

UNCLASSIFIED

AD NUMBER
AD049027
NEW LIMITATION CHANGE
TO Approved for public release, distribution unlimited
FROM Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; SEP 1954. Other requests shall be referred to Commanding Officer, Wright Air Development Center, Attn: ARDC, Wright-Patterson AFB, OH 45433.
AUTHORITY
AFAL ltr, 17 Aug 1979

THIS PAGE IS UNCLASSIFIED

CATALOGED BY WCOSI-3

20,085
TI-WADC TECHNICAL REPORT 54-28

✓
DO NOT DESTROY
RETURN TO
TECHNICAL DOCUMENT
CONTROL SECTION
WCOSI-3

49007
FILE COPY

A METHOD FOR PREDICTING DYNAMIC LANDING LOADS

(This report supersedes Memorandum Report
MCREXA-5-4595-8-2, 20 February 1948)

RICHARD L. EISENMAN, 1ST LT., USAF
EDWARD H. KRAMER

AIRCRAFT LABORATORY

SEPTEMBER 1954

Statement C
U.S. Govt and Their Contractors

WRIGHT AIR DEVELOPMENT CENTER

20050713181

NOTICE

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

XXXXXXXXXXXXXXXXXXXX

A METHOD FOR PREDICTING DYNAMIC LANDING LOADS

Richard L. Eisenman, 1st Lt., USAF

Edward H. Kramer

Aircraft Laboratory

September 1954

Project 1367

**Wright Air Development Center
Air Research and Development Command
United States Air Force
Wright-Patterson Air Force Base, Ohio**

FOREWORD

This report supersedes Air Materiel Command Memorandum Report MCREXA5-4595-8-2, "A Method for Predicting Dynamic Landing Loads", prepared by Lee S. Wasserman under date of 20 February 1948. The purpose of re-writing this report is to expand and revise details, to present an additional derivation of the theory and to present a new computation form.

This report was prepared in the Dynamic Loads Section, Dynamics Branch, Aircraft Laboratory, Directorate of Laboratories, Wright Air Development Center under Research and Development Project 1367, Structural Design Criteria.

ABSTRACT

This report supersedes Memorandum Report MCREXA5-4595-8-2, "A Method for Predicting Dynamic Landing Loads", 20 February 1948. The purpose of rewriting is to correct minor errors and make several refinements.

Dynamic responses may be computed as the sum of the rigid body response and the vibratory responses in each normal mode. The rigid body response is determined first from basic airplane parameters and in this report is assumed trapezoidal in shape. This trapezoid is then applied to the equation of motion of the elastic system to determine the vibratory response.

The vibratory response of an elastic system to a trapezoidal forcing function can be computed algebraically or graphically. The algebraic computation method is motivated by two distinct principles; discontinuity and superposition; and the graphical computation method is motivated by the superposition principle. New computation forms are provided for both the algebraic and the graphical methods.

Three particular problems are solved to compare theoretical and measured results, to serve as a computation guide and to illustrate the flexibility of the approach. In the first problem the effect of varying basic parameters is discussed with a flow chart.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:

Ed Schwartz
for DANIEL D. McKEE
Colonel, USAF
Chief, Aircraft Laboratory
Directorate of Laboratories

TABLE OF CONTENTS

	<u>Page</u>
Introduction	v
Section I Basic Theory.	1
Section II Derivation of the Acceleration Response to a Trapezoidal Forcing Function.	4
Section III Example 1. Vertical Incremental Accelerations of the Wing Tip of an F-80A Airplane with Full Wing Tip Tanks.	16
Section IV Example 2. Vertical Incremental Accelerations of the Tail Boom of an F-61 Airplane	25
Section V Example 3. B-17G Landing Gear Drag Load (Normal to Strut)	35
References	43
Distribution List	44
Blank Computational Forms for Vibratory Acceleration Response to a Trapezoidal Forcing Function	47

INTRODUCTION

The evolution of vastly stepped up performance of airplanes has placed increased importance on designing structures to close tolerances to minimize weight penalties. The problem of predicting and analyzing dynamic landing loads provides a fertile field for replacing empirical criteria by rational computations.

This report presents an acceptable method for the computation of dynamic landing loads. The basic assumption of a trapezoidal shape for the rigid body loads was suggested by Mr. Lee Wasserman. This assumption is in agreement with test results. Fortunately it is also simple to handle as a forcing function for the differential equation of motion of the vibratory system.

The present paper supersedes MCREXA5-4595-8-2, same title, dated 20 February 1948. The object of this revision is to:

1. Correct the following errors in the examples:

<u>Page in</u> <u>MCREXA5-4595-8-2</u>	<u>Reads</u>	<u>Should Read</u>
13, 26 and 33, column 10	$\frac{(9) - (4)}{\omega_n}$	$\frac{(9) + (4)}{\omega_n}$
33, column 12, row "A" to "B"	∞	$-\infty$

These errors had little effect on the final results.

2. Expand and revise the details of the original theory for easier reading.
3. Provide an additional derivation of the acceleration response to a trapezoidal forcing function.
4. Present a new self-contained computation form which eliminates a significant amount of superfluous arithmetic.
5. Present a flow-chart analysis of the basic parameters in the first example. The effect of changing basic parameters is shown to be intricate but predictable.

In the examples of Sections III through V experimental data were used to determine fuselage and landing gear frequencies and the relevant modes of vibration. For other aircraft experimental values of frequencies may not be available. The computation of fuselage frequencies presents no problem (see Section IV); but the computation of landing gear frequencies requires further investigation.

If it is not known which natural vibration modes are relevant, calculations may be necessary beginning with the mode of lowest frequency and continuing until the computed responses are no longer significant. Reference 9 discusses the difficult question of selecting relevant modes.

SECTION I

BASIC THEORY

1. It is assumed that at the moment of landing wing lift exactly counterbalances the weight of the airplane, so that the vertical velocity is constant. Consequently the response is due entirely to dissipation of kinetic energy at impact and may be considered in two stages:

- a. Rigid body response of the whole structure.
- b. Vibratory response within the structure in each normal mode.

2. Energy equilibrium conditions must be satisfied:

- a. For the structure as a whole (determining rigid body response) i.e., Kinetic Energy = Potential Energy.
- b. For each particle of mass (determining vibratory response) i.e., Inertial Work - Elastic Work = External Work.

3. a. The rigid body response is determined from the equilibrium conditions for the structure as a whole. Briefly, the kinetic energy at impact is determined from the rate of descent and gross weight. But this is equal to the potential energy of the tire and strut work. The tire deflection vs load curves determine the tire work, and the strut work is determined assuming isothermal expansion and quasi-adiabatic compression.

Assuming a trapezoidal shape for rigid body load, the time history is now easy to evaluate. For further detail of this method see Reference 3.

b. The vibratory response in each mode is determined by the local equilibrium conditions:

$$\sum \text{Inertia work} - \sum \text{elastic work} = \sum \text{external work} \quad (1)$$

In particular, let the entire mass be represented by a finite number of elements m_i (e.g. gear, body, tail, wing, etc.) each located at a point in space. Then the vertical displacement and acceleration of the element m_i in each mode depend on the element's location and can be written $c_i x$ and $c_i \ddot{x}$ respectively, where c_i is a constant determined by the position and mode.

Now consider incremental displacements $c_i dx$ of the elements m_i caused by external forces $f_i F(t)$ where $F(t)$ is the time history of the external forcing function with unit amplitude. Temporarily neglecting damping:

<u>Force</u>	<u>x</u>	<u>Distance</u>	=	<u>Work</u>
Inertia: $m_i c_i \ddot{x}$	x	$c_i dx$	=	$(m_i c_i \ddot{x})(c_i dx)$
Elastic: $K_i c_i x$	x	$c_i dx$	=	$(K_i c_i x)(c_i dx)$
External: $f_i F(t)$	x	$c_i dx$	=	$(f_i \cdot F(t))(c_i dx)$

And so equation (1) can be written:

$$\sum_i (m_i c_i^2 \ddot{x})(dx) - \sum_i (K_i c_i^2 x)(dx) = \sum_i (f_i c_i) dx \cdot F(t) \quad (2-a)$$

or

$$(\sum m_i c_i^2) \ddot{x} - (\sum K_i c_i^2) x = (\sum f_i c_i) \cdot F(t) \quad (2-b)$$

It will now be shown that $\sum K_i c_i^2 = -\omega^2 \sum m_i c_i^2$. For in the particular case $F(t)=0$: $(\sum m_i c_i^2) \ddot{x} = (\sum K_i c_i^2) x$ and there is simple harmonic motion so that: $\ddot{x} = -\omega^2 x$ where ω is the mode frequency. Thus $-\omega^2 (\sum m_i c_i^2) = \sum K_i c_i^2$
Substituting

$$(\sum m_i c_i^2) \ddot{x} + (\sum m_i c_i^2) \omega^2 x = (\sum f_i c_i) \cdot F(t) \quad (2-c)$$

or

$$\ddot{x} + \omega^2 x = \frac{\sum f_i c_i}{\sum m_i c_i^2} F(t) \quad (2-d)$$

4. The effect of structural damping can be approximated using a dimensionless "damping coefficient" $\bar{g}$ acting on the displacement so that:

$$\ddot{x} + \omega^2 (1 + \bar{g} j) x = \frac{\sum f_i c_i}{\sum m_i c_i^2} F(t) \quad (3)$$

It is this equation (3) which gives the vibratory acceleration response $\ddot{x}$. In this report $\bar{g}$ is rather arbitrarily assumed to be .10.

5. It is sometimes convenient to by-pass the (constant) coefficient of $F(t)$ by defining:

$$\text{GAF} = \text{Generalized Acceleration factor} = \frac{\text{Generalized force}}{\text{Generalized mass}} \quad (4)$$

(In "g" units.)

$$= \frac{\sum f_i c_i}{\sum m_i c_i^2}$$

* K_i = spring constant

6. This equation (3) can be solved separately for each normal mode because of the original assumption that normal modes do not feed energy to each other or to the rigid body modes. The assumption is validated by the excitation of reasonably pure natural modes during ground vibration tests.

7. The next problem is to determine which normal modes are important. (See Reference 9 for further discussion.) The experience of AMC dynamic tests shows that only the first few modes are important unless there is appreciable coupling between landing gear fore and aft vibrations and higher structural modes.

8. An outline of the computational procedure:

Step I: Compute rigid body vertical load time history from basic airplane parameters.

Step II: Compute rigid body drag load time history if appropriate. Assume (empirically) the coefficient of friction is .55 until the wheel gets up to speed, after which the rigid body drag load falls to zero in one-quarter the spinup time.

Step III: Determine which modes of vibration are important. Experimental data of frequencies will be used if available; If data is not available equations of motion of the structure can be used, but care is required in selecting the appropriate degrees of freedom.

Step IV: Compute generalized acceleration factor if appropriate.

Step V: Compute vibratory response in each mode. The theory for this computation is discussed in Section II.

Step VI: Obtain the time history of total acceleration or structural force by appropriate combination of rigid and vibratory components. Notice that the trapezoid considered as rigid body component may have a different ordinate from the trapezoid considered as forcing function for vibrations.

SECTION II

DERIVATION OF THE ACCELERATION RESPONSE TO A TRAPEZOIDAL FORCING FUNCTION (Solution of Equation (3))

1. Given forcing function *

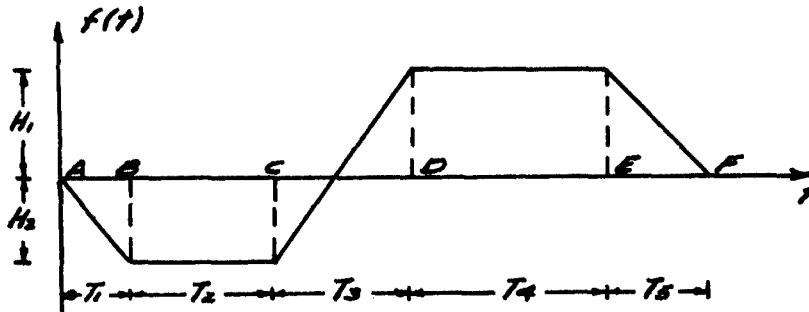


Fig. 1. Generalized Trapezoidal Forcing Function

2. It is required to determine explicitly the vibratory acceleration response for each interval AB, BC....In symbols, find $\ddot{x}$ explicitly where $\ddot{x}$ for each interval is given implicitly by:

$$\ddot{x} + \omega^2(1 + \bar{g}j)x = f(T) \quad (5)$$

with T defined as follows:

AB:	$T = t$	and	$0 \leq t \leq T_1$	
BC:	$T = t - T_1 = t'$		$0 \leq t' \leq T_2$	
CD:	$T = t - (T_1 + T_2) = t''$		$0 \leq t'' \leq T_3$	(6)
DE:	$T = t - (T_1 + T_2 + T_3) = t'''$		$0 \leq t''' \leq T_4$	
EF:	$T = t - (T_1 + T_2 + T_3 + T_4) = t''''$		$0 \leq t'''' \leq T_5$	
→ F:	$T = t - (T_1 + T_2 + T_3 + T_4 + T_5) = t'''''$		$0 \leq t''''' \leq 0$	

* The function has been sketched in sufficiently general terms to satisfy all common problems. In some cases, e.g., $H_1 = 0$ and $H_2 = GAF$ (pos. or neg.).

3. It is apparent * that (5) has an explicit solution of the form:

$$\ddot{x} = e^{-\bar{g}\omega T/2} [a \sin \omega T + b \cos \omega T] \quad (7)$$

For example in AB the initial conditions at $T=0$ are $\dot{x}=0$ and $\ddot{x}=f'(t)$

Then:
$$\ddot{x} = \frac{f'(t)}{\omega} e^{-\bar{g}\omega t/2} \sin \omega t \quad (8)$$

The problem is thus solved in the interval AB.

4. To get the solution in each successive interval two approaches may be taken:

a. Discontinuity Derivation: Evaluate the constants a and b in (7) by applying the initial conditions.

b. Superposition Derivation: Resolve trapezoid into the sum of straight lines all of which emanate from points on the t -axis.

Each of these approaches will now be demonstrated. Of course they must lead to identical results.

Discontinuity Method (Paragraphs 5-9)

5. It is required to determine the constants a and b in (7) in each interval from the initial conditions $\dot{x}_0$ and $\ddot{x}_0$. (The values of $\dot{x}$ and $\ddot{x}$ when $T=0$.)

6. The solution in AB, according to paragraph 3, is (8). Thus the value of $\dot{x}$ and $\ddot{x}$ will be known at the end of AB. Similarly once the interval BC is solved, $\dot{x}$ and $\ddot{x}$ will be known at the end of BC. So if each interval is solved in turn it is fair to assume in all cases that $\dot{x}$ and $\ddot{x}$ are known at the end of the previous interval.

* For since $\ddot{x} + \omega^2(1+\bar{g}j)\dot{x} = f(T)$ is to be solved for $\ddot{x}$ (not x), in present form it is an integral equation (involving x which is $\iint \ddot{x}$). To get a differential equation in $\ddot{x}$ differentiate (5) twice with respect to T to get

$$\frac{d^2}{dT^2} \ddot{x} + \omega^2(1+\bar{g}j)\ddot{x} = 0$$

since $f(T)$ is always linear in T so that $\frac{d^2}{dT^2} f(T) = 0$

This is a standard homogeneous equation in $\ddot{x}$ with the general solution: (approximate)

$$\ddot{x} = e^{-\bar{g}\omega T/2} [a \sin \omega T + b \cos \omega T]$$

7. According to paragraphs 5 and 6, it will be sufficient to evolve $\ddot{x}_0$ and $\ddot{x}_e$ from $\dot{x}_e$ and $\ddot{x}_e$ (where "e" denotes evaluation at the end of the previous interval). Since $\dot{x}$ is continuous from one interval to the next,

$$\dot{x}_0 = \dot{x}_e \quad (9)$$

But $\ddot{x}$ is not continuous, so that $\ddot{x}_0 \neq \ddot{x}_e$

So a gimmick will be introduced to evaluate $\ddot{x}_0$. Basically, we transport the discontinuous variable $\ddot{x}$ through the medium of the continuous variable $\dot{x}$.

First consider equation (5) when differentiated once:

$$\ddot{x} + \omega^2(1 + \bar{g}j)\dot{x} = f'(T) \quad (10)$$

Evaluating (10) at $T=0$

$$\ddot{x}_0 = f'_0 - \omega^2(1 + \bar{g}j)\dot{x}_0 \quad (11)$$

$$\text{From continuity, } \dot{x}_0 = \dot{x}_e \quad (12)$$

So evaluating (10) at the end of the previous interval:

$$\omega^2(1 + \bar{g}j)\dot{x}_e = f'_e - \ddot{x}_e \quad (13)$$

$$\text{or, from (12): } \omega^2(1 + \bar{g}j)\dot{x}_0 = f'_e - \ddot{x}_e \quad (14)$$

Finally, from (11) and (14),

$$\ddot{x}_0 = \ddot{x}_e + [f'_0 - f'_e] \quad (15)$$

8. Solving (7) and its derivative at $T=0$, and applying (9) and (15), the constants a and b can be determined:

$$\begin{aligned} \ddot{x}_0 &= b & a &= \frac{\ddot{x}_e + f'_0 - f'_e}{\omega} + \bar{g} \frac{\dot{x}_e}{2} & (17) \\ \ddot{x}_0 &= a\omega - \bar{g} \frac{\omega b}{2} & \text{or} & & b = \dot{x}_e \end{aligned}$$

9. In particular, the solution in each interval is:

a. In AB:

$$\ddot{x} = \frac{H_2}{\omega T_1} e^{-\bar{g}\omega t/2} \sin \omega t \quad (18)$$

at B:

$$\ddot{x}_B = \frac{H_2}{\omega T_1} e^{-\bar{g}\omega T_1/2} \sin \omega T_1 \quad (19)$$

$$\ddot{\ddot{x}}_B = \frac{H_2 e^{-\bar{g}\omega T_1/2}}{T_1} \left[\cos \omega T_1 - \frac{\bar{g}}{2} \sin \omega T_1 \right] \quad (20)$$

b. In BC:

$$\ddot{x}_0 = \ddot{x}_B \quad \ddot{\ddot{x}}_0 = \ddot{\ddot{x}}_B - \frac{H_2}{T_1} \quad (21)$$

So from (17)

$$q = \frac{H_2 e^{-\bar{g}\omega T_1/2}}{\omega T_1} \left[\cos \omega T_1 - \frac{\bar{g}}{2} \sin \omega T_1 \right] - \frac{H_2}{\omega T_1} + \frac{\bar{g} H_2 e^{-\bar{g}\omega T_1/2}}{2\omega T_1} \sin \omega T_1$$

$$q = \frac{H_2 e^{-\bar{g}\omega T_1/2}}{\omega T_1} \cos \omega T_1 - \frac{H_2}{\omega T_1} \quad (22)$$

So that (7) becomes:

$$\ddot{x} = e^{-\bar{g}\omega t/2} \left[\left(\frac{H_2 e^{-\bar{g}\omega T_1/2}}{\omega T_1} \cos \omega T_1 - \frac{H_2}{\omega T_1} \right) \sin \omega t + \ddot{x}_0 \cos \omega t \right] \quad (23)$$

This may be rewritten:

$$\ddot{x} = e^{-\bar{g}\omega t'/2} \sqrt{\ddot{x}_0^2 + R_1^2} \sin(\omega t' + \phi_1) \quad (24)$$

where

$$\begin{cases} R_1 = \frac{H_2 e^{-\bar{g}\omega T_1/2} \cos \omega T_1 - \frac{H_2}{\omega T_1}}{\omega T_1} \\ \phi_1 = \arctan \frac{\ddot{x}_0}{R_1} \end{cases} \quad (25)$$

$$(26)$$

Note: The choice of the correct quadrant for ϕ is crucial.
The quadrant must be selected so that:

$$\begin{cases} \sin \phi_1 \\ \cos \phi_1 \end{cases} \text{ has the algebraic sign of } \begin{cases} \ddot{x}_0 \\ R_1 \end{cases} \quad (27)$$

i.e.: $\sin \phi_1$ has the algebraic sign of $\ddot{x}_0$
and $\cos \phi_1$ has the algebraic sign of R_1

Finally at C:

$$\begin{aligned} \ddot{x}_c &= e^{-\bar{g}\omega T_2/2} \sqrt{\ddot{x}_0^2 + R_1^2} \sin(\omega T_2 + \phi_1) \\ \ddot{x}_c &= e^{-\bar{g}\omega T_2/2} \sqrt{\ddot{x}_0^2 + R_1^2} \left[-\frac{\bar{g}\omega}{2} \sin(\omega T_2 + \phi_1) + \omega \cos(\omega T_2 + \phi_1) \right] \quad (28) \end{aligned}$$

$$c. \text{ In CD: } \ddot{x}_0 = \ddot{x}_c$$

$$\ddot{x}_0 = \ddot{x}_c + \frac{H_1 - H_2}{T_3} \quad (29)$$

from (15).

So from (17)

$$a = \frac{H_1 - H_2}{\omega T_3} + e^{-\bar{g}\omega T_2/2} \sqrt{\ddot{x}_0^2 + R_1^2} \cos(\omega T_2 + \phi_1) \quad (30)$$

And (after simplification):

$$\ddot{x} = e^{-\bar{g}\omega t''/2} \sqrt{\ddot{x}_c^2 + R_2^2} \sin(\omega t'' + \phi_2) \quad (31)$$

where

$$\begin{cases} R_2 = \frac{H_1 - H_2}{\omega T_3} + e^{-\bar{g}\omega T_3/2} \sqrt{\ddot{X}_0^2 + R_1^2} \cos(\omega T_3 + \phi_1) \\ \phi_2 = \arctan \frac{\ddot{X}_0}{R_2} \end{cases} \quad (32)$$

Note: ϕ_2 must be determined as follows:

(a) Quadrant:

$$\begin{cases} \sin \phi_2 \\ \cos \phi_2 \end{cases} \text{ has the algebraic sign of } \begin{Bmatrix} \ddot{X}_0 \\ R_2 \end{Bmatrix} \quad (33)$$

(b) Angle: $\tan \phi_2 = \frac{\ddot{X}_0}{R_2}$

Finally

$$\ddot{X}_0 = e^{-\bar{g}\omega T_3/2} \sqrt{\ddot{X}_0^2 + R_2^2} \sin(\omega T_3 + \phi_2) \quad (34)$$

d. In DE the process is similar, resulting in:

$$\ddot{X} = e^{-\bar{g}\omega t'''/2} \sqrt{\ddot{X}_0^2 + R_3^2} \sin(\omega t''' + \phi_3) \quad (35)$$

where:

$$\begin{cases} R_3 = e^{-\bar{g}\omega T_3/2} \sqrt{\ddot{X}_0^2 + R_2^2} \cos(\omega T_3 + \phi_2) - \frac{H_1 - H_2}{\omega T_3} \\ \phi_3 = \arctan \frac{\ddot{X}_0}{R_3} \end{cases} \quad (36)$$

Note: ϕ_3 must be determined as follows:

(a) Quadrant:

$$\begin{cases} \sin \phi_3 \\ \cos \phi_3 \end{cases} \text{ Has the algebraic sign of } \begin{Bmatrix} \ddot{X}_0 \\ R_3 \end{Bmatrix} \quad (37)$$

(b) Angle: $\tan \phi_3 = \frac{\ddot{X}_0}{R_3}$

and

$$\ddot{X}_E = e^{-\bar{g}\omega T_4/2} \sqrt{\ddot{X}_0^2 + R_3^2} \sin(\omega T_4 + \phi_3) \quad (38)$$

e. And in EF:

$$\ddot{X} = e^{-\bar{g}\omega t''''/2} \sqrt{\ddot{X}_E^2 + R_4^2} \sin(\omega t'''' + \phi_4) \quad (39)$$

where
$$\begin{cases} R_4 = \frac{-H_1}{\omega T_5} + e^{-\gamma \omega T_4/2} \sqrt{\ddot{X}_D^2 + R_3^2} \cos(\omega T_4 + \phi_3) \\ \phi_4 = \arctan \frac{\ddot{X}_E}{R_4} \end{cases} \quad (40)$$

Note: ϕ_4 must be determined as follows:

(a) Quadrant:

$$\begin{cases} \sin \phi_4 \\ \cos \phi_4 \end{cases} \text{ has the algebraic sign of } \begin{cases} \ddot{X}_E \\ R_4 \end{cases} \quad (41)$$

(b) Angle: $\tan \phi_4 = \frac{\ddot{X}_E}{R_4}$

and
$$\ddot{X}_F = e^{-\gamma \omega T_5/2} \sqrt{\ddot{X}_E^2 + R_4^2} \sin(\omega T_5 + \phi_4) \quad (42)$$

As stated in paragraph 4, the derivation of paragraphs 5-9 could be replaced by paragraphs 10-13.

Superposition Method (Paragraphs 10-13)

10. According to paragraph 3, it is easy to find the response to any straight line emanating from the time axis. The forcing function of figure 1 can be resolved into such lines:

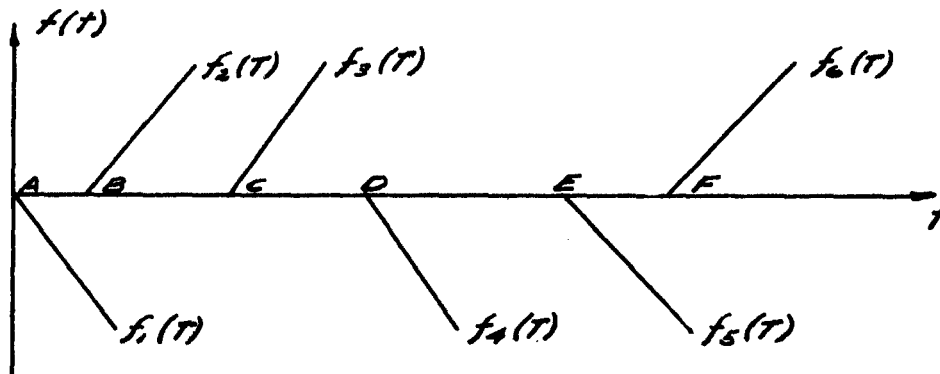


Fig. 2. Linear Decomposition of the Trapezoidal Forcing Function of Figure 1

where the slope

$$\begin{aligned}
 f_1'(T) & \text{ is } \frac{H_2}{T_1} \\
 f_2'(T) & \frac{-H_2}{T_1} \\
 f_3'(T) & \frac{H_1 - H_2}{T_3} \\
 f_4'(T) & \frac{-H_1 - H_2}{T_3} \\
 f_5'(T) & \frac{-H_1}{T_5} \\
 f_6'(T) & \frac{H_1}{T_5}
 \end{aligned} \tag{43}$$

11. Then, according to paragraph 3, the response to $f_1(T)$ is $\ddot{x}_1 = \frac{H_2}{\omega T_1} e^{-\bar{g}\omega t'/2} \sin \omega t' \quad t' \geq 0$

$$f_2(T) \quad \ddot{x}_2 = \frac{-H_2}{\omega T_1} e^{-\bar{g}\omega t'/2} \sin \omega t' \quad t' \geq 0$$

$$f_3(T) \quad \ddot{x}_3 = \frac{H_1 - H_2}{\omega T_3} e^{-\bar{g}\omega t''/2} \sin \omega t'' \quad t'' \geq 0 \tag{44}$$

$$f_4(T) \quad \ddot{x}_4 = \frac{-H_1 - H_2}{\omega T_3} e^{-\bar{g}\omega t'''/2} \sin \omega t''' \quad t''' \geq 0$$

$$f_5(T) \quad \ddot{x}_5 = \frac{-H_1}{\omega T_5} e^{-\bar{g}\omega t''''/2} \sin \omega t'''' \quad t'''' \geq 0$$

12. But since $f = f_1 + f_2 + f_3 + f_4 + f_5 + f_6$ (45)
 The response to $f(t)$ must be the sum of the responses to f_1, f_2 etc.

13. That is:

a. In AB:

$$\ddot{X} = \frac{H_2}{\omega T_1} e^{-\bar{g}\omega t/2} \sin \omega t \quad (46)$$

and

$$\ddot{X}_B = \frac{H_2}{\omega T_1} e^{-\bar{g}\omega T_1/2} \sin \omega T_1 \quad (47)$$

b. In BC: $\ddot{X} = \ddot{X}_1 + \ddot{X}_2$

$$\ddot{X} = \frac{H_2}{\omega T_1} e^{-\bar{g}\omega t/2} \sin \omega t - \frac{H_2}{\omega T_1} e^{-\bar{g}\omega t'/2} \sin \omega t' \quad (48)$$

But since $t = t' + T_1$,

$$\ddot{X} = e^{-\bar{g}\omega t'/2} \left[\frac{H_2}{\omega T_1} e^{-\bar{g}\omega T_1/2} \sin \omega(t' + T_1) - \frac{H_2}{\omega T_1} \sin \omega t' \right] \quad (49)$$

Expanding and collecting:

$$\ddot{X} = e^{-\bar{g}\omega t'/2} \left[\left(\frac{H_2}{\omega T_1} e^{-\bar{g}\omega T_1/2} \cos \omega T_1 - \frac{H_2}{\omega T_1} \right) \sin \omega t' \right. \quad (50)$$

$$\left. + \left(\frac{H_2}{\omega T_1} e^{-\bar{g}\omega T_1/2} \sin \omega T_1 \right) \cos \omega t' \right]$$

and finally:

$$\ddot{X} = e^{-\bar{g}\omega t'/2} \sqrt{\frac{H_2^2}{\omega^2 T_1^2} + R_1^2} \sin(\omega t' + \phi_1) \quad (51)$$

where

$$\begin{cases} R_1 = \frac{H_2}{\omega T_1} e^{-\bar{g}\omega T_1/2} \cos \omega T_1 - \frac{H_2}{\omega T_1} \\ \phi_1 = \arctan \frac{\ddot{X}_B}{R_1} \end{cases} \quad (52)$$

Note: ϕ_1 must be determined as follows:

(a) Quadrant

$$\begin{cases} \sin \phi_1 \\ \cos \phi_1 \end{cases} \text{ has the algebraic sign of } \begin{cases} \ddot{X}_B \\ R_1 \end{cases} \quad (53)$$

(b) Angle: $\tan \phi_1 = \frac{\ddot{X}_B}{R_1}$

And

$$\ddot{X}_C = e^{-\bar{g}\omega T_2/2} \sqrt{\ddot{X}_B^2 + R_1^2} \sin(\omega T_2 + \phi_1) \quad (54)$$

c. In CD

$$\ddot{X} = \ddot{X}_1 + \ddot{X}_2 + \ddot{X}_3 = (\ddot{X}_1 + \ddot{X}_2) + \ddot{X}_3 \quad (55)$$

$$\ddot{X} = e^{-\bar{g}\omega t'/2} \sqrt{\ddot{X}_B^2 + R_1^2} \sin(\omega t' + \phi_1) + \frac{H_1 - H_2}{\omega T_3} e^{-\bar{g}\omega t''/2} \sin \omega t'' \quad (56)$$

But since $t' = t'' + T_2$

$$\ddot{X} = e^{-\bar{g}\omega t''/2} \left[e^{-\bar{g}\omega T_2/2} \sqrt{\ddot{X}_B^2 + R_1^2} \sin[\omega(t'' + T_2) + \phi_1] + \frac{H_1 - H_2}{\omega T_3} \sin \omega t'' \right] \quad (57)$$

Expanding and collecting:

$$\ddot{X} = e^{-\bar{g}\omega t''/2} \left[\left(e^{-\bar{g}\omega T_2/2} \sqrt{\ddot{X}_B^2 + R_1^2} \cos[\omega T_2 + \phi_1] + \frac{H_1 - H_2}{\omega T_3} \right) \sin \omega t'' \right. \quad (58)$$

$$\left. + \left(e^{-\bar{g}\omega T_2/2} \sqrt{\ddot{X}_B^2 + R_1^2} \sin[\omega T_2 + \phi_1] \right) \cos \omega t'' \right]$$

and finally:

$$\ddot{X} = e^{-\bar{g}\omega t''/2} \sqrt{\ddot{X}_C^2 + R_2^2} \sin(\omega t'' + \phi_2) \quad (59)$$

where

$$\begin{cases} R_2 = e^{-\bar{g}\omega T_2/2} \sqrt{\ddot{X}_B^2 + R_1^2} \cos(\omega T_2 + \phi_1) + \frac{H_1 - H_2}{\omega T_3} \\ \phi_2 = \arctan \frac{\ddot{X}_C}{R_2} \end{cases} \quad (60)$$

Note: ϕ_2 must be determined as follows:

(a) Quadrant:

$$\begin{Bmatrix} \sin \phi_2 \\ \cos \phi_2 \end{Bmatrix} \text{ has the algebraic sign of } \begin{Bmatrix} \ddot{X}_C \\ R_2 \end{Bmatrix} \quad (61)$$

(b) Angle: $\tan \phi_2 = \frac{\ddot{X}_c}{R_2}$

and

$$\ddot{X}_D = e^{-\bar{g}\omega T_3/2} \sqrt{\ddot{X}_c^2 + R_2^2} \sin(\omega T_3 + \phi_2) \quad (62)$$

d. In DE: $\ddot{X} = (\ddot{X}_1 + \ddot{X}_2 + \ddot{X}_3) + \ddot{X}_4$

$$\ddot{X} = e^{-\bar{g}\omega t''/2} \sqrt{\ddot{X}_c^2 + R_2^2} \sin(\omega t'' + \phi_2) - \frac{H_1 - H_2}{\omega T_3} e^{-\bar{g}\omega T_3/2} \sin \omega t''' \quad (63)$$

But since $t'' = t''' + T_3$

by the same process as before:

$$\ddot{X} = e^{-\bar{g}\omega t'''/2} \sqrt{\ddot{X}_D^2 + R_3^2} \sin(\omega t''' + \phi_3) \quad (65)$$

where
$$\begin{cases} R_3 = e^{-\bar{g}\omega T_3/2} \sqrt{\ddot{X}_c^2 + R_2^2} \cos(\omega T_3 + \phi_2) - \frac{H_1 - H_2}{\omega T_3} \\ \phi_3 = \arctan \frac{\ddot{X}_D}{R_3} \end{cases} \quad (66)$$

Note: ϕ_3 must be determined as follows:

(a) Quadrant:

$$\begin{cases} \sin \phi_3 \\ \cos \phi_3 \end{cases} \text{ has the algebraic sign of } \begin{cases} \ddot{X}_D \\ R_3 \end{cases} \quad (67)$$

(b) Angle: $\tan \phi_3 = \frac{\ddot{X}_D}{R_3}$

and

$$\ddot{X}_E = e^{-\bar{g}\omega T_4/2} \sqrt{\ddot{X}_D^2 + R_3^2} \sin(\omega T_4 + \phi_3) \quad (68)$$

e. In EF,

$$\ddot{X} = (\ddot{X}_1 + \ddot{X}_2 + \ddot{X}_3 + \ddot{X}_4) + \ddot{X}_5 \quad (69)$$

$$\ddot{X} = e^{-\bar{g}\omega t'''/2} \sqrt{\ddot{X}_D^2 + R_3^2} \sin(\omega t''' + \phi_3) - \frac{H_1}{\omega T_5} e^{-\bar{g}\omega t'''/2} \sin \omega t''' \quad (70)$$

Again since $t''' = t'''' + T_4$

$$\ddot{x} = e^{-\zeta \omega t''''/2} \sqrt{\dot{x}_E^2 + R_4^2} \sin(\omega t'''' + \phi_4) \quad (71)$$

Where

$$\begin{cases} R_4 = e^{-\zeta \omega T_4/2} \sqrt{\dot{x}_E^2 + R_3^2} \cos(\omega T_4 + \phi_3) - \frac{H_1}{\omega T_5} \\ \phi_4 = \arctan \frac{\ddot{x}_E}{R_4} \end{cases} \quad (72)$$

Note: ϕ_4 must be determined as follows:

(a) Quadrant:

$$\begin{Bmatrix} \sin \phi_4 \\ \cos \phi_4 \end{Bmatrix} \text{ has the algebraic sign of } \begin{Bmatrix} \ddot{x}_E \\ R_4 \end{Bmatrix} \quad (73)$$

$$(b) \text{ Angle: } \tan \phi_4 = \frac{\ddot{x}_E}{R_4}$$

The results of paragraph 13 (that is, $\ddot{x}$ in each interval and at the end of each interval) are identical with the results of paragraph 9. From these the zeros, peaks and discontinuities of the acceleration response are easily found and plotted. A self-explanatory form for the cumbersome computations will be used in Sections III - V.

SECTION III

VERTICAL INCREMENTAL ACCELERATION OF THE WING TIP OF AN F-80A AIRPLANE WITH FULL WING TIP TANKS

This problem uses the parameters of landing 2, flight 37, of the AMC F-80A tests reported in reference 4.

Comparisons (but not computations) are also shown for the cases of half-empty (landing 30-1) and empty (landing 28-2) tanks in Figure 3.

These particular landings were selected because their values of the rigid body incremental acceleration correlated better than any others with the theoretical

$$\frac{P_{MAX}}{\frac{1}{2} \text{ Airplane Weight}}$$

Step I. Vertical Load Time History:

1. Basic airplane data:

Gross weight = 14,000 lbs

V_0 = Rate of descent = 6 ft/sec (assumed)

M = Mass per main gear = 217 slugs (2 wheel landing)

W = Static load per main gear = 6250 lbs (3 point position)

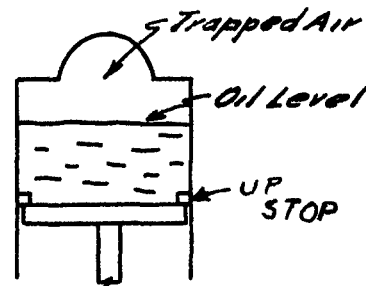
KE = Kinetic energy per main gear = 3906 ft-lbs (2 wheel landing)

ω = Natural frequency of wing = 16.75 rad/sec

2. Basic oleo data:

E_r = Total extension plus latent air column* = (7.95 + 2.00) in. = .8292 ft

* Latent air column = $\frac{\text{trapped air volume}}{\text{piston cross section area}}$



$$E_s = \text{Static extension plus latent air column} = (3.00 + 2.00) \text{ in.} = .4167 \text{ ft.}$$

Assuming isothermal expansion from static to fully extended position, the "load factor" at total extension is:

$$n = \frac{E_s}{E_r} = 0.5025 \quad (74)$$

Then assuming quasi-adiabatic compression from the fully extended position during impact, the extension at any load factor n is:

$$E_n = \frac{E_r}{\left(\frac{n}{n_r}\right)^{\frac{1}{\gamma}}} = E_r \left(\frac{n_r}{n}\right)^{\frac{1}{\gamma}} \quad (75)$$

where n = load/6250 lbs and γ = 1.3 (from reference 8).

3. Basic tire data: (Manufacturer's data)

① Load (Lbs)	② Tire Deflection (Ft)	③ Incremental Tire Work * (Ft-Lbs)	④ Tire Work = Σ ③ (Ft-Lbs)
2500	.058	73	73
6500	.125	302	375
9000	.166	318	693

4. Total work:

⑤ $n = \frac{\text{①}}{6250}$	⑥ $\frac{n_r - 0.5025}{n}$	⑦ ⑥ $\frac{1}{1.3}$	⑧ $1 - \text{⑦}$	⑨ $E_r - E_n = 0.8292$	⑩ ⑨ * ①	⑪ TOTAL WORK = ② + ⑩
.40	1.3565				0**	73
1.04	.4832	.5715	.4285	.3553	2309	2684
1.44	.3490	.4455	.5545	.4598	4138	4831

* ③ = (average value of load during increment) (deflection in increment); Approximating the area under the tire curve by trapezoids.

** The load is not yet sufficient to compress the strut.

5. Tire deflection, oleo deflection and load when kinetic energy per gear = total work:

Total Work (Ft-Lbs)	Load (Lbs)	Tire Deflections (Ft)	Oleo Deflection (Ft)
2684	6500	.125	.355
KE = 3906	P _{MAX} = 7923	X _T = .148	X _O = .415
4831	9000	.166	.460

6. The time for tire compression T_T oleo compression T_O and tire-oleo expansion T_{OT} are determined from the formulas of reference 3, assuming a trapezoidal time history for the strut axial load:

$$T_T = 3 \left(\frac{MV_0}{P_{MAX}} \right) - \frac{1}{2} \sqrt{\left(\frac{6MV_0}{P_{MAX}} \right)^2 - \frac{24MX_T}{P_{MAX}}} = 0.025 \text{ sec.} \quad (76)$$

$$T_O = \sqrt{\frac{2MX_O}{P_{MAX}}} = 0.151 \text{ sec.} \quad (77)$$

$$T_{OT} = \sqrt{\frac{3M(X_O + X_T)}{P_{MAX}}} = 0.215 \text{ sec.} \quad (78)$$

Step II. Drag Load Time History:

The drag load is not used since fore and aft forces do not put appreciable energy into the first uncoupled bending mode.

Step III: Important Modes:

Since this example involves the vertical acceleration at the elastic axis, the torsional mode of the wing is expected to have little effect. But if the dynamic torque were to be predicted the torsional mode and the torque caused by the drag load would be considered.

Step IV: Generalized Acceleration Factor for Vibratory Response:

Reference 4 gives the computed frequencies and mode shapes for the first bending mode for full tanks, half full tanks, and empty tanks. GAF (per "g" load at gear) = $\frac{W h_{LG}}{\sum d m_i h_i^2}$ (g units)

Where:

W = static load per wheel, assuming all loads taken by the main gear

h_{LG} = $\frac{\text{vertical deflection of wing at gear}}{\text{vertical deflection of wing tip}}$ (first bending mode)

$d m_i$ = mass of wing element at station "i" (slugs)

h_i = $\frac{\text{vertical deflection of wing at station "i"}}{\text{vertical deflection of wing tip}}$ (first bending mode)

The results per "g" landing load: $GAF = -1.03$ (full)

= -1.06 (half-full)

= -1.15 (empty)

Step V. Vibratory Acceleration Response:

1. Table 1-a computes the vibratory response by the desk-calculator method. The form is designed to make the cumbersome computations as mechanical and well-grouped as possible. It is meant to be self-explanatory after following the theory in Section II.

2. Table 1-b computes the vibratory response by the graphical method.

Step VI. Total Acceleration Time History:

Figure 3 shows the total tip acceleration as the sum of the rigid body and vibratory accelerations. The results for half-full and empty wing tip tanks are also shown.

The measured and computed results agree well for full tanks, but for empty tanks additional modes should probably be included in the computations.

I. THE PROBLEM: $\frac{1}{10}$

GIVEN FORCING FUNCTION $f(t)$

FIND THE RESPONSE x FROM:

$\ddot{x} + \omega_n^2 x = f(t)$

II. BASIC DATA

COMPUTER: *102*

TIME: *102*

1	T ₁	2	T ₂	3	T ₃	4	T ₄	5	T ₅
6	H ₁	7	H ₂	8	H ₃	9	H ₄	10	H ₅
11	0	12	1.133	13	1.675	14	1.0	15	0

C. RADIANS/SEC.

III. INITIALLY COMPUTABLE CONSTANTS

16	T ₁ - 0.01	17	T ₂ - 0.02	18	T ₃ - 0.03	19	T ₄ - 0.04	20	T ₅ - 0.05
21	H ₁ - 0.01	22	H ₂ - 0.02	23	H ₃ - 0.03	24	H ₄ - 0.04	25	H ₅ - 0.05
26	T ₁ - 0.01	27	T ₂ - 0.02	28	T ₃ - 0.03	29	T ₄ - 0.04	30	T ₅ - 0.05
31	H ₁ - 0.01	32	H ₂ - 0.02	33	H ₃ - 0.03	34	H ₄ - 0.04	35	H ₅ - 0.05

IV. IN AB $\ddot{x} = \frac{1}{10} e^{-\frac{1}{2}t} \sin \omega t$

AT B. $\ddot{x}_0 = \frac{1}{10} e^{-\frac{1}{2}t} \sin \omega t$

36	10 X 2.5	37	10 X 10	38	10 X 31
39	2.6993	40	1.007	41	1.0775
42	1.007	43	1.007	44	1.007
45	1.007	46	1.007	47	1.007

V. IN BC $\ddot{x} = \frac{1}{10} e^{-\frac{1}{2}t} \sin(\omega t + \phi)$

AT C. $\ddot{x}_0 = \frac{1}{10} e^{-\frac{1}{2}t} \sin(\omega t + \phi)$

WHERE $\phi = \arctan \frac{\ddot{x}_0}{\dot{x}_0}$ (SEE BELOW)

39	34.16	40	32.43	41	30.70
42	28.49	43	26.76	44	25.03
45	23.35	46	21.62	47	19.89
48	18.70	49	16.97	50	15.24
51	11.70	52	10.07	53	8.44
54	6.10	55	4.47	56	2.84
57	1.79	58	0.59	59	0.19

VI. IN CD $\ddot{x} = \frac{1}{10} e^{-\frac{1}{2}t} \sin(\omega t + \phi)$

AT D. $\ddot{x}_0 = \frac{1}{10} e^{-\frac{1}{2}t} \sin(\omega t + \phi)$

WHERE $\phi = \arctan \frac{\ddot{x}_0}{\dot{x}_0}$ (SEE BELOW)

60	45.10	61	43.47	62	41.84
63	36.50	64	34.87	65	33.24
66	27.90	67	26.27	68	24.64
69	19.30	70	17.67	71	16.04
72	11.70	73	10.07	74	8.44
75	6.10	76	4.47	77	2.84
78	1.79	79	0.59	80	0.19

VII. IN DE $\ddot{x} = \frac{1}{10} e^{-\frac{1}{2}t} \sin(\omega t + \phi)$

AT E. $\ddot{x}_0 = \frac{1}{10} e^{-\frac{1}{2}t} \sin(\omega t + \phi)$

WHERE $\phi = \arctan \frac{\ddot{x}_0}{\dot{x}_0}$ (SEE BELOW)

81	56.10	82	54.47	83	52.84
84	47.50	85	45.87	86	44.24
87	38.90	88	37.27	89	35.64
90	30.30	91	28.67	92	27.04
93	21.70	94	20.07	95	18.44
96	14.10	97	12.47	98	10.84
99	8.90	100	7.27	101	5.64

VIII. IN EF $\ddot{x} = \frac{1}{10} e^{-\frac{1}{2}t} \sin(\omega t + \phi)$

AT F. $\ddot{x}_0 = \frac{1}{10} e^{-\frac{1}{2}t} \sin(\omega t + \phi)$

WHERE $\phi = \arctan \frac{\ddot{x}_0}{\dot{x}_0}$ (SEE BELOW)

102	67.10	103	65.47	104	63.84
105	58.50	106	56.87	107	55.24
108	49.90	109	48.27	110	46.64
111	41.30	112	39.67	113	38.04
114	32.70	115	31.07	116	29.44
117	24.10	118	22.47	119	20.84
120	16.50	121	14.87	122	13.24
123	8.90	124	7.27	125	5.64

IX. INSTRUCTIONS FOR SKETCHING

IN EACH INTERVAL THE RESPONSE IS OF THE FORM $\ddot{x} = K e^{-\frac{1}{2}t} \sin(\omega t + \phi)$. THE VALUES AT DISCONTINUITIES HAVE BEEN FOUND IN IX. FOR SKETCHING, THE ZEROS $(\omega t + \phi) = 0$ AND PEAKS $(\omega t + \phi) = \frac{\pi}{2}$ ARE SUFFICIENT ADDITIONAL POINTS. SOME VALUES OF $\omega t + \phi$ FOR WHICH $\sin(\omega t + \phi) = 0$ ARE IN XI. BUT SINCE $\omega t + \phi$ IN EACH INTERVAL ONLY ARE USED, THESE LIMITS OF $\omega t + \phi$ AND $\omega t + \phi$ CAN BE USED. THESE LIMITS OF $\omega t + \phi$ ARE LISTED IN X. PEAKS AND ZEROS ARE COMPUTED PLOTTING POINTS.

X. LIMITS OF $(\omega t + \phi)$

INTERVAL	AB	BC	CD	DE	EF
FROM	0	0	0	0	0
TO	1.0	1.0	1.0	1.0	1.0

XI. VALUES OF $\sin(\omega t + \phi)$ FOR COMPUTING ZEROS & PEAKS

$\omega t + \phi$	$\sin(\omega t + \phi)$	$\sin(\omega t + \phi)$	$\sin(\omega t + \phi)$
0.0	0.0000	0.0000	0.0000
0.1	0.0998	0.0998	0.0998
0.2	0.1985	0.1985	0.1985
0.3	0.2963	0.2963	0.2963
0.4	0.3932	0.3932	0.3932
0.5	0.4892	0.4892	0.4892
0.6	0.5848	0.5848	0.5848
0.7	0.6796	0.6796	0.6796
0.8	0.7732	0.7732	0.7732
0.9	0.8660	0.8660	0.8660
1.0	0.9589	0.9589	0.9589

XII. ZEROS AND PEAKS BETWEEN A & B

NOTE: $\phi = 0$

IN A-B

POINT	$\omega t + \phi$	$\sin(\omega t + \phi)$	TYPE
a	0.0	0.0000	Zero
b	0.1	0.0998	Peak
c	0.2	0.1985	Zero
d	0.3	0.2963	Peak
e	0.4	0.3932	Zero
f	0.5	0.4892	Peak
g	0.6	0.5848	Zero
h	0.7	0.6796	Peak
i	0.8	0.7732	Zero
j	0.9	0.8660	Peak
k	1.0	0.9589	Zero

XIII. ZEROS & PEAKS BETWEEN B & C

NOTE: $\phi = 0$

IN B-C

POINT	$\omega t + \phi$	$\sin(\omega t + \phi)$	TYPE
a	0.0	0.0000	Zero
b	0.1	0.0998	Peak
c	0.2	0.1985	Zero
d	0.3	0.2963	Peak
e	0.4	0.3932	Zero
f	0.5	0.4892	Peak
g	0.6	0.5848	Zero
h	0.7	0.6796	Peak
i	0.8	0.7732	Zero
j	0.9	0.8660	Peak
k	1.0	0.9589	Zero

XIV. ZEROS & PEAKS BETWEEN C & D

NOTE: $\phi = 0$

IN C-D

POINT	$\omega t + \phi$	$\sin(\omega t + \phi)$	TYPE
a	0.0	0.0000	Zero
b	0.1	0.0998	Peak
c	0.2	0.1985	Zero
d	0.3	0.2963	Peak
e	0.4	0.3932	Zero
f	0.5	0.4892	Peak
g	0.6	0.5848	Zero
h	0.7	0.6796	Peak
i	0.8	0.7732	Zero
j	0.9	0.8660	Peak
k	1.0	0.9589	Zero

XV. ZEROS & PEAKS BETWEEN D & E

NOTE: $\phi = 0$

IN D-E

POINT	$\omega t + \phi$	$\sin(\omega t + \phi)$	TYPE
a	0.0	0.0000	Zero
b	0.1	0.0998	Peak
c	0.2	0.1985	Zero
d	0.3	0.2963	Peak
e	0.4	0.3932	Zero
f	0.5	0.4892	Peak
g	0.6	0.5848	Zero
h	0.7	0.6796	Peak
i	0.8	0.7732	Zero
j	0.9	0.8660	Peak
k	1.0	0.9589	Zero

XVI. ZEROS AND PEAKS BETWEEN E & F

NOTE: $\phi = 0$

IN E-F

POINT	$\omega t + \phi$	$\sin(\omega t + \phi)$	TYPE
a	0.0	0.0000	Zero
b	0.1	0.0998	Peak
c	0.2	0.1985	Zero
d	0.3	0.2963	Peak
e	0.4	0.3932	Zero
f	0.5	0.4892	Peak
g	0.6	0.5848	Zero
h	0.7	0.6796	Peak
i	0.8	0.7732	Zero
j	0.9	0.8660	Peak
k	1.0	0.9589	Zero

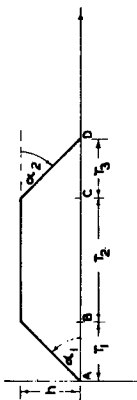
TABLE I-0

TABLE I-b

GRAPHICAL SOLUTION OF VIBRATORY ACCELERATION RESPONSE TO A TRAPEZOIDAL FORCING FUNCTION

I. THE PROBLEM:

GIVEN FORCING FUNCTION $f(t)$.
FIND THE RESPONSE $\ddot{x}$ FROM:
 $\ddot{x} + \omega^2 x = f(t)$

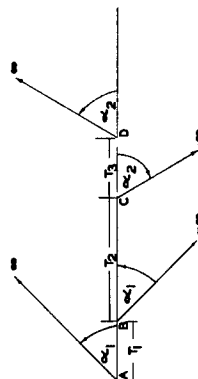


II. DECOMPOSITION:

$$f(t) = f_1(t) + f_2(t) + f_3(t)$$

WHERE

$$\begin{aligned} f_1(t) &= t - T_1 \\ f_2(t) &= t - T_2 \\ f_3(t) &= t - T_3 \end{aligned}$$



III. APPLYING SUPERPOSITION PRINCIPLE:

$$\begin{aligned} \ddot{x} &= \ddot{x}_1 + \ddot{x}_2 + \ddot{x}_3 + \ddot{x}_4 \\ \ddot{x}_1 &= \omega^2 (1 + \frac{1}{\omega^2}) x_1 = f_1(t) \\ \ddot{x}_2 &= \omega^2 (1 + \frac{1}{\omega^2}) x_2 = f_2(t) \\ \ddot{x}_3 &= \omega^2 (1 + \frac{1}{\omega^2}) x_3 = f_3(t) \\ \ddot{x}_4 &= \omega^2 (1 + \frac{1}{\omega^2}) x_4 = f_4(t) \end{aligned}$$

WHERE:

$$\begin{aligned} t &\geq 0 \\ t &\geq 0 \\ t &\geq 0 \\ t &\geq 0 \end{aligned}$$

IV. SOLUTIONS TO III:

$$\begin{aligned} \ddot{x}_1 &= \frac{h}{\omega^2} e^{-\frac{1}{2}\omega t} \sin \omega t & (t \geq 0) \\ \ddot{x}_2 &= \frac{h}{\omega^2} e^{-\frac{1}{2}\omega(t-T_1)} \sin \omega(t-T_1) & (t \geq T_1) \\ \ddot{x}_3 &= \frac{h}{\omega^2} e^{-\frac{1}{2}\omega(t-T_2)} \sin \omega(t-T_2) & (t \geq T_2) \\ \ddot{x}_4 &= \frac{h}{\omega^2} e^{-\frac{1}{2}\omega(t-T_3)} \sin \omega(t-T_3) & (t \geq T_3) \end{aligned}$$

V. BASIC DATA & BASIC COMPUTATIONS:

TITLE: Full Tanks - F80A - Vib. Acc. adding Tips
COMPUTER: A K P

1. T_1	2. T_2	3. T_3	4. $\frac{h}{\omega^2}$	5. $\frac{\omega}{\text{RAD/SEC}}$
.025	.151	.215	.05	16.775
6. h	7. $\omega T_1 = 5 \times 11.8$	8. $\omega T_2 = 5 \times 3.9$	9. $\frac{h}{\omega^2} = \frac{1}{10}$	10. $\frac{h}{\omega^2} = 9\%$
-1.133	1.188	3.6013	-2.7057	-2.146

VI. COMPUTATIONS OF IV AT PEAKS:

	A.	B.	C.	D.	E.	F.	G.
11. ωT^n	0	1.5708	4.7124	7.8540	10.9956	14.1372	17.2788
12. $\sin \omega t$	0	+	-1	+	-1	+	-1
13. $\frac{h}{\omega^2} e^{-\frac{1}{2}\omega t}$	—	.0785	.2356	.3921	.5198	—	—
14. $e^{-\frac{1}{2}\omega(t-T_1)}$	—	—	.9445	.7901	.6592	.5771	—
15. $\frac{h}{\omega^2} e^{-\frac{1}{2}\omega(t-T_2)}$	—	—	—	.8504	.71378	.5816	—
16. $\frac{h}{\omega^2} e^{-\frac{1}{2}\omega(t-T_3)}$	—	—	—	—	.2997	.2486	.2124
17. $t = 1/5$	0	.0938	.2819	.4689	.6565	—	—
18. $t + T_1 = 17 + 1$	—	.025	.1188	.3063	.4939	.6815	—
19. $t + T_2 = 18 + 2$	—	—	.176	.2678	.4573	.6449	—
20. $t + T_3 = 19 + 3$	—	—	—	.391	.5848	.6823	—

$T = t, t', \text{ or } t''$ DEPENDING ON INTERVAL CONSIDERED

VII. INSTRUCTIONS FOR SKETCHING:

IN VIII PLOT AND CONNECT BY EYE AS DAMPED SINE WAVES.

ORDINATE

ABSCISSA

- A. $\ddot{x}_1 =$ 15 17
- B. $\ddot{x}_2 =$ -15 18
- C. $\ddot{x}_3 =$ -16 19
- D. $\ddot{x}_4 =$ 16 20

NOW ADD ALL ORDINATES TO GET:

$$\ddot{x} = \ddot{x}_1 + \ddot{x}_2 + \ddot{x}_3 + \ddot{x}_4$$

VIII. $\ddot{x} = \ddot{x}_1 + \ddot{x}_2 + \ddot{x}_3 + \ddot{x}_4$

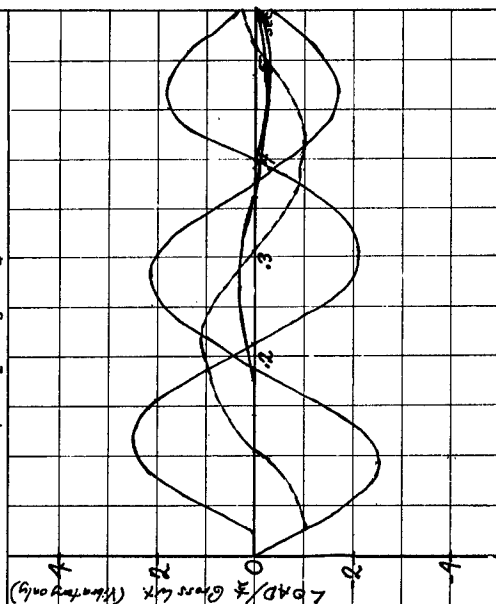
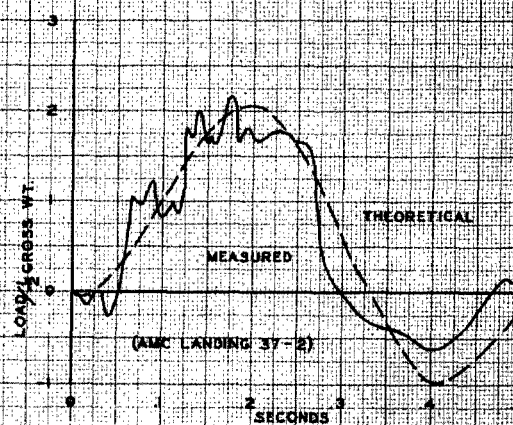
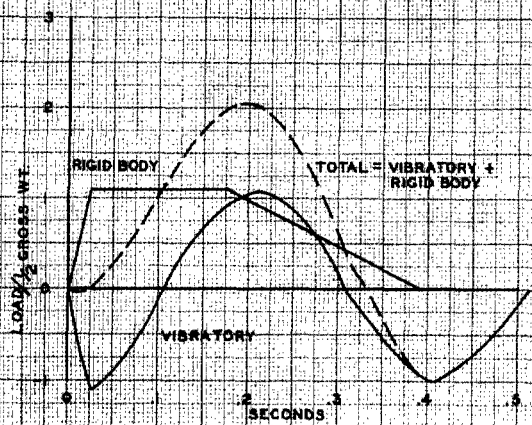


FIGURE 3
F-80A INCREMENTAL WING TIP ACCELERATION IN LANDING
VS TIME

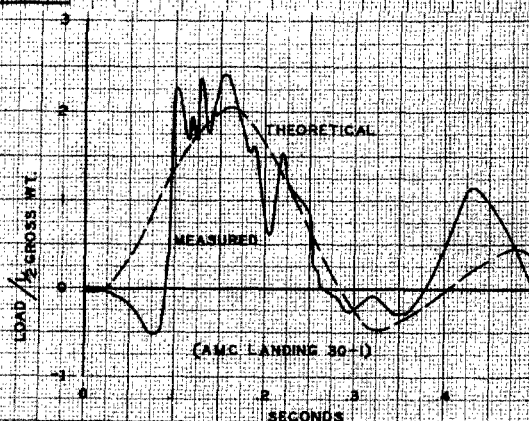
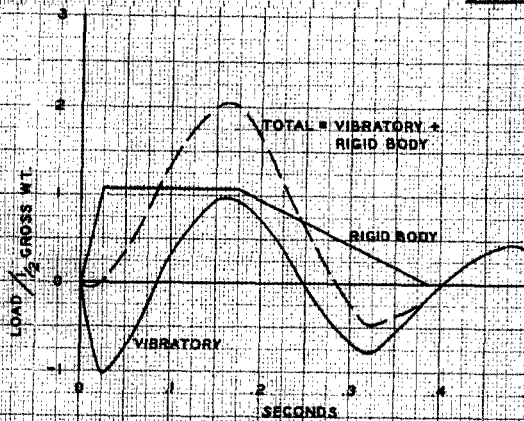
COMPOSITION OF THEORETICAL

COMPARISON OF THEORETICAL
AND MEASURED

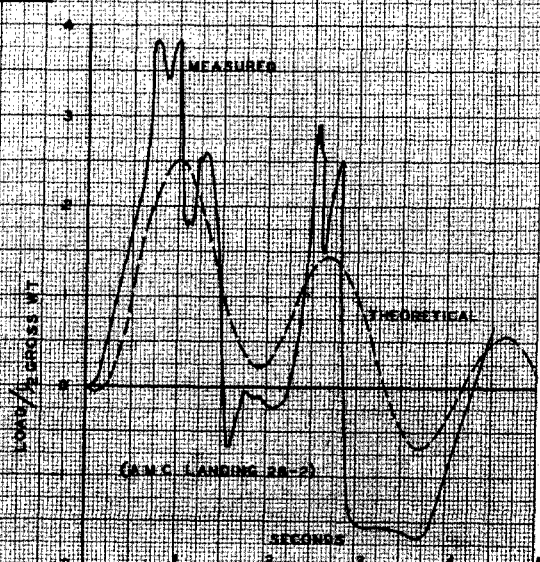
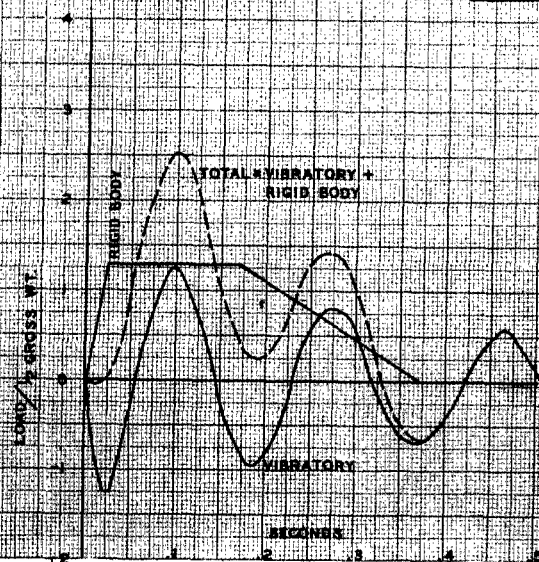
FULL TANKS



$\frac{1}{2}$ FULL TANKS



EMPTY TANKS



Step VII. Variation of Basic Parameters:

1. In this example many parameters were involved - about twelve either previously measured or assumed and the others computed. Will it be possible to predict (without completely new computations) the effect on the final results caused by changing any of these parameters?

2. To facilitate this discussion, a flow chart (Table 1-c) of the computations is shown. Each parameter is oriented vertically by its order of convenient computation and horizontally according to the number of previous parameters which are used to compute this one. Lines emanating upward from any block lead to all of the other parameters which effect this one. The chart gives little if any quantitative help in particular cases.

3. But the following general conclusions seem worthwhile:

a. Change in a basic parameter has an intricate effect, and in many cases the best procedure is to recompute from scratch.

b. There are a few parameters (ω , $\bar{g}$, GAF) which effect only the vibratory response. The effect of altering any of these is rather easily predicted.

c. The problem is conveniently considered in three stages (separated by broken horizontal lines in Table 1-c).

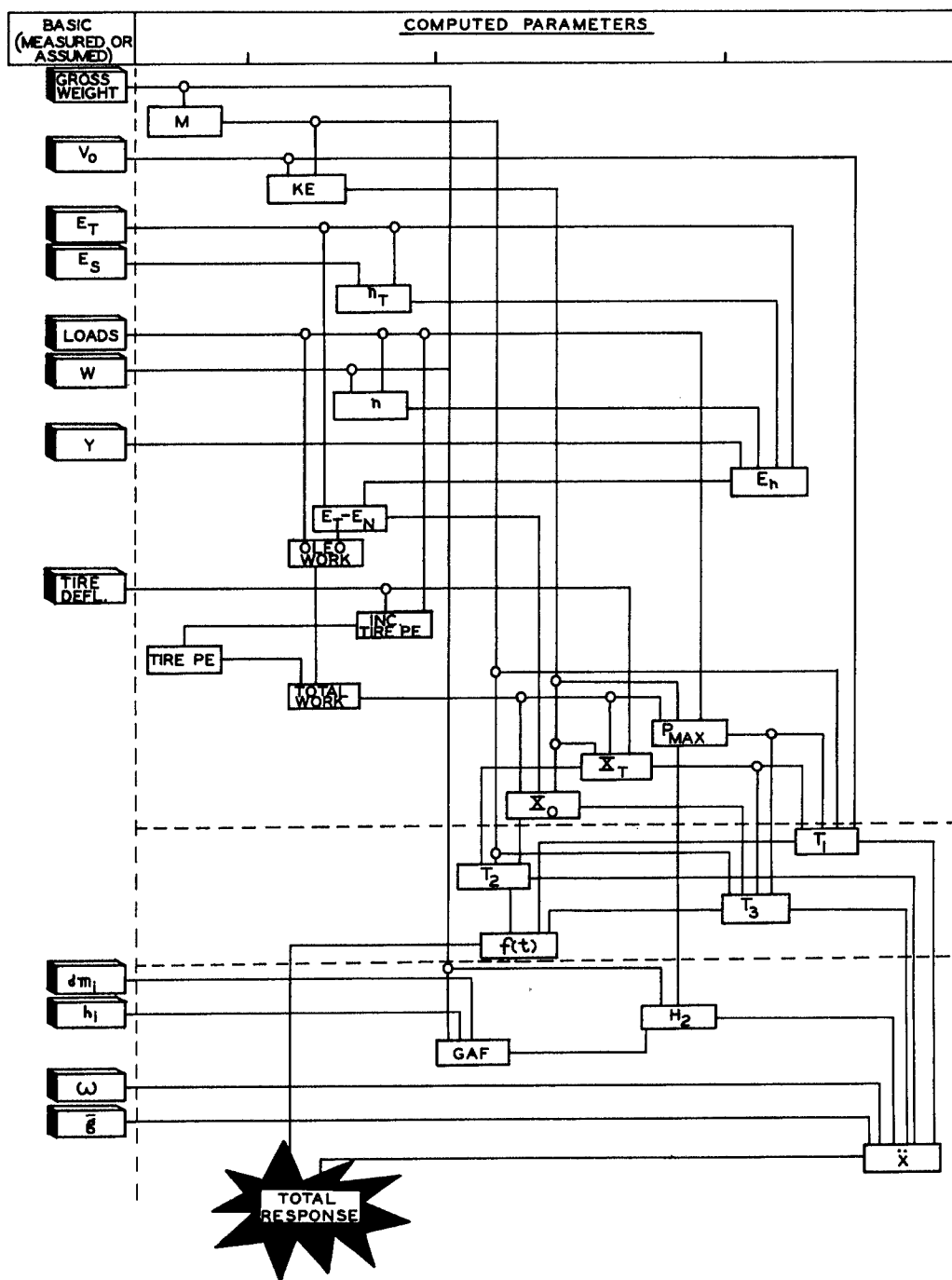
(1) Equating energies.

(2) The rigid-body trapezoid.

(3) Vibratory response and summation.

4. If many changes were contemplated in the first section, it may be profitable to solve the second and third sections only once in general terms, then plug in the particular numbers computed from Part I. See reference 10 for some computations on altering basic parameters.

TABLE 1-C: PARAMETER FLOW - CHART (EXAMPLE 1)



SECTION IV

TAIL BOOM INCREMENTAL ACCELERATIONS OF AN F-61 AIRPLANE

This problem uses the parameters of landing eighteen of the AMC tests reported in reference 5. This particular landing is selected because the maximum rigid body vertical load is closest to the computed value.

Step 1. Vertical Load Time History:

1. Basic airplane data:

Gross weight = 25,000 pounds (app.)

M = Mass per main gear = 388 slugs (2 wheel landing)

V_0 = Rate of descent = 8 ft/sec (assumed)

W = Kinetic energy per gear = 12,400 ft-lbs

2. Basic oleo data:

E_T = Total extension * = 10 in. = .833 ft

E_S = Static extension * = 2.87 in. = .239 ft.

Assuming isothermal expansion from static to fully extended position, the "load factor" at total extension =

$$n_T = \frac{E_S}{E_T} = 0.287 \quad (79)$$

Then assuming quasi-adiabatic compression from the fully extended position during impact, the extension at any load factor "n" during impact is:

$$E_n = \frac{E_T}{\left(\frac{n}{n_T}\right)^{1/\gamma}} = E_T \left(\frac{n_T}{n}\right)^{1/\gamma} \quad (80)$$

where n = load/12500 lbs and γ = 1.3.

* Assuming latent air column = 0

3. Basic tire data:

(1) Load (Lbs)	(2) Tire Deflection (Ft)	(3) Incremental Tire Work * (Ft-lbs)	(4) Tire Work = Σ (3) (Ft-Lbs)
6000	.125	375	375
13000	.250	1188	1563
22000	.333	1453	3016

4. Total work:

(5) $n = \frac{(1)}{12500}$	(6) $\frac{n \cdot 0.287}{\pi \cdot (5)}$	(7) $(6) \cdot 1.3$	(8) $1 - (7)$	(9) $E_T - E_n$.833 x (8)	(10) OLEO WK. (1) x (9)	(11) TOTAL WORK (4) + (10)
.48	.5979	.6733	.3267	.272	1644	2019
1.04	.2760	.3715	.6285	.523	6199	8362
1.76	.1631	.2478	.7522	.627	13794	16810

5. Tire deflection, oleo deflection and load when kinetic energy per gear = total work:

Total Work (Ft-Lbs)	Load (Lbs)	Tire Deflection (Ft)	Oleo Deflection (Ft)
8362	13000	.250	.523
KE = 12400	P _{MAX} = 17302	X _T = .290	X _O = .573
16810	22000	.333	.627

6. The times for tire compression T_o , oleo compression T_T and tire-oleo expansion T_{OT} are determined from the formulas of reference 3.

$$T_T = \frac{3MV_o}{P_{MAX}} - \frac{1}{2} \sqrt{\left(\frac{6MV_o}{P_{MAX}}\right)^2 - \frac{24MX_T}{P_{MAX}}} = 0.0376 \text{ sec} \quad (81)$$

$$T_o = \sqrt{\frac{2MX_o}{P_{MAX}}} = 0.1603 \text{ sec.} \quad (82)$$

$$T_{OT} = \sqrt{\frac{3M(X_o + X_T)}{P_{MAX}}} = 0.2409 \text{ sec.} \quad (83)$$

* (3) = (Average load during increment) (deflection in increment)
Approximating the area under the tire curve by trapezoids

Step II. Drag Load Time History

1. Basic parameters:

I_A = moment of inertia of each landing gear rolling assembly = 12.3 slug ft²

R_R = rolling radius of wheel = 1.65 ft.

V_L = landing speed = 147 ft/sec.

Tire coefficient of friction = 0.55 (assumed)

2. Computation:

$$\theta = \text{angular velocity of wheel after spin up} = \frac{V_L}{R_R} = \frac{147}{1.05} \text{ rad/sec} = 89.1 \text{ rad/sec} \quad (84)$$

$$\theta_T = \text{angular velocity after tire compression} = \frac{0.55 P_{\max} R_R T_T}{2 I_A} = 24.0 \text{ rad/sec.} \quad (85)$$

Assuming (empirically) that peak drag load = .55 $P_{\max}$

$$\theta_0 = \text{angular velocity required for spin up during oleo compression} = \theta - \theta_T = 65.1 \text{ rad/sec} \quad (86)$$

$$T_S = \text{duration of skid during oleo compression} = \frac{\theta_0 I_A}{0.55 P_{\max} R_R} = .0510 \text{ sec.} \quad (87)$$

$$T_T + T_S = \text{total spin up time} = .0886 \text{ sec.}$$

$$T_Z = \text{time to drop to zero drag load} = \frac{T_T + T_S}{4} = .0221 \text{ sec. (Empirical formula)} \quad (88)$$

Step III. Important Modes

1. The vibratory acceleration of the tail boom is caused partially by the vertical load and partially by the drag load. It is assumed here that the vertical load acts directly to set up a vertical vibration in the tail at the natural frequency of the tail.

2. In order to determine the important vibratory modes set up by the drag load, the airplane is assumed to have the configuration shown in figures 4 and 5 with the three degrees of freedom illustrated.

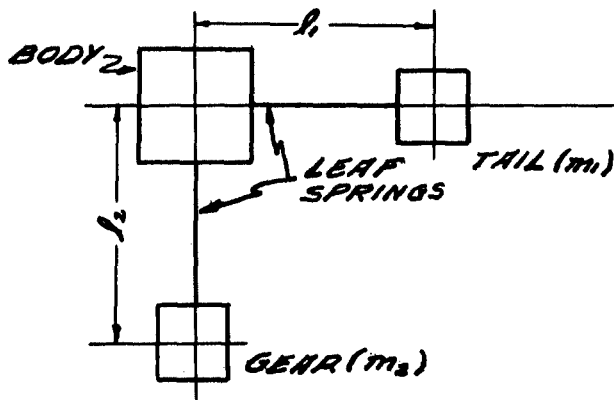


Fig. 4. Geometric Representation

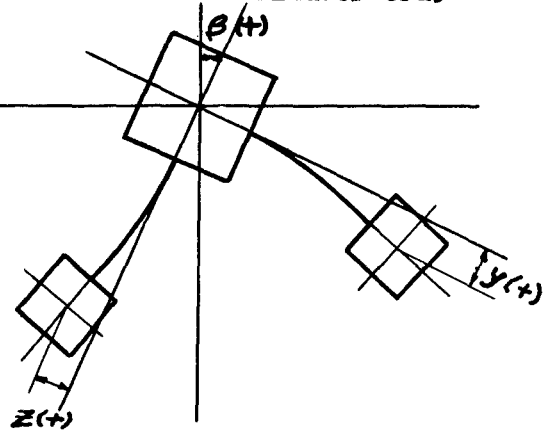


Fig. 5. Positive Direction of Generalized Coordinates

Then (Lagrange's equation for zero external torque)

$$\frac{d}{dt} \left(\frac{\partial K}{\partial \dot{g}_i} \right) + \frac{\partial P}{\partial g_i} = 0 \quad (89)$$

$$g_i = \beta, \gamma, \text{ or } \zeta$$

With:

$$K = \frac{1}{2} I \dot{\beta}^2 + \frac{1}{2} m_1 (l_1 \dot{\beta} + \dot{\gamma})^2 + \frac{1}{2} m_2 (l_2 \dot{\beta} + \dot{\zeta})^2 \quad (90)$$

$$P = \frac{1}{2} m_1 \omega_1^2 \gamma^2 + \frac{1}{2} m_2 \omega_2^2 \zeta^2$$

for which I is the moment of inertia of the body and ω_1 and ω_2 are natural frequencies of tail and gear respectively.

Directly from (89) and (90)

$$\begin{aligned} (I + m_1 l_1^2 + m_2 l_2^2) \ddot{\beta} + m_1 l_1 \ddot{\gamma} + m_2 l_2 \ddot{\zeta} &= 0 \\ m_1 \ddot{\gamma} + m_1 \omega_1^2 \gamma + m_1 l_1 \ddot{\beta} &= 0 \\ m_2 \ddot{\zeta} + m_2 \omega_2^2 \zeta + m_2 l_2 \ddot{\beta} &= 0 \end{aligned} \quad (91)$$

These equations are unwieldy because they contain both the parameters β, γ, ζ and their second derivatives $\ddot{\beta}, \ddot{\gamma}, \ddot{\zeta}$.

But the spring action is assumed simple harmonic motion. Then:

$$\beta = a_1 e^{i\omega(t+b_1)} \quad y = a_2 e^{i\omega(t+b_2)} \quad z = a_3 e^{i\omega(t+b_3)} \quad (92)$$

where a_i and b_i are constants and ω is the response frequency of the system. The object is to determine the value of ω for each of the possible modes of action.

Directly from (92):

$$\ddot{\beta} = -\omega^2 \beta \quad \ddot{y} = -\omega^2 y \quad \ddot{z} = -\omega^2 z \quad (93)$$

Rewriting (91) by substituting (93):

$$\begin{aligned} I_p \beta + m_1 l_1 y + m_2 l_2 z &= 0 \\ l_1 \beta + \left[1 - \left(\frac{\omega_1}{\omega}\right)^2\right] y &= 0 \\ l_2 \beta + \left[1 - \left(\frac{\omega_2}{\omega}\right)^2\right] z &= 0 \end{aligned} \quad (94)$$

This system of three equations in the coordinates β, y, z has all constant terms zero. Therefore it has only the trivial solution $\beta = y = z = 0$ unless the determinant of the coefficients of β, y, z is zero. That is:

$$\begin{vmatrix} I_p & m_1 l_1 & m_2 l_2 \\ l_1 & \left[1 - \left(\frac{\omega_1}{\omega}\right)^2\right] & 0 \\ l_2 & 0 & \left[1 - \left(\frac{\omega_2}{\omega}\right)^2\right] \end{vmatrix} = 0 \quad (95)$$

The only parameter unknown in (95) is ω . For the F-61 airplane, the other parameters have the values (for the complete airplane)

$$I_p = \frac{1.11 \times 10^6}{g} \text{ ft}^2 \text{ slugs}$$

$$m_1 = \frac{650}{g} \text{ slugs} \quad m_2 = \frac{700}{g} \text{ slugs}$$

$$\omega_1 = \omega_2 = 47.1 \text{ rad/sec}$$

$$l_1 = 24 \text{ ft} \quad l_2 = 7.7 \text{ ft}$$

Expanding (95) and solving for ω gives two solutions, and corresponding values of the ratios $\frac{\beta}{y}$, $\frac{z}{y}$. These are:

1st Coupled Mode

$$\omega = \omega_1 = 47.1 \text{ rad/sec.}$$

$$\beta/y = 0$$

$$z/y = -\frac{m_1 l_1}{m_2 l_2} = -2.895$$

2nd Coupled Mode

$$\omega = \omega_2 = \sqrt{\frac{I_p}{I_p - m_1 l_1^2 - m_2 l_2^2}} = 59.56 \text{ rad/sec} \quad (96)$$

$$\beta/y = \left[\left(\frac{\omega_1}{\omega}\right)^2 - 1 \right] / l_1 = -0.03993 \text{ rad/ft.}$$

$$z/y = \frac{l_1}{l_2} = 3.117$$

Step IV. Generalized Acceleration Factors.

1. For the vertical load:

$$GAF = \frac{(17302) \left(-\frac{650}{24350}\right)}{\frac{650(1)^2}{29} + \frac{24350}{29} \left(-\frac{650}{24350}\right)^2} = -1.3842 \quad (97-a)$$

per foot of tail deflection.

2. For the drag load:

- a. In the first mode:

$$GAF = \frac{(0.55)(17302)(-2.898)}{\frac{650(1)^2}{29} + \frac{700}{29}(-2.898)^2} = -8.4449 \quad (97-b)$$

per foot of tail deflection.

- b. In the second mode:

$$\text{Tail deflection} = 1 \text{ ft} = y + l_1 \beta$$

$$\text{Thus gear deflection} = z + l_2 \beta = .3208 \text{ ft.}$$

$$\text{And } \beta = -.02497 \text{ rad.}$$

So that:

$$GAF = \frac{(0.55)(17302)(.3208)}{\left(\frac{6940.97}{29}\right)(.02497)^2 + \frac{650(1)^2}{29} + \frac{700(.3208)^2}{29}} = 5.2879 \quad (97-c)$$

per foot of tail deflection

Step V. Vibratory Acceleration Response.

1. Since the mode for the vertical load has the same frequency as the first mode for drag load, the two rigid body forcing functions can be combined into one trapezoidal forcing function as shown in Figure 6.

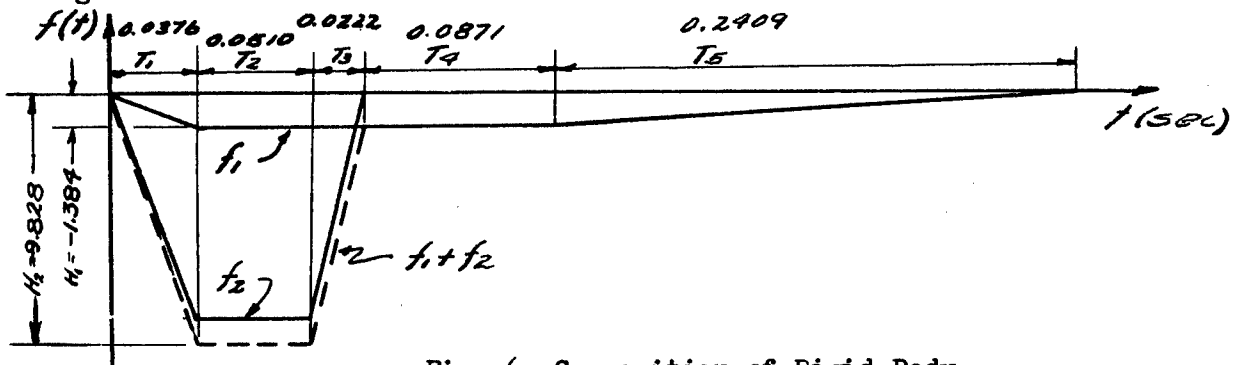


Fig. 6 Composition of Rigid Body Forcing Functions

The response to this function is computed in table 2-a.

2. The response in the second mode due to drag load is computed in table 2-b.

Step VI. Total Acceleration Time History.

1. The total acceleration is the sum of:

a. Rigid body response to vertical load. This and all other components of the final response are nondimensionalized in units of 1/2 airplane weight. Thus the maximum value of the rigid body response will be scaled as $\frac{17302}{12500} = 1.384$

b. Rigid body response to drag load. The maximum value is determined from the definition of torque:

$$\text{Torque} = (.55) (17302) (7.7) \text{ ft-lbs}$$

$$\text{But: Torque} = I \omega = \frac{(1.11 \times 10^6)}{2} [\text{Max. vertical acceleration}/24] \text{ ft lbs}$$

So:

$$\text{Max vertical acceleration} = \frac{(.55)(17302)(7.7)(24)}{(1.11)(10^6)/2} \text{ g} = 3.169 \text{ g} \quad (98)$$

c. Vibratory responses to vertical and drag loads.

2. All the above components are summed and compared with experimental results in figure 7.

I. THE PROBLEM

GIVEN FORCING FUNCTION $f(t)$
FIND THE RESPONSE x FROM:
 $\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = f(t)$

II. BASIC DATA

TITLE: $\frac{1}{2} A \cos F \cos T_0 / B \cos (2nd Mode)$
COMPUTER: $q - 2$

1	T ₁	2	T ₂	3	T ₃	4	T ₄	5	T ₅
1	0.376	0.510	0.222						
6	0	0	0	0	0	0	0	0	0

III. INITIALLY COMPUTABLE CONSTANTS

10	W ₁ EX	11	W ₂ EX	12	W ₃ EX	13	W ₄ EX	14	W ₅ EX
1	1.2395	2	0.376	3	0.510	4	0.222	5	0.000
6	0.000	7	0.000	8	0.000	9	0.000	10	0.000

IV. IN AB

$x = \frac{H_1}{\omega_1} e^{-\frac{c}{2m} \omega_1 t} \sin \omega_1 t$

16	X ₁	17	X ₂	18	X ₃	19	X ₄	20	X ₅
1	1.106	2	0.786	3	0.510	4	0.222	5	0.000
6	0.000	7	0.000	8	0.000	9	0.000	10	0.000

IV. IN AB

$\ddot{x} = \frac{H_2}{m} e^{-\frac{c}{2m} \omega_2 t} \sin \omega_2 t$

26	X ₁	27	X ₂	28	X ₃	29	X ₄	30	X ₅
1	1.106	2	0.786	3	0.510	4	0.222	5	0.000
6	0.000	7	0.000	8	0.000	9	0.000	10	0.000

V. IN BC

$x = \frac{H_1}{\omega_1} e^{-\frac{c}{2m} \omega_1 t} \sin(\omega_1 t + \phi_1)$

31	X ₁	32	X ₂	33	X ₃	34	X ₄	35	X ₅
1	1.106	2	0.786	3	0.510	4	0.222	5	0.000
6	0.000	7	0.000	8	0.000	9	0.000	10	0.000

V. IN BC

$\ddot{x} = \frac{H_2}{m} e^{-\frac{c}{2m} \omega_2 t} \sin(\omega_2 t + \phi_2)$

41	X ₁	42	X ₂	43	X ₃	44	X ₄	45	X ₅
1	1.106	2	0.786	3	0.510	4	0.222	5	0.000
6	0.000	7	0.000	8	0.000	9	0.000	10	0.000

VI. IN CD

$x = \frac{H_1}{\omega_1} e^{-\frac{c}{2m} \omega_1 t} \sin(\omega_1 t + \phi_1)$

51	X ₁	52	X ₂	53	X ₃	54	X ₄	55	X ₅
1	1.106	2	0.786	3	0.510	4	0.222	5	0.000
6	0.000	7	0.000	8	0.000	9	0.000	10	0.000

VI. IN CD

$\ddot{x} = \frac{H_2}{m} e^{-\frac{c}{2m} \omega_2 t} \sin(\omega_2 t + \phi_2)$

61	X ₁	62	X ₂	63	X ₃	64	X ₄	65	X ₅
1	1.106	2	0.786	3	0.510	4	0.222	5	0.000
6	0.000	7	0.000	8	0.000	9	0.000	10	0.000

VII. IN DE

$x = \frac{H_1}{\omega_1} e^{-\frac{c}{2m} \omega_1 t} \sin(\omega_1 t + \phi_1)$

71	X ₁	72	X ₂	73	X ₃	74	X ₄	75	X ₅
1	1.106	2	0.786	3	0.510	4	0.222	5	0.000
6	0.000	7	0.000	8	0.000	9	0.000	10	0.000

VII. IN DE

$\ddot{x} = \frac{H_2}{m} e^{-\frac{c}{2m} \omega_2 t} \sin(\omega_2 t + \phi_2)$

81	X ₁	82	X ₂	83	X ₃	84	X ₄	85	X ₅
1	1.106	2	0.786	3	0.510	4	0.222	5	0.000
6	0.000	7	0.000	8	0.000	9	0.000	10	0.000

VIII. IN EF

$x = \frac{H_1}{\omega_1} e^{-\frac{c}{2m} \omega_1 t} \sin(\omega_1 t + \phi_1)$

91	X ₁	92	X ₂	93	X ₃	94	X ₄	95	X ₅
1	1.106	2	0.786	3	0.510	4	0.222	5	0.000
6	0.000	7	0.000	8	0.000	9	0.000	10	0.000

IX. INSTRUCTIONS FOR SKETCHING

IN EACH INTERVAL THE RESPONSE IS OF THE FORM $x = \frac{H_1}{\omega_1} e^{-\frac{c}{2m} \omega_1 t} \sin(\omega_1 t + \phi_1)$

AND PEAKS $\{e^{-\frac{c}{2m} \omega_1 t} \sin(\omega_1 t + \phi_1)\}$ ARE SUFFICIENT ADDITIONAL POINTS. SOME VALUES OF $\omega_1 t + \phi_1$ FOR WHICH $\sin(\omega_1 t + \phi_1) = 0$ ARE IN XI. BUT SINCE ϕ_1 IS IN EACH INTERVAL ONLY THE VALUES OF $\omega_1 t + \phi_1$ BETWEEN ϕ_1 AND ϕ_2 CAN BE USED. THESE LIMITS OF $\omega_1 t + \phi_1$ ARE LISTED IN X. PARTS XII-XXI THEN COMPUTE PLOTTING POINTS AT PEAKS AND ZEROS.

X. LIMITS OF $(\omega_1 t + \phi_1)$

INTERVAL	AB	BC	CD	DE	EF
FROM	0	0	0	0	0
TO	0.376	0.510	0.222	0.000	0.000

XI. VALUES OF $\sin(\omega_1 t + \phi_1)$ FOR COMPUTING ZEROS & PEAKS

$\omega_1 t + \phi_1$	$\sin(\omega_1 t + \phi_1)$	$\omega_1 t + \phi_1$	$\sin(\omega_1 t + \phi_1)$
0.376	0.376	0.510	0.510
0.510	0.510	0.222	0.222
0.222	0.222	0.000	0.000

XII. ZEROS & PEAKS BETWEEN D & E

1	0.376	2	0.510
---	-------	---	-------

3	0.222	4	0.000
5	0.000	6	0.000
7	0.000	8	0.000

XIII. ZEROS & PEAKS BETWEEN B & C

9	0.376
---	-------

11	0.510	12	0.222
13	0.000	14	0.000
15	0.000	16	0.000

XIV. ZEROS & PEAKS BETWEEN C & D

17	0.376
----	-------

19	0.510	20	0.222
21	0.000	22	0.000
23	0.000	24	0.000

XV. ZEROS & PEAKS BETWEEN E & F

25	0.376
----	-------

27	0.510	28	0.222
29	0.000	30	0.000
31	0.000	32	0.000

XVI. ZEROS & PEAKS BETWEEN A & B

33	0.376
----	-------

35	0.510	36	0.222
37	0.000	38	0.000
39	0.000	40	0.000

XVII. ZEROS & PEAKS BETWEEN D & E

41	0.376
----	-------

43	0.510	44	0.222
45	0.000	46	0.000
47	0.000	48	0.000

XVIII. ZEROS & PEAKS BETWEEN B & C

49	0.376
----	-------

51	0.510	52	0.222
53	0.000	54	0.000
55	0.000	56	0.000

XIX. ZEROS & PEAKS BETWEEN C & D

57	0.376
----	-------

59	0.510	60	0.222
61	0.000	62	0.000
63	0.000	64	0.000

XX. ZEROS & PEAKS BETWEEN E & F

65	0.376
----	-------

67	0.510	68	0.222
69	0.000	70	0.000
71	0.000	72	0.000

XXI. ZEROS & PEAKS BETWEEN A & B

73	0.376
----	-------

75	0.510	76	0.222
77	0.000	78	0.000
79	0.000	80	0.000

XXII. ZEROS & PEAKS BETWEEN D & E

81	0.376
----	-------

83	0.510	84	0.222
85	0.000	86	0.000
87	0.000	88	0.000

XXIII. ZEROS & PEAKS BETWEEN B & C

89	0.376
----	-------

91	0.510	92	0.222
93	0.000	94	0.000
95	0.000	96	0.000

XXIV. ZEROS & PEAKS BETWEEN C & D

97	0.376
----	-------

99	0.510	100	0.222
101	0.000	102	0.000
103	0.000	104	0.000

XXV. ZEROS & PEAKS BETWEEN E & F

105	0.376
-----	-------

107	0.510	108	0.222
109	0.000	110	0.000
111	0.000	112	0.000

XXVI. ZEROS & PEAKS BETWEEN A & B

113	0.376
-----	-------

115	0.510	116	0.222
117	0.000	118	0.000
119	0.000	120	0.000

XXVII. ZEROS & PEAKS BETWEEN D & E

121	0.376
-----	-------

123	0.510	124	0.222
125	0.000	126	0.000
127	0.000	128	0.000

XXVIII. ZEROS & PEAKS BETWEEN B & C

129	0.376
-----	-------

131	0.510	132	0.222
133	0.000	134	0.000
135	0.000	136	0.000

XXIX. ZEROS & PEAKS BETWEEN C & D

137	0.376
-----	-------

139	0.510	140	0.222
141	0.000	142	0.000
143	0.000	144	0.000

XXX. ZEROS & PEAKS BETWEEN E & F

145	0.376
-----	-------

147	0.510	148	0.222
149	0.000	150	0.000
151	0.000	152	0.000

XXXI. ZEROS & PEAKS BETWEEN A & B

153	0.376
-----	-------

155	0.510	156	0.222
157	0.000	158	0.000
159	0.000	160	0.000

XXXII. ZEROS & PEAKS BETWEEN D & E

161	0.376
-----	-------

163	0.510	164	0.222
165	0.000	166	0.000
167	0.000	168	0.000

XXXIII. ZEROS & PEAKS BETWEEN B & C

169	0.376
-----	-------

171	0.510	172	0.222
173	0.000	174	0.000
175	0.000	176	0.000

XXXIV. ZEROS & PEAKS BETWEEN C & D

177	0.376
-----	-------

179	0.510	180	0.222
181	0.000	182	0.000
183	0.000	184	0.000

XXXV. ZEROS & PEAKS BETWEEN E & F

185	0.376
-----	-------

||
||
||

TABLE 2-b

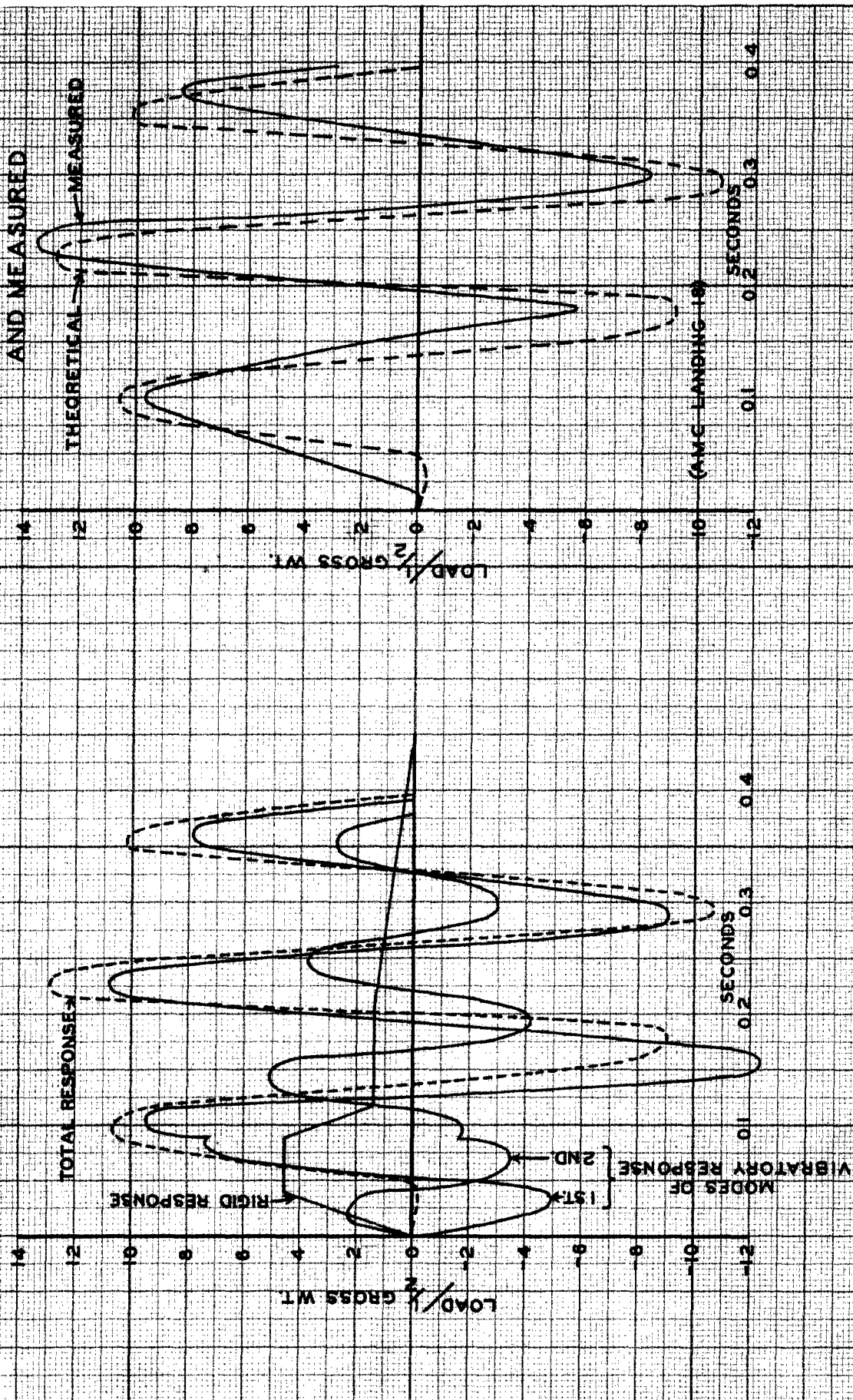
FIGURE 7

F-61 INCREMENTAL TAIL BOOM ACCELERATION DURING LANDING

VS TIME

COMPOSITION OF THEORETICAL

COMPARISON OF THEORETICAL
AND MEASURED



SECTION V

B-17G LANDING GEAR DRAG LOAD (NORMAL TO STRUT)

Parameters are taken from landing number 1 of flight 3 of the B-17G landing load tests of AMC, reported in reference 6.

This example is a particularly good criteria of the accuracy of the predictions based on the trapezoidal theory, because V_0 was actually measured.

Step I. Vertical Load Time History:

1. Basic airplane data:

Gross weight = 48,137 pounds

M = Mass per main gear = 750 slugs (two wheel landing)

W = Static load per gear = 21,775 lbs (3 point position)

V_0 = Rate of descent = 7 ft/sec (measured)

KE = Kinetic energy per gear = 18,380 ft-lbs

ω = Natural frequency of gear fore and aft = 73.2 rad/sec
(measured from landing records)

2. Basic oleo data:

E_T = Total extension + latent air column = (9.6 + 1.56) in. =
.930 ft.

E_S = Static extension + latent air column = (1.44 + 1.56) in. =
.250 ft.

Assuming isothermal expansion from static to fully loaded position, the "load factor" at total extension =

$$n_T = \frac{E_S}{E_T} = 0.2688 \quad (99)$$

Then assuming quasi-adiabatic compression from the fully extended position during impact, the extension at any load factor "n" during impact is:

$$E_n = \frac{E_T}{\left(\frac{n}{n_T}\right)^{1/4}} = E_T \left(\frac{n_T}{n}\right)^{1/4} \quad (100)$$

where η = load/21,775 lbs and γ = 1.3 (assumed).

3. Basic tire data:

① Load (Lbs)	② Tire Deflection (Ft)	③ Incremental Tire Work * (Ft-lbs)	④ Tire Work = $\sum$ ③ (Ft-Lbs)
10000	.160	800	800
15000	.223	792	1592
20000	.283	1050	2642
25000	.337	1219	3861

4. Total Work:

⑤ $\eta = \frac{①}{21775}$	⑥ $\frac{\eta_r}{\eta} = \frac{0.2628}{⑤} \left(\frac{\eta_r}{\eta} \right)^{1/4} = ⑥^{1/4}$	⑦ $⑥^{1/4} \cdot ⑦$	⑧ $E_T - E_r$ 0.930 · ⑧	⑨ OLEO WK. ① · ⑨	⑩ TOTAL WORK ④ + ⑨
.4592	.5854	.8339	.1545	1545	2345
.6889	.3902	.4849	.4790	7185	8777
.9185	.2927	.3886	.5686	11372	14014
1.1481	.2341	.3273	.6256	15640	19501

5. Tire deflection, oleo deflection and load when kinetic energy per gear = total work:

Total Work (Ft-Lbs)	Load (Lbs)	Tire Deflection (Ft)	Oleo Deflection (Ft)
14014	20000	.283	.569
KE = 18380	P _{MAX} = 23978	X _T = .326	X _O = .614
19501	25000	.337	.626

6. The times for tire compression T_r oleo compression T_o and tire-oleo expansion T_{or} are determined from the formulas of reference 3.

$$T_r = \frac{3MV_0}{P_{MAX}} - \frac{1}{2} \sqrt{\left(\frac{6MV_0}{P_{MAX}} \right)^2 - \frac{24MX_T}{P_{MAX}}} = .0484 \text{ sec (101)}$$

* ③ = (average load during increment) (deflection in increment)
(Approximating the area under the tire curve by trapezoids)

$$T_0 = \sqrt{\frac{2MX_0}{P_{MAX}}} = 0.1968 \text{ sec.} \quad (102)$$

$$T_{0T} = \sqrt{\frac{3M(X_0 + X_T)}{P_{MAX}}} = 0.2970 \text{ sec.} \quad (103)$$

Step II. Drag Load Time History:

1. Basic parameters:

I_A = moment of inertia of each landing gear rolling assembly = 31.23 slug ft²

R_R = rolling radius of wheel = 1.942 ft.

V_L = landing speed = 147 ft/sec.

Tire coefficient of friction = .55 (assumed)

2. Computation:

$$\begin{aligned} \theta &= \text{angular velocity of wheel after} & (104) \\ \text{spin up} &= \frac{V_L}{R_R} = \frac{147}{1.942} \text{ rad/sec} = \\ &75.70 \text{ rad/sec.} \end{aligned}$$

$$\begin{aligned} \theta_T &= \text{angular velocity after tire compression} = & (105) \\ &\frac{R_R 0.55 P_{MAX} T_T}{2 I_A} = 19.68 \text{ rad/sec (peak} \\ &\text{drag load} = .55 P_{MAX}). \end{aligned}$$

$$\begin{aligned} \theta_0 &= \text{angular velocity for spin up during oleo} & (106) \\ \text{compression} &= \theta - \theta_T = 56.02 \text{ rad/sec} \end{aligned}$$

$$\begin{aligned} T_S &= \text{duration of skid during oleo compression} = & (107) \\ &\frac{\theta_0 I_A}{0.55 P_{MAX} R_R} = .0683 \text{ sec.} \end{aligned}$$

$$T_T + T_S = \text{total spin up time} = .1167 \text{ sec.}$$

$$\begin{aligned} T_Z &= \text{time to drop to zero drag load} = \frac{T_T + T_S}{4} & (108) \\ &= .0292 \text{ sec. (empirical formula).} \end{aligned}$$

Step III. Important Modes:

The first mode only, (frequency = natural frequency of gear fore and aft = 73.2 rad/sec) is assumed to be excited by the vertical load component normal to the strut and by the drag load component normal to the strut.

Step IV. Generalized Acceleration Factor of Vibratory Response:

1. It will first be shown that it is not necessary to compute the vibratory response to the normal component of vertical load and the vibrating response to the normal component of drag load separately. Rather the entire vibratory response can be computed from a "double" trapezoid combination of the two forcing functions. Consider:

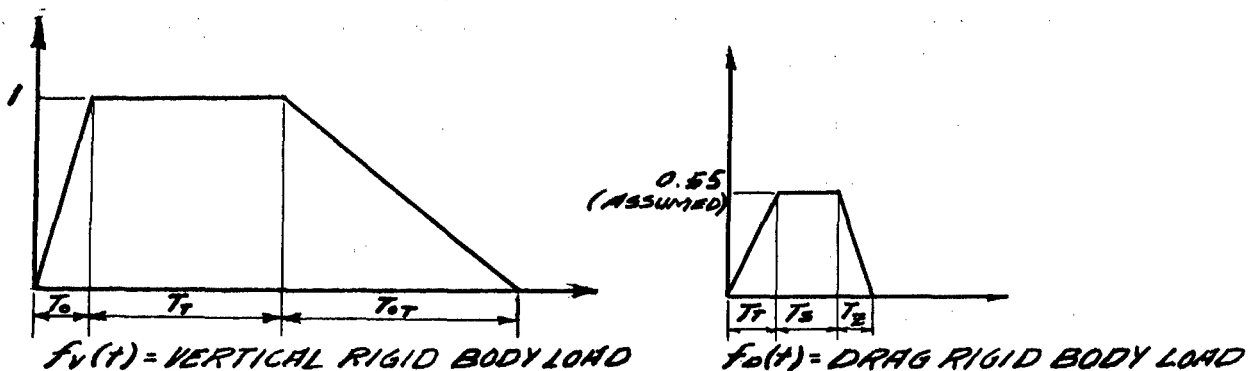


Fig. 8. Time Histories of Vertical Load and of Drag Load in Units of P_{MAX}

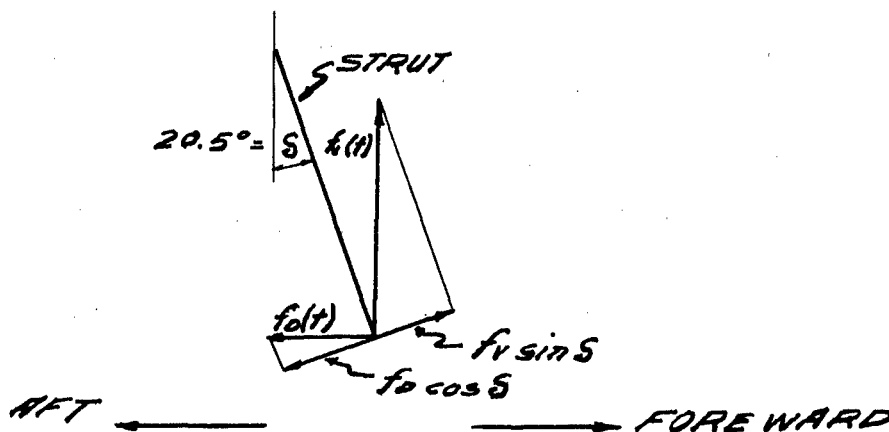


Fig. 9. Load Normal to Strut at Time t
Due to Vertical Load and Drag Load

2. Thus the load normal to the strut due to rigid body motion is:

$$Q_R(t) = \sin \delta \cdot f_p(t) - \cos \delta \cdot f_\theta(t) \quad (109)$$

3. It is therefore possible to consider a combined function to represent rigid body acceleration and as forcing function for the vibratory acceleration:

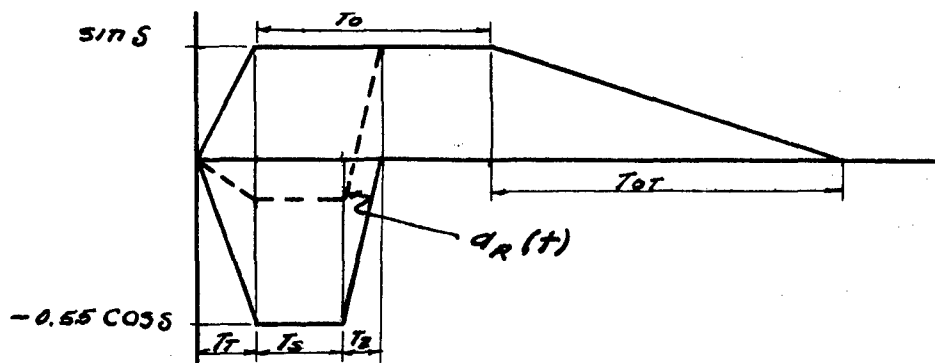


Fig. 10. The Composition of the Trapezoidal Forcing Function

4. In Examples I and II the vibratory acceleration $\ddot{X}$ and rigid body acceleration $F(t)$ are found and added to find the total acceleration response.

5. But in this example the structural force rather than the total acceleration is to be computed. That is, in this example find structural force normal to the strut at the axle due to a one-foot vertical deflection of the axle. Writing equation (1) with $dx = 1$ ft:

$$(\sum c_i^2 dm_i) \ddot{X} + (\sum c_i^2 m_i) \omega^2 (1 + \bar{g}_j) X = (\sum c_i f_i) \cdot F(t) \quad (110)$$

where: $(\sum c_i^2 m_i) \omega^2 (1 + \bar{g}_j) X$ is the required structural force

$(\sum c_i^2 m_i) \ddot{X}$ is the inertial force

$(\sum c_i f_i) \cdot F(t)$ is the external force

Then *: Structural force = $(\sum c_i f_i) \cdot F(t) - (\sum c_i^2 m_i) \ddot{x}$ (111)

But $\sum c_i f_i = P_{max}$ and $\ddot{x}$ is obtained in terms of $\frac{\sum c_i f_i}{(\sum c_i^2 m_i)} f(t)$
 Therefore $\sum c_i^2 m_i$ can be eliminated from equation (111).

6. It is clear that the generalized acceleration factor does not appear as such.

7. Using the "double" trapezoid derived above, the right side of equation (110) is already scaled relative to P_{max} . In the computation of Table 3, the ordinate of the trapezoid will be multiplied by $\frac{P_{max}}{1/2 \text{ Airplane Weight}}$ so that the structural force is finally non-dimensionalized in units of 1/2 airplane weight.

Step V. Vibratory Acceleration Response:

Table 3 computes the vibratory response by the desk-calculator method.

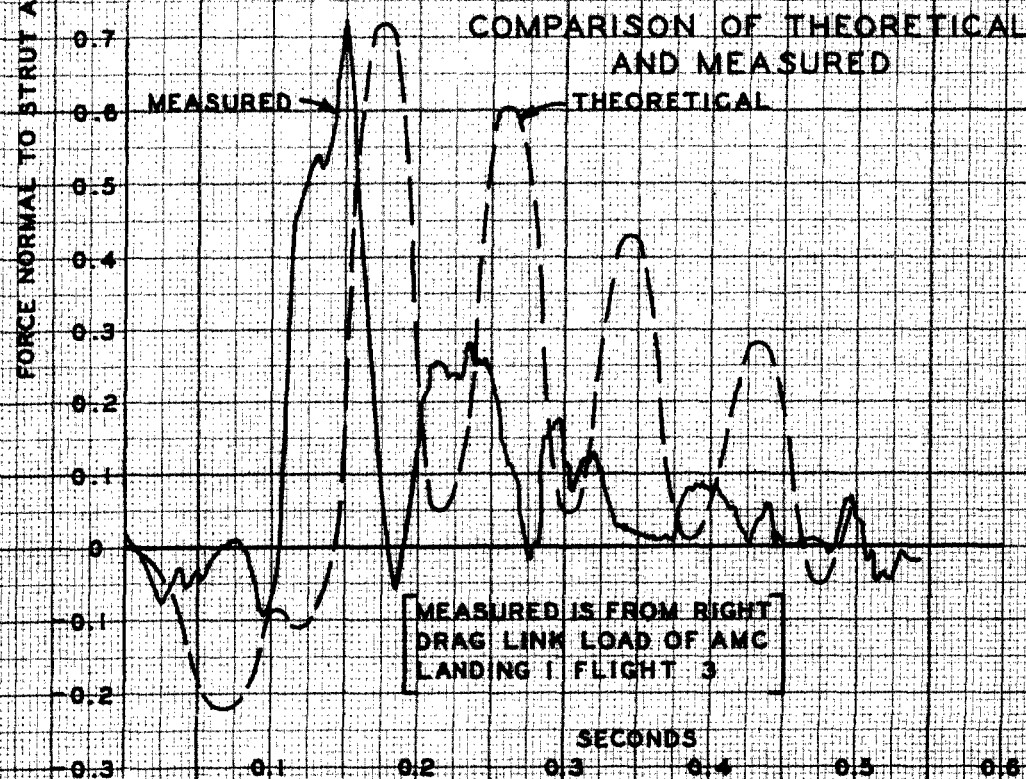
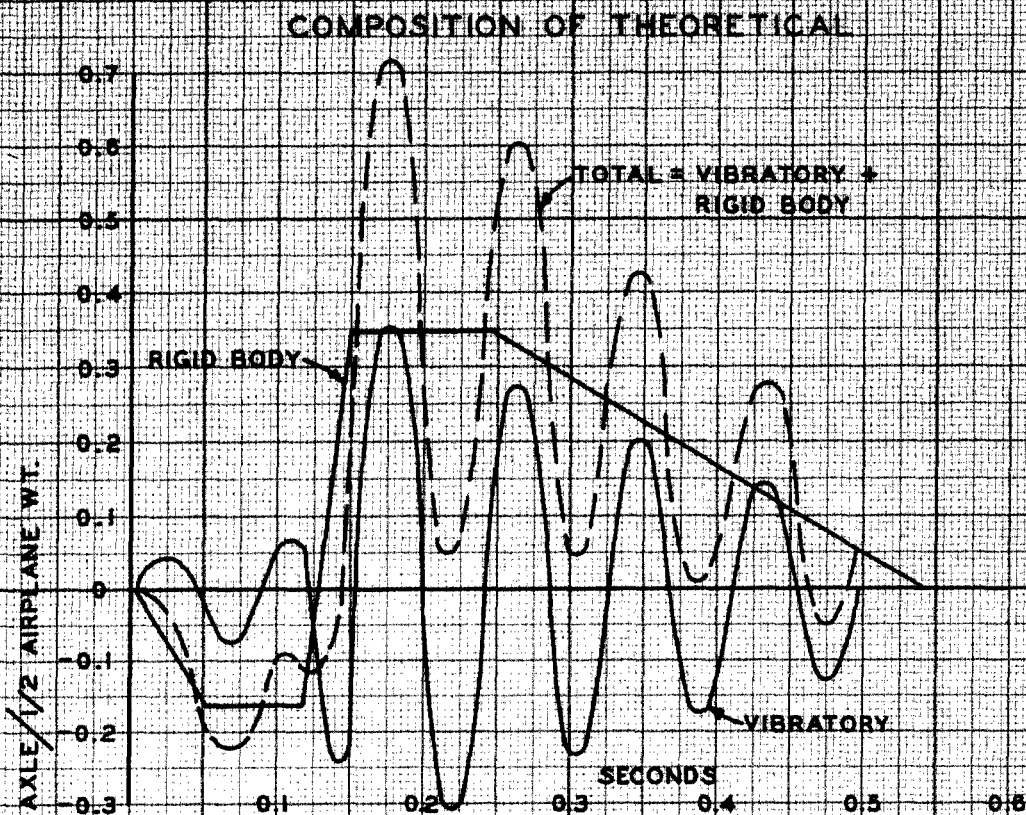
Step VI. Total Acceleration Time History:

The final dimensionless result of $\frac{\text{Force normal to strut at axle}}{\text{One-half weight of airplane}}$ is composed and compared with experimental results in Figure 11. The experimental force normal to the strut at the axle was computed from the measured drag link load by taking a summation of moments about the top of the strut assuming the oleo to be fully extended.

* Of course the differential equation (5) could be solved this time for x rather than $\ddot{x}$, but it is convenient to utilize the methods of Section II.

I. THE PROBLEM: GIVEN FORCING FUNCTION $f(t)$ FIND THE RESPONSE x FROM: $\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = f(t)$	II. BASIC DATA (used) TITLE: <i>Dr. J. K. Joshi's Normal to Start at Aile</i> COMPUTER: <i>ALB</i> <table border="1"> <tr> <td>1</td><td>T₁</td><td>2</td><td>T₂</td><td>3</td><td>T₃</td><td>4</td><td>T₄</td><td>5</td><td>T₅</td></tr> <tr> <td>6</td><td>H₁</td><td>7</td><td>H₂</td><td>8</td><td>w</td><td>9</td><td>5</td><td></td><td></td></tr> </table> 0.484 0.683 0.094 0.093 0.2770 3.489 1.644 73.2 10 C. RADIAN/SEC.	1	T ₁	2	T ₂	3	T ₃	4	T ₄	5	T ₅	6	H ₁	7	H ₂	8	w	9	5			III. INITIALLY COMPUTABLE CONSTANTS <table border="1"> <tr> <td>10</td><td>W₁ = 0.81</td><td>11</td><td>W₂ = 0.82</td><td>12</td><td>W₃ = 0.83</td><td>13</td><td>W₄ = 0.84</td><td>14</td><td>W₅ = 0.85</td></tr> </table> 3.5429 4.9996 1.1974 7.2688 1.7404 15 3.66 1.064 -5.733 -2.402 -0.160 20 3.151 2.1 2.152 2.2 2.254 2.3 2.356 2.4 2.458 25 1.771 2.500 1.069 3.634 1.0870 30 1.827 1.788 3.986 6.953 3.3372	10	W ₁ = 0.81	11	W ₂ = 0.82	12	W ₃ = 0.83	13	W ₄ = 0.84	14	W ₅ = 0.85	IV. IN AB $\ddot{x} = \frac{H_1}{\omega_1} e^{-\frac{\zeta}{\omega_1} \sin \omega t}$ AT B $\ddot{x}_0 = \frac{H_2}{\omega_2} e^{-\frac{\zeta}{\omega_2} \sin \omega t}$ <table border="1"> <tr> <td>31</td><td>10x25</td><td>32</td><td>sin 10</td><td>33</td><td>30x31</td></tr> </table> 0.0389 -3.006 -0.152 -3.006 -0.0389	31	10x25	32	sin 10	33	30x31	VIII. IN EF $\ddot{x} = \frac{H_3}{\omega_3} e^{-\frac{\zeta}{\omega_3} \sin(\omega t + \phi_3)}$ AT F $\ddot{x}_0 = \frac{H_4}{\omega_4} e^{-\frac{\zeta}{\omega_4} \sin(\omega t + \phi_4)}$ WHERE $\phi_3 = \arctan \frac{H_3}{R_3}$ (SEE BELOW) <table border="1"> <tr> <td>34</td><td>67-19</td><td>35</td><td>63x40</td><td>36</td><td>765</td><td>37</td><td>29x70</td></tr> </table> 7.760 0.0813 1.04 0.990 78 65/60 79 60/60 80 14+73 81 75 sin 74 82 71x75 83 7.8159 84 3.7022 85 0.874 86 0.862 87 0.095 88 0.095 89 0.095 90 0.095 91 0.095 92 0.095 93 0.095 94 0.095 95 0.095 96 0.095 97 0.095 98 0.095 99 0.095 100 0.095 101 0.095 102 0.095 103 0.095 104 0.095 105 0.095 106 0.095 107 0.095 108 0.095 109 0.095 110 0.095 111 0.095 112 0.095 113 0.095 114 0.095 115 0.095 116 0.095 117 0.095 118 0.095 119 0.095 120 0.095 121 0.095 122 0.095 123 0.095 124 0.095 125 0.095 126 0.095 127 0.095 128 0.095 129 0.095 130 0.095 131 0.095 132 0.095 133 0.095 134 0.095 135 0.095 136 0.095 137 0.095 138 0.095 139 0.095 140 0.095 141 0.095 142 0.095 143 0.095 144 0.095 145 0.095 146 0.095 147 0.095 148 0.095 149 0.095 150 0.095 151 0.095 152 0.095 153 0.095 154 0.095 155 0.095 156 0.095 157 0.095 158 0.095 159 0.095 160 0.095 161 0.095 162 0.095 163 0.095 164 0.095 165 0.095 166 0.095 167 0.095 168 0.095 169 0.095 170 0.095 171 0.095 172 0.095 173 0.095 174 0.095 175 0.095 176 0.095 177 0.095 178 0.095 179 0.095 180 0.095 181 0.095 182 0.095 183 0.095 184 0.095 185 0.095 186 0.095 187 0.095 188 0.095 189 0.095 190 0.095 191 0.095 192 0.095 193 0.095 194 0.095 195 0.095 196 0.095 197 0.095 198 0.095 199 0.095 200 0.095 201 0.095 202 0.095 203 0.095 204 0.095 205 0.095 206 0.095 207 0.095 208 0.095 209 0.095 210 0.095 211 0.095 212 0.095 213 0.095 214 0.095 215 0.095 216 0.095 217 0.095 218 0.095 219 0.095 220 0.095 221 0.095 222 0.095 223 0.095 224 0.095 225 0.095 226 0.095 227 0.095 228 0.095 229 0.095 230 0.095 231 0.095 232 0.095 233 0.095 234 0.095 235 0.095 236 0.095 237 0.095 238 0.095 239 0.095 240 0.095 241 0.095 242 0.095 243 0.095 244 0.095 245 0.095 246 0.095 247 0.095 248 0.095 249 0.095 250 0.095 251 0.095 252 0.095 253 0.095 254 0.095 255 0.095 256 0.095 257 0.095 258 0.095 259 0.095 260 0.095 261 0.095 262 0.095 263 0.095 264 0.095 265 0.095 266 0.095 267 0.095 268 0.095 269 0.095 270 0.095 271 0.095 272 0.095 273 0.095 274 0.095 275 0.095 276 0.095 277 0.095 278 0.095 279 0.095 280 0.095 281 0.095 282 0.095 283 0.095 284 0.095 285 0.095 286 0.095 287 0.095 288 0.095 289 0.095 290 0.095 291 0.095 292 0.095 293 0.095 294 0.095 295 0.095 296 0.095 297 0.095 298 0.095 299 0.095 300 0.095 301 0.095 302 0.095 303 0.095 304 0.095 305 0.095 306 0.095 307 0.095 308 0.095 309 0.095 310 0.095 311 0.095 312 0.095 313 0.095 314 0.095 315 0.095 316 0.095 317 0.095 318 0.095 319 0.095 320 0.095 321 0.095 322 0.095 323 0.095 324 0.095 325 0.095 326 0.095 327 0.095 328 0.095 329 0.095 330 0.095 331 0.095 332 0.095 333 0.095 334 0.095 335 0.095 336 0.095 337 0.095 338 0.095 339 0.095 340 0.095 341 0.095 342 0.095 343 0.095 344 0.095 345 0.095 346 0.095 347 0.095 348 0.095 349 0.095 350 0.095 351 0.095 352 0.095 353 0.095 354 0.095 355 0.095 356 0.095 357 0.095 358 0.095 359 0.095 360 0.095 361 0.095 362 0.095 363 0.095 364 0.095 365 0.095 366 0.095 367 0.095 368 0.095 369 0.095 370 0.095 371 0.095 372 0.095 373 0.095 374 0.095 375 0.095 376 0.095 377 0.095 378 0.095 379 0.095 380 0.095 381 0.095 382 0.095 383 0.095 384 0.095 385 0.095 386 0.095 387 0.095 388 0.095 389 0.095 390 0.095 391 0.095 392 0.095 393 0.095 394 0.095 395 0.095 396 0.095 397 0.095 398 0.095 399 0.095 400 0.095 401 0.095 402 0.095 403 0.095 404 0.095 405 0.095 406 0.095 407 0.095 408 0.095 409 0.095 410 0.095 411 0.095 412 0.095 413 0.095 414 0.095 415 0.095 416 0.095 417 0.095 418 0.095 419 0.095 420 0.095 421 0.095 422 0.095 423 0.095 424 0.095 425 0.095 426 0.095 427 0.095 428 0.095 429 0.095 430 0.095 431 0.095 432 0.095 433 0.095 434 0.095 435 0.095 436 0.095 437 0.095 438 0.095 439 0.095 440 0.095 441 0.095 442 0.095 443 0.095 444 0.095 445 0.095 446 0.095 447 0.095 448 0.095 449 0.095 450 0.095 451 0.095 452 0.095 453 0.095 454 0.095 455 0.095 456 0.095 457 0.095 458 0.095 459 0.095 460 0.095 461 0.095 462 0.095 463 0.095 464 0.095 465 0.095 466 0.095 467 0.095 468 0.095 469 0.095 470 0.095 471 0.095 472 0.095 473 0.095 474 0.095 475 0.095 476 0.095 477 0.095 478 0.095 479 0.095 480 0.095 481 0.095 482 0.095 483 0.095 484 0.095 485 0.095 486 0.095 487 0.095 488 0.095 489 0.095 490 0.095 491 0.095 492 0.095 493 0.095 494 0.095 495 0.095 496 0.095 497 0.095 498 0.095 499 0.095 500 0.095 501 0.095 502 0.095 503 0.095 504 0.095 505 0.095 506 0.095 507 0.095 508 0.095 509 0.095 510 0.095 511 0.095 512 0.095 513 0.095 514 0.095 515 0.095 516 0.095 517 0.095 518 0.095 519 0.095 520 0.095 521 0.095 522 0.095 523 0.095 524 0.095 525 0.095 526 0.095 527 0.095 528 0.095 529 0.095 530 0.095 531 0.095 532 0.095 533 0.095 534 0.095 535 0.095 536 0.095 537 0.095 538 0.095 539 0.095 540 0.095 541 0.095 542 0.095 543 0.095 544 0.095 545 0.095 546 0.095 547 0.095 548 0.095 549 0.095 550 0.095 551 0.095 552 0.095 553 0.095 554 0.095 555 0.095 556 0.095 557 0.095 558 0.095 559 0.095 560 0.095 561 0.095 562 0.095 563 0.095 564 0.095 565 0.095 566 0.095 567 0.095 568 0.095 569 0.095 570 0.095 571 0.095 572 0.095 573 0.095 574 0.095 575 0.095 576 0.095 577 0.095 578 0.095 579 0.095 580 0.095 581 0.095 582 0.095 583 0.095 584 0.095 585 0.095 586 0.095 587 0.095 588 0.095 589 0.095 590 0.095 591 0.095 592 0.095 593 0.095 594 0.095 595 0.095 596 0.095 597 0.095 598 0.095 599 0.095 600 0.095 601 0.095 602 0.095 603 0.095 604 0.095 605 0.095 606 0.095 607 0.095 608 0.095 609 0.095 610 0.095 611 0.095 612 0.095 613 0.095 614 0.095 615 0.095 616 0.095 617 0.095 618 0.095 619 0.095 620 0.095 621 0.095 622 0.095 623 0.095 624 0.095 625 0.095 626 0.095 627 0.095 628 0.095 629 0.095 630 0.095 631 0.095 632 0.095 633 0.095 634 0.095 635 0.095 636 0.095 637 0.095 638 0.095 639 0.095 640 0.095 641 0.095 642 0.095 643 0.095 644 0.095 645 0.095 646 0.095 647 0.095 648 0.095 649 0.095 650 0.095 651 0.095 652 0.095 653 0.095 654 0.095 655 0.095 656 0.095 657 0.095 658 0.095 659 0.095 660 0.095 661 0.095 662 0.095 663 0.095 664 0.095 665 0.095 666 0.095 667 0.095 668 0.095 669 0.095 670 0.095 671 0.095 672 0.095 673 0.095 674 0.095 675 0.095 676 0.095 677 0.095 678 0.095 679 0.095 680 0.095 681 0.095 682 0.095 683 0.095 684 0.095 685 0.095 686 0.095 687 0.095 688 0.095 689 0.095 690 0.095 691 0.095 692 0.095 693 0.095 694 0.095 695 0.095 696 0.095 697 0.095 698 0.095 699 0.095 700 0.095 701 0.095 702 0.095 703 0.095 704 0.095 705 0.095 706 0.095 707 0.095 708 0.095 709 0.095 710 0.095 711 0.095 712 0.095 713 0.095 714 0.095 715 0.095 716 0.095 717 0.095 718 0.095 719 0.095 720 0.095 721 0.095 722 0.095 723 0.095 724 0.095 725 0.095 726 0.095 727 0.095 728 0.095 729 0.095 730 0.095 731 0.095 732 0.095 733 0.095 734 0.095 735 0.095 736 0.095 737 0.095 738 0.095 739 0.095 740 0.095 741 0.095 742 0.095 743 0.095 744 0.095 745 0.095 746 0.095 747 0.095 748 0.095 749 0.095 750 0.095 751 0.095 752 0.095 753 0.095 754 0.095 755 0.095 756 0.095 757 0.095 758 0.095 759 0.095 760 0.095 761 0.095 762 0.095 763 0.095 764 0.095 765 0.095 766 0.095 767 0.095 768 0.095 769 0.095 770 0.095 771 0.095 772 0.095 773 0.095 774 0.095 775 0.095 776 0.095 777 0.095 778 0.095 779 0.095 780 0.095 781 0.095 782 0.095 783 0.095 784 0.095 785 0.095 786 0.095 787 0.095 788 0.095 789 0.095 790 0.095 791 0.095 792 0.095 793 0.095 794 0.095 795 0.095 796 0.095 797 0.095 798 0.095 799 0.095 800 0.095 801 0.095 802 0.095 803 0.095 804 0.095 805 0.095 806 0.095 807 0.095 808 0.095 809 0.095 810 0.095 811 0.095 812 0.095 813 0.095 814 0.095 815 0.095 816 0.095 817 0.095 818 0.095 819 0.095 820 0.095 821 0.095 822 0.095 823 0.095 824 0.095 825 0.095 826 0.095 827 0.095 828 0.095 829 0.095 830 0.095 831 0.095 832 0.095 833 0.095 834 0.095 835 0.095 836 0.095 837 0.095 838 0.095 839 0.095 840 0.095 841 0.095 842 0.095 843 0.095 844 0.095 845 0.095 846 0.095 847 0.095 848 0.095 849 0.095 850 0.095 851 0.095 852 0.095 853 0.095 854 0.095 855 0.095 856 0.095 857 0.095 858 0.095 859 0.095 860 0.095 861 0.095 862 0.095 863 0.095 864 0.095 865 0.095 866 0.095 867 0.095 868 0.095 869 0.095 870 0.095 871 0.095 872 0.095 873 0.095 874 0.095 875 0.095 876 0.095 877 0.095 878 0.095 879 0.095 880 0.095 881 0.095 882 0.095 883 0.095 884 0.095 885 0.095 886 0.095 887 0.095 888 0.095 889 0.095 890 0.095 891 0.095 892 0.095 893 0.095 894 0.095 895 0.095 896 0.095 897 0.095 898 0.095 899 0.095 900 0.095 901 0.095 902 0.095 903 0.095 904 0.095 905 0.095 906 0.095 907 0.095 908 0.095 909 0.095 910 0.095 911 0.095 912 0.095 913 0.095 914 0.095 915 0.095 916 0.095 917 0.095 918 0.095 919 0.095 920 0.095 921 0.095 922 0.095 923 0.095 924 0.095 925 0.095 926 0.095 927 0.095 928 0.095 929 0.095 930 0.095 931 0.095 932 0.095 933 0.095 934 0.095 935 0.095 936 0.095 937 0.095 938 0.095 939 0.095 940 0.095 941 0.095 942 0.095 943 0.095 944 0.095 945 0.095 946 0.095 947 0.095 948 0.095 949 0.095 950 0.095 951 0.095 952 0.095 953 0.095 954 0.095 955 0.095 956 0.095 957 0.095 958 0.095 959 0.095 960 0.095 961 0.095 962 0.095 963 0.095 964 0.095 965 0.095 966 0.095 967 0.095 968 0.095 969 0.095 970 0.095 971 0.095 972 0.095 973 0.095 974 0.095 975 0.095 976 0.095 977 0.095 978 0.095 979 0.095 980 0.095 981 0.095 982 0.095 983 0.095 984 0.095 985 0.095 986 0.095 987 0.095 988 0.095 989 0.095 990 0.095 991 0.095 992 0.095 993 0.095 994 0.095 995 0.095 996 0.095 997 0.095 998 0.095 999 0.095 1000 0.095 1001 0.095 1002 0.095 1003 0.095 1004 0.095 1005 0.095 1006 0.095 1007 0.095 1008 0.095 1009 0.095 1010 0.095 1011 0.095 1012 0.095 1013 0.095 1014 0.095 1015 0.095 1016 0.095 1017 0.095 1018 0.095 1019 0.095 1020 0.095 1021 0.095 1022 0.095 1023 0.095 1024 0.095 1025 0.095 1026 0.095 1027 0.095 1028 0.095 1029 0.095 1030 0.095 1031 0.095 1032 0.095 1033 0.095 1034 0.095 1035 0.095 1036 0.095 1037 0.095 1038 0.095 1039 0.095 1040 0.095 1041 0.095 1042 0.095 1043 0.095 1044 0.095 1045 0.095 1046 0.095 1047 0.095 1048 0.095 1049 0.095 1050 0.095 1051 0.095 1052 0.095 1053 0.095 1054 0.095 1055 0.095 1056 0.095 1057 0.095 1058 0.095 1059 0.095 1060 0.095 1061 0.095 1062 0.095 1063 0.095 1064 0.095 1065 0.095 1066 0.095 1067 0.095 1068 0.095 1069	34	67-19	35	63x40	36	765	37	29x70
1	T ₁	2	T ₂	3	T ₃	4	T ₄	5	T ₅																																							
6	H ₁	7	H ₂	8	w	9	5																																									
10	W ₁ = 0.81	11	W ₂ = 0.82	12	W ₃ = 0.83	13	W ₄ = 0.84	14	W ₅ = 0.85																																							
31	10x25	32	sin 10	33	30x31																																											
34	67-19	35	63x40	36	765	37	29x70																																									

FIGURE II
B-17C LOAD NORMAL TO STRUT AT AXLE
VS TIME



REFERENCES

1. MCREXA5-4595-8-2, "A Method for Predicting Dynamic Landing Loads", 20 February 1948.
2. Memorandum Report TSEAC5-4595-5-4, "A Simplified Procedure for Computing Transient Responses of Linear Systems to Landing Gear Impulses", 8 August 1947.
3. Memorandum Report TSEAC5-4595-5-5, "Estimation of Vertical Loads in Landing Gears", 8 August 1947.
4. Memorandum Report TSEAC5-45128-2-15, "Dynamic Loads on Wings with Tip Weights During Landing, Taxiing and Takeoff", 1 August 1947.
5. Memorandum Report TSEAL-2-7140, "Dynamic Landing Loads on P-61 Booms", 24 March 1945.
6. Memorandum Report MCREXA5-45128-2-18, "Rate of Descent Measurements During Hard Landing Tests of a B-17G Airplane", 24 March 1945.
7. Memorandum Report ENG-51-4595-1-6, "Summary of Recent Data Concerning Dynamic Loads, Particularly in Airplane Landing", 11 July 1944.
8. British Report SME 3298, "Prediction of Undercarriage Reactions", September 1948.
9. Technical Report 5815, "Prediction of Dynamic Landing Loads", 30 January 1949.
10. (Unpublished) Midwest Research Institute Technical Report, "The Effect of Several Variables Upon the Landing Gear Drag Load Time History", 25 May 1953.
11. Zahorski, Adam, "Remarks on Dynamic Loads in Landing", Journal of the Aeronautical Sciences, Vol. 19, No. 4, April, 1952.

DISTRIBUTION LIST

WCAPP (2 cys)
ASTIA (5 cys)
BAGR (4 cys)
WCLSY-3 (8 cys)
WCLSS-1
WCLSM-1

AF Development Field Representative
NACA Langley Memorial Aeronautical Laboratory
Langley AFB, Virginia

Civil Aeronautics Administration
Washington, D.C.

Beech Aircraft Corporation
6600 East Central Ave.
Wichita 1, Kansas

Bell Aircraft Company
Buffalo, New York

Boeing Aircraft Company
Seattle 14, Washington

Boeing Aircraft Company
Wichita, Kansas

California Institute of Technology
Pasadena, California

Cessna Aircraft Company
RFD #5
Wichita, Kansas

Chance Vought Aircraft Division
Dallas, Texas

Consolidated Vultee Aircraft Corporation
3156 Pacific Highway
San Diego 12, California

Consolidated Vultee Aircraft Corporation
Fort Worth, Texas

Cornell Aeronautical Laboratory
4455 Genesee St.
Buffalo 21, New York

DISTRIBUTION LIST

Douglas Aircraft Company, Inc.
3855 Lakewood Blvd
Long Beach 1, California

EDO Corporation
13-10 111th Street
College Point, L.I., New York

Fairchild Engine & Airplane Corporation
Hagerstown, Maryland

Goodyear Aircraft Corporation
Goodyear Tire & Rubber Co., Inc.
1210 Massillon Road
Akron 15, Ohio

Grumman Aircraft Engineering Corporation
Bethpage, L.I., New York

Hughes Aircraft Company
Florence and Teale Streets
Culver City, California

Lockheed Aircraft Corporation
Burbank, California

Massachusetts Institute of Technology
77 Massachusetts Ave.
Cambridge 39, Massachusetts

Glenn L. Martin Company
Baltimore, Maryland

McDonald Aircraft Corporation
St. Louis 3, Missouri

University of Michigan
Ann Arbor, Michigan

Midwest Research Institute
4049 Pennsylvania
Kansas City 2, Missouri

Northrop Aircraft, Inc.
Hawthorne, California

North American Aviation, Inc.
Los Angeles International Airport
Los Angeles, California

DISTRIBUTION LIST

Republic Aviation Corporation
Farmingdale, L.I., New York

Ryan Aeronautical Company
Lindberg Drive
San Diego 12, California

Sikorsky Aircraft Division
United Aircraft Corporation
Bridgeport 1, Connecticut

Stanford Research Institute
Stanford, California

Director
Air University Library
Maxwell AFB, Alabama