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NSF-tr-106
November, 1953

Physics Translated from Doklady Akademii Nauk SSSR, 89, 809-12 (1953)

Theory of Displacement of Domain Boundaries

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Extensive experimental material has been accumulated on the process of displacement of domain boundaries in ferromagnetic substances.[1-3] The rate of displacement of boundaries has been measured over a wide range of variation of sample thickness, chemical composition, heat and mechanical treatment, temperature, and so forth. However, the regularities that were found experimentally have as yet received not even a qualitative theoretical explanation, despite the importance of this question in theory and practice. Here, we shall examine the problem of the displacement of the domain boundary in a ferromagnet under the simultaneous action of the external magnetic field and the opposing fields: (a) eddy macrocurrents, (b) eddy microcurrents caused by thermal fluctuations of the boundary layer, and (c) eddy microcurrents caused by "fluctuations" of the critical field at various points of the ferromagnet.

We shall use the following model as a basis for our study: potential barriers are distributed throughout the ferromagnet along the path of displacement of the boundary at an average distance l. The barrier height $\mathcal{H}_0$ can vary from one barrier to another; it is given by a distribution function $f(\mathcal{H}_0)$. A region of the boundary layer of area σ cm^2, after having passed the potential barriers in its path, advances the distance l cm in one jump. Magnetic reversal in an average volume $q = \sigma l$ cm^3 is thus produced by such an elementary Barkhausen jump. A boundary layer of 1 cm^2 advances an average of l cm after $N = 1/\sigma$ elementary jumps. Hence we obtain the expression for the average rate of displacement of the boundary:

$$\frac{dx}{dt} = v = \frac{l}{\tau}\frac{\Delta N}{N}, \qquad (1)$$

where τ is the transition time of the boundary inside the volume q (from a given potential minimum to its neighbor), and $\Delta N/N$ is the relative number of such transitions in time τ, i.e., the "transition probability." This elementary transition can also occur in the absence of a magnetic field under the action of thermal fluctuations. The effect of such fluctuations is equivalent to the action of a certain magnetic field h_T.

According to the Boltzmann principle, the probability of occurrence of the field h_T in the volume q as a result of thermal fluctuations is given by a distribution function of the following form:

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$$w(h_T) = \left(\frac{p}{\pi kT}\right)^{1/2} e^{-ph_T^2/kT}, \quad (2)$$

where kT/p is equal to the mean-square value of h_T in the volume $q : \overline{h_T^2}$. A section of the boundary can go over the potential barrier under the action of thermal fluctuations if the field $h_T > \mathcal{H}_0$ occurs in the volume q, where $\mathcal{H}_0$ is the height of the highest barrier. Therefore, the number of sections of the boundary capable of overcoming the potential barriers is

$$N \int_0^{\infty} f(\mathcal{H}_0) \int_{\mathcal{H}_0}^{\infty} w(h_T)\, dh_T\, d\mathcal{H}_0.$$

However, since in the absence of an external magnetic field the jumps of sections of the boundary are equally probable in both directions, the average displacement rate of the boundary will be zero. When a magnetic field h is applied, the height of the potential barrier, which hinders the jump in the direction of an increase in the magnetization of the sample, decreases to $\mathcal{H}_0 - h$, while in the opposite direction the barrier height increases to $\mathcal{H}_0 + h$. This produces a rate of displacement of the boundary in the direction of increase in magnetization of the sample: first, because of barrier "fluctuations" (since under the action of the field h there is a displacement of those sections of the boundary which have potential barriers $\mathcal{H}_0 < h$ on their path; the number of these sections is found from the equation

$$\Delta N = N \int_0^{h} f(\mathcal{H}_0)\, d\mathcal{H}_0); \quad (3)$$

and secondly, because of the thermal fluctuations, since the number of elementary jumps which increase the magnetization of the sample is larger than the number of backward jumps. The number of the former is

$$\Delta N_+ = N \int_0^{\infty} f(\mathcal{H}_0) \int_{\mathcal{H}_0 - h}^{\infty} w(h_T)\, dh_T\, d\mathcal{H}_0; \quad (4)$$

the number of backward jumps is correspondingly

$$\Delta N_- = N \int_h^{\infty} f(\mathcal{H}_0) \int_{\mathcal{H}_0 + h}^{\infty} w(h_T)\, dh_T\, d\mathcal{H}_0. \quad (5)$$

If we take the difference between (4) and (5), add (3) and substitute the result into (1), we obtain the following equation for the boundary displacement rate:

$$v = \frac{l}{\tau} \int_h^{\infty} f(\mathcal{H}_0) \int_{\mathcal{H}_0 - h}^{\mathcal{H}_0 + h} w(h_T)\, dh_T\, d\mathcal{H}_0 + \frac{l}{\tau} \int_0^{h} f(\mathcal{H}_0)\, d\mathcal{H}_0. \quad (7) \text{ [sic]}$$

The ratio l/τ is the average velocity of transition of a section of the boundary from a given potential minimum to the adjoining one. This speed is determined by the retarding force hindering the motion of the given section of the boundary. On the basis of several articles,[4-6] it can be shown that in the case of free motion the rate of displacement of a section of the boundary in the presence of eddy microcurrents is, apart from a constant factor, approximately equal to unity:

$$\frac{l}{\tau} = v_0 = \frac{\rho C^2}{\pi^2 \sqrt{\sigma}\, I_s} h_s, \quad (8)$$

where ρ is the specific electric resistance of the substance, I_s is the saturation magnetization, and h_s is the strength of the magnetic field, under whose action the section of the boundary is displaced. The average value of h_s is $\mathcal{H}_0\alpha$, where α is a coefficient depending on the shape of the potential barrier. The magnetic field h in the region of the moving boundary is

$$h = H - C_1 v, \tag{9}$$

where C_1 is a coefficient which is proportional to the electric conductivity of the substance and to the thickness of the sample, and which depends on the shape of the boundary layer.

Substituting (9) and (2) in (7), we obtain the following functional expression for the boundary displacement rate for steady motion:

$$v = v_0\Big[\left(\frac{p}{\pi kT}\right)^{1/2} \int\limits_{H-C_1v}^{\infty} f(\mathcal{H}_0) \int\limits_{\mathcal{H}_0-(H-C_1v)}^{\mathcal{H}_0+(H-C_1v)} e^{-ph_T^2/kT}\, dh_T\, d\mathcal{H}_0 + \int\limits_{0}^{H-C_1v} f(\mathcal{H}_0)\, d\mathcal{H}_0\Big]. \tag{10}$$

For small h's, we can take the average value of the integrals: in the first term we set $h_T = \mathcal{H}_0$ in the integrand, and in the second term we evaluate the integrand at $\mathcal{H}_0 = 0$:

$$v = v_0\Big[2\left(\frac{p}{\pi kT}\right)^{1/2} (H - C_1v) \int\limits_{H-C_1v}^{\infty} f(\mathcal{H}_0)\, e^{-p\mathcal{H}_0^2/kT} d\mathcal{H}_0 + f(0)(H - C_1v)\Big]. \tag{11}$$

In the obtained expression, the integral represents an average over all $\mathcal{H}_0$ larger than h_T. If this integral depends weakly on h and is therefore equal to

$$e^{-p\overline{\mathcal{H}}_0^2/kT},$$

where $\overline{\mathcal{H}}_0$ is the mean value of the critical field and is between 0 and $\mathcal{H}_{0\,max}$, Eq. (11) becomes

$$v = v_0\Big[2\left(\frac{p}{\pi kT}\right)^{1/2} (H - C_1v)\, e^{-p\overline{\mathcal{H}}_0^2/kT} + f(0)(H - C_1v)\Big]. \tag{12}$$

From this we find v explicitly:

$$v = C^{-1}H = \frac{1}{C_1 + C_2} H, \tag{13}$$

where

$$C_2^{-1} = v_0\Big[2\left(\frac{p}{\pi kT}\right)^{1/2} e^{-p\overline{\mathcal{H}}_0^2/kT} + f(0)\Big]. \tag{14}$$

The formulas obtained are valid on condition that the boundary moves in an arbitrarily small external field. However, in several cases, experiment shows that the boundary begins to move with a constant speed only when the external magnetic field reaches a value H_0, which is called the critical field. From the viewpoint of the theory formulated here, this is caused by the fact that until the external field reaches the value H_0, the probability of thermal jumps of sections of boundaries is negligible. Expressed mathematically, this condition means that we can disregard

$$\int_{h_0+(\Delta H - C_1 v)}^{h_0+2H_0} w(h_T)\,dh_T$$

compared to

$$\int_{h_0-(\Delta H - C_1 v)}^{h_0+(\Delta H - C_1 v)} w(h_T)\,dh_T,$$

where

$$\Delta H = H - H_0, \quad h_0 = \mathcal{H}_0 - H_0.$$

Taking this condition into account, we can generalize (13) and (14) to the case $H_0 \neq 0$ as follows:

$$v = C^{-1}\Delta H = \frac{1}{C_1 + C_2}\Delta H, \tag{15}$$

where

$$C_2^{-1} = v_\sigma\left[2\left(\frac{p}{\pi kT}\right)^{1/2} e^{-p\overline{h}_0^2/kT} + f(0)\right]. \tag{16}$$

Eq. (15) is in good agreement with the experimental data. First, the equation results in a peculiar dependence of C on the thickness of the samples since C_1 is proportional to the thickness of the sample and C_2 does not depend on the thickness. Dijkstra and Snoek[2] have investigated the dependence of C on the thickness of the sample (for a 10-fold change of thickness) and found empirically the dependence on d which follows from our formula. As for the temperature dependence of v, it is easy to show that v/ρ does not depend on T. Indeed, averaging with respect to h_T between the limits $h_0 - h$ and $h_0 + h$, we find that $\overline{h_T^2} = \overline{h}_T^2$; so $\overline{h}_T^2$ is proportional to T. The nondependence of v/ρ on T, which follows from (16) under this condition, agrees quantitatively with the results of the detailed experiments of Dijkstra and Snoek.

The experimental fact that during the application of external elastic stresses the value of C does not change also confirms (16). This is accounted for by the fact that $f(h_0)$ and v_σ are independent of the external elastic stresses. On the other hand, under plastic deformation of the samples, under different heat treatments, and under variation of the composition of the alloy, when $f(h_0)$ and v_σ change considerably, experiment shows a sharp change in C. Thus the formulated theory makes it possible to interpret the extensive experimental material available.

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Received February 11, 1953