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To all holders of NAVORD Report 2101
insert change; write on cover 'Change 1 inserted'
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Change 1
20 Sep 1954
2 pages

R. Z. Lighter
By direction

NAVORD Report 2101, "Statistical Methods Appropriate for Evaluation of Fuze Explosive-Train Safety and Reliability", is changed as follows:

- Item 1. Page 1, line 21, Replace "references (a) and (b)" with "references (b) and (c)".
- Item 2. Insert Page vii attached.

Insert this change sheet between the cover and the title page of your copy.

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REFERENCES

- a. NAVORD Report No. 29-44, "Notes on Statistical Methods Applied to Torpedo Test Results".
- b. NOL S-5892, "A Statistical Analysis for a New Procedure in Sensitivity Experiments". OSRD AMP Report No. 101.1R SRG-P No. 40.
- c. "Probit Analysis-- A Statistical Treatment of the Sigmoid Response Curve", by D. J. Finney, Cambridge University Press, 1947 (2nd Ed. 1952).
- d. NOLM 9910, "Comparison of the Probit Method and the Bruceton Up-and-Down Method as Applied to Sensitivity Data."
- e. Deleted.
- f. NAMTC Technical Report No. 75, "A Study of Methods for Achieving Reliability of Guided Missiles". 10 July 1950.
- g. NAMTC Technical Report No. 84, "General Specification for the Safety Margins Required for Guided Missile Components." 10 July 1951.

STATISTICAL METHODS APPROPRIATE FOR EVALUATION OF FUZE
EXPLOSIVE-TRAIN SAFETY AND RELIABILITY

Prepared by:

H. P. Culling

ABSTRACT: The statistical problems of evaluating fuze explosive trains for safety and reliability are presented. The Bruceton and Probit procedures of testing are illustrated by examples of actual tests and the interpretation of the results is discussed.

Appendix A presents a step-by-step description of the calculations necessary to determine the "50% reliable distance", the standard deviation, errors of estimate, and "degree of reliability" by the Bruceton Method of Analysis. Tables and graphs necessary for the calculations are appended.

Appendix B presents the Probit Method calculations necessary to determine the Probit Line Equation from which estimates of the "50% unsafe distance", standard deviation, errors of estimate, and "degree of safety" may be obtained. A test for "goodness of fit" of the Probit Line is presented. Tables necessary for the analysis are appended.

Appendices A and B thus provide a working handbook for the two methods of analysis, which are both appropriate for either safety or reliability tests.

The methods presented and tables appended have been extracted from references (a) and (b) and adapted to the problems of estimating fuze safety and reliability. Physical considerations governing the applicability of the test methods described to various configurations of explosive trains, and

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the use of the results in the evaluation of ordnance items as a whole, are the subject of a separate report and are not considered herein.

Many similar problems in ordnance evaluation and testing in which the data must be of a "go-no go" nature may profitably be investigated by the methods presented in this report. For example, the minimum and average firing energy characteristics of electric primers are often so determined.

Since the test points are distributed over the center of the distribution rather than being concentrated at the end points (where all or none of a small sample may be expected to function and which usually correspond to nominal part-dimensions, firing voltages or whatever), these methods are relatively efficient in affording a good basis for estimating the effects of later design or manufacturing changes or proposals. The value of the data from tests conducted in accordance with these plans is therefore less likely to be lost in case of such changes.

The applicability of these methods is dependent upon an understanding, based upon experience with large numbers of previous tests or demonstration of the validity on a theoretical basis, of the nature of the physical processes involved. In particular, since these methods are aimed at giving an overall estimate of the population from a very limited number of samples, the analysis cannot, in itself, check the degree to which the assumptions of normal distribution and linearity (or other assumed relationship) between the two variables are justified. The greatest value of the test methods is in quickly confirming the suitability of a design or pointing out a marginal condition requiring correction or further investigation by the use of more samples.

13 October 1953

This report has been prepared under Task NOL-B2b-41-1, for the following purposes:

1. To illustrate the statistical philosophy of evaluating fuze-explosive trains for safety and reliability,
2. To present the application of two methods of analysis which deal with "go-no go" results as a function of a controlled variable, and
3. To provide a working handbook for using these methods in the design, analysis and presentation of explosive-train safety and reliability evaluations.

These procedures have been developed by the Trains and Mechanisms Branch, Mechanical Evaluation Division, in the course of the evaluation of ordnance items under various Bureau of Ordnance tasks. Tables appearing in this report have been taken or adapted from the references cited.

The Naval Ordnance Laboratory is indebted to Professor Sir Ronald A. Fisher, Cambridge, to Dr. Frank Yates, Rothamsted, and to Messrs. Oliver and Boyd Ltd., Edinburgh, for permission to reprint Table No. IX from their book "Statistical Tables for Biological, Agricultural, and Medical Research".

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STATISTICAL METHODS APPROPRIATE FOR EVALUATION OF FUZE
EXPLOSIVE-TRAIN SAFETY AND RELIABILITY

INTRODUCTION

1. Present-day emphasis on highly reliable and safe weapons calls for increasingly accurate methods of estimating the probabilities of reliable and safe functioning of components. The usefulness of statistical procedures in determining reliability is discussed in references (f) and (g). The general procedure, however, is based on a large sample of quantitative data which is obtained for a given test condition. This report illustrates the statistical methods which are applicable to "go-no go" data (such as the firing or non-firing of an explosive element) obtained at various levels of a test condition.

2. Organizations engaged in the evaluation of fuzes to determine their suitability for release to production have the problem of determining the reliability and safety of fuze explosive trains. This report, which is limited to the statistical aspects of the problem, has the following objectives:

- a. To present the statistical philosophy of evaluating fuze explosive trains for safety and reliability,
- b. To illustrate the application of two methods of analysis (the Bruceton and Probit) which deal with "go-no go" results as a function of a controlled variable, and
- c. To provide a working handbook for using these methods in the design, analysis and presentation of explosive-train safety and reliability evaluations.

Inadequacy of Confidence Limit Estimates

3. In order to meet the explosive-safety requirement, an explosive train usually contains an out-of-line safety device which rotates or slides from the unarmed to the armed position. The safety or reliability of the train may be estimated roughly by testing in the unarmed and armed positions, respectively.

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But if the sample size is limited to, say, 20 for each test, the statistical interpretation of the results cannot establish the high assurance of safety or reliability necessary for release. For example, given a sample of 20, all of which fire reliably in the armed position, one may expect from future samples a reliability as low as 83% (or 17% failures). If the sample is increased to 100, one may still expect as many as 4% failures. These estimates of percentage failures are based on the lower 95% confidence limit (reference (a)). Since an explosive train is probably more reliable than indicated by such a test, a more accurate estimate is necessary.

Method of Testing Between Armed and Unarmed Positions

4. If the safety device were placed half way between the armed and unarmed positions and the explosive train still functioned reliably, one would expect the reliability of the train in armed position to be considerably better than shown in the previous example. Then, if more samples were tested at various distances of partial arming and in such a manner that an array of high to low percentage-functioning distances was obtained, a graph could be plotted of percentage reliable versus distance from armed position. By methods of statistical extrapolation, a percent-reliable curve could be calculated from which an estimate of the 99%-reliable distance could be read. If the 99%-reliable distance occurs when the safety device is still considerably out of line, it is probable that the explosive train will function consistently in the armed position.

5. The safety test could be run in a similar fashion, where an array of high to low percent-unsafe conditions versus distance was obtained. The calculated percent-unsafe curve would give the estimated 1%-unsafe distance.

Methods of Analysis and Calculated Parameters

6. Since the estimates of the 99%-reliable distance and 1%-unsafe distance are the bases of conclusions concerning reliability and safety of the train, the method of extrapolation for obtaining these distances should be as accurate as possible. The method should also be applicable for small

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samples. These two restrictions prohibit the use of several established methods which employ relatively simple test procedures. Two established methods which meet these requirements are the Bruceton Method of Sensitivity Testing (reference (b)) and Probit Analysis (reference (c)). Both methods may be employed to estimate the 50%-reliable distance, $\bar{X}_R$, or the 50%-unsafe distance, $\bar{X}_U$, and the respective standard deviations, σ_R and σ_U . Each $\bar{X}$ minus its appropriate σ gives an estimate of the 84% distance. If σ is added to $\bar{X}$, an estimate of the 15% distance is obtained. By using the appropriate constants obtained from a table of normal curve areas, one may estimate any percentage distance. Therefore, the estimates of $\bar{X}_R$ and $\bar{X}_U$ and σ_R and σ_U are the only measurements necessary to determine a measure of reliability or safety. In appendices A and B, detailed descriptions of the calculation of these measurements for the Bruceton Method and the Probit Method are presented.

Specific Applications for Each Method

7. Although both of the methods provide a means of estimating the same measures ($\bar{X}_R$, $\bar{X}_U$, σ_R , σ_U), there are specific reasons for the application of one instead of the other, depending on the purpose of the test.

a. The Bruceton Method of testing centers the testing in the region of the 50% distance, while the Probit Method of testing includes the high and low percentage points. Thus, the Bruceton Method is more accurate for estimates of $\bar{X}_R$ and $\bar{X}_U$, while the Probit Method is more accurate for estimates of σ_R and σ_U .

b. The Bruceton Method requires fewer samples to determine $\bar{X}_R$ or $\bar{X}_U$ and a rough estimate of σ_R and σ_U , so it may be used for small samples (not less than 20). It should be held in mind that the Bruceton estimates of σ_R and σ_U tend to be slightly less than the Probit estimates; and, in connection with safety or reliability tests, such estimates are overly optimistic.

c. The Bruceton Analysis may not be used except when the Bruceton scheme has been followed, using equally spaced test distances and the up-and-down procedure of testing (described

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in appendix A). On the other hand, the Probit Analysis is adaptable to data of unequal sample sizes at each test distance and the test distances need not be equally spaced.

d. The Bruceton Method of Analysis is based on the assumption that the percentage functioning is related to the test distances by means of the normal error function, while the Probit method may be analyzed by other appropriate transformations, if necessary.

Thus, the Probit Method may be considered more adaptable to most test data, except when the sample sizes are too small, or when $\bar{X}_R$ or $\bar{X}_U$ are the most important measures. In some cases the Bruceton Method of Analysis is preferred because the computations involved are relatively simple, but this fact alone should not determine the choice of method. See reference (d) for a more detailed discussion of the two methods.

Comparison of Reliability or Safety Probabilities

8. The purpose of an explosive-train reliability or safety test is to determine just how reliable or safe the train is in the armed or unarmed positions, respectively. It is possible by means of the proposed methods to determine the probability of each; but, in most cases, this probability is in the range of 99% to 99.999999%. Obviously, small samples do not warrant so many significant figures, yet the trains represented by many nines after the decimal point are certainly more reliable than those with only one or two. Another disadvantage of the probability figure is that it does not indicate the amount of variation due to sample differences in the explosive components and mechanical parts (or inconsistency in manufacture of the design). In order to determine a difference between two high probabilities, a measure which gives the ratio of $\bar{X}$ to σ should be used.

A Measure of the Degree of Reliability or Safety

9. Two measures termed the "degree of reliability", D_R , and the "degree of safety", D_S , which are not subject to the discrepancies of the probability values, may be calculated.

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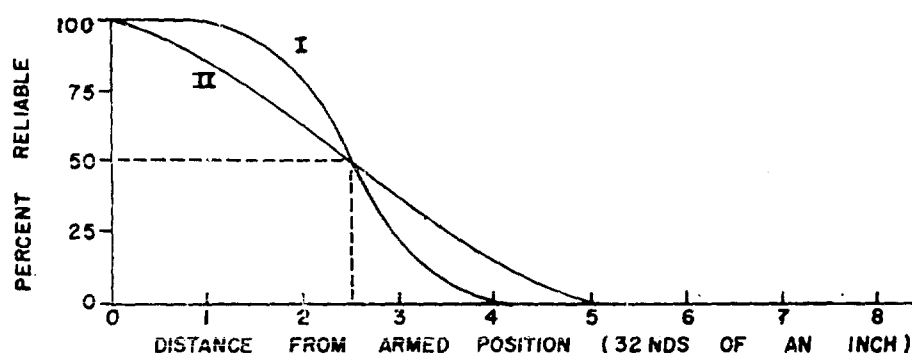
D_R is defined as $\bar{X}_R$ in terms of σ_R , or

$$D_R = \frac{\bar{X}_R}{\sigma_R}$$

where $\bar{X}_R$ is measured from the armed position for reliability. D_S is obtained in a similar manner by

$$D_S = \frac{\bar{X}_U}{\sigma_U}$$

where $\bar{X}_U$ is measured from unarmed position. The values for $\bar{X}$ and σ are obtained from the Bruceton or Probit estimates of the reliability curve and safety curve, respectively. A comparison of two reliability functions which have identical $\bar{X}_R$'s but different values of σ_R may best illustrate the usefulness of this measure.



The above graph shows $\bar{X}_R$ for I and II is at 2.5/32 of an inch. The reliability curve I has a σ_R equal to 0.5/32 inches, and curve II has a σ_R equal to 1/32 inches. The reliability probability (P) in the armed position for curve I equals 99.99997% while curve II has a P = 99.4%. In a practical sense, these two values of P do not indicate any significant difference when estimated from small samples. But by means of the degree measure, we find that curve I has a $D_R = 5.0$ while curve II gives a $D_R = 2.5$. By comparing the two D_R values, one can conclude that D_R for the train of curve I is twice the value of D_R for the train of curve II, while the probability figures might lead to the conclusion that the two trains are almost similar.

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10. A more difficult comparison between probability values occurs when two curves have slightly different values of $\bar{X}_R$ and σ_R , but the probability values occur in the 99 to 100% region. Here again the D_R of each will indicate which is the more reliable.

Lower Limit for the Degree of Reliability or Safety

11. Consideration must be given to the degree of reliability or safety that is necessary to insure consistent operation of the explosive train. A lower limit for D_R and D_S has been designated as 5.0 to insure better than 99.5% operation. This lower limit was calculated to allow for the errors of estimates from small samples of not less than 20. If the exact 50% distance, $\bar{X}_E$, and the exact standard deviation, σ_E , were known, 99.5% operation would be assured with a degree equal to 2.6. (In other words, 99.5% reliability is insured if $\bar{X}_E/\sigma_E = 2.6$ where 2.6 is the normal-deviate constant corresponding to 99.5%.) But the small sample estimates of $\bar{X}$ and σ are subject to sampling errors, so these must be allowed for in addition to the degree of 2.6.

12. The error in $\bar{X}$ (sample estimate) may be calculated by:

$\sigma_{\bar{X}} = \sigma/\sqrt{n}$, where n is the number of results that responded, but not the total sample size, and σ is the standard deviation estimated from the sample.

If we consider samples of 20 or more with at least 9 favorable results, then $\sigma_{\bar{X}}$ will be less than or equal to $\sigma/\sqrt{9}$ or $\sigma/3$. The 95% lower limit for $\bar{X}$ can be found by $\bar{X} - 1.86\sigma_{\bar{X}}$, or $\bar{X} - \frac{1.86\sigma}{3}$ which equals $\bar{X} - 0.6\sigma$. Therefore, to insure that

the exact $\bar{X}_E$ has been included in the measure of D , an allowance of 0.6σ must be subtracted from the sample $\bar{X}$.

This 95% lower limit for $\bar{X}$ may be explained by the following short discussion. Confidence limits may be computed for a mean by the expressions,

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$$\bar{X} \pm t\sigma_{\bar{X}} \quad \text{or} \quad \bar{X} \pm t \frac{\sigma}{\sqrt{n}} \quad (\text{since } \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}),$$

where the value for t may be found in table 2, appendix A using $n - 1$ degrees of freedom and that probability or "risk" of excluding $\bar{X}$ which we are willing to allow. Thus, if $n = 9$ (as in the example above) and we wished to be 90% sure of including the real $\bar{X}$ (i.e., we desired to know the 90% confidence limits), we should enter the t table with $9 - 1$ or 8 degrees of freedom and a probability level of $100\% - 90\%$ or 10% . This would give a t value of 1.86, and so the 90% confidence limits for $n = 9$ are

$$\bar{X} \pm 1.86 \frac{\sigma}{\sqrt{9}} \quad \text{or} \quad \bar{X} \pm 0.6\sigma.$$

The other way of looking at these 90% confidence limits is that the probability is 10% that the real $\bar{X}$ has been excluded. This 10% probability is the sum of the 5% probability that the true $\bar{X}$ lies above the greater 90% confidence limit, $\bar{X} + 0.6\sigma$, and the 5% probability that the true $\bar{X}$ lies below the smaller 90% confidence limit, $\bar{X} - 0.6\sigma$. In other words, the probability is 95% that the true $\bar{X}$ lies above the smaller 90% confidence limit, $\bar{X} - 0.6\sigma$, which is the value given above for the 95% lower limit for $\bar{X}$.

13. The sampling error of σ may be determined by obtaining an upper limit of σ which will include the true σ_E with 95% confidence. A relation between σ_E and the sample σ is given by the F-ratio, defined as

$$F = \sigma_E^2 / \sigma^2.$$

Tables are available in most statistical texts which give values of F for specific confidence levels and for various sample sizes used for estimating σ . The 95% upper limit of σ is calculated using the F -value given at the 5% level under $N_1 = \infty$ (representing the size of the population) and $N_2 = 8$ (representing $n - 1$ which was used to obtain σ). This value of F is 2.9 which gives the 95% upper limit of σ as:

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$$\sigma_E^2/\sigma^2 = 2.9 \quad \text{or} \quad 1.7\sigma = \sigma_E.$$

Therefore, the error in σ is allowed for by estimating σ_E as 1.7σ .

14. It was previously stated that 99.5% reliability was insured if

$$\bar{X}_E/\sigma_E = 2.6.$$

By substituting the errors for the sample estimates in the above relation, it becomes,

$$\frac{\bar{X} - 0.6\sigma}{1.7\sigma} = 2.6 \quad \text{or} \quad \bar{X}/\sigma = 5.0.$$

Thus D , obtained from a sample of 20 or more, must be greater than 5.0, if 99.5% operation is desired. (It should be noted that the samples tested must be representative of the production lots for these figures to be meaningful.)

Looking back on this problem, it can be stated that if D_R (which is the ratio of $\bar{X}$ to σ) is 5.0 or greater, then even if the sample estimates of σ and $\bar{X}$ were both so overly optimistic that they might be exceeded only 5% of the time from a sample as small as 9 the ratio $\bar{X}_E/\sigma_E$ (the true $\bar{X}$ to the true σ) should be equal to or greater than 2.6. It was stated above that a value of 2.6 would insure at least 99.5% operation (or safety). The requirement implied here is that the ratio of the pessimistic 95% limits for σ and $\bar{X}$ be greater than 2.6. The value of 5.0 was merely found to be the necessary lower limit to the ratio of the estimates of $\bar{X}$ to σ from a sample as small as 9 to be certain that the 2.6 requirement for the ratio of the pessimistic limits for $\bar{X}$ and σ was met. It can be seen that while a D_R of 5.0 always implies 99.5% operation, the failure of a set of data to yield a D_R of 5.0 does not mean that the 2.6 value for the ratio of the pessimistic 95% limits for $\bar{X}$ and σ (and hence 99.5% operation)

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has not been met, since the sample size may have been larger than 9 and this would result in the pessimistic 95% limits for $\bar{X}$ and σ being closer to the sample estimates of $\bar{X}$ and σ . The ratio of these pessimistic 95% limits for $\bar{X}$ and σ should be computed whenever D_R or D_S is found to be less than 5.0, since if this ratio is equal to or greater than 2.6, the statement that the item is 99.5% safe or operable may still be made.

Degree of Reliability Versus Degree of Safety

15. Since an explosive train must be both reliable and safe, these two opposing requirements must be balanced. In other words, D_R must not exceed D_S to such an extent that the safety is endangered, and conversely. In a theoretically ideal train, the difference between D_R and D_S would be minimized. In practice, the difference may be large so long as both values exceed the lower limit of 5.0. Tables 1 through 7 present actual test results determined by the Bruceton and Probit Methods. Graphs 1 through 7 are the respective graphs of percent-reliable and percent-unsafe conditions estimated from the data given in the tables. The calculated D_R and D_S for each of the samples are as follows:

		<u>D_R</u> <u>Degree of</u> <u>Reliability</u>	<u>D_S</u> <u>Degree of</u> <u>Safety</u>
Table 1,	Graph 1	10.4	14.7
" 2,	" 2	6.8	2.0*
" 3,	" 3	3.4*	12.3
" 4,	" 4	12.2	12.0
" 5,	" 5	8.5	23.1
" 6,	" 6	6.6	31.8
" 7,	" 7	3.6*	----

The tabulation indicates that the device of Graph 2 was unsafe and those of Graphs 3 and 7 were unreliable. The other devices may be compared by their values of D_R and D_S which show that the devices for which Graphs 1 and 4 were plotted contain the best combination of safety and reliability.

CONCLUSIONS

16. Fuze explosive-train reliability or safety estimates obtained by means of confidence limits cannot establish the high assurance necessary for release. Therefore, tests conducted with the explosive-safety device in the armed or unarmed position are, for the most part, inadequate for determining statistical estimates of reliability or safety, respectively.

17. The Bruceton or Probit methods of testing at partial-arming positions provide results from which a statistical estimate of a high assurance of reliability or safety can be established. If the degree of reliability (the ratio of the 50%-reliable distance to the standard deviation) is greater than 5.0, the explosive-train reliability will be at least 99.5%. Similarly, if the degree of safety (the ratio of the 50%-unsafe distance to the standard deviation) is greater than 5.0, the explosive-train safety will be at least 99.5%.

18. The reliability of an explosive train may exceed the safety to such an extent that the safety requirement is endangered, or conversely. A comparison of the degree of safety, D_S , with the degree of reliability, D_R , will indicate whether the train is over-designed in favor of either the safety or reliability requirement.

TABLE 1
EXPLOSIVE-TRAIN SAFETY AND RELIABILITY TESTS OF A GUIDED MISSILE FUZE
PROBIT TEST

RELIABILITY TEST - ELECTRIC PRIMER TO RELAY DETONATOR									
Distance From Armed Position (in 32nd's of an inch)		50% Reliable at 3.65/32" Standard Deviation = 0.35/32"						FIRE	FAILURES
3.0	X	X	X	X	X	X	X	9	0
3.5	X	X	X	X	X	X	X	6	3
3.75	0	0	0	0	0	0	0	3	7
4.0	0	0	0	0	0	0	0	4	16
5.0	0	0	0	0	0	0	0	0	10
6.0	0	0	0	0	0	0	0	0	10
8.0	0	0	0	0	0	0	0	0	2
								22	48
SAFETY TEST - RELAY DETONATOR TO LEAD									
Distance From Armed Position (in 32nd's of an inch)		50% Unsafe at 5.90/32" Standard Deviation = 0.55/32"						UNSAFE	SAFE
4	X	X	X	X	X	X	X	10	0
5	X	X	X	X	X	X	X	10	0
5.5	X	X	X	X	X	X	X	6	0
6	0	0	0	0	0	0	0	2	10
6.5	0	0	0	0	0	0	0	2	4
7	0	0	0	0	0	0	0	0	4
8	0	0	0	0	0	0	0	0	2
								30	20

Distance from armed to safe position equals 14/32 of an inch.

TABLE 2
BRUCETON ANALYSIS
SAFETY AND RELIABILITY OF AN EXPLOSIVE TRAIN

RELIABILITY TEST		50% Reliable at 0".63 Standard Deviation = 0".10		FIRES		FAILURES	
Distance From Armed Position (in inches)							
0.5	X	X	X	X	6		0
0.75	X	0	0	0	1		7
1.00	0				0		1
					7		8

SAFETY TEST		50% Unsafe at 1".52 Standard Deviation = 0.24		UNSAFE		SAFE	
Distance From Armed Position (in inches)							
1.25	X	X	X	X	5		0
1.50	X	0	0	0	2		4
1.75	0	X	0		1		2
2.00		0			0		1
					8		7

Distance from armed to safe position equals 2 inches.

TABLE 3

EXPLOSIVE-TRAIN SAFETY AND RELIABILITY TESTS OF A BOMB FUZE

BRUCETON TEST

RELIABILITY TEST - ELECTRIC PRIMER TO SLIDER DETONATOR

Distance From Armed Position (in 32nd's of an inch)	50% Reliable at 3.5/32" Standard Deviation = 1.04/32"	FIRES	FAILURES
2.5	X	X	4
3.0	0	0	2
3.5	X	X	4
4.0	0	0	2
4.5	0	0	0
		12	10

SAFETY TEST - SLIDER DETONATOR TO LEAD

Distance From Armed Position (in 32nd's of an inch)	50% Unsafe at 9.6/32" Standard Deviation = 0.6/32"	UNSAFE	SAFE
8.0	X	2	0
9.0	X	6	1
10.0	0	2	6
11.0	0	0	2
		10	9

Distance from armed to safe position equals 17/32 of an inch.

TABLE 4
BRUCETON ANALYSIS
EX LOSIVE TRAIN SAFETY AND RELIABILITY OF A TORPEDO FIRING DEVICE

RELIABILITY TEST		50% Reliable at 9.8/32" Standard Deviation = 0.8/32"		FIRES		FAILURES	
Distance From Armed Position (in 32nd's of an inch)							
8	X					1	0
9	X	X	X	X	X	10	1
10	X	0	0	0	0	3	10
11	0			0	X	1	3
12				0	0	0	1
						15	15
SAFETY TEST		50% Unsafe at 24.8/32" Standard Deviation = 0.6/32"		SAFE		UNSAFE	
Distance From Armed Position (in 32nd's of an inch)							
23	X					0	2
24	0	X	X	X	X	1	9
25	0	0	0	X	0	8	5
26		0	0	0	0	5	0
						14	16

Distance from armed to safe position equals 1 inch.

TABLE 5
EXPLOSIVE-TRAIN SAFETY AND RELIABILITY TESTS OF A TORPEDO ARMING MECHANISM
BRUCETON TEST

RELIABILITY TEST		50% Reliable at 22.0° Standard Deviation = 2.6°		FIRES		FAILURES	
No. of Degrees From Armed Position							
17.5	X				1		0
20.0	X	X	X	X	6	X	1
22.5	0	0	0	X	3	0	6
25.0				0	1	0	3
27.5				0	0	0	1
					11		11
SAFETY TEST		50% Unsafe at 39.1° Standard Deviation = 2.2°		UNSAFE		SAFE	
No. of Degrees From Armed Position							
35.0	X				3		0
37.5	0	X	X	0	6	X	3
40.0	0	0	X	X	5	0	6
42.5				0	0	0	5
					14		14

Number of degrees from armed to safe position equals 90°.

TABLE 6

EXPLOSIVE-TRAIN SAFETY AND RELIABILITY OF A REMOTE CONTROLLED FUZE

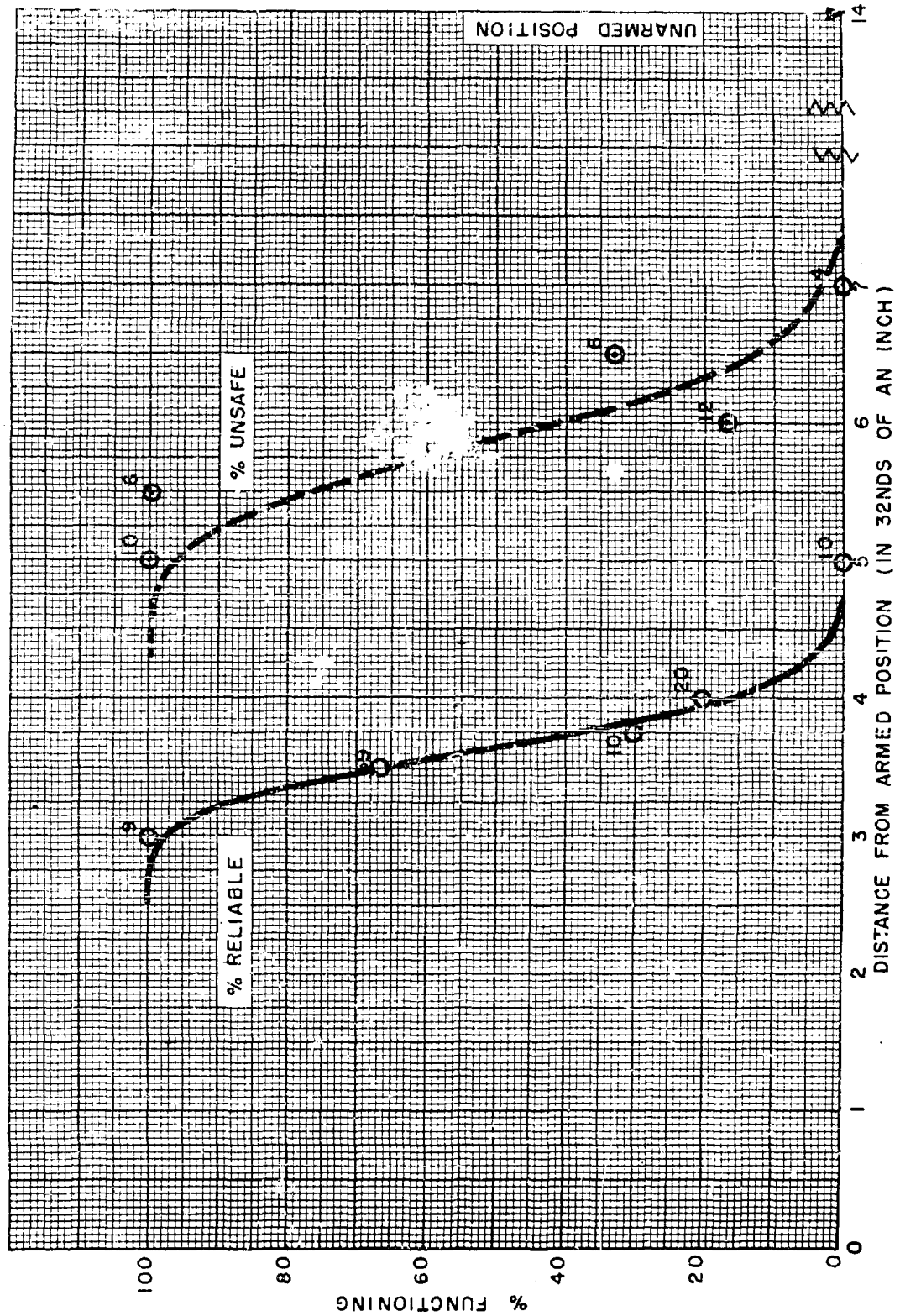
RELIABILITY TEST			50% Reliable at 0°1877 Standard Deviation = 0°0283		
No. of Degrees From Armed Position and Corresponding Chord Lengths				FIRES	FAILURES
23.77° 0°16265		X	X	4	0
26.83° 0°17828	X		0	2	4
28.42° 0°19390	0	X	X	3	3
30.77° 0°20952	0	X	0	1	4
33.12° 0°22515	0	0	0	0	2
				10	13
SAFETY TEST			50% Unsafe at 0°3963 Standard Deviation = 0°0124		
No. of Degrees From Armed Position and Corresponding Chord Lengths				UNSAFE	SAFE
42.70° 0°36411	X			1	0
45.13° 0°38389	0	X	X	6	1
47.60° 0°40367		0	0	2	5
50.27° 0°42345			0	0	2
				9	8

Number of degrees from armed to safe position equals 180°.
 Distance between armed and safe position equals 0.79 inches.
 Chord lengths were used as the equispaced variable.

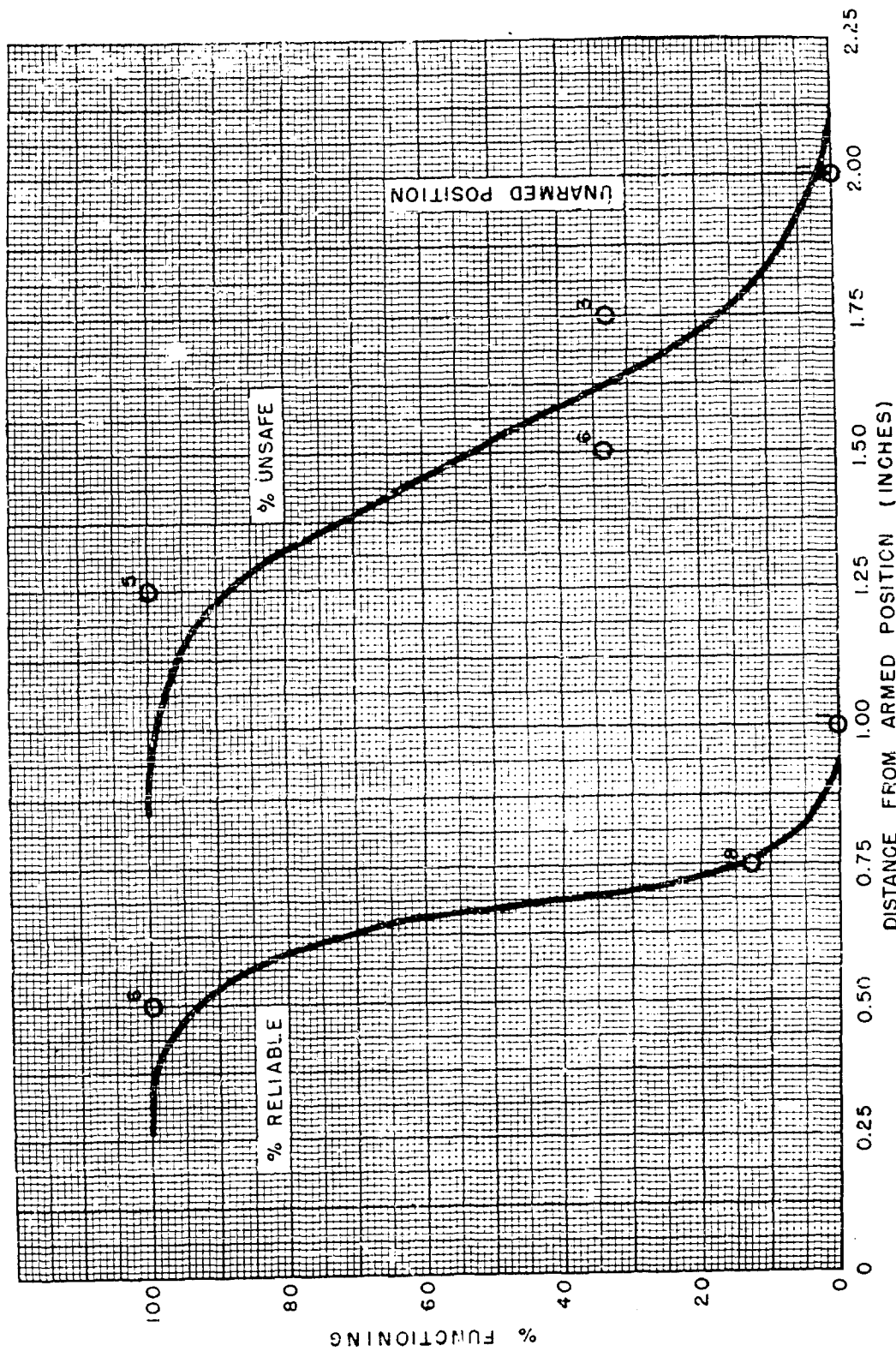
TABLE 7
 PROBIT ANALYSIS
 PRIMACORD PROPAGATION
 ACROSS PLASTIC SHEET BARRIERS

NUMBER OF SHEETS OF PLASTIC	FIRES	FAILURES
2	X X X X	4 0
3	X X X X X X X	9 1
3-1/2	X O X X X O X X X	7 3
4	O X X X X X X X X X X	11 5
4-1/2	O X X X X X X X X X X X X	9 8
5	O O O O X O O O X O O O X O	3 13
5-1/2	O X O O X O X X O O O	4 7
6	O O O O O O O O O	0 9
8	O	0 1
		47 47

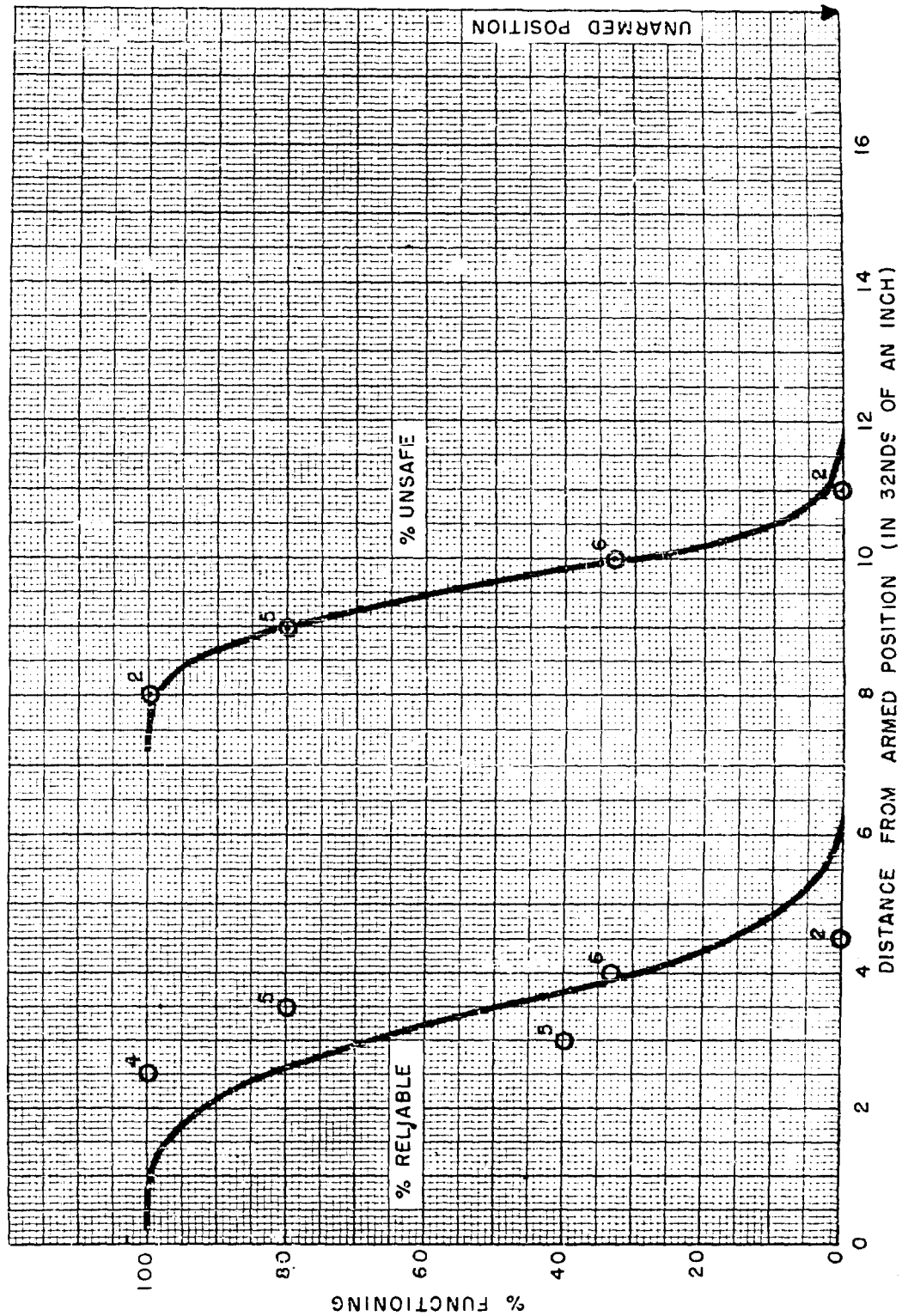
50% Reliable at 4-1/2 sheets.
 Standard Deviation equals 1-1/4 sheets.



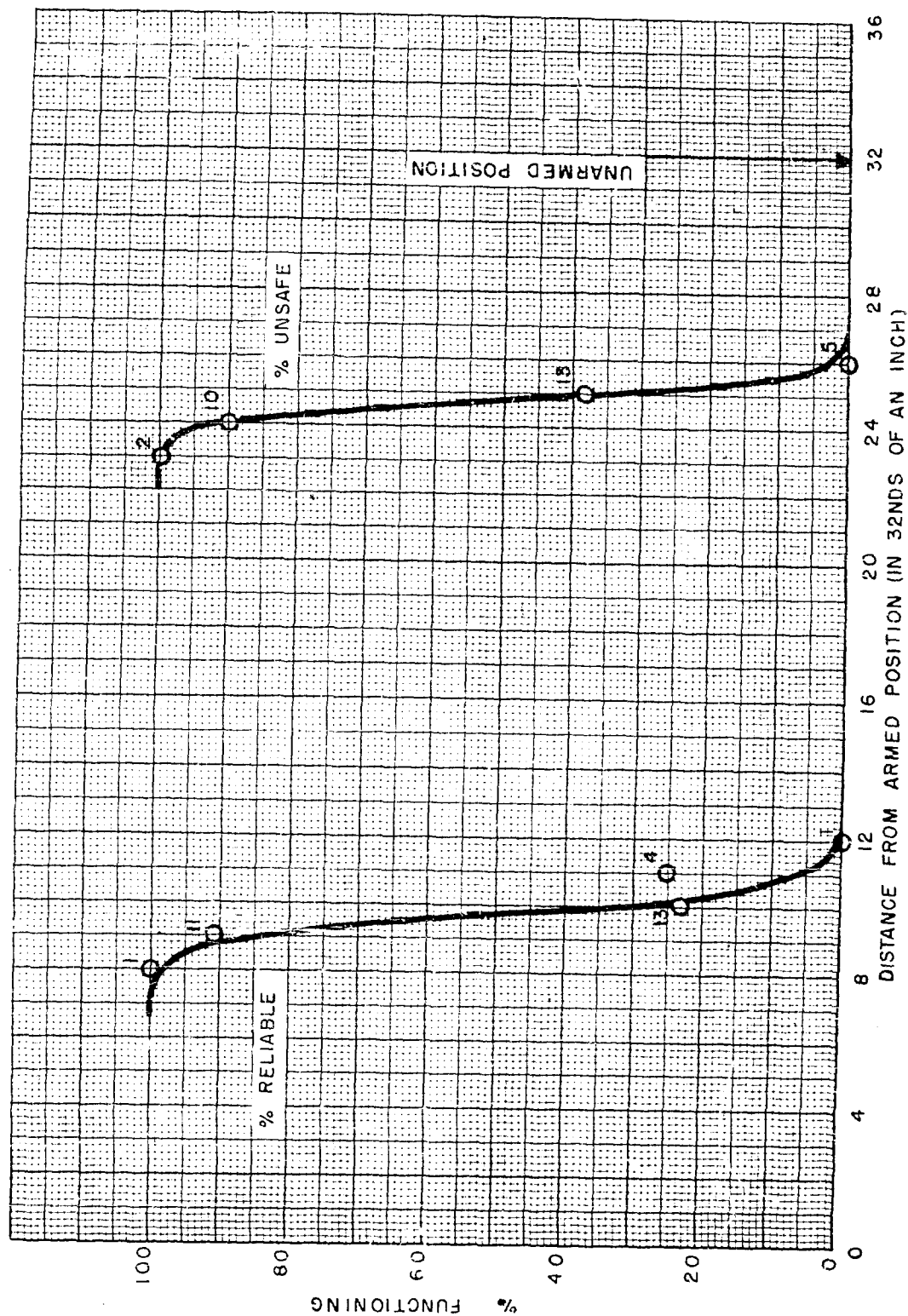
GRAPH I PROBIT ANALYSIS OF EXPLOSIVE TRAIN SAFETY AND RELIABILITY
OF A GUIDED MISSILE FUZE



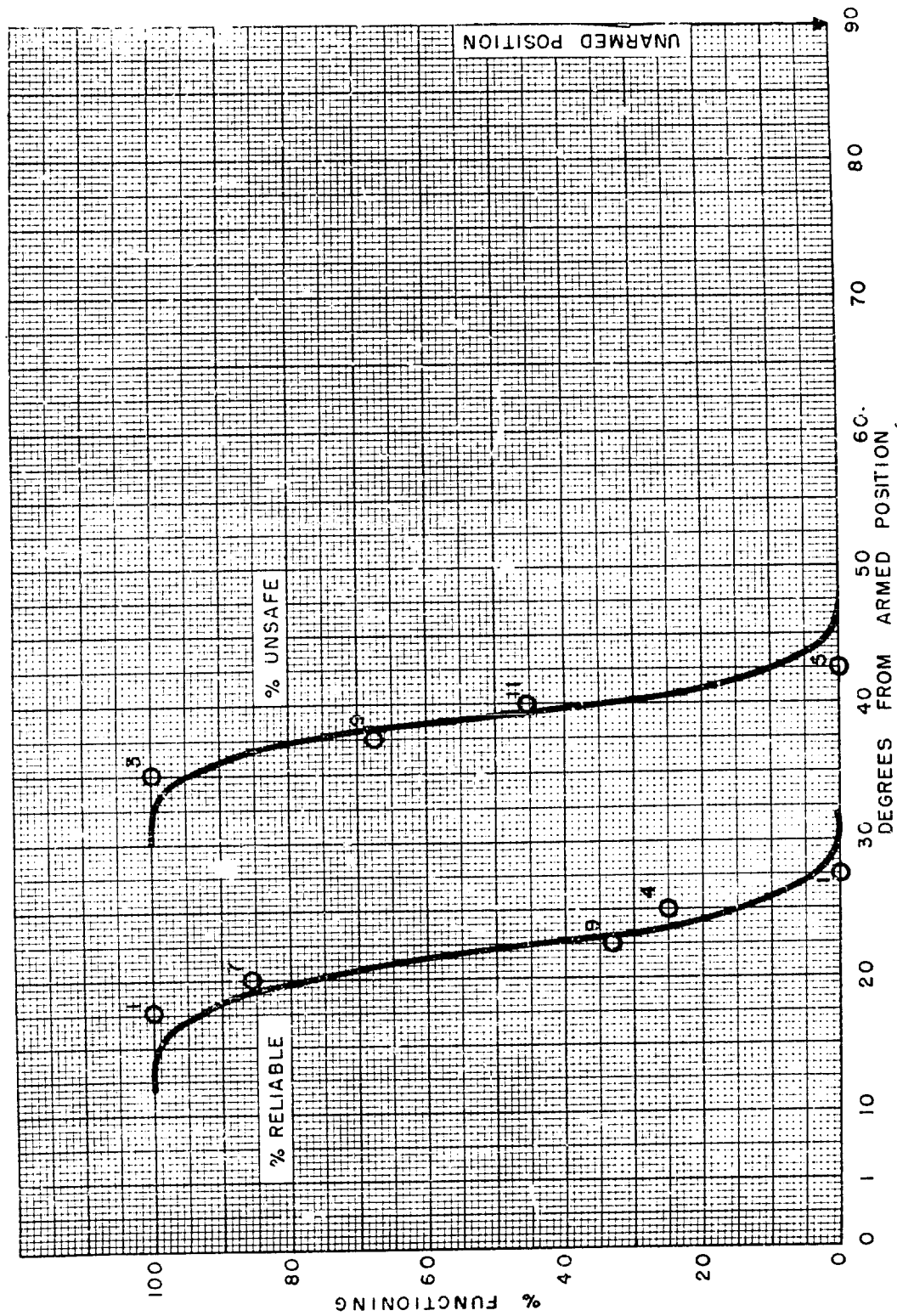
GRAPH 2 BRUCETON ANALYSIS OF EXPLOSIVE TRAIN SAFETY AND RELIABILITY
OF A MINE FIRING DEVICE



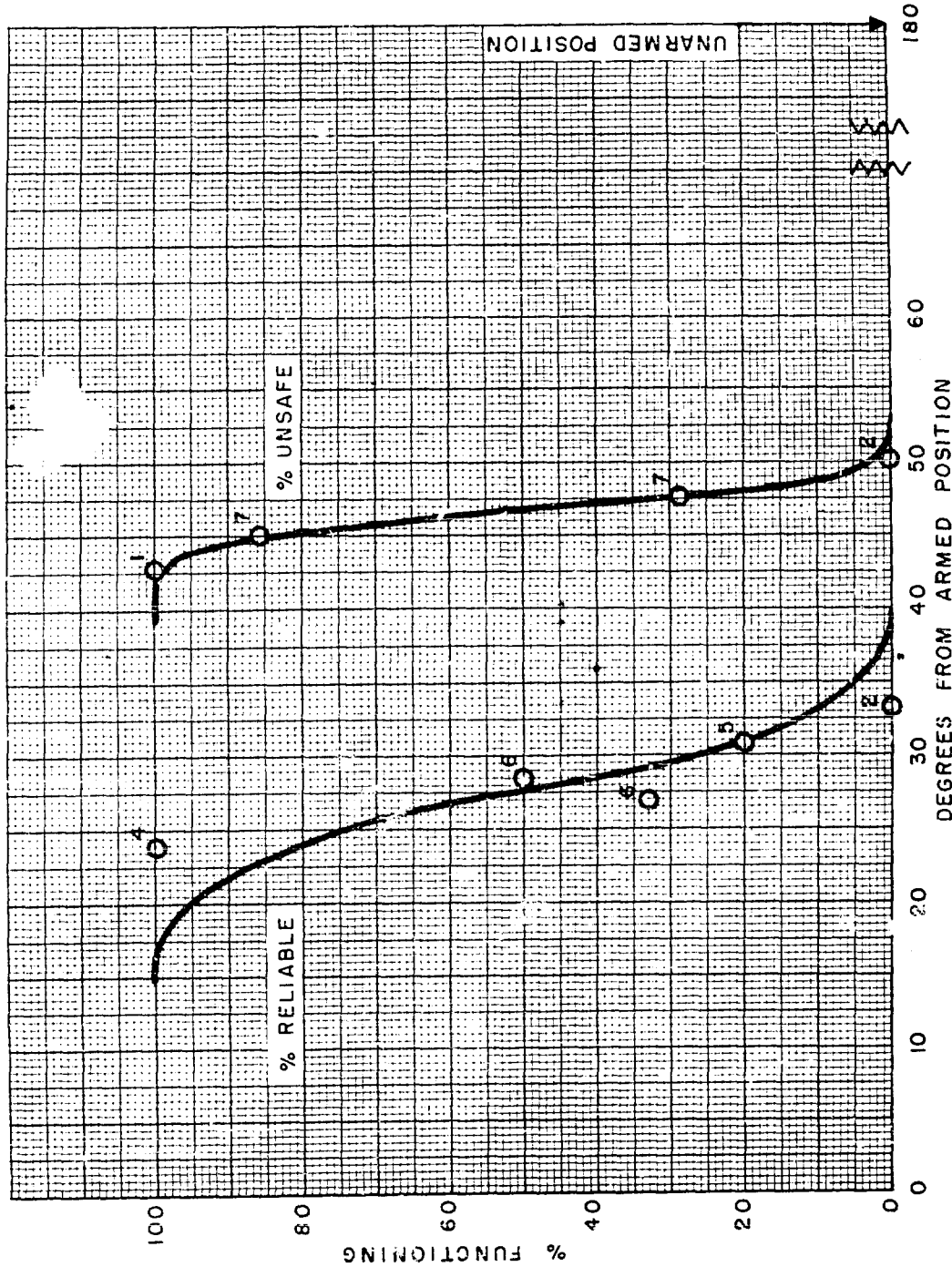
GRAPH 3 BRUCETON ANALYSIS OF EXPLOSIVE TRAIN SAFETY AND RELIABILITY OF A BOMB FUZE



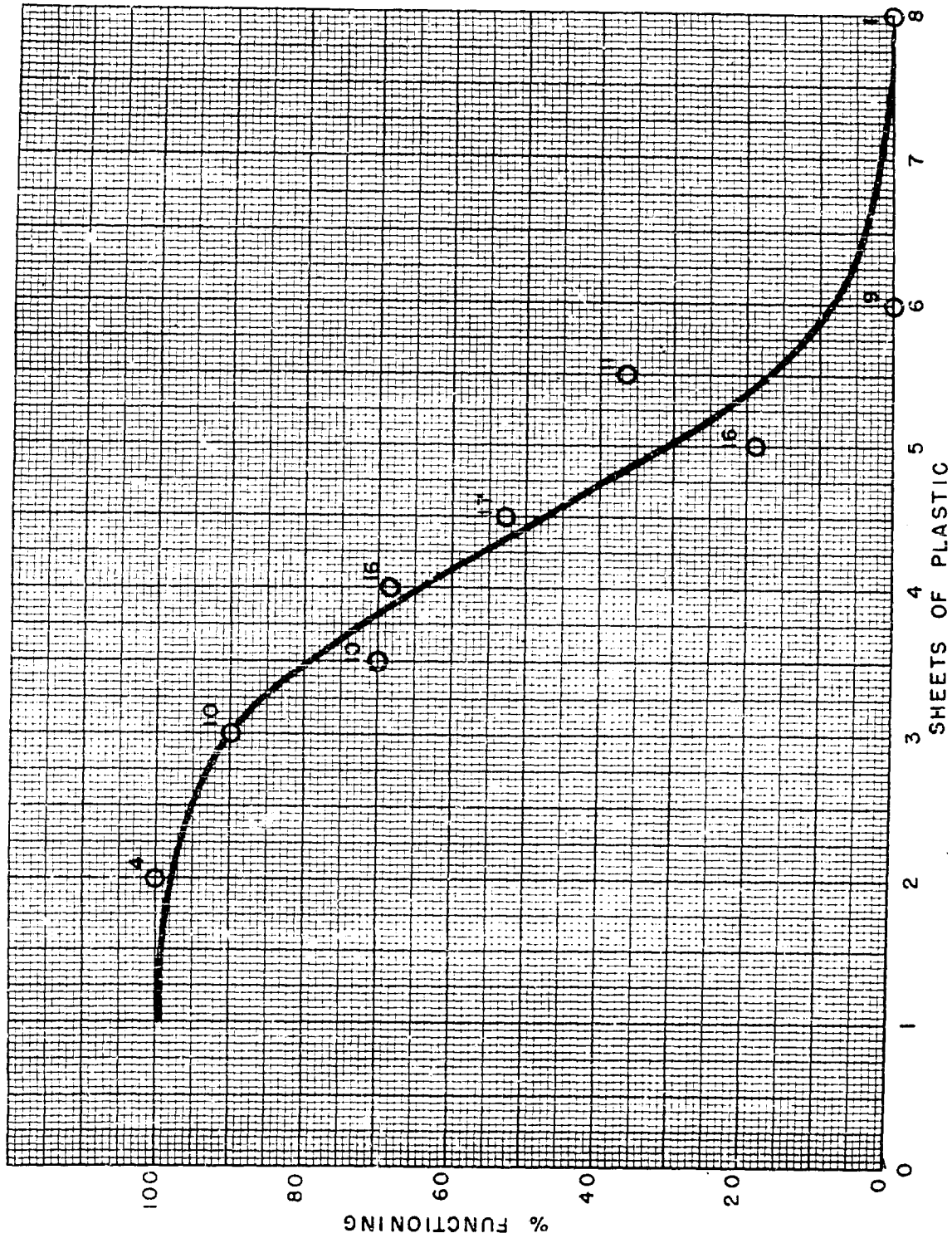
GRAPH 4 BRUCETON ANALYSIS OF EXPLOSIVE TRAIN SAFETY AND RELIABILITY OF A TORPEDO FIRING DEVICE



GRAPH 5 BRUCETON ANALYSIS OF EXPLOSIVE TRAIN SAFETY AND RELIABILITY
OF A TORPEDO ARMING DEVICE



GRAPH 6 BRUCETON ANALYSIS OF EXPLOSIVE TRAIN SAFETY AND RELIABILITY OF A GUIDED MISSILE FUZE



GRAPH 7 PROBIT ANALYSIS OF PRIMACORD PROAGATION
ACROSS PLASTIC SHEET BARRIERS

APPENDIX A

THE BRUCETON METHOD OF SENSITIVITY TESTING

INTRODUCTION

1. In order to illustrate the basic statistical application of the Bruceton Method, an example of an out-of-line safety device arrangement, consisting of a primer, a detonator housed in a slider, and a lead, will be used. Given a sample size of 30 units with a distance of one inch between the "armed" and "safe" positions, the following procedure may be used to determine reliability estimates.

Test Procedure

2. Choose a distance, k , measured from "armed" position, at which the slider will be placed for the first sample. This distance should be the position at which roughly $1/2$ of the samples will fire.

3. If the first unit fires when tested at a distance k , the second one should be tested at a distance $k + d$, where d is a preassigned increment between test distances. If the first unit does not fire, the second one should be tested at a distance $k - d$. Each succeeding unit will be tested at a distance dependent on the firing behavior of the previous unit: if the previous unit fired, the next test distance will be d greater than the previous distance; or, if the previous unit failed to fire, the next test distance will be d less than the previous distance. Thus, the test results will be a sequence of fires (X) and failures (O) centered about the 50%-firing distance, $\bar{X}_R$.

4. For the example described, a distance of $10/32$ of an inch was chosen for the first unit and increments of $1/32$ of an inch for d . The test results formed the following sequence:

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Distance From
Armed Position
(in 32nd's of an inch)

k - 2d = 8 X

k - d = 9 X X X X X X X X X X

k = 10 X X X X X X X X X X

k + d = 11 X X X X X X X X X X

k + 2d = 12 X X X X X X X X X X

Calculation of the 50%-Reliable Distance

5. The data are tabulated by recording the number of X's and the number of O's for each test distance. The test distances are assigned levels, i (of equal unit increments), with the level at which no failures were obtained set equal to zero. (Caution: If no failures were obtained at more than one level, the $i = 0$ level is that distance which is only d less than the level at which failures were obtained. Data from test distances less than the $i = 0$ level should be disregarded.) The calculations of $\bar{X}_R$ and σ_R involve only the zeros or only the X's; i.e., the set which contains the fewer number of results, n , is used. The tabulation for the example cited is as follows:

Distance From Armed Position (32nd's of an inch)	i	n_1 x	0	$i n_1$	$i^2 n_1$
8	0	1	0	0	0
9	1	10	1	10	10
10	2	3	10	6	12
11	3	1	3	3	9
12	4	0	1	0	0
		$n=15$	15	$A=19$	$B=31$

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Since the number of X's and number of O's were equal, the X's were chosen arbitrarily for the calculation.

6. The $\bar{X}_R$ is obtained by the formula,

$$\bar{X}_R = c + d (A/n + 1/2),$$

where the plus sign is taken when the X's are used and the minus sign when the O's are used. The smallest distance tested is 0.001, the $1 = 0$ level in terms of inches. For the example above, the values are,

$$c = 8 \quad A = 19$$

$$d = 1 \quad n = 15,$$

so $\bar{X}_R$ is,

$$\begin{aligned}\bar{X}_R &= 8 + 1(19/15 + 1/2) \\ &= 8 + 1.3 = 9.3 \text{ (in 32nd's of an inch)}\end{aligned}$$

Calculation of the Standard Deviation (σ_R)

7. The standard deviation is determined by the following steps.

a. Find a value $M = (nB - A^2)/n^2$.

b. Find the value of s corresponding to the calculated value M . Table 1 or Graph 1 give s for values of $M > .40$, or graph 2 gives s for values of $M < .40$.

c. Determine $\sigma_R = sd$.

For the example the calculations are as follows:

$$\begin{aligned}M &= (15 \times 31 - (19)^2)/(15)^2 \\ &= (465 - 361)/225 = 0.46, \text{ then}\end{aligned}$$

from table 1,

$$s = .79, \text{ and}$$

$$\sigma_R = .79 (1) = 0.8 \text{ (in 32nd's of an inch).}$$

Calculation of Percentage-Reliable Curve

8. Distances at which a given percentage of the devices will function may be estimated by adding to or subtracting from $\bar{X}_R$ certain multiples of σ_R . Percentage-reliable distances may be estimated from $\bar{X}_R \pm t\sigma_R$ by choosing the proper value of t shown below for specified percent points. The percentage distances less than 50 are obtained by using the plus sign in the above expression, and those greater than 50 (given in the parentheses below) by using the minus sign.

Percent, p	t
50 (50)	0
25 (75)	0.675
10 (90)	1.282
5 (95)	1.645
1 (99)	2.326
0.1 (99.9)	3.090
0.01 (99.99)	3.719

The percentage-reliable curve for the particular example cited is:

Percent, p	$\pm t\sigma_R$	Percentage- Distance (32nd's of an inch) $\bar{X}_R \pm t\sigma_R$
99.99	-2.98	6.8
99.9	-2.47	7.3
99	-1.86	7.9
95	-1.32	8.5
90	-1.03	8.8
75	-0.54	9.3
50	0	9.8
25	+0.54	10.3
10	+1.03	10.8
5	+1.32	11.1
1	+1.86	11.7
0.1	+2.47	12.3
0.01	+2.98	12.8

Sampling Error, $\sigma_{\bar{X}}$, for the 50%-Reliable Estimate

9. The estimate of $\bar{X}_R$ is subject to sampling error which may be calculated from σ_R by the formula,

$$\sigma_{\bar{X}} = \sigma_R G / (n)^{1/2},$$

generally referred to as the standard error of $\bar{X}_R$, where n is the number of O's (or X's) used in the calculation of $\bar{X}_R$ and σ_R , and G is read from Graph 3 at the value s determined in computing σ_R . For the example the error of $\bar{X}_R$ is,

$$\begin{aligned}\sigma_{\bar{X}} &= (0.8)(1.04)/(15)^{1/2} \\ &= 0.22\end{aligned}$$

Sampling Error (σ_{σ}) of the Standard Deviation

10. The estimate of σ_R is also subject to sampling error, which is measured by its standard error, σ_{σ} , given by the relation

$$\sigma_{\sigma} = \sigma_R H / (n)^{1/2},$$

where H is read from Graph 4 using the value of s determined in computing σ_R . This measure calculated for the example is

$$\begin{aligned}\sigma_{\sigma} &= 0.8(1.31)/(15)^{1/2} \\ &= 0.27\end{aligned}$$

Confidence Intervals Determined From $\sigma_{\bar{X}}$ and σ_{σ}

11. The standard errors, $\sigma_{\bar{X}}$ and σ_{σ} , provide measures of precision of the $\bar{X}_R$ estimate and the σ_R estimate. Since $\bar{X}_R$ and σ_R are only estimates of their true values, confidence limits for these estimates which include the true value will indicate the precision of the estimates. If y denotes an estimate and σ_y its standard error, confidence limits may be obtained by

$$y \pm t\sigma_y,$$

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where t is a value given in table 2 using $N = (n - 1)$ and $P = 1$ minus the level of confidence. For the illustrated example, the 90% confidence limits of $\bar{X}_R$ and σ_R may be calculated as follows:

$$N = 15 - 1 = 14,$$

$$P = 1 - .90 = .10,$$

which gives a $t = 1.76$;

therefore, the limits for $\bar{X}_R$ are

$$\begin{aligned}\bar{X}_R \pm 1.76\sigma_{\bar{X}} &= (9.8) \pm 1.76(0.22) \\ &= 9.4 \text{ to } 10.2 \text{ (in 32nd's of an inch)}\end{aligned}$$

and the limits for σ_R are

$$\begin{aligned}\sigma_R \pm 1.76\sigma_{\sigma} &= (0.8) \pm 1.76(0.27) \\ &= 0.3 \text{ to } 1.3 \text{ (in 32nd's of an inch)}.\end{aligned}$$

The probability is 10% that a value lies outside of its 90% confidence limits. Since this 10% probability is the sum of the 5% probability that a value lies above its greater 90% confidence limit and the 5% probability that a value lies below its smaller 90% confidence limit, it can be seen that the probability is 95% that a value will lie above its smaller 90% confidence limit. In other words, a smaller 90% confidence limit is a 95% lower limit since the probability is 95% that the true value lies above it. Similarly, the larger 90% confidence limit is a 95% upper limit.

Conclusions Concerning the Degree of Reliability

12. When only a few samples are used to estimate the reliability of an explosive train, the above confidence limits may be used to determine the "most pessimistic" D_R to be expected. By using the lower limit for $\bar{X}_R$ and the upper limit of σ_R to calculate D_R , the "most pessimistic" value will be obtained. The example used gave a lower limit for $\bar{X}_R$ equal to 9.4 with 1.3 as the upper limit for σ_R . The "most pessimistic" degree of reliability is, then,

$$9.4/1.3 = 7.2,$$

as compared to its average degree of reliability,

$$9.8/0.8 = 12.2.$$

If the "most pessimistic" D_R is greater than the normal deviate of 2.58 which corresponds to 99.5%, then the 99.5% reliability is insured.

The Previous Methods Applied to Safety Tests

13. The Bruceton Method, as outlined, may be readily adapted to safety tests, by recording "unsafe" and "safe" results rather than fires and misfires. If the distances are measured from "armed" position for the safety tests, the method of calculation will be identical to the preceding example. But to obtain D_S , $\bar{X}_U$ must be converted into units from "safe" position. Also, the "most pessimistic" D_S would be the upper limit of $\bar{X}_U$ converted into units from safe position, divided by the upper limit of σ_U .

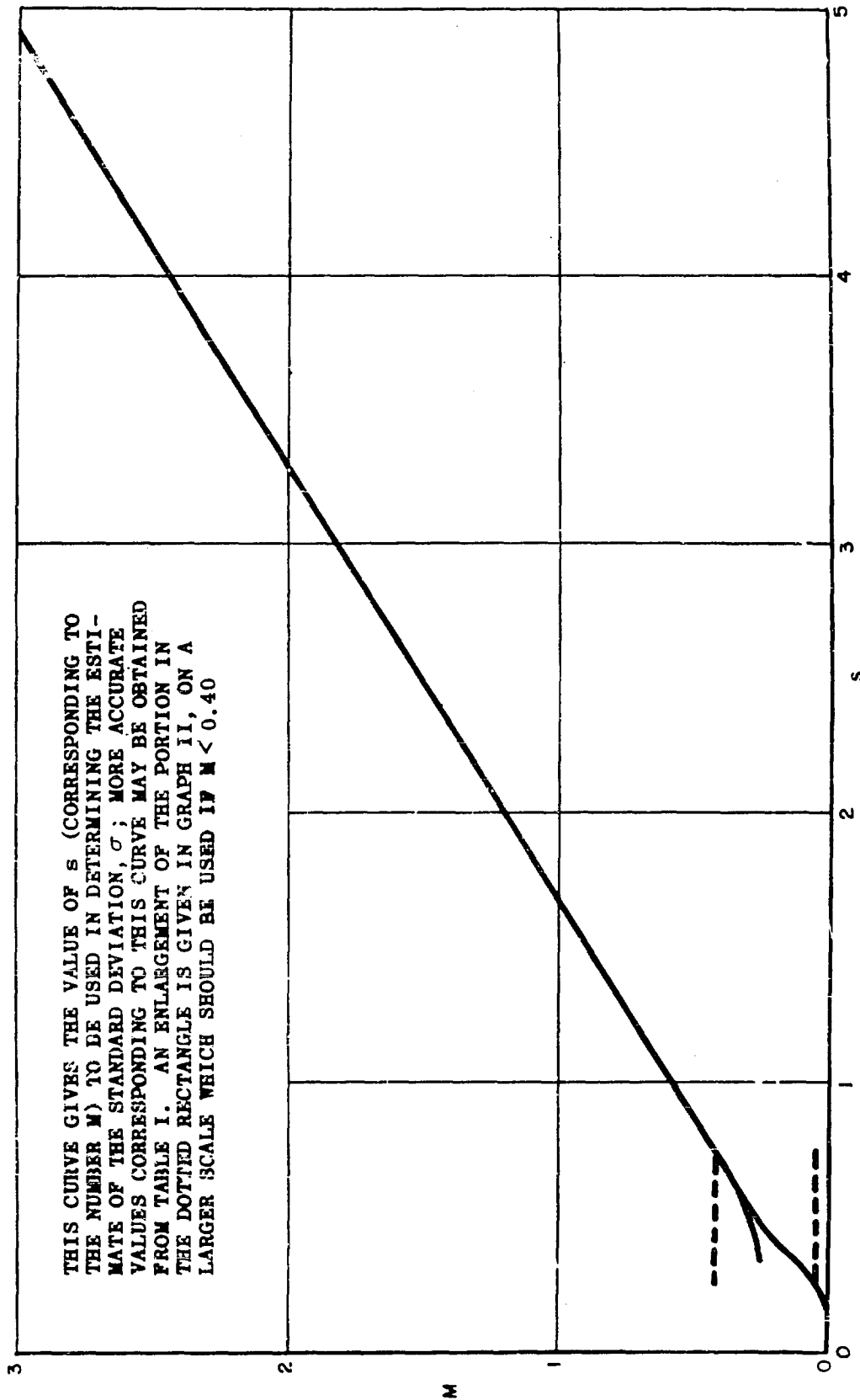
TABLE 1

[illegible]

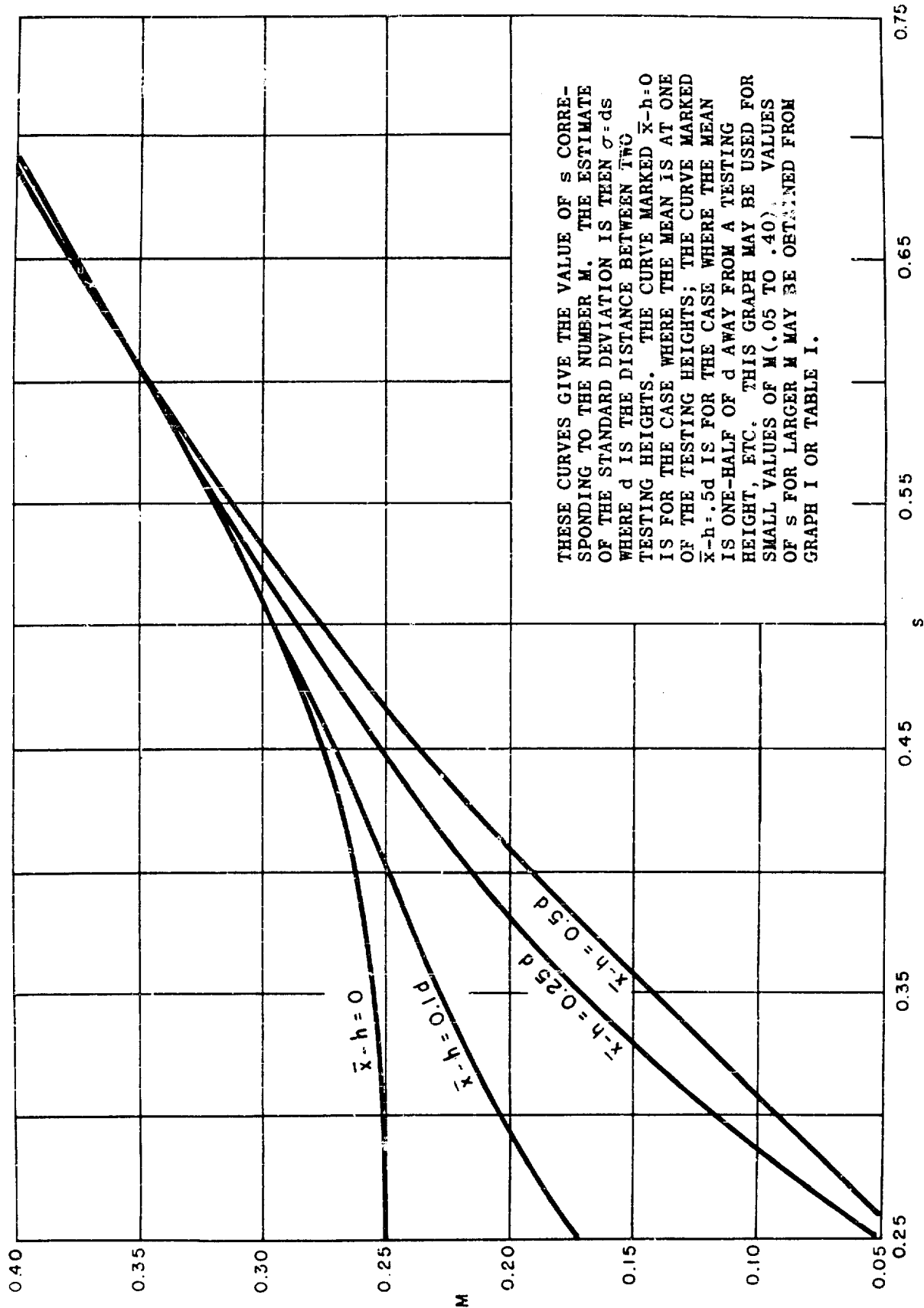
TABLE 2

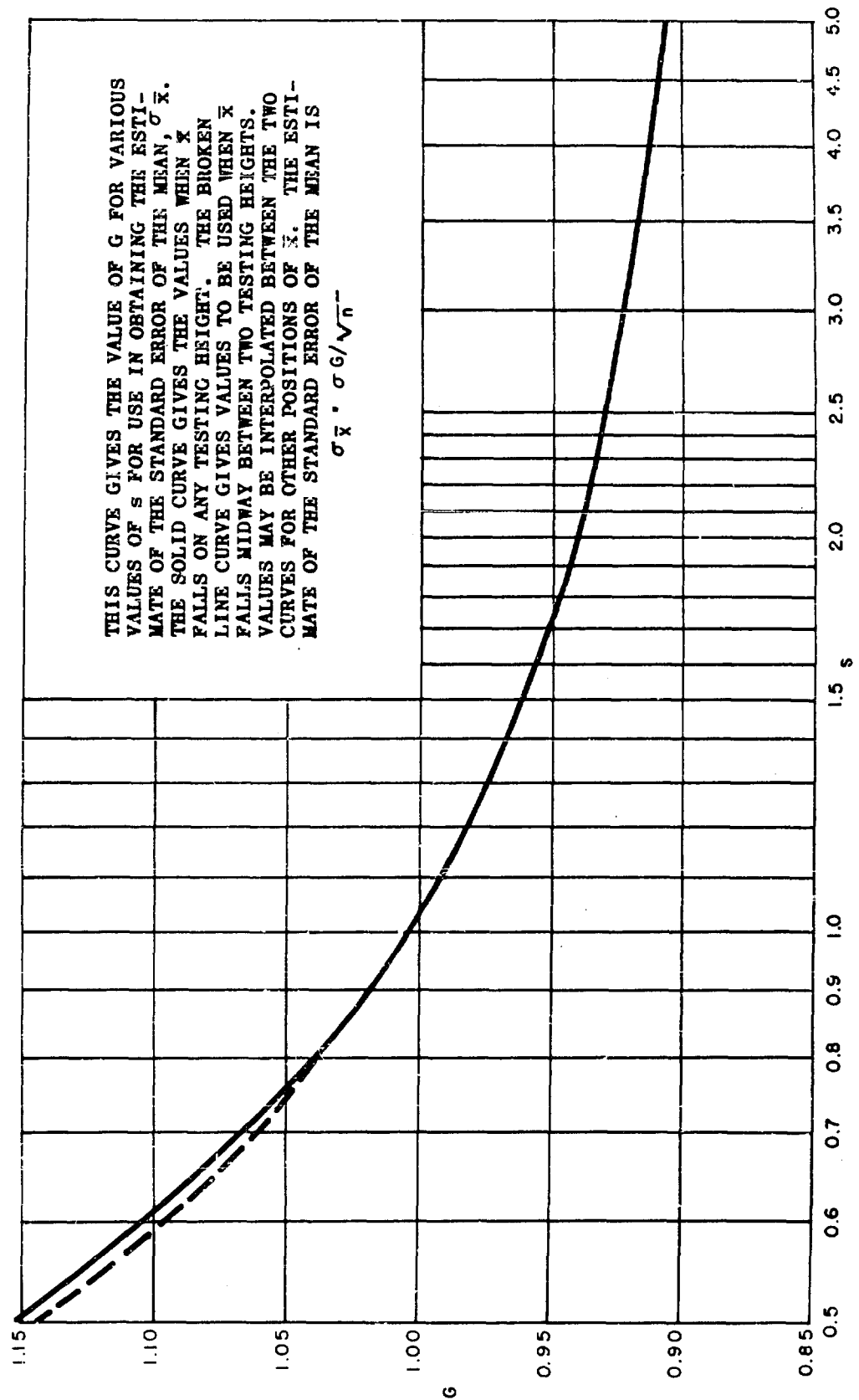
Values of t for $N = (n - 1)$
versus $P = 1 - \text{level of confidence}$

	(P)							
N	.90	.70	.50	.30	.10	.05	.02	.01
1	.16	.51	1.00	1.96	6.31	12.7	31.8	63.7
2	.14	.44	.82	1.39	2.92	4.30	6.96	9.92
3	.14	.42	.76	1.25	2.35	3.18	4.54	5.84
4	.13	.41	.74	1.19	2.13	2.78	3.75	4.60
5	.13	.41	.73	1.16	2.02	2.57	3.36	4.03
6	.13	.40	.72	1.13	1.94	2.45	3.14	3.71
7	.13	.40	.71	1.12	1.90	2.36	3.00	3.50
8	.13	.40	.71	1.11	1.86	2.31	2.90	3.36
9	.13	.40	.70	1.10	1.83	2.26	2.82	3.25
10	.13	.40	.70	1.09	1.81	2.23	2.76	3.17
12	.13	.40	.70	1.08	1.78	2.18	2.68	3.06
14	.13	.39	.69	1.08	1.76	2.14	2.62	2.98
16	.13	.39	.69	1.07	1.75	2.12	2.58	2.92
18	.13	.39	.69	1.07	1.73	2.10	2.55	2.88
20	.13	.39	.69	1.06	1.72	2.09	2.53	2.84
22	.13	.39	.69	1.06	1.72	2.07	2.51	2.82
24	.13	.39	.68	1.06	1.71	2.06	2.49	2.80
26	.13	.39	.68	1.06	1.71	2.06	2.48	2.78
28	.13	.39	.68	1.06	1.70	2.05	2.47	2.76
30	.13	.39	.68	1.06	1.70	2.04	2.46	2.75
40	.13	.39	.68	1.05	1.68	2.02	2.42	2.70
60	.13	.39	.68	1.05	1.67	2.00	2.39	2.66
120	.13	.39	.68	1.04	1.66	1.98	2.36	2.62
∞	.126	.385	.674	1.036	1.645	1.960	2.326	2.576

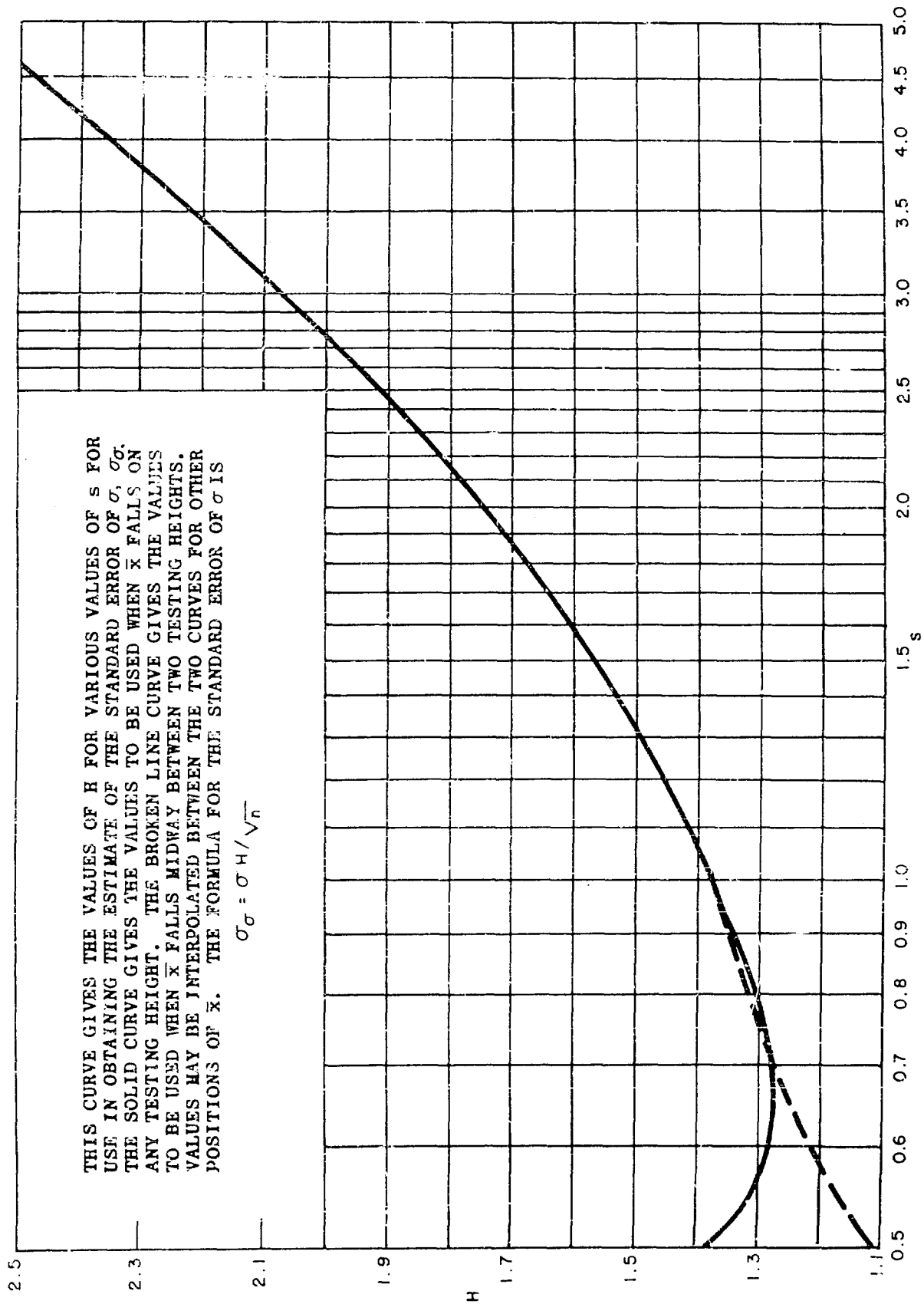


GRAPH 1 CURVES FOR ESTIMATING THE STANDARD DEVIATION ($M > 0.40$)

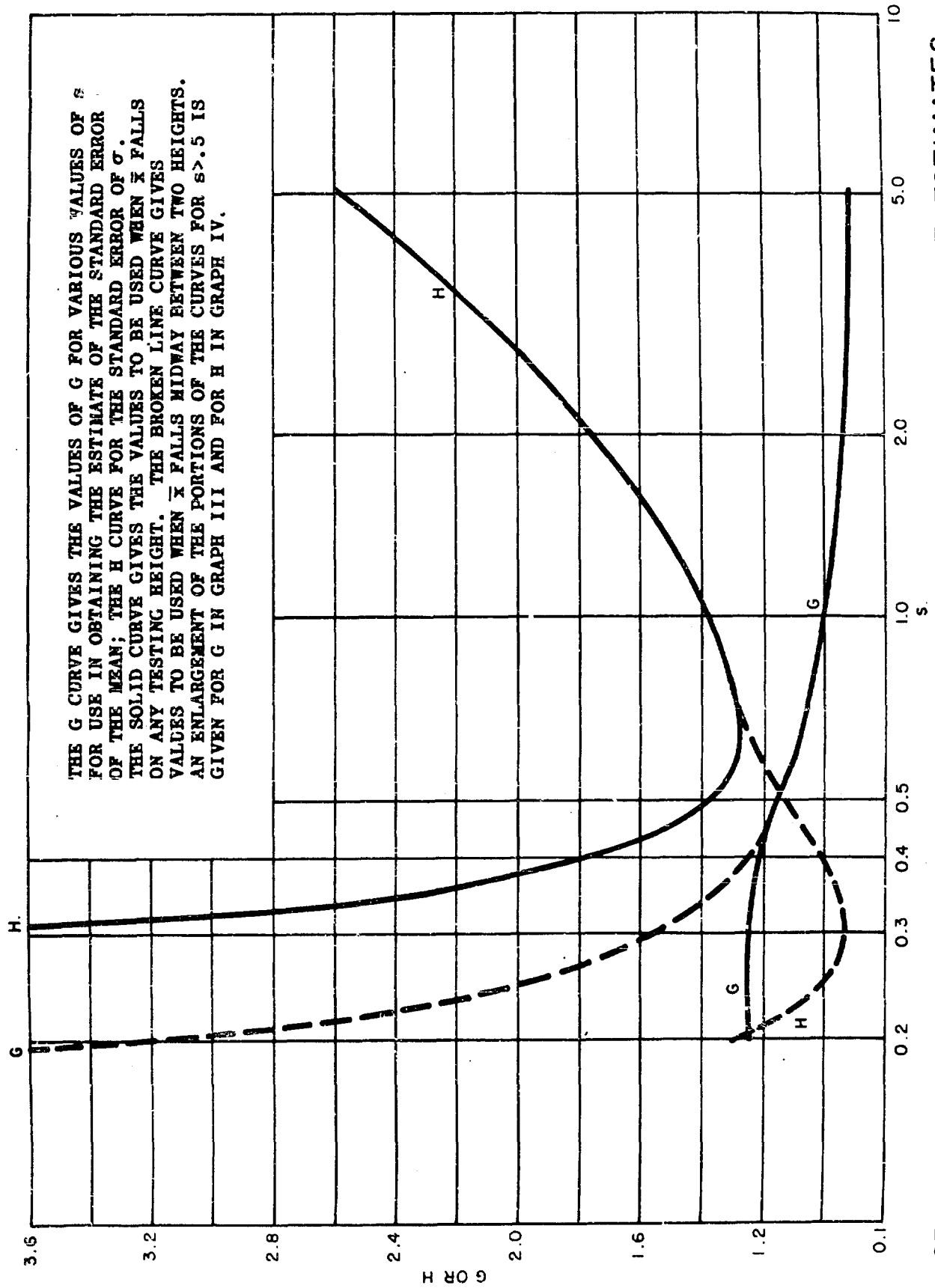

GRAPH 2 CURVES FOR ESTIMATING THE STANDARD DEVIATION ($M < 0.40$)



GRAPH 3 CURVE FOR ESTIMATING THE STANDARD ERROR OF THE MEAN ($s > 0.5$)



GRAPH 4 CURVE FOR ESTIMATING THE STANDARD ERROR OF σ ($s > 0.5$)



GRAPH 5 CURVE FOR ESTIMATING STANDARD ERRORS OF THE ESTIMATES OF THE MEAN AND STANDARD DEVIATION

APPENDIX B

THE PROBIT METHOD OF SENSITIVITY TESTING

INTRODUCTION

1. In order to illustrate the Probit Method, consider a safety test on an explosive train which contains a primer, a slider detonator, and a lead. The slider safety device travels a distance of $16/32$ of an inch between "armed" and "safe" positions. Assume that, when the slider detonator is in any position in which it can be initiated by the primer, it will always initiate the lead. Thus, the safety of the train is dependent on the slider-detonator to lead transfer, since the most unsafe condition exists at this transfer. Therefore, estimates of safety will be obtained by purposely initiating the slider detonator at greater distances than the primer transfer allows and recording the effect on the lead as "safe" (O) or "unsafe" (X). In this manner, if spurious initiation of the slider detonator occurs, it may be determined whether the safety of the train is adequate or not, as the case may be.

Test Procedure for the Probit Method

2. Determine, roughly, the distance from armed position which produces an unsafe condition about 50% of the time when the slider detonator is purposely initiated. The 50% distance may be roughly estimated by testing a small sample of 5 by means of the up-and-down procedure. For illustration, the following results were obtained from an actual test:

Distance from
Armed Position

5/32 ——— X —————
6/32 — O ——— X ——— X ———
7/32 ————— O —————

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3. Samples of at least 10 each, including the first 5 tested above, should be tested at distances around the 50% distance, thus obtaining a range of percentage-distances. Such percentage distances may be obtained in any order and do not require an equal number of samples for each distance tested; nor do the distances tested need to be equally spaced. The high and low percentage distances should be estimated by larger samples than those near the 50% distance, since they are less accurate when estimated from small samples. The results obtained for the illustrative example are tabulated as follows:

Distance from Armed Position (in 32nd's of an inch)		Unsafe X	Safe O	Percent Unsafe P
5	XXXXOXXXXXXXXXXXXX	17	1	94.7
5-3/4	XXXXXOXOXO	7	3	70
6	OXXXXXOXXX	8	2	80
6-1/4	OXOXXOXOXX	3	7	30
7	OXXXXXXOXX	1	9	10
	Total	36	22	

Description of a "Probit"

4. The Probit Analysis is based on the transformation of the percentage unsafe into "Probits". To illustrate this step clearly, some understanding of the theoretical normal curve is necessary. The theoretical normal curve is determined by its two parameters, the 50% unsafe distance, $\bar{X}_U$, and the standard deviation, σ_U . Any specified percentage-unsafe distance may be obtained by integration of the normal curve equation. Tables of this integration are available in most statistical texts which give the percentage of the area under the curve for various distances, x , measured from $\bar{X}_U$ in terms of σ_U , or defined as,

$$t = \frac{x - \bar{X}_U}{\sigma_U}$$

Thus, any desired percentage-unsafe distance, x , may be obtained from $\bar{X}_U$ and σ_U by $x = \bar{X}_U \pm t\sigma_U$. Now if σ_U and $\bar{X}_U$ are not known but, say, a 30% distance is obtained, the table of the normal curve areas will give a t value that represents the difference between $\bar{X}_U$ and the 30% distance in terms of σ_U . Thus, the "Probit" of a percentage-unsafe distance, x , is the number of standard deviations that x is removed from the 50% distance, and, for simplicity in calculation, the probit of $\bar{X}_U$ is designated as 5.0. For percentages below 50% the probits are determined by subtracting the number of standard deviations from 5.0, and for percentages above 50%, the number of standard deviations are added to 5.0. A probit, Y , may be defined as,

$$Y = 5 + \frac{(x - \bar{X}_U)}{\sigma_U}.$$

Table 1 presents a complete tabulation of the transformation of percentages to probits.

Analysis of Test Data by the Provisional Probit Line

5. Step 1 of the analysis is the transformation of the observed percentages to probits, designated as "empirical probits". By using Table 1, these values may be read directly.

6. Step 2 involves plotting a graph of the empirical probits as a function of the distances tested. A provisional straight line is then drawn that best fits the empirical probits, placing the line by eye. For each of the distances tested, the value of the ordinate of the provisional line is read. These are termed the "provisional probits", Y .

7. If the provisional line is drawn so the difference between the empirical probits and the provisional probits is a minimum, and no other line will fit the empirical probits more accurately, the more precise mathematical procedures for determining the probit line may not be needed.

8. The equation for the provisional line may be obtained directly from the graph. The estimate of $\bar{X}_U$ is the x value which corresponds to the probit 5.0. An estimate of σ_U is the increase in x for a unit increase of the probit values.

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The slope of the line, b , is given by $1/\sigma_U$. The equation of the line can then be obtained by substituting the values of $\bar{X}_U$ and b in the equation

$$Y = 5 + b(x - \bar{X}_U).$$

9. As an illustration, the test results presented in paragraph 3 may be analyzed by the methods of paragraphs 5 through 8 as follows:

Distance from Armed Position (in 32nd's of an inch) x	Number Tested n	Percentage Unsafe p	Empirical Probits	Provisional Probits Y
5	18	94.7	6.62	6.6
5-3/4	10	70	5.52	5.5
6	10	80	5.84	5.2
6-1/4	10	30	4.48	4.8
7	10	10	3.72	3.7

The empirical probits were plotted in Graph 1 and the provisional line estimated. The provisional probits were obtained from the provisional line of Graph 1. The $\bar{X}_U$ is the x value corresponding to the probit 5.0, which is 6.12 measured in 32nd's of an inch. A unit increase in the probit value of the provisional line provides a decrease in the value of x equal to 0.69, measured in 32nd's of an inch. This value is an estimate of σ_U , but since it represents a decrease in x rather than an increase, it must be considered as a negative value in determining b . Thus, the slope, b , of the provisional line is $1/\sigma_U$ or $1/-0.69$, which is equal to -1.45 . By substituting $\bar{X}_U = 6.12$, and $b = -1.45$ in the equation

$$Y = 5 + b(x - \bar{X}_U),$$

one may obtain the equation of the provisional probit line in terms of x , or distance from armed position. Thus,

$$Y = 5 - 1.45(x - 6.12)$$

$$\text{or, } Y = 13.87 - 1.45x$$

10. The equation of the provisional line provides a means for estimating the distance at which a specified expected probit occurs. If the distances related to probits between 3.0 and 7.0 are calculated, the percentages corresponding to these probits may be obtained from Table 1, and then the relationship of distances versus percentage-unsafe may be plotted.

11. The foregoing method does not provide a means for determining the error of the $\bar{X}_U$ or σ_U , so it should not be used as the complete analysis unless D_S is considerably larger than 5.0. D_S is obtained by dividing the $\bar{X}_U$, measured from safe position, by σ_U obtained from the provisional line. For the illustrative example:

safe position is 16/32 of an inch from armed position,
and $\bar{X}_U$ is 6.12/32 of an inch from armed position; therefore,

$$D_S = \frac{16 - 6.12}{0.69} = 9.88/.69 = 14.3$$

which indicates that the safety is more than adequate and that further calculation is not necessary. But for illustration and comparison, this example will be used to demonstrate the calculation of the Probit Line.

The Method of Calculating the Probit Line

12. The method of calculating the Probit Line, based on the maximum likelihood theory of reference (c), has been simplified considerably by use of tables which are appended to this report. The explanation of the theoretical basis of the method has been omitted, so the following example is only an illustration of the calculations and application of the tables.

13. The preceding tabulation in paragraph 9 for estimating the provisional line is a necessary part of the calculations for the probit line. Thus, steps 1, 2, and 3 are: (1) transforming the percentages to empirical probits by means of Table 1, (2) plotting the empirical probits and estimating

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the provisional probit line by eye, and, (3) reading the provisional probits, Y , from the provisional line. Step 4 incorporates the weighting coefficient, w , which is read from Table 2 for each provisional probit value. The total number of trials at each distance is n , which is multiplied by w and entered in the tabulation under nw . Step 5 determines a "working probit", y , for each empirical percentage versus its provisional probit. This value is read from Table 3. For the example cited, the tabulation of the data is as follows:

Distance from Armed Position (32nd's of an inch)	Number Tested	Percent Unsafe	Empirical Probits	Provi- sional Probits	Weighting Factor	Working Probits
x	n	p		Y	nw	y
5	18	94.7	6.62	6.6	4.28	6.62
5.75	10	70	5.52	5.5	5.81	5.52
6	10	80	5.84	5.2	6.27	5.76
6.25	10	30	4.48	4.8	6.27	4.49
7	10	10	3.72	3.7	3.36	3.72
					$\Sigma nw = 25.99$	

14. The products nwx and nwy are tabulated and added; division of the totals by Σnw gives the means, $\bar{x}$ and $\bar{y}$. The values nwx are multiplied by x and summed to give Σnwx^2 . Also, they are multiplied by y and added to give $\Sigma nwx y$. The values nwy are multiplied by y and added to give Σnwy^2 . The sums of squares and products about the mean are then obtained as follows:

$$(a) S_{xx} = \Sigma nwx^2 - (\Sigma nwx)^2 / \Sigma nw,$$

$$(b) S_{xy} = \Sigma nwx y - (\Sigma nwx)(\Sigma nwy) / \Sigma nw, \text{ and}$$

$$(c) S_{yy} = \Sigma nwy^2 - (\Sigma nwy)^2 / \Sigma nw.$$

15. The equation of the Probit Line may be obtained by substituting in the equation $Y = \bar{y} + b(x - \bar{x})$ the calculated values

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$$\bar{y} = \frac{\sum nwy}{\sum nw}$$

$$\bar{x} = \frac{\sum nwx}{\sum nw}$$

and $b = S_{xy}/S_{xx}$, which give a linear relationship between Y, in probits, and x, distance from armed position.

16. The previous example provides the following calculations as illustration of paragraphs 14 and 15:

<u>nwx</u>	<u>nwy</u>	<u>nwx²</u>	<u>nwy²</u>	<u>nwxy</u>
21.4000	28.3336	107.000000	187.568432	141.668000
33.4075	32.0712	192.093125	177.033024	184.409400
37.6200	36.1152	225.720000	208.023552	216.691200
39.1875	28.1523	244.921875	126.403827	175.951875
23.5200	12.4992	164.640000	46.497024	87.494400
$\sum nwx = 155.1350 \quad \sum nwy = 137.1715 \quad \sum nwx^2 = 934.375000 \quad \sum nwy^2 = 745.525859 \quad \sum nwxy = 806.214875$				

$$\sum nwx^2 = 934.375000 \quad (\sum nwx)(\sum nwy)/\sum nw = 818.78032$$

$$\sum nwy^2 = 745.525859 \quad (\sum nwx)^2/\sum nw = 926.00493$$

$$\sum nwxy = 806.214875 \quad (\sum nwy)^2/\sum nw = 723.97154$$

thus,

$$(a) \quad S_{xx} = 934.375000 - 926.00493 = 8.37007,$$

$$(b) \quad S_{yy} = 745.525859 - 723.97154 = 21.55432, \text{ and}$$

$$(c) \quad S_{xy} = 806.214875 - 818.78032 = 12.56544.$$

The preceding measures allow the calculations of b, $\bar{x}$ and $\bar{y}$ for substitution in the equation for the Probit Line, $Y = \bar{y} + b(x - \bar{x})$. The calculated values for the example are

$$\bar{y} = 137.1715/25.99 = 5.278$$

$$\bar{x} = 155.1350/25.99 = 5.969$$

$$\text{and,} \quad b = -12.56544/8.37007 = -1.5012.$$

Therefore, the Probit Line equation for the example is

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$$Y = 5.278 - 1.501(x - 5.969)$$

$$\text{or } Y = 14.237 - 1.501x$$

Calculation of the 50%-Unsafe Distance, $\bar{X}_U$, and Standard Deviation, σ_U

17. The estimated $\bar{X}_U$ is obtained by substituting in the Probit Line equation the value 5.0 for the probit value Y and solving for x. For the example

$$5.0 = 14.237 - 1.501x$$

$$\text{or } x = 9.237/1.501 = 6.15.$$

Thus $\bar{X}_U$ equals 6.15, measured in 32nd's of an inch. The estimated σ_U is equal to $1/b$, or for the example,

$$1/1.501 \text{ which equals } 0.67 \text{ (in 32nd's of an inch).}$$

The Degree of Safety, D_S , and Comparison Between the Provisional Estimates and Calculated Estimates

18. The degree of safety, D_S , is obtained by dividing $\bar{X}_U$, measured from safe position, by σ_U , obtained from the calculated probit line. From paragraph 17, $\bar{X}_U$ for the example is 6.15, measured from armed position in 32nd's of an inch, which gives 9.85 from safe position ($16 - 6.15$), and σ_U is 0.67. Therefore, D_S is found to be $9.85/0.67 = 14.7$.

19. The method of calculating the probit line is an approximation that results in convergence; therefore, if the provisional line differs considerably from the calculated one, it is advisable to perform another cycle of computations and compare the first cycle with the second to see if convergence has been approximated. Thus, a comparison between the eye estimates and the calculated estimates seems in order. For the example, the following measures may be compared:

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Provisional Estimates

$$\bar{X}_U = 6.12$$

$$\sigma_U = 0.69$$

Probit Line,

$$Y = 13.87 - 1.45x$$

$$D_S = 14.3$$

Calculated Estimates

$$\bar{X}_U = 6.15$$

$$\sigma_U = 0.67$$

Probit Line,

$$Y = 14.24 - 1.50x$$

$$D_S = 14.7$$

The difference between the two estimates is so small that no further calculation of the Probit Line is needed. Should there be any doubt concerning the convergence of the provisional line and the calculated line, the calculations described in paragraphs 20 and 21 may be performed to determine whether another cycle of computations is necessary.

Sampling Errors of the 50%-Unsafe Distance and Slope, b, of the Calculated Probit Line

20. The error, $\sigma_{\bar{X}}$, to be expected in the calculated estimate of $\bar{X}_U$ may be obtained by

$$\sigma_{\bar{X}} = \frac{1}{b(\sum n_w)^{1/2}},$$

or for the example

$$\sigma_{\bar{X}} = \frac{1}{1.50(25.99)^{1/2}} = \frac{1}{1.50(5.10)} = 0.13.$$

If the limits obtained from $\bar{X}_U \pm \sigma_{\bar{X}}$ include the provisional $\bar{X}_U$, a further cycle of computations is not necessary. But if the provisional $\bar{X}_U$ is outside these limits another cycle should be performed and comparison made between cycle 1 and cycle 2 estimates. The provisional $\bar{X}_U$ of 6.12 is well within the limits of 6.02 and 6.28, indicating that sufficient convergence of the $\bar{X}_U$ estimates has been obtained.

21. The error, σ_b , to be expected in the calculated slope, b, is determined from

$$\sigma_b = \left(\frac{1}{S_{xx}} \right)^{1/2}.$$

The error for the example is:

$$\sigma_b = \left(\frac{1}{8.37} \right)^{1/2} = (0.1194)^{1/2} = 0.35$$

If the limits obtained from $b \pm \sigma_b$ include the provisional estimate of b , a further cycle of computations is not necessary. But if these limits do not include the provisional b , another cycle should be performed. The provisional b of 1.45 for the example is well within the limits of 1.15 and 1.85, which indicates that sufficient convergence has been obtained.

22. The 90% confidence limits of $\bar{X}_U$ may be determined by

$$\bar{X}_U \pm t\sigma_{\bar{X}},$$

where t is obtained from Table 2 of Appendix A and is the normal deviate for $P = .10$, or $t = 1.65$. The example gives

$$6.15 \pm 1.65(0.13)$$

or, 6.15 ± 0.21 , which gives limits of

$$5.94 \text{ to } 6.36.$$

23. Now, the 90% confidence limits of σ_b , using $t = 1.65$, are obtained by

$$b \pm t\sigma_b,$$

or, for the example:

$$1.50 \pm 1.65(0.35) = 1.50 \pm 0.58,$$

which gives limits of 0.92 to 2.08. By obtaining the reciprocals of the confidence limits of b , one may determine the limits of σ_U , which are

$$1/0.92 \text{ to } 1/2.08,$$

or, $1.09 \text{ to } 0.48$

24. The "most pessimistic" D_S may be calculated from the 90% upper limit of $\bar{X}_U$, measured from safe position, and the 90% upper limit of σ_U as follows, where X_T represents the total distance from armed to safe position:

$$D_S = \frac{X_T - (\bar{X}_U + t\sigma_{\bar{X}})}{1/(b + t\sigma_b)}.$$

For the example, $\bar{X}_U + t\sigma_{\bar{X}}$ is 6.36, and $1/(b + t\sigma_b)$ is 1.09; therefore, D_S is calculated as follows:

$$D_S = \frac{16 - 6.36}{1.09} = 8.8$$

This D_S is considerably better than 2.58, the normal deviate corresponding to 99.5%, therefore, the device would be considered safe.

A Test for Goodness of Fit

25. In some cases, it may be doubtful whether the probit line is actually a measure of the relationship between percentage-unsafe and distance tested. If the empirical probits are scattered in such a fashion that the probit line could be drawn in several positions by eye estimate, it is advisable to calculate the probit line and obtain a measure of "goodness of fit" termed the chi-squared test.

26. A chi-squared value, χ^2 , is calculated as follows:

$$\chi^2_{(k-2)} = S_{yy} - (S_{xy}^2/S_{xx})$$

where $(k - 2)$ refers to the "number of degrees of freedom", or the number of test distances minus 2. By referring to a chi-squared table (available in reference (c) or other standard statistical references), one may ascertain the probability of obtaining such a value with $(k - 2)$ degrees of freedom, providing the probit line is the true relationship between percentage unsafe and distance tested. For the example:

$$\begin{aligned}\chi^2_{(k-2)} &= 21.55432 - (-12.56544)^2/8.37 \\ &= 21.55432 - 18.86383 = 2.69049\end{aligned}$$

with $(k - 2)$ degrees of freedom equal to 3. The chi-squared table gives a probability of 0.47, which means that practically half the time one would expect a larger χ^2 value than

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2.69 from samples explained by the calculated probit line. In other words, the probit line is a "good fit".

27. A probit line which gave a chi-squared value corresponding to a probability less than .05 is considered a poor fit and would not be acceptable as the true relationship between percentage unsafe and distance tested. For such cases, one might determine some function of x for which a calculated probit line would fit more closely, such as $\log x$, x^2 , or $1/x$, etc. The $\log x$ transformation is used to some extent; in which case the probit equation takes the form

$$Y = \bar{y} + b(\log x - \overline{\log x}),$$

and similarly for other transformations.

TABLE 1
TRANSFORMATION OF PERCENTAGES TO PROBITS

%	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	(-∞)	1.9098	2.1218	2.2522	2.3479	2.4242	2.4879	2.5427	2.5911	2.6344
1	2.6737	2.7096	2.7429	2.7738	2.8027	2.8299	2.8556	2.8799	2.9031	2.9251
2	2.9463	2.9665	2.9859	3.0046	3.0226	3.0400	3.0569	3.0732	3.0890	3.1043
3	3.1192	3.1337	3.1478	3.1616	3.1750	3.1881	3.2009	3.2134	3.2256	3.2376
4	3.2493	3.2608	3.2721	3.2831	3.2940	3.3046	3.3151	3.3254	3.3354	3.3454
5	3.3551	3.3648	3.3742	3.3836	3.3928	3.4018	3.4107	3.4195	3.4282	3.4368
6	3.4452	3.4536	3.4618	3.4699	3.4780	3.4859	3.4937	3.5015	3.5091	3.5167
7	3.5242	3.5316	3.5389	3.5462	3.5534	3.5605	3.5675	3.5745	3.5813	3.5882
8	3.5949	3.6016	3.6083	3.6149	3.6213	3.6278	3.6342	3.6405	3.6468	3.6531
9	3.6592	3.6654	3.6715	3.6775	3.6835	3.6894	3.6953	3.7012	3.7070	3.7127
10	3.7184	3.7241	3.7298	3.7354	3.7409	3.7464	3.7519	3.7574	3.7628	3.7681
11	3.7735	3.7788	3.7840	3.7893	3.7945	3.7996	3.8048	3.8099	3.8150	3.8200
12	3.8250	3.8300	3.8350	3.8399	3.8448	3.8497	3.8545	3.8593	3.8641	3.8689
13	3.8736	3.8783	3.8830	3.8877	3.8923	3.8969	3.9015	3.9061	3.9107	3.9152
14	3.9197	3.9242	3.9286	3.9331	3.9375	3.9419	3.9463	3.9506	3.9550	3.9593
15	3.9636	3.9678	3.9721	3.9763	3.9806	3.9848	3.9890	3.9931	3.9973	4.0014
16	4.0055	4.0096	4.0137	4.0178	4.0218	4.0259	4.0299	4.0339	4.0379	4.0419
17	4.0458	4.0498	4.0537	4.0576	4.0615	4.0654	4.0693	4.0731	4.0770	4.0808
18	4.0846	4.0884	4.0922	4.0960	4.0998	4.1035	4.1073	4.1110	4.1147	4.1184
19	4.1221	4.1258	4.1295	4.1331	4.1367	4.1404	4.1440	4.1476	4.1512	4.1546
20	4.1584	4.1619	4.1655	4.1690	4.1726	4.1761	4.1796	4.1831	4.1866	4.1901
21	4.1936	4.1970	4.2005	4.2039	4.2074	4.2108	4.2142	4.2176	4.2210	4.2244
22	4.2278	4.2312	4.2345	4.2379	4.2412	4.2446	4.2479	4.2512	4.2546	4.2579
23	4.2612	4.2644	4.2677	4.2710	4.2743	4.2775	4.2808	4.2840	4.2872	4.2905
24	4.2937	4.2969	4.3001	4.3033	4.3065	4.3097	4.3129	4.3160	4.3192	4.3224

TABLE 1 (CONTINUED)

x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
25	4.3255	4.3287	4.3318	4.3349	4.3380	4.3412	4.3443	4.3474	4.3505	4.3536
26	4.3567	4.3597	4.3628	4.3659	4.3689	4.3720	4.3750	4.3781	4.3811	4.3842
27	4.3872	4.3902	4.3932	4.3962	4.3992	4.4022	4.4052	4.4082	4.4112	4.4142
28	4.4172	4.4201	4.4231	4.4260	4.4290	4.4319	4.4349	4.4378	4.4408	4.4437
29	4.4466	4.4495	4.4524	4.4554	4.4583	4.4612	4.4641	4.4670	4.4698	4.4727
30	4.4756	4.4785	4.4813	4.4842	4.4871	4.4899	4.4928	4.4956	4.4985	4.5013
31	4.5041	4.5070	4.5098	4.5126	4.5155	4.5183	4.5211	4.5239	4.5267	4.5295
32	4.5323	4.5351	4.5379	4.5407	4.5435	4.5462	4.5490	4.5518	4.5546	4.5573
33	4.5601	4.5628	4.5656	4.5684	4.5711	4.5739	4.5766	4.5793	4.5821	4.5848
34	4.5875	4.5903	4.5930	4.5957	4.5984	4.6011	4.6039	4.6066	4.6093	4.6120
35	4.6147	4.6174	4.6201	4.6228	4.6255	4.6281	4.6308	4.6335	4.6362	4.6389
36	4.6415	4.6442	4.6469	4.6495	4.6522	4.6549	4.6575	4.6602	4.6628	4.6655
37	4.6681	4.6708	4.6734	4.6761	4.6787	4.6814	4.6840	4.6866	4.6893	4.6919
38	4.6945	4.6971	4.6998	4.7024	4.7050	4.7076	4.7102	4.7129	4.7155	4.7181
39	4.7207	4.7233	4.7259	4.7285	4.7311	4.7337	4.7363	4.7389	4.7415	4.7441
40	4.7467	4.7492	4.7518	4.7544	4.7570	4.7596	4.7622	4.7647	4.7673	4.7699
41	4.7725	4.7750	4.7776	4.7802	4.7827	4.7853	4.7879	4.7904	4.7930	4.7955
42	4.7981	4.8007	4.8032	4.8058	4.8083	4.8109	4.8134	4.8160	4.8185	4.8211
43	4.8236	4.8262	4.8287	4.8313	4.8338	4.8363	4.8389	4.8414	4.8440	4.8465
44	4.8490	4.8516	4.8541	4.8566	4.8592	4.8617	4.8642	4.8668	4.8693	4.8718
45	4.8743	4.8769	4.8794	4.8819	4.8844	4.8870	4.8895	4.8920	4.8945	4.8970
46	4.8996	4.9021	4.9046	4.9071	4.9096	4.9122	4.9147	4.9172	4.9197	4.9222
47	4.9247	4.9272	4.9298	4.9323	4.9348	4.9373	4.9398	4.9423	4.9448	4.9473
48	4.9498	4.9524	4.9549	4.9574	4.9599	4.9624	4.9649	4.9674	4.9699	4.9724
49	4.9749	4.9774	4.9799	4.9825	4.9850	4.9875	4.9900	4.9925	4.9950	4.9975

TABLE 1 (CONTINUED)

%	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
50	5.0000	5.0025	5.0050	5.0075	5.0100	5.0125	5.0150	5.0175	5.0201	5.0226
51	5.0251	5.0276	5.0301	5.0326	5.0351	5.0376	5.0401	5.0426	5.0451	5.0476
52	5.0502	5.0527	5.0552	5.0577	5.0602	5.0627	5.0652	5.0677	5.0702	5.0728
53	5.0753	5.0778	5.0803	5.0828	5.0853	5.0878	5.0904	5.0929	5.0954	5.0979
54	5.1004	5.1030	5.1055	5.1080	5.1105	5.1130	5.1156	5.1181	5.1206	5.1231
55	5.1257	5.1282	5.1307	5.1332	5.1358	5.1383	5.1408	5.1434	5.1459	5.1484
56	5.1510	5.1535	5.1560	5.1586	5.1611	5.1637	5.1662	5.1687	5.1713	5.1738
57	5.1764	5.1789	5.1815	5.1840	5.1866	5.1891	5.1917	5.1942	5.1968	5.1993
58	5.2019	5.2045	5.2070	5.2096	5.2121	5.2147	5.2173	5.2198	5.2224	5.2250
59	5.2275	5.2301	5.2327	5.2353	5.2378	5.2404	5.2430	5.2456	5.2482	5.2508
60	5.2533	5.2559	5.2585	5.2611	5.2637	5.2663	5.2689	5.2715	5.2741	5.2767
61	5.2793	5.2819	5.2845	5.2871	5.2898	5.2924	5.2950	5.2976	5.3002	5.3029
62	5.3055	5.3081	5.3107	5.3134	5.3160	5.3186	5.3213	5.3239	5.3266	5.3292
63	5.3319	5.3345	5.3372	5.3398	5.3425	5.3451	5.3478	5.3505	5.3531	5.3558
64	5.3585	5.3611	5.3638	5.3665	5.3692	5.3719	5.3745	5.3772	5.3799	5.3826
65	5.3853	5.3880	5.3907	5.3934	5.3961	5.3989	5.4016	5.4043	5.4070	5.4097
66	5.4125	5.4152	5.4179	5.4207	5.4234	5.4261	5.4289	5.4316	5.4344	5.4372
67	5.4399	5.4427	5.4454	5.4482	5.4510	5.4538	5.4565	5.4593	5.4621	5.4649
68	5.4677	5.4705	5.4733	5.4761	5.4789	5.4817	5.4845	5.4874	5.4902	5.4930
69	5.4959	5.4987	5.5015	5.5044	5.5072	5.5101	5.5129	5.5158	5.5187	5.5215
70	5.5244	5.5273	5.5302	5.5330	5.5359	5.5388	5.5417	5.5446	5.5476	5.5505
71	5.5534	5.5563	5.5592	5.5622	5.5651	5.5681	5.5710	5.5740	5.5769	5.5799
72	5.5828	5.5858	5.5888	5.5918	5.5948	5.5978	5.6008	5.6038	5.6068	5.6098
73	5.6128	5.6158	5.6189	5.6219	5.6250	5.6280	5.6311	5.6341	5.6372	5.6403
74	5.6433	5.6464	5.6495	5.6526	5.6557	5.6588	5.6620	5.6651	5.6682	5.6713

TABLE 1 (CONTINUED)

λ	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
75	5.6745	5.6776	5.6808	5.6840	5.6871	5.6903	5.6935	5.6967	5.6999	5.7031
76	5.7063	5.7095	5.7128	5.7160	5.7192	5.7225	5.7257	5.7290	5.7323	5.7356
77	5.7388	5.7421	5.7454	5.7488	5.7521	5.7554	5.7588	5.7621	5.7655	5.7688
78	5.7722	5.7756	5.7790	5.7824	5.7858	5.7892	5.7926	5.7961	5.7995	5.8030
79	5.8064	5.8099	5.8134	5.8169	5.8204	5.8239	5.8274	5.8310	5.8345	5.8381
80	5.8416	5.8452	5.8488	5.8524	5.8560	5.8596	5.8633	5.8669	5.8705	5.8742
81	5.8779	5.8816	5.8853	5.8890	5.8927	5.8965	5.9002	5.9040	5.9078	5.9116
82	5.9154	5.9192	5.9230	5.9269	5.9307	5.9346	5.9385	5.9424	5.9463	5.9502
83	5.9524	5.9581	5.9621	5.9661	5.9701	5.9741	5.9782	5.9822	5.9863	5.9904
84	5.9945	5.9986	6.0027	6.0069	6.0110	6.0152	6.0194	6.0237	6.0279	6.0322
85	6.0364	6.0407	6.0450	6.0494	6.0537	6.0581	6.0625	6.0669	6.0714	6.0758
86	6.0803	6.0848	6.0893	6.0939	6.0985	6.1031	6.1077	6.1123	6.1170	6.1217
87	6.1264	6.1311	6.1359	6.1407	6.1455	6.1503	6.1552	6.1601	6.1650	6.1700
88	6.1750	6.1800	6.1850	6.1901	6.1952	6.2004	6.2055	6.2107	6.2160	6.2212
89	6.2265	6.2319	6.2372	6.2426	6.2481	6.2536	6.2591	6.2646	6.2702	6.2759
90	6.2816	6.2873	6.2930	6.2988	6.3047	6.3106	6.3165	6.3225	6.3285	6.3346
91	6.3408	6.3469	6.3532	6.3595	6.3658	6.3722	6.3787	6.3852	6.3917	6.3984
92	6.4051	6.4118	6.4187	6.4255	6.4325	6.4395	6.4466	6.4538	6.4611	6.4684
93	6.4758	6.4833	6.4909	6.4985	6.5063	6.5141	6.5220	6.5301	6.5382	6.5464
94	6.5548	6.5632	6.5718	6.5805	6.5893	6.5982	6.6072	6.6164	6.6258	6.6352
95	6.6449	6.6546	6.6646	6.6747	6.6849	6.6954	6.7060	6.7169	6.7279	6.7392
96	6.7507	6.7624	6.7744	6.7866	6.7991	6.8119	6.8250	6.8384	6.8522	6.8653
97	6.8808	6.8957	6.9110	6.9268	6.9431	6.9600	6.9774	6.9954	7.0141	7.0335

TABLE 1 (CONTINUED)

%	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
98.0	7.0537	7.0558	7.0579	7.0600	7.0621	7.0642	7.0663	7.0684	7.0706	7.0727
98.1	7.0749	7.0770	7.0792	7.0814	7.0836	7.0858	7.0880	7.0902	7.0924	7.0947
98.2	7.0969	7.0992	7.1015	7.1038	7.1061	7.1084	7.1107	7.1130	7.1154	7.1177
98.3	7.1201	7.1224	7.1248	7.1272	7.1297	7.1321	7.1345	7.1370	7.1394	7.1419
98.4	7.1444	7.1469	7.1494	7.1520	7.1545	7.1571	7.1596	7.1622	7.1648	7.1675
98.5	7.1701	7.1727	7.1754	7.1781	7.1808	7.1835	7.1862	7.1890	7.1917	7.1945
98.6	7.1973	7.2001	7.2029	7.2058	7.2086	7.2115	7.2144	7.2173	7.2203	7.2232
98.7	7.2262	7.2292	7.2322	7.2353	7.2383	7.2414	7.2445	7.2476	7.2508	7.2539
98.8	7.2571	7.2603	7.2636	7.2668	7.2701	7.2734	7.2768	7.2801	7.2835	7.2869
98.9	7.2904	7.2938	7.2973	7.3009	7.3044	7.3080	7.3116	7.3152	7.3189	7.3226
99.0	7.3263	7.3301	7.3339	7.3378	7.3416	7.3455	7.3495	7.3535	7.3575	7.3615
99.1	7.3656	7.3698	7.3739	7.3781	7.3824	7.3867	7.3911	7.3954	7.3999	7.4044
99.2	7.4089	7.4135	7.4181	7.4228	7.4276	7.4324	7.4372	7.4422	7.4471	7.4522
99.3	7.4573	7.4624	7.4677	7.4730	7.4783	7.4838	7.4893	7.4949	7.5006	7.5063
99.4	7.5121	7.5181	7.5241	7.5302	7.5364	7.5427	7.5491	7.5556	7.5622	7.5690
99.5	7.5758	7.5828	7.5899	7.5972	7.6045	7.6121	7.6197	7.6276	7.6356	7.6437
99.6	7.6521	7.6606	7.6693	7.6783	7.6874	7.6968	7.7065	7.7164	7.7266	7.7370
99.7	7.7478	7.7589	7.7703	7.7822	7.7944	7.8070	7.8202	7.8338	7.8480	7.8627
99.8	7.8782	7.8943	7.9112	7.9290	7.9478	7.9677	7.9889	8.0115	8.0357	8.0618
99.9	8.0902	8.1214	8.1559	8.1947	8.2389	8.2905	8.3528	8.4316	8.5401	8.7190
100	∞									

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TABLE 2

[illegible]

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TABLE 3

WORKING PROBITS (y)

Obs.

[illegible]

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TABLE 3 (CONTINUED)

Obs. %	provisional probits, Y									
p	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9
0	2.579	2.662	2.745	2.826	2.906	2.984	3.061	3.135	3.207	3.277
1	2.764	2.815	2.872	2.932	2.995	3.061	3.127	3.193	3.259	3.323
2	.949	.967	.998	3.039	3.086	.139	.194	.252	.310	.369
3	3.134	3.120	3.125	.145	.176	.216	.261	.310	.362	.415
4	.319	.272	.252	.251	.267	.293	.328	.362	.413	.461
5	.505	.424	.378	.358	.357	.370	.395	.427	.465	.507
6	3.690	3.577	3.505	3.464	3.447	3.447	3.461	3.485	3.516	3.553
7	.875	.729	.632	.570	.537	.525	.528	.544	.568	.599
8	4.060	.882	.758	.677	.627	.602	.595	.602	.619	.645
9	.246	4.034	.885	.783	.717	.679	.662	.660	.671	.690
10	.431	.186	4.012	.889	.808	.756	.728	.719	.722	.736
11	4.616	4.339	4.138	3.996	3.898	3.834	3.795	3.777	3.774	3.782
12	.801	.491	.265	4.102	.988	.911	.862	.835	.825	.828
13	.986	.644	.391	.208	4.078	.988	.929	.894	.877	.874
14	5.172	.796	.518	.315	.168	4.065	.996	.952	.928	.920
15	.357	.948	.645	.421	.258	.142	4.062	4.010	.980	.966
16	5.542	5.101	4.771	4.527	4.348	4.220	4.129	4.069	4.031	4.012
17	.727	.253	.898	.634	.439	.297	.196	.127	.083	.058
18	.913	.406	5.025	.740	.529	.374	.263	.185	.134	.104
19	6.098	.558	.151	.846	.619	.451	.330	.244	.186	.149
20	.283	.710	.278	.953	.709	.528	.396	.302	.237	.195
21	6.468	5.863	5.405	5.059	4.799	4.606	4.463	4.361	4.289	4.241
22	.653	6.015	.531	.165	.889	.683	.530	.419	.340	.287
23	.839	.168	.658	.272	.979	.760	.597	.477	.392	.333
24	7.024	.320	.785	.378	5.070	.837	.664	.536	.443	.379
25	.209	.472	.911	.484	.160	.914	.730	.594	.495	.425
26	7.394	6.625	6.038	5.591	5.250	4.992	4.797	4.652	4.546	4.471
27	.580	.777	.165	.697	.340	5.069	.864	.711	.598	.517
28	.765	.930	.291	.803	.430	.146	.931	.769	.649	.563
29	.950	7.082	.418	.910	.520	.223	.997	.827	.701	.608
30	8.135	.234	.545	6.016	.610	.300	5.064	.886	.752	.654
31	8.320	7.387	6.671	6.122	5.701	5.378	5.131	4.944	4.804	4.700
32	.506	.539	.798	.229	.791	.455	.198	5.002	.855	.746
33	.691	.692	.925	.335	.881	.532	.265	.061	.907	.792
34	.876	.844	7.051	.441	.971	.609	.331	.119	.958	.838
35	9.061	.996	.178	.548	6.061	.687	.398	.177	5.010	.884
36	9.247	8.149	7.305	6.654	6.151	5.764	5.465	5.236	5.061	4.930
37	.432	.301	.431	.760	.242	.841	.532	.294	.113	.976
38	.617	.454	.558	.867	.332	.918	.599	.353	.164	5.022
39	.802	.606	.685	.973	.422	.995	.665	.411	.216	.068
40	.987	.758	.811	7.079	.512	6.073	.732	.469	.267	.113
41		8.911	7.938	7.186	6.602	6.150	5.799	5.528	5.319	5.159
42		9.063	8.065	.292	.692	.227	.866	.586	.370	.205
43		.216	.191	.398	.782	.304	.932	.644	.422	.251
44		.368	.318	.505	.873	.381	.999	.703	.473	.297
45		.520	.445	.611	.963	.459	6.066	.761	.525	.343
46		9.673	8.571	7.717	7.053	6.536	6.133	5.819	5.576	5.389
47		.825	.698	.824	.143	.613	.200	.878	.628	.435
48		.978	.825	.930	.233	.690	.266	.936	.679	.481
49			.951	8.036	.323	.767	.333	.994	.731	.527
50			9.078	.143	.414	.845	.400	6.053	.78	.572

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TABLE 3 (CONTINUED)

Obs.

y	provisional probits, Y									
	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9
0	3.344	3.408	3.469	3.525	3.577	3.624	3.664	3.698	3.724	3.741
1	3.386	3.446	3.503	3.557	3.607	3.652	3.691	3.724	3.750	3.766
2	.427	.487	.538	.589	.637	.680	.719	.751	.775	.791
3	.468	.521	.572	.621	.667	.709	.746	.777	.801	.816
4	.510	.559	.607	.653	.697	.737	.773	.803	.826	.841
5	.551	.596	.641	.685	.727	.766	.800	.829	.852	.867
6	3.592	3.634	3.676	3.717	3.757	3.794	3.827	3.856	3.878	3.892
7	.634	.671	.710	.749	.787	.822	.854	.882	.903	.917
8	.675	.709	.745	.781	.817	.851	.882	.908	.929	.942
9	.716	.747	.779	.813	.847	.879	.909	.934	.954	.967
10	.758	.784	.814	.845	.877	.908	.936	.960	.980	.993
11	3.799	3.822	3.848	3.877	3.907	3.936	3.963	3.987	4.005	4.018
12	.840	.859	.883	.909	.937	.964	.990	4.013	.031	.043
13	.882	.897	.917	.941	.967	.993	4.017	.039	.057	.068
14	.923	.934	.952	.973	.997	4.021	.044	.065	.082	.093
15	.964	.972	.986	4.005	4.027	.050	.072	.092	.108	.119
16	4.006	4.010	4.021	4.038	4.057	4.078	4.099	4.118	4.133	4.144
17	.047	.047	.056	.070	.087	.106	.126	.144	.159	.169
18	.088	.085	.090	.102	.117	.135	.153	.170	.184	.194
19	.130	.122	.125	.134	.147	.163	.180	.196	.210	.219
20	.171	.160	.159	.166	.177	.192	.207	.223	.236	.245
21	4.212	4.198	4.194	4.198	4.207	4.220	4.235	4.249	4.261	4.270
22	.253	.235	.228	.230	.237	.248	.262	.275	.287	.295
23	.295	.273	.263	.262	.267	.277	.289	.301	.312	.320
24	.336	.310	.297	.294	.297	.305	.316	.327	.338	.345
25	.377	.348	.332	.326	.327	.334	.343	.354	.363	.370
26	4.419	4.385	4.366	4.358	4.357	4.362	4.370	4.380	4.389	4.396
27	.460	.423	.401	.390	.387	.391	.397	.406	.415	.421
28	.501	.461	.435	.422	.417	.419	.425	.432	.440	.446
29	.543	.498	.470	.454	.447	.447	.452	.459	.466	.471
30	.584	.536	.504	.486	.477	.476	.479	.485	.491	.496
31	4.625	4.573	4.539	4.518	4.507	4.504	4.506	4.511	4.517	4.522
32	.667	.611	.573	.550	.537	.533	.533	.537	.542	.547
33	.708	.649	.608	.582	.567	.561	.560	.563	.568	.572
34	.749	.686	.642	.614	.597	.589	.588	.590	.594	.597
35	.791	.724	.677	.646	.627	.618	.615	.616	.619	.622
36	4.832	4.761	4.711	4.678	4.657	4.646	4.642	4.642	4.645	4.648
37	.873	.799	.746	.710	.687	.675	.669	.668	.670	.673
38	.915	.836	.780	.742	.717	.703	.696	.695	.696	.698
39	.956	.874	.815	.774	.747	.731	.723	.721	.721	.723
40	.997	.912	.849	.806	.777	.760	.750	.747	.747	.748
41	5.039	4.949	4.884	4.838	4.807	4.788	4.778	4.773	4.773	4.774
42	.080	.987	.918	.870	.837	.817	.805	.799	.798	.799
43	.121	5.024	.953	.902	.867	.845	.832	.826	.824	.824
44	.163	.062	.988	.934	.897	.873	.859	.852	.849	.849
45	.204	.099	5.022	.966	.927	.902	.886	.878	.875	.874
46	5.245	5.137	5.057	4.998	4.957	4.930	4.913	4.904	4.900	4.900
47	.287	.175	.091	5.030	.937	.959	.941	.931	.926	.925
48	.328	.212	.126	.062	5.017	.987	.968	.957	.952	.950
49	.369	.250	.160	.094	.047	5.015	.995	.983	.977	.975
50	.411	.287	.195	.126	.078	.044	5.022	5.009	5.003	5.000

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TABLE 3 (CONTINUED)

Obs.	TABLE 5 (CONTINUED)									
%	provisional probits, y									
p	5.0	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9
0	3.747	3.740	3.719	3.680	3.620	3.536	3.422	3.272	3.079	2.834
1	3.772	3.765	3.744	3.706	3.647	3.564	3.452	3.304	3.114	2.871
2	.797	.790	.770	.732	.675	.593	.482	.336	.148	.909
3	.822	.816	.795	.758	.702	.621	.512	.368	.183	.946
4	.847	.841	.821	.785	.729	.650	.542	.400	.217	.984
5	.872	.866	.846	.811	.756	.678	.572	.433	.252	3.021
6	3.897	3.891	3.872	3.837	3.783	3.706	3.602	3.465	3.287	3.059
7	.922	.916	.898	.863	.810	.735	.632	.497	.321	.097
8	.947	.942	.923	.890	.838	.763	.662	.529	.356	.134
9	.972	.967	.949	.916	.865	.792	.692	.561	.390	.172
10	.997	.992	.974	.942	.892	.820	.722	.593	.425	.209
11	4.022	4.017	4.000	3.968	3.919	3.848	3.752	3.625	3.459	3.247
12	.047	.042	.025	.994	.946	.877	.782	.657	.494	.284
13	.073	.068	.051	4.021	.973	.905	.812	.689	.528	.322
14	.098	.093	.077	.047	4.000	.934	.842	.721	.563	.360
15	.123	.118	.102	.073	.028	.962	.872	.753	.597	.397
16	4.148	4.143	4.128	4.099	4.055	3.990	3.902	3.785	3.632	3.435
17	.173	.168	.153	.126	.082	4.019	.932	.817	.666	.472
18	.198	.194	.179	.152	.109	.047	.962	.849	.701	.510
19	.223	.219	.204	.178	.136	.076	.992	.881	.735	.548
20	.248	.244	.230	.204	.163	.104	4.022	.913	.770	.585
21	4.273	4.269	4.256	4.230	4.191	4.132	4.052	3.945	3.804	3.623
22	.298	.294	.281	.257	.218	.161	.082	.977	.839	.660
23	.323	.320	.307	.283	.245	.189	.112	4.009	.873	.698
24	.348	.345	.332	.309	.272	.218	.142	.041	.908	.735
25	.373	.370	.358	.335	.299	.246	.172	.073	.942	.773
26	4.398	4.395	4.383	4.362	4.326	4.275	4.202	4.105	3.977	3.811
27	.423	.420	.409	.388	.353	.303	.232	.137	4.011	.848
28	.449	.445	.435	.414	.381	.331	.262	.169	.046	.886
29	.474	.471	.460	.440	.408	.360	.292	.201	.080	.923
30	.499	.496	.486	.466	.435	.388	.322	.233	.115	.961
31	4.524	4.521	4.511	4.493	4.462	4.417	4.352	4.265	4.149	3.999
32	.549	.546	.537	.519	.489	.445	.382	.297	.184	4.036
33	.574	.571	.563	.545	.516	.473	.412	.329	.219	.074
34	.599	.597	.588	.571	.544	.502	.442	.361	.253	.111
35	.624	.622	.614	.598	.571	.530	.472	.393	.288	.149
36	4.649	4.647	4.639	4.624	4.598	4.559	4.502	4.425	4.322	4.186
37	.674	.672	.665	.650	.625	.587	.532	.457	.357	.224
38	.699	.697	.690	.676	.652	.615	.562	.489	.391	.262
39	.724	.723	.716	.702	.679	.644	.592	.521	.426	.299
40	.749	.748	.742	.729	.706	.672	.622	.553	.460	.337
41	4.774	4.773	4.767	4.755	4.734	4.701	4.652	4.585	4.495	4.374
42	.799	.798	.793	.781	.761	.729	.682	.617	.529	.412
43	.825	.823	.818	.807	.788	.757	.712	.649	.564	.450
44	.850	.849	.844	.833	.815	.786	.742	.682	.598	.487
45	.875	.874	.869	.860	.842	.814	.772	.714	.633	.525
46	4.900	4.899	4.895	4.886	4.869	4.843	4.802	4.746	4.667	4.562
47	.925	.924	.921	.912	.897	.871	.832	.778	.702	.600
48	.950	.949	.946	.938	.924	.899	.862	.810	.736	.637
49	.975	.975	.972	.965	.951	.928	.892	.842	.771	.675
50	5.000	5.000	.997	.991	.978	.956	.922	.874	.805	.713

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TABLE 3 (CONTINUED)

Obs.	provisional probits, Y									
% p	6.0	6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9
0	2.523	2.132	1.643	1.030	0.261					
1	2.564	2.178	1.694	1.088	0.327					
2	.606	.224	.746	.146	.394					
3	.647	.270	.797	.205	.461					
4	.688	.316	.849	.263	.528					
5	.730	.362	.900	.321	.595					
6	2.771	2.408	1.952	1.380	0.661					
7	.812	.454	2.003	.438	.728					
8	.854	.500	.055	.496	.795					
9	.895	.546	.106	.555	.862					
10	.936	.591	.158	.613	.928	0.067				
11	2.978	2.637	2.209	1.671	0.995	0.144				
12	3.019	.683	.261	.730	1.062	.221				
13	.060	.729	.312	.788	.129	.299				
14	.102	.775	.364	.846	.196	.376				
15	.143	.821	.415	.905	.262	.453				
16	3.184	2.867	2.467	1.963	1.329	0.530				
17	.226	.913	.518	2.022	.396	.607				
18	.267	.959	.570	.080	.463	.685				
19	.308	3.005	.621	.138	.530	.762				
20	.350	.050	.673	.197	.596	.839				
21	3.391	3.096	2.724	2.255	1.663	0.916				
22	.432	.142	.776	.313	.730	.993	0.062			
23	.474	.188	.827	.372	.797	1.071	.152			
24	.515	.234	.879	.430	.864	.148	.243			
25	.556	.280	.930	.488	.930	.225	.333			
26	3.598	3.326	2.982	2.547	1.997	1.302	0.423			
27	.639	.372	3.033	.605	2.064	.379	.513			
28	.680	.418	.085	.663	.131	.457	.603			
29	.721	.464	.136	.722	.197	.534	.693			
30	.763	.509	.188	.780	.264	.611	.784			
31	3.804	3.555	3.239	2.838	2.331	1.688	0.874			
32	.845	.601	.291	.897	.398	.766	.964			
33	.887	.647	.342	.955	.465	.843	1.054	0.050		
34	.928	.693	.394	3.014	.531	.920	.144	.156		
35	.969	.739	.445	.072	.598	.997	.234	.262		
36	4.011	3.785	3.497	3.130	2.665	2.074	1.324	0.369		
37	.052	.831	.548	.189	.732	.152	.415	.475		
38	.093	.877	.600	.247	.799	.229	.505	.581		
39	.135	.923	.651	.305	.865	.306	.595	.688		
40	.176	.969	.703	.364	.932	.383	.685	.794		
41	4.217	4.014	3.754	3.422	2.999	2.460	1.775	0.900		
42	.259	.060	.806	.480	3.066	.538	.865	1.007		
43	.300	.106	.857	.539	.132	.615	.955	.113	0.035	
44	.341	.152	.909	.597	.199	.692	2.046	.219	.162	
45	.383	.198	.960	.655	.266	.769	.136	.326	.289	
46	4.424	4.244	4.012	3.714	3.333	2.846	2.226	1.432	0.415	
47	.465	.290	.063	.772	.400	.924	.316	.538	.542	
48	.507	.336	.115	.830	.466	3.001	.406	.645	.669	
49	.548	.382	.166	.889	.533	.078	.496	.751	.795	
50	.589	.428	.218	.947	.600	.155	.586	.857	.922	

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TABLE 3 (CONTINUED)

Obs. %	provisional probits, Y									
p	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9
51	—	—	9.205	8.249	7.504	6.922	6.467	6.111	5.834	5.618
52			.331	.355	.594	.999	.534	.170	.885	.664
53			.458	.462	.684	7.076	.600	.228	.937	.710
54			.585	.568	.774	.154	.667	.286	.988	.756
55			.711	.674	.864	.231	.734	.345	6.040	.802
56			9.838	8.781	7.954	7.308	6.801	6.403	6.091	5.848
57			.965	.887	8.045	.385	.868	.461	.143	.894
58				.993	.135	.462	.934	.520	.194	.940
59				9.100	.225	.540	7.001	.578	.246	.986
60				.206	.315	.617	.068	.636	.297	6.031
61				9.312	8.405	7.694	7.135	6.695	6.349	6.077
62				.419	.495	.771	.201	.753	.400	.123
63				.525	.585	.848	.268	.811	.452	.169
64				.631	.676	.926	.335	.870	.503	.215
65				.738	.766	8.003	.402	.928	.555	.261
66				9.844	8.856	8.080	7.469	6.986	6.606	6.307
67				.950	.946	.157	.535	7.045	.658	.353
68					9.036	.234	.602	.103	.709	.399
69					.126	.312	.669	.162	.761	.445
70					.216	.389	.736	.220	.812	.491
71					9.307	8.466	7.803	7.278	6.864	6.536
72					.397	.543	.869	.337	.915	.582
73					.487	.621	.936	.395	.967	.628
74					.577	.698	8.003	.453	7.018	.674
75					.667	.775	.070	.512	.070	.720
76					9.757	8.852	8.136	7.570	7.121	6.766
77					.848	.929	.203	.628	.173	.812
78					.938	9.007	.270	.687	.224	.858
79						.084	.337	.745	.276	.904
80						.161	.404	.803	.327	.950
81						9.238	8.470	7.862	7.379	6.995
82						.315	.537	.920	.430	7.041
83						.393	.604	.978	.482	.087
84						.470	.671	8.037	.533	.133
85						.547	.738	.095	.585	.179
86						9.624	8.804	8.154	7.636	7.225
87						.701	.871	.212	.688	.271
88						.779	.938	.270	.739	.317
89						.856	9.005	.329	.791	.363
90						.933	.072	.387	.842	.409
91							9.138	8.445	7.894	7.454
92							.205	.504	.945	.500
93							.272	.562	.997	.546
94							.339	.620	8.048	.592
95							.405	.679	.100	.638
96							9.472	8.737	8.151	7.684
97							.539	.795	.203	.730
98							.606	.854	.254	.776
99							.673	.912	.306	.822
100							.739	.970	.357	.868

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TABLE 3 (CONTINUED)

Obs. %	provisional probits, Y									
P	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9
51	5.452	5.325	5.229	5.158	5.108	5.072	5.049	5.035	5.028	5.025
52	.493	.363	.264	.190	.138	.101	.076	.062	.054	.051
53	.535	.400	.298	.222	.168	.129	.103	.088	.079	.076
54	.576	.438	.333	.254	.198	.157	.131	.114	.105	.101
55	.617	.475	.367	.286	.228	.186	.158	.140	.131	.126
56	5.659	5.513	5.402	5.318	5.258	5.214	5.185	5.167	5.156	5.151
57	.700	.550	.436	.351	.288	.243	.212	.193	.182	.177
58	.741	.588	.471	.383	.318	.271	.239	.219	.207	.202
59	.783	.626	.505	.415	.348	.299	.266	.245	.233	.227
60	.824	.663	.540	.447	.378	.328	.294	.271	.258	.252
61	5.865	5.701	5.574	5.479	5.408	5.356	5.321	5.298	5.284	5.277
62	.907	.738	.609	.511	.438	.385	.348	.324	.310	.303
63	.948	.776	.643	.543	.468	.413	.375	.350	.335	.328
64	.989	.814	.678	.575	.498	.441	.402	.376	.361	.353
65	6.031	.851	.712	.607	.528	.470	.429	.402	.386	.378
66	6.072	5.889	5.747	5.639	5.558	5.498	5.456	5.429	5.412	5.403
67	.113	.926	.781	.671	.588	.527	.484	.455	.437	.429
68	.155	.964	.816	.703	.613	.555	.511	.481	.463	.454
69	.196	6.001	.851	.735	.648	.583	.538	.507	.489	.479
70	.237	.039	.885	.767	.678	.612	.565	.534	.514	.504
71	6.279	6.077	5.920	5.799	5.708	5.640	5.592	5.560	5.540	5.529
72	.320	.114	.954	.831	.738	.669	.619	.586	.565	.555
73	.361	.152	.989	.863	.768	.697	.647	.612	.591	.580
74	.402	.189	6.023	.895	.798	.725	.674	.638	.617	.605
75	.444	.227	.058	.927	.828	.754	.701	.665	.642	.630
76	6.485	6.265	6.092	5.959	5.858	5.782	5.728	5.691	5.668	5.655
77	.526	.302	.127	.991	.888	.811	.755	.717	.693	.680
78	.568	.340	.161	6.023	.918	.839	.782	.743	.719	.706
79	.609	.377	.196	.055	.948	.868	.809	.770	.744	.731
80	.650	.415	.230	.087	.978	.896	.837	.796	.770	.756
81	6.692	6.452	6.265	6.119	6.008	5.924	5.864	5.822	5.796	5.781
82	.733	.490	.299	.151	.038	.953	.891	.848	.821	.806
83	.774	.528	.334	.183	.068	.981	.918	.874	.847	.832
84	.816	.565	.368	.215	.098	6.010	.945	.901	.872	.857
85	.857	.603	.403	.247	.128	.038	.972	.927	.898	.882
86	6.898	6.640	6.437	6.279	6.158	6.066	6.000	5.953	5.923	5.907
87	.940	.678	.472	.311	.188	.095	.027	.979	.949	.932
88	.981	.716	.506	.343	.218	.123	.054	6.006	.975	.958
89	7.022	.753	.541	.375	.248	.152	.081	.032	6.000	.983
90	.064	.791	.575	.407	.278	.180	.108	.058	.026	6.008
91	7.105	6.828	6.610	6.439	6.308	6.208	6.135	6.084	6.051	6.033
92	.146	.866	.644	.471	.338	.237	.162	.110	.077	.058
93	.188	.903	.679	.503	.368	.265	.190	.137	.102	.084
94	.229	.941	.713	.535	.398	.294	.217	.163	.128	.109
95	.270	.979	.748	.567	.428	.322	.244	.189	.154	.134
96	7.312	7.016	6.783	6.600	6.458	6.350	6.271	6.215	6.179	6.159
97	.353	.054	.817	.632	.488	.379	.298	.242	.205	.184
98	.394	.091	.852	.664	.518	.407	.325	.263	.230	.210
99	.436	.129	.886	.696	.548	.436	.353	.294	.256	.235
100	.477	.166	.921	.728	.578	.464	.380	.320	.281	.260

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TABLE 3 (CONTINUED)

Obs. %	provisional probits, Y									
p	5.0	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9
51	5.025	5.025	5.023	5.017	5.005	4.985	4.953	4.906	4.840	4.750
52	.050	.050	.048	.043	.032	5.013	.983	.938	.874	.788
53	.075	.075	.074	.069	.059	.041	5.013	.970	.909	.825
54	.100	.100	.100	.096	.087	.070	.043	5.002	.943	.863
55	.125	.126	.125	.122	.114	.098	.073	.034	.978	.901
56	5.150	5.151	5.151	5.148	5.141	5.127	5.103	5.066	5.012	4.938
57	.175	.176	.176	.174	.168	.155	.133	.098	.047	.976
58	.201	.201	.202	.201	.195	.183	.163	.130	.082	5.013
59	.226	.226	.227	.227	.222	.212	.193	.162	.116	.051
60	.251	.252	.253	.253	.250	.240	.223	.194	.151	.088
61	5.276	5.277	5.279	5.279	5.277	5.269	5.253	5.226	5.185	5.126
62	.301	.302	.304	.305	.304	.297	.283	.258	.220	.164
63	.326	.327	.330	.332	.331	.325	.313	.290	.254	.201
64	.351	.352	.355	.358	.358	.354	.343	.322	.289	.239
65	.376	.378	.381	.384	.385	.382	.373	.354	.323	.276
66	5.401	5.403	5.406	5.410	5.412	5.411	5.403	5.386	5.358	5.314
67	.426	.428	.432	.437	.440	.439	.433	.418	.392	.351
68	.451	.453	.458	.463	.467	.467	.463	.450	.427	.389
69	.476	.478	.483	.489	.494	.496	.493	.482	.461	.427
70	.501	.504	.509	.515	.521	.524	.523	.514	.496	.464
71	5.526	5.529	5.534	5.541	5.548	5.553	5.553	5.546	5.530	5.502
72	.551	.554	.560	.568	.575	.581	.583	.578	.565	.539
73	.577	.579	.585	.594	.603	.609	.613	.610	.599	.577
74	.602	.604	.611	.620	.630	.638	.643	.642	.634	.615
75	.627	.630	.637	.646	.657	.666	.673	.674	.668	.652
76	5.652	5.655	5.662	5.673	5.684	5.695	5.703	5.706	5.703	5.690
77	.677	.680	.688	.699	.711	.723	.733	.738	.737	.727
78	.702	.705	.713	.725	.738	.752	.763	.770	.772	.765
79	.727	.730	.739	.751	.765	.780	.793	.802	.806	.802
80	.752	.755	.764	.777	.793	.808	.823	.834	.841	.840
81	5.777	5.781	5.790	5.804	5.820	5.837	5.853	5.866	5.875	5.878
82	.802	.806	.816	.830	.847	.865	.883	.898	.910	.915
83	.827	.831	.841	.856	.874	.894	.913	.930	.944	.953
84	.852	.856	.867	.882	.901	.922	.943	.962	.979	.990
85	.877	.881	.892	.908	.928	.950	.973	.995	6.014	6.028
86	5.902	5.907	5.918	5.935	5.956	5.979	6.003	6.027	6.048	6.066
87	.927	.932	.943	.961	.983	6.007	.033	.059	.083	.103
88	.953	.957	.969	.987	6.010	.036	.063	.091	.117	.141
89	.978	.982	.995	6.013	.037	.064	.093	.123	.152	.178
90	6.003	6.007	6.020	.040	.064	.092	.123	.155	.186	.216
91	6.028	6.033	6.046	6.066	6.091	6.121	6.153	6.187	6.221	6.253
92	.053	.058	.071	.092	.118	.149	.183	.219	.255	.291
93	.078	.083	.097	.118	.146	.178	.213	.251	.290	.329
94	.103	.108	.122	.144	.173	.206	.243	.283	.324	.366
95	.128	.133	.148	.171	.200	.234	.273	.315	.359	.404
96	6.153	6.159	6.174	6.197	6.227	6.263	6.303	6.347	6.393	6.441
97	.178	.184	.199	.223	.254	.291	.333	.379	.428	.479
98	.203	.209	.225	.249	.281	.320	.363	.411	.462	.517
99	.228	.234	.250	.276	.309	.348	.393	.443	.497	.554
100	.253	.259	.276	.302	.336	.376	.423	.475	.531	.592

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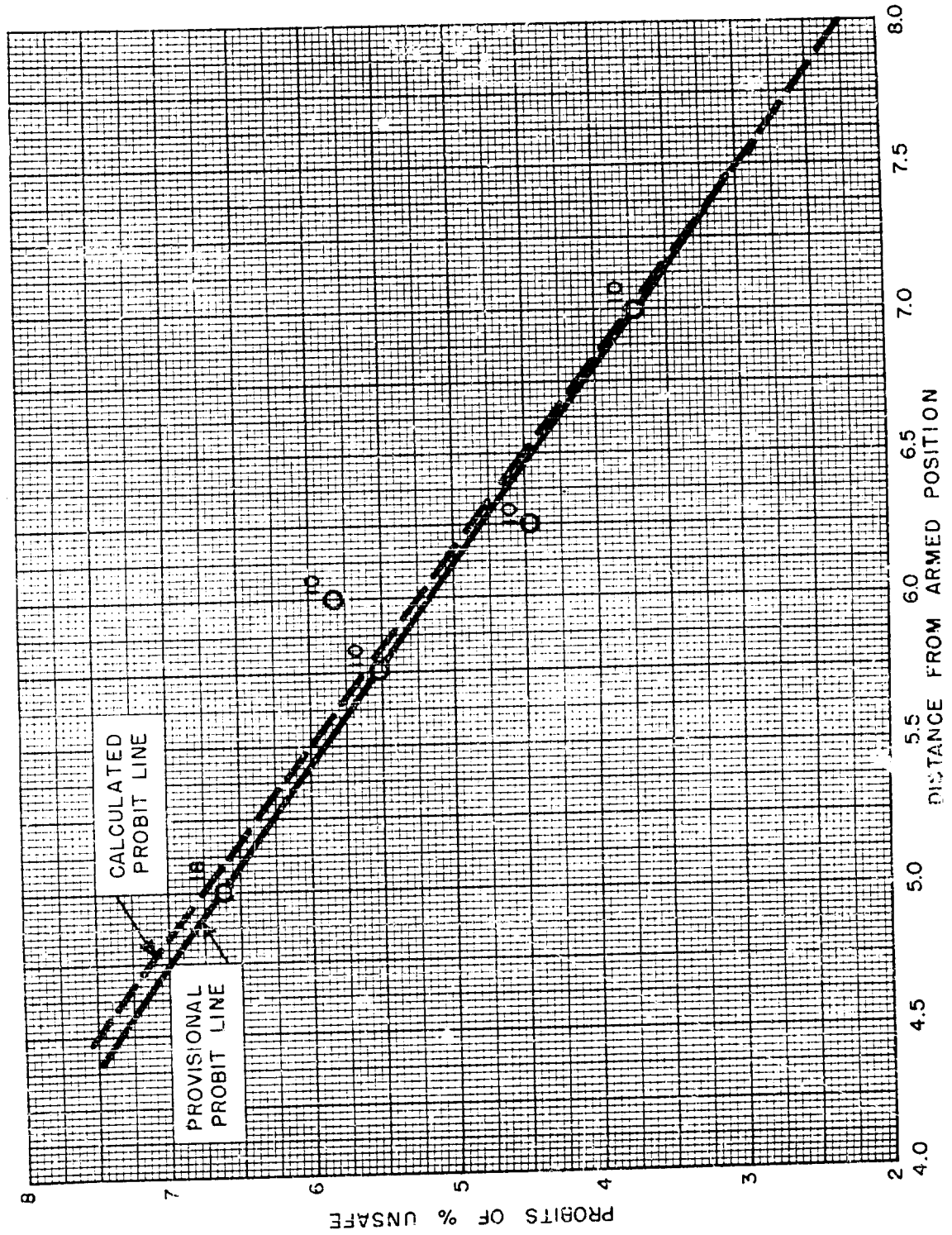
TABLE 3 (CONTINUED)

Obs. % p	provisional probits, Y									
	6.0	6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9
51	4.631	4.473	4.269	4.006	3.667	3.233	2.677	1.964	1.049	
52	.672	.519	.311	.064	.734	.310	.767	2.070	.175	0.022
53	.713	.565	.372	.122	.800	.387	.857	.176	.302	.175
54	.755	.611	.424	.181	.867	.464	.947	.283	.429	.327
55	.796	.657	.475	.239	.934	.541	3.037	.389	.555	.480
56	4.837	4.703	4.527	4.297	4.001	3.619	3.127	2.495	1.682	0.632
57	.879	.749	.578	.356	.068	.696	.218	.602	.809	.784
58	.920	.795	.630	.414	.134	.773	.308	.708	.935	.937
59	.961	.841	.681	.472	.201	.850	.398	.814	2.062	1.089
60	5.003	.887	.733	.531	.268	.927	.488	.921	.189	.242
61	5.044	4.932	4.784	4.589	4.335	4.005	3.578	3.027	2.315	1.394
62	.085	.978	.836	.647	.401	.082	.668	.133	.442	.546
63	.127	5.024	.887	.706	.468	.159	.758	.240	.569	.699
64	.168	.070	.939	.764	.535	.236	.849	.346	.695	.851
65	.209	.116	.990	.823	.602	.313	.939	.452	.822	2.004
66	5.251	5.162	5.042	4.881	4.669	4.391	4.029	3.559	2.949	2.156
67	.292	.208	.093	.939	.735	.468	.119	.665	3.075	.308
68	.333	.254	.145	.998	.802	.545	.209	.771	.202	.451
69	.375	.300	.196	5.056	.869	.622	.299	.878	.329	.613
70	.416	.346	.248	.114	.936	.700	.390	.984	.455	.766
71	5.457	5.392	5.299	5.173	5.003	4.777	4.480	4.090	3.582	2.918
72	.499	.437	.351	.231	.069	.854	.570	.197	.709	3.070
73	.540	.483	.402	.289	.136	.931	.660	.303	.835	.223
74	.581	.529	.454	.348	.203	5.008	.750	.409	.962	.375
75	.623	.575	.505	.406	.270	.086	.840	.516	4.089	.528
76	5.664	5.621	5.557	5.464	5.336	5.163	4.930	4.622	4.215	3.680
77	.705	.667	.608	.523	.403	.240	5.021	.728	.342	.832
78	.747	.713	.660	.581	.470	.317	.111	.835	.469	.985
79	.788	.759	.711	.639	.537	.394	.201	.941	.595	4.137
80	.829	.805	.763	.698	.604	.472	.291	5.047	.722	.290
81	5.870	5.851	5.814	5.756	5.670	5.549	5.381	5.154	4.849	4.442
82	.912	.896	.866	.815	.737	.626	.471	.260	.975	.594
83	.953	.942	.917	.873	.804	.703	.561	.366	5.102	.747
84	.994	.988	.969	.931	.871	.780	.652	.473	.229	.899
85	6.036	6.034	6.020	.990	.938	.858	.742	.579	.355	5.052
86	6.077	6.080	6.072	6.048	6.004	5.935	5.832	5.685	5.482	5.204
87	.118	.126	.123	.106	.071	6.012	.922	.792	.609	.356
88	.160	.172	.175	.165	.138	.089	6.012	.898	.735	.509
89	.201	.218	.226	.223	.205	.166	.102	6.004	.862	.661
90	.242	.264	.278	.281	.272	.244	.192	.111	.988	.814
91	6.284	6.310	6.329	6.340	6.338	6.321	6.283	6.217	6.115	5.966
92	.325	.355	.381	.398	.405	.398	.373	.323	.242	6.118
93	.366	.401	.432	.456	.472	.475	.463	.430	.368	.271
94	.408	.447	.484	.515	.539	.553	.553	.536	.495	.423
95	.449	.493	.535	.573	.605	.630	.643	.642	.622	.576
96	6.490	6.539	6.587	6.631	6.672	6.707	6.733	6.749	6.748	6.728
97	.532	.585	.638	.690	.739	.784	.824	.855	.875	.880
98	.573	.631	.690	.748	.806	.861	.914	.961	7.002	7.033
99	.614	.677	.741	.807	.873	.939	7.004	7.068	.128	.185
100	.656	.723	.793	.865	.939	7.016	.094	.174	.255	.338

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TABLE 3 (CONTINUED)

Obs. %	provisional probits, Y									
p	7.0	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9
51										
52										
53										
54										
55										
56										
57										
58										
59										
60	0.013									
61	0.198									
62	.383									
63	.568									
64	.753									
65	.939									
66	1.124									
67	.309	0.003								
68	.494	.231								
69	.680	.458								
70	.865	.685								
71	2.050	0.913								
72	.235	1.140								
73	.420	.367								
74	.606	.595	0.263							
75	.791	.822	.545							
76	2.976	2.050	0.827							
77	3.161	.277	1.108							
78	.347	.504	.390							
79	.532	.732	.672	0.265						
80	.717	.959	.954	.618						
81	3.902	3.186	2.236	0.971						
82	4.087	.414	.518	1.324						
83	.273	.641	.800	.677	0.175					
84	.458	.868	3.082	2.030	.621					
85	.643	4.096	.364	.383	1.068					
86	4.828	4.323	3.645	2.736	1.514					
87	5.014	.551	.927	3.089	.961	0.438				
88	.199	.778	4.209	.442	2.408	1.008				
89	.384	5.005	.491	.795	.854	.579				
90	.569	.233	.773	4.148	3.301	2.149	0.581			
91	5.754	5.460	5.055	4.501	3.747	2.720	1.317			
92	.940	.687	.337	.854	4.194	3.290	2.054	0.356		
93	6.125	.915	.619	5.207	.640	.861	.790	1.316		
94	.310	6.142	.901	.560	5.087	4.431	3.526	2.275	0.542	
95	.495	.369	6.182	.914	.533	5.002	4.262	3.235	1.806	
96	6.681	6.597	6.464	6.267	5.980	5.572	4.998	4.194	3.069	1.493
97	.866	.824	.746	.620	6.426	6.143	5.735	5.154	4.333	3.173
98	7.051	7.051	7.028	.973	.873	.713	6.471	6.114	5.596	4.853
99	.236	.279	.310	7.326	7.319	7.284	7.207	7.073	6.859	6.533
100	.421	.506	.592	.679	.766	.854	.943	8.033	8.123	8.213



GRAPH 1 PROBITS OF % UNSAFE vs DISTANCE FROM ARMED POSITION

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