A STUDY OF SPEED FACTORS IN TESTS AND ACADEMIC GRADES

A Technical Report

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FREDERIC M. LORD

Office of Naval Research Contract Nonr-564(00)
Project Designation NR 150-089

EDUCATIONAL TESTING SERVICE
PRINCETON, NEW JERSEY

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Abstract

Speeded and unspeeded tests of vocabulary, spatial relations, and arithmetic reasoning were factorially analyzed, together with certain reference tests and academic grades. Lawley's maximum-likelihood method was used, the computations being carried out on the Whirlwind electronic computer. Four different speed factors were isolated, together with a second-order general speed factor. Consistent small positive correlations between the academic grades and the speed factors were found.

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A STUDY OF SPEED FACTORS IN TESTS AND ACADEMIC GRADES

The speed with which an examinee responds to the items in a test is an ability or characteristic of the examinee that frequently affects his test score. Almost all published achievement and aptitude tests are to some extent measures of "speed" for at least some of the examinees. Tests specifically designed for factor analysis studies are frequently highly speeded in view of the necessity for administering many tests in a limited time.

Much remains to be learned about the trait, or traits, called "speed," in spite of the fact that it is commonly an element in reported test scores. Is speed on cognitive tests a unitary trait? Or are there different kinds of speed corresponding to different kinds of tasks? In the latter case, how highly correlated with each other are these different kinds of speed? How highly correlated are speed and level on the same task? How do various criteria, such as course grades relate to speed, and how speeded should tests designed to predict these criteria be? These are the questions which the present study attempts to answer, at least with respect to the tests and examinees studied.

Some Previous Results

Factor analytic studies in the past have often isolated a "perceptual-speed factor," usually measured by tests requiring simple, rapid visual discriminations. According to French [6], "This factor is characterized by the task of [quickly] finding in a mass of distracting material a given configuration which is borne in mind during the search." Any speed test composed entirely of very easy items is likely to have a loading on this factor. A more recent publication [7] breaks down "perceptual speed" into at least two factors, "speed of symbol discrimination" and "form perception," the former relating to familiar symbols, the latter to unfamiliar figures.

Other factors obviously related to speed, identified by French [6] in more than one published study, include finger dexterity, fluency of expression, ideational fluency, reaction time, speed of association, speed of judgment, tapping, word fluency. Speed of closure and motor speed are included in [7]. Rimoldi [20] finds a "speed of judgment," a "speed of cognition," and a second-order "personal tempo" factor; but in his study, as in that of many earlier workers, the subjects were asked to work on their tests and tasks at a "natural, congenial" speed rather than at the maximal speed required in most actual testing situations.

Since so many of the tests used in factor analytic studies are speeded, many of the factors obtained are actually speed factors, although not always explicitly described as such. An important example of this is the "number" factor, which is commonly measured by highly speeded tests of addition, subtraction, multiplication, and division. As a reminder of its true nature, this factor will here be referred to as the <u>number-speed factor</u>.

In spite of the presence of both speeded and unspeeded tests in most factor analysis batteries, it has not been exactly customary for the analyst to find a general intellectual speed factor. Factorial studies specially designed to investigate the existence of both general and specific speed factors in mental test batteries have been limited in number and have yielded some seemingly conflicting evidence.

DuBois [4], using tetrad and triad analysis, found a speed factor common to speeded arithmetic, vocabulary, analogies, directions, and space tests; all his test items were chosen so as to be extremely easy, however. Sutherland [21], also using tetrad analysis, found no general speed factor in a variety of speeded tasks at different difficulty levels selected from various intelligence tests. Myers [18] analyzed the scores on differently speeded, but otherwise parallel, parts of a nonverbal reasoning test and found two orthogonal factors, one for speed and one for level.

Tate [22], using a carefully controlled, complex analysis of variance design, clearly demonstrated the existence of a general speed factor common to rate-of-work scores on arithmetic-reasoning, number-series, sentence-completion, and spatial-relations items. He also obtained indications of speed factors specific to each of these types of items. Rate of work was found to be uncorrelated with independent measures of "altitude" in the ability measured.

Davidson and Carroll [3] obtained and factor-analyzed time-limit, work-limit (level), and rate-of-work scores on the following tests: Letter Grouping, Scattered X's, Phrase Completion, Disarranged Morphemes, paragraph reading, and the eight subtests of the Revised Alpha Examination. The Scattered X's test, which is a measure of perceptual speed, was found to have so little correlation with all other measures that it was dropped from the analysis. They found a reasoning-speed factor (as well as a reasoning-level factor) and also a "general speed factor" involving nearly all of the rate-of-work scores. In spite of the presence of several typically verbal tests, they did not find a clear-cut verbal factor.

Mahan [17] administered three differently speeded tests for each of the following five factors: verbal, numerical, spatial, perceptual speed, inductive reasoning. By the multiple-group method he then extracted five factors corresponding to the five types of tests. No speed factor was found. The foregoing procedure was independently carried through once with moderately and highly speeded tests and once with moderately speeded and unspeeded tests. The five factors obtained in the two situations were found to differ somewhat from each other.

For further discussion of "speed factors," the reader is referred to a discussion by Vernon [24, pp. 80-85] and to the 33 references listed by Goheen and Kavruck [8].

Data for the Present Study

The Subjects

All measures used in this study were obtained on 649 students in the entering class at the United States Naval Academy at Annapolis. This unusually large number of cases was secured with a view toward obtaining clearly interpretable factorial results.

The Tests

The study centers around tests of the verbal factor, of spatial ability, and of arithmetic reasoning. This choice was made because of the widespread use of tests in these three areas for actual selection and guidance. Almost all entering students at Annapolis take an admissions battery that includes tests in these three areas.

In each of these three areas, seven tests were administered to the examinees. One of these was the regular admissions examination, which will be denoted by the letter A, administered to applicants before admission. The admissions tests are only slightly speeded. The remaining six were short experimental tests administered at the beginning of the school year. These six tests were constructed so as to be wholly parallel to each other in content, but different in degree of speededness. Two of the six tests were "level" tests, to be denoted by L, involving virtually no speed at all. One of the six tests was moderately speeded (M). The remaining three tests were highly speeded (S). In order to confound practice effect insofar as possible, the tests were administered in scrambled order, as follows: LSMSLS. In advance of taking each test, the examinee was told the degree of speededness that would be required by the time allowance given.

Six reference-factor tests (number, perceptual speed, word fluency) were administered at the beginning of the school year in addition to the foregoing. These are designated by the letter R. A more complete description of the tests follows.

1. Word Fluency (R). The examinee is instructed to write down as many words and their opposites as he can in four minutes. This test was included so as to determine its relation to the verbal factor and to the verbal-speed factor, if the latter were actually found to exist.

2. Verbal (A). This test contained both word-analogies items and "double-definitions" items. The latter item type is essentially a sentence with two missing words which the examinee must select from alternative pairs of words provided, thus producing a completed sentence that is a simple definition of one of the missing words.

3,4. Vocabulary (L). These tests require the examinee to find among the choices a word opposite in meaning to the given key word. Also 5. Vocabulary (M) and 6,7,8. Vocabulary (S). (9. Vocabulary (LIA) is merely the "last-item-attempted score" on test 7.)

10. Spatial Relations (A) contained both block-counting and "identical-blocks" items. The latter requires the examinee to indicate which of five drawings represents a key block drawn from a different angle.

11,12. Intersections (L). These tests consist of a type of spatial relations item requiring the examinee to visualize the two-dimensional outline of the intersection of a solid geometric object cut by a plane. Also 13. Intersections (M) and 14,15,16. Intersections (S). (17. Intersections (LIA) is merely the "last-item-attempted score" on test 15.)

18. Mathematics (A) is composed of arithmetic reasoning, algebra, and geometry items.

19,20. Arithmetic Reasoning (L) consist entirely of the usual arithmetic-reasoning items. Also 21. Arithmetic Reasoning (M) and 22,23,24. Arithmetic Reasoning (S). (25. Arithmetic Reasoning (LIA) is merely the "last-item-attempted score" on test 23.)

26,27. Number Speed (R) are typical reference tests for the number-speed factor. 26 is a test of addition and division; 27 is a test of subtraction and multiplication. Both are highly speeded tests involving only very easy computations.

28,29,30. Perceptual Speed (R) are typical reference tests for the perceptual-speed factor. 28. Cancellation requires the examinee to cross out as many letter A's in a paragraph as he can in two minutes. 29. Picture Discrimination requires him to indicate which of three very sketchily drawn faces is different from the other two. 30. Number Checking requires him to indicate whether two multi-digit numbers are the same or different.

Table I summarizes the background information about the "experimental" tests and shows the proportion of examinees who answered the last item in each test. It is apparent that the speeded tests were in fact very highly speeded, as intended. Actually, there is reason to believe that many or all of the examinees who answered the last item of the speeded tests could have done so only by skipping many items or by responding to them at random.

Table 1

Background Information and Data
on Speededness for the "Experimental" Tests

Tests		Speed-edness	Number of Items	Testing Time	Items per Hour	Per cent* of Examinees Finishing
3,4	Vocabulary	L	15	7	129	97
5	"	M	30	5	360	71
6,7,8	"	S	75	5	900	2
11,12	Intersections	L	15	20	45	98
13	"	M	20	12	100	75
14,15,16	"	S	35	9	233	11
19,20	Arithmetic Reasoning	L	10	20	30	94
21	" "	M	15	15	60	50
22,23,24	" "	S	30	10	180	4

*The mean of two values in the case of the level tests, of three in the case of the speed tests.

Scoring

The three admissions tests are composed of multiple-choice items having five (in a few cases, eight) alternative responses. The score obtained for each test was the number of items answered correctly, this being the standard procedure for scoring the admissions examinations.

The eighteen experimental vocabulary, intersections, and arithmetic reasoning tests were all composed of five-alternative multiple-choice items and were scored number-right minus one-fourth-number-wrong. It was felt necessary to make this "correction for guessing" in order that any speed factor that might be found should not be open to the challenge that it was merely a willingness-to-guess-wildly factor. It would, of course, have been wrong to include both corrected and uncorrected scores on the same test in a straightforward factor analysis, because of their experimental dependence. Some further investigation of the effect of the correction for guessing was nevertheless planned. For this purpose, number-right (NR) scores were obtained for tests 7, 15, and 23, these new scores being designated as variables 37, 38, and 39.

The score on each of the six reference tests was taken to be the number of right answers. This method of scoring was used simply because this is the usual method for scoring these tests. If they were scored in any other way, they might no longer represent the same reference factors.

In addition to the regular score already mentioned, a "last-item-attempted score" (LIA) was obtained for one speeded test in each of the three areas, as noted in the preceding section. The LIA score is simply the serial number of the last item to which the examinee responded. This score is a crude measure of rate-of-work. It was considered to be highly desirable to include such scores in the present study, although in general the study is primarily concerned with the type of scores normally used in practical work with aptitude tests. The statistical method used to deal with the experimental dependence of these scores and of the NR scores on the other scores obtained from the same tests will be outlined in a later section.

School Grades

During their first year, all entering students at Annapolis normally receive grades in each of the following:

31. English Composition and Literature.

32. Foreign Language. (Each student selects one out of the several available foreign languages.)

33. Engineering Drawing and Descriptive Geometry.

34. Chemistry.

35. Mathematics. (Plane trigonometry, college algebra, plane and solid analytic geometry, and calculus.)

36. Conduct. (The method by which grades in conduct are assigned need not concern us here, since no factor loadings of interest were found for this variable.)

In the present study, each grade is a numerical grade averaged over two semesters. Each course grade for each semester represents a combination of day-to-day course work and final-examination performance weighted in the ratio of three-to-two. It may be noted that the instructors could not possibly have had knowledge of any of their students' test scores, with the possible exception of the three admissions tests.

The final examinations were virtually totally unspeeded, almost every student finishing in the time allowed. The day-to-day work in class was of varied kinds but was not in general compulsorily speeded. It is not known whether students felt pressed for time while doing their homework assignments.

Statistical Analysis

Normalizing

All variables used in the present study were normalized before computing Pearson product-moment correlations. This was considered desirable since otherwise any speed factors that might be found might conceivably have been attributable to certain common features in the shape of the score distribution of the speeded tests (e.g., skewness), rather than to the existence of a real speed factor.

The Correlations

The use of product-moment correlations, rather than tetrachoric correlations, is required for the significance tests to be described later. The matrix of the correlations is presented as Table 2. Variables 9, 17, 25, 37, 38, and 39 are experimentally dependent on variables 7, 15, and 23. The consequent spuriously high correlations are placed in parentheses in the table. Variables 9, 17, 25, 37, 38, and 39 were not used in the factor-extraction process for this reason.

Lawley's Maximum Likelihood Method of Factor Analysis

Factors were extracted by the maximum-likelihood method developed by Lawley. Since this extremely important method has not often been mentioned in the literature in this country, and since this method or a modification of it is likely to become widely used in the near future, it will be worth-while to list some of the available references. The basic development of the method was given in [12]. Extensions and further developments appeared in [13, 15, 1, 2, and 23]. (A maximum-likelihood Method II, avoiding the

Table 2

Matrix of Intercorrelations (decimal point omitted)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
1	---	264	200	262	249	231	341	321	314	056	111	100	064	136	080	090	122	181	156	165	154	183	174	216	233	253	200	223	212	122	303	184	170	249	207	-082	346	097	216
2	264	---	720	720	790	666	732	679	383	173	174	135	165	153	138	176	063	413	341	324	363	335	329	364	195	051	050	058	121	-062	560	210	184	230	156	-051	707	135	325
3	200	720	---	669	706	620	693	641	328	134	150	055	102	119	093	079	087	287	300	251	285	279	275	276	160	047	044	052	054	-038	497	172	084	172	119	-106	665	099	257
4	262	720	669	---	690	643	697	650	343	125	181	144	134	171	128	121	054	310	316	311	339	255	286	301	117	013	013	043	065	-023	568	205	186	238	145	-017	673	124	271
5	249	790	706	690	---	660	745	700	393	133	111	032	115	093	082	075	046	326	265	258	299	300	297	314	194	086	088	097	100	-056	537	192	138	196	128	-064	724	082	295
6	231	666	620	643	660	---	775	757	531	176	162	137	176	138	174	161	140	323	276	290	323	357	333	380	226	133	165	211	245	093	519	186	247	270	213	-034	770	181	341
7	341	732	693	697	745	775	---	855	(671)	135	122	061	103	127	107	100	130	337	291	270	313	353	350	364	294	196	178	223	254	105	590	220	192	258	211	-038	(936)	120	362
8	321	679	641	650	700	757	855	---	609	133	094	067	117	135	104	114	126	334	273	283	348	334	347	368	290	205	202	233	250	112	540	226	221	248	210	-019	856	118	362
9	314	383	328	343	393	531	(671)	609	---	078	060	026	031	127	086	109	291	290	192	176	208	303	282	316	455	339	358	316	354	249	373	204	132	223	258	-020	(745)	139	368
10	056	173	134	125	133	176	135	133	078	---	480	541	548	543	534	553	236	349	290	271	331	309	321	275	172	027	058	068	247	-016	120	082	476	238	218	055	124	522	326
11	111	174	150	181	111	162	122	094	060	480	---	722	714	696	701	696	270	312	337	357	295	289	337	257	124	-046	-018	103	215	-028	143	136	546	321	328	019	111	678	329
12	100	135	055	144	032	137	061	067	026	541	722	---	767	714	738	750	246	255	295	302	297	280	230	230	112	-065	-022	076	243	-049	093	125	582	310	268	025	049	698	272
13	064	165	102	134	115	176	103	117	031	548	714	767	---	748	760	770	262	274	337	362	344	334	333	252	105	-092	-045	065	231	-080	102	100	605	318	300	056	087	722	317
14	136	153	119	171	093	138	127	135	127	543	696	714	748	---	796	738	428	266	322	300	309	310	345	266	192	-018	012	114	292	-043	073	110	574	327	308	023	127	790	352
15	080	138	093	128	082	174	107	104	086	534	701	738	760	796	---	833	(481)	253	312	286	289	291	338	254	177	-028	-025	072	283	-022	037	100	568	292	283	023	102	(977)	340
16	090	126	079	121	075	161	100	114	109	553	696	750	770	738	833	---	449	295	335	314	324	361	370	307	194	004	037	123	277	005	058	106	584	320	326	037	099	825	366
17	122	063	087	054	046	140	130	126	291	236	270	246	262	428	(481)	449	---	132	105	090	121	174	192	152	394	102	070	150	231	052	003	019	225	141	143	-054	171	(625)	297
18	181	413	287	310	326	323	337	334	290	349	312	255	274	266	253	295	132	---	555	596	633	623	602	596	355	260	351	160	194	113	302	177	308	376	435	059	334	256	597
19	156	341	300	316	265	276	291	273	192	290	337	295	337	322	312	335	105	555	---	538	606	551	532	514	211	155	193	100	115	026	309	192	355	424	411	072	277	295	507
20	165	324	251	311	252	290	270	283	176	271	357	302	362	300	286	314	090	596	538	---	560	544	548	534	229	207	276	083	129	039	281	223	328	429	475	029	258	270	518
21	154	363	285	339	299	323	313	348	208	331	295	297	344	309	289	324	121	638	606	560	---	632	578	571	284	137	310	147	155	090	318	177	313	378	376	047	304	278	554
22	183	335	279	255	300	357	353	334	303	309	289	280	334	310	291	361	174	628	551	544	632	---	632	639	393	320	409	178	195	120	278	190	338	330	416	083	351	293	629
23	174	329	275	286	297	333	350	347	282	321	337	280	353	345	338	370	192	602	532	548	578	632	---	610	(454)	241	321	178	242	143	272	225	350	420	430	095	347	332	(949)
24	216	364	276	301	314	380	364	368	316	275	257	230	252	266	254	307	152	596	514	534	571	639	610	---	349	329	407	175	199	137	300	213	302	364	400	084	360	252	596
25	233	195	160	147	194	226	294	290	455	171	124	112	105	192	177	194	394	355	211	229	284	393	(454)	349	---	322	304	245	280	137	178	157	183	230	300	-012	340	237	(670)
26	253	051	047	013	086	133	196	205	339	027	-046	-068	-092	-015	-028	004	102	260	155	207	137	320	241	329	322	---	646	347	316	443	160	222	081	236	342	007	227	000	286
27	200	050	044	013	088	165	178	202	358	058	-018	-022	-045	012	-028	037	070	351	193	276	310	409	321	407	304	646	---	322	264	464	167	198	090	234	359	067	212	-013	348
28	223	058	052	043	097	211	233	233	316	068	103	076	065	114	072	123	150	160	100	083	147	178	175	175	245	347	322	---	370	428	152	175	207	194	259	-033	248	094	213
29	212	121	054	065	100	245	254	252	354	247	215	243	231	292	283	277	231	194	115	129	155	195	242	199	280	316	264	370	---	352	109	084	331	201	216	021	275	291	290
30	122	-062	-038	-023	-056	093	105	112	249	-016	-028	-049	-080	-043	-022	005	052	113	026	039	090	120	143	137	137	443	464	428	352	---	082	127	123	165	237	047	135	-009	170
31	303	560	497	568	537	519	590	540	373	120	143	093	102	073	037	058	003	302	309	281	318	278	272	300	178	160	167	152	109	082	---	467	300	448	386	081	584	036	277
32	184	210	172	205	192	136	220	226	204	082	136	125	100	110	100	106	019	177	192	223	177	190	225	213	157	222	198	175	084	127	467	---	302	497	504	147	230	094	230
33	170	184	084	186	138	247	192	221	132	476	546	582	605	574	568	584	225	308	355	328	313	338	350	302	183	081	090	207	331	123	300	302	---	604	570	177	194	540	351
34	249	230	172	238	196	270	258	248	223	233	321	310	318	327	292	320	141	376	424	429	378	330	420	364	280	236	234	194	201	165	448	497	604	---	806	125	259	283	427
35	207	156	119	145	123	213	211	210	258	218	323	288	300	308	283	326	143	435	411	475	376	416	430	400	300	342	359	259	216	237	386	504	570	806	---	134	221	275	438
36	-082	-051	-106	-017	-064	-034	-038	-019	-020	055	019	025	056	023	023	037	-054	059	072	029	047	083	095	084	-012	007	067	-033	021	047	081	147	177	125	134	---	-040	007	082
37	346	707	665	673	724	770	(936)	856	(745)	124	111	049	087	127	102	099	171	334	277	258	304	351	347	360	340	227	212	248	275	135	584	230	194	259	221	-040	---	126	381
38	097	135	099	124	082	181	120	118	139	522	678	698	722	790	(977)	825	(625)	256	295	270	278	293	332	252	237	000	-013	094	291	-009	036	094	540	283	275	007	126	---	359
39	216	325	257	271	295	341	362	362	368	326	329	272	317	352	340	366	297	597	507	518	554	629	(949)	596	(670)	286	348	218	290	170	277	230	351	427	438	082	381	359	---

assumption of multivariate normality, was developed by Lawley in [13]. This second method will not be considered further here, since the usual optimum properties of maximum likelihood estimates do not appear to hold in the situation under consideration [1, 11]. Whittle [25] derived a relatively simple and satisfactory solution for a similar situation in the special case where the variables are of known reliability.) Henrysson [10] reported an empirical sampling study supporting Lawley's test of significance. Bartlett [1, 2] gave a significance test that is presumably superior to Lawley's whenever the number of examinees is not large compared to the number of variables. Some recent and interesting papers (in English) by Bartlett, Lawley, and others have appeared in [26]. Very recently Rao [19] discussed the basic differences between Hotelling's principal-component analysis and common-factor analysis and described further developments related to those of Lawley.

Lawley's maximum-likelihood method of factor analysis and Thurstone's centroid method of analysis are both concerned with estimating common-factor loadings, specific-factor variance being systematically set aside. Certain characteristics of the maximum-likelihood method may be listed briefly:

1. The number of common factors is tentatively hypothesized in advance.

2. The procedure in effect determines the population correlation matrix, having the hypothesized rank, for which the likelihood of occurrence of the observed sample in the course of random sampling is a maximum. The matrix of factor loadings exactly reproducing this matrix of population correlation coefficients is the basic result obtained by the maximum-likelihood method. The result is obtained by iterative computational procedures.

3. The usual matrix of residuals is computed; a rigorous large-sample significance test is made to determine whether or not the residuals may

plausibly be attributed solely to sampling fluctuations in the correlation coefficients.

4. If the residuals are statistically significant, the research worker repeats the foregoing process, starting with different tentative hypotheses as to the number of common factors required to explain the data, until he is ready finally to accept one of them.

5. The usual problem of estimating the communalities ceases to be a serious cause for concern, since the maximum-likelihood estimates of the communalities are one of the outcomes of the procedure.

The practical application of the maximum-likelihood method is discussed in [5, 16, 14]. Until now, the method has not been applied to other than very small correlation matrices becuase of the large amount of computations required. From a computational point of view, Lawley's method can be described essentially as equivalent to the task of finding the latent roots and vectors of a modified correlation matrix, the correlations being modified by dividing them by a simple function of the unknown latent vectors.

Extraction of Factors

The application of Lawley's method to the actual data was carried out on Whirlwind I*, a high-speed digital electronic computer. The computing program was written by the author with a view to minimizing the use of computer time so as to be prepared in case convergence proved to be so slow as to require hundreds of iterations. A single iteration with this program required roughly 12 seconds, the time varying somewhat with the number (m) of factors hypothesized.

*Whirlwind I is sponsored by the Office of Naval Research. The author wishes to express his appreciation to Dr. P. Youtz and Dr. C. W. Adams for the opportunity to use this computer and to Dr. H. Denman for his help in programming and in putting the program on the computer.

The original hypothesis of the author suggested that m should be at least 9 for the 33-variable matrix analyzed. However, it was found that application of Lawley's method to the initial set of trial values selected for iteration failed because the computations generated imaginary numbers. It became apparent that extremely close initial approximations to the solution were necessary whenever m was at all large if failure of the procedure was to be avoided.

The problem was dealt with as follows. Computations were first carried out on the assumption that $m = 4$. Initial trial values of the factor loadings were arbitrary except that (a) calculated loadings on the first centroid factor were used for the first column of the trial values, (b) the remaining trial values were selected so that the sum of squares of the trial values for any one variable was equal to the highest correlation between that variable and any other variable. When the iterations were successfully completed for $m = 4$, the resulting estimates of the factor loadings were used as the first four columns of the trial values needed to start the iterations with $m = 5$; the fifth column of these trial values was set up in accordance with informed guesses based on a survey of the matrix of residuals. The trial values for $m = 6$ were set up in the same way from the results obtained with $m = 5$, and so forth. In every case after $m = 4$, the initial trial values, except for the last column, proved to be close approximations to the final factor loadings. No further imaginary numbers were encountered under this procedure.

The matrix of residuals obtained with each value of m was tested for significance by means of Lawley's chi-square test. Information about the progress of the computations and about the chi-square significance tests is given in Table 3. Although arguments could be advanced for extracting an eleventh factor, it was finally decided to stop when just ten factors had been extracted.

Table 3

Tests of Significance and Other Information According to the Number (m) of Factors Hypothesized

Number of Factors Hypothesized (m)	Number of Iterations Required for Convergence	Sum of Latent Roots	Chi-Square Calculated from Residuals	Degrees of Freedom for Chi-Square	Probability Level for Chi-Square
4	35	61	2,605	402	< .01
5	22	69	893	373	< .01
6	23	75	662	345	< .01
7	28	78	530	318	< .01
8	26	80	436	292	< .01
9	25	83	357	267	< .01
10	28	88	284	243	.07

The orthogonal unrotated matrix of the maximum-likelihood estimates of the factor loadings is given in Table 4. The communality for each test and the latent root for each factor are also shown. Each latent root is the weighted sum of the squares of the loadings on the corresponding factor, the weight for the squared loading of each test being the reciprocal of the uniqueness of that test.

Estimation of Unrotated Factor Loadings for Experimentally Dependent Variables

The six variables in Table 4 with loadings enclosed in parentheses were not included in the 33-variable correlation matrix from which the factors were extracted, as already mentioned. The loadings in parentheses were estimated by the method briefly outlined in the following paragraphs.

The usual factor equation, $R = F F'$ ($\cong$ is used to indicate approximate equality), may be written

$$\left[\begin{array}{c:c} P & Q \\ \hdashline Q' & S \end{array}\right] \cong \left[\begin{array}{c} G \\ \hdashline H \end{array}\right] \left[\begin{array}{c:c} G' & H' \end{array}\right] \qquad (1)$$

where P is the 33-variable matrix of correlations used for extracting the factors, Q is the matrix of the correlations of these thirty-three variables with the six variables that were omitted from P, S is the matrix of the intercorrelations of these six variables, G is the matrix of the factor loadings of the thirty-three variables and H is the matrix of the factor

Table 4

Unrotated Factor Coefficients
(decimal points omitted)

Variable No.	I	II	III	IV	V	VI	VII	VIII	IX	X	Communality
1	34	16	09	[illegible]	17	08	-04	-04	-07	-02	192
2	65	51	-25	-03	-17	18	01	12	10	-02	832
3	56	51	-24	-04	-10	12	12	04	-06	-03	672
4	61	46	-25	-10	-12	08	-05	00	-12	-02	691
5	60	55	-24	-04	-07	14	05	04	12	05	774
6	66	46	-16	-03	13	-10	00	01	-02	03	699
7	68	59	-15	-04	17	-09	04	-02	-04	00	875
8	67	56	-12	00	19	-19	01	-06	01	06	853
9	(48)	(34)	(11)	(06)	(38)	(-11)	(03)	(-04)	(02)	(-02)	(519)
10	49	-36	-16	11	01	02	-14	05	13	01	446
11	57	-52	-22	-03	-02	11	-02	02	-15	07	689
12	55	-59	-26	-03	02	10	-11	-02	-03	13	765
13	60	-58	-29	-01	-02	-01	-07	-03	-03	12	790
14	60	-57	-26	01	10	01	06	-03	-01	-04	761
15	58	-61	-31	02	11	01	12	-02	02	-12	847
16	60	-61	-24	06	11	01	08	-05	03	-05	826
17	(30)	(-24)	(-12)	(07)	(25)	(-05)	(28)	(-02)	(05)	(-31)	(406)
18	62	02	24	38	-25	02	-04	14	00	01	669
19	59	-07	16	22	-31	-06	-03	03	-10	-12	556
20	59	-07	23	24	-29	-01	04	01	-13	12	580
21	62	-02	17	37	-27	-06	-12	00	-08	-04	642
22	64	-02	23	43	-13	-11	-02	-08	08	02	684
23	63	-05	21	33	-14	-11	02	03	-01	-11	601
24	61	04	24	39	-11	-05	-01	-06	06	-04	604
25	(39)	(04)	(21)	(19)	(16)	(-04)	(11)	(03)	(08)	(-13)	(297)
26	24	12	52	26	39	22	06	-08	06	01	618
27	28	10	55	38	32	21	00	-13	06	09	696
28	26	03	25	05	42	02	-11	13	-12	-02	358
29	36	-09	08	09	48	00	-14	26	-01	-09	483
30	13	04	43	09	49	09	-09	19	-14	-08	531
31	58	39	12	-28	-09	14	-24	-19	-08	-11	718
32	38	04	34	-32	-03	14	-06	-21	01	-11	446
33	64	-41	10	-26	07	-11	-22	08	15	01	754
34	62	-15	48	-35	-12	-05	02	01	03	-04	781
35	60	-20	62	-28	-05	-01	13	04	-01	05	887
36	05	-09	15	-05	-05	-11	-20	-10	15	-09	127
37	(66)	(57)	(-11)	(-03)	(20)	(-07)	(02)	(-02)	(-01)	(00)	(827)
38	(57)	(-58)	(-29)	(03)	(14)	(01)	(14)	(-02)	(02)	(-13)	(807)
39	(64)	(-04)	(23)	(32)	(-07)	(-09)	(02)	(04)	(02)	(-14)	(600)
Latent Roots:	40.92	21.90	11.28	5.10	4.09	1.34	1.08	0.83	0.65	0.58	

loadings of the six variables. Assuming that the entire matrix, R , has the same common factors as does P , it follows that G is the matrix of factor loadings obtained by analyzing P . H can then be determined from the equations:

$$HG' \cong Q' \quad , \tag{2}$$

$$HH' \cong S \quad . \tag{3}$$

Since equations 1 and 2 never hold exactly in practice, (2) represents an inconsistent set of simultaneous linear equations, there being more equations than unknowns. In practice, equation 3 is totally ignored. A least-squares (but not a maximum likelihood) approximate solution for (2) can be obtained [9] by postmultiplying both sides by $G(G'G)^{-1}$, the result being

$$H = Q'G(G'G)^{-1} \quad . \tag{4}$$

It seemed more appropriate, however, and also computationally easier, in the present case where maximum likelihood procedures had been employed, to postmultiply (2) by $S^{-2}G(G'S^{-2}G)^{-1}$; S^2 being the 33 x 33 diagonal matrix whose elements are the uniquenesses of the 33 variables in P . The result is

$$H = Q'S^{-2}G(G'S^{-2}G)^{-1} \quad . \tag{5}$$

A rigorous justification for (5) is not immediately available. Sufficient justification for its use is apparent, however, when it is pointed out that in Lawley's method of analysis $G'S^{-2}G$ is the diagonal matrix whose elements are the latent roots, and further that the basic equation of Lawley's method can be written

$$G = (P - S^2)S^{-2}G(G'S^{-2}G)^{-1} \quad . \tag{6}$$

Rotation

The rotation of the original factor matrix toward psychologically meaningful oblique factors was carried out with the help of the matrix rotator at The Adjutant General's Office*. Extensive careful final rotations were made by desk calculator. Variables 37, 38, and 39 (scores not "corrected for guessing") were not available and were not used in determining the rotations.

The main guiding principle in all rotations was psychological meaningfulness, as interpreted according to the notions of the writer. The facility with which rotations could be made on the matrix rotator encouraged persistence in the ultimately unsuccessful attempt to find an arithmetic-reasoning speed factor. A total of 497 rotations were carried out, each rotation involving the shift of only one axis.

Table 5 gives the orghogonal projections of the thirty-nine variables on the reference axes -- frequently referred to as the "loadings on the rotated factors." Since the term "factor loading" has been used with various meanings in oblique analyses, these projections will hereafter be referred to as "factor coefficients." Table 6 gives the transformation matrix for rotating Table 4 into Table 5. Table 7 gives the intercorrelations among the primary vectors.

If the last factor (or possibly the last two factors) is excluded from consideration, the clarity of the factor structure in Table 5 is made apparent by the visually obvious distinction between 2-digit and 1-digit coefficients. The 1-digit coefficients may be conveniently dismissed as insignificant. Each 2-digit coefficient without exception has an obvious realistic interpretation.

*The writer wishes to express his appreciation to Dr. Hubert Brogden and to Miss Bertha Harper for the opportunity to use the matrix rotator and for their helpful guidance in its operation.

Table 5

Rotated Factor Coefficients

(decimal points and initial zeros omitted)

		V I	S II	M III	N IV	P V	v VI	s VII	G VIII	H IX	X X
1.	Word Fluency (R)	25	8	-3	6	7	7	0	9	11	-5
2.	Verbal (A)	75	2	3	2	5	-9	0	6	-6	12
3.	Vocabulary (L)	67	1	2	-3	3	3	6	2	2	-8
4.	" (L)	65	4	5	-9	7	8	-4	4	13	-6
5.	" (M)	77	2	-7	9	-5	2	-3	3	-9	10
6.	" (S)	63	4	2	-5	2	27	-3	0	-1	1
7.	" (S)	72	-1	-1	-4	2	30	1	0	2	-3
8.	" (S)	67	-3	0	-1	-5	39	-5	-2	-2	2
9.	" (LIA)	38	1	-3	7	4	28	5	0	0	1
10.	Spatial Relations (A)	4	43	5	6	2	-6	1	-5	-5	20
11.	Intersections (L)	-2	63	5	-3	4	-2	3	2	6	-11
12.	" (L)	-2	71	-4	5	-4	0	-4	-1	2	3
13.	" (M)	-2	67	3	-0	-7	7	-1	-2	-0	2
14.	" (S)	-1	67	0	0	-2	2	20	-1	2	-1
15.	" (S)	-2	69	-1	-2	-1	-3	30	-2	1	1
16.	" (S)	-4	68	0	4	-5	1	23	-3	0	2
17.	" (LIA)	3	29	-2	-4	5	-2	49	-2	2	-3
18.	Mathematics (A)	2	-4	50	6	9	-9	-2	-2	-9	0
19.	Arithmetic Reasoning (L)	-2	-1	50	-9	9	-5	8	7	8	-8
20.	" " (L)	-4	4	45	4	-4	4	-7	6	-3	-17
21.	" " (M)	-1	0	51	1	7	0	-1	-6	7	-3
22.	" " (S)	1	2	40	17	-9	9	3	-8	-5	5
23.	" " (S)	-1	0	46	0	6	2	13	0	0	-2
24.	" " (S)	6	-1	38	16	-4	3	7	-4	-1	4
25.	" " (LIA)	8	1	16	8	5	2	20	4	-5	3
26.	Number Speed (R)	3	1	-3	40	2	-1	6	4	1	-1
27.	" " (R)	-3	2	3	48	-4	2	-3	-4	0	0
28.	Cancellation (R)	4	9	2	0	27	10	-3	2	5	-5
29.	Picture Discrimination (R)	8	23	1	-5	35	2	4	-5	-4	10
30.	Number Checking (R)	-7	-1	5	2	36	0	1	6	6	-8
31.	English (G)	54	-3	2	1	6	2	-8	36	35	1
32.	Foreign Language (G)	20	2	-1	9	-5	-5	5	51	24	0
33.	Eng'g. Draw. & Des. Geom. (G)	6	36	8	-8	5	4	-6	38	-4	23
34.	Chemistry (G)	6	1	26	-8	-1	0	2	67	2	-2
35.	Mathematics (G)	-5	2	28	-1	-4	2	0	66	-7	-12
36.	Conduct (G)	-8	-7	6	2	-3	-1	-2	15	8	21
37.	Vocabulary (NR)	70	-1	-3	0	3	28	0	1	2	1
38.	Intersections (NR)	-1	67	-2	-1	0	-2	32	-2	1	0
39.	Arithmetic Reasoning (NR)	2	2	42	2	8	0	16	2	-1	1

TABLE 6

Transformation Matrix

	I	II	III	IV	V	VI	VII	VIII	IX	X
I	415	324	192	020	009	109	053	181	011	007
II	647	-561	-047	008	030	141	-119	-056	015	003
III	-389	-448	352	129	038	-060	-078	497	000	-086
IV	-308	-036	337	295	035	-031	082	-784	-082	002
V	132	362	-510	161	167	294	091	-192	005	034
VI	273	408	-416	525	112	-684	030	051	172	-020
VII	007	002	-029	056	-350	059	532	090	-392	-432
VIII	-049	-189	357	-497	650	-345	-112	026	-540	089
IX	261	-042	-336	474	-409	-259	081	183	-520	888
X	013	207	-223	346	-493	468	-811	-145	-497	-092

Table 7

Correlations between Primary Vectors

		V I	S II	M III	N IV	P V	v VI	s VII	G VIII	H IX	X X
V	I	1.00	.14	.44	.00	-.11	-.08	-.05	-.11	.03	-.13
S	II	.14	1.00	.49	-.08	-.08	.05	-.02	.17	-.01	.20
M	III	.44	.49	1.00	.29	-.04	.13	.03	.13	.09	.19
N	IV	.00	-.08	.29	1.00	.71	.56	.28	.40	-.18	-.09
P	V	-.11	-.08	-.04	.71	1.00	.66	.28	.40	-.31	.01
v	VI	-.08	.05	.13	.56	.66	1.00	.44	.42	-.14	.27
s	VII	-.05	-.02	.03	.28	.28	.44	1.00	.12	-.22	.12
G	VIII	-.11	.17	.13	.40	.40	.42	.12	1.00	-.12	.01
H	IX	.03	-.01	.09	-.18	-.31	-.14	-.22	-.12	1.00	.27
X	X	-.13	.20	.19	-.09	.01	.27	.12	.01	.27	1.00

In most factor analyses it is customary to ignore coefficients less than .30 or .20, say, as not reliably different from zero. Standard errors for individual factor coefficients have not been computed for the present study. However, with correlations based on 649 cases, as in the present study, the standard error of a correlation coefficient is about .04 for correlations in the neighborhood of zero and about .01 for correlations in the neighborhood of .80. In view of these standard errors, it is to be expected that factor coefficients will have some meaning even in the low range from .10 to .20. This will be seen to be actually the case.

Interpretation of Factors

The first three factors of Table 5 correspond to the three aptitude areas about which the present study is centered. These three factors may be considered to be "level" factors, in contrast to the next four, all of which are speed factors. The eighth and to a large extent the ninth are factors determined by academic grades. The tenth and last factor seems to have no simple interpretation. All these factors will now be discussed in more detail.

Factor I (V) is the verbal factor. In addition to the experimental vocabulary tests, the following variables have two-digit coefficients for this factor, as would be expected:

2.	Verbal Test (A)	.75
31.	English Grade	.54
1.	Word Fluency (R)	.25
32.	Foreign Language Grade	.20

Factor II (S) is a space factor. In addition to the experimental intersections tests, the following variables have two-digit coefficients for this factor, as would be expected:

10.	Spatial Relations (A)	.43
33.	Engineering Drawing and Descriptive Geometry Grades	.36
29.	Picture Discrimination (R)	.23

The picture discrimination test is a reference test for the perceptual speed factor, but the test obviously requires also the ability to perceive and discriminate spatial patterns.

Factor III (M) is a mathematical-reasoning factor. In addition to the experimental arithmetic-reasoning tests, the following variables have two-digit coefficients for this factor, as would be expected:

18.	Mathematics (A)	.50
35.	Mathematics grade	.28
34.	Chemistry grade	.26

Factor IV (N) is the number-speed factor, determined by the two reference tests included for this purpose. The only other variables with two-digit coefficients for this factor are two of the speeded arithmetic-reasoning tests, which obviously require some speed in numerical operations:

22.	Arithmetic-reasoning (speeded)	.17
24.	Arithmetic-reasoning (speeded)	.16

Factor V (P) is the perceptual-speed factor determined by the three reference tests included for this purpose. No ther variables have two-digit coefficients for this factor.

Factor VI (v) is clearly the verbal-speed factor that the present analysis was designed to isolate (if it actually existed) and to study. All the two-digit coefficients for this factor are listed below:

8.	Vocabulary (speeded)	.39
7.	Vocabulary (speeded)	.30
9.	Vocabulary (last item attempted)	.28
37.	Vocabulary (speeded; number-right score)	.28
6.	Vocabulary (speeded)	.27
28.	Cancellation (R)	.10

The cancellation test is a reference test for the perceptual-speed factor. The coefficient of .10 for this test on the verbal-speed factor is not large enough to be of great interest; a positive coefficient might be expected, however, in view of the fact that this test requires rapid work with alphabetical and verbal symbols.

Factor VII (s) is clearly the spatial-speed factor that the present study was designed to isolate (if it actually existed) and to study. All the two-digit coefficients for this factor are listed below:

17.	Intersections (last item attempted)	.49
38.	Intersections (speeded; number-right score)	.32
15.	Intersections (speeded)	.30
16.	Intersections (speeded)	.23
14.	Intersections (speeded)	.20
25.	Arithmetic Reasoning (last item attempted)	.20
39.	Arithmetic Reasoning (speeded; number-right score) .	.16
23.	Arithmetic Reasoning (speeded)	.13

The fact that all of the speed scores on the arithmetic-reasoning tests have small positive loadings on the spatial-speed factor is consistent with the fact that the arithmetic-reasoning tests contain a considerable proportion of simple geometry and other items that involve diagrams and other graphic illustrations, these being printed in the test booklets alongside the items.

Factor VIII (G) is an academic-grades factor. No variables other than the six academic grades have two-digit coefficients for this factor.

Factor IX (H) appears to be some sort of verbal-academic-grade factor, as indicated by its two-digit coefficients, which are as follows:

31.	English Grade	.35
32.	Foreign Language Grade	.24
4.	Vocabulary (level)	.13
1.	Word Fluency (R)	.11

Factor X (X) does not suggest any ready interpretation. Two-digit coefficients for this bipolar factor are

33.	Engineering Drawing and Descriptive Geometry grade	.23
36.	Conduct grades	.21
10.	Spatial Relations (A)	.20
20.	Arithmetic Reasoning (level)	-.17
2.	Verbal (A)	-.12
35.	Mathematics grade	-.12
11.	Intersections (level)	-.11
5.	Vocabulary (M)	.10
29.	Picture Discrimination (R)	.10

<u>The Correlations Among Factors</u>

The correlations among the primary vectors in Table 7 are of paramount interest. First, it should be pointed out that the reference axis for the verbal factor was arbitrarily set approximately orthogonal to the reference axis for the verbal-speed factor. The reason for doing this was that it was felt that interpretation would be hindered by a choice of reference axes that would give the speeded verbal tests zero loadings on the verbal factor. For the same reason, the spatial-factor axis was set roughly orthogonal to the spatial-speed axis. In each of these cases, the correlations between the speeded tests and the corresponding level factor are therefore represented approximately by the factor coefficients of the speeded tests for the level factor, and not by the corresponding near-zero correlation in Table 7.

The verbal-factor axis and the academic-grades axis were both set approximately orthogonal to the axis for the ninth factor. This was done because the considerable indeterminacy as to the proper position of the primary vector for the ninth factor required some arbitrary decision to be made.

The mathematical-reasoning factor shows correlations of .44 and .49 with the verbal and spatial factors, respectively. These correlations are quite reasonable in view of the fact that the arithmetic-reasoning tests include verbally presented problems, geometry problems, and other graphically presented problems.

The only other correlations as large as these in Table 7 are between various speed factors In fact, the striking thing about Table 7 is the consistently positive intercorrelations of the four speed factors that have been isolated. In general, these correlate much more highly with each other than they do with the three "level" factors. This result clearly demonstrates the existence of a second-order general speed factor.

The Relation of Grades to Speed.

The academic-grade factor is seen from Table 7 to be positively correlated with all four of the speed factors. The ninth factor, however, which is determined mainly by grades in English and in Foreign Language, has negative correlations with each of the four speed tests, and also, incidentally, with the academic-grade factor itself. In order to interpret the relation of course grades to the various speed factors with any degree of clarity, it is necessary to obtain the actual correlations of each of the grades with the primary vectors for each of the speed factors. The correlations of the course grades with all ten of the primary vectors is given in Table 8.

With one minor exception, each of the course grades is positively correlated with each of the four speed factors. Although these relationships are not high, there seems to be conclusive evidence of some positive relation between grades at Annapolis and speed.

Discussion

No speed factor for the arithmetic-reasoning tests could be isolated, although the attempt was persistently made. An examination of the factor loadings indicates certain small systematic differences between the speeded and the unspeeded arithmetic-reasoning tests. The speeded tests tend to involve the number-speed factor, the verbal-speed factor, and the spatial-speed factor to a slightly greater extent than do the unspeeded tests, as

Table 8

Correlations between Course Grades and Primary Vectors
(decimal points omitted)

		V	S	M	N	P	v	s	G	H	X
		I	II	III	IV	V	VI	VII	VIII	IX	X
31.	English (G)	61	12	39	20	10	15	-10	36	35	04
32.	Foreign Language (G)	18	13	24	26	12	17	04	54	20	02
33.	Eng'g. Draw. & Des. Geom. (G)	09	68	46	09	15	30	02	54	00	41
34.	Chemistry (G)	17	36	51	28	16	28	07	76	-02	08
35.	Mathematics (G)	08	35	51	43	27	35	11	84	-15	-05

might reasonably be expected. The picture is somewhat confused by the fact that test 23 behaves slightly differently from the parallel tests 22 and 24, and test 19, from the parallel test 20. It may be that an arithmetic-reasoning factor exists in the data but is so very unimportant that it could not be separated from "noise" in the analysis. It would be interesting to see if the extraction of an eleventh factor would have changed this picture.

The results obtained for the last-item-attempted scores are of particular interest. It will be remembered that these scores were not used to determine the common-factor space, the reason being that it was desired to center the present study primarily around the usual types of scores such as are normally used for aptitude tests. In spite of the fact that these scores were not used to determine the common-factor space, it is nevertheless found, for both the verbal and the spatial tests, that the last-item-attempted score is a purer measure of the corresponding speed factor than are the corrected-for-guessing scores on any of the three speeded tests. This is particularly true for the spatial test, where the last-item-attempted score has a much higher loading on the spatial-speed factor than on the space factor.

The communalities of the last-item-attempted scores are .519 for verbal, .406 for spatial, and .297 for arithmetic reasoning. The conclusion that the common-factor space did not include a clear-cut dimension for an arithmetic-reasoning speed factor is to some extent substantiated by the very low communality of the arithmetic-reasoning last-item-attempted score.

It is noteworthy that in all three cases the moderately speeded tests (M) are like the level tests and not like the speeded tests, even though only 50 to 75 per cent of the examinees responded to the last item on these "moderately" speeded tests.

Variables 37, 38, and 39 -- the number-right scores corresponding to variables 7, 13, and 23 -- have loadings so similar to the "corrected-for-guessing" scores on the same tests as to be virtually indistinguishable from the latter. It may be noted, however, that in each case the number-right score has a smaller loading on the level factor for the ability tested than does the "corrected-for-guessing" score.

It is interesting to note that the word-fluency test had very little loading on the verbal-speed factor; even its loading on the verbal factor is quite small. This test has a very low communality (.192); its reliability is not known.

The communalities of the perceptual-speed tests are lower than those of most of the other tests. This is reflected in the fact that the factor coefficients that determine the perceptual-speed factor are rather small (.27 to .36). The factor is nevertheless very clearly determined because all other coefficients for the factor are less than .10.

One variable with extremely low communality (.127) is the grade for conduct. This variable cannot be predicted effectively by any of the tests in the battery. The remaining academic grades, with the exception of English, all have higher loadings on the academic-grade factor than they do on any of the aptitude tests. This situation clearly shows that the course grades have a very sizable reliable, and therefore theoretically predictable, variance over and above that actually predicted by the aptitude tests. Whether this variance is attributable to personality factors or to other causes cannot be determined from the present study.

Summary and Conclusions

The present study was designed to investigate the existence and interrelations of various speed factors, and their relation to academic course grades.

Speeded and unspeeded, but otherwise parallel, tests of vocabulary, spatial ability, and arithmetic reasoning were administered to 649 entering students at the U. S. Naval Academy at Annapolis. Also included in the factorial analysis were scores on certain regular admissions examinations, scores on certain specially prepared reference tests, and end-of-year course grades at Annapolis.

Extraction of factors from the 33-variable correlation matrix was carried out by Lawley's maximum-likelihood method, the calculations being done on the Whirlwind, a high-speed electronic computer. Factoring was continued until, after the extraction of the tenth factor, a significance test on the matrix of residuals showed them to be no longer statistically significant.

Rotation to psychologically meaningful oblique axes was carried out with the help of the matrix rotator at The Adjutant General's Office. The tenth rotated factor was found to be difficult or impossible to interpret. With this exception, the structure of the factor matrix was found to be so clear that a ready interpretation existed for every factor coefficient above .10.

As would be expected, three of the factors obtained were verbal, spatial, and mathematical-reasoning factors. The reference tests included in the battery, again as expected, yielded the usual number-speed factor (ordinarily called simply the number factor) and the usual perceptual-speed factor. The academic grades in the battery were found to define not only a general academic-grade factor but also what appeared to be a verbal-academic-grade factor. Finally, a verbal-speed factor and a spatial-speed factor were clearly identified and distinguished from the number-speed and the perceptual-speed factors. No arithmetic-reasoning speed factor was isolated.

The primary vectors for all four speed factors were found to be positively intercorrelated with each other to such an extent as to demonstrate clearly the existence of a general speed factor at the second-order level.

All correlations between course grades and the four speed factors, with one small exception, were found to be positive, although not large. It is to be concluded that speed of various kinds plays some part in the course grades at Annapolis, and that speededness in the admissions examinations is to this extent justified. The exact degree of speededness that will result in the highest validity for the admissions examinations can only be determined by an experimental study of the admissions examinations themselves. It would seem, however, that tests that are "moderately" speeded, so that only 50 to 75 per cent of the examinees respond to the last item, do not involve the speed factors needed. Apparently, only very highly speeded tests involve these factors to a useful extent.

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