On Stress Functions for Elastokinetics and the Integration

of the Repeated Wave Equation

by

E. Sternberg and R. A. Eubanks

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by

E. Sternberg
Illinois Institute of Technology

and

R. A. Eubanks
American Machine and Foundry Company

1.   Introduction.

The general solution due to Galerkin [1][1] of the field equations in

classical elastostatics, was extended to elastokinetics by Iacovache [2]

on the basis of a formal operational scheme originated by Gr. C. Moisil [3].

The method of derivation employed in [2], however, does not assure the com-

pleteness of the solution so obtained. In the present paper we supply a

completeness proof for the generalized Galerkin solution and relate it to

a generalization of Papkovich's [4] solution of the equilibrium equations.

With a view toward the fact that the Galerkin vector of elastokinetics, in

the absence of body forces, satisfies a repeated wave equation, we prove

further that every solution of such an equation may be represented as the

sum of two wave functions. This result is a generalization of a theorem

due to Almansi [5] regarding the integration of the biharmonic equation.

2.   Completeness of the generalized Galerkin solution.

The displacement equations of motion in the linear theory of homo-

geneous and isotropic elastic media are given by

[1]Numbers in brackets refer to the bibliography at the end of the paper.

$$\nabla^2 \bar{u} + \frac{1}{1-2\nu} \nabla\nabla \cdot \bar{u} - \frac{\rho}{\mu} \frac{\partial^2 \bar{u}}{\partial t^2} + \frac{\bar{F}}{\mu} = 0 \quad , \tag{1}$$

where $\bar{u}$ is the displacement vector,[2] t stands for the time, $\bar{F}$ is the body force density, ρ the mass density, while ν and μ denote Poisson's ratio and the shear modulus, respectively. Let $\bar{G}(P,t)$ be[3] four times continuously differentiable for P in an arbitrary region of space D and $a < t < b$. Substitution into (1) confirms that every displacement field of the form

$$\bar{u} = \frac{1}{2\mu} \left[2(1-\nu) \,\square_1^2\, \bar{G} - \nabla\nabla \cdot \bar{G} \right] \tag{2}$$

satisfies (1) provided

$$\square_1^2 \square_2^2 \bar{G} = - \frac{\bar{F}}{1-\nu} \quad , \tag{3}$$

where

$$\square_i^2 = \nabla^2 - \frac{1}{c_i^2} \frac{\partial^2}{\partial t^2} \qquad (i = 1,\, 2) \tag{4}$$

and

$$c_1^2 = \frac{2(1-\nu)}{1-2\nu} \frac{\mu}{\rho} \quad , \quad c_2^2 = \frac{\mu}{\rho} \quad . \tag{5}$$

Equations (2), (3) constitute the solution of (1) discovered by Iacovache [2], when the body forces are absent. If $\bar{G}$ is independent of t, this solution becomes identical with Galerkin's solution [1] of the equilibrium equations.[4] We now prove the following completeness theorem:

Let D, with the boundary B, be a bounded (not necessarily simply connected) region of space. Let $\bar{u}(P,t)$ be continuous in $D + B$, piecewise

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[2] Letters carrying bars designate vectors; the symbols "$\cdot$" and "$\times$" indicate scalar and vector multiplication of two vectors, respectively. ∇ is the usual del-operator.

[3] We shall write consistently $f(P,t)$ in place of $f(x,y,z,t)$, P being a point with rectangular cartesian coordinates (x,y,z).

[4] See also Mindlin [6] and Westergaard [7].

four times continuously differentiable for P in D and $-\infty < t < \infty$. If $\bar{u}(P,t)$ is a solution of (1), then there exists a function $\bar{G}(P,t)$, of the same degree of smoothness, such that the representation (2), (3) holds.

By virtue of the Stokes-Helmholtz resolution,[5] there exist $\phi(P,t)$ and $\bar{A}(P,t)$, both having the degree of smoothness of $\bar{u}(P,t)$, such that

$$2\mu\bar{u} = \nabla\phi + \nabla \times \bar{A} \tag{6}$$

Substitution of (6) into (1) yields, with the aid of (4) and (5),

$$\frac{c_1^2}{c_2^2} \Box_1^2 \nabla\phi + \Box_2^2 \nabla \times \bar{A} + 2\bar{F} = 0 \quad . \tag{7}$$

Next define a function $\bar{G}(P,t)$ by means of

$$\bar{G}(P,t) = \frac{-1}{8\pi(1-\nu)}\left[\frac{c_1^2}{c_2^2}\nabla\phi_2 + \nabla \times \bar{A}_1\right] \quad , \tag{8}$$

in which $\phi_2(P,t)$ and $\bar{A}_1(P,t)$ are the retarded potentials given by

$$\phi_2(P,t) = \int_D \frac{\phi(Q,t-R/c_2)}{R} \, d\tau \quad , \quad \bar{A}_1(P,t) = \int_D \frac{\bar{A}(Q,t-R/c_1)}{R} \, d\tau \quad . \tag{9}$$

Here R is the distance from P to Q and the integrations are with respect to Q. Clearly, $\bar{G}(P,t)$ has the required smoothness. Moreover, from the theory of retarded potentials,[6]

$$\Box_2^2 \phi_2 = -4\pi\phi \quad , \qquad \Box_1^2 \bar{A}_1 = -4\pi\bar{A} \quad , \tag{10}$$

whence (8) implies

$$\Box_1^2 \bar{G} = \frac{1}{2(1-\nu)} \nabla \times \bar{A} - \frac{c_1^2}{8\pi(1-\nu)c_2^2} \Box_1^2 \nabla\phi_2 \tag{11}$$

By (10), (11),

$$\Box_1^2 \Box_2^2 \bar{G} = \frac{1}{2(1-\nu)}\left(\frac{c_1^2}{c_2^2}\Box_1^2 \nabla\phi + \Box_2^2 \nabla \times \bar{A}\right) \quad , \tag{12}$$

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[5]See, for example, Phillips [8], p. 187.

[6]See Phillips [8], Chapter IX.

and (7), (12) imply (3).

In view of (8),

$$\nabla \cdot \bar{G} = - \frac{c_1^2}{8\pi(1-\nu)c_2^2} \nabla^2 \phi_2 \ , \tag{13}$$

while (6), (11) lead to

$$2\mu\bar{u} = \nabla \phi + 2(1-\nu) \Box_1^2 \bar{G} + \frac{c_1^2}{4\pi c_2^2} \Box_1^2 \nabla \phi_2 \ . \tag{14}$$

It follows from (14), (10), and (4) that

$$2\mu\bar{u} = 2(1-\nu) \Box_1^2 \bar{G} + \frac{1}{4\pi}\left(\frac{c_1^2}{c_2^2} - 1\right) \nabla \nabla^2 \phi_2 \tag{15}$$

and (15), (13), because of (5), yield (2). This completes the proof of the theorem.

If we define functions $\varphi(P,t)$ and $\overline{\Psi}(P,t)$ through

$$\varphi(P,t) = -\nabla \cdot \bar{G} - \bar{r} \cdot \overline{\Psi} \ , \quad \overline{\Psi}(P,t) = -\frac{1}{2} \Box_1^2 \bar{G} \ , \tag{16}$$

in which $\bar{r}$ is the position vector of P, (2) becomes

$$2\mu\bar{u} = \nabla(\varphi + \bar{r} \cdot \overline{\Psi}) - 4(1-\nu) \overline{\Psi} \ . \tag{17}$$

On the other hand, we conclude from (16), with the aid of (3) and (4), that

$$\Box_1^2 \varphi + \bar{r} \cdot \Box_1^2 \overline{\Psi} = 0 \ , \quad \Box_2^2 \overline{\Psi} = \frac{\bar{F}}{2(1-\nu)} \ . \tag{18}$$

If φ and $\overline{\Psi}$ are independent of t, Equations (17), (18) reduce to Papkovich's [4] general solution of the equilibrium equations which was independently discovered by Neuber [9].[7]

3. A theorem on the integration of the repeated wave equation.

According to (3), the generalized Galerkin vector $\bar{G}(P,t)$ satisfies an inhomogeneous repeated wave equation. If D is bounded while $F(P,t)$ is continuous in $D + B$ and piecewise twice continuously differentiable

[7]See also Mindlin [6], [10]. The body forces are assumed to vanish in [4] and [9]; they are included in [10].

for P in D, $-\infty < t < \infty$, a particular solution of (3) is obtainable through an iterated retarded potential. Indeed, such a solution is given by

$$\bar{G}(P,t) = \int_D \frac{\bar{E}(Q,t - R/c_1)}{R}\, d\tau \quad ,$$

$$\bar{E}(P,t) = \frac{-1}{16\pi^2(1-\nu)} \int_D \frac{\bar{\bar{F}}(Q,t - R/c_2)}{R}\, d\tau \quad ,$$

(19)

where R is again the distance from P to Q. We now turn to the homogeneous repeated wave equation and prove the theorem:

Let $G(P,t)$ be four times continuously differentiable for P in an arbitrary region D and $a < t < b$. If $G(P,t)$ is a solution of

$$\Box_1^2 \Box_2^2\, G = 0 \quad , \qquad c_1^2 \neq c_2^2 \quad , \tag{20}$$

then it admits the representation

$$G = G_1(P,t) + G_2(P,t) \quad , \tag{21}$$

where

$$\Box_i^2\, G_i = 0 \qquad (i = 1,2) \tag{22}$$

and $G_i(P,t)$ is twice continuously differentiable in D, $a < t < b$.

It suffices to show that there exists a $G_1(P,t)$ which has the desired smoothness and simultaneously meets the equations

$$\Box_1^2\, G_1 = 0 \quad , \qquad \Box_2^2\, (G - G_1) = 0 \quad . \tag{23}$$

Once the existence of such a G_1 has been demonstrated, we define $G_2(P,t)$ by

$$G_2(P,t) = G - G_1 \quad , \tag{24}$$

and the proof is complete.

Equations (23), in view of (4), are equivalent to

$$\Box_1^2\, G_1 = 0 \quad , \qquad \frac{\partial^2 G_1}{\partial t^2} = g \quad , \tag{25}$$

where

$$g(P,t) = \frac{c_1^2 \, c_2^2}{c_2^2 - c_1^2} \, \Box_2^2 \, G \qquad (26)^8$$

and, from (20),

$$\Box_1^2 \, g = 0 \quad . \qquad (27)$$

Thus, we need only exhibit a function $G_1(P,t)$ satisfying (25), subject to (27).

Consider the function $H(P,t)$ defined by

$$H(P,t) = \int_0^t \int_0^\lambda g(P,\xi) \, d\xi \, d\lambda \quad . \qquad (28)$$

Clearly, $G_1 = H$ conforms to the second of (25). Moreover, by (27), (28),

$$\frac{\partial^2}{\partial t^2} \, \Box_1^2 \, H = \Box_1^2 \, \frac{\partial^2 H}{\partial t^2} = \Box_1^2 \, g = 0 \quad , \qquad (29)$$

whence

$$\Box_1^2 \, H = \alpha(P) + t\beta(P) \quad . \qquad (30)$$

Next, consider the function $G_1(P,t)$ defined by

$$G_1(P,t) = H(P,t) + A(P) + tB(P) \quad , \qquad (31)$$

where A and B are particular solutions of the Poisson equations

$$\nabla^2 A = -\alpha \quad , \qquad \nabla^2 B = -\beta \quad . \qquad (32)$$

By virtue of (28), G_1 so defined still meets the second of (25), and according to (30), (32), it also satisfies the first of (25). Finally, $G_1(P,t)$ has the desired smoothness. The proof is now complete.

The foregoing theorem is the counterpart for the repeated wave equation, of Almansi's [5] theorem regarding the representability of biharmonic functions in terms of harmonic functions. A different kind of extension

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[8]Recall that $c_1^2 \neq c_2^2$.

of Almansi's theorem was given in [11]. It should be noted that the present theorem, in contrast to those established in [5] and [11], does not involve any convexity restrictions on the region D.

As a consequence of the two theorems proved in this paper, and with reference to (2), (3), (4), the general solution of the equations of motion (1), in the absence of body forces, admits the alternative representation

$$2\mu\bar{u} = \frac{\rho}{\mu} \frac{\partial^2 \bar{G}_2}{\partial t^2} - \nabla\nabla \cdot (\bar{G}_1 + \bar{G}_2) \quad , \tag{33}$$

$$\Box_i^2 \bar{G}_i = 0 \qquad (i = 1,2) \quad . \tag{34}$$

This reduces the problem of elastokinetics to the determination of appropriate solutions of the wave equation.

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Professor L. E. Goodman
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Stanford, California (1)

Professor B. G. Johnston
Dept. of Civil Engineering
University of Michigan
Ann Arbor, Michigan (1)

Dr. T. R. Kane
25-2 Valley Road
Drexel Hill, Pennsylvania (1)

Professor K. Klotter
Division of Engineering Mechanics
Stanford University
Stanford, California (1)

Professor W. T. Koiter
Technische Hogeschool
Nieuwe Laan 76
Delft, NETHERLANDS (1)

Professor B. J. Lazan
Dept. of Mechanics and Materials
University of Minnesota
Minneapolis 14, Minnesota (1)

Professor E. H. Lee
Division of Applied Mathematics
Brown University
Providence 12, Rhode Island (1)

Professor G. H. Lee
Rensselaer Polytechnic Institute
Troy, New York (1)

Professor J. J. Lessells
Lessells and Assoc., Inc.
916 Commonwealth Avenue
Boston 15, Massachusetts (1)

Library, Engineering Foundation
29 West 39th Street
New York, New York (1)

Professor P. Lieber
Rensselaer Polytechnic Institute
Troy, New York (1)

Professor Hsu Lo
School of Aeronautics
Purdue University
Lafayette, Indiana (1)

Professor C. T. G. Looney
Dept. of Civil Engineering
Yale University
New Haven, Connecticut (1)

Professor J. F. Ludloff
School of Aeronautics
New York University
New York 53, New York (1)

Professor J. H. Marchant
Brown University
Providence 12, Rhode Island (1)

Professor K. Marguerre
Technische Hochschule
Darmstadt, GERMANY (1)

Professor J. Marin
Dept. of Engineering Mechanics
Pennsylvania State University
State College, Pennsylvania (1)

Dr. W. P. Mason
Bell Telephone Laboratories
Murray Hill, New Jersey (1)

Professor E. Masur
Dept. of Engineering Mechanics
University of Michigan
Ann Arbor, Michigan (1)

Mr. E. L. McDowell
Armour Research Foundation
Chicago 16, Illinois (1)

Professor Ernst Melan
Technische Hochschule
Vienna, AUSTRIA (1)

Dr. J. Miklowitz
U. S. Naval Ordnance Test Station
3202 East Foothill Blvd
Pasadena 8, California (1)

Professor R. D. Mindlin
Dept. of Civil Engineering and
    Engineering Mechanics
Columbia University
New York 27, New York (1)

Dr. A. Nadai
136 Cherry Valley Road
Pittsburgh 21, Pennsylvania (1)

Professor P. M. Naghdi
Dept. of Engineering Mechanics
University of Michigan
Ann Arbor, Michigan (1)

Professor N. M. Newmark
207 Talbot Laboratory
University of Illinois
Urbana, Illinois (1)

Professor J. Ormondroyd
Dept. of Engineering Mechanics
University of Michigan
Ann Arbor, Michigan (1)

Professor W. R. Osgood
Rensselaer Polytechnic Institute
Troy, New York (1)

Mr. R. E. Peterson
Westinghouse Research Labs
East Pittsburgh, Pennsylvania (1)

Professor A. Phillips
School of Engineering
Yale University
New Haven 11, Connecticut (1)

Professor G. Pickett
Dept. of Mechanics
University of Wisconsin
Madison 6, Wisconsin (1)

Dr. H. Poritsky
General Engineering Laboratory
General Electric Company
Schenectady, New York (1)

Professor W. Prager
Graduate Division of Applied
    Mathematics
Brown University
Providence 12, Rhode Island (1)

RAND Corporation
1700 Main Street
Santa Monica, California
    Attn: Dr. E. H. Plesset (1)

Professor R. S. Rivlin
Division of Applied Mathematics
Brown University
Providence 12, Rhode Island   (1)

Professor E. Reissner
Dept. of Mathematics
Massachusetts Institute of
    Technology
Cambridge 39, Massachusetts   (1)

Professor L. Rongved
Dept. of Engineering Mechanics
Pennsylvania State University
State College, Pennsylvania   (1)

Dr. Felix Rosenthal
Clevite-Brush Company
Cleveland, Ohio   (1)

Professor M. A. Sadowsky
Dept. of Mechanics
Rensselaer Polytechnic Institute
Troy, New York   (1)

Mr. E. Salame
Armour Research Foundation
Chicago 16, Illinois   (1)

Professor M. G. Salvadori
Dept. of Civil Engineering and
    Engineering Mechanics
Columbia University
New York 27, New York   (1)

Dr. F. S. Shaw, Superintendent
Structures & Materials Division
Aeronautical Research Laboratories
Box 4331, GPO Melbourne
Victoria, AUSTRALIA   (1)

Professor C. B. Smith
Dept. of Mathematics
Walker Hall
University of Florida
Gainesville, Florida   (1)

Professor C. R. Soderberg
Dept. of Mechanical Engineering
Massachusetts Institute of
    Technology
Cambridge 39, Massachusetts   (1)

Professor I. S. Sokolnikoff
Dept. of Mathematics
University of California
    at Los Angeles
Los Angeles 24, California   (1)

Sir Richard V. Southwell
The Old House
Trumpington
Cambridge, ENGLAND   (1)

Professor E. Sternberg
Illinois Institute of Technology
Chicago 16, Illinois   (15)

Professor J. J. Stoker
New York University
Washington Square
New York, New York   (1)

Mr. R. A. Sykes
Bell Telephone Laboratories
Murray Hill, New Jersey   (1)

Professor P. S. Symonds
Division of Applied Mathematics
Brown University
Providence 12, Rhode Island   (1)

Professor J. L. Synge
Dublin Institute for Advanced
    Studies
School of Theoretical Physics
64-65 Merrion Square
Dublin, IRELAND   (1)

Professor F. K. Teichmann
Dept. of Aeronautics
New York University
Bronx 53, New York   (1)

Professor S. P. Timoshenko
School of Engineering
Stanford University
Stanford, California   (1)

Professor C. A. Truesdell
The Graduate Institute for
    Mathematics and Mechanics
Indiana University
Bloomington, Indiana   (1)

Dr. I. Vigness
Naval Research Laboratory
Anacostia Station
Washington, D. C.   (1)

Professor E. Volterra
Dept. of Mechanics
Rensselaer Polytechnic Institute
Troy, New York   (1)

Mr. A. M. Wahl
Westinghouse Research Labs
East Pittsburgh, Pennsylvania   (1)

Dr. Conrad Wan
3422 Shorecraft Drive
Dallas, Texas   (1)

Professor E. E. Weibel
Dept. of Mechanical Engineering
University of Colorado
Boulder, Colorado   (1)

Professor J. Weiner
Dept. of Civil Engineering and
    Engineering Mechanics
Columbia University   (1)

Professor A. Weinstein
Institute for Fluid Dynamics and
    Applied Mathematics
University of Maryland
College Park, Maryland   (1)

Professor D. Young
Yale University
New Haven 11, Connecticut   (1)