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NAVORD REPORT 3577

EVALUATION OF INERTIA SENSITIVITY

24 NOVEMBER 1953

U. S. NAVAL ORDNANCE LABORATORY
WHITE OAK, MARYLAND

NAVORD Report 3577

EVALUATION OF INERTIA SENSITIVITY

Prepared by

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ABSTRACT: The main objective of this report is to discuss the determination, presentation and interpretation of inertia sensitivity data. Theoretical analysis of the mass-spring system for response to acceleration-time pulses, amplification factors, characteristic delay times, and inertia sensitivity are used as a basis for discussion of actual devices. Effects of deviations from the ideal mass-spring system are considered. Practical use of sensitivity data is discussed with regard to the reliability of laboratory methods, the accuracy of field measurements and variability of service conditions.

U. S. NAVAL ORDNANCE LABORATORY
White Oak, Maryland

In the course of evaluating the suitability of various inertia-actuated ordnance devices from the standpoint of their sensitivity to actuation by field conditions such as launching and target impact, or by undesirable influences such as missile maneuvers and countermining, various methods (experimental and theoretical) of testing and analyzing the performance of these devices have evolved. It is the purpose of this report to present these techniques in a unified form with the theoretical background necessary for their intelligent application. The subject matter is intended primarily as an information source for those interested in design and evaluation of inertia-operated devices. The information contained herein is the product of many design and evaluation tasks; the work of presenting this report having been done under Task C6-256-11-54. The opinions and judgments expressed are those of the Technical Evaluation Department.

EDWARD L. WOODYARD
Captain, USN
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By direction

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INTRODUCTION

1. The inertia device considered herein may be any mechanism which must move some distance against a restraining force in order to perform its mission, in response to a transient forcing function. The forcing function is an acceleration-time pulse acting on a mass and the restraining force is exerted by some elastic member. A simple example of such a device may be an inertia firing switch for a torpedo, consisting of a small mass on a spring, Figure 1.

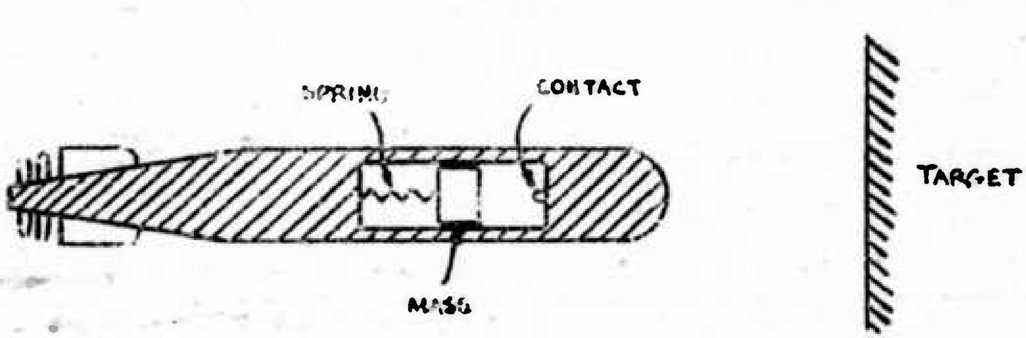


Figure 1
An Inertia Firing Switch

In the activation of the switch, the acceleration-time pulse is generated by the deceleration of the torpedo as it strikes the target, and the mass stretches the spring (Figure 1) under its own inertia force until an electrical firing contact is made. Inertia devices are seldom as idealized as this simple mass-spring system. However, many of the conclusions reached for the behavior of the mass-spring system will be valid for more complex devices. It should be noted, however, that these analyses are not applicable to point-detonating switches, which are actuated by direct forces which displace the parts.

2. In describing the performance of an inertia mechanism, it is sometimes deemed adequate to determine whether or not the device will function properly under one given set of operating conditions. For the example cited above Figure 1, a go or no-go test of the torpedo striking a given target at a certain angle at a speed of three knots may be considered an evaluation of the firing switch. However, this type of test does not explore the full potentialities of the switch in that application--namely, the "quantitative" sensitivity or minimum striking velocity to fire; nor does it examine the performance of the switch under other operating circumstances, such as other types of target or other torpedoes. It is a peculiarity of development work that most devices originally conceived for one specific purpose end up being used for various other applications. For this reason, and also for the reason that the designer can rarely define the service conditions closely, it is highly desirable to know the performance of the mechanism under any set of circumstances.

3. As to the question of what circumstances could influence the action of an inertia device, the acceleration-time history of the device entirely describes the input pulse, the most basic parameters being the acceleration, velocity change, and duration. (Note that the activating pulse consists of the local acceleration-time history at the housing of the inertia device, and that the response of the device is independent of the initial velocity of the weapon, i.e., the device cannot differentiate between a sudden change in velocity from 1500 to 1490 ft/sec and one from 10 ft/sec to rest.) The sensitivity curve for a single-degree-of freedom system (such as the mass-spring system of Figure 1) in response to simple acceleration-time pulses in a given direction will generally be similar to Figure 2.

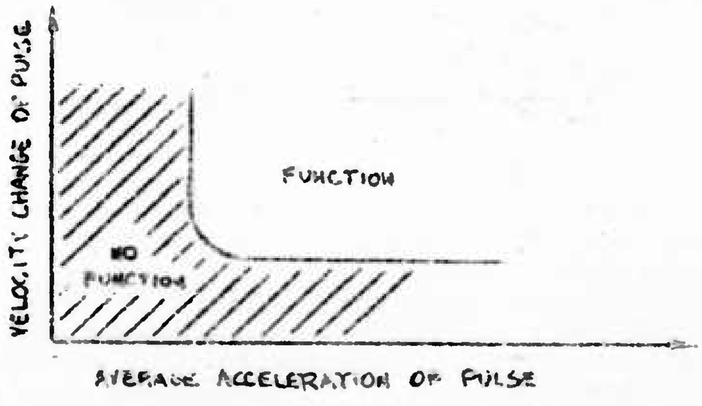


Figure 2
A Typical Sensitivity Curve

This curve makes the prediction that when the velocity change and average acceleration of the input pulse can combine to define a point in the cross-hatched region, the device will not function. All points above and to the right of the curve represent points at which the device will function. Basically, this single curve completely defines the sensitivity of the mechanism to inertial actuation. The sensitivity curve for a torpedo firing switch (such as Figure 1) would indicate what minimum velocities would be required for firing at all angles of impact (individual sensitivity curves are usually indicated for each direction of acceleration, since few mechanisms have identical properties in all directions) and all degrees of target rigidity (which affects the peak acceleration of the pulse). Figure 3 indicates, for one angle of impact, how the properties of the target are typically reflected in the sensitivity curve for the torpedo firing switch of Figure 1.

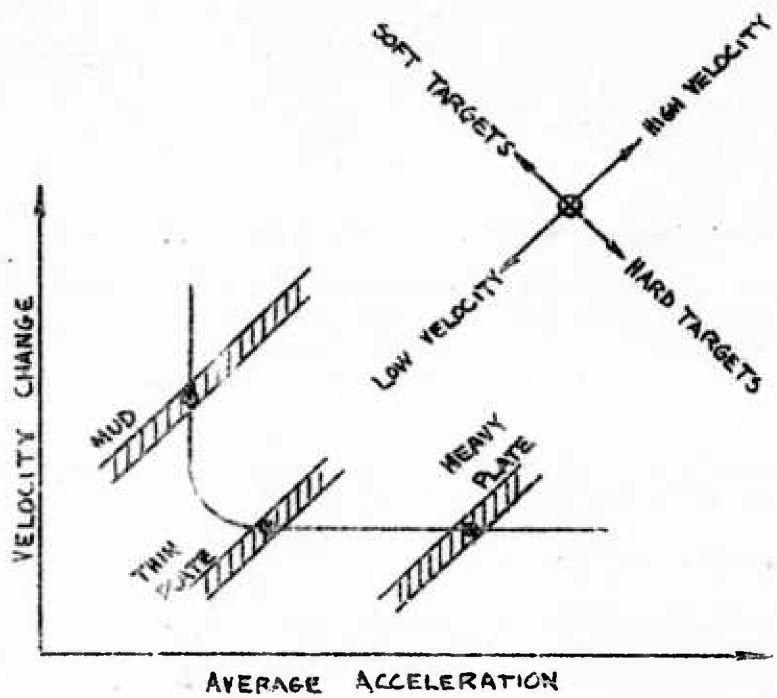


Figure 3
Typical Impacts Located on a Sensitivity Curve

4. Unfortunately, there are other phenomena which affect the performance of inertia mechanisms and therefore shift or distort the sensitivity curve. One factor which makes difficult any attempt to employ a single sensitivity curve or narrow band for all service conditions is the shape of the acceleration-time pulse. That is, a square pulse, a half-sine pulse, and a triangular pulse (See Figure 11 for plots of these pulses) will all produce sensitivity curves different in some regions, even though the velocity changes and average accelerations are identical. This problem represents an inherent difficulty in the accurate but general presentation and application of inertia sensitivity data, and is considered in detail later in this report.

APPLICABILITY OF THE THEORY

5. Theoretical prediction has an important role in design and development work. It should not be used as a substitute for laboratory evaluation, as is sometimes done by those theoretically inclined; nor should it be ignored in favor of the build-it and test-it method. Some happy balance of theory and empiricism should result in an efficient development process. In such a combined process, theory should have the following purposes:

a. Provide a general background of knowledge on which to draw for proper qualitative design and for proper interpretation of results. Theoretical experience must supplement practical experience in order to appreciate fully the significance of the actual results.

b. Considerably improve the closeness of performance with design specifications on the first attempt at design. Blind selection of the first model of a mechanism is most inefficient.

c. Shorten the development process by predicting the effects of changes, rather than creeping up on the desired result with small changes.

d. In the process of making comparison between theory and performance, and while making the subsequent search for sources of discrepancy, it is not unusual to discover rare "bugs" which may plague the mechanism, or even to come upon novel design ideas. An example of this was afforded recently, when a laboratory test produced a sensitivity curve such as Figure 4:

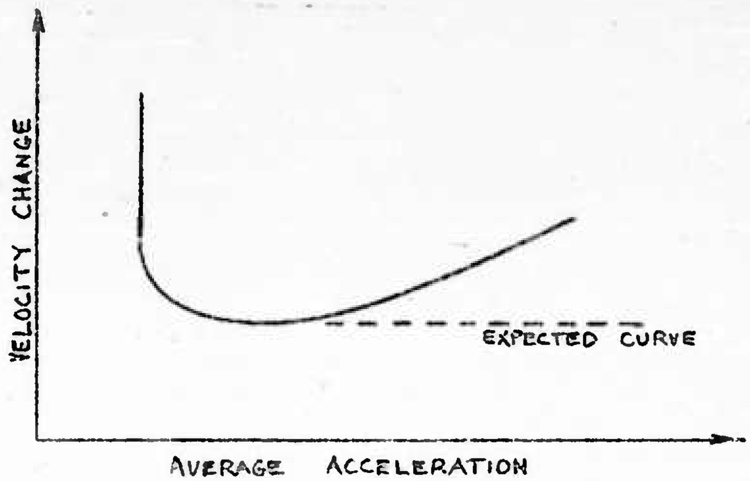


Figure 4
An Unusual Sensitivity Curve

After some speculation and further test, it was found that air was being trapped under the moving mass, which acted as an extra spring; the "spring constant" being a function of the velocity of the mass and how fast the air could leak past the mass. Although undesirable in this design, the unusual principle showed promise in other applications.

6. Some factors which make theoretical predictions of sensitivity curves difficult, but which do not render laboratory-determined sensitivity curves invalid, are as follows:

a. Some systems require a certain amount of kinetic energy at the end of their travel in order to perform their function. In contrast to an electrical switch which must merely make contact, many practical devices must perform mechanical work other than moving their prescribed distance against spring force. Some mechanisms are required to move mechanical parts such as detents or cams, while some firing switches must penetrate the surface of an explosive with sufficient energy to initiate detonation. In the latter case it is generally found that mere penetration of an explosive does not necessarily cause detonation, and that a minimum striking velocity is needed.

b. Many systems are more complex than the single mass-spring system. Figure 5 shows some simple cantilever configurations which may represent many degrees of freedom.

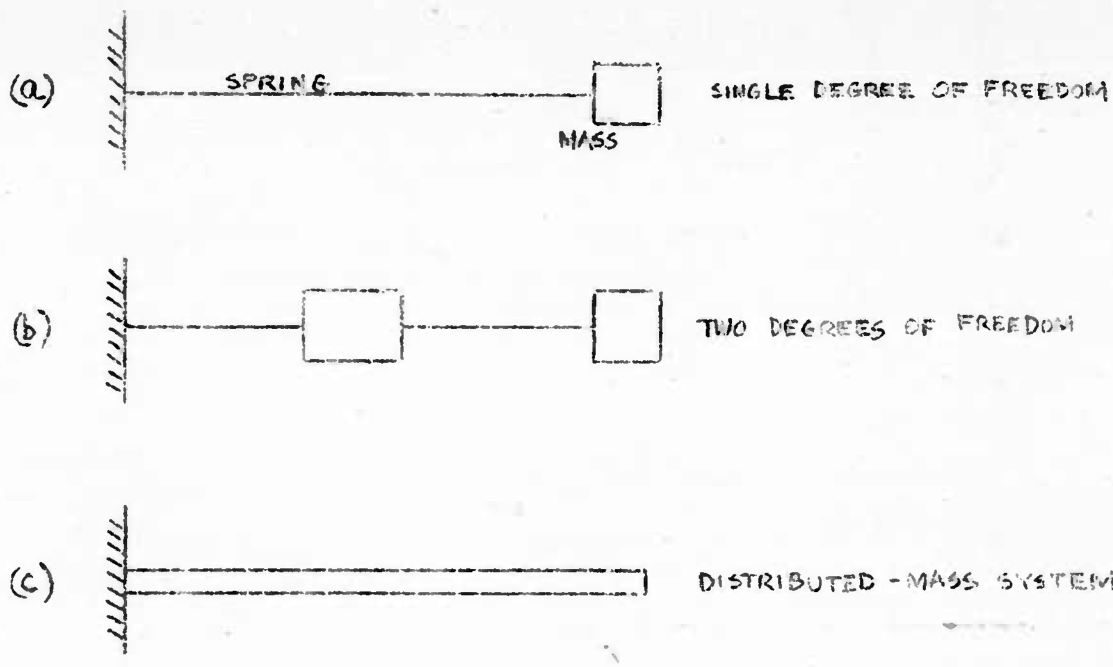


Figure 5
Degrees of Freedom of Cantilever Systems

For example, consider the distributed-mass system (Figure 5c) which has an infinite number of natural periods. In this case, many of the higher modes of vibration may be excited by an input pulse and the resulting motion is far more complex and difficult to analyze than that of the single mass-spring system. Compare, in Figure 6 the motions of the tip of a uniform cantilever beam (relative to the built-in end) (Figure 5c) with those of a mass-spring system (Figure 5a), both responding to an excitation consisting of an impulsive upward velocity of the base or fixed end:

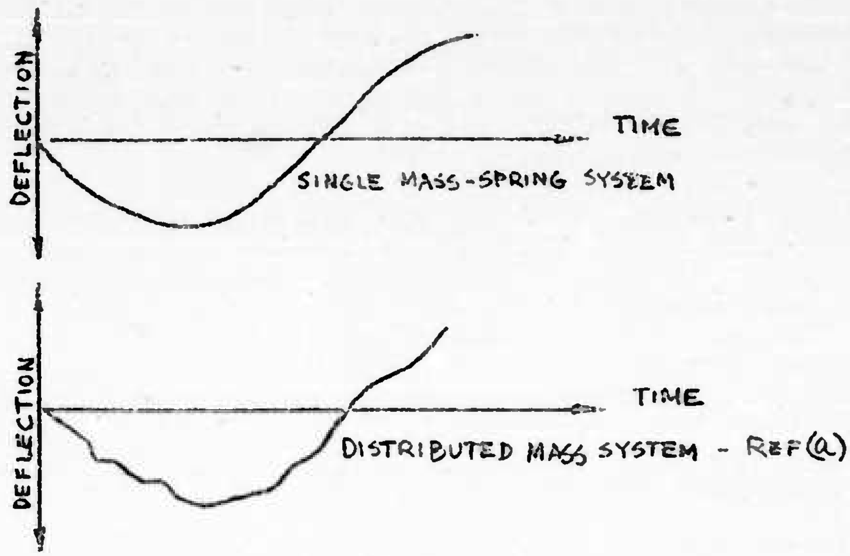


Figure 6
Comparison of Motions of Mass-Spring System and Distributed-Mass System

The mass-spring system responds with simple harmonic motion; whereas the uniform cantilever tip remains motionless until the disturbance traverses the length of the cantilever, begins to move in a direction opposite to that of the motion of the base and finally executes a vibration which contains its higher modes. Apparently, an analysis of the inertia sensitivity of the uniform cantilever would be rather difficult and laborious. However, under certain conditions of loading*, the cantilever will respond approximately as a single-degree-of-freedom system, and the results of a mass-spring analysis will be directly applicable.

*Reference (b) suggests the following restrictions:

(1) The duration of impact exceeds 0.1 or more of the fundamental period.
(2) The impact load is distributed over the structure fairly uniformly.
(3) The fundamental mode of the structure is uncoupled with the higher modes; in other words the momentary deflection profile of the structure in its extreme position agrees with its profile under static load.

c. There are miscellaneous divergences from the ideal mass-spring system which influence the behavior of the inertia device. Effects such as non-linearity of the "spring," preset of the spring, friction, damping, restriction of motion to only one direction from the neutral position, etc. may be of considerable importance in altering the shape of the sensitivity curve.

d. Some devices, such as the Magnetic Impulse Generator (an electro-mechanical transducer), have force-deflection characteristics in no way similar to the mass-spring system and their sensitivity curves may not resemble Figure 2 too closely. Unless it is found by experience that the performance does permit deductions and extrapolations valid for the sensitivity curves of mass-spring systems, such unusual devices are better evaluated on a wholly experimental basis (sensitivity curve applied only to field conditions very similar to the laboratory conditions).

7. It must be emphasized at this point that only when absolutely necessary should quantitative theoretical predictions be used in lieu of actual performance data. The above sources of discrepancy, and possibly many others, indicate that any calculated sensitivity curve should be checked in the laboratory or in the field. How closely the theoretical sensitivity curve conforms to test results will depend on the skil of the personnel, but there is always the "bug" in the system which is not forseen by even the experienced calculator. However, the qualitative theoretical conclusions will be valid. That is, the general shape of the curve will be correct. And the general conclusions as to the action of the device in response to shocks of short or long duration will also be valid.

8. In view of the importance of theoretical considerations, much of the remainder of this report is concerned with performance of the mass-spring system. The detailed mathematical analyses which provide the basis for the following discussions have been relegated to the Appendix. The Appendix also contains useful information on characteristic delay times (time between actuating pulse and switch closure) and on simple design relationships.

DETERMINATION OF A THEORETICAL SENSITIVITY CURVE

9. In order to avoid being confused by a maze of mathematical formulae, it appears profitable to discuss qualitatively the response of a mass-spring system to a pulse of simple shape. After an understanding of the physical problem has been attained, the mathematical details may be examined by reference to the Appendix.

10. Consider (Figure 7) the free end of the spring of a mass-spring system subjected to a rectangular acceleration pulse:

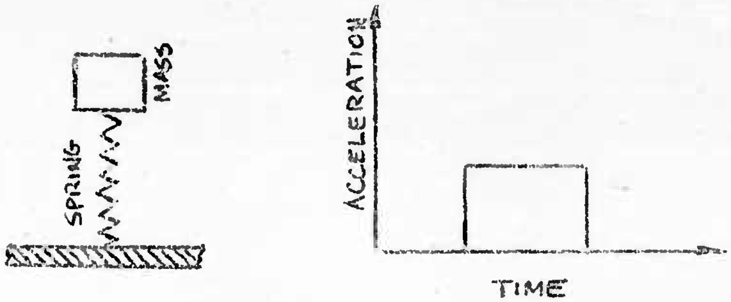


Figure 7
Mass-Spring System Subjected to Rectangular Pulse

The mass does not follow the forcing function because of the flexibility of the spring, but executes a vibration such as Figure 8.

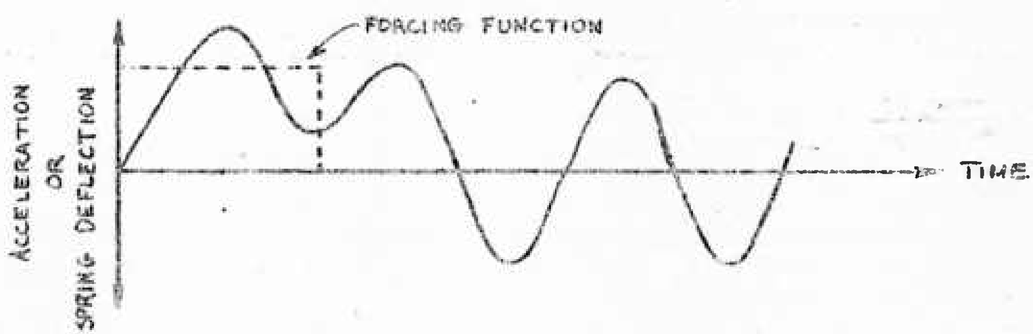


Figure 8
Response of Mass-Spring System to Rectangular Pulse

The peak deflection attained by the mass is different from the deflection which could have been achieved by very slowly applying the maximum value of acceleration of the rectangular pulse. The ratio of maximum dynamic deflection to maximum static deflection (See Equation 10b) is termed the amplification factor.

11. Had the ratio of pulse duration to natural period of the mass-spring system been different, the amplification factor would have been different. A plot (Figure 9) of amplification factors for a mass-spring system subjected to rectangular pulses shows (reference (b)) this dependence on duration.

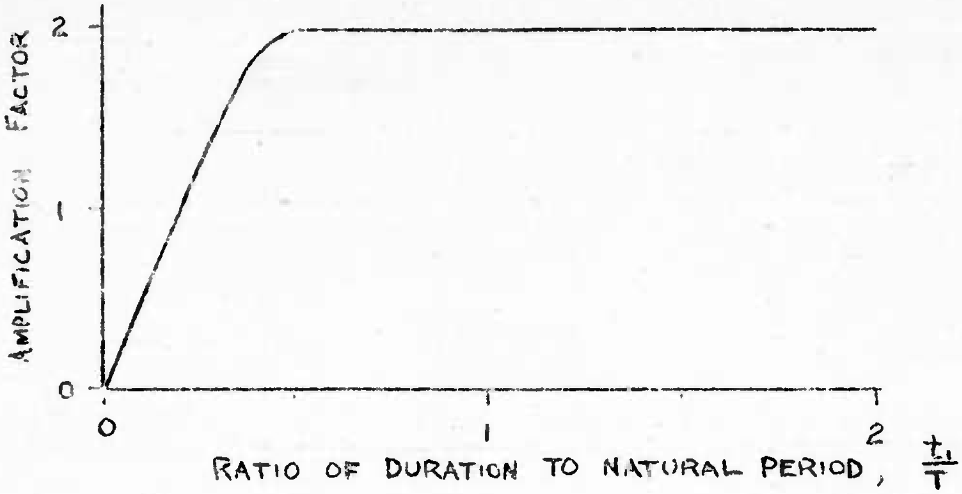


Figure 9
Amplification Factors for Mass-Spring System Subjected to Rectangular Pulses

12. Since the mass must move a fixed distance, and a fixed deflection is equivalent to a fixed acceleration (the peak response in Figure 8) the higher the amplification factor the lower the required applied acceleration. For a rectangular pulse, the amplification factor increases as duration increases soon becoming constant. With regard to velocity change which is equal to average acceleration times duration, it would appear that longer duration (higher than 0.5 on Figure 9) signifies greater velocity change. A graphical presentation of these statements is the inertia sensitivity curve of Figure 10.

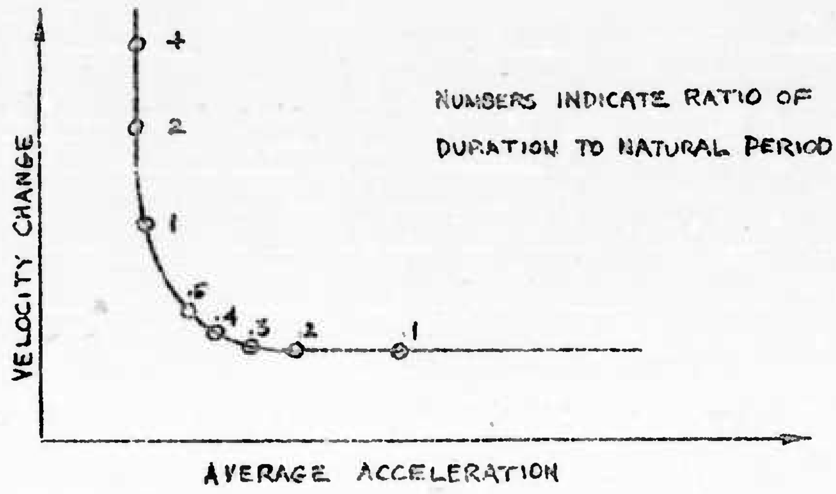


Figure 10
Inertia Sensitivity Curve for Mass-Spring System
Subjected to Rectangular Pulses

13. Exception could be taken to the choice of coordinates of Figure 10, since the basic parameters are acceleration and duration. However, the use of velocity change is found to be more practical, since it leads to a curve which quickly approaches two asymptotes. (The word asymptote is used loosely here, since for some shape pulses the curve crosses the "asymptote." and the rectangular pulse curve actually reaches its "asymptote.") Velocity change is also an index of the kinetic energy required for actuation. The reason for the choice of average acceleration instead of peak acceleration, aside from the fact that duration may be readily determined as velocity change divided by average acceleration, is that most service pulses have high frequency "hash" superimposed on the main pulse. The high acceleration spikes associated with the hash have little effect on the inertia mechanism, and a plot of peak acceleration would produce very erratic and unreliable sensitivity curves.

Interpretation of a Sensitivity Curve

14. Figure 10 represents the inertia sensitivity of a mass-spring system to any pulse of rectangular shape. In general, for a given velocity change, it predicts the minimum acceleration necessary to actuate the mechanism. Also, for a given acceleration it predicts the minimum velocity change necessary to actuate the mechanism.

15. Other conclusions may be drawn from this curve due to the presence of the asymptotes (which also exist for other pulse shapes). Take, for example, the left hand portion of the curve. In this region of relatively long pulse durations, the velocity change or duration may increase considerably without appreciably changing the acceleration. The value of this fact may be illustrated by the example of the designer of an inertia mechanism which must be actuated by a catapult which attains a peak acceleration of 10 gravitational units. If he designs the mechanism with too long a natural period, the catapult pulse will effectively act as an impulsive velocity change, and the operation of the mechanism will depend on the velocity delivered by the catapult. However, by examination of the inertia sensitivity curve, he may note that so long as the natural period of the mechanism is about 1/2 or less of the duration of the catapult pulse, the mechanism operation is virtually independent of velocity. Thus, as long as he keeps that restriction on natural period in mind, he may design the mechanism solely on the basis of static deflection under 10 gravitational units.

16. Similar considerations apply to the right hand portion of Figure 10. In this region, as long as the duration of the pulse is about 1/3 or less of the natural period, the velocity change remains relatively constant. Therefore, a designer who is dealing with short durations (relative, of course, to the natural period of the mechanism) has merely to stay below a certain duration in order to achieve a constant result. Within this region, the peak acceleration may be unknown or highly variable without affecting the result.

17. The above remarks on asymptotes apply to any sensitivity curve. The quantitative results, however, are different for different pulse shapes. That is, the asymptotes exist but are parallel. (See Figure 11). In the following sections a comparison will be made of sensitivity curves for several types of pulses, in an attempt to reach some quantitative conclusions which may apply to a large number of cases.

EFFECT OF PULSE SHAPE ON THEORETICAL SENSITIVITY CURVES

18. Figure 11 presents sensitivity curves for a mass-spring system subjected to pulses of various shapes. For purposes of comparison, the sensitivity curve for the rectangular pulse is repeated on each plot as a dashed curve. Figure 11(f) is composed of all the sensitivity curves plotted together.

(See Appendix for Definition of Symbols)

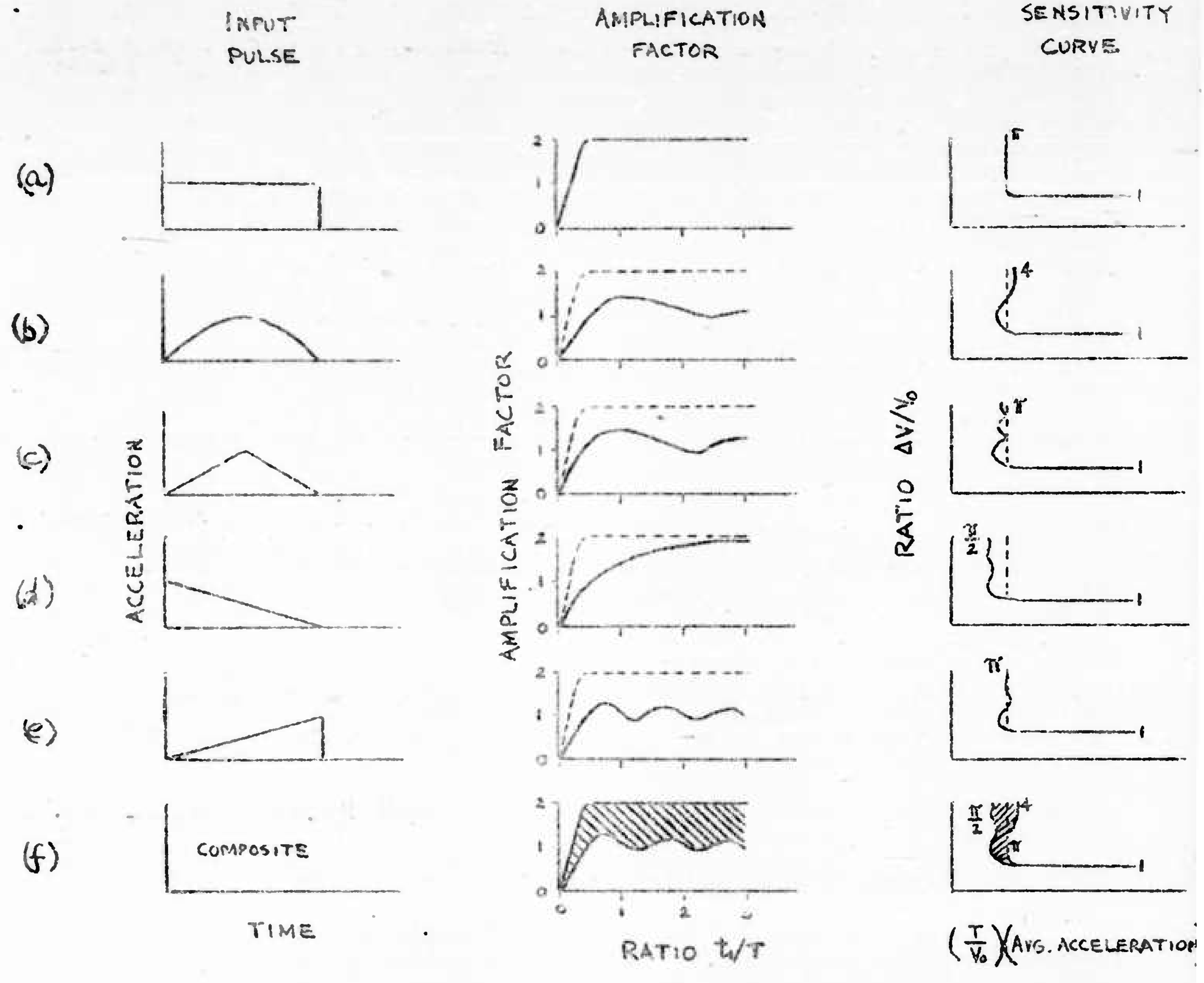


Figure 11
Amplification Factors and Sensitivity Curves for Mass-Spring System Subjected to Single Pulses

19. It would be extremely advantageous to be able to employ a single curve, or narrow band, which would be reasonably accurate in many applications. Unfortunately, Figure 11(f) reveals that the low acceleration asymptotes are spread over a range of about 2.5 to 1. The variation for the single pulses shown (systems which can move in either direction from the neutral position show behavior less desirable than Figure 11 when subjected to multiple pulses--see reference (d)) is from $\pi/2$ for the triangle with very short build-up time to 4 for the half-sine pulse. Since either of these pulses is likely to be encountered in practice, it appears that the use of a single sensitivity curve would be subject to errors up to about 125% maximum either way.

20. On the brighter side of the picture, Figure 11 shows that the high acceleration (short duration) asymptote exhibits negligible dependence upon pulse shape. This means that any shock of duration shorter than 1/3 or 1/4 of the natural period, regardless of pulse shape, will require the same velocity change, or energy, to actuate the inertia mechanism.

21. Without going into the questions of which pulse shapes are produced in the laboratory and which pulse shapes may be expected in the field, the following general conclusions may be drawn:

a. The low-acceleration asymptote depends a good deal on pulse shape, and is therefore unreliable for general quantitative use. Unless the use of the sensitivity curve is narrowly restricted as to pulse shape, this asymptote is useful only for order-of-magnitude.

b. The high-acceleration asymptote is quite independent of pulse shape and therefore very reliable.

LABORATORY DETERMINATION OF INERTIA SENSITIVITY

22. The obvious answer to the problem of pulse shape is a laboratory-determined sensitivity curve for each pulse shape likely to be encountered by the inertia device. This, however, would lead to large expense, both in testing time and in equipment. The practical approach is to use the best equipment available and make the best attempt at interpretation.

23. Figure 12 gives a general picture of the equipment in current use at the Naval Ordnance Laboratory. The full practical ranges of duration, acceleration and velocity change are covered by this equipment, with some variation in pulse shape. See reference (j).

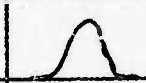
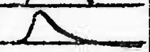
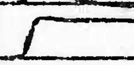
Equipment	Peak Accelerations, g	Duration, Milliseconds	Velocity Change, ft/sec	Pulse Shape
Drop Testers, Steel	200 - 25,000	.040 - .400	0 - 60	
Drop Testers, Lead Pads	10 - 1000	.40 - 20	0 - 60	
Sine Wave Tester	20 - 200	3 - 6	0 - 10	
Square Wave Tester	0 - 60	5 - 50	0 - 25	
Air Guns	5 - 100,000	1 - 200	0 - 1000	
Rotary Accelerator	10 - 650	Long	--	
Centrifuges	0 - 30,000	Long	--	—

Figure 12
Inertia Sensitivity Equipment at the
Naval Ordnance Laboratory

24. The general procedure in testing a specific mechanism is first to make a rough estimate of the natural frequency of the device and the minimum velocity change to effect actuation. At this stage one may select the test equipment which will be used. For example, if an inertia mechanism has a natural period of 30 milliseconds and a minimum velocity change of 5 ft/sec (see Appendix E), then any equipment with a duration of less than about 10 milliseconds would give test points out on the high-acceleration asymptote (see Figure 10). Therefore, the equipment to be chosen from Figure 12 would be drop testers for the high-acceleration region, the Square-Wave Tester for the bend of the sensitivity curve, and the centrifuge for the low-acceleration asymptote.*

25. Proper mounting of the test specimen is most important for the sensitivity curve to have any significance. Whenever possible, the device should be mounted exactly as in the service application. In this way, the only simulation involved is in the input acceleration pulse. Unfortunately, however, many of the service vehicles carrying inertia devices are physically too large to be accommodated in the testing machines, and the question arises as to the effect of interposing a substitute elastic system between the source of shock and the component to be tested. This problem of transmission of shock through complex structures has not yet attained the stature of a science, due to the dearth of information on even the more simple mechanical systems responding to transients.

*See Appendix C, page 26, for discussion of presentation of centrifuge data.

At some future date, when the literature contains the solutions of a sufficient number of classical problems, it may be practical to design an elastic mount equivalent to the actual mounting structure.

26. In some cases after it has been determined that the actual mounting structure is too large, it is possible to select an intermediate mounting structure which "sees" the input pulse (in service) and is of suitable size. This method, of course, depends entirely on the judgment of the individual who decides on the relative rigidities of the structural members.

27. A simple, and also determinate, method of testing inertia devices is to use a relatively rigid mount which faithfully transmits the input (test) pulse. In this way, whenever the entire service mount or some intermediate mount cannot be tested as a unit, the performance of the mechanism on a determinate mount is ascertained. Although application of this information to the service mounting condition may be difficult, the procedure is much more satisfactory than making indeterminate attempts at simulation of mounting. It may be observed that the usual ordnance application of inertia switches corresponds to the rigid mount (switch frequency considerably lower than structure frequency), and that many of the cases which do not fit in this category are of a size suitable for full-scale testing of the entire weapon.

28. After the machines have been selected and the specimen mounted, testing for sensitivity is done by finding the minimum velocity change for actuation of the device. For example, on the drop tester the method is to find the height of drop just sufficient for operation. The sensitivity levels for various pulse durations are determined and the inertia sensitivity curve plotted. The procedure is repeated for representative oblique orientations of impact. When the inertia device fires an explosive element directly or for some other reason can only be tested once and at a pre-determined input level, a statistical approach is essential for efficient, economical testing. The Bruceton and Probit methods of approach to the analysis of go-no go data of this type are explained in reference (i).

PRESENTATION OF LABORATORY SENSITIVITY DATA

29. Laboratory tests produce an inertia sensitivity curve composed of points representing several pulse shapes. Unless each point accidentally coincides with the field application as regards pulse shape, it would appear that the sensitivity curve may not be very useful. It must be recalled, however, that the high acceleration asymptote shows negligible

dependence on pulse shape, which makes part of the curve quite reliable.

30. There are two approaches to the problem of the lack of reliability of the low-acceleration region of the curve. The generally accepted practice is to present the data "as is," based on the reasoning that the test machines yield results midway between the possible spread of values, and also based on the lack of accurate knowledge of field conditions. This approach realistically does not try to produce information on a level higher than the purpose for which it is intended. Data in the low-acceleration region are unreliable for close calculation, and should be regarded as such.

31. In the rare instances which warrant accurate simulation, based on precise information on field shocks, it is possible to get better answers by adjusting the sensitivity data. For example consider a set of data obtained with laboratory equipment which gave the shape pulse and duration as indicated in Figure 13.

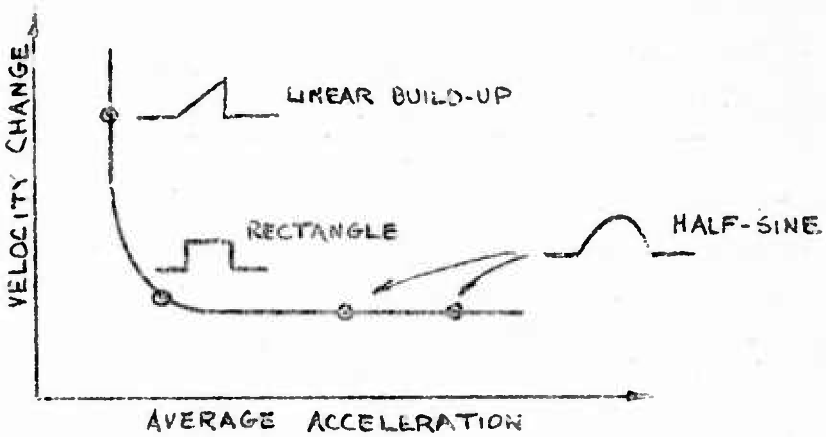


Figure 13
Sensitivity Curve Obtained with Typical Laboratory Equipment

This curve is valid only for the pulse shapes indicated, except in the high-acceleration region where all pulse shapes yield almost identical results. Now let us say that the mechanism tested above is to be used in an application which calls for a half-sine pulse in place of the linear build-up, a symmetrical triangle instead of the rectangle, and rectangles instead of the half-sines. A rule of thumb approximation of the answer is to apply the theoretical values of acceleration (for each portion of the curve) to the test data of Figure 13. Take, for illustration, the point which was obtained with a linear build-up and which must be presented as a half-sine pulse. If the data were found as 12 gravitational units, the approximate point would be $12 \frac{4}{\pi} = 15.3$ gravitational units, based on the theoretical value of 4 for the half sine and π for the linear build-up (See Figure 11). Using this approximate method, the desired sensitivity curve would be presented as the solid curve of Figure 14.

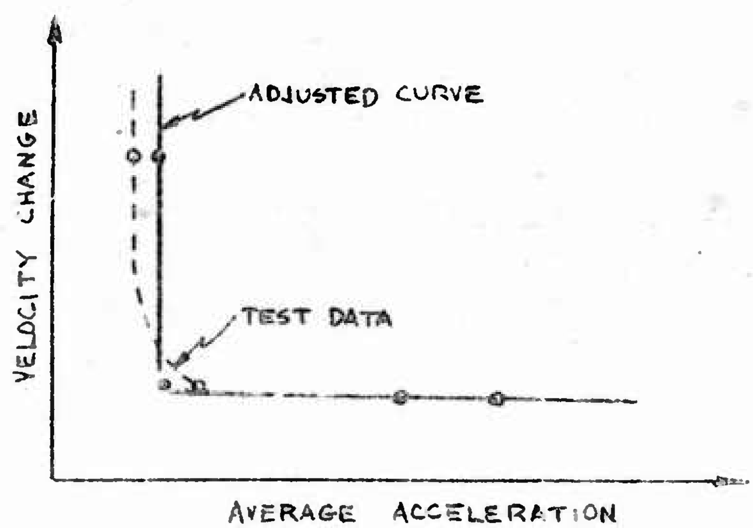


Figure 14
Adjustment of Sensitivity Data

PRACTICAL USE OF A SENSITIVITY CURVE

32. There are powerful reasons for not depending too closely on inertia sensitivity curves in the low-acceleration region. One of these sources of difficulty lies in the statistical nature of the physical problem. For example, if one considers the shock on entry into water, the important factor of water surface configuration leads to statistical variability of results. If it is a target impact problem, the target structure and exact location of impact are rarely well defined. In problems of this type, any carefully reasoned estimate should be presented with a probable spread of values.

33. Another source of error is the lack of enough good field or service information. Gathering data in the field is an extremely expensive and time-consuming task, considering the instrumentation involved in measuring accelerations. For this reason, few measurements are made on any given field vehicle in a given service application. One or two measurements applied to a statistical problem do not lead to an answer which can be regarded with a good deal of confidence.

34. The designer's answer to the sources of error mentioned above is to make provisions for satisfactory operation of the mechanism over a wide spread of service conditions. The mechanism must be quite flexible in its range of operation, with variability of field conditions and errors in field measurements adding up to some ±50%. This approach of recognizing, and meeting, the difficult design specifications is far superior to that of attempting to predict the probability of operation of a mechanism which operates somewhere within the spread of service conditions. An added advantage is the likelihood that a design which can cope with the spread of service requirements will also be adequate for the spread of the low-acceleration asymptotes due to the variation in pulse shapes.

35. Upon consideration of the variability of field conditions, and the unreliability of typical field data, it becomes apparent that the laboratory sensitivity data are probably the most carefully controlled and reliable components of the overall problem. It is therefore concluded that use of an "average" sensitivity curve (the laboratory test data) in the low-acceleration range is justified until such time as extensive and reliable field data are obtained (on each problem) to lead to solutions in terms of statistical spreads.

APPENDIX A - RESPONSE OF THE MASS-SPRING SYSTEM

The following nomenclature is used throughout the analysis:

K - Spring constant

L - Free length of spring

M - Mass

t - Time

t_1 - Duration of pulse

t_m - Time at maximum amplitude of response

T - Natural period $= 2\pi\sqrt{\frac{M}{K}}$

V_o - Impulsive velocity required to produce X_o

ΔV - Velocity change of the applied acceleration-time pulse

X - Deflection of mass on the spring

X_o - Specified travel of mass, measured from initial position

X_m - Maximum deflection

X_p - Present deflection of mass, measured from the neutral position

y - Deflection of free end of spring from a fixed reference axis

$\ddot{y}_m$ - Peak value of applied acceleration

α - Dummy variable of integration

β - Pulse shape factor equal to average acceleration divided by peak acceleration

ω - Natural circular frequency $=\sqrt{\frac{K}{M}}$

Figure 15 shows the position of the mass-spring system relative to some fixed reference system:

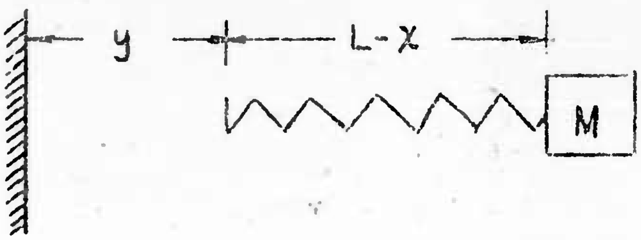


Figure 15
The Mass-Spring System

An analysis similar to that of reference (b) is followed:

$$M\frac{d^2}{dt^2}(y+L-x) - kx = 0$$

$$M\ddot{x} + kx = M\ddot{y}$$

$$\ddot{x} + \omega^2 x = \ddot{y} \qquad (1)$$

Multiplying by $\sin\omega t$:

$$\ddot{x}\sin\omega t + \omega^2 x \sin\omega t = \ddot{y}\sin\omega t$$

$$\frac{d}{dt}(\dot{x}\sin\omega t) - \frac{d}{dt}(\omega x\cos\omega t) = \ddot{y}\sin\omega t$$

Integrating

$$\dot{x}\sin\omega t - \omega x\cos\omega t = \int_0^t \frac{d^2y}{d\alpha^2}\sin\omega\alpha\, d\alpha \qquad (2)$$

Where α is a dummy variable of integration. Multiplying Equation (1) by $\cos\omega t$:

$$\ddot{x}\cos\omega t + \omega^2 x\cos\omega t = \ddot{y}\cos\omega t$$

$$\frac{d}{dt}(\dot{x}\cos\omega t) + \frac{d}{dt}(\omega x\sin\omega t) = \ddot{y}\cos\omega t$$

Integrating: $$\dot{x}\cos\omega t + \omega x\sin\omega t = \int_0^t \frac{d^2y}{d\alpha^2}\cos\omega\alpha\, d\alpha \qquad (3)$$

The above integrations assume the boundary conditions of zero displacement and zero velocity at zero time. Eliminating terms in $\dot{x}$ from Equations (2) and (3):

$$\omega x = \sin\omega t\int_0^t \frac{d^2y}{d\alpha^2}\cos\omega\alpha\, d\alpha - \cos\omega t\int_0^t \frac{d^2y}{d\alpha^2}\sin\omega\alpha\, d\alpha$$

$$\omega x = \int_0^t \frac{d^2y}{d\alpha^2}\sin\omega(t-\alpha)\, d\alpha$$

Dimensionlessly: $$\frac{\omega^2 x}{\ddot{y}_m} = \omega\int_0^t \left(\frac{1}{\ddot{y}_m}\frac{d^2y}{d\alpha^2}\right)\sin(\omega t-\omega\alpha)\, d\alpha \qquad (4)$$

The procedure for determining the amplification factor is to substitute $\left(\frac{1}{\ddot{y}_m}\frac{d^2y}{d\alpha^2}\right)$ as a function of α (shape of the input pulse) into Equation (4), solve for the maximum value of $\frac{\omega^2 x}{\ddot{y}_m}$, then:

$$\text{Amplification factor} = \frac{k x_m}{M\ddot{y}_m} = \frac{\omega^2 x_m}{\ddot{y}_m} \qquad (5)$$

APPENDIX B - THE RECTANGULAR PULSE

For the rectangular pulse (Figure 10)

$$t < 0 \qquad \ddot{y}/\ddot{y}_m = 0$$

$$0 \leq t \leq t_1 \qquad \ddot{y}/\ddot{y}_m = 1$$

$$t > t_1 \qquad \ddot{y}/\ddot{y}_m = 0$$

Substituting into Equation (4) for the period $0 \leq t \leq t_1$

$$\frac{\omega^2 x}{\ddot{y}_m} = \omega \int_0^t \sin(\omega t - \omega\alpha)\,d\alpha = \cos(\omega t - \omega\alpha)\Big|_0^t = 1 - \cos\omega t \qquad (6)$$

To find the maximum, differentiate and equate to zero:

$$\sin\omega t_m = 0 \longrightarrow \omega t_m = 2\pi \frac{t_m}{T} = 0, \pi, 2\pi, \ldots\ldots$$

$$\frac{t_m}{T} = 0, \tfrac{1}{2}, 1, 1\tfrac{1}{2}, \ldots\ldots$$

$$\text{for } \frac{t_m}{t_1} \leq 1 \quad , \quad \frac{t_1}{T} \geq \frac{1}{2}$$

Amplification factor = 1-(-1) = 2 (7)

For the period $t \geq t_1$

$$\frac{\omega^2 x}{\ddot{y}_m} = \omega \int_0^{t_1} \sin(\omega t - \omega\alpha)\,d\alpha + \omega \int_{t_1}^{t} 0$$

$$= \cos(\omega t - \omega\alpha)\Big|_0^{t_1} = \cos(\omega t - \omega t_1) - \cos\omega t$$

$$= \sin\omega t \sin\omega t_1 - \cos\omega t\,(1 - \cos\omega t_1) \qquad (8)$$

Differentiating for the maximum:

$$\cos\omega t_m \sin\omega t_1 + \sin\omega t_m (1 - \cos\omega t_1) = 0$$

$$\tan\omega t_m = \frac{\sin\omega t_1}{\cos\omega t_1 - 1} = -\operatorname{ctn}\frac{\omega t_1}{2}$$

$$\omega t_m = 2\pi \frac{t_m}{T} = \frac{\pi}{2} + \frac{\omega t_1}{2}$$

$$\frac{t_m}{T} = \frac{1}{4} + \frac{1}{2}\frac{t_1}{T}$$

$$\text{for } \frac{t_m}{t_1} \geq 1 \quad , \quad \frac{1}{4}\frac{T}{t_1} + \frac{1}{2} \geq 1 \quad , \quad \frac{t_1}{T} \leq \frac{1}{2}$$

$$\text{Amplification factor} = \sin 2\pi\left(\frac{1}{4}+\frac{1}{2}\frac{t_1}{T}\right)\sin 2\pi\frac{t_1}{T} - \cos 2\pi\left(\frac{1}{4}+\frac{1}{2}\frac{t_1}{T}\right)\left(1-\cos 2\pi\frac{t_1}{T}\right)$$

$$= \cos\pi\frac{t_1}{T}\sin 2\pi\frac{t_1}{T} + \sin\pi\frac{t_1}{T}\left(1-\cos 2\pi\frac{t_1}{T}\right)$$

$$= \sin\left(2\pi\frac{t_1}{T}-\pi\frac{t_1}{T}\right) + \sin\pi\frac{t_1}{T}$$

$$= 2\sin\pi\frac{t_1}{T} \qquad (9)$$

Figure 9 is a plot of Equation (9) for $\frac{t_1}{T} \leq \frac{1}{2}$ and Equation (7) for $\frac{t_1}{T} \geq \frac{1}{2}$. Figure 11 shows amplification factor curves for several pulse shapes other than the rectangular pulse.

APPENDIX C - SENSITIVITY CURVES FOR THE MASS-SPRING SYSTEM

In order to express inertia sensitivity parameters in dimensionless form, the quantity V_0 is introduced. This is the velocity of the mass as it passes through the neutral position (zero spring deflection) sufficient to produce X_0, which is the specified travel of the mechanism. Equating potential and kinetic energy (no preset):

$$\int_0^{X_0} kx\,dx = \frac{1}{2}MV_0^2$$

$$V_0 = \sqrt{\frac{k}{M}}\,X_0$$

$$V_0 = \omega X_0 \qquad (10a)$$

The steady acceleration needed to produce $X_0 = \frac{kX_0}{M} = \omega^2 X_0 = \omega V_0$ (10b)

Peak applied acceleration $= \frac{\text{steady acceleration}}{\text{Amplification factor}} = \frac{V_0\omega}{\text{Amp. Factor}}$

Average applied acceleration = β (peak acceleration)

Pulse shape	β
rectangle	1
triangle	1/2
half-sine	$\frac{2}{\pi}$

Average acceleration = $\beta \frac{\omega V_o}{\text{Amp. Factor}}$

dimensionlessly, $$\frac{\text{Average Acceleration}}{V_o/T} = \frac{2\pi\beta}{\text{Amp. Factor}} \qquad (11)$$

Velocity change ΔV = average acceleration x t_1

dimensionlessly, $$\frac{\Delta V}{V_o} = \left(\frac{2\pi\beta}{\text{Amp. Factor}}\right)\left(\frac{t_1}{T}\right) \qquad (12)$$

By selecting values of amplification factor from Figure 11, the sensitivity plots of $\frac{\Delta V}{V_o}$ versus $\frac{\text{Average Acceleration}}{V_o/T}$ (also Figure 11) were easily calculated.

At small values of t_1/T, $\frac{\Delta V}{V_o}$ approaches 1, or Amp. Factor $= 2\pi\beta\frac{t_1}{T}$ that is, the initial slope of the Amp. Factor versus t_1/T curve = $2\pi\beta$. Consequently, Average Acceleration/$\frac{V_o}{T} = \frac{T}{t_1}$

At large values of t_1/T, as indicated on Figure 11

Shape	Amplification Factor	$\frac{\text{Average Accel.}}{V_o/T}$
	2	π
	1	4
	1	π
	2	$\pi/2$
	1	π
Centrifuge	1	2π

Note that half the centrifuge value of 2π would fall within the crosshatched region of Figure 11(f), and therefore one-half of the indicated centrifuge reading is used on laboratory sensitivity curves. This is in line with the policy of using an "average" sensitivity curve to represent most service conditions with an accuracy warranted by such applications. One could perhaps reason that the centrifuge test consists of a gradual build-up of acceleration with maximum deflection of the test item occurring at the instant of reaching peak applied acceleration, and therefore the average applied acceleration is half the peak. This analysis would be appropriate, however, only for a linear build-up, and it may be better to rely on the mathematical results which indicate the use of half the centrifuge reading.

APPENDIX D - CHARACTERISTIC DELAY TIMES

Figure 16 is a plot of time to reach maximum deflection (characteristic delay time) versus pulse duration $\left(\frac{t_m}{T} \text{ vs } \frac{t_1}{T}\right)$. Some general conclusions may be drawn:

(1) A short-duration pulse causes actuation in 1/4 period. This is evident when one considers that the response to an impulse is a free vibration at the natural period, and that the first peak in a free vibration occurs at 1/4 period.

(2) For long-duration pulses with finite build-up times (or, more precisely, for large ratios of build-up time to natural period) the maximum deflection is reached at about the same time as the peak acceleration of the applied pulse. This may be checked by noting (Figure 16) that values of $t_m/t_1 = \left(\frac{t_m}{T}/\frac{t_1}{T}\right)$ at large ratios of $\frac{t_1}{T}$ (>1) agree roughly with the ratio of build-up time to pulse duration. This statement is equivalent to saying that the mass-spring system of short natural period follows the input pulse closely.

(3) For long-duration pulses with instantaneous build-up (a step function, which can be approximated in the laboratory and in service) the peak deflection is attained in 1/2 period.

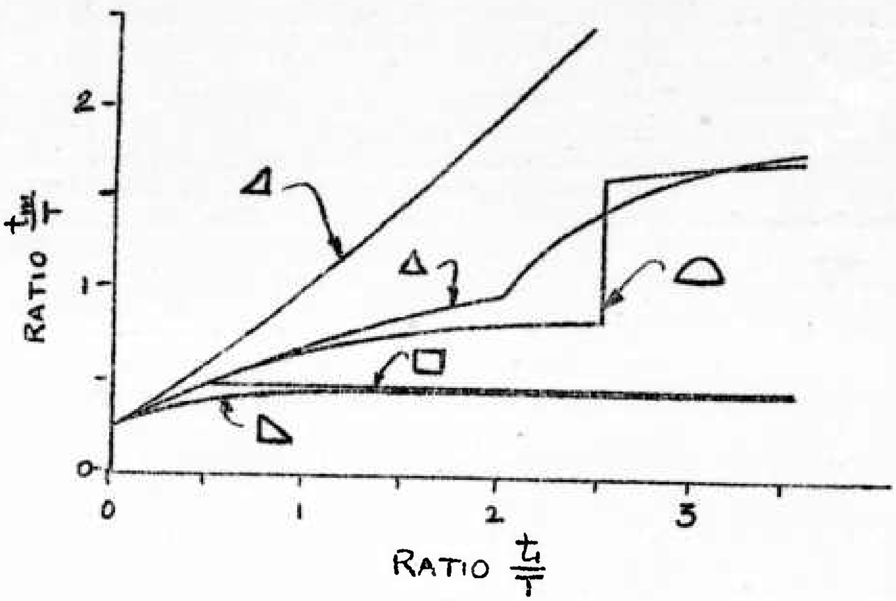


Figure 16
Characteristic Delay Times

The discontinuities in the curves of Figure 16 indicate the points at which the first positive motion of the mass is no longer the maximum deflection. This raises the question of applicability of these theoretical sensitivity curves (obtained for a mass-spring system which could operate in either direction from the neutral position) to one-directional mass-spring systems. In response to multiple pulses, such as a full sine wave (reference (d)) or a blast pulse (reference (b)), the negative motion may exceed the positive. Therefore, the one-directional system is less sensitive than the two-directional system. For single positive pulses, however, the mass may reverse direction after the first positive peak, but it reaches its maximum peak before passing through neutral, and both types of system will have identical sensitivities.

APPENDIX E - SOME SIMPLE DESIGN RELATIONSHIPS

The designer who must estimate a sensitivity curve has some simple formulae at his disposal. For the high-acceleration asymptote, the velocity change is close to the V_o of Equation (10). Rewriting Equation (10a) to include the effect of preset of the spring (X_p):

$$V_o = \omega X_o \sqrt{1 + 2\frac{X_p}{X_o}} \qquad (13)$$

Since the total deflection at the point of maximum travel is equal to $X_p + X_o$, the peak acceleration applied by centrifuge = $\frac{k(X_o + X_p)}{M} = \omega^2 (X_p + X_o)$.

Using the "average" acceleration of half the centrifuge value:

$$\text{Average acceleration} = 1/2\,\omega^2 (X_p + X_o) \qquad (14)$$

Figure 17 uses Equations (13) and (14) in estimating the sensitivity curve.

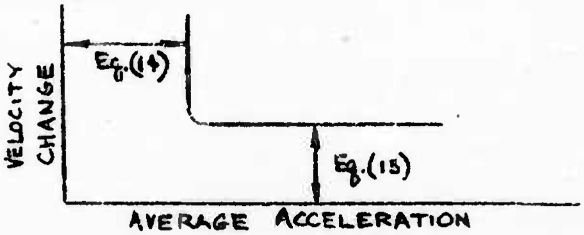


Figure 17
An Estimated Sensitivity Curve

An example of the accuracy of these simple equations, when applied to a simple determinate system, is the following comparison of test results and theory for a small cantilever beam used as a switch:

measurements { centrifuge setting of switch = 4790g
natural frequency - measured on vibrator = 2085 cps }

Using Equation (10): $$V_o = \frac{\text{static acceleration}}{\text{natural frequency}} = \frac{(4790)(32.2)}{(2\pi)(2085)}$$

$$= 11.78 \text{ fps}$$

The measured value of V_o, using a 50 microsecond input pulse, was 11.35 fps.

The basic design parameters (Figure 17) are velocity change average acceleration, natural frequency, preset of the spring, and travel distance. Since, in general, the restrictions on more than one or two of these quantities are not too severe, the designer has considerable latitude in his design. It is because of this freedom of design that it is possible to overcome the liability of the expected spread in operating conditions which usually occurs in service applications of inertia devices.

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