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ON THE HEAT BALANCE AND MAINTENANCE OF CIRCULATION IN THE TRADES

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ON THE HEAT BALANCE AND MAINTENANCE OF CIRCULATION IN THE TRADES

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Abstract

The heat balance is calculated for the Pacific northeast trade during summer and it is shown that the trade exports sensible as well as latent heat. The sensible heat gain is related to the surface pressure distribution; it is found that the low-level circulation in the region considered is self-driving.

Several years ago (1) the authors and collaborators attempted (a) to describe the structure of the northeast trade in the Pacific Ocean between the surface and 3 km, (b) to calculate the heat, moisture and momentum balance in this region, and (c) to find a mechanism for the breakdown of the inversion. This paper will extend the former heat balance computations, followed by a discussion of the maintenance of the trade winds.

The following symbols are used in the paper.

c_p specific heat of air at constant pressure

c_v specific heat of air at constant volume

L latent heat of condensation

p pressure

P precipitation

Q substantial change in heat content

Q_s sensible heat exchange between ocean and atmosphere

R net loss of heat by radiation

R^* gas constant for air

s horizontal space coordinate along air trajectory

t time

T temperature

v component of mean motion along s

w vertical component of velocity

z vertical space coordinate

θ potential temperature

κ R/c_p

ρ density

τ shearing stress in s,z plane

Φ geopotential

Heat Balance

In the previous paper it had been assumed that sensible heat sources and sinks cancel, an assumption made because of difficulties in calculating the sensible heat export from the region considered. The sink is the net radiation cooling, the sources are the flow of sensible heat from ocean to atmosphere and the condensation heating. Integrating over the mean trajectory from 32°N, 136°W to Hawaii (fig. 1),

$$\iint (Q_s + LP - R)\, ds\, dz = 0 \qquad (1)$$

for balance. Equation (1) really represents a volume integral. However, the turning of wind with height through the layer was very small so that, with some approximation (cf. 1), the mean motion may be regarded as two-dimensional and unit distance considered along the axis normal to the trajectory in all calculations.

The data satisfied equation (1) fairly well. If completely true, the trades act only as accumulator and exporter of latent heat. However, as evident from a crossection of potential temperature (fig. 2) equation

(1) cannot be entirely correct because the mean trajectories cross from low to high potential temperatures, especially below the trade inversion. This indicates some export of sensible heat. Our previous attempt to compute this export failed; a slightly different approach will now be taken by calculating an energy budget through integration of the first law of thermodynamics over the section. Per unit mass this law may be stated in the form

$$Q = c_p \frac{T}{\theta} \frac{d\theta}{dt} = c_p \frac{dT}{dt} - \frac{1}{\rho} \frac{dp}{dt}. \qquad (2)$$

This equation can be integrated directly using the data given in (1) because trajectories have been computed. However, a few well known transformations will be made first in order to facilitate the later analysis. In the steady state considered here

$$\frac{1}{\rho} \frac{dp}{dt} = \frac{v}{\rho} \frac{\partial p}{\partial s} + \frac{w}{\rho} \frac{\partial p}{\partial z}. \qquad (3)$$

Because of the hydrostatic relation,

$$\frac{w}{\rho} \frac{\partial p}{\partial z} = - \frac{d\Phi}{dt}.$$

The first equation of motion is given by

$$\frac{dv}{dt} = - \frac{1}{\rho} \frac{\partial p}{\partial s} + \frac{1}{\rho} \frac{\partial \tau}{\partial z}. \qquad (4)$$

Using this equation

$$\frac{v}{\rho} \frac{\partial p}{\partial s} = - \left(\frac{1}{2} \frac{dv^2}{dt} - \frac{v}{\rho} \frac{\partial \tau}{\partial z} \right),$$

where $\frac{1}{2}\frac{dv^2}{dt} = \frac{dKE}{dt}$, the change of kinetic energy per unit mass.
The first law now transforms to

$$\rho Q = \rho \frac{d}{dt}\left(\bar{\Phi} + c_p T + KE\right) - v\frac{\partial I}{\partial z} . \qquad (5)$$

Table 1 gives the value of all terms, integrated over the section.

Table 1

	10^9 cal/day	
	+	-
Release of potential energy		7.0
Increase of enthalpy	15.2	
Increase of kinetic energy	.004	
Work against friction	.50	
	15.7	7.0
Gain of sensible heat	8.7	

Since a sensible heat increase is calculated, we must amend equation (1) to read

$$H = \iint (Q_s + LP - R)\, ds\, dz . \qquad (6)$$

Since the weather ships did not make rainfall measurements, equation (6) cannot furnish a complete check on all calculations but must be solved for the precipitation warming. Table 2 shows the calculation, using values for radiation cooling and flow of heat from ocean to air obtained previously.

Table 2

	10^9 cal/day		
		+	-
Increase of sensible heat		9	
Flow of sensible heat from ocean			3
Net radiation cooling		18	
		27	3
Precipitation heating	24	24	

Computation of the moisture balance will indicate to what extent the result of Table 2 can be considered valid. The moisture balance is presented in Table 3, again using values from the previous paper.

Table 3

	10^9 cal/day		
		+	-
Evaporation		49	
Lateral export by mean motion			26
Vertical export through turbulence			5
		49	31
Precipitation heating (from Table 2)	24	18	

Agreement has been attained within the limits of accuracy of the computations. It should be pointed out that the vertical export through turbulence has not been measured. It is a transport assumed in the first paper for moisture balance. If we exclude this amount, the difference of evaporation minus lateral export is 23 units or almost exactly the value demanded for heat balance in Table 2. Since all calculated values are subject to sizeable error, however, one must not expect precise balances and the result of Table 3 may be considered quite satisfactory. Certainly it does not follow that there is no vertical export of moisture through the 3-km level.

Because of the steady state the sensible heat acquired must be exported. The export of sensible heat is 8 units, that of latent heat 26-31 units, adding to a total of 34-39 units. Thus the sensible heat export contributes 20-25 per cent to the total, and the statement that the trades act mainly to store latent heat, while still largely true, must be somewhat modified.

Maintenance of the Trade Winds

Along the mean trade wind trajectories balance of forces is provided largely by a large pressure gradient force directed downstream acting against friction. The heat balance calculation just completed raises the question whether we must consider the trade as a driven member of the general circulation sustained from the equatorial zone or perhaps even from the polar westerlies, or whether the maintenance of pressure gradients and winds can be explained at least partly from more local energy transformations. This subject will be studied with two approaches: (1) we shall compute whether the Pacific trade is an energy exporting or importing circulation branch; and (2) we shall relate heat source and pressure field to determine whether the heat source can be utilized for maintaining the force field.

The trades as an energy exporting circulation: In view of Table 1 the first law of thermodynamics may be stated to a high degree of accuracy in the form

$$Q = \frac{d}{dt}\left(\bar{\Phi} + c_p T\right). \tag{7}$$

For present purposes the change in enthalpy will be divided into the change in internal energy and the work done by pressure forces on the mass considered. Thus,

$$c_p \frac{dT}{dt} = c_v \frac{dT}{dt} + \frac{d}{dt}\left(\frac{p}{\rho}\right), \tag{8}$$

and

$$Q - \frac{d}{dt}\left(\frac{p}{\rho}\right) = \frac{d}{dt}\left(\bar{\Phi} + c_v T\right), \tag{9}$$

where the right hand side now contains the changes of potential and internal energy explicitly. Utilizing the data of Table 1, the computation of the energy change is given in Table 4.

The Pacific northeast trade gains energy. It must therefore be regarded as an energy exporting circulation branch since for steady state the energy must be transferred to other portions of the globe to help maintain the general circulation there.

It is of interest to compute the energy change also with the assumption made in the first paper, namely that the heat source is zero. The

Table 4

	10^9 cal/day			
	+	-	+	-
Increase in heat content	8.2			
Work done by pressure force		4.4		
Increase in internal energy			10.8	
Loss of potential energy				7.0
Balance	3.8		3.8	

increase in enthalpy then must balance the potential energy release, hence its value must be 7.0 units instead of 15.2 units. This implies slightly lower temperatures, and also pressures, at the downstream and compared to the actual situation; and both internal energy and work terms will be smaller. The energy transformations for this case are shown in Table 5.

Table 5

	10^9 cal/day		
	-	+	-
Work done by pressure force	2.0		
Increase in internal energy		5.0	
Loss of potential energy			7.0
Balance	2.0		2.0

When the trade acts as accumulator of latent heat only, it cannot maintain itself but is dependent on energy releases in distant portions of the general circulation. The heat acquired by the trade wind air in the course of its equatorward trajectory therefore becomes important for understanding the general circulation, at least in low latitudes, and probably also for its irregular fluctuations.

The trade as a driving circulation: Excepting the case of Table 5, the energy transformations do not of themselves permit us to deduce whether the trade is a "driving" member of the general circulation. This term, which has no generally accepted meaning, will here be understood to denote a circulation which locally maintains itself and which exports energy. As already demonstrated, the trade exports both sensible and latent heat. It remains to determine whether the circulation is locally self-sustaining; this problem will be approached by considering relations between heat source and pressure gradient.

As seen from fig. 2, the heat source is strongest near the ground and decreases upward. Above 800 mb the trajectories parallel the isentropes; hence the heat source is confined to the air below 800 mb. Similarly, the horizontal pressure drop along the section (cf. 1) is largest at the surface (5 mb) and becomes zero at 2 km (800 mb). Thus both heat source and pressure gradient vanish at the same altitude, a suggestive coincidence. For the purpose of relating these two quantities we can restrict the consideration to the layer below 800 mb which consists mainly of the subcloud and cloud layers.

From equation (2)

$$Q \approx c_p \frac{d\theta}{dt} \approx c_p \left(v \frac{\partial\theta}{\partial s} + w \frac{\partial\theta}{\partial z} \right). \tag{10}$$

Inspecting fig. 2 the vertical gradient of potential temperature is zero in the subcloud layer. In the cloud layer the horizontal advection of potential temperature exceeds the vertical advection by an order of magnitude as can be shown using the profiles of horizontal and vertical wind speed published in (1). Therefore the vertical advection may be omitted from equation (10) so that

$$Q = c_p v \frac{\partial\theta}{\partial s},$$

or

$$\frac{Q}{c_p v} = \frac{\partial\theta}{\partial s}.$$

Integrating over the section and denoting vertical averages with a bar,

$$\frac{1}{c_p} \iint \frac{Q}{v}\, ds\, dz = (\bar{\theta}_d - \bar{\theta}_u)\, D, \tag{11}$$

where D is the depth of the layer with heat source and "d" and "u" denote downstream and upstream respectively.

Introduction of Poisson's equation into the hydrostatic formula yields

$$p_0^{\,k} - p^{\,k} = \frac{gD}{c_p} \frac{1000^k}{\bar{\theta}}, \tag{12}$$

where p and p_o are the pressures at top and bottom of the layer of depth D. Differentiating equation (12) with respect to s (symbol δ),

$$\delta p_o = \left(\frac{p}{p_o}\right)^{k-1} \delta p - \frac{gD\, 1000^k}{R^* \bar{\theta}^2 p_o^{\,k-1}}\, \delta\bar{\theta}.$$

Since

$$\delta\bar{\theta} = \bar{\theta}_d - \bar{\theta}_u,$$

$$\delta p_0 = \left(\frac{p}{p_0}\right)^{k-1} \delta p - K \iint \frac{Q}{v}\, ds\, dz, \tag{13}$$

where

$$K = \frac{g\, 1000^k}{c_p R^* \bar{\theta}^2 p_0^{k-1}} = 1.4 \times 10^{-2}\ g^2/cm^2\ sec^2\ cal.$$

Equation (13) establishes an explicit relation between the surface pressure drop, the heat absorbed over the section and the wind speed, provided δp, the pressure gradient imposed by extraneous circulations at the top of heat source, vanishes. Now we have already seen from observations that this is the case. Thus,

$$\delta p_0 = - K \iint \frac{Q}{v}\, ds\, dz. \tag{14}$$

All quantities are known; calculation shows that the correct surface pressure drop is indeed obtained from the right hand side of the equation. However, in order to make the equation fully acceptable, an argument should be given which demonstrates that δp must vanish. At present this cannot be done quantitatively but the following qualitative hypothesis is presented.

The heat transmitted from ocean to atmosphere is distributed upward through the cloud layer by convection. In the mean, the upper limit of this transport is situated near the level at which the atmosphere is in equilibrium with the underlying surface, i. e. the pressure at which air rising from the surface as an isolated parcel will attain the environment temperature. This limit insures that the heat source remains mostly restricted to the subcloud and cloud layers as seen from fig. 2. In addition, a vertical distribution of friction is introduced which is quite different from that often assumed, though not always observed, in higher latitudes.

On account of the turbulence in the cloud layer a "gradient wind level" is not found at 2,000-3,000 feet altitude. As indicated earlier, the balance of forces in the turbulent layer is essentially given by

$$\frac{\partial p}{\partial s} - \frac{\partial \tau}{\partial z} = 0. \tag{15}$$

Above the top of this layer the balance of forces must change drastically since, in accord with expectations, the frictional drag vanishes almost entirely (cf. 1, Table 3). Here the complete first equation of motion (eq. 5) becomes applicable. Only the large-scale eddy stresses $\rho v \frac{\partial v}{\partial s}$ and $\rho w \frac{\partial v}{\partial z}$ are available to balance any large cross-isobar flow. There is every indication that the magnitude of these stresses is small (1, Table 3) and that the whole system is not strongly accelerated. Thus at the top of the turbulent layer the balance of forces is similar to that found farther poleward in broad currents with little acceleration. The flow is laminar and parallels the isobars, therefore is quasi-geostrophic. The influence of the heat source on the pressure field <u>must be confined to the turbulent layer itself</u> in which the heat accumulates. <u>It is the dual role of the heat source -- production of downstream warming and maintenance of vertical turbulence in a layer of limited thickness -- which accounts for</u>

the fact that the heat gained is utilized directly to maintain the trade winds. The portion of the trade wind cell that has been analyzed thus forms a driving link of the general circulation.

The preceding deductions depend specifically on two characteristics of the region studied: existence of a trade inversion, and a vertical wind profile with a curvature in the turbulent layer such that turbulence must act to retard the flow (fig. 3). This kind of profile is typical of those portions of the trade wind belts where the meridional temperature gradient is directed poleward. The geostrophic wind is strongest at the ground and decreases throughout the trade wind layer. On account of ground friction however, the maximum wind is situated some distance above the ground and the vertical wind profile assumes the shape depicted in fig. 3. Thus the conclusions drawn cannot be readily applied to the tropical and equatorial regions in their entirety. A separate assessment of the role of the heat source must be made for those regions in which instability extends through a deep layer and where the easterlies do not decrease, perhaps even increase, with height.

It is finally of interest to consider the maintenance of the trades assuming no heat source. Then the last term of equation (13) would be zero and $\delta p_0 = \left(\frac{p_0}{p}\right)^{k-1} \delta p$. Using δp_0 = 5 mb, p = 800 mb and p_0 = 1000 mb, δp = 4.3 mb. A large downstream pressure drop would still exist at the top of the turbulent layer. This pressure drop may be interpreted as the means by which distant circulations would maintain the trades which, as shown earlier, become an energy consuming branch of the general circulation when there is no heat source. If large accelerations are to be avoided above the turbulent layer, the wind must veer sharply with height in the trade inversion.

Conclusion

This report has shown that the Pacific northeast trade is a source of sensible as well as of latent heat for the general circulation. Latent heat contributes about 75%, sensible heat 25% to the total heat export to other parts of the world. The trade current gains and exports energy. Under the specific conditions that prevail with regard to the vertical wind and temperature structure, the heat source is utilized directly to maintain the trade. We may therefore consider the flow in the area studied as a driving branch of the general circulation.

The results suggest extension of the present inquiry in several directions; Malkus (2) has already taken up some of these. The calculations performed may be applied to other portions of the tropical and equatorial regions and to the low latitude meridional circulation cell as a whole. Since a local heat source within the trade has turned out to be an important link in maintaining the current, one may consider the role of varying heat source intensity and wind speed on fluctuations of the general circulation, perhaps following the outline of a tropical "index cycle" suggested by Riehl (3).

One of the striking features of the trade is the lack of turning of wind with height through the inversion, noted early by Ficker (4) in the Atlantic. Since the depth of the turbulent layer is much greater in the trades than in the stable atmosphere of the temperate zone, one cannot assume in any derivation of the vertical wind profile that the isobars are independent of height in the friction layer. In the trade section it is warmer at the downstream than at the upstream end. Since the pressure decreases from right to left across the section looking downstream, the isobars will rotate counterclockwise with height under the influence of the temperature field and thus assume more and more the direction of the surface wind. The foregoing calculations contain a suggestion that under certain conditions the flow should be two-dimensional in a driving circulation branch. Considering also the results of Sheppard (5, 6) it appears evident that the theory of the wind structure in friction layers is due for revision.

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References

(1) Riehl, H. et al, 1951, Quart. J. Roy. Meteor. Soc., 77, p. 598.

(2) Malkus, J. S., 1956 (to be published in Tellus).

(3) Riehl, H., 1954, Weather, 9, p. 335.

(4) Ficker, H., 1936, Veroeff, Preuss. Akad. Wiss., Phys. Math. Kl., Berlin, No. 11.

(5) Sheppard, P. A., H. Charnok and J. R. D. Francis, 1952, Quart. J. Roy. Meteor. Soc., 78, p. 563.

(6) Sheppard, P. A. and M. H. Omar, 1952, Quart. J. Roy. Meteor. Soc., 78, p. 583.

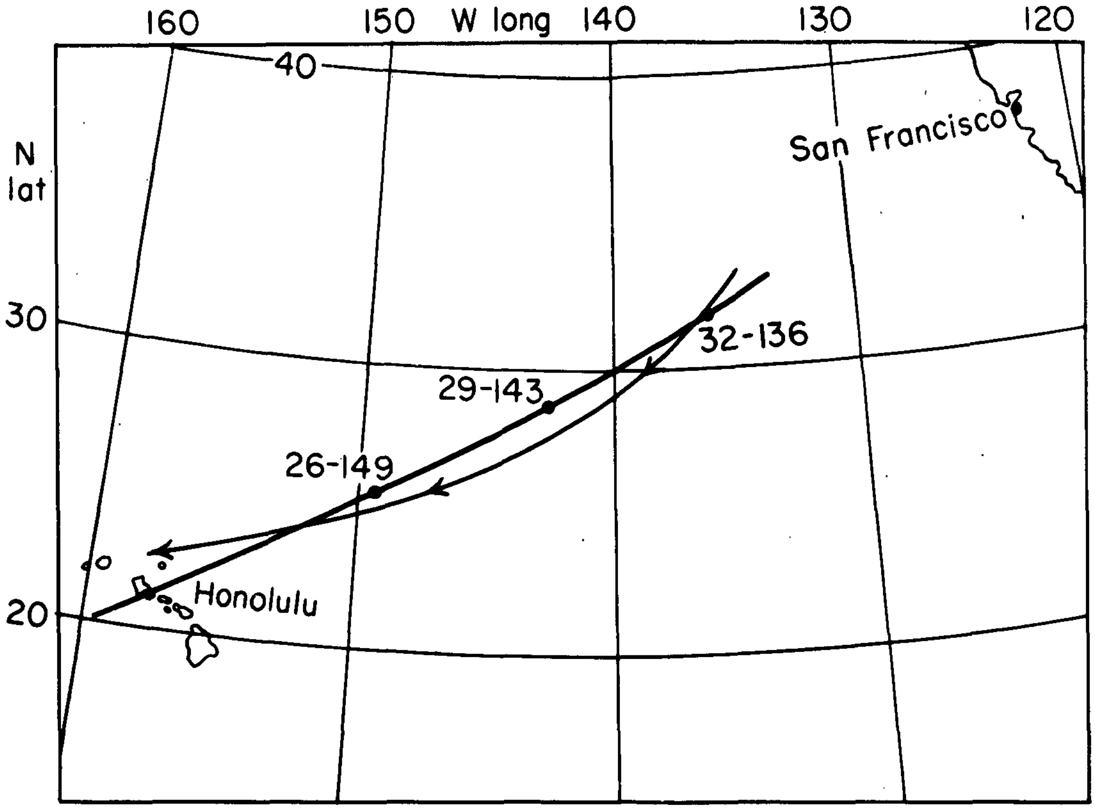

Fig. 1. Location of weather ships July-October 1945, and mean air trajectory for period (1).

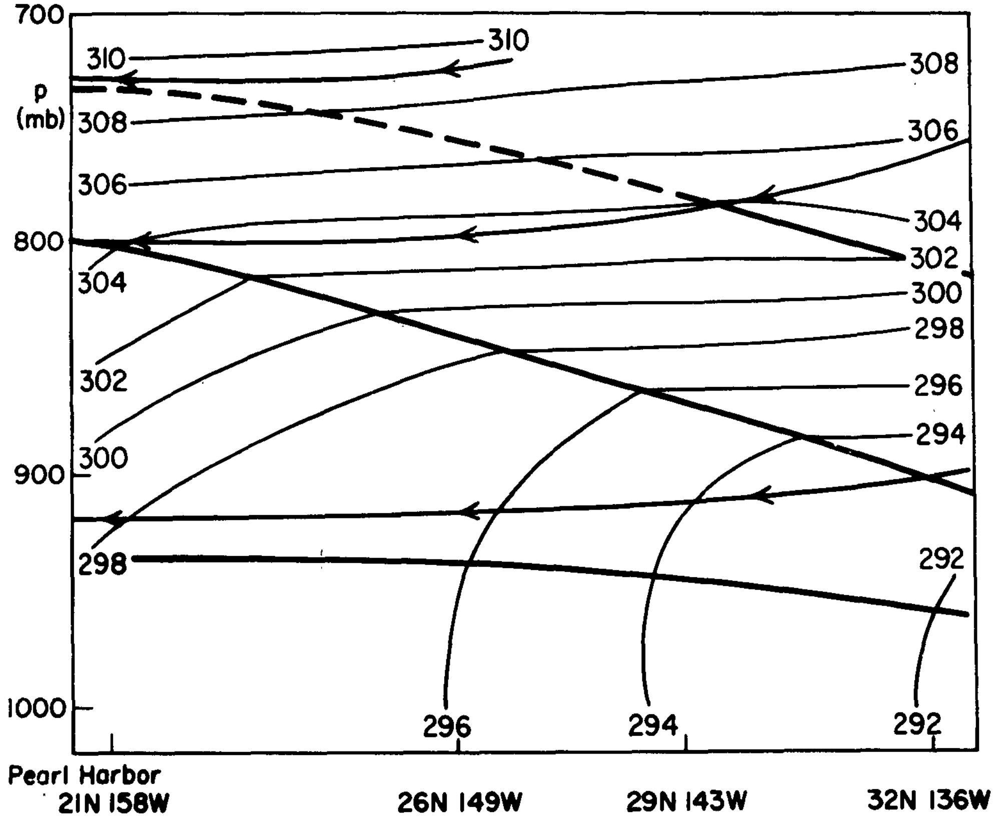

Fig. 2. Vertical crossection of potential temperature (°A) in relation to sample air trajectories (1). The heavy lines denote the top of the subcloud layer, and base and top of the trade wind inversion.

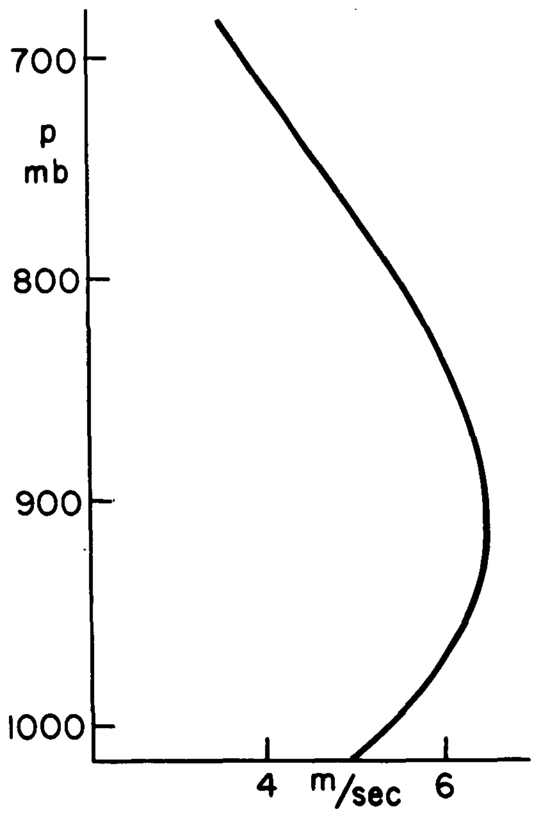

Fig. 3. Mean vertical distribution of wind speed (1).