=1}^{\infty} (-1)^{\frac{NA}{2}} J_{NA+1} (C) \sin \left(\frac{a2\pi}{A}\right) \cos (NA + 1) \varphi_1$$
$$+ A \sum_{n=1}^{\infty} (-1)^{\frac{NA-2}{2}} J_{NA-1} (C) \sin \left(\frac{a2\pi}{A}\right) \cos (NA - 1) \varphi_1 . \qquad (8.4)$$

The lowest order Bessel function that can appear after J_1 is J_{A-1}. For example J_{15} is the lowest order Bessel function for a 16-element system. When $C < \pi$, $J_{A-1}(C)$ will be reasonably small for a large number of antennas. Therefore to a first approximation the magnitude of the sine and cosine components and hence the resultant vector will be very small when $J_1(C)$ is zero. The first zero occurs when $C \cong 3.83$. For a 16-element system this corresponds to a radius of about 3.1 π electrical radians, or a system diameter of about 3.1 wavelengths.

When an interfering signal is present, similar action takes place and the resultant vector again has zero amplitude for a 16-element system* at apertures near 3.1 π. This is illustrated by Plate 20 which shows the resultant vector magnitude as a function of aperture for systems with both sinusoidal phase detection and linear phase detection. The first order Bessel function of the spacing between antennas (C) is also shown for reference. For a 16-element system this Bessel function curve represents a good approximation to the case of sinusoidal phase detection where no interfering signal is present.

It is seen by examination of the wave interference error curves of Plates 11, 12, and 13 that the error for systems with practical phase detectors becomes very great once an aperture of about 2.5π radians is exceeded. A large part of this error would be due to system error reflected in the calculations. As the small resultant vector is produced from the partial cancellation of many larger vectors, any small variation or error in one of the larger vectors can have a considerable effect on the phase of the resultant vector. An attempt was made to simulate possible variations by using only three place accuracy in the calculations. Probably a more important effect occurs as a result of the reduced vector magnitude. The second incoming signal can be thought of as producing a small spurious vector adding on to the desired

* Identical effects take place in a switched-Doppler system where the magnitude of the fundamental switching frequency is equivalent to the magnitude of the resultant vector. See reference No. 7.



RELATIVE MAGNITUDE OF RESULTANT VECTOR VS. SYSTEM APERTURE FOR A 16 ELEMENT VECTOR - PHASE SYSTEM WITH ONE INTERFERING SIGNAL.

$\phi = \pi/4$, $h = 0.4$, $I = 0$, $A = 16$

PLATE 20

DEPT. ELECTRICAL ENGR. - UNIVERSITY OF ILLINOIS vector resulting from the first signal. In the case where the original vector is reasonably large the resultant vector is at almost the same angle as the original (i.e. a sort of capture effect is in operation). For apertures such that $J_1(C)$ becomes small, the magnitude of the desired vector is apparently reduced faster than that of the spurious vector and the capture effect is lessened. Aside from the effect on interfering signal discrimination for apertures such that $J_1(C)$ is near zero, the signal to noise ratio would be very unfavorable.

Thus it is seen that for the systems discussed previously, a definite upper limit on the system diameter exists. For a 16-element system it appears that the upper limit would be about two and one-half wavelengths regardless of the type of detector used.

One method of extending this operating range might be to use phase compression by means of delay line techniques at the intermediate frequencies of alternate receivers.[7]

A second method to be considered is the use of a "triple-angle" vector-phase system. This system has the vector summing plates rotated at three times the normal angle as described in Appendix A and is analogous to a switch-Doppler system using the third harmonic of the switching frequency. As in the case of the "third-harmonic" Doppler system,[7] the triple-angle vector-phase can be made to operate with apertures where the single angle vector-phase, or fundamental Doppler system, "blows up". For a triple-angle system the sine and cosine components of the resultant vector are

$$\sum_{a=1}^{A} \sin\left[2\beta \sin\frac{\pi}{A} \cos\left(\frac{a2\pi}{A} - \varphi_1\right) \sin\left(\frac{3a2\pi}{A}\right)\right] \tag{8.5}$$

for the sine component, and

$$\sum_{a=1}^{A} \sin\left[2\beta \sin\frac{\pi}{A} \cos\left(\frac{a2\pi}{A} - \varphi_1\right) \cos\left(\frac{3a2\pi}{A}\right)\right] \tag{8.6}$$

for the cosine component. When expanded in Bessel functions as was done for Equation 8.1 and 8.2, the first and most significant terms are

$$A J_3(C) \sin(3\varphi_1) \tag{8.7}$$

for the sine component, and

$$A J_3(C) \cos(3\varphi_1) \tag{8.8}$$

for the cosine component.

Thus the magnitude of the resultant vector approximates the third order Bessel function of the spacing between antennas. When $J_1(C)$ is near the first crossing (zero), $J_3(C)$ is very large, hence the triple-angle systems shown in Plates 11, 12, and 13 give much better operation in the vicinity of $\beta = 3\pi$ than the single-angle systems. There is a triple ambiguity with a triple-angle vector-phase system as indicated by Equations 8.7 and 8.8 as the indicated bearing with such a system is nearly $3\varphi_1$.

If $J_1(C)$ were not exactly zero, a rough unambiguous bearing could be taken with a single-angle system and then the detectors could be switched into a triple-angle system for a more accurate bearing.

It is noted from the curves of Plates 12 and 13 that the triple-angle systems do not have too high an order of bearing error suppression even in their most favorable ranges.

C. Equivalent Circuits for a Multiplate Vector Summing Scope

Although the vector summing oscilloscopes shown in Plates 1, 2, 3, and 4 make convenient illustrative devices, there is no need to use such a specialized scope tube in a practical system. With proper circuits, it is possible to duplicate the action of any of these multiple deflection plate tubes with a common four plate tube possessing a single pair of vertical and a single pair of horizontal deflection plates. As the angle of any electric force vector in a multiple plate tube is known in advance for a given type system, the vertical and horizontal components of this vector are known and the vector can be duplicated in its action by applying these vertical and horizontal components to the vertical and horizontal deflection plates of a conventional tube. If this procedure is carried out for all vectors, the horizontal and vertical components of the vectors can be summed external to the scope and only one horizontal and one vertical vector need be summed in the scope.

An examination of the eight-pair vector summing scope shown in Plate 6, Fig. 2 used with an eight-element continuous circular system shows that all vectors are at angles which are multiples of 45° to the reference axis. The voltages corresponding to vectors at angles of 0° to 90° can be applied to the vertical and horizontal plates of a two-pair scope tube, while the voltages corresponding to vectors at ±45° to the reference axis can have $\frac{1}{\sqrt{2}}$ times their magnitude applied (in the correct polarity) to both the vertical and horizontal plates of the two-pair tube. A resistor ring network for applying these voltages to a two-pair tube is shown in Plate 21, Fig. 1 together with the eight-element circular continuous system for which it is applicable.

A resistor network which is capable of duplicating the action of an oscilloscope tube with three pairs of plates (as shown in Plate 1, Fig. 2) can be used with a tube with two pair of plates as shown in Plate 21, Fig. 2 together with the three element system to which it is applicable.

With similar techniques the action of any vector summing tube can be duplicated with a conventional oscilloscope tube. The devices for obtaining the sine and cosine components of the vectors and summing them need not necessarily be resistor ring networks. For greater accuracy vacuum tube circuits might prove more satisfactory.

If the two resultant voltages representing the sine and cosine vector components were applied directly to the vertical and horizontal deflection plates, the beam spot would deflect to the correct angular position corresponding to the azimuth of the arriving signal. However a more readable bearing would result if the direct voltages were converted to co-phasal alternating voltages which were applied to these oscilloscope plates to produce a radial-line type of bearing indication.

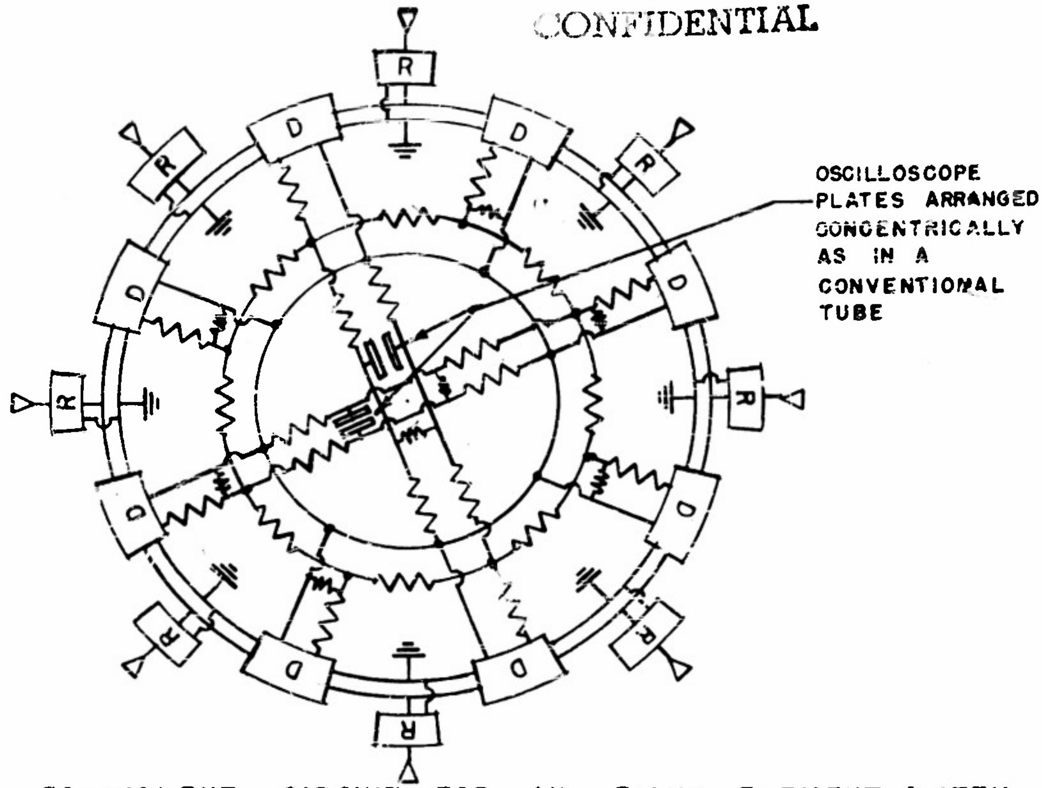


FIG. 1 EQUIVALENT CIRCUIT FOR AN EIGHT ELEMENT SYSTEM

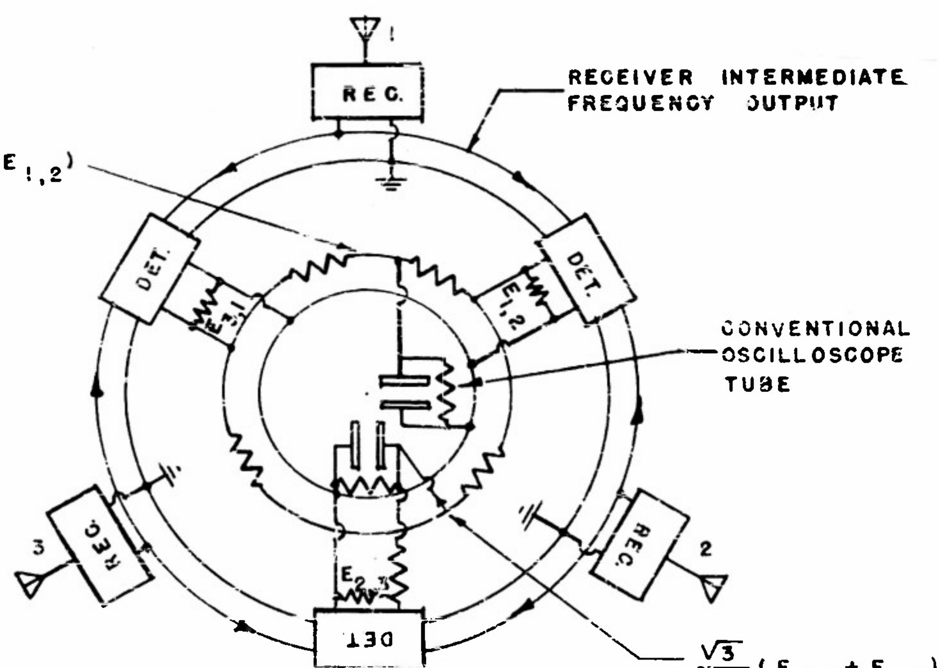


FIG. 2 EQUIVALENT CIRCUIT FOR A THREE ELEMENT SYSTEM

EQUIVALENT CIRCUITS FOR A MULTI-PLATE VECTOR SUMMING OSCILLOSCOPE

PLATE 21

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D. Phase-Balanced Receivers

Assuming that the injection signal techniques with audio-phase comparison as described in Section VIII are not used, most vector-phase systems will require a number of receivers equal to the number of antennas, each receiver having no phase shift over its tuning range as compared with any other receiver in the system. This phase balancing alignment of receivers is probably the greatest practical problem to be overcome in a circular continuous vector-phase system where the number of antennas and receivers is large.

The problem can be attacked in at least three ways:

1. Operator adjustment (for each frequency) of each receiver in the system by phase trimmers on a test signal
2. Automatic alignment of each receiver in the system on a periodic checking signal of the same frequency as the desired operating frequency
3. Perfect phase tracking of all receivers over the tuning range with no trim adjustments.

The first method requiring manual phase trimming adjustments is probably too slow for a system which is potentially instantaneous in its operation.

The second scheme could use the outputs from the vector-phase detectors to bring about automatic alignment on a reference signal fed in phase to every receiver input of the system. This alignment would have to be held for some duty cycle over which the bearing of the desired signal could be taken. Without modification this scheme hinders the instantaneous operation of the system because it renders the system inoperative at periodic intervals.

The third method requires perfect phase tracking over a wide tuning range which necessitates a large number of aligning adjustments together with exceptional electrical and mechanical stability. Perfect alignment has not been realized to date even with two receivers, but it is probable that it can be practically approached.

The overall problem is not different from that being faced at the present time in the alignment of two receivers for a dual-channel narrow-diameter system such as a Watson-Watt system.

IX. CONCLUSIONS

Three types of vector-phase systems are suggested as being of potential practical merit:

1. Three-element narrow-aperture system
2. Wide-aperture distribution of narrow-aperture vector phase systems with vector-average combination of bearing information
3. Wide-aperture continuous circular system

All three are instantaneous and omnidirectional in the sense of being able to give a bearing on a transmission of very short duration from an unknown direction.

The three-element narrow aperture system offers very little improvement over many existing narrow-aperture systems. It offers no improvement in interfering signal suppression over existing systems. However, it does offer an unambiguous bearing indication on signals of short duration. Perhaps its greatest merit is that its bearing information results in direct voltages suitable for combination in a wide-aperture distribution with other identical systems.

A wide-aperture distribution of narrow-aperture vector-phase systems with vector-average combination of bearing information from each of the unit systems offers a relatively large amount of interference error reduction under the particular conditions resulting from multipath propagation. Under these conditions the angular separation will probably be small and the relative magnitude large. Although there is some interfering signal suppression under site error conditions with large angular separations and small relative magnitudes, the relative guaranteed maximum suppression is rather small when compared to theoretical true-Doppler systems and circular continuous vector-phase systems. This system is basically no more complicated than a single narrow-aperture system, since phase tracking of only three receivers at a time is required.

The wide-aperture continuous circular systems have exactly the same bearing error suppression characteristics as a switched-Doppler system with the same antenna configuration, and hence have characteristics resembling those of a true-Doppler system. It should be emphasized that most of the analyses and curves published in the report for circular systems are applicable without modification to the analogous rectangular switched-Doppler systems. As compared to a switched-Doppler system which uses only two receivers, the circular continuous vector-phase system offers the advantage of being instantaneous, but has the formidable disadvantage of requiring a large number of phase-balanced receivers. A practical circular continuous vector-phase system could not be made too large in aperture because of phase detector limitations, and therefore for the multipath propagation conditions previously mentioned such a system would not have a very large amount of error suppression. For site error conditions however, the wide-aperture circular continuous system should give reasonably good error suppression.

There is at least one other wide-aperture instantaneous system to which the wide-aperture vector-phase systems should be compared. This is the wide aperture distribution of narrow-aperture Watson-Watt systems discussed in reference No. 2. This system is capable, in most cases, of actually resolving the separate angles of arrival and relative magnitudes of two or more incoming plane waves The wide-aperture vector-phase systems might be criticized as "throwing away" amplitude information, because the bearing is in effect largely captured by the strongest incoming signal, and there is no hope of obtaining bearings on the weaker signals. For the distribution of Watson-Watts, however, site errors resulting from relatively nearby sources such that the second wave is not a plane wave would cause unresolved bearings and possibly considerable bearing error. The same type of site error for a vector-phase system would probably be no worse than a site error condition resulting from a plane wave.

X. REFERENCES

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3. "A Symposium on Countermeasures Intercept", The National Military Establishment Research and Development Board, July 15, 1948.
4. U. S. Patent No. 2,415,088, E. N. Dingley Jr., Feb. 4, 1947.
5. Technical Report No. 9, "A Comparison of Radio Direction Finding Systems", Radio Direction Finding Section, University of Illinois Dec. 1, 1949.
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7. "Radio Direction Finding by the Cyclical Differential Measurement of Phase", C. W. Earp and R. M. Godfrey, Jour. I.E.E., Vol. 94, Part III A, No. 15, Mar-Apr., 1947.
8. "An Analysis of Phase Discriminators", Masters' Thesis by R. A. Campbell, University of Illinois.
9. Report No. 5155, "The Study of Alternative to Inverse-Loan", Hazeltine Electronics Corporation, Mar. 1, 1948.
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APPENDIX A

Bearing Equivalence Between A Switched-Doppler System and a Vector-Phase System

BEARING EQUIVALENCE BETWEEN A SWITCHED-DOPPLER SYSTEM AND A VECTOR-PHASE SYSTEM

A switched-Doppler system of the type shown in Plate 10, Fig. 2 gives under any electromagnetic field conditions exactly the same theoretical bearing as an "equivalent" continuous circular vector-phase system with the same number of antenna elements as shown in Plate 10, Fig. 1. It is easiest to show this by realizing the equivalence between the switching schemes of Plate 10, Fig. 2 and Plate 22, Fig. 1. Neglecting transient effects, it makes no difference to the Doppler system whether the switching is done at the antennas or at the output of identical phase detectors. The detector output going to the filter is the same in either scheme. If $\psi_{a,a+1}$ is the phase difference between the voltages induced in antennas a and a + 1 and hence the difference between the voltages applied to any detector, then $E_{a,a+1}$ will represent the output of this detector going to the switch. The successive outputs $E_{a,a+1}$ as a function of the switching time are shown in Fig. 2 of Plate 22.

This step function of time will in general have an alternating fundamental component at a period equal to the switching period and this fundamental component can be removed by filtering. This component of filtered output is also shown in Fig. 1 of Plate 22. The phase of the fundamental component is a function of the angle of arrival and the indicated direction of arrival is taken to be this phase referred to any field reference.

It is feasible to build switched-Doppler systems which operate on harmonics of the switching frequency other than the first. The phase of this Nth harmonic is then taken as indicating the direction of arrival when referred to a reference, for example the Nth harmonic of the switching voltage. N-1 ambiguities in the indicated bearing will result in a system using the Nth harmonic of the switching frequency.

It is also possible to build a vector-phase system similar to the type shown in Plate 10, Fig. 1 except that the angles which the perpendiculars to the scope plates make with the horizontal axis are multiplied by an integer N. If this integer N corresponds to the N representing the Nth harmonic in a switched-Doppler system, then the Nth-angle vector-phase system and the Nth-harmonic Doppler system are called equivalent systems.

It remains to be shown that these equivalent systems give the same indicated bearing. Consider first the indicated bearing for a switched-Doppler system given by the phase of the Nth harmonic of a series of rectangular pulses similar to those shown in Fig. 2 of Plate 21. This series of pulses can be expanded in a Fourier series giving

$$E(t) = \frac{a_o}{2} + \sum_{N=1}^{\infty} (A_N \cos N\omega t + B_N \sin N\omega t) \qquad (A.1)$$

where

$$\omega = \frac{2\pi}{T}.$$

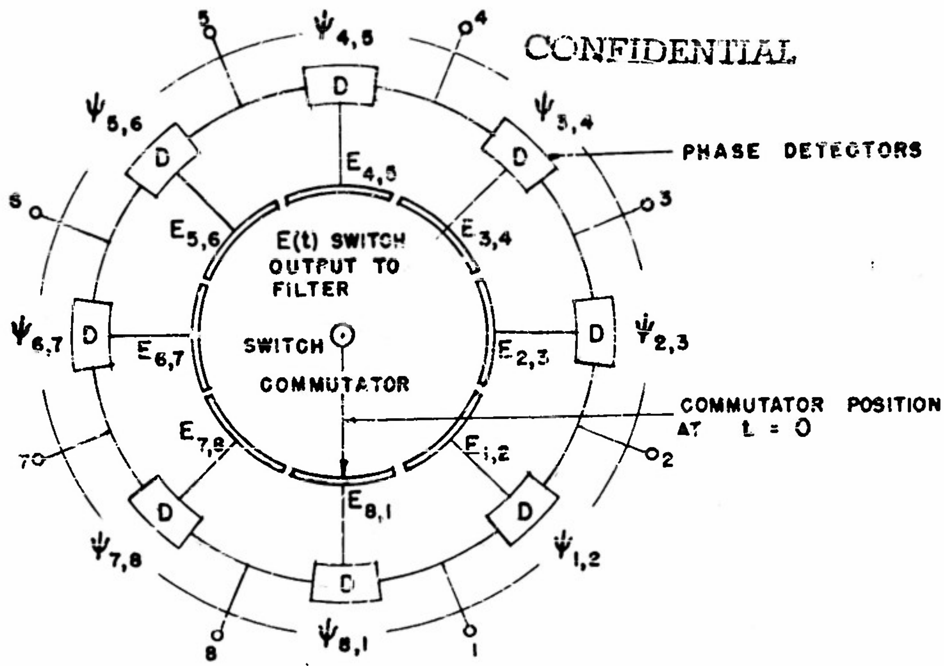

FIG. 1 SWITCHING ACTION FOR A SWITCHED DOPPLER SYSTEM

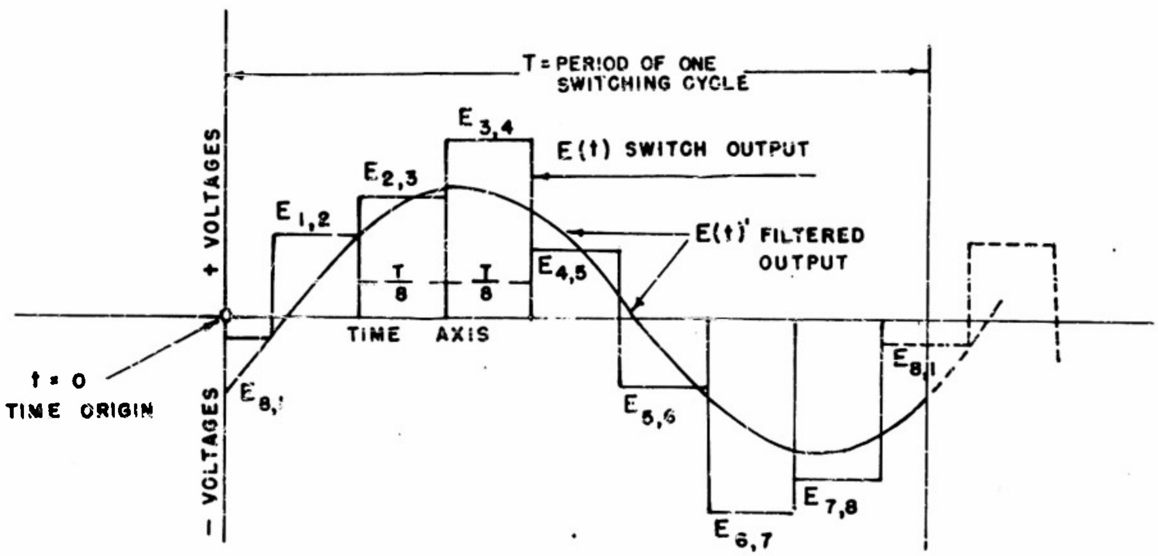

FIG. 2 VOLTAGE OUTPUT OF THE SWITCH AS A FUNCTION OF TIME AND FILTERED OUTPUT

THE VOLTAGE OUTPUT FROM A SWITCHED DOPPLER SWITCH AS A FUNCTION OF TIME

PLATE 22

After passing through a bandpass filter all terms except one particular harmonic of the switching frequency are eliminated and the phase of this harmonic gives the indicated bearing. If the Nth harmonic is selected, then the filtered output which remains is expressed by

$$E_f = A_N \cos N\omega t + B_N \sin N\omega t$$
$$= \sqrt{A_N^2 + B_N^2} \sin [N\omega t + \tan^{-1} \frac{B_N}{A_N}], \qquad (A.2)$$

where the phase angle $\tan^{-1} \frac{B_N}{A_N}$ of E_f gives the indicated bearing. Now

$$A_N = \frac{2}{T} \int_0^T E(t) \cos N\omega t \, dt.$$

Therefore for a system with eight antennas and eight d-c pulses,

$$A_N = \frac{2}{T} \sum_{a=1}^{8} \int_{\left(\frac{2a-1}{16}\right)T}^{\left(\frac{2a+1}{16}\right)T} [E_{a,a+1} \cos (N\omega t)] \, dt,$$

and for a system with A antennas and A d-c pulses

$$A_N = \frac{2}{T} \sum_{a=1}^{A} \int_{\left(\frac{2a+1}{2A}\right)T}^{\left(\frac{2a+1}{2A}\right)T} [E_{a,a+1} \cos (N\omega t)] \, dt. \qquad (A.3)$$

Consequently for eight antennas

$$A_N = \frac{2}{TN\omega} \sum_{a=1}^{8} E_{a,a+1} \sin N\omega T\Big]_{\left(\frac{2a-1}{16}\right)T}^{\left(\frac{2a+1}{16}\right)T}$$

$$= \frac{2}{TN\omega} \sum_{a=1}^{8} E_{a,a+1} [\sin (\frac{2a+1}{16} 2N\pi) - \sin (\frac{2a-1}{16} 2N\pi)],$$

and for A antennas

$$A_N = \frac{2}{TN\omega} \sum_{a=1}^{A} E_{a,a+1} [\sin (\frac{2a+1}{2A} 2N\pi) - \sin (\frac{2a-1}{2A} 2N\pi)].$$

Thus for A antennas

$$A_N = \frac{1}{N\pi} \sum_{a=1}^{A} E_{a,a+1} [2 \sin(\frac{1}{2A} 2N\pi) \cos (\frac{a}{A} 2N\pi)]$$

(See Pierce, "A Short Table of Integrals," 595)

$$= \frac{2 \sin(\frac{N\pi}{A})}{N\pi} \sum_{a=1}^{A} E_{a,a+1} \cos (\frac{aN2\pi}{A}). \qquad (A.4)$$

Similarly evaluating B_N for a system with A antennas and A d-c pulses gives

$$B_N = \frac{2}{T}\int_o^T E(t)\sin N\omega t\, dt$$

$$= \frac{2}{T}\sum_{a=1}^{A}\int_{\left(\frac{2a-1}{2A}\right)T}^{\left(\frac{2a+1}{2A}\right)T} [E_{a,a+1}\sin(N\omega t)]\, dt. \qquad (A.5)$$

Consequently

$$B_N = \frac{2}{TN\omega}\sum_{a=1}^{A} E_{a,a+1} - \cos N\omega t\Big]_{\left(\frac{2a-1}{2A}\right)T}^{\left(\frac{2a+1}{2A}\right)T}$$

$$= \frac{1}{N\pi}\sum_{a=1}^{A} - E_{a,a+1}\left[\cos\left(\frac{2a+1}{2A}2N\pi\right) - \cos\left(\frac{2a-1}{2A}2N\pi\right)\right]$$

$$= \frac{1}{N\pi}\sum_{a=1}^{A} - E_{a,a+1}\left[-2\sin\left(\frac{2a}{2A}2N\pi\right)\sin\left(\frac{1}{2A}2N\pi\right)\right]$$

(See Pierce, "A Short Table of Integrals", 597)

$$= \frac{2\sin\left(\frac{N\pi}{A}\right)}{N\pi}\sum_{a=1}^{A} E_{a,a+1}\sin\left(\frac{aN2\pi}{A}\right). \qquad (A.6)$$

From Equation A.2 the indicated bearing B_I is given by

$$B_I = \tan^{-1}\left[\frac{B_N}{A_N}\right]$$

where B_N and A_N have the values given in A.6 and A.4. Therefore

$$B_I = \tan^{-1}\left[\frac{\frac{2\sin\left(\frac{N\pi}{A}\right)}{N\pi}\sum_{a=1}^{A} E_{a,a+1}\sin\left(\frac{aN2\pi}{A}\right)}{\frac{2\sin\left(\frac{N\pi}{A}\right)}{N\pi}\sum_{a=1}^{A} E_{a,a+1}\cos\left(\frac{aN2\pi}{A}\right)}\right]$$

$$= \tan^{-1}\left[\frac{\sum_{a=1}^{A} E_{a,a+1}\sin\left(\frac{aN2\pi}{A}\right)}{\sum_{a=1}^{A} E_{a,a+1}\cos\left(\frac{aN2\pi}{A}\right)}\right]. \qquad (A.7)$$

If $N = 1$, then it is seen that the indicated bearing for a switched-Doppler as given by A.7 is identical to the indicated bearing for a circular continuous vector-phase system as given by equation 4.13. If the scope plates shown in Plate 4, Fig. 2 are rotated by N times their pictured angle with the horizontal axis, rotating the resultant vectors shown in Plate 6, Fig. 2 to an angle N times their indicated direction of arrival for an eight-element circular system will be expressed by Equation A.7. Equation A.7 will give the indicated bearing for any circular continuous vector-phase system with a rotation angle H regardless of the number of antennas.