UNCLASSIFIED

AD NUMBER
AD048455
LIMITATION CHANGES
TO: **Approved for public release; distribution is unlimited.**
FROM: **Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; 08 DEC 1954. Other requests shall be referred to Office of Naval Research, 875 North Randolph Street, Arlington, VA 22203-1995.**
AUTHORITY
ONR ltr, 9 Nov 1977

THIS PAGE IS UNCLASSIFIED

THIS REPORT HAS BEEN DELIMITED
AND CLEARED FOR PUBLIC RELEASE
UNDER DOD DIRECTIVE 5200.20 AND
NO RESTRICTIONS ARE IMPOSED UPON
ITS USE AND DISCLOSURE.

DISTRIBUTION STATEMENT A

APPROVED FOR PUBLIC RELEASE;
DISTRIBUTION UNLIMITED.

Armed Services Technical Information Agency

Because of our limited supply, you are requested to return this copy WHEN IT HAS SERVED YOUR PURPOSE so that it may be made available to other requesters. Your cooperation will be appreciated.

AD 48455

NOTICE: WHEN GOVERNMENT OR OTHER DRAWINGS, SPECIFICATIONS OR OTHER DATA ARE USED FOR ANY PURPOSE OTHER THAN IN CONNECTION WITH A DEFINITELY RELATED GOVERNMENT PROCUREMENT OPERATION, THE U. S. GOVERNMENT THEREBY INCURS NO RESPONSIBILITY, NOR ANY OBLIGATION WHATSOEVER; AND THE FACT THAT THE GOVERNMENT MAY HAVE FORMULATED, FURNISHED, OR IN ANY WAY SUPPLIED THE SAID DRAWINGS, SPECIFICATIONS, OR OTHER DATA IS NOT TO BE REGARDED BY IMPLICATION OR OTHERWISE AS IN ANY MANNER LICENSING THE HOLDER OR ANY OTHER PERSON OR CORPORATION, OR CONVEYING ANY RIGHTS OR PERMISSION TO MANUFACTURE, USE OR SELL ANY PATENTED INVENTION THAT MAY IN ANY WAY BE RELATED THERETO.

UNCLASSIFIED

NUMERICAL ANALYSIS RESEARCH

UNPUBLISHED STATISTICAL TABLES

by
D. Teichroew

A Technical Report

Prepared under the sponsorship of the
Office of Naval Research
Project NR-044-144 and Contract Nonr-233(24),
the
Office of Ordnance Research
Project TB2-0001(1210) and Contract DA 04-495-Ord-559
"Research in Numerical Analysis"
and the
School of Aviation Medicine, USAF
Contract AF-18(600)-931, "Asymptotic Expansion Research"

Numerical Analysis Research
Department of Mathematics
University of California
Los Angeles 24, California

December 8, 1954

Numerical Analysis Research

Unpublished Statistical Tables[1]

by

D. Teichroew
Numerical Analysis Research,
University of California, Los Angeles

This note lists tables computed at what is now Numerical Analysis Research, University of California, Los Angeles and before July 1, 1954 was the Institute for Numerical Analysis, National Bureau of Standards.

Publication of tables is expensive and funds for the publication of statistical tables are extremely hard to find. It is therefore unlikely that many of the tables listed below will be published. Also, many of the tables are at present not ready for publication and some of the tables are too special to warrant publication. It seems worthwhile to report the existence of these tables and indicate their present state.

The following list contains tables connected with probability and statistics which have been computed at NAR. Some of these tables were computed as an end in themselves, others were by-products in computations or were computed because the machine codes were available from other problems. In general, the tables have not checked beyond the accuracy required for the purpose for which the table was intended. It seems worthwhile to list tables which are not completely verified because they may simplify the checking procedures if and when the functions are recomputed.

---

[1]The preparation of this table was sponsored (in part) by the Office of Naval Research, USN and the School of Aviation Medicine, USAF.

The number of digits given in the tables may seem excessive to many statisticians. The number of digits used is a consequence of the fact that the tables were computed on the SWAC (Bureau of Standards Western Automatic Computer). This machine operates on a basic number length of 36 binary digits (10.8 decimal digits) and there is no point in using less than an integral multiple of the basic length and, in general, the tables retain most of the digits used in the computation.

Publication appears probable for the following three tables and they are therefore not listed elsewhere in this note.

1. "Tables of the bivariate normal distribution function and related functions." Collated by the National Bureau of Standards. The introduction is by G. Blanch.

The functions given are

$$L(h, k, r) = \frac{1}{2\pi\sqrt{1 - r^2}} \int_h^\infty dx \int_k^\infty \exp\left[-\frac{1}{2}\left(\frac{x^2 + y^2 - 2rxy}{1 - r^2}\right)\right] dy$$

and

$$V(h, \lambda h) = \frac{1}{2\pi} \int_0^h dx \int_0^{\lambda x} \exp\left[-\frac{1}{2}(x^2 + y^2)\right] dy \quad .$$

2. "Table of salvo kill probabilities for square targets" Applied Mathematics Series 44, National Bureau of Standards. Introduction by A. D. Hestenes.

This table gives values of the function

$$P = \frac{1}{2\pi\sigma_{A_x}\sigma_{A_y}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Q(\xi,\eta)\, \exp\left[-\frac{1}{2}\frac{(\xi - x_o)^2}{\sigma_{A_x}^2} - \frac{1}{2}\frac{(\eta - y_o)^2}{\sigma_{A_y}^2}\right] d\xi\, d\eta$$

where $Q(\xi, \eta) = 1 - [1 - P_k P_R(\xi, \eta)]^N \; ; \; 0 \leq P_k \leq 1$

and

$$P_R(\xi, \eta) = \frac{1}{2\pi\sigma_{R_x}\sigma_{R_y}} \int_{-a}^{a}\int_{-a}^{a} \exp\left[-\frac{1}{2}\frac{(x-\xi)^2}{\sigma_{R_x}^2} - \frac{1}{2}\frac{(y-\eta)^2}{\sigma_{R_y}^2}\right] dxdy \quad .$$

3. "Empirical power functions for non parametric two-sample tests for small samples" D. Teichroew. Submitted to a technical journal.

This paper gives the empirical frequencies of all possible rankings which are obtained when a sample of m from N(0, 1) and a sample of n from N(δ , 1) are ranked in order of size, for (m, n) = (3, 2) (3, 3) (4, 2) and (4, 3) and various values of δ .

All the tables listed below exist on punched cards. In addition, two tables mentioned in section II, namely, those containing $y(p; \lambda)$ and $Q_1(p)$, have been multilithed and a limited number of copies are available for distribution.

The tables are listed under five categories:

I. Tables associated with the normal distribution.

II. Tables associated with the Gamma distribution.

III. Tables associated with the t-distribution.

IV. Tables for selecting samples from certain distributions.

V. Miscellaneous tables.

I. Tables Associated with the Normal Distribution

Let $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$, $F(x) = \int_{-\infty}^{x} f(t)\, dt$,

$$H(x; b) = \int_{-\infty}^{x} [F(t)]^b \, dt ,$$

$$\psi(a, b) = \int_{-\infty}^{\infty} [F(x)]^a \int_{x}^{\infty} [1 - F(y)]^b \, dy\, dx ,$$

$$E(x_j; N) = \frac{N!}{(j-1)!(N-j)!} \int_{-\infty}^{\infty} x f(x)\, F(x)^{j-1} [1 - F(x)]^{N-j} \, dx ,$$

$$E(x_j^2; N) = \frac{N!}{(j-1)!(N-j)!} \int_{-\infty}^{\infty} x^2 f(x)\, [F(x)]^{j-1} [1 - F(x)]^{N-j} \, dx ,$$

$$E(x_i, x_j; N) = \frac{N!}{(i-1)!(j-i-1)!(N-j)!} G(i-1, N-j, j-i-1) ,$$

$$G(m, n, p) = \int_{-\infty}^{\infty} \int_{-\infty}^{y} xy\, f(x)\, f(y)\, [F(x)]^m [1 - F(y)]^n [F(y) - F(x)]^p \, dxdy ,$$

and

$$K(\delta; \alpha, \beta) = (\beta + 1) \int_{-\infty}^{\infty} [F(x+\delta)]^{\alpha} [1 - F(x)]^{\beta} f(x)\, dx .$$

The range and size of the tables is as follows:

Function	Range	Decimal Places Tabulated	Accuracy
$F(x)$	$x = -12.00(.02)12.00$	32	≥ 27
$[F(x)]^k$	$x = -12.00(.02)12.00$ $k = 1(1)19$	32	≥ 25
$H(x; b)$	$x = -12.00(.02)12.00$ $b = 1(1)19$	32	≥ 25
$\Psi(a, b)$	$a, b = 1(1)19$	32	≥ 25
$E(x_j; N)$	$j = 1(1)N$ $N = 1(1)22$	21	≥ 19
$E(x_j^2; N)$	$j = 1(1)N$ $N = 1(1)21$	21	≥ 19
$E(x_i, x_j; N)$	$i, j = 1(1)N$ $N = 1(1)20$	21	≥ 19
$K(\delta; \alpha, \beta)$	$\delta = -3.2(.1)0(.01)6.4$ $\alpha = 1(1)9$ $\beta = 0(1)4$	8	8

II. Tables Associated with the Gamma Distribution

$$\text{Let } p = \frac{1}{\Gamma(\lambda)} \int_0^{y(p; \lambda)} e^{-t}\, t^{\lambda-1}\, dt \;.$$

Two SWAC routines have been developed.

1. The first computes p when y is given for λ an integer by summing the series

$$p(y, \lambda) = 1 - e^{-y}\left(1 + y + \frac{y^2}{2!} + \frac{y^3}{3!} + \cdots \frac{y^{\lambda-1}}{(\lambda-1)!}\right)$$

The routine is also used to compute y for a given p by inverse interpolation.

2. The second routine computes y for a given p by the asymptotic series (using terms up to and including Q_{11})

$$y(p; \lambda) = c^2 + Q_1(p)\, c + Q_2(p) + \frac{Q_3(p)}{c} + \cdots$$

where $\lambda = c^2$.

These routines have been used to compute the following tables:

Function	Range	Decimal Places Tabulated	Accuracy
$p(y; \lambda)$	$y = 0(.5)$ varying; ≤ 45.0 $\lambda = 2(1)15$	10	5-10
$y(p; \lambda)$	$p = .000(.001).999$ $\lambda = 2(1)15,\ 20(10)50,\ 100$	8	4-8
$2y(p; \lambda)$	$p = .000(.001).999$ $2\lambda = 3(1)5(2)29$	7	4-7
$Q_i(p)$	$p = .500(.001).999$ $i = 1(1)11$	8	8

III. Tables Associated with the t distribution

The tables are concerned with solving the equation

$$p = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{\pi n}\, \Gamma\left(\frac{n}{2}\right)} \int_{-\infty}^{t(p;\, n)} \frac{d\tau}{\left(1 + \frac{\tau^2}{n}\right)^{\frac{n+1}{2}}}$$

for $t(p; n)$ when p and n are given. An approximation to $t(p; n)$ is obtained by summing the first 8 terms of the asymptotic series

$$t(p; n) = x + \frac{H_1(p)}{n} + \frac{H_2(p)}{n^2} + \cdots .$$

The tables available are the following:

Function	Range	Decimal Places Tabulated	Accuracy
$H_i(p)$	$p = .500(.001).999$ $i = 1(1)8$	8	8
$t(p; n)$	$p = .500(.001)999$ $n = 2(1)\ 16, 20, 25, 50, 100, 200$	8	3-8

IV. Tables for Selecting Samples from Certain Distributions

One method of selecting samples from distributions consists of the following steps:

1. Compute, $\theta(k)$, the sum of k variates uniformly distributed on (0,1).

2. Let $y = \sum_{i=o}^{m} a_i \theta^i$.

The a_i are chosen so that y has the required distribution. These coefficients have been computed for the case where k = 8 and for the following distributions:

1. The normal distribution

2. $\frac{t}{\sqrt{n}}$; where t has a normal distribution, for n = 50(1)200

3. The Gamma distribution, for $\lambda = 2(1)\ 15, 20, 25, 50, 100$.

4. $\frac{1}{\sqrt{y}}$ where y has a Gamma distribution for

$$\lambda = 2(1)\ 15, 20, 25, 50, 100.$$

The variates $\frac{t}{\sqrt{n}}$ and $\frac{1}{\sqrt{y}}$ are used to generate random values of Wishart matrices.

V. Miscellaneous Tables

1. Multinomial coefficients.

This table gives the function

$$\frac{(m + n + p + 2)!}{m!\, n!\, p!}$$

for all combinations of m, n, and p such that $m + n + p \leq 18$. The function occurs in the expression for the covariances of order statistics.

2. Coefficients for curve fitting by Chebyshev polynomials.

This table gives 10 decimal digit values of

$$\cos \frac{k\pi}{2n} (2\alpha + 1)$$

for $n = 2(2)\ 12, 16, 18, 20, 25, 30, 40$; $k = 1(1)\ n - 1$, $\alpha = 0(1)\ n - 1$. This function appears in the curve fitting method described, for example, in "Tables of Chebyshev Polynomials $S_n(x)$ and $C_n(x)$," National Bureau of Standards, Applied Mathematics Series 9. Introduction by Cornelius Lanczos.

3. Tables for Probit Analysis with Poisson Error Models.

Three functions have been tabulated:

$$U(\alpha, \beta; h) = \sum_{d=1}^{\infty} \frac{e^{-h} h^d}{d!} \int_{-\infty}^{\alpha+\beta \log_{10} d} \frac{e^{-t^2/2}}{\sqrt{2\pi}} \, dt$$

$$Z(\alpha, \beta; h) = \sum_{d=1}^{\infty} \frac{e^{-h} h^d}{d!} \frac{e^{-\frac{1}{2}(\alpha+\beta \log_{10} d)^2}}{\sqrt{2\pi}}$$

$$T(\alpha; \beta; h) = \sum_{d=1}^{\infty} \frac{e^{-h} h^d}{d!} (\log_{10} d) \frac{e^{-\frac{1}{2}(\alpha+\beta \log_{10} d)^2}}{\sqrt{2\pi}}$$

for $h = 1(1)\ 17$; $\beta = 0(.5)\ 10$ $\alpha = -5(.5)\ 5$. The accuracy varies from 3 to 8 decimal places.

December 8, 1954