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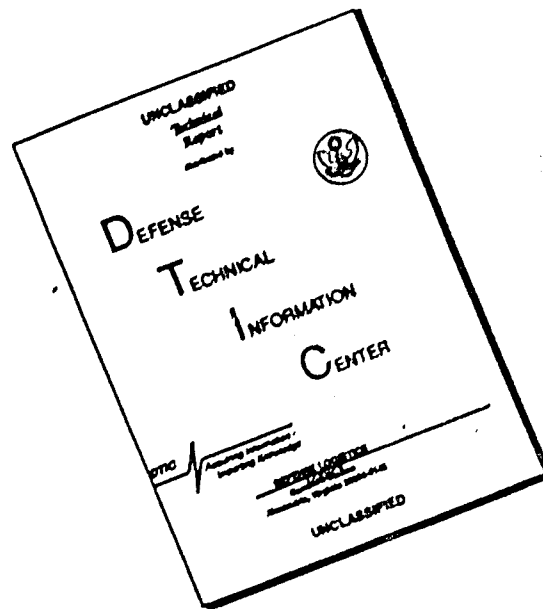
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RESEARCH MEMORANDUM

(Unclass) STATISTICAL THEORY OF NAVIGATION EMPLOYING
INDEPENDENT INERTIAL AND VELOCITY MEASUREMENTS:
MINIMUM RMS ERROR IN COMPUTED POSITION

P. Swerling
E. Reich

RM-1321

17 August 1954

Copy No. 16

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Correction to RM-1321: Statistical Theory of Navigation Employing
Independent Inertial and Velocity Measurements:
Minimum RMS Error in Computed Position

by P. Swerling and E. Reich

The fourth line of Eq. (II.5), p. 4, should read:

$$\Delta = L_{11}L_{22} - L_{12}^2$$

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SUMMARY

Analysis of navigation systems employing independent inertial and velocity measurements, begun in Ref. 1, is continued. Explicit formulas are given for minimum rms error in computed position as a function of time of flight. Curves based on these formulas are presented, showing the results for a number of illustrative cases.

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LIST OF SYMBOLS

<u>Symbol</u>	<u>Definition</u>	<u>Page First Used</u>
$x(t)$	vehicle position at time t	1
$\hat{x}(t;T)$	optimum computer's estimate of $x(t)$ based on all the dial readings up to time T	1
$\hat{x}(T)$	$\hat{x}(T;T)$	1
T	elapsed time since beginning of flight	1
$\gamma_1, \gamma_2, \gamma_3, \gamma_4,$ $\alpha_2, \beta_2, \alpha_1, \beta_1$	parameters describing instrument error statistics	1,2,5,16
$\xi_{\hat{x}}(t;T)$	$\hat{x}(t;T) - x(t)$	2
$\xi_{\hat{x}}(T)$	$\hat{x}(T;T) - x(T)$	2
ξ_{ij}	defined by Eq. (II.1)	2
η_{ij}	defined by Eq. (III.1)	15
$\phi_B(s,t)$	accelerometer error autocorrelation function	1
$\phi_V(s,t)$	velocity dial error autocorrelation function	1
Ω	frequency of earth's radius pendulum	3
μ	defined by Eq. (II.4)	4

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1. INTRODUCTION

This report is an extension of the work begun in Ref. 1. This is not a self-contained report; rather, a reading of Ref. 1 is a necessary basis for the understanding of the following material. Repetition of expository material and results obtained in Ref. 1 will be kept to a minimum.

The major results of Ref. 1 were obtained for two cases, viz:

Case 1

$$\phi_B(\tau, t) = \gamma_1$$

Case 2

$$\phi_V(s, t) = \gamma_2$$

where $\phi_B(s, t)$ and $\phi_V(s, t)$ are the autocorrelation functions of the accelerometer dial error and the velocity dial error, respectively. Formulas were derived for the optimum method of position computation in each case; also, formulas were given by means of which the rms error in computed position as a function of time of flight could be derived.

In the following pages, the actual formulas for rms error in computed position will be given; also, computations based on these formulas will be given for a number of illustrative cases.

As in Ref. 1, let, for $0 \leq t \leq T$,

$x(t)$ = true position at time t

$\hat{x}(t; T)$ = optimum computer's estimate of $x(t)$, based on all the dial readings up to time T .

Denote $\hat{x}(T; T)$ simply by $\hat{x}(T)$.

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Also let

$$\begin{aligned} \hat{x}(t;T) &= \hat{x}(t;T) - x(t) \\ (I.1) \quad \hat{x}(T) &= \hat{x}(T;T) - x(T) \end{aligned}$$

and

$$(I.2) \quad \overline{\hat{x}^2(t;T)} = \overline{[x(t;T) - x(t)]^2}$$

the mean being taken over an ensemble of flights.

As in Ref. 1, the following notation is used for description of the dial error statistics:

$$\begin{aligned} \xi_B(t) &= \text{accelerometer dial error} \\ \xi_V(t) &= \text{velocity dial error} \\ \xi_{X_0} &= \text{independent initial position dial error} \\ \xi_{X'_0} &= \text{independent initial velocity dial error} \end{aligned}$$

The ensemble means of all these errors are zero; also

$$(I.3) \quad \overline{\xi_{X_0}^2} = \gamma_3, \quad \overline{\xi_{X'_0}^2} = \gamma_4$$

and

$$\begin{aligned} (I.4) \quad \phi_B(s,t) &= \overline{\xi_B(s) \xi_B(t)} \\ \phi_V(s,t) &= \overline{\xi_V(s) \xi_V(t)} \end{aligned}$$

(All averages are taken over an ensemble of flights.)

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II. CASE 1: $\delta_B(s, t) = \gamma_1$

Let $0 = t_0 < t_1 < \dots < t_{n-1} = T$ be n equally spaced time points in the interval $(0, T)$. Let

$$(II.1) \quad [\xi_{1j}] = [\phi_V(t_1, t_j)]^{-1} \quad (\text{matrix inverse})$$

Also let

$$(II.2) \quad [A_{11}]_n = \Omega^2 \sum_{i,j=0}^{n-1} \xi_{1j} \sin \Omega t_i \sin \Omega t_j$$

$$[A_{12}]_n = \Omega \sum_{i,j=0}^{n-1} \xi_{1j} \sin \Omega t_i \cos \Omega t_j$$

$$[A_{22}]_n = \sum_{i,j=0}^{n-1} \xi_{1j} \cos \Omega t_i \cos \Omega t_j$$

and

$$A_{11} = \lim_{n \rightarrow \infty} [A_{11}]_n$$

$$(II.3) \quad A_{12} = \lim_{n \rightarrow \infty} [A_{12}]_n$$

$$A_{22} = \lim_{n \rightarrow \infty} [A_{22}]_n$$

The quantities A_{11} , A_{12} , A_{22} are functions of T .

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reference 1 gave formulas (Eqs. III.16, III.17', III.20 of Ref. 1)
from which could be derived the formula for $\overline{\xi_{\hat{x}}^2(t;T)}$. The result can be
expressed as follows:

Let

$$(II.4) \quad \mu = \frac{\frac{\gamma_1}{\Omega^4} \gamma_3}{\frac{\gamma_1}{\Omega^4} + \gamma_3}$$

and

$$(II.5) \quad \left\{ \begin{array}{l} L_{11} = A_{11} + \frac{1}{\frac{\gamma_1}{\Omega^4} + \gamma_3} \\ L_{12} = A_{12} \\ L_{22} = A_{22} + \frac{1}{\gamma_4} \\ \Delta = L_{11} L_{12} - L_{12}^2 \end{array} \right.$$

Then, for $0 \leq t \leq T$,

$$(II.6) \quad \overline{\xi_{\hat{x}}^2(t;T)} = \mu + \frac{1}{\Delta} \left\{ L_{22} \left(\cos \Omega t - \frac{\mu}{\gamma_3} \right)^2 \right. \\ + 2L_{12} \left(\cos \Omega t - \frac{\mu}{\gamma_3} \right) \frac{\sin \Omega t}{\Omega} \\ \left. + L_{11} \frac{\sin^2 \Omega t}{\Omega^2} \right\}$$

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The quantity $\overline{\{\hat{x}(T) - x(T)\}^2}$ is equal to $\overline{\xi_x^2(T;T)}$. Thus

$$(II.6') \quad \overline{\xi_x^2(T)} = \mu \cdot \frac{1}{3} \left\{ L_{22} \left(\cos \Omega T - \frac{\mu}{\gamma_3} \right)^2 + 2L_{12} \left(\cos \Omega T - \frac{\mu}{\gamma_3} \right) \frac{\sin \Omega T}{\Omega} + L_{11} \frac{\sin^2 \Omega T}{\Omega^2} \right\}$$

These formulas hold for any $\phi_V(s,t)$.

In Ref. 1, explicit expressions were obtained for A_{11} , A_{12} , A_{22} for the case:

$$(II.7) \quad \phi_V(s,t) = \gamma_2 + \beta_2 e^{-a_2 |s-t|}$$

The results were

$$(II.81) \quad A_{11} = \frac{\Omega^2}{2\beta_2} \left\{ \frac{a_2 T}{2} \left(1 + \frac{\Omega^2}{a_2^2} \right) - \frac{a_2}{4\Omega} \left(1 - \frac{\Omega^2}{a_2^2} \right) \sin 2\Omega T + \sin^2 \Omega T - \frac{\left[\frac{a_2}{\Omega} (1 - \cos \Omega T) + \sin \Omega T \right]^2}{2 + a_2 T + \frac{2\beta_2}{\gamma_2}} \right\}$$

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(II.811)

$$A_{12} = \frac{1}{2\beta_2} \left[\frac{a_2}{\gamma_2} \left(1 - \frac{\gamma_2^2}{a_2^2} \right) \sin^2 \Omega T + \frac{1}{2} \sin 2\Omega T \right]$$

$$- \frac{\left[\frac{a_2}{\gamma_2} (1 - \cos 2\Omega T) + \sin 2\Omega T \right] \left[\frac{a_2}{\gamma_2} \sin 2\Omega T + 1 + \cos 2\Omega T \right]}{2 + a_2 T + \frac{2\beta_2}{\gamma_2}}$$

(II.8111)

$$A_{22} = \frac{1}{2\beta_2} \left\{ \frac{a_2}{2} \left(1 + \frac{\gamma_2^2}{a_2^2} \right) + \frac{a_2}{4\gamma_2} \left(1 - \frac{\gamma_2^2}{a_2^2} \right) \sin 2\Omega T + 1 + \cos^2 \Omega T \right. \\ \left. - \frac{\left[\frac{a_2}{\gamma_2} \sin 2\Omega T + 1 + \cos 2\Omega T \right]^2}{2 + a_2 T + \frac{2\beta_2}{\gamma_2}} \right\}$$

In Figs. 1-7 are shown curves of the quantity $\left[\hat{\xi}_{\mathbf{x}(T)}^2 \right]^{1/2}$ as a function of T for $\phi_v(s,t) = \gamma_2 + \beta_2 e^{-a_2|s-t|}$. Each curve is determined by specification of the six statistical parameters $\gamma_1, \gamma_2, \gamma_3, \gamma_4, a_2, \beta_2$.

The following table gives the parameter values associated with each curve.

One word of explanation about the table is in order: an interesting case is the case in which the velocity dial error contains a "white noise" component. This can be obtained by letting $a_2 \rightarrow \infty$ and $\frac{\beta_2}{a_2} \rightarrow \kappa_2$ in Eq. (II.7). The exponential component of the velocity dial error then approaches white noise with spectral density κ_2 per cycle.

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In the following table, the units for the various parameters are

as follows:

$$\gamma_1 : (\text{nautical miles})^2 (\text{hours})^{-4}$$

$$\gamma_2 : (\text{nautical miles})^2 (\text{hours})^{-2}$$

$$\gamma_3 : (\text{nautical miles})^2$$

$$\gamma_4 : (\text{nautical miles})^2 (\text{hours})^{-2}$$

$$a_2 : (\text{hours})^{-1}$$

$$\beta_2 : (\text{nautical miles})^2 (\text{hours})^{-2}$$

The individual curves in Figs. 1-7 are identified by circled numbers.

The parameter values associated with each curve are as follows:

Curve No.	γ_1	γ_2	γ_3	γ_4	a_2	β_2	$\lim_{a_2} \frac{\beta_2}{a_2^2}$
1	50.0	1.0	.0010	9.0	$\rightarrow \infty$	$\rightarrow \infty$	.00010
2	50.0	1.0	.0010	∞	$\rightarrow \infty$	$\rightarrow \infty$	.0010
3	50.0	.10	.10	9.0	$\rightarrow \infty$	$\rightarrow \infty$	.0010
4	0.00	.10	.10	9.0	$\rightarrow \infty$	$\rightarrow \infty$	.0010
5	0.00	.10	.10	9.0	$\rightarrow \infty$	$\rightarrow \infty$	.10
6	10.0	1.0	.010	1.0	$\rightarrow \infty$	$\rightarrow \infty$	1.0
7	0.00	1.0	.10	1.0	$\rightarrow \infty$	$\rightarrow \infty$	.10
8	50.0	200	1.0	25.0	.60	2000	
9	50.0	200	1.0	25.0	.60	800	
10	50.0	200	1.0	25.0	1.80	800	
11	50.0	200	∞	∞	1.80	800	
12	50.0	200	9.0	400	1.80	2000	
13	50.0	200	9.0	400	.20	2000	
14	50.0	200	9.0	400	10.0	2000	
15	50.0	200	9.0	400	1000	2000	

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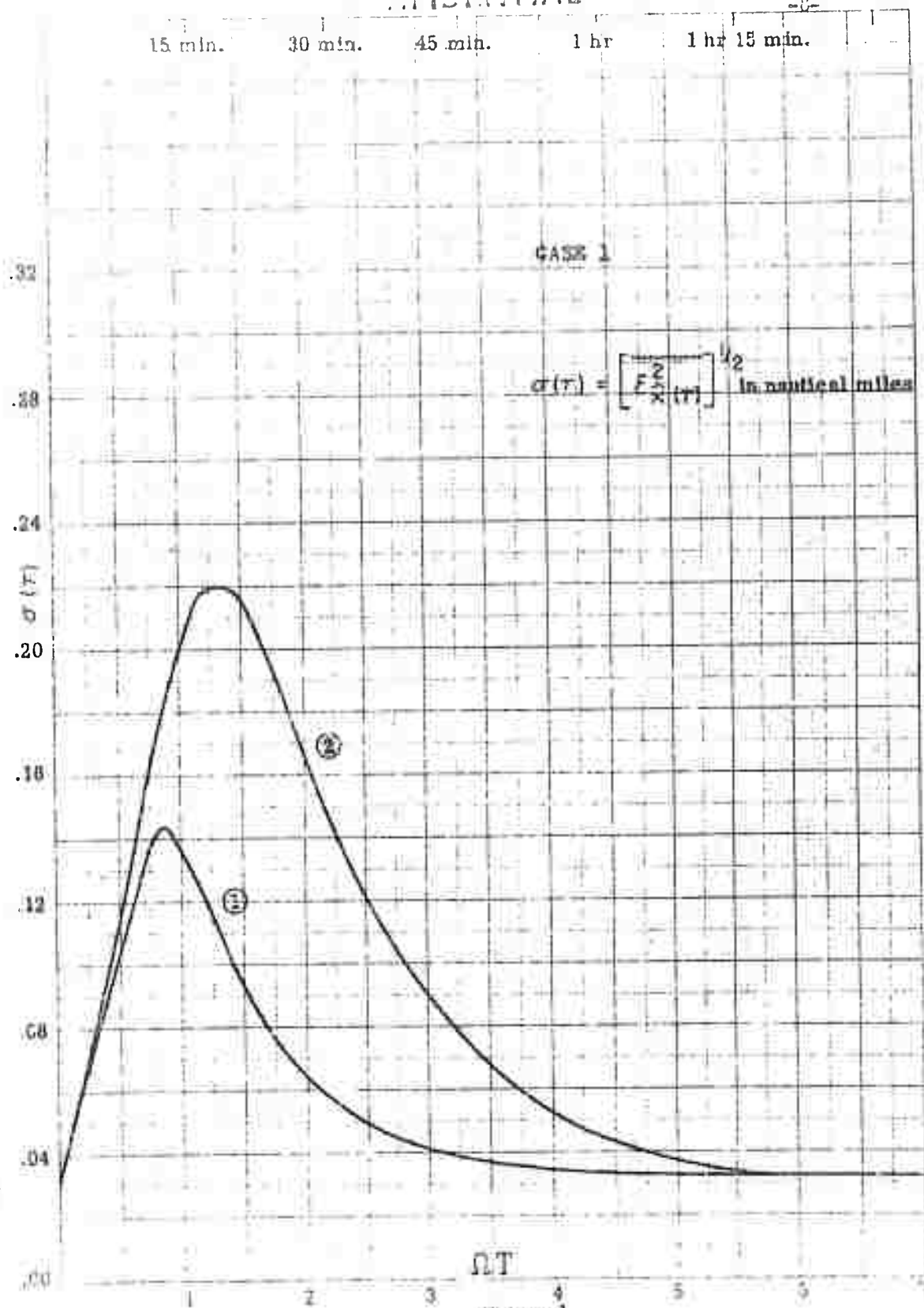


Figure 1

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40

15 min

30 min

45 min

1 hr

1 hr 15 min

(3)

GASB 1

$$\sigma(\tau) = \left[\frac{\epsilon^2}{\tau(\tau)} \right]^{1/2} \text{ in nautical miles}$$

.32

.28

.24

.20

.16

.12

.08

.04

.00

.00

.00

.00

.00

.00

(4)

ΩT

Figure 2

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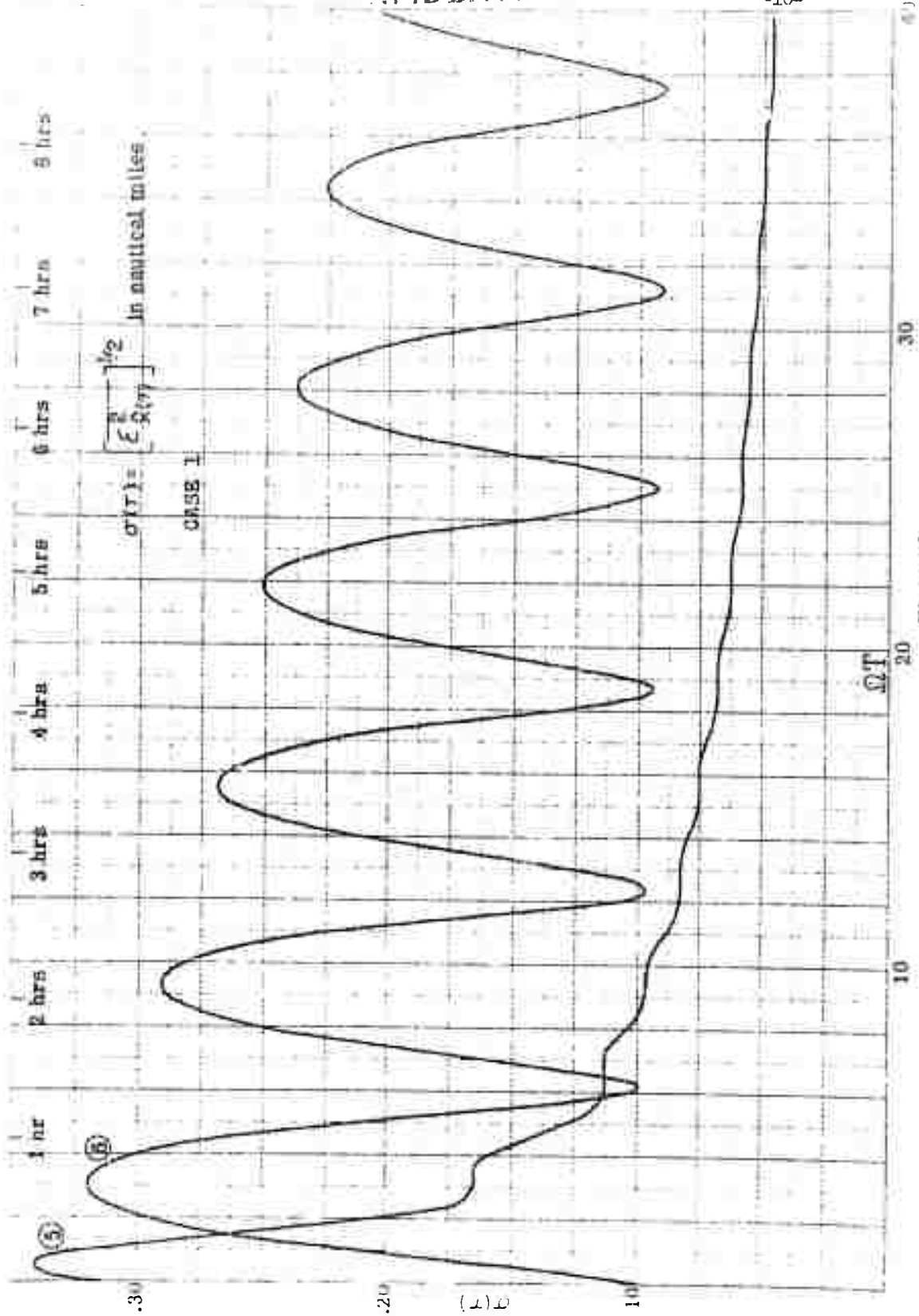


Figure 3

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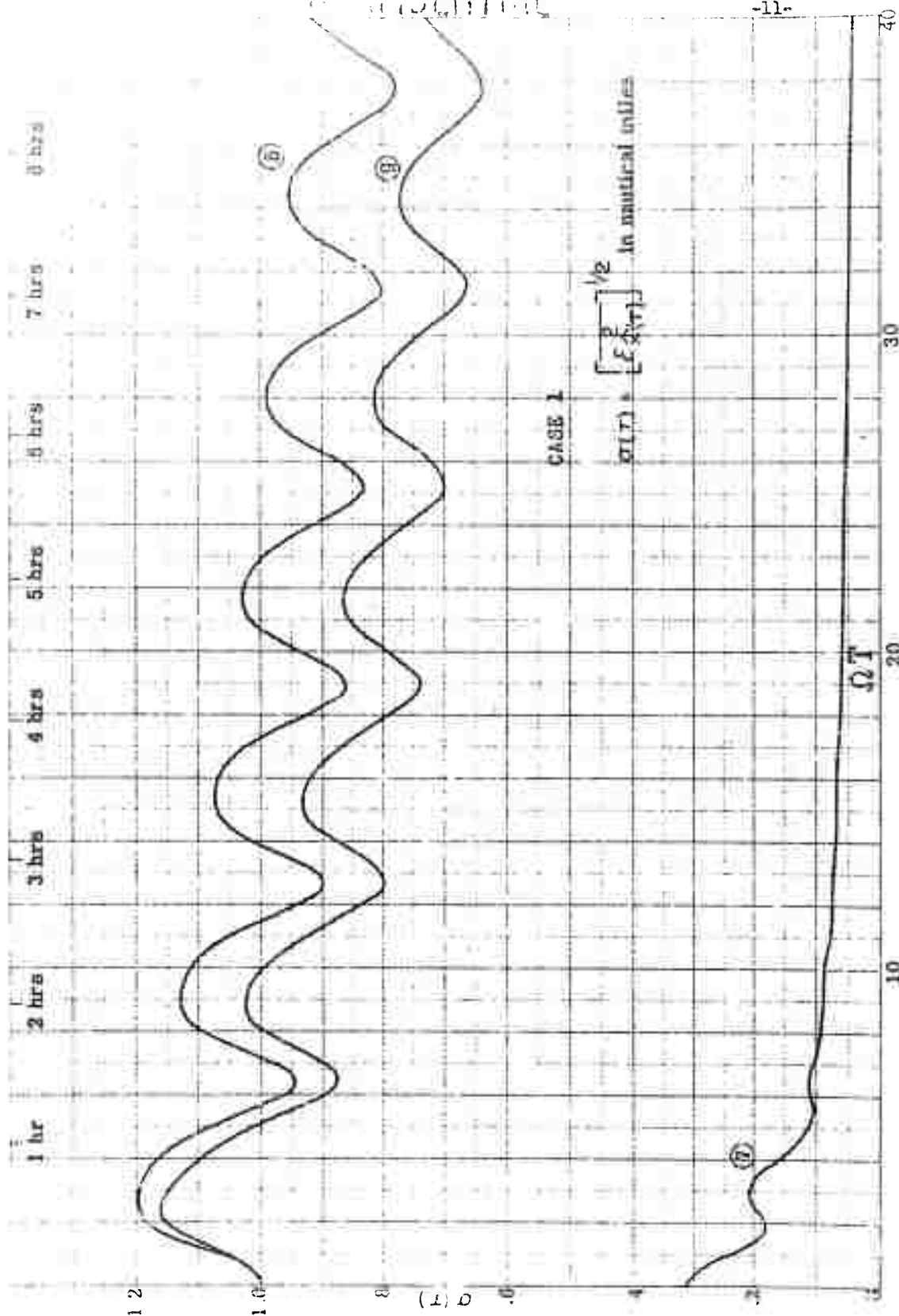


Figure 4

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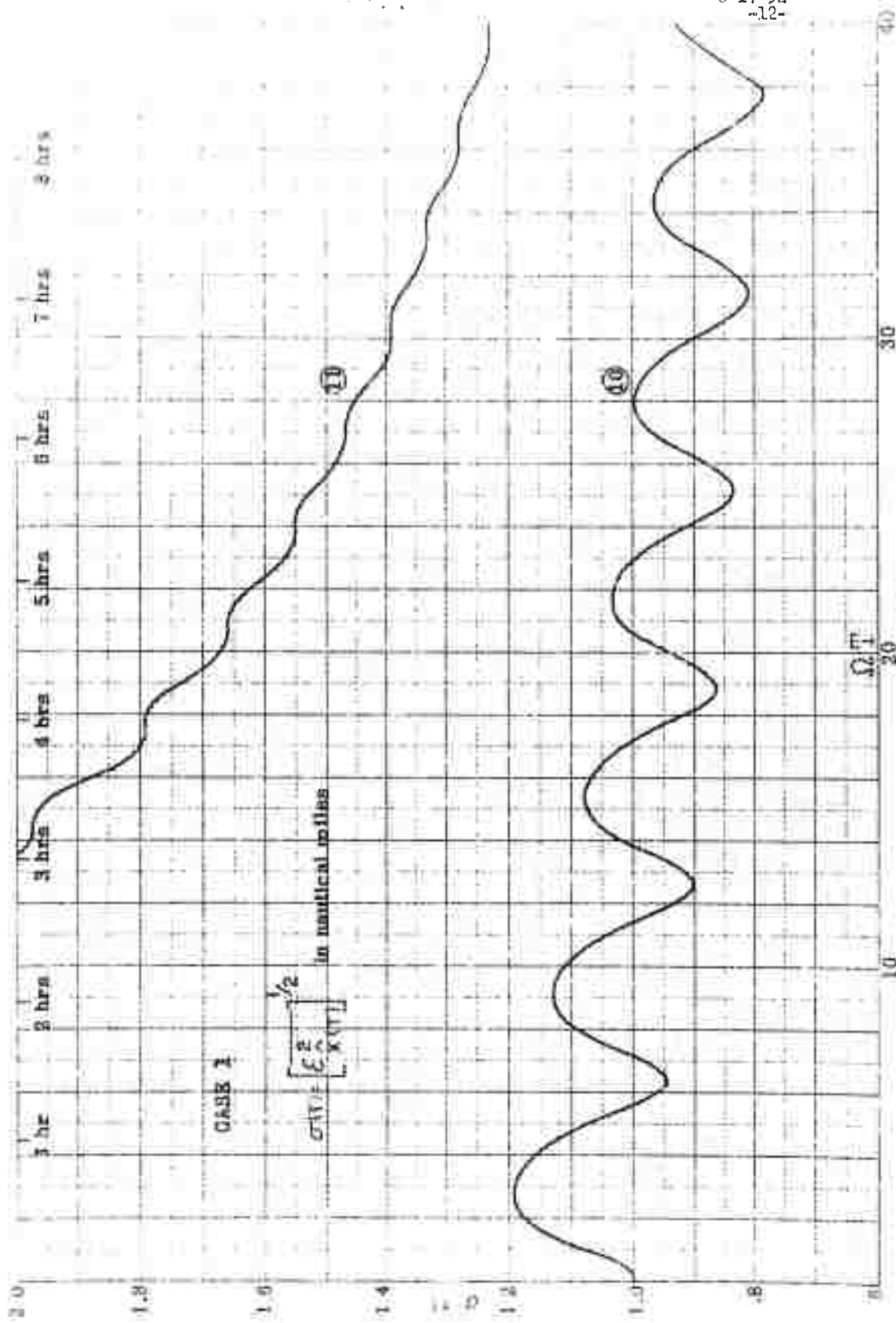


Figure 5

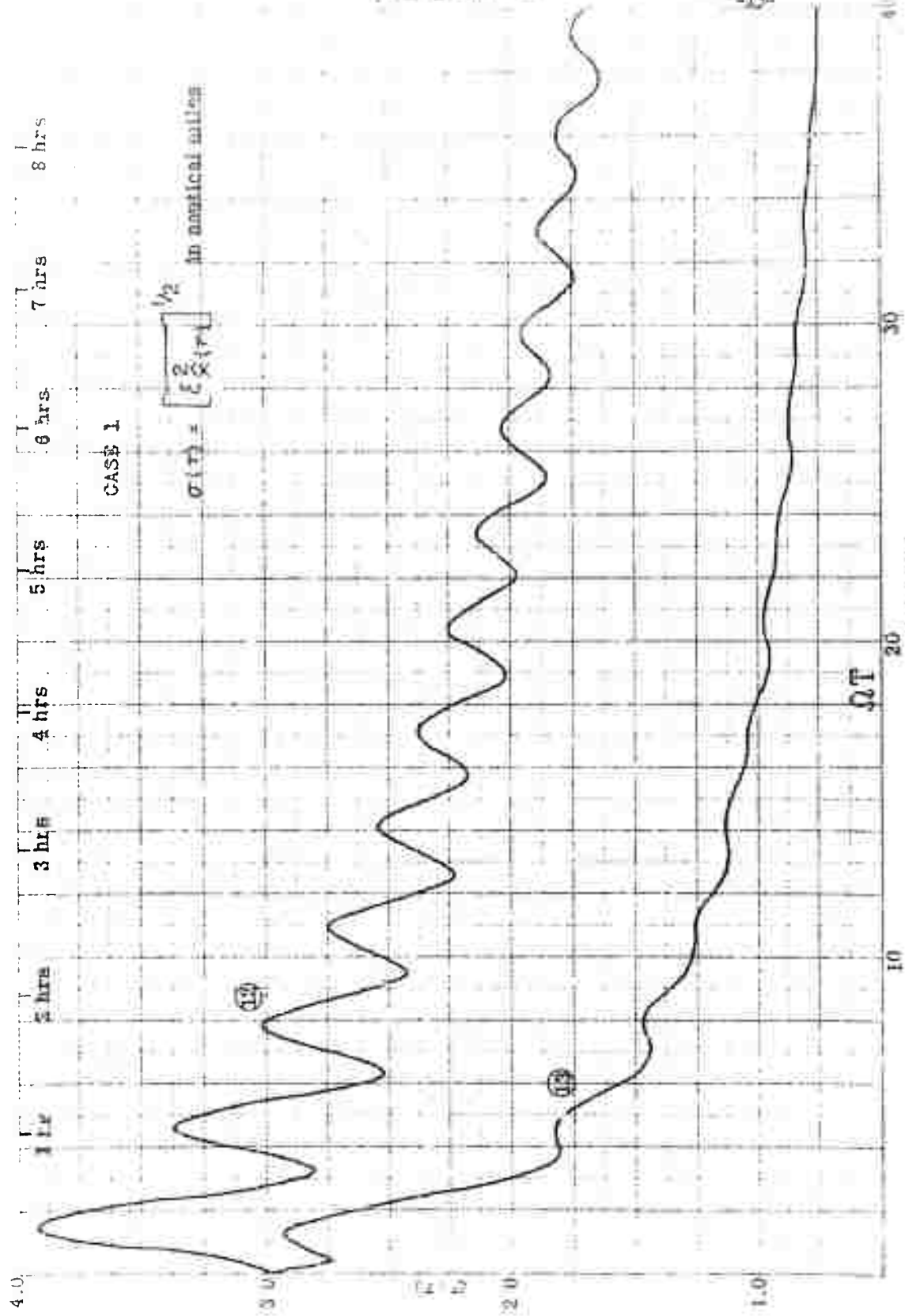
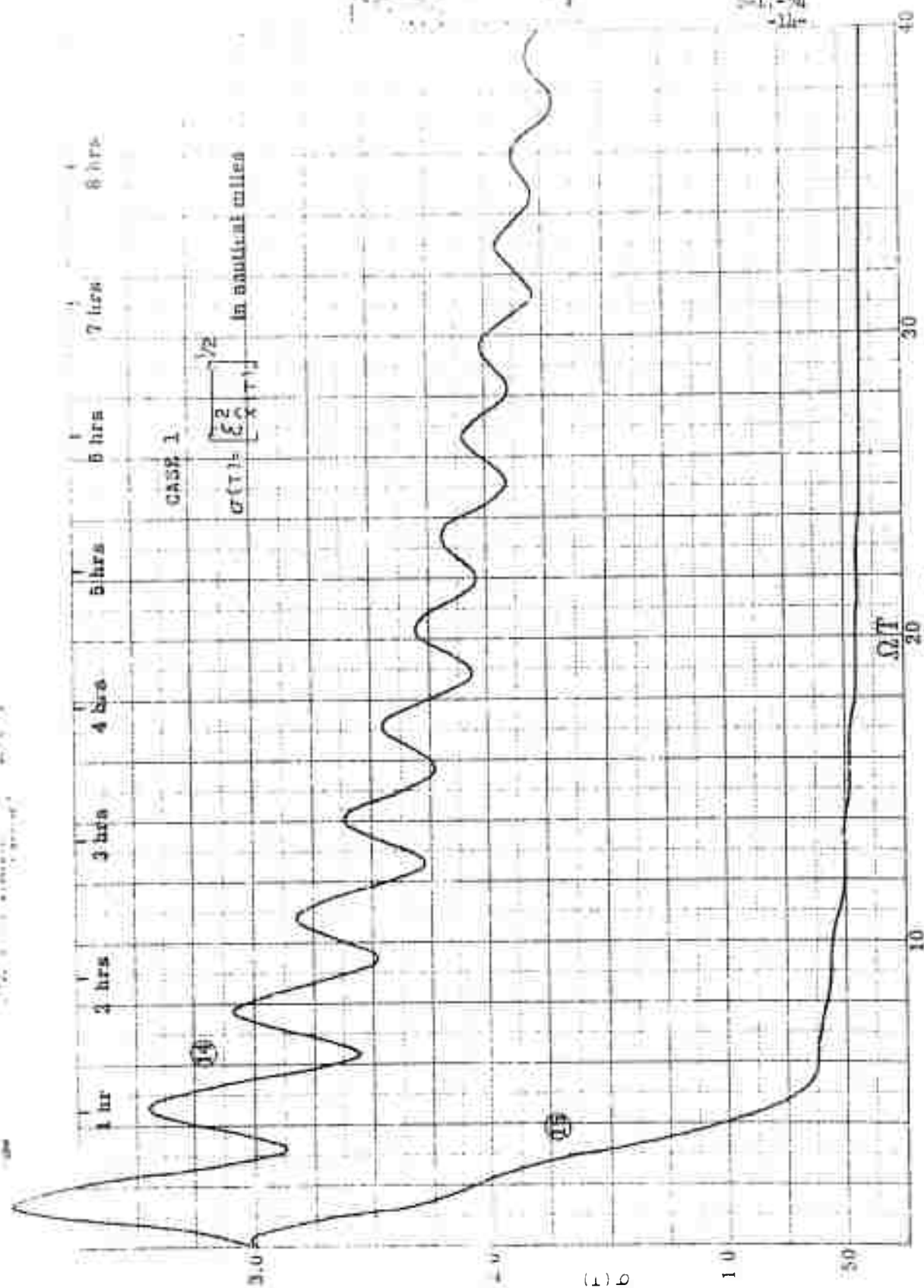


Figure 6

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3000 - 4000 Hz
 1000 - 2000 Hz
 100 - 200 Hz
 10 - 20 Hz
 1 - 2 Hz
 0.1 - 0.2 Hz



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III. CASE 2: $\phi_V(n, t) = \gamma_2$

As in Case 1, let $0 = t_0 < t_1 \cdots < t_{n-1} = T$ be n equally spaced time points in $(0, T)$.

Let

$$(III.1) \quad [\eta_{ij}] = [\phi_B(t_i, t_j)]^{-1} \quad (\text{matrix inverse})$$

Also let

$$[A_{11}]_n = \Omega^4 \sum_{i,j=0}^{n-1} \eta_{ij}$$

$$(III.2) \quad [A_{12}]_n = \Omega^4 \sum_{i,j=0}^{n-1} \eta_{ij} t_i$$

$$[A_{22}]_n = \Omega^4 \sum_{i,j=0}^{n-1} \eta_{ij} t_i t_j$$

and

$$A_{11} = \lim_{n \rightarrow \infty} [A_{11}]_n$$

$$(III.3) \quad A_{12} = \lim_{n \rightarrow \infty} [A_{12}]_n$$

$$A_{22} = \lim_{n \rightarrow \infty} [A_{22}]_n$$

The quantities A_{11} , A_{12} , A_{22} are functions of T .

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Reference 1 gave formulas (eqs. IV.14, IV.15, IV.17 of ref. 1) from which could be derived the formula for $\overline{\xi_x^2}(t;T)$. The result can be expressed as follows:

Let

$$(III.4) \quad \left\{ \begin{array}{l} K_{11} = A_{11} + \frac{1}{\gamma_3} \\ K_{12} = A_{12} \\ K_{22} = A_{22} + \frac{1}{\gamma_2} + \frac{1}{\gamma_4} \\ \Delta = K_{11} K_{22} - K_{12}^2 \end{array} \right.$$

Then, for $0 \leq t \leq T$,

$$(III.5) \quad \overline{\xi_x^2}(t;T) = \frac{1}{\Delta} \left\{ K_{22} - 2 K_{12} t + K_{11} t^2 \right\}$$

and therefore

$$(III.5') \quad \overline{\xi_x^2}(T) = \frac{1}{\Delta} \left\{ K_{22} - 2 K_{12} T + K_{11} T^2 \right\}$$

These formulas hold for any $\phi_B(s,t)$.

In Ref. 1, explicit expressions were obtained for A_{11} , A_{12} , A_{22} for the case

$$(III.6) \quad \phi_B(s,t) = \gamma_1 + \beta_1 e^{-\alpha_1 |s-t|}$$

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The results were

$$(III.71) \quad A_{11} = \frac{\Omega^4(2+a_1T)}{2\beta_1} \left[1 - \frac{1}{1 + \frac{2\beta_1}{\gamma_1(2+a_1T)}} \right]$$

$$(III.711) \quad A_{12} = \frac{\Omega^4(2+a_1T)T}{4\beta_1} \left[1 - \frac{1}{1 + \frac{2\beta_1}{\gamma_1(2+a_1T)}} \right]$$

$$(III.7111) \quad A_{22} = \frac{\Omega^4T^2}{2\beta_1} \left[1 + \frac{a_1T}{3} + \frac{1}{a_1T} - \frac{\frac{1}{4}(2+a_1T)}{1 + \frac{2\beta_1}{\gamma_1(2+a_1T)}} \right]$$

In Figs. 8-10 are shown curves of $\left[\frac{\xi^2}{\hat{x}(T)} \right]^{1/2}$ as a function of T for $\phi_B(s,t) = \gamma_1 + \beta_1 e^{-a_1|s-t|}$. Each curve is determined by specification of the six statistical parameters $\gamma_1, \gamma_2, \gamma_3, \gamma_4, a_1, \beta_1$.

An interesting case is the case in which the accelerometer dial error contains a "white noise" component. This can be obtained by letting $a_1 \rightarrow \infty$ and $\frac{\beta_1}{a_1} \rightarrow \kappa_1$ in Eq. (III.6). The exponential component of the accelerometer dial error then approaches white noise with spectral density κ_1 per cycle.

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In the following table, the units for the various parameters are as follows:

$$\gamma_1 : (\text{nautical miles})^2 (\text{hours})^{-4}$$

$$\gamma_2 : (\text{nautical miles})^2 (\text{hours})^{-2}$$

$$\gamma_3 : (\text{nautical miles})^2$$

$$\gamma_4 : (\text{nautical miles})^2 (\text{hours})^{-2}$$

$$a_1 : (\text{hours})^{-1}$$

$$\beta_1 : (\text{nautical miles})^2 (\text{hours})^{-4}$$

The parameter values associated with each curve in Figs. 8-10 are as follows:

Curve No.	γ_1	γ_2	γ_3	γ_4	a_1	β_1	$\lim_{t \rightarrow \infty} \frac{\beta_1}{\epsilon_1}$
1	50.0	1.0	.10	1.0	$\rightarrow \infty$	$\rightarrow \infty$	.20
2	0.00	1.0	.10	9.0	$\rightarrow \infty$	$\rightarrow \infty$	.20
3	0.00	1.0	.10	9.0	10	800	
4	50.0	1.0	.10	1.0	10	100	
5	25.0	1000	1.0	25.0	10	25.0	
6	0.00	1000	1.0	25.0	10	50.0	

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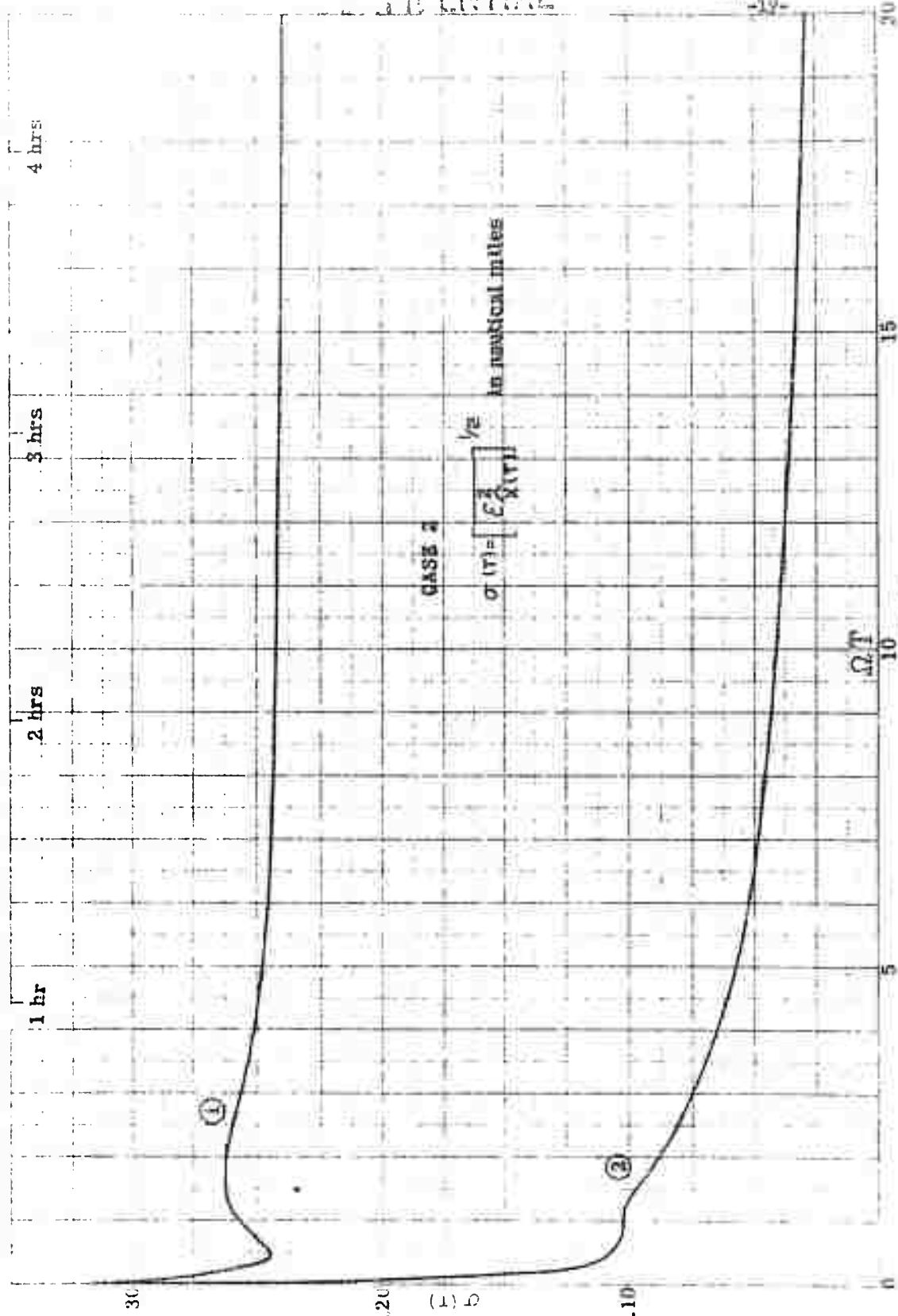
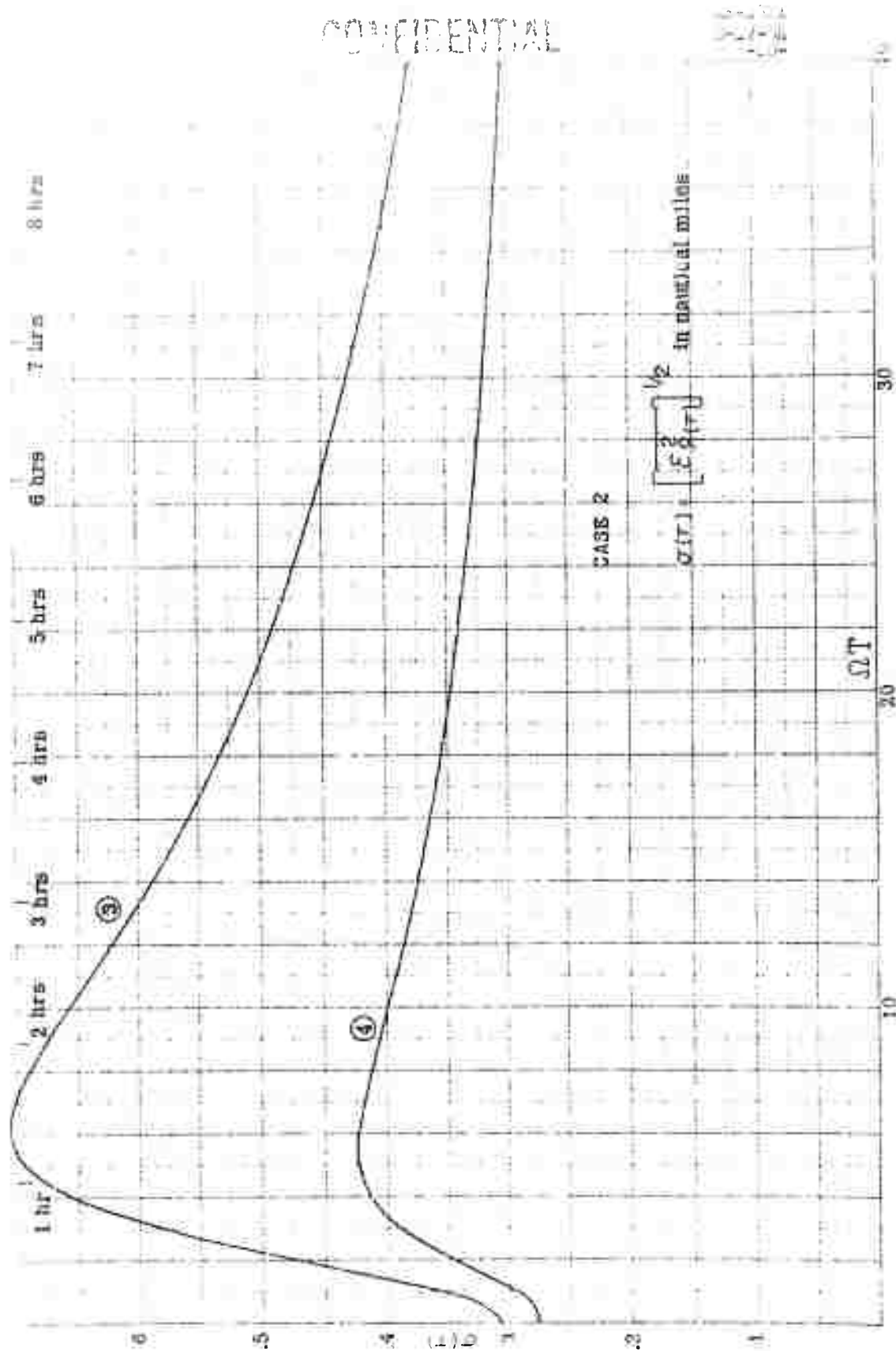


Figure 8



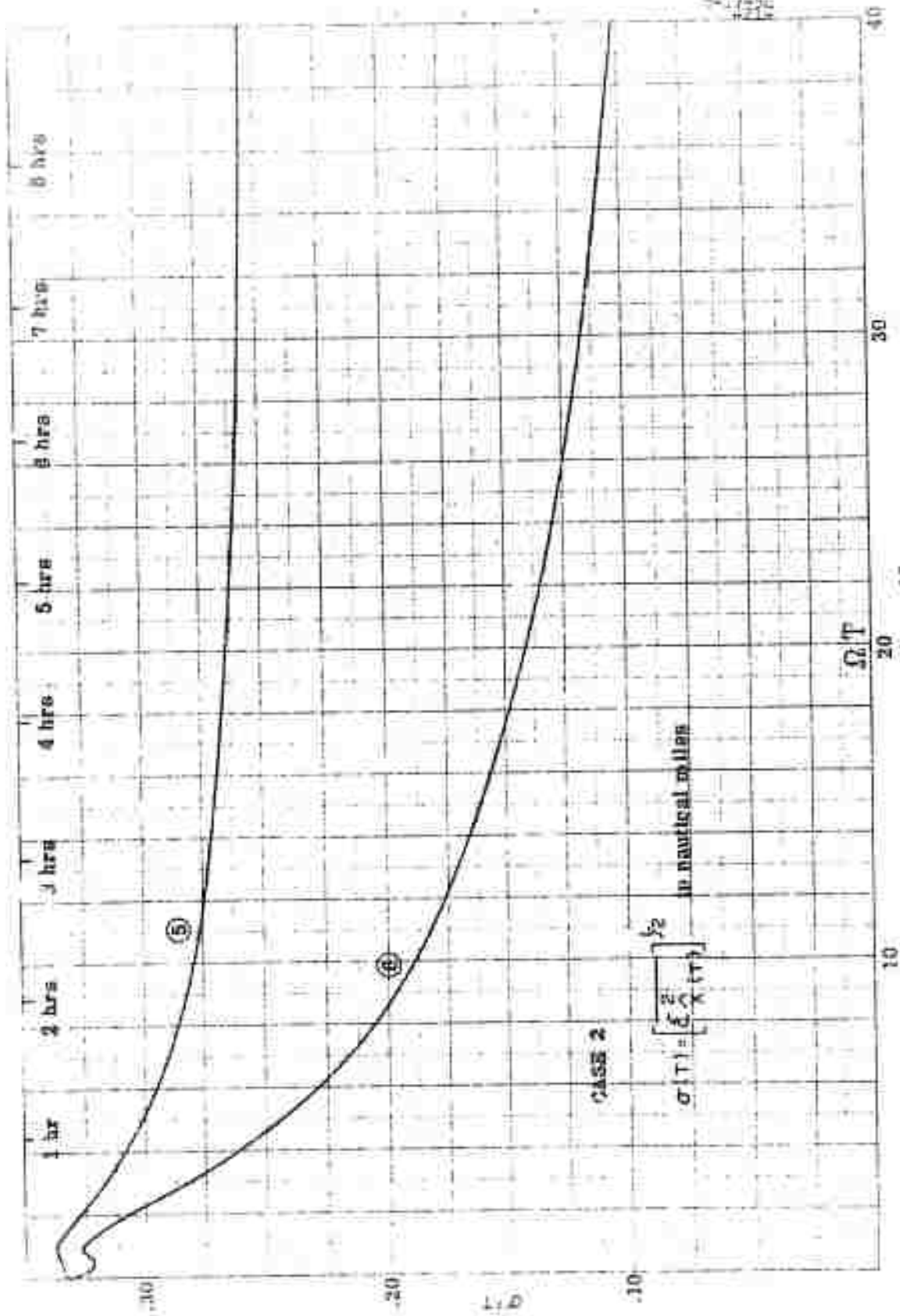


Figure 10

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